Discrete Structure and Combinatorics

Submitted To:
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Course- MCA
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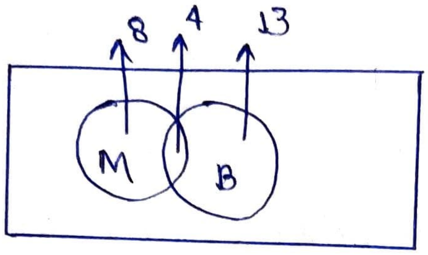
1. In a class of 25 student, 12 have taken Mathematics, 8 have taken Mathematics but not Biology. Find the no. of students who have taken Mathematics and Biology. And also find the no. of students those who have taken Biology but not Mathematics.



Students who taken mathematics = 12

Students who taken mathematics but not biology = 8

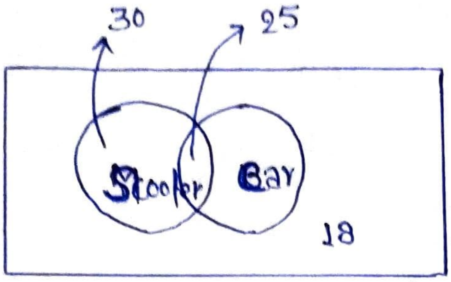
$\therefore$ student who taken mathematics & biology = 12 - 8 = 4

$\therefore$ Student who taken biology but not mathematics

$= 25 - 12$

$= 13.$

2. It is found that out of 100 students 18 can drive neither a scooter nor a car. While 25 can drive both these and 55 of them can drive a scooter. Use Venn diagram and find how many students can drive a car.



Total student = 100

Given, 18 student can't drive

$\therefore$ student $= 100 - 18$

$= 82$

30, student can drive scooter & 25 can both

$\therefore$ student who drives only scooter $= 55 - 25 = 30$

$\therefore$ student who drives car $= 82 - 30 = 52$.

3. Prove that $(A\times B)\cap(P\times Q)=(A\cap P)\times(B\cap Q)$

L.H.S $=(A\times B)\cap(P\times Q)$

Let $(a,b)\in(A\times B)\cap(P\times Q)$

$(a,b)\in A\times B$ and $(a,b)\in P\times Q$

$a\in A$ and $b\in B$ and $a\in P$ and $b\in Q$

$a\in A$ and $a\in P$ and $b\in B$ and $b\in Q$

$a\in(A\cap P)$ and $b\in(B\cap Q)$

$(a,b)\in(A\cap P)\times(B\cap Q)$

$(A\cap P)\times(B\cap Q)\subseteq(A\times B)\cap(P\times Q)$ — 1.

RHS $=(A\cap P)\times(B\cap Q)$

$(a,b)\in(A\cap P)\times(B\cap Q)$

$a\in A\cap P$ and $b\in B\cap Q$

$a\in A$ and $a\in P$ and $b\in B$ and $b\in Q$

$a\in A$ and $b\in B$ and $a\in P$ and $b\in Q$

$(a,b)\in(A\times B)$ and $(a,b)\in(P\times Q)$

$(a,b)\in(A\times B)\cap(P\times Q)$

$(A\times B)\cap(P\times Q)\subseteq(A\cap P)\times(B\cap Q)$ — 2.

From 1 & 2.,

$(A\times B)\cap(P\times Q)=(A\cap P)\times(B\cap Q)$ proved.

4. In a survey of 100 students, it was found that 50 student liked tea, 40 liked coffee and 30 liked cold drinks, Out of these 20 liked both tea and coffee, 15 liked coffee and cold drink and 10 liked tea and cold drink. How many student like all i.e., tea, coffee and cold drinks (each student one or more of the 3 items definitely).

$n(\text{Tea} \cup \text{Coffee} \cup \text{Colddrink}) = 100$

$n(\text{Tea}) = 50,$

$n(\text{Coffee}) = 40,$

$n(\text{Colddrink}) = 30,$

$n(\text{Tea} \cap \text{Coffee}) = 20,$

$n(\text{Coffee} \cap \text{Cold drink}) = 15,$

$n(\text{Tea} \cap \text{Cold drink}) = 10$

$n(\text{Tea} \cap \text{Coffee} \cap \text{colddrink}) = ?$

$\because n(A \cap B \cap C) = n(A \cup B \cup C) - n(A) - n(B) - n(C) + n(A \cap B) + n(A \cap C) + n(B \cap C)$

$n(\text{Tea} \cap \text{Coffee} \cap \text{Cold drink}) = 100 - 50 - 40 - 30 + 20 + 15 + 10$

$= 100 - 120 + 45$

$= 145 - 120$

$= 25.$

5. Given that $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ and $C = \{p, q, r\}$ and R is a relation from A to B, S is the relation from B to C defined as $R = \{(1, a), (2, c), (3, b), (4, a), (4, d)\}$ and $S = \{(a, q), (b, p), (d, r), (c, p)\}$. Verify that $(R o S)^{-1} = S^{-1} o R^{-1}$

$A = \{1, 2, 3, 4\}$

$B = \{a, b, c, d\}$

$C = \{p, q, r\}$

$R = \{(1, a), (2, c), (3, b), (4, a), (4, d)\}$

$R^{-1} = \{(a, 1), (c, 2), (b, 3), (a, 4), (d, 4)\}$

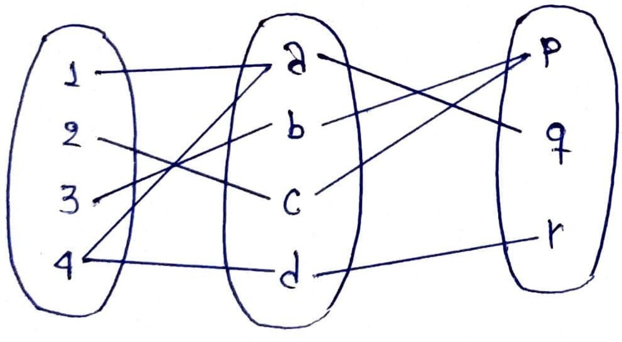
$S = \{(a, q), (b, p), (d, r), (c, p)\}$

$S^{-1} = \{(q, a), (p, b), (r, d), (p, c)\}$

$R o S = \{(1, q), (2, p), (3, p), (4, q), (4, r)\}$

$(R o S)^{-1} = \{(q, 1), (p, 2), (p, 3), (q, 4), (r, 4)\}$

$S^{-1} o R^{-1} = \{(q, 1), (p, 2), (q, 4), (r, 4), (p, 3)\}$



Hence, $(R o S)^{-1} = S^{-1} o R^{-1}$

proved.

