MODULE 1
OSCILLATIONS AND WAVES

Periodic motion:

Periodic motion is defined as a motion in which the body describes the same path in the same way again and again in equal interval of time. Periodic motion is also called harmonic motion.

Simple harmonic motion:

It is the periodic or oscillatory motion executed by a body such that its acceleration is proportional to its displacement from a fixed point and is always directed towards the fixed point.

SHM is the oscillatory motion of a body where the restoring force is proportional to the negative of the displacement.

Characteristics of the SHM

1. It is a particular type of periodic motion.
2. The oscillating system must have inertia which in turn means mass.
3. When displaced from the fixed point or the mean position, a restoring force acts on the particle, tending to bring it to the mean position.
4. The acceleration developed in the motion due to the restoring force is directly proportional to the displacement.
5. The direction of the acceleration is opposite to that of the displacement.
6. It can be represented by sine or cosine function such as $x = a \sin \omega t$. Where x is the displacement at the instant t, a is the amplitude and ω is the angular frequency.

Examples of SHM

1. A mass suspended to spring when pulled down and left free executes SHM vertically.
2. A pendulum set for oscillation.
3. Excited tuning fork.
4. Swings with which the children play.
5. Vibrating guitar string.

Differential equation of SHM

Consider a body of mass 'm' undergoing simple harmonic motion. For simple harmonic motion, restoring force is directly proportional to the displacement. If F is the restoring force and x is the displacement of the body from the mean position, then from Hooke's law

$F \propto -x$ the negative sign indicates that force is opposite to the displacement.

$F = -kx$

Where k is force constant, which represents the force required to displace the mass by unit distance.

As per Newton's second law of motion

$$F = ma = m\frac{d^2x}{dt^2}$$

$$\therefore m\frac{d^2x}{dt^2} = -kx$$

Or $$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Or $$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad \text{put } \frac{k}{m} = \omega^2$$

Or $\frac{d^2x}{dt^2} + \omega^2 x = 0$
The above equation represents the differential equation or the body executing simple harmonic motion (SHM).
The solution for the above equation is given as x = A sin(ωt-ϕ)
Where x is the displacement of the body from mean position, 'A' is the amplitude, ϕ is the initial phase and ω is the angular frequency and is given by $\omega = \sqrt{\frac{k}{m}}$

Terminologies in SHM

Displacement: The distance of the particle from its mean position at the given instant is called displacement
Displacement x = A Sin(wt-ϕ)

Amplitude:
The maximum displacement of the particle during the simple harmonic motion is called amplitude A.

Phase Angle and Initial Phase :
The value (wt+ϕ) represents the state of the system and is called phase angle. The angle ϕ is called initial phase.

Angular velocity or Frequency (ω)
It is the rate of change of angular displacement and is given by $\omega = \sqrt{\frac{k}{m}}$

Frequency (f):
Frequency of oscillations is defined as the no. of oscillations per second and is given by $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Time Period (T):
It is the time taken to complete one oscillation and is given by T = 1/f .
Time period T is given by $T = 2\pi\sqrt{\frac{Displacement}{Acceleration}}$

Velocity (v):
The velocity of the particle in simple harmonic motion is given by
$v = \frac{dx}{dt} = A\omega\cos\omega t = \sqrt{\omega^2 - x^2}$

Acceleration (a):
The acceleration of the particle in simple harmonic motion is given by
a=$-A\omega^2 Sin\,\omega t = -\omega^2 x$

