Probability Distributions Part 2

Presented By
Aditya Joshi
Assistant Professor
Graphic Era Deemed to be University

Bernoulli Distribution

A Bernoulli distribution is a discrete probability distribution for a Bernoulli trail.

A random experiment has only two outcomes (usually called a success or failure).

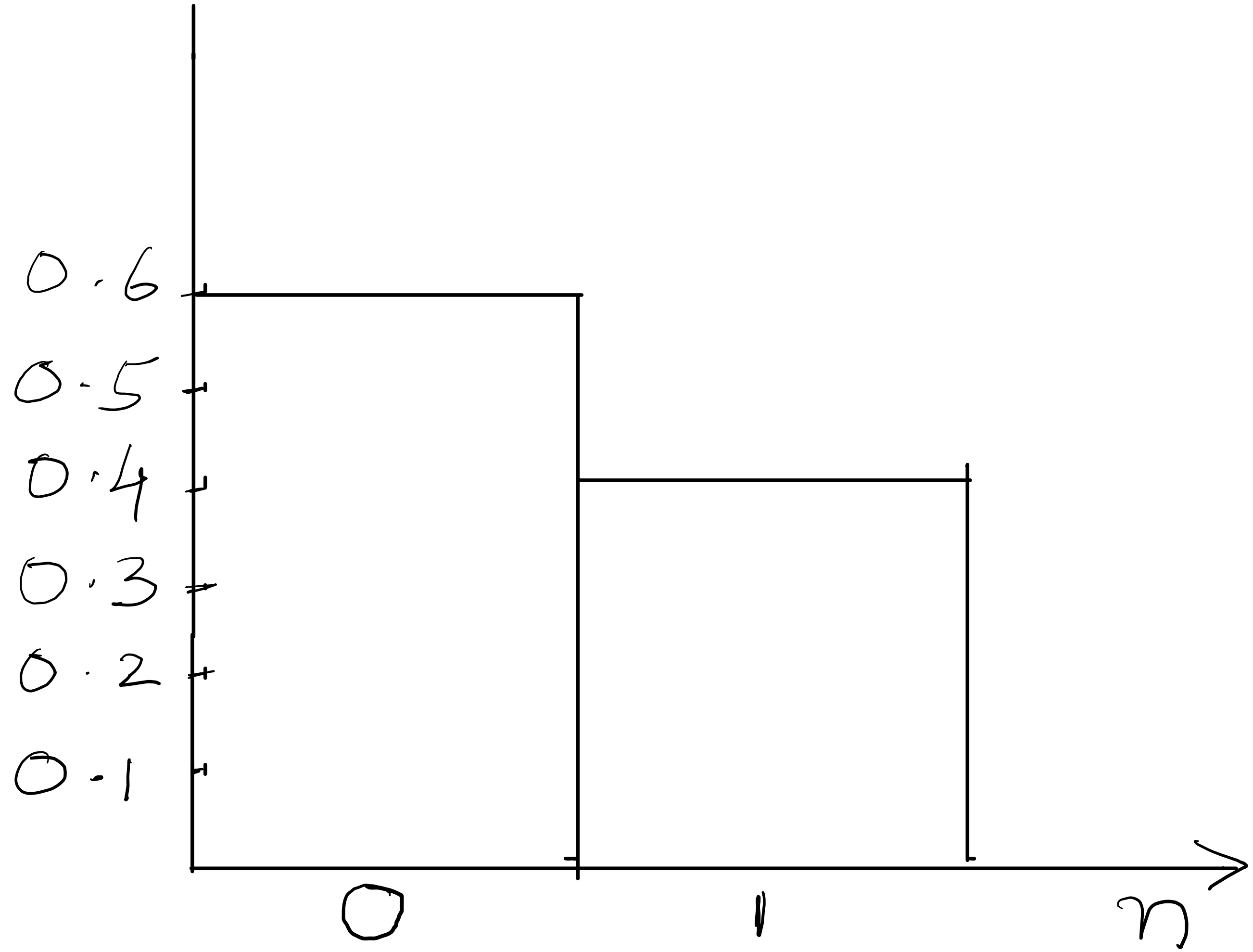
E.g. Probability of getting heads p is success while flipping a coin is 0.5

Probability of failure is 1-p (1 minus the probability of success) which is also equals to 0.5

It is a special case of binomial distribution for n=1, in other words it is a binomial distribution with a single trail.

