Probability Distributions

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Probability Distribution

Random Variable: It is a variable that contains the outcome of a probability experiment. It is assumed as a numerical value associated with the random outcome of an experiment.

There are two types of random variable

1. Discrete Random Variable (countable number of values, finite number of values, whole number)

Ex. In an experiment which counts the number of customers entering a shop, the outcome can vary from 0 to n customer i.e. 0,1,2,3,4,5,…….,n

Outcome of experiment can be represented by x this random variable x is a countable and finite is a discrete random variable

2. Continuous Random Variable (any value along a given interval of a number line, can be fraction)

In an experiment which counts the time taken to buy a good in minutes i.e. 0, 1.5, 2.75, 3.2, ……, n

Outcome of experiment can be represented by x the random variable, which can be assumed any value in an interval (0-n minutes)

Discrete Random Variable	Continuous Random Variable
• No of steps to the CS/IT Lab 6 • No of Sales • No of Sales • People in Line • Mistakes Per Page	• The time student stays at lab 6 when he/she get there • Length • Depth • Time • Weight

Probability Distribution:

Probability Distribution Defined how probabilities occur for different values of the random variable.

Defined by a probability function f(x) or P(x).

The probability function provides the probability of each value of the random variable.

If the random value of x is discrete, the f(x) of P(x) is discrete probability function and if x is continuous then f(x) is a continuous probability function.

Key Point about probability distribution

Probability is always between 0 & 1 therefore P(X) will be between 0 & 1

$$P(X=x)$$

Generally X= random variable x=represent one value

$$F(X)=2X+5$$

This is a function of X, for different input value of X, F(X) will take on different output

X	F(X)
0	5
1	7
2	9

This can be drawn on graph

A probability distribution tells you what the probability of an event happening is. Probability distributions can show simple events, like tossing a coin or picking a card. They can also show much more complex events, like the probability of a certain drug successfully treating cancer.

There are many different types of probability distributions in statistics including:

- Basic probability distributions which can be shown on a probability distribution table.
- Binomial distributions, which have “Successes” and “Failures.”
- Normal distributions, sometimes called a Bell Curve.

The sum of all the probabilities in a probability distribution is always 100% (or 1 as a decimal).

Ways of Displaying Probability Distributions

Probability distributions can be shown in tables and graphs or they can also be described by a formula. For example, the binomial formula is used to calculate binomial probabilities.

What is a Probability Distribution Table?

A probability distribution table links every outcome of a statistical experiment with the probability of the event occurring. The outcome of an experiment is listed as a random variable, usually written as a capital letter (for example, X or Y). For example, if you were to toss a coin three times, the possible outcomes are:

TTT, TTH, THT, HTT, THH, HTH, HHT, HHH

- You have a 1 out of 8 chance of getting no heads at all if you throw TTT. The probability is 1/8 or 0.125,
- a 3/8 or 0.375 chance of throwing one head with TTH, THT, and HTT,
- a 3/8 or 0.375 chance of throwing two heads with either THH, HTH, or HHT,
- and a 1/8 or .125 chance of getting three heads.

The following table lists the random variable (the number of heads) along with the probability of you getting either 0, 1, 2, or 3 heads.

Number of heads (X)	Probability P(X)
0	0.125
1	0.375
2	0.375
3	0.125

Probabilities are written as numbers between 0 and 1;

0 means there is no chance at all, while 1 means that the event is certain.

The sum of all probabilities for an experiment is always 1, because if you conduct and experiment, something is bound to happen!

For the coin toss example, 0.125 + 0.375 + 0.375 + 0.125 = 1.

Discrete Probability Distribution and continuous probability distribution examples:

A discrete probability distribution is made up of discrete variables. Specifically, if a random variable is discrete, then it will have a discrete probability distribution.

For example, let’s say you had the choice of playing two games of chance at a fair.

Game 1: Roll a die. If you roll a six, you win a prize.

Game 2: Guess the weight of the man. If you guess within 10 pounds, you win a prize.

One of these games is a discrete probability distribution and one is a continuous probability distribution. Which is which?

For the guess the weight game, you could guess that the mean weighs 75 kgs. Or 100 kgs. Or 85.5 kgs. Or any fraction of a kg (85.566 kgs). Even if you stick to, say, between 75 and 100 pounds, the possibilities are endless:

85.1 kgs.