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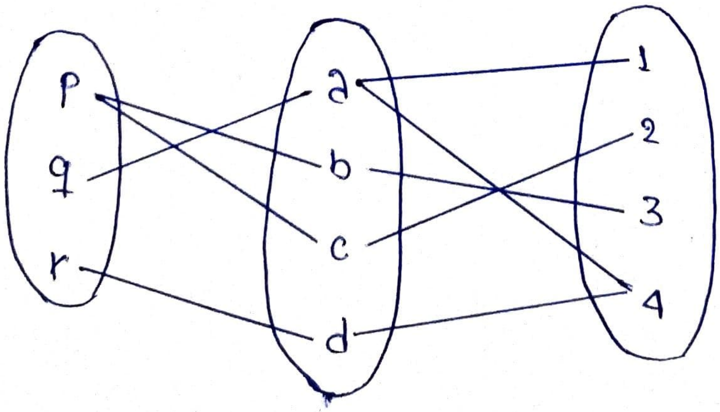


$S^{-1} o R^{-1}$

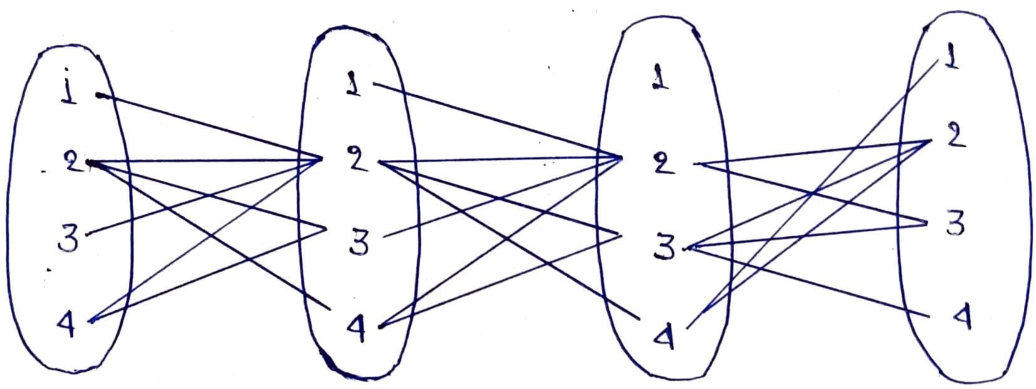
6. Let P and Q be the relations defined on set $A = \{1, 2, 3, 4\}$ defined by $P = \{(1,2), (2,2), (2,3), (2,4), (3,2), (4,2), (4,3)\}$ and $Q = \{(2,2), (2,3), (3,2), (3,3), (3,4), (4,1), (4,2)\}$. Find $P \circ P$ & $P \circ P \circ Q$.

$A = \{1, 2, 3, 4\}$

$P = \{(1,2), (2,2), (2,3), (2,4), (3,2), (4,2), (4,3)\}$

$Q = \{(2,2), (2,3), (3,2), (3,3), (3,4), (4,1), (4,2)\}$

$P \circ P = \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4),$
$(3,2), (3,3), (3,4), (4,2), (4,3), (4,4)\}$



$P \circ P \circ Q = \{(1,2), (1,3), (1,4), (1,1), (2,2), (2,3), (2,4) (2,1),$
$(3,2), (3,3), (3,4), (3,1), (4,2), (4,3), (4,4), (4,1)\}$

1. Let $A = \{7, 2, 5, 4, 12\}$ and consider the partial order of divisibility on A, i.e., if a and $b \in A$, $a \leq b$ if and only $a \backslash b$. Draw the Hasse diagram of the poset $(A, \leq)$.

$A = \{7, 2, 5, 4, 12\}$

$R = \{(7,7), (2,2), (5,5), (4,4), (12,12),$
$(2,4), (2,12),$
$(4,12)\}$

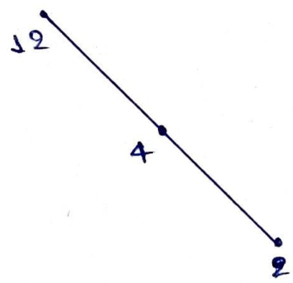
Step1. Remove the self loop (reflexive pair)

$R = \{(2,4), (2,12), (4,12)\}$

Step2. Remove the transitive pair.

$R = \{(2,4), (4,12)\}$

Step3. Draw Hasse diagram.



8. Draw the Hasse diagram of $(A, \leq)$, where $A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ and a relation $\leq$ be such that $a \leq b$ if a divides b.

$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (6,6), (9,9), (12,12), (18,18), (36,36),$
$(1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (1,18), (1,36),$
$(2,4), (2,6), (2,12), (2,18), (2,36),$
$(3,6), (3,9), (3,12), (3,18), (3,36),$
$(4,12), (4,36),$
$(6,12), (6,18), (6,36),$
$(9,18), (9,36),$
$(12,36),$
$(18,36)\}$

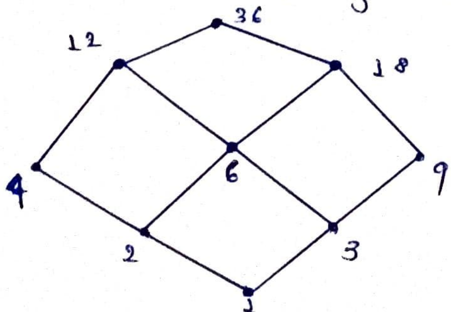
Step 1. Remove self loop (reflexive pair)

$R = \{(1,2), (1,3), (1,4), (1,6), (1,9), (1,12), (1,18), (1,36),$
$(2,4), (2,6), (2,12), (2,18), (2,36),$
$(3,6), (3,9), (3,12), (3,18), (3,36),$
$(4,12), (4,36),$
$(6,12), (6,18), (6,36),$
$(9,18), (9,36),$
$(12,36), (18,36)\}$

Step 2. Remove transitive pair

$R = \{(1,2), (1,3), (2,4), (2,6), (3,6), (3,9), (4,12), (6,12),$
$(6,18), (9,18), (12,36), (18,36)\}$

Step 3. Draw Hasse Diagram



9. Draw the Hasse diagram of $(A, \leq)$, where $A = \{3, 4, 12, 24, 48, 72\}$ and a relation $\leq$ be such that $a \leq b$ if a divides b.

