Assignment - 1

Set Theory:

1. Prove that (i) $A - B = A \cap B'$ (ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$.
2. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$ find
   (i) $(A \cup B)\, \Delta\, A$ (ii) $A\, \Delta\, (A \Delta B)$ (iii) $B - B$
3. Find the following sets in set builder form.
   i. A = {3, 6, 9, 12, 15}
   ii. B = {-4, -3, -2, -1, 0, 1, 2, 3}
   iii. C = {1, ½, 1/3, 1/4, …}
   iv. D = {2, 4, 6, 8, 10}
4. Convert in tabular form.
   i. $A = \{x: x^2 - 3x + 2 = 0\}$
   ii. $B = \{x: x \text{ is an integer and } 1 \leq x \leq 7\}$
   iii. $C = \{x: (x \geq 0) \text{ and } (x^2 \leq 10), x \in Z\}$
   iv. $D = \{x: x \in R, x^2 - 1 = 0\}$
5. $S_1 = \{1, 2, 3\}$, $S_2 = \{x: x^2 - 2x + 1 = 0\}$ and $S_3 = \{x: x^3 - 6x^2 + 11x - 6 = 0\}$. Show that which sets are equal.
6. $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6\}$ and $C = \{x \in Z \text{ where x is even}\}$. Find
   i. $A - B$ ii. $B \cap A$ iii. $C - B$ iv. $(A \cap B) \cup C$
7. Write all the subsets of sets.
   i. $A = \{1, 2, 3\}$ ii. $B = \{1, \{2, 3\}\}$ iii. $C = \{\{1, 2, 3\}\}$

Relation:

1. Let A = {2, 3, 4} and B = {3, 4, 5}. List the elements of each relation R defined below and the domain and range.
   (i) $a \in A$ is related to $b \in B$, that is, a R b if, and only if $a < b$.
   (ii) $a \in A$ is related to $b \in B$, that is, a R b if a and b are both odd numbers.
2. Let S = {x, y} and S^2 is the set of all words of length 2, find the elements of S^2.
3. Let A = {2, 3, 5} and B = {6, 8, 10} and define a binary relation R from A to B as follows:
   For all $(x, y) \in A \times B, (x, y) \in R \Leftrightarrow \frac{x}{y}(x \text{ divides } y)$.
   Write each R and R^{-1} as a set of ordered pairs.
4. Let R = {(1, 1), (2, 1), (3, 2)}, compute R^2.
5. Consider a relation R defined on A = {1, 2, 3} whose matrix representation is given below. Determine its inverse R^{-1} and complement R'.

$$M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

6. Consider the following relation on {1, 2, 3, 4, 5, 6}

$$R = \{(i, j): |i - j| = 2\}$$

Is R reflexive? Is R symmetric? Is R transitive?

7. Let R be a binary relation defined as

$$R = \{(a, b) \in R^2 : (a - b) \leq 3\}$$

Determine whether R is reflexive, symmetric, antisymmetric and transitive.

8. Give an example of a relation which is:

(i) Reflexive and transitive but not symmetric.

(ii) Symmetric and transitive but not reflexive.

(iii) Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but neither symmetric nor antisymmetric.

9. If A is a set of positive integers and a relation R is defined on A as follows: $(a, b)R(c, d) \Leftrightarrow a + d = b + c \ \forall\ a, b, c, d \in R$, then prove that R is an equivalence relation.

10. Let N = {1, 2, 3 …} and a Relation is defined in $N \times N$ as follows: (a, b) is related to (c, d) iff ad = bc, then show whether R is a equivalence relation or not.

Function:

1. Let A = {1,2,3,4,5}, B = {0,1,2,3,5,7,9,12,13} and

i) f_1 = {(1,1), (2,0), (3,7), (4,9), (5,12)}

ii) f_2 = {(1,3), (2,3), (3,5), (4,9), (5,9)}

iii) f_3 = {(1,1), (2,3), (4,7), (5,12)}

iv) f_4 = {(1,1), (2,3), (3,5), (3,7), (4,9)}

Check which of the notation represents a function and write down the range of such function.

2. Determine the nature (type) of the functions f: R → R (one-one, onto, one-one onto etc…)

i) f(x) = 3x -1

ii) f(x) = x^2

iii) f(x) = x^3

iv) f(x) = x^3+1

v) f(x) = x+1

3. Let A = {1, 2, 3} and B = {a, b, c, d}. In each case, state whether the given function (if defined) is injective, surjective, bijective.

i) f = {(1,a), (2,d), (3,b)}

ii) g = {(1,a), (2,a), (3,d)}

iii) h = {(1,a), (1,b), (2,d), (3,c)}

iv) j = {(1,a), (2,b)}

4. A function defined over the set of integers as follows:

$$f(x) = \begin{cases} x & if\ 0 \leq x < 1 \\ x + 2 & if\ 1 \leq x < 3 \\ 4x - 5 & if\ 3 \leq x < 5 \end{cases}$$

i) Find the domain of f

ii) Find the range of f

iii) State whether f is one-one or many one function.

5. Consider A = B = C = R and let f: A → B and g: B → C be defined by f(x) = x + 9 and g(y) = y^2 + 3. Find the following composite functions:

i) (fof)(a)
ii) (gog)(a)
iii) (fog)(b)
iv) (gof)(b)
v) (gof)(3)
vi) (fog)(-3)

6. Find the inverse function of the following functions $f : R \rightarrow R$ (if exists)

i) $f(x) = x^3$
ii) $f(x) = ax + b \ (a \neq 0)$
iii) $f(x) = x^2+1$
iv) $f(x) = x^2+3$

7. If f and g are two functions defined over the sets of real numbers such that $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. Show that $fog \neq gof$.

8. Find the domain of the real valued function $f(x) = \sqrt{81 - x^2}$.

9. Show that the function $f(x) = x^3$ and $g(x) = x^{1/3}$ for $x \in R$ are inverses of one another.

10. Let $A = \{1,2,3,4\}$ and let $f = \{(1,3), (2,1), (3,4), (4,3)\}$ and $g = \{(1,2), (2,3), (3,1), (4,1)\}$. Find

i) fog
ii) gof
iii) fof
iv) gog