

Basic Probability Theory: Rules and Formulas Part 2

Presented By

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Probability Rules

- $P(A)$ is read as 'the probability of A', where A is an event we are interested in.
- $P(A|B)$ is read as 'the probability of A given B occurs'.
- $P(\text{not } A)$ is read as 'the probability of not A ', or 'the probability that A does not occur'.

There are three main rules associated with basic probability: the addition rule, the multiplication rule, and the complement rule. You can think of the complement rule as the 'subtraction rule'

1.) **The Addition Rule:** or is called union \cup and is called intersection \cap

If A and B are **not Mutually Exclusive** event can occur together

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

probability of both A and B ($P(A \text{ and } B)$)

If A and B are **mutually exclusive** events, or those that cannot occur together, then the third term is 0, and the rule reduces to

$$P(A \text{ or } B) = P(A) + P(B).$$

For example, you can't flip a coin and have it come up both heads and tails on one toss.



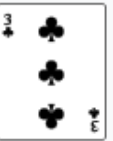





























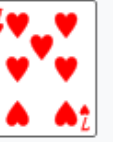



















Disjoint Outcomes: two outcomes are disjoint if they cannot occur at the same time.

Draw both Ace or king in a single deck of card in single draw.

Draw both Heart or Spade

Addition rules for disjoint outcomes $P(A \text{ or } B) = P(A) + P(B)$.

Example set of 52 playing cards; 13 of each suit: clubs, diamonds, hearts, and spades

	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Clubs													
Diamonds													
Hearts													
Spades													

2.) The Multiplication Rule:

If A and B are **dependent Event** The term dependent refers to any event whose outcome is affected by the outcome of another event.

$$P(A \text{ and } B) = P(A) * P(B | A) \text{ or } P(B) * P(A | B)$$

If A and B are **independent events**, we can reduce the formula to

$P(A \text{ and } B) = P(A) * P(B)$. The term independent refers to any event whose outcome is not affected by the outcome of another event.

For instance, consider the second of two coin flips, which still has a .50 (50%) probability of landing heads, regardless of what came up on the first flip. What is the probability that, during the two coin flips, you come up with tails on the first flip and heads on the second flip?

$$P = P(\text{tails}) * P(\text{heads}) = (0.5) * (0.5) = 0.25$$

Independent outcomes: two outcomes are independent if knowing that one outcome occurs does not change the probability of other outcome occurs.

Multiplication rules for independent outcomes

$$P(A \text{ and } B) = P(A) * P(B).$$

Two coins are tossed what is the probability of head on first and tail on second

Two packets of playing card probability of getting ace from first and king from second

3.) The Complement Rule:

$$P(\text{not } A) = 1 - P(A)$$

We can also be thought of as the subtraction rule. This rule builds upon the mutually exclusive nature of $P(A)$ and $P(\text{not } A)$. These two events can never occur together, but one of them always has to occur.

Therefore $P(A) + P(\text{not } A) = 1$.

For example, if the weatherman says there is a 0.3 chance of rain tomorrow, what are the chances of no rain?

$$P(\text{no rain}) = 1 - P(\text{rain}) = 1 - 0.3 = 0.7$$

Example : a study on the current marital status of woman aged 25- 29 years old was conducted on a large group of women between ages of 25 and 29.

The table below summarized the finding of this study

Marital Status	Never Married	Married	Divorced	Widowed
Proportion	0.468	0.459	0.06	0.013

What is the probability that a randomly selected woman aged 25 to 29 years old is currently either never married or divorced?

Outcome 1= Never Married

Outcome 2= Divorced

Are disjoint outcomes no women can be both never married and divorced at the same time

$P(\text{Never Married or Divorced}) = P(\text{Never Married}) + P(\text{Divorced})$

$0.468 + 0.06$

$= 0.528$

What is the probability of when the first woman is never married and second woman is divorced.

Example: An automobile manufacturer buys computer chips from a supplier. Each chip chosen from this shipment has probability 0.05 of being defective. Each automobile uses 7 computer chips that are both selected independently and work independently of each other.

What is the probability that all 7 chips in an automobile will work properly?

What is the probability that at least 1 of the 7 chips in an automobile is defective?