$A = \{3, 4, 12, 24, 48, 72\}$

$R = \{(3,3), (4,4), (12,12), (24,24), (48,48), (72,72),$
$(3,12), (3,24), (3,48), (3,72),$
$(4,12), (4,24), (4,48), (4,72)$
$(12,24), (12,48), (12,72),$
$(24,48), (24,72)\}$

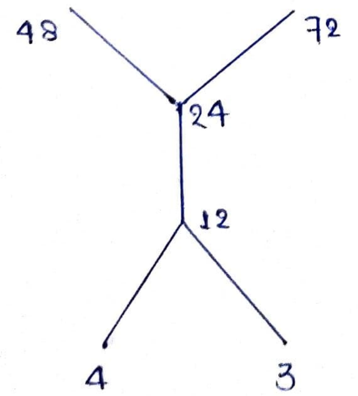
step 1. Remove the self loop (reflexive)

$R = \{(3,12), (3,24), (3,48), (3,72),$
$(4,12), (4,24), (4,48), (4,72),$
$(12,24), (12,48), (12,72),$
$(24,48), (24,72)\}$

step 2. Remove the transitive pair.

$R = \{(3,12), (4,12), (12,24), (24,48), (24,72)\}$

step 3. Draw Hasse diagram.



10. Discuss the bijectivity of the following function: $f: R \to R$, given by $f(x) = x^3 + 2 \ \forall \ x \in R$.

$f(x) = x^3 + 2$

For one-one,

$f(x_1) = f(x_2)$

$\Rightarrow x_1^3 + 2 = x_2^3 + 2$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

$\therefore f(x)$ is one-one — (1)

For onto,

$f(x) = x^3 + 2$

$\Rightarrow y = x^3 + 2$

$\Rightarrow y - 2 = x^3$

$\Rightarrow x = (y-2)^{1/3}$

$\because y \in R \Rightarrow x \in R$

$\therefore f(x)$ is onto — (2)

From (1) & (2)

$f(x)$ is one-one and onto.

Hence, $f(x)$ is bijective.

11. Show that the function $f: R \to R$ defined by $f(x) = 3x^2+5$ for all $x \in R$ is bijection.

$f(x) = 3x^2+5$

For one-one,

$f(x_1) = f(x_2)$

$3x_1^2+5 = 3x_2^2+5$

$x_1^2 = x_2^2$

$\Rightarrow x_1 = -x_2$ or $x_1 = x_2$

$\therefore$ $f(x)$ is not one-one.

Since, for Bijectivity $f(x)$ must be one-one as well as onto.

Therefore,

$f(x)$ is not bijective.

12. If $f, g : R \to R$, are defined respectively by $f(x) = x^2+3x+1$, $g(x) = 2x+3$, Find the following composition

(i) $f \circ g$ (ii) $g \circ g$ (iii) $g \circ f$

$f(x) = x^2+3x+1$, $g(x) = 2x+3$

(i) $f \circ g$

$= f \circ g(x)$

$= f(g(x))$

$= f(2x+3)$

$= (2x+3)^2 + 3(2x+3)+1$

$= 4x^2+12x+9+6x+9+1$

$= 4x^2+18x+19$

(ii) $g \circ g$

$= g \circ g(x)$

$= g(g(x))$

$= g(2x+3)$

$= 2(2x+3)+3$

$= 4x+6+3$

$= 4x+9$

(iii) $g \circ f$

$= g \circ f(x)$

$= g(f(x))$

$= g(x^2+3x+1)$

$= 2(x^2+3x+1)+3$

$= 2x^2+6x+2+3$

$= 2x^2+6x+5$

13. Calculate Truth table for given statements, check which is contradiction and tautology.

(i) $p \vee \sim q \Rightarrow p$

(ii) $((\sim(p \wedge q) \vee r) \Rightarrow \sim p)$

(iii) $(p \vee q) \wedge (\sim p \wedge \sim q)$

(iv) $p \rightarrow (p \vee q)$

(v) $\neg(p \wedge q) \vee q$

(vi) $((\neg(p \wedge q) \vee r) \Rightarrow \neg p)$

(i) $p \vee \sim q \Rightarrow p$

p	q	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \rightarrow p$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	F

$(p \vee \sim q) \rightarrow p$ is neither tautology nor contradiction.

(ii) $((\sim(p \wedge q) \vee r) \Rightarrow \sim p)$

p	q	r	$(p \wedge q)$	$\sim(p \wedge q) \vee r$	$\sim p$	$(\sim(p \wedge q) \vee r) \rightarrow \sim p$
T	T	T	T	T	F	F
T	T	F	T	F	F	T
T	F	T	F	T	F	F
T	F	F	F	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T.

$(\sim(p \wedge q) \vee r) \rightarrow \sim p$ is neither tautology nor contradiction

(iii) $(p \vee q) \wedge (\sim p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim p \wedge \sim q$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

$(p \vee q) \wedge (\sim p \wedge \sim q)$ is contradiction.

(iv.) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$p \rightarrow (p \vee q)$ is tautology.

(v.) $\sim(p \wedge q) \vee q$

p	q	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

$\sim(p \wedge q) \vee q$ is tautology.

14. Represent the argument symbolically and determine whether the argument is valid or not.

If I study, then I will not fail Mathematics
If I do not have play basketball, then I will study
But I failed Mathematics

Therefore, I played basketball.

Let us consider,

$p \rightarrow$ I study

$q \rightarrow$ fail in mathematics

$r \rightarrow$ play basketball

$S_1 = p \rightarrow \sim q$

$S_2 = (\sim r \rightarrow p) \wedge q$

$S_3 = r$

p	q	r	$\sim q$	$\sim r$	$p \rightarrow \sim q$	$\sim r \rightarrow p$	$(\sim r \rightarrow p) \wedge q$	$(p \rightarrow \sim q) \wedge ((\sim r \rightarrow p) \wedge q)$	$(p \rightarrow \sim q) \wedge ((\sim r \rightarrow p) \wedge q) \rightarrow r$
T	F	T	F	F	F	T	T	F	T
T	T	F	F	T	F	T	T	F	T
T	F	T	T	F	T	T	F	F	T
T	F	F	T	T	T	T	F	F	T
F	T	T	F	F	T	T	T	T	T
F	T	F	F	T	T	F	F	F	T
F	F	T	T	F	T	T	F	F	T
F	F	F	T	T	T	F	F	F	T

$\because (p \rightarrow \sim q) \wedge ((\sim r \rightarrow p) \wedge q) \rightarrow r$ is tautology

Hence, Given statement is valid.

15. Represent the argument

If it rain today, then we will not have a party today.
If we do not have party today, then we will have party tomorrow.