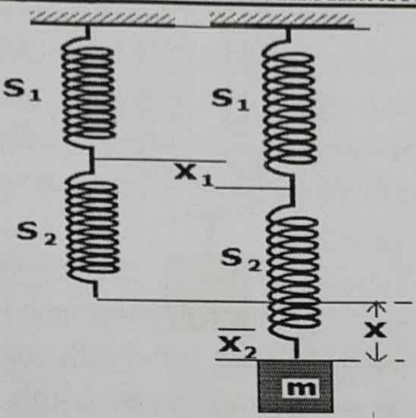
Force constant

Consider a spring whose upper end is fixed to rigid support and mass is attached to the lower end. If the mass is pulled down by a distance x, restoring force F acting is
F = −kx where k is force constant
$\therefore k = \frac{-F}{x}$ Restoring force is equal and opposite to applied force
When x = 1 k = F
Thus **Force constant is defined as the magnitude of the applied force that produces unit extension in the spring while it is loaded within the elastic limit.**

Physical significance of force constant

Physically force constant means the stiffness. In the case of springs, it represents how much force it takes to stretch the spring over the unit length. Thus the springs with larger value of spring constant will be stiffer.

Equivalent force constant for spring in series combination:



Consider two idealised springs S_1 and S_2 connected in series. Let k_1 and k_2 are the spring constants of springs S_1 and S_2 respectively. When mass 'm' is attached to lower end of the combination extension in spring S_1 is x_1 and that in spring S_2 is x_2. Total extension of spring combination is x

$\therefore x = x_1+x_2$ -------------(1)

Now same force (F=mg) cause extension in both the springs.

From Hooke's law

For S_1 spring $F = mg = -k_1x_1$

$\therefore x_1 = -\frac{mg}{k_1}$ --------------(2)

and for S_2 spring $F= mg = -k_2x_2$

$\therefore x_2 = -\frac{mg}{k_2}$ --------(3)

If k_S is the equivalent spring constant of the combination, we have from Hooke's law

$F = mg = -k_S x$

$x = -\frac{mg}{k_S}$ ----------------(4)

Substituting equations (2) (3) and (4) in equation (1), we get

$$-\frac{mg}{k_1} - \frac{mg}{k_2} = -\frac{mg}{k_S}$$

Removing the common factor –mg and rearranging we have

$$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_S = \frac{k_1k_2}{k_1+k_2}$$

If there are n number of springs in series, then

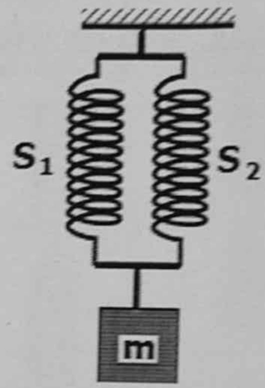
$$\frac{1}{k_S} = \frac{1}{k_1} + \frac{1}{k_2} \cdots \frac{1}{k_n}$$

If a mass m is attached to the bottom of such a series combination of springs and set for oscillations, its period of oscillation will be

$$T = 2\pi\sqrt{\frac{m}{k_S}}$$

Equivalent force constant for springs in parallel combination:

Consider two idealised springs S_1 and S_2 with spring constants k_1 and k_2 respectively. Let two springs are connected in parallel as shown in the figure. Each spring will share the total load and will have equal elongation x. Let k_P be the equivalent spring constant for the combination.



Total restoring force $F = mg = -k_P x$

Restoring force in spring S_1 is $F_1 = -k_1 x$

Restoring force in spring S_2 is $F_2 = -k_2 x$

Now total restoring force = Restoring force in spring S_1+ Restoring force in spring S_2

$F = F1 + F2$

$-k_P x = -k_1 x - k_2 x$

$k_P x = k_1 x + k_2 x$

or $\mathbf{k_P = k_1 + k_2}$

If k_P is the equivalent force constant for the parallel combination of n springs

$k_P = k_1 + k_2 \cdots + k_n$

If a mass m is attached to the bottom of such a parallel combination of springs and set for oscillations, its period of oscillation will be

$$T = 2\pi\sqrt{\frac{m}{k_P}}$$

Free oscillation

If no resistive or damping force is acting on the oscillating body it will continue its oscillation indefinitely without decreasing its amplitude. This is called free oscillations. ***Free oscillations are one in which, an oscillating body oscillates with undiminished amplitude at its own natural frequency with no external influence.***

Examples of free oscillations:

Since all oscillations in nature are subjected to one or the other kind of resistive forces,free oscillations are ideal and do not exist in nature in reality. However, for small displacements and for negligible damping, the oscillations of mass suspended to spring, simple pendulum etc. represents free oscillations.