85.11 kgs.

85.111 kgs.

85.1111 kgs.

85.111111 kgs.

so that’s why it’s a continuous probability distribution.

Types of discrete probability distributions commonly used in statistics:

- Binomial distribution
- Bernoulli distribution
- Geometric Distribution
- Negative binomial distribution
- Poisson distribution
- Hypergeometric distribution
- Multinomial Distribution

Types of continuous probability distribution used in statistics

Normal, Student’s T, Chi-square, Exponential, etc.

Binomial Distribution:

A binomial distribution can be thought of as simply the probability of a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times.

The binomial is a type of distribution that has two possible outcomes (the prefix “bi” means two, or twice).

For example, a coin toss has only two possible outcomes: heads or tails and taking a test could have two possible outcomes: pass or fail

The first variable in the binomial formula, n, stands for the number of times the experiment runs.

The second variable, p, represents the probability of one specific outcome.

For example, let’s suppose you wanted to know the probability of getting a 1 on a die roll. if you were to roll a die 20 times, the probability of rolling a one on any throw is 1/6.

Roll twenty times and you have a binomial distribution of (n=20, p=1/6). SUCCESS would be “roll a one” and FAILURE would be “roll anything else.”

If the outcome in question was the probability of the die landing on an even number, the binomial distribution would then become (n=20, p=1/2).

That’s because your probability of throwing an even number is one half.

Criteria

Binomial distributions must also meet the following three criteria:

1. The number of observations or trials is fixed. In other words, you can only figure out the probability of something happening if you do it a certain number of times. This is common sense—if you toss a coin once, your probability of getting a tails is 50%. If you toss a coin a 20 times, your probability of getting a tails is very, very close to 100%.
2. Each observation or trial is independent. In other words, none of your trials have an effect on the probability of the next trial.
3. The probability of success (tails, heads, fail or pass) is exactly the same from one trial to another.

Once you know that your distribution is binomial, you can apply the binomial distribution formula to calculate the probability.

The binomial distribution formula is:

$$b(x; n, P) = {}_nC_x * P^x * (1 - P)^{n - x}$$

Where:

b = binomial probability

x = total number of "successes" (pass or fail, heads or tails etc.)

P = probability of a success on an individual trial

n = number of trials

The binomial distribution formula can also be written in a slightly different way, because ${}_nC_x = n! / x!(n - x)!$ (this binomial distribution formula uses factorials "q" in this formula is just the probability of failure (subtract your probability of success from 1).

$$P(X)= n! / X!(n - X)! * p^x * (q)^{n - x}$$

Q. A coin is tossed 10 times. What is the probability of getting exactly 6 heads?

$$b(x; n, P) = {}_{n}C_{x} * P^{x} * (1 - P)^{n - x}$$

The number of trials (n) is 10

The odds of success ("tossing a heads") is 0.5 (So 1-p = 0.5)

x = 6

$= {}_{10}C_{6}$ * 0.5^6 * 0.5^4 = 210 * 0.015625 * 0.0625 = 0.205078125

80% of people who purchase pet insurance are women. If 9 pet insurance owners are randomly selected, find the probability that exactly 6 are women.

$P(X)= n! / X!(n – X)! * p^x * (q)^{n-x}$

- Step 1: Identify ‘n’ from the problem. Using our example question, n (the number of randomly selected items) is 9.

- Step 2: Identify ‘X’ from the problem. X (the number you are asked to find the probability for) is 6.

- **Step 3:** Work the first part of the formula. The first part of the formula is

$n! / (n – X)!\ \ X!$

Substitute your variables:

$9! / ((9 – 6)! \times 6!)$

Which equals 84.

Step 4: Find p and q. p is the probability of success and q is the probability of failure. We are given p = 80%, or .8. So the probability of failure is 1 – .8 = .2 (20%).

- **Step 5:** Work the second part of the formula.

p^X
$= .8^6$
$= .262144$

Step 6: Work the third part of the formula.

$q^{(n-X)}$
$= .2^{(9-6)}$
$= .2^3$
$= .008$

Step 7: Multiply your answer from step 3, 5, and 6 together.
$84 \times .262144 \times .008 = 0.176.$

Q. 60% of people who purchase sports cars are men. If 10 sports car owners are randomly selected, find the probability that exactly 7 are men.