Therefore, if it rains today, then we will have a party tomorrow. Symbolically and determine whether the argument is valid.

Let us consider,

p = it rains

q = party today

r = party ~~tommorrow~~ tomorrow

$S_1 = p \rightarrow \sim q$

$S_2 = \sim q \rightarrow r$

$S_3 = p \rightarrow r$

p	q	r	$\sim q$	$p \rightarrow \sim q$	$\sim q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow \sim q) \wedge (\sim q \rightarrow r)$	$(p \rightarrow \sim q) \wedge (\sim q \rightarrow r) \rightarrow (p \rightarrow r)$
T	T	T	F	F	T	T	F	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	F	F	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	T	F	T

$\because (p \rightarrow \sim q) \wedge (\sim q \rightarrow r) \rightarrow (p \rightarrow r)$ is tautology.

Hence, Given, statement is valid.

16. Express the following using quantifiers if
$K(x)$: x is student, $M(x)$: x is clever, $N(x)$: x is successful
(i) There exists a student,
(ii) Some students are clever
(iii) Some students are not successful.

$K(x)$: x is student

$M(x)$: x is clever

$N(x)$: x is successful

(i) There exists a student

$$\forall x\ (K(x))$$

(ii) Some students are clever

$$\exists x\ (K(x) \rightarrow M(x))$$

(iii) Some students are not successful

$$\exists x\ (K(x) \rightarrow \sim N(x))$$

17. Write the negation of quantified proposition

$$\forall x(\neg p(x)) \wedge \exists y\, q(y)$$

$$\sim(\forall x(\sim p(x)) \wedge \exists y\, q(y))$$

$$\exists x(p(x)) \wedge \forall y(\sim q(y))$$

18. Use Mathematical Induction. To show that

$3 + \cancel{3.5} + 3\cdot5 + 3\cdot5^2 + 3\cdot5^3 + \cdots + 3\cdot5^n = 3(5^{n+1}-1)/4$,

whenever n is non-negative integer.

$P(n)$: $3 + 3\cdot5 + 3\cdot5^2 + 3\cdot5^3 + \cdots + 3\cdot5^n = 3(5^{n+1}-1)/4$

Put $n=0$,

$$3\cdot5^0 = 3(5^{0+1}-1)/4$$

$$3\cdot1 = 3(5-1)/4$$

$$3 = 3\cdot4/4$$

$$3 = 3$$

Hence, $P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=K$

$P(K)$: $3 + 3\cdot5 + 3\cdot5^2 + 3\cdot5^3 + \cdots + 3\cdot5^K = 3(5^{K+1}-1)/4$ — (1)

Now, we have to prove $P(n)$ is true for $n = K+1$

$P(K+1)$: $3 + 3\cdot5 + 3\cdot5^2 + 3\cdot5^3 + \cdots + 3\cdot5^K + 3\cdot5^{K+1} = 3(5^{K+2}-1)/4$

LHS $= 3 + 3\cdot5 + 3\cdot5^2 + 3\cdot5^3 + \cdots + 3\cdot5^K + 3\cdot5^{K+1}$

$= 3(5^{K+1}-1)/4 + 3\cdot5^{K+1}$; [From 1]

$= 3\left[\dfrac{5^{K+1} - 1 + 4\cdot5^{K+1}}{4}\right]$

$= 3\left[\dfrac{5\cdot5^{K+1} - 1}{4}\right]$

$= 3\left[\dfrac{5^{K+2} - 1}{4}\right]$

$= 3(5^{K+2}-1)/4$

$=$ RHS

Hence, $P(n)$ is true for $n = K+1$

i.e., By the principle of mathematical induction,

$P(n)$ is true $\forall\, n \in$ Non-negative integer.

proved.

19. Prove that $5^{2n+1} + 3^{n+2} \cdot 2^{n-1}$ is divisible by 19 $\forall\ n \in N$, by the principal of mathematical induction.

$P(n)$: $5^{2n+1} + 3^{n+2} \cdot 2^{n-1}$ is divisible by 19

$\Rightarrow P(n): 5^{2n+1} + 3^{n+2} \cdot 2^{n-1} = 19m$

Put $n = 1$

$P(1): 5^{2(1)+1} + 3^{1+2} \cdot 2^{1-1} = 19m$

$5^{2+1} + 3^{3} \cdot 2^{0} = 19m$

$125 + 27 = 19m$

$152 = 19m$

$19 \times 8 = 19m$

Hence, $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = k$

$P(k): 5^{2k+1} + 3^{k+2} \cdot 2^{k-1} = 19m$ — (1)

Now, we have to prove $P(n)$ is true for $n = k+1$

$P(k+1): 5^{2(k+1)+1} + 3^{(k+1)+2} \cdot 2^{(k+1)-1} = 19m$

$5^{2k+3} + 3^{k+3} \cdot 2^{k} = 19m$

$LHS = 5^{2k+3} + 3^{k+3} \cdot 2^{k}$

$= 5^{2} \cdot 5^{2k+1} + 3 \cdot 2 \cdot 3^{k+2} \cdot 2^{k-1}$

$= 25 \cdot 5^{2k+1} + 6\, 3^{k+2} \cdot 2^{k-1}$

$$5^{2k+1} + 3^{k+2} \cdot 2^{k-1} \Big) \; 25 \cdot \cancel{5^{2k+1}} + 6 \cdot 3^{k+2} \cdot 2^{k-1} \; \Big(25$$

$$25 \cdot \cancel{5^{2k+1}} + 25 \cdot 3^{k+2} \cdot 2^{k-1}$$

$$- 19 \cdot 3^{k+2} \cdot 2^{k-1}$$

$= 25\,(5^{2k+1} + 3^{k+2} \cdot 2^{k-1}) - 19 \cdot 3^{k+2} \cdot 2^{k-1}$

$= 25 \cdot 19m - 19 \cdot 3^{k+2} \cdot 2^{k-1}$; [From 1]

$= 19\,(25m - 3^{k+2} \cdot 2^{k-1})$

$= 19K$

Hence, $P(n)$ is true for $n = k+1$

i.e. By the principle of Mathematical induction, $P(n)$ is true $\forall\ n \in N$.

proved.