Equation of motion for free oscillations:

The general equation of motion of SHM itself represents the equation of motion for free oscillations. The equation of motion for free oscillation can be written as

$\frac{d^2x}{dt^2} + \omega^2 x = 0$ Where $\omega = \sqrt{\frac{k}{m}}$, k is spring constant and x is the displacement at the instant t.

Natural frequency:

When a body exhibits free oscillations the frequency with which the oscillations occur is called Natural Frequency.The natural frequency is given by $\boxed{\mathbf{n} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$

Damped Oscillations:

In nature one or the other type of resistive or damping force is continuasly acting on the oscillating syste. Therefore amplitude of scillation decreases continuously and hence the oscillations die out after some time. This is called damped oscillations.

The periodic oscillations of gradually decreasing amplitude produced by an oscillator due to the presence of resistive forces are called damped oscillations.

Examples of damped oscillations:

1. Mechanical oscillations of a simple pendulum,
2. A swing left free to oscillate after being pushed once.
3. A mass suspended by the spring set to vibrate.

Theory of damped vibrations:

Consider a body of mass 'm' executing vibrations in resistive medium. The vibrations are damped due to the resistance offered by the medium.

The damped system is subjected to

1. Resistive force (damping force) which is proportional to the velocity of the body and act in the direction opposite to its movement, we can write

   Resistive force = $-r\frac{dx}{dt}$ ----------------(1)

   Where r is the damping constant and (dx/dt) is the velocity of the body
2. Restoring force which is proportional to displacement but oppositely directed

   Restoring force = $-kx$ ----------------(2)

   Where x is the displacement and k is force constant.

The net force acting on the body is the resultant of the two forces and is given by sum of the right side of (1) and (2)

Resultant force = $-r\frac{dx}{dt} - kx$ ------------(3)

As per the Newton's second law of motion resultant force on the body is given by

Resultant force = $m\frac{d^2x}{dt^2}$ ----------------(4)

From equation (3) and (4)

$$m\frac{d^2x}{dt^2} = -r\frac{dx}{dt} - kx$$

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = 0 \text{ ------------------(5)}$$

This is the equation of motion for the damped oscillations.

Dividing throughout by m we get

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0 \text{ -------------------(6)}$$

Natural frequency of oscillation ω is given by $\omega = \sqrt{\frac{k}{m}}$ we have $\frac{k}{m} = \omega^2$

Let (r/m) = 2b

Equation (6) becomes

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = 0 \text{ -------------------(7)}$$

Let the solution of the above equation is

$x = Ae^{\alpha t}$ ----------------------------------(8)

Where A and α are constants

Differentiating with respect to t we get

$\frac{dx}{dt} = A\alpha e^{\alpha t}$ ----------------------------------(9)

Differentiating again with respect to t we get

$\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t}$ ----------------------------------(10)

Substituting (10), (9)and (8) in equation (7)

$A\alpha^2 e^{\alpha t} + 2bA\alpha e^{\alpha t} + \omega^2 A e^{\alpha t} = 0$

$Ae^{\alpha t}[\alpha^2 + 2\alpha b + \omega^2] = 0$

For the above equation to be satisfied either $Ae^{\alpha t} = 0$ or $\alpha^2 + 2\alpha b + \omega^2 = 0$

But $Ae^{\alpha t} \neq 0$ therefore $\alpha^2 + 2\alpha b + \omega^2 = 0$

The standard solution of the above equation is given by

$\alpha = -b \pm \sqrt{b^2 - \omega^2}$ $\{\because$ if $ax^2 + bx + c = 0$ then $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}\}$

Substituting for α in equation (8), general solution can be given by

$\boxed{x = \boldsymbol{Ce}^{\left(-b+\sqrt{b^2-\omega^2}\right)t} + \boldsymbol{De}^{\left(-b-\sqrt{b^2-\omega^2}\right)t}}$ --------(11)

Where C and D are constants to be evaluated depending on the initial condition of motion.

