ASSIGNMENT

MCA / M.Sc.(CS/IT)- IST SEM

Discrete Structure and Combinatorics (TMC 104_TMI 102)

1. In a class of 25 student ,12 have taken Mathematics ,8 have taken Mathematics but not Biology. Find the number of students who have taken Mathematics and biology. And also Find the number of students those who have taken biology but not mathematics.
2. It is found that out of 100 students 18 can drive neither a scooter nor a car. While 25 can drive both these and 55 of them can drive a scooter. Use Venn diagram and Find many can drive a car.
3. Prove that $(A\times B)\cap(P\times Q)=(A\cap P)\times(B\cap Q)$.
4. In a survey of 100 students , it was found that 50 student liked tea ,40 liked coffee and 30 liked cold drinks, Out of these 20 liked both tea and coffee,15 liked coffee and cold drinks and 10 liked tea and cold drink. How many student liked all i.e tea , coffee and cold drinks (each student one or more of the 3 items definitely).
5. Given that A={1,2,3,4}and B={a, b, c, d} and C={p, q, r} and R is a relation from A to B, S is the relation from B to C defined as **R=**{(1,a),(2,c),(3,b),(4,a),(4,d)} and **S=**{(a, q),(b, p),(d, r),(c, p)}**.**verify that $(RoS)^{-1} = S^{-1} o R^{-1}$.
6. Let P and Q be the relations defined on set A = {1,2,3,4} defined by.P = {(1,2), (2,2),(2,3),(2,4),(3,2),(4,2),(4,3)}and Q = {(2,2),(2,3),(3,2),(3,3),(3,4),(4,1,(4,2)}, find PoP & PoP0Q
7. Let A = {7, 2, 5, 4 12} and consider the partial order of divisibility on A, i.e., if a and $b \in A$, $a \leq b$ if and only a \ b. Draw the Hasse diagram of the poset (A, ≤).
8. Draw the Hasse diagram of (A, ≤), where A= {1, 2, 3, 4, 6. 9, 12, 18, 36} and a relation ≤ be such that a≤ b if a divides b.
9. Draw the Hasse diagram of (A, ≤), where A={3,4,12,24,48,72}and a relation ≤ be such that a≤ b if a divides b .
10. Discuss the bijectivity of the following function : $f: R \rightarrow R$, given by $f(x) = x^3 + 2\forall x \in R$
11. Show that the function $f: R \rightarrow R$ defined by $f(x) = 3x^2 + 5$ for all $x \in R$ is bijection.
12. If $f, g: R \rightarrow R$, are defined respectively by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$,

    Find the following composition *(i) fog* *(ii) gog* *(iii) gof*
13. Calculate Truth table for given statements, check which is contradiction and tautology

    (i) $p \vee \sim q \Rightarrow p$

    (ii) $((\sim (p \wedge q) \vee r) \Rightarrow \sim p)$

    (iii) $(p \vee q) \wedge (\sim p \wedge \sim q)$

    (iv) $p \rightarrow (p \vee q)$

    (vii) $\neg(p \wedge q) \vee q$

    viii) $((\neg (p \wedge q) \vee r) \Rightarrow \neg p)$

14. Represent the argument symbolically and determine whether the argument is Valid.

    If I Study, then I will not fail Mathematics.
    If I do not have play basketball, then I will study.
    But I failed Mathematics

....
Therefore, I played basketball.

15. Represent the argument
If it rain today, then we will not have a party today.
If we do not have party today, then we will have a party tomorrow.
....
Therefore, if it rains today, then we will have a party tomorrow.
Symbolically and determine whether the argument is Valid.
16. Express the followings using quantifiers if K(x): x is student , M(x): x is clever, N(x): x is successful
(i) There exists a student (ii) Some Students are clever (iii) Some students are not successful .
17. Write the negation of quantified proposition $\forall x (\neg p(x)) \wedge \exists y\, q(y)$

18. Use Mathematical Induction. To show that $3+3.5+3.5+3.5^2+3.5^3+...+3.5^n = 3(5^{n+1}-1)/4$, whenever n is a nonnegative integer.
19. Prove that $5^{2n+1}+3^{n+2}.2^{n-1}$ is divisible by 19 $\forall n \in N$, by the principal of mathematical induction.
20. Use Mathematical Induction. To show that $1+2+2^2+$--------$+2^n = (2^{n+1}-1)$ for all non-negative integers of n.
21. $1^2+2^2+3^2+$-------------$+n^2 = n(n+1)(2n+1)/6,\ n \geq 1$.

22. In how many ways can the integer 1, 2, 3, 4, 5, 6, 7, 8 be permuted such no odd integer will be in its natural place.
23. Find the number of integers from 1 to 300 that are not divisible by 3, 5 or 7.
24. In how many way a 11 football player can be chosen out of 17 player when (i) five particular player are to be always include. (ii) two particular player are to be always excluded.
25. In how Many ways 12 thing can be divided equally among 4 people.
26. In how many ways can we get an even sum when two distinguishable dice are rolled?
27. If 30 book in school contain of total 61327 pages then at least how many pages in one book.
28. How many different permutation can be made out of the letters of the word "ASSASSINATION" taken all together?
29. How many words can be formed by using the letters of the word 'MISSISSPPI'.
30. How many five-letter PALINDROMES (a word that reads the same when read in forward or backward) can be formed from English alphabets.
31. Find The minimum number of student in a class so that three of them are born in the same month.
32. Consider three different cities P, Q, and R. There are 4 railway track between P and Q , 3 railways track between Q and R, and 2 railway track that go directly from P to R. Find Out
(i) Total number of ways to go from P to R altogether (i.e. P to R via Q and P to R directly)
(ii) Total number of way to go from P to R and then back to P.
(iii) Total number of ways that go from P to R via Q and return directly from R to P.
12. Prove that ${}^{n}C_{r} = {}^{n-1}C_{r} + {}^{n-1}C_{r-1}$
33. Solve the recurrence relation**:** $a_{n+2} = 5a_{n+1} - 6a_n + 7^n$.
34. Solve the recurrence relation: $u_{n+2} = 4u_{n+1} - 4u_n + 2^n$
35. Solve $a_n - 7a_{n-1} + 12a_{n-2} = n.4^n$
36. Solve $a_n = 3a_{n-1} + n^2$.
37. Solve $a_n - 2a_{n-1} - 3a_{n-2} = 0; n \geq 2$ with $a_0 = 3$ and $a_1 = 3$.

38. Solve the recurrence relation:

$a_n - 4a_{n-1} + 4a_{n-2} = (n+1)2^n ; n \geq 2$.

39. Define Abelian Group .

40. Show that the matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ form a multiplicative abelian group.

41. Prove that the Fourth root of unity G={1, -1, i, -i}form an group under multiplication.

42. Show that the set of matrices $A_\alpha = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ form a finite abelian group with respect to matrix multiplication (using composition table).

43. Prove that the set {0,1,2,3,4,5}is a finite abelian group under addition modulo 6.

44. To show that the order of each sub group of a finite group G is a divisor of the order of the group.

45. Find the product of two permutations and show that it is not commutative $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$.

46. If G = {f_1, f_2, f_3, f_4} where $f_1(x) = x,\ f_2(x) = -x,\ f_3(x) = \frac{1}{x},\ f_4(x) = -\frac{1}{x}$ then show (G,*) is abelian group where * show the composition of function.

47. A graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in G.

48. Prove that in a simple graph no two vertices have same degree.

49. Prove that the number of vertices having odd degree in a graph G is always even.

50. Show that the maximum degree of any vertex in a simple graph of n vertices is n-1.