20. Use Mathematical Induction. To show that $1+2+2^2+\cdots+2^n=(2^{n+1}-1)$ for all non-negative integers of n.

$P(n): 1+2+2^2+\cdots+2^n = (2^{n+1}-1)$

Put $n=0$

$2^0 = 2^{1+0}-1$

$2^0 = 2^1-1$

$1 = 2-1$

$1 = 1$

Hence, $P(n)$ is true for $n=1$

Let $P(n)$ is true for $n=k$

$P(k): 1+2+2^2+\cdots+2^k = 2^{k+1}-1$ — (1)

Now,

we have to prove $P(n)$ is true for $n=k+1$

$P(k+1): 1+2+2^2+\cdots+2^k+2^{k+1} = 2^{k+2}-1$

$LHS = 1+2+2^2+\cdots+2^k+2^{k+1}$

$= 2^{k+1}-1+2^{k+1}$; [from 1]

$= 2\,2^{k+1}-1$

$= 2^{k+2}-1$

$= RHS$

Hence, $P(n)$ is true for $n=k+1$

i.e., By the principle of mathematical induction, $P(n)$ is true for every integer including 0.

proved.

21. $1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6,\ n \geq 1.$

$P(n):\ 1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)(2n+1)/6$

Put $n = 1$,

$$1^2 = 1(1+1)(2(1)+1)/6$$

$$1 = 1(2)(3)/6$$

$$1 = 1$$

Hence, $P(n)$ is true for $n = 1$

Let $P(n)$ is true for $n = K$

$P(K) = 1^2 + 2^2 + 3^2 + \cdots + K^2 = K(K+1)(2K+1)/6 \quad — ①$

Now, we have to prove $P(n)$ is true for $n = K+1$

$P(K+1):\ 1^2 + 2^2 + 3^2 + \cdots + K^2 + (K+1)^2 = (K+1)(K+2)(2K+3)/6$

$LHS = 1^2 + 2^2 + 3^2 + \cdots + K^2 + (K+1)^2$

$= K(K+1)(2K+1)/6 + (K+1)^2$; [From 1]

$= (K+1)/6\ [K(2K+1) + 6(K+1)]$

$= (K+1)/6\ [2K^2 + K + 6K + 6]$

$= (K+1)/6\ [2K^2 + 7K + 6]$

$= (K+1)/6\ [2K^2 + 4K + 3K + 6]$

$= (K+1)/6\ [2K(K+2) + 3(K+2)]$

$= (K+1)/6\ [(K+2)(2K+3)]$

$= (K+1)(K+2)(2K+3)/6$

$= RHS$

Hence, $P(n)$ is true for $n = K+1$

i.e., By The Principle of Mathematical Induction,

$P(n)$ is true $\forall\ n \geq 1$

proved.

22. In how many ways can the integer 1, 2, 3, 4, 5, 6, 7, 8 be permuted such no odd integer will be in its natural place.

Total no. = 8

Total odd no. = 4

$$n = {}^4C_0\, 8! - {}^4C_1\, 7! + {}^4C_2\, 6! - {}^4C_3\, 5! + {}^4C_4\, 4!$$

$$= 1 \cdot 8! - 4 \cdot 7! + 6 \cdot 6! - 4 \cdot 5! + 1 \cdot 4!$$

$$= 4!\,(8 \times 7 \times 6 \times 5 - 4 \times 7 \times 6 \times 5 + 6 \times 6 \times 5 - 4 \times 5 + 1)$$

$$= 4!\,(1680 - 840 + 30 - 20 + 1)$$

$$= 4!\,(1711 - 860)$$

$$= 24 \times 851$$

$$= 20424$$

23. Find the no. of integers from 1 to 300 that are not divisible by 3, 5 or 7.

$n(3) = 100,$

$n(5) = 60,$

$n(7) = 42,$

$n(3 \cap 5) = 20,$

$n(5 \cap 7) = 8,$

$n(3 \cap 7) = 14,$

$n(3 \cap 5 \cap 7) = 2$

$$n(3 \cup 5 \cup 7) = n(3) + n(5) + n(7) - n(3 \cap 5) - n(5 \cap 7) - n(3 \cap 7) + n(3 \cap 5 \cap 7)$$

$$= 100 + 60 + 42 + 20 - 8 - 14 + 2$$

$$= 202 - 40$$

$$= 162$$

$$n(3 \cup 5 \cup 7)' = U - n(3 \cup 5 \cup 7)$$

$$= 300 - 162$$

$$= 138.$$

24. In how many way a 11 football player can be chosen out of 17 player when -

(i) five particular player are to be always included

(ii) Two particular player are to be always excluded

(i) $n = 17,\ r = 11,$

$\because$ five particular player are to be always included

$n = 12$

& $r = 6$

$$\therefore\ {}^{n}C_{r} = {}^{12}C_{6}$$

$$= \frac{\lfloor 12}{\lfloor 6 \cdot \lfloor 12-6} \quad ; \left[\because {}^{n}C_{r} = \frac{\lfloor n}{\lfloor r \cdot \lfloor n-r}\right]$$

$$= \frac{\lfloor 12}{\lfloor 6 \cdot \lfloor 6}$$

$$= \frac{\cancel{12}\times 11\times \cancel{10}\times 9\times \overset{2}{\cancel{8}}\times 7}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}$$

$$= 11\times 9\times 2\times 7$$

$$= 1386 \text{ ways}$$

$n=17,\ r=11$

(ii) Two particular player are to be always excluded

$n = 15,$ & $r = 11$

$$\therefore\ {}^{n}C_{r} = {}^{15}C_{11}$$

$$= \frac{\lfloor 15}{\lfloor 11 \cdot \lfloor 15-11} \quad ; \left[\because {}^{n}C_{r} = \frac{\lfloor n}{\lfloor r \cdot \lfloor n-r}\right]$$

$$= \frac{\lfloor 15}{\lfloor 11 \cdot \lfloor 4}$$

$$= \frac{15\times \overset{7}{\cancel{14}}\times 13\times \cancel{12}}{\cancel{4}\times\cancel{3}\times\cancel{2}\times 1}$$

$$= 15\times 7\times 13$$

$$= 1365 \text{ ways}$$

25. In how many ways 12 things can be divided equally among 4 people.

$$12 \rightarrow 3,\ 3,\ 3,\ 3$$

$$= {}^{12}C_3 \cdot {}^{9}C_3 \cdot {}^{6}C_3 \cdot {}^{3}C_3$$

$$= \frac{\lfloor 12}{\lfloor 3 \cdot \lfloor 12-3} \cdot \frac{\lfloor 9}{\lfloor 3 \cdot \lfloor 9-3} \cdot \frac{\lfloor 6}{\lfloor 3 \cdot \lfloor 6-3} \cdot \frac{\lfloor 3}{\lfloor 3 \cdot \lfloor 3-3}$$