Let the time be counted from the maximum displacement position for which the vale of displacement is x_o.

i.e. $x = x_o$ when t=0

From equation (11) we get

$x_o = C + D$ ----------------------------------(12)

At maximum displacement position velocity (dx/dt) is zero. i.e. dx/dt=0 when t=0

Differentiating equation (11)

$\frac{dx}{dt} = \left(-b + \sqrt{b^2 - \omega^2}\right)Ce^{(-b+\sqrt{b^2-\omega^2})t} + \left(-b - \sqrt{b^2 - \omega^2}\right)De^{(-b-\sqrt{b^2-\omega^2})t}$

Since t = 0 when dx/dt = 0 above equation becomes

$\left(-b + \sqrt{b^2 - \omega^2}\right)C + \left(-b - \sqrt{b^2 - \omega^2}\right)D = 0$

Rearranging and substituting for C+D from equation (12)

$-bx_0 + \sqrt{b^2 - \omega^2}(C - D) = 0$

$\therefore \frac{bx_0}{\sqrt{b^2-w^2}} = (\boldsymbol{C} - \boldsymbol{D})$ ----------------------------(13)

Adding equations (12) and (13)

$2C = x_0\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right]$

$C = \frac{x_0}{2}\left[1 + \frac{b}{\sqrt{b^2-\omega^2}}\right]$ ----------------------------(14)

By subtracting (13) from (12)

$2D = x_0\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right]$

$D = \frac{x_0}{2}\left[1 - \frac{b}{\sqrt{b^2-\omega^2}}\right]$ ----------------------------(15)

Substituting (14) and (15) in (11) we get

$$x = \frac{x_0}{2}\left[1+\frac{b}{\sqrt{b^2-\omega^2}}\right] e^{\left(-b+\sqrt{b^2-\omega^2}\right)t} + \frac{x_0}{2}\left[1-\frac{b}{\sqrt{b^2-\omega^2}}\right] e^{\left(-b-\sqrt{b^2-\omega^2}\right)t} \text{------(16)}$$

This is the general solution for damped vibrations

As t varies x also varies but the nature of variation depends upon the term $\sqrt{b^2-\omega^2}$. The three domains of the variation are (1) $b^2 > \omega^2$ (2) $b^2 = \omega^2$ and (3) $b^2 < \omega^2$

Case (1) $b^2 > \omega^2$, the over damping or dead-beat case:

If the damping is high so that $b^2 > \omega^2$ then $\sqrt{b^2-\omega^2}$ is real and less than b

Now the powers $\left(-b+\sqrt{b^2-\omega^2}\right)$ and $\left(-b-\sqrt{b^2-\omega^2}\right)$ in equation (16) are both negative. Thus displacement x consists of two terms, both dying off exponentially to zero without performing any oscillations. The body after passing through maximum displacement simply slumps to its equilibrium position and rests there. This is referred to as **over damped**, or **deadbeat** or **aperiodic** motion.

Examples: The motion of pendulum in highly viscous liquid, Dead beat moving coil galvanometer etc.