P(all 7 chips work properly)

Chip 1 Chip 2 Chip 3 Chip 4 Chip 5 Chip 6 Chip 7

works properly 7 outcomes and independent

$P(\text{all 7 chips work properly}) = P(\text{chip 1 works properly and chip 2 works properly and chip 7 works properly})$

$P(\text{chip 1 works}) = 1 - P(\text{chip 1 does not work}) = 1 - 0.05 = 0.95$ (compliment rule)

$= 0.95 * 0.95 * 0.95 * 0.95 * 0.95 * 0.95 * 0.95 = .6983$

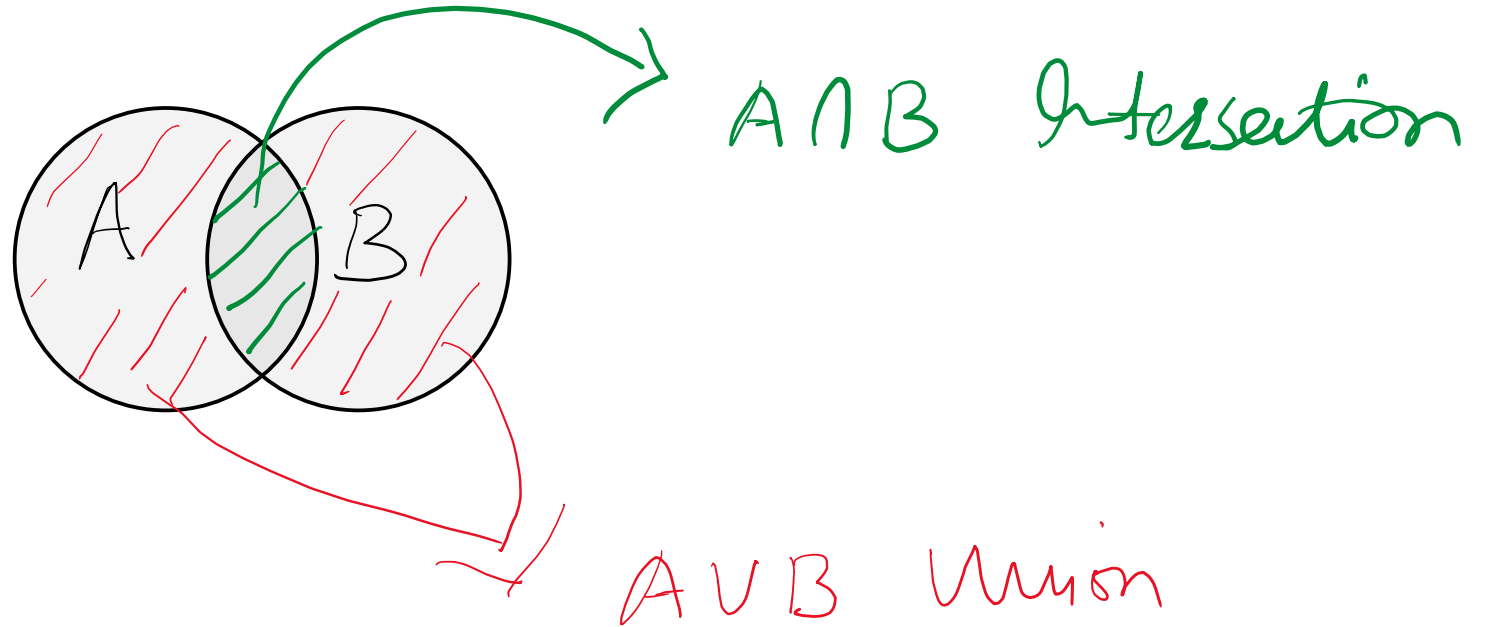
$$\begin{aligned}
 P(\text{At least one chip is defective}) &= 1 - P(\text{none of the chip is defective}) \\
 &= 1 - P(\text{all 7 chips working properly}) \\
 &= 1 - 0.6983 \\
 &= 0.3017
 \end{aligned}$$

$$A = \{1, 2, 3, 4\}$$

$$B = \{4, 5, 6, 7\}$$

$$S = \{1, 2, 3, 4, 5, 6, 7\}$$

$$A \cap B = \{4\}$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{4}{7} + \frac{4}{7} - \frac{1}{7} = 1$$