$$= \frac{\lfloor 12}{\lfloor 3 \cdot \cancel{\lfloor 9}} \cdot \frac{\cancel{\lfloor 9}}{\lfloor 3 \cdot \cancel{\lfloor 6}} \cdot \frac{\cancel{\lfloor 6}}{\lfloor 3 \cdot \cancel{\lfloor 3}} \cdot \frac{\cancel{\lfloor 3}}{\lfloor 3 \cdot \lfloor 0}$$

$$= \frac{\lfloor 12}{(\lfloor 3)^4}$$

26. In how many ways can we get an even sum when two distinguishable dice are rolled?

Total sample space = $6 \times 6 = 36$

Even sum = Any of the dice must be even

$\boxed{3}$ $\boxed{6}$

even Any digit

$\therefore$ Total possibility $= 3 \times 6$

$= 18$

27. If 30 book in school contain of 61327 pages then atleast how many pages in one book.

$n = 30$

$m = 61327$

$p = \dfrac{m-1}{n}$

$= \dfrac{61326}{30}$

No. of pages per books $= \dfrac{61326}{30} + 1$

$= 2045$ pages

28. How many different permutation can be made out of the letters of the word "ASSASSINATION" taken all S together.

$A \rightarrow 3$

$S \rightarrow 4$

$I \rightarrow 2$

$N \rightarrow 2$

ASSASSINATION $\longrightarrow 13 \Rightarrow$ AAINATION$\boxed{SSSS}$ $\rightarrow 10$

Total no. of ways when all S together $= \dfrac{10!}{3! \cdot 2! \cdot 2!}$

$= \dfrac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4}}{\cancel{2 \times 2}}$

$= 151200$ ways.

29. How many different words can be formed by using the letters of the word 'MISSISSPPI'.

MISSISSPPI $\rightarrow$ 10

I $\rightarrow$ 3

S $\rightarrow$ 4

P $\rightarrow$ 2

No. of ways = $\dfrac{\lfloor 10}{\lfloor 3 \cdot \lfloor 4 \cdot \lfloor 2}$

$= \dfrac{10 \times 9 \times \overset{4}{\cancel{8}} \times 7 \times \cancel{6} \times 5}{\cancel{3 \times 2} \times \cancel{2}}$

$= 12600$ ways

30. How many five-letters pallindrome can be formed from english alphabets.

$\underline{26} \quad \underline{26} \quad \underline{26} \quad \underline{1} \quad \underline{1}$

Total no. of ways = $26 \times 26 \times 26$

$= 17576$ ways.

31. Find the minimum no. of student in a class so that three of them are born in the same month.

$n = 12$ (pigeon holes)

$k + 1 = 3$

$\Rightarrow k = 2$

$\therefore kn + 1 = 2 \times 12 + 1$

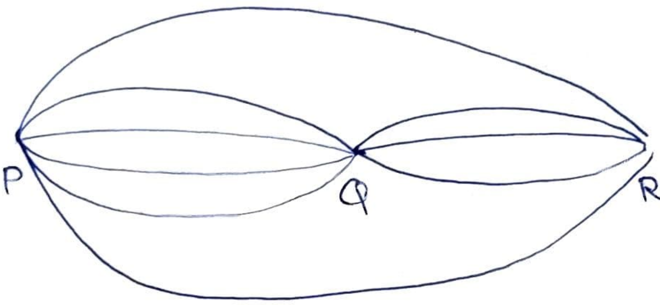
$= 25$ students

32. Consider three different cities P, Q, R. There are 4 railway track between P and Q, 3 railway track between Q and R, and 2 railway track ~~between~~ that go directly from P to R. Find out -

(i) Total no. of ways to go from P to R altogether
(i.e. P to R via Q and P to R directly)

(ii) Total no. of ways to go from P to R and then back to P.

(iii) Total no. of ways that go from P to R via Q and return directly from R to P.



(i) Total no. of ways to go from P to R altogether

$= (P \text{ to } R \text{ via } Q) + (P \text{ to } R \text{ directly})$

$= (3 \times 3 \times 3 \times 3) + 2$

$= 81 + 2$

$= 83$ ways

(ii) Total no. of ways to go from P to R and then back to P.

$= (\text{ways to go from } P \text{ to } R) \times (\text{ways to go from } R \text{ to } P)$

$= [(3 \times 3 \times 3 \times 3) + 2] \times [(4 \times 4 \times 4) + 2]$

$= 83 \times 66$

$= 5478$ ways

(iii) Total no. of ways that go from P to R via Q and return directly from R to P.

$= (\text{ways to go from } P \text{ to } R \text{ via } Q) \times (\text{ways to go from } R \text{ to } P \text{ directly})$

$= (3 \times 3 \times 3 \times 3) * (2)$

$= 81 \times 2$

$= 162$ ways.

33. Prove that ${}^{n}C_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$

R.H.S $= {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$$= \frac{\lfloor n-1}{\lfloor r \cdot \lfloor n-1-r} + \frac{\lfloor n-1}{\lfloor r-1 \ \lfloor n-1-r+1} \quad ; \left[{}^{n}C_r = \frac{\lfloor n}{\lfloor r \lfloor n-r} \right]$$

$$= \frac{\lfloor n-1}{r \cdot \lfloor r-1 \cdot \lfloor n-r-1} + \frac{\lfloor n-1}{\lfloor r-1 \ \lfloor n-r}$$

$$= \frac{\lfloor n-1}{r \cdot \lfloor r-1 \ \lfloor n-r-1} + \frac{\lfloor n-1}{\lfloor r-1 \cdot (n-r) \cdot \lfloor n-r-1}$$

$$= \frac{\lfloor n-1}{\lfloor r-1 \cdot \lfloor n-r-1} \left[\frac{1}{r} + \frac{1}{n-r} \right]$$

$$= \frac{\lfloor n-1}{\lfloor r-1 \ \lfloor n-r-1} \left[\frac{n-r+r}{r(n-r)} \right]$$

$$= \frac{n \lfloor n-1}{r \lfloor r-1 \ (n-r) \lfloor n-r-1}$$

$$= \frac{\lfloor n}{\lfloor r \ \lfloor n-r}$$

$$= {}^{n}C_r$$

$= \text{LHS}$ proved.