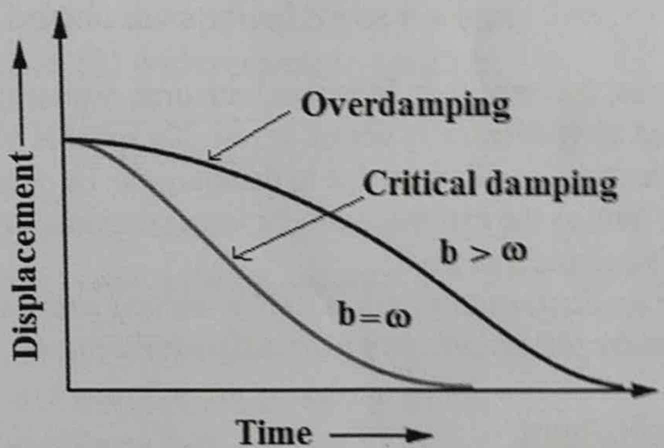
Case (2) $b^2 = \omega^2$, critical damping case:

If we put $b^2 = \omega^2$ in equation (16) becomes

$$x = e^{-bt}[p+qt]$$

It is clear from the above equation that, x increases due to the factor p+qt but at the same time x decreases due to the factor e^{-bt}. Since e^{-bt} has a predominant effect x decreases throughout with increase in t. It decrease slowly in the beginning and then rapidly approaches to zero. It is also clear that particle tend to acquire its equilibrium position much more rapidly than in case of over damping. Such a motion is called **critical damped** motion.

Examples:This type of motion is exhibited by many pointer instruments such as voltmeter, ammeter etc. in which the pointer moves to the correct position and comes to rest without any oscillation.



Case (3) $b^2 < \omega^2$, under damping case:

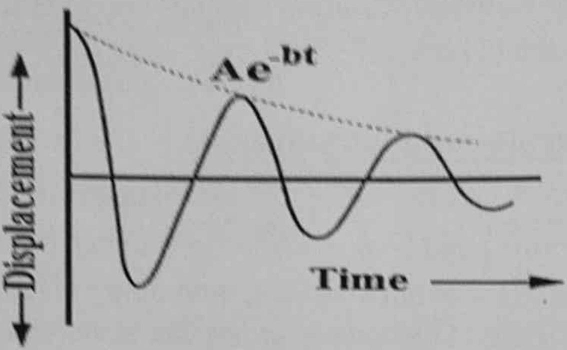
If damping is small such that $b^2 < \omega^2$,

Then equation(16) becomes $x = Ae^{-bt}\sin(nt+\phi)$ where $n = \sqrt{[\omega^2 - b^2]}$

This equation which represents SHM with amplitude Ae^{-bt} and time period $T = \frac{2\pi}{\sqrt{\omega^2-b^2}}$

The amplitude of the motion is continuously decreasing because of the factor e^{-bt} which is called the damping factor. Since the value $\sin(nt+\phi)$ varies between +1 and -1 the amplitude also varies between $+A e^{-bt}$ and $-Ae^{-bt}$. The decay of the amplitude depend upon the damping coefficient b. It is called **under damped motion**.

Example: Any real physical system which is set to vibrate and left free exhibits this kind of decay in its vibratory motion.



Quality factor:

The quality factor is the dimensionless quantity that describes the nature of damping in underdamped oscillations.

It is defined as 2π times the ratio of energy stored in the system to the energy lost per period.

$$Q = 2\pi \frac{Energy\ stored}{Energy\ loss\ per\ period}$$

$$Q = 2\pi \frac{E}{P \times T} = \frac{E\omega}{P} \text{ or } Q = \omega\tau$$

Where τ is the relaxation time. It is defined as the time taken for total mechanical energy to decay to a value 1/ e times the original value.

Q is also given by

$$\mathbf{Q = \frac{\omega}{2b}}$$

This Q is the measure of the extent to which oscillator is free from damping.

Thus high value of Q means that the damping of the oscillating system is low and energy loss per period is relatively small.

Forced Oscillations:

In the damped oscillations, the amplitude decreases with time exponentially due to dissipation of energy and body eventually comes to rest. If a suitable periodic external force is applied to the body, decay of oscillations due to damping can be overcome, and body executes oscillations so long as the external periodic force continues to act on it. Such oscillations are called forced oscillations.