E.g. A Single coin toss

The Probability of a failure is labeled on x axis as 0 and success is labeled as 1. in the following Bernoulli distribution, the probability of success is 0.4 and probability failure is 0.6



The Probability density function (pdf) for this distribution is

$$P(n) = p^{X} (1-p)^{1-X}$$

Which can also be written as

$$P(n) = \begin{cases} 1-p & \text{for } x=0 \text{ failure} \\ p & \text{for } x=1 \text{ success} \end{cases}$$

A Bernoulli trial is one of the simplest experiments you can conduct. It’s an experiment where you can have one of two possible outcomes. For example, “Yes” and “No” or “Heads” and “Tails.” A few examples:

- Coin tosses: record how many coins land heads up and how many land tails up.
- Births: how many boys are born and how many girls are born each day.
- Rolling Dice: the probability of a roll of two die resulting in a double six

An important part of every Bernoulli trial is that each action must be independent. That means the probabilities must remain the same throughout the trials; each event must be completely separate and have nothing to do with the previous event.

Q. Approximately 1 out of 42 births is a twin. One set of new parent is chosen. What is the probability they are parents of twins?

$P=1/42$

1 out of every 42 births

Pdf is $P(X=x) = (1/42)^x(1-1/42)^{1-x}$

For $x=0,1$

For success

$P(X=1)=(1/42)^1(1-1/42)^{1-1}$

$=1/42$

For failures

$P(X=0)=(1/42)^0(1-1/42)^{1-0}$

$=1-1/42$

Negative Binomial Distribution

The negative binomial experiment is almost the same as a binomial experiment with one difference: a binomial experiment has a fixed number of trials.

- The number of trials, n is not fixed.
- A random variable Y= the number of trials needed to make r successes

A negative binomial distribution (also called the Pascal Distribution) is a discrete probability distribution for random variables in a negative binomial experiment.

The random variable is the number of repeated trials, X, that produce a certain number of successes, r. In other words, it's the number of failures before a success. This is the main difference from the binomial distribution: with a regular binomial distribution, you're looking at the number of successes. With a negative binomial distribution, it's the number of failures that counts.

Probability mass function for negative binomial distribution is

$$nb(x; r, p) = \binom{x + r - 1}{r - 1} p^r (1 - p)^x \quad x = 0, 1, 2, \ldots$$

Where:

r is the number of successes and

p = the probability of success.

Q. You are surveying people exiting from a polling booth and asking them if they voted independent. The probability (p) that a person voted independent is 20%. What is the probability that 15 people must be asked before you can find 5 people who voted independent?

Step 1: Find p, r and X.

We are given (in the question) that p = 20%(.2) and r = 5. The number of failures, X, is 15 – 5 = 10.

Step 2: Insert those values from Step 1 into the formula:

$$nb(10; 5, .2) = \binom{14}{4} (.2)^5(.8)^{10}$$

Step 3: Solve. The first part (14 over 4) is a combination

$1001 * .2^5 * .8^{10} = 0.034.$

The probability you'll have to ask 15 people to get 5 votes for independent is .034, or 3.4%.

Geometric Distribution

The geometric distribution is a special case of the negative binomial distribution. The geometric distribution represents the number of failures before you get a success in a series of Bernoulli trials. This discrete probability distribution is represented by the probability density function:

$$f(x) = (1 - p)^{x - 1}p$$

If your probability of success is 0.2, what is the probability you meet an independent voter on your third try?

Inserting 0.2 as p and with X = 3, the probability density function becomes:

$f(x) = (1 - p)^{x - 1}*p$

$P(X = 3) = (1 - 0.2)^{3 - 1}(0.2)$

$P(X = 3) = (0.8)^2*0.2 = 0.128.$

Poisson Distribution

A Poisson distribution is a tool that helps to predict the probability of certain events happening when you know how often the event has occurred. It gives us the probability of a given number of events happening in a fixed interval of time.

The Poisson Distribution pmf is: $P(x; \mu) = (e^{-\mu} * \mu^{x}) / x!$

The symbol "!" is a factorial.

μ (the expected number of occurrences) is sometimes written as λ. Sometimes called the event rate or rate parameter.

Q. The average number of major storms in your city is 2 per year. What is the probability that exactly 3 storms will hit your city next year?

Step 1: Figure out the components you need to put into the equation.

$\mu = 2$ (average number of storms per year, historically)
$x = 3$ (the number of storms we think might hit next year)
$e = 2.71828$ (e is Euler's number, a constant)
Step 2: Plug the values from Step 1 into the Poisson distribution formula:

$P(x; \mu) = (e^{-\mu}) (\mu^{x}) / x!$
$= (2.71828^{-2}) (2^{3}) / 3!$
$= (0.13534) (8) / 6$
$= 0.180$
The probability of 3 storms happening next year is 0.180, or 18%