34. Solve the recurrence relation: $a_{n+2} = 5a_{n+1} - 6a_n + 7^n$.

$$a_{n+2} = 5a_{n+1} - 6a_n + 7^n$$

$$\Rightarrow a_{n+2} - 5a_{n+1} + 6a_n = 7^n$$

CF:

$$a_{n+2} - 5a_{n+1} + 6a_n = 0$$

Auxillary equ^n, put $a_n = E^r$

$$\phi(E): E^2 - 5E + 6 = 0$$

$$E^2 - 3E - 2E + 6 = 0$$

$$E(E-3) - 2(E-3) = 0$$

$$(E-3)(E-2) = 0$$

$$\therefore a_n = C_1(3)^n + C_2(2)^n$$

Now,

$$PI = a_n = \frac{1}{\phi(E)} \cdot 7^n$$

$$= \frac{1}{(E-3)(E-2)} 7^n$$

$$= \frac{1}{(7-3)(7-2)} \cdot 7^n$$

$$= \frac{7^n}{4 \cdot 5}$$

$$= \frac{7^n}{20}$$

Hence, General sol^n: $C_1(3)^n + C_2(2)^n + \frac{7^n}{20}$.

35. Solve the recurrence relation: $u_{n+2} = 4u_{n+1} - 4u_n + 2^n$

$$u_{n+2} = 4u_{n+1} - 4u_n + 2^n$$

$$\Rightarrow u_{n+2} - 4u_{n+1} + 4u_n = 2^n$$

CF:

$$u_{n+2} - 4u_{n+1} + 4u_n = 0$$

Auxillary equ^n, put $u_n = E^r$

$$\phi(E) : E^2 - 4E + 4 = 0$$

$$(E-2)^2 = 0$$

$$u_n = (C_1 + nC_2)2^n$$

Now,

$$PI = u_n = \frac{1}{\phi(E)} \cdot 2^n$$

$$= \frac{1}{(E-2)^2} \cdot 2^n$$

$$= \frac{n(n-1)2^{n-2}}{\lfloor 2}$$

General sol^n: $u_n = (C_1 + nC_2)2^n + \frac{n(n-1)2^{n-2}}{2}$

36. Solve $a_n - 7a_{n-1} + 12a_{n-2} = n \cdot 4^n$

CF:

$$a_n - 7a_{n-1} + 12a_{n-2} = 0$$

Auxillary equ^n, put $a_n = E^K$

$$E^K - 7E^{K-1} + 12E^{K-2} = 0$$

Put $K = 2$,

$$\phi(E): E^2 - 7E + 12 = 0$$

$$E^2 - 4E - 3E + 12 = 0$$

$$E(E-4) - 3(E-4) = 0$$

$$(E-4)(E-3) = 0$$

$$a_n = c_1(4)^n + c_2(3)^n$$

Now,

$$PI = \frac{1}{\phi(E)} \cdot n \cdot 4^n$$

$$= \frac{1}{(E-4)(E-3)} \cdot n \cdot 4^n$$

$$= \frac{1}{(E-4)(4-3)} \cdot n \cdot 4^n$$

$$= \frac{1}{(E-4)} \cdot n \cdot 4^n$$

$$= n \cdot n \cdot 4^{n-1}$$

$$= n^2 4^{n-1}$$

General sol^n: $a_n = c_1(4)^n + c_2(3)^n + n^2 4^{n-1}$

34. Solve $a_n = 3a_{n-1} + n^2$

Auxillary equⁿ put $a_n = z^k$

$$z^k = 3z^{k-1} + n^2$$

$$z^k - 3z^{k-1} = 0$$

Put $k = 1$

$$\phi(E) : z - 3z^0 = 0$$

$$z - 3 = 0$$

$$z = 3$$

$$a_n = c_1 (3)^n$$

Now,

$$PI = a_n = \frac{1}{\phi(E)} \cdot n^2$$

$$= \frac{1}{(z-3)} n^2$$

$$= n(n-1)^2$$

General solⁿ: $a_n = c_1(3)^n + n(n-1)^2$.

38. Solve $a_n - 2a_{n-1} - 3a_{n-2} = 0;\ n \geq 2$ with $a_0 = 3$ and $a_1 = 3$

Auxillary equ^n; put $a_n = z^K$

$$z^K - 2 \cdot z^{K-1} - 3 \cdot z^{K-2} = 0$$

Put $z = 2$

$$z^2 - 2z - 3 \cdot = 0$$

$$z^2 - 3z + z - 3 = 0$$

$$z(z-3) + 1(z-3) = 0$$

$$(z-3)(z+1) = 0$$

$$a_n = c_1(3)^n + c_2(-1)^n \quad \text{— (1)}$$

Put $n = 0$ in equ^n-1

$$a_0 = c_1(3)^0 + c_2(-1)^0$$

$$3 = c_1 + c_2 \quad \text{— (2)}$$

Put $n = 1$, in equ^n-2

$$a_1 = c_1(3)^1 + c_2(-1)^1$$

$$3 = 3c_1 - c_2 \quad \text{— (3)}$$

Adding (2) & (3)

$$6 = 4c_1$$

$$c_1 = 3/2$$

$$\& \ c_2 = 3/2$$

From (1),

$$a_n = \frac{3}{2}(3)^n + \left(\frac{3}{2}\right)(-1)^n$$

39. Solve the recurrence relation:

$$a_n - 4a_{n-1} + 4a_{n-2} = (n+1)2^n ;\ n \geq 2.$$

Auxillary equ^n, Put $a_n = z^k$

$$z^k - 4z^{k-1} + 4z^{k-2} = (n+1)2^n$$

$$\Rightarrow z^k - 4z^{k-1} + 4z^{k-2} = 0$$

Put $z = 2$

$$\phi(E): z^2 - 4z + 4 = 0$$

$$(z-2)^2 = 0$$

$$a_n = (c_1 + nc_2)2^n$$

Now,

$$PI = \frac{1}{\phi(E)} \cdot (n+1)2^n$$

$$= \frac{1}{(z-2)^2} \cdot (n+1)2^n$$

$$= \frac{(n+1)(n)(n-1)2^{n-2}}{\lfloor 2}$$

General sol^n: $a_n = (c_1 + nc_2)2^n + \dfrac{n(n-1)(n+1)2^{n-2}}{2}$

40. Define Abelian Group.

Abelian Group : A Group $(G, *)$ is said to be Abelian Group if Group $(G, *)$ satisfy the commutative law.

Abelian Group satisfy the following property -

- Closure property
- Associative property
- There exist an identity element
- There exist an inverse element of each element.
- Commutative Property.

41. Show that the matrices $\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}-1&0\\0&1\end{bmatrix}\begin{bmatrix}1&0\\0&-1\end{bmatrix}\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ from a multiplication Abelian group.