Forced oscillations are oscillations in which the body vibrates with frequency other than its natural frequency under the action of an external periodic force.

Examples of forced oscillations:

1. Oscillations of swing which is pushed periodically by a person.
2. The periodic variation of current in an LCR circuit driven by an ac source.
3. The vibrations of the ear drum caused by sound from a sounding body
4. The motion of hammer in a calling bell.

Theory of forced oscillation:

Consider a body mass 'm' executing oscillation in damping medium acted upon by external periodic force. The forces acted upon the body are

1. Restoring force proportional to displacement but oppositely directed given by – kx where k is known as the force constant
2. The damping force which is proportional to the velocity but oppositely directed and is given by $-r\frac{dx}{dt}$. Where r is the damping constant.
3. The external periodic force represented by F sin pt, where F is the maximum value of this force and p is the angular frequency of the external force.

So the total force acting on the particle is given by

$$-kx - r\frac{dx}{dt} + F\sin pt$$

By Newton's law of motion this must be equal to the product of mass m of the body and acceleration d^2x/dt^2 i.e. m d^2x/dt^2 hence

$$m\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt} + F\sin pt$$

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = F\sin pt$$

Dividing by m

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F}{m}\sin pt$$

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega^2 x = \frac{F}{m}\sin pt \text{ ----------------------(1)}$$

Where $\frac{r}{m} = 2b,\ natural\ frequency\ \omega = \sqrt{\frac{k}{m}}$ therefore $\frac{k}{m} = \omega^2$

Equation (1) is the differential equation of the motion of the body.

When the steady state is setup, the body vibrates with frequency of applied force and not with its own frequency. The solution of the equation (1) is must be of the type

$$x = a\sin(pt-\alpha) \text{ ---------------------------------------(2)}$$

Where 'a' is the amplitude of vibration and 'α' is the angle with which the displacement x lags behind the applied force F sin pt.

Differentiating equation (2) with respect to t we get

$$\frac{dx}{dt} = a\,p\cos(pt-\alpha) \text{ -------------------------------------(3)}$$

Differentiating again

$$\frac{d^2x}{dt^2} = -a\,p^2\sin(pt-\alpha) \text{ ----------------------------------(4)}$$

Substituting equation(2), (3) and (4) in Equation (1)

$$-a\,p^2\sin(pt-\alpha) + 2ba\,p\cos(pt-\alpha) + \omega^2 a\sin(pt-\alpha) = \frac{F}{m}\sin pt \text{ -----(5)}$$

The right side of the above equation can be written as

$$\frac{F}{m}\sin pt = \frac{F}{m}\sin[(pt-\alpha)+\alpha]$$

$$= \frac{F}{m}\sin(pt-\alpha)\cos\alpha + \frac{F}{m}\cos(pt-\alpha)\sin\alpha$$

Substituting in (5)

$$-a\,p^2\sin(pt-\alpha) + \omega^2 a\sin(pt-\alpha) + 2ba\,p\cos(pt-\alpha) = \frac{F}{m}\cos\alpha\sin(pt-\alpha) + \frac{F}{m}\sin\alpha\cos(pt-\alpha)$$

Rearranging

$$(\omega^2 - p^2)a\sin(pt-\alpha) + 2bap\cos(pt-\alpha) = \frac{F}{m}\cos\alpha\sin(pt-\alpha) + \frac{F}{m}\sin\alpha\cos(pt-\alpha) \quad \text{-------(6)}$$

By equating coefficient of sin(pt-α) on both sides

$$(\omega^2 - p^2)a = \frac{F}{m}\cos\alpha \quad \text{-------(7)}$$

Similarly, by equating coefficient of cos(pt-α) on both sides

$$2bap = \frac{F}{m}\sin\alpha \quad \text{-------(8)}$$

Squaring and adding equation (7) and (8)

$$[(\omega^2 - p^2)a]^2 + (2bap)^2 = \left(\frac{F}{m}\right)^2(\sin^2\alpha + \cos^2\alpha)$$

$$a^2[(\omega^2 - p^2)^2 + 4b^2p^2] = \left(\frac{F}{m}\right)^2$$

$$\boldsymbol{a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2 + (\omega^2 - p^2)^2}}} \quad \text{-------(9)}$$

Substituting (9) in equation (2) the solution of the equation for the forced vibration can be written as

$$\boldsymbol{x = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2 + (\omega^2 - p^2)^2}}\sin(pt-\alpha)} \quad \text{-------(10)}$$

Dividing equation (8) by (7)

$$\tan\alpha = \frac{2bap}{a(\omega^2 - p^2)} = \frac{2bp}{(\omega^2 - p^2)}$$

Therefore phase α of the forced vibration is given by

$$\boldsymbol{\alpha = \tan^{-1}\left[\frac{2bp}{(\omega^2 - p^2)}\right]} \quad \text{-------(11)}$$

Equations (9) and (11) represent amplitude and phase of the forced vibrations respectively Depending upon the relative values of p and ω the following three cases are possible

Case(1). When the driving frequency is low i.e. $p \ll \omega$. In this case amplitude of vibration is given by

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2 + (\omega^2 - p^2)^2}} \quad \text{since } p \ll \omega,\ \omega^2 - p^2 \approx \omega^2 \text{ and } 2bp \approx 0$$

$$a = \frac{F}{m\omega^2} = \text{constant}$$

This shows that the amplitude of vibration is independent of frequency of the force and depends on the magnitude of the applied force F. For constant F amplitude a is constant. Phase is given by

$$\alpha = \tan^{-1}\left[\frac{2bp}{(\omega^2 - p^2)}\right] = \tan^{-1}(0) = 0 \quad \text{because } \omega^2 - p^2 \approx \omega^2 \text{ and } 2bp \approx 0$$

α=0 i.e. displacement and force will be in the same phase.

Case(2) When $p = \omega$.

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2 + (\omega^2 - p^2)^2}} = \frac{\left(\frac{F}{m}\right)}{2bp} = \frac{\left(\frac{F}{m}\right)}{\left(\frac{r}{m}\right)p} = \frac{F}{r\omega}$$

The amplitude of vibration is maximum for low damping in this case.

Phase is given by $\alpha = \tan^{-1}\left[\frac{2bp}{(\omega^2-p^2)}\right] = \tan^{-1}(\infty) = \frac{\pi}{2}$

Displacement has a phase lag of $(\pi/2)$ with respect to applied force in this case.

Case(3) $p >> \omega$

In this case amplitude of vibration is given by

$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2+(\omega^2-p^2)^2}}$ since $p >> \omega$, $(\omega^2 - p^2)^2 = p^4$ and $4b^2p^2 << p^2$

$a = \frac{F}{mp^2}$

In this case amplitude goes on decreasing as p goes on increasing

Phase is given by $\alpha = \tan^{-1}\left[\frac{2bp}{(\omega^2-p^2)}\right] = \tan^{-1}\left(\frac{2b}{-p}\right)$

Since b is small $\frac{2b}{p} \approx 0$ and $\alpha = \tan^{-1}(-0) = \pi$

As p increases phase difference between the displacement and applied force approaches π

Resonance:

When the frequency of the periodic force acting on the vibrating body is equal to the natural frequency of vibration of the body, the energy transfer from periodic force to the body becomes maximum because of which body will vibrate with maximum amplitude. This phenomenon is called resonance.

Examples of resonance:

1. Helmholtz resonator.
2. A radio receiver is tuned to the broadcast frequency of a transmitting station.
3. The vibrations caused by an excited tuning fork in another nearby identical tuning fork.
4. Soldiers crossing the suspension bridge are prