12/15

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HW05

1. In class we proved the first version of DeMorgans laws for sets, namely that $\overline{A \cup B} = \overline{A} \cap \overline{B}$. Prove the other version, specifically that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

$\cap = \land$
$\cup = \lor$

As written, this proves $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$; Need the other direction.

$\equiv$ Expression	Reason
$\overline{A \cap B}$	Assumption
$x \notin A \cap B$	Definition of ~~Intersection~~ complement
$\neg(x \in A) \lor \neg(x \in B)$	DeMorgan ok
$(x \notin A) \lor (x \notin B)$	Negation ✓
$(x \in \overline{A}) \lor (x \in \overline{B})$	Definition of Complement ✓
$x \in \overline{A} \cup \overline{B}$	Definition of Union ✓
$\overline{A} \cup \overline{B}$	Final Expression ok

Break it down further

We conclude that DeMorgan's Second Law is true since when factoring the expression on the left $\overline{A \cap B}$, through Set Operations and Set Propositional Logic, it results to $\overline{A} \cup \overline{B}$.

2. Prove that $A - B = A \cap \overline{B}$.

Same comment as above

$\equiv$ Expression	Reason
$A - B$	Assumption
$(x \in A) \land \neg(x \in B)$	Definition of Difference ok
$(x \in A) \land (x \notin B)$	Negation ✓
$(x \in A) \land (x \in \overline{B})$	Definition of Complement ✓
$x \in A \cap \overline{B}$	Definition of Intersection ✓
$A \cap \overline{B}$	Final Expression

We conclude that $A - B = A \cap \overline{B}$ is true, since when factoring the expression $A - B$, through Set Operations and Set Proposition Logic, it results in $A \cap \overline{B}$.

3. Let A, B, and C be sets. Prove that if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. In other words, we are proving that the relation "is a subset of" is transitive.

Assume that $x \in A$.
By the definition of a subset, since x is an element in Set A then it must also be an element in Set B $(x \in B)$.

Since $x \in B$, then by the definition of a subset x must also be in Set $C(x \in C)$.

We can conclude that since x is in Set A, ~~and that~~ then x is in Set C, that Set A is a subset of Set C $(A \subseteq C)$. QED