Let $A=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $B=\begin{bmatrix}-1&0\\0&1\end{bmatrix}$, $C=\begin{bmatrix}1&0\\0&-1\end{bmatrix}$ & $D=\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$

×	A	B	C	D
A	A	B	C	D
B	B	A	D	C
C	C	D	A	B
D	D	C	B	A

Closure Property:

Since, every entries of the composition table belongs to A.
Therefore, Closure property satisfy.

Associative Law:

Since, multiplication on matrix is associative & A is the set of matrices.
Therefore, A holds associative law on matrix multiplication.

Identity element:

Since, row headed by 1 is same as initial row.
Therfore, $\boxed{e = A}$

Inverse of element:

$A^{-1} = A$

$B^{-1} = B$

$C^{-1} = C$

$D^{-1} = D$

Therefore, $(A, \times)$ is a Group.

Commutative Law:

Since, Transpose of the composition table is same as original table
Hence, $(A, \times)$ is Abelian Group.

42. Prove that the fourth root of unity $G = \{1, -1, i, -i\}$ form a group under multiplication.

$A = \{1, -1, i, -i\}$

$\times$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

; $[\because i^2 = -1]$

Closure Property:

Since, All the entries in the composition table are belongs to A

Hence, Closure property satisfy on A.

Associative Law:

Since, Multiplication on complex no. is associative & A is the set of complex no.

Hence, Multiplication on A is associative.

Identity element:

Since, Row headed by one is same as initial row.

Therefore $\boxed{e = 1}$

Inverse element:

$1^{-1} = 1$

$-1^{-1} = -1$

$i^{-1} = -i$

$-i^{-1} = i$

Hence, $(A, \times)$ is Group.

44. Prove that the set {0, 1, 2, 3, 4, 5} is a finite abelian group under addition modulo 6.

Let A = {0, 1, 2, 3, 4, 5}

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Closure Property:

Since, All the entries in the composition table belongs to A
Hence, Closure property satisfy on A.

Associative law:

Since, addition is associative on integer & set A is the set of integers
Hence, $+_6$ is associative on set A.

Identity element:

Since, Row headed by one is same as initial row.
Therefore, $\boxed{e = 0}$

Inverse element:

$0^{-1} = 0$, $3^{-1} = 3$

$1^{-1} = 5$, $4^{-1} = 2$

$2^{-1} = 4$, $5^{-1} = 1$

Hence, $(A, +_6)$ is Group.

Commutative law;

Since, addition is commutative on integer. and transpose of composition table is same as original table.
Hence, $+_6$ is commutative on A.
Therefor $(A, +_6)$ is Abelian Group.

45. To show that the order of each sub-group of a finite group G is a divisor of the order of the group.

Proof: Let H be any sub-group of order m of a finite group G of order n.

$$O(H) = m, \quad O(G) = n$$

Since, The union of all different right coset of G (left coset) is equal to the G

Therefore, $G = \cup gH$ where $g \in G$

Therefore, All the left cosets of H have only K different cosets.

$$G = g_1H \cup g_2H \cup g_3H \cup \dots \cup g_kH$$

$$O(G) = O(g_1H) + O(g_2H) + O(g_3H) + \dots + O(g_kH)$$

Since, $O(gH) = O(H) \; \forall \; g \in G$

$$O(G) = O(H) + O(H) + O(H) + \dots + O(H) \text{ k times}$$

$$n = m + m + m + \dots + m \text{ k times}$$

$$n = km$$

$$\Rightarrow k = n/m$$

Hence, Order of H is a divisor of order of G.

proved.

46. Find the product of two permutations and show that it is not commutative $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$$

$f \cdot g(1) = g \circ f(1) = 1$

$f \cdot g(2) = g \circ f(2) = 2$

$f \cdot g(3) = g \circ f(3) = 3$

$f \cdot g(4) = g \circ f(4) = 4$

$g \cdot f(1) = f \circ g(1) = 1$

$g \cdot f(2) = f \circ g(2) = 2$

$g \cdot f(3) = f \circ g(3) = 3$

$g \cdot f(4) = f \circ g(4) = 4$

$$f \cdot g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$g \cdot f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Since, $f \cdot g = g \cdot f$

Hence, it is commutative.

47. If $G = \{f_1, f_2, f_3, f_4\}$ where $f_1(x) = x$, $f_2(x) = -x$, $f_3(x) = \frac{1}{x}$, $f_4(x) = -\frac{1}{x}$ then show $(G, *)$ is abelian group where $*$ show the composition of function.

$*$	f_1	f_2	f_3	f_4
f_1	f_1	f_2	f_3	f_4
f_2	f_2	f_1	f_4	f_3
f_3	f_3	f_4	f_1	f_2
f_4	f_4	f_3	f_2	f_1

Closure Property

Since, All the entries of composition table belongs to G
Hence, Closure property satisfy on G.

Associative Law,

Since function follow associative law & G is the set of function.
Hence G is associative.

Identity element.

Since, row headed by one is same as initial row.
Therefore $\boxed{e = f_1}$

Inverse element.

$f_1^{-1} = f_1$, $f_3^{-1} = f_3$,
$f_2^{-1} = f_2$, $f_4^{-1} = f_4$.

Hence, $(G, *)$ is Group.

Commutative Law.

Since, transpose of the composition table is same as original ~~and~~ table.
Hence, G is commutative.

Therefore, $(G, *)$ is Abelian Group.

48. A Graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the no. of vertices in G.

Given, $e = 21$

Let total vertices $= n$

Since, By Handsaking lema,

$$d \cdot n = 2e$$

$$\Rightarrow 4 \times 3 + 3(n-3) = 2 \times 21$$

$$\Rightarrow 12 + 3(n-3) = 42$$

$$\Rightarrow 3(n-3) = 30$$

$$\Rightarrow n-3 = 10$$

$$\Rightarrow n = 13$$

Hence total no. of vertices is $\underline{\underline{13}}$.

50. Prove that the no. of vertices having odd degree in a Graph G is always even.

Proof:
$$\sum_{i=1}^{n} d(V_i) = 2e$$

$$\sum_{even} d(V_i) + \sum d(V_i) = 2e$$

$$\sum d(V_i) = 2e - \sum_{even} 2e$$

$$\sum d(V_i) = even \; ; \quad [\because \text{Diff of even always even}]$$

proved

51. Show that the maximum degree of any vertex in a simple graph of n vertices is $n-1$.

Since edge $= n$

$\therefore$ Maximum edges $= \frac{n(n-1)}{2}$

$$e = \frac{n(n-1)}{2}$$

$$2e = n(n-1)$$

Here $n =$ vertices

$n-1 =$ degree

Hence maximum degree of n edges in a simple graph is $n-1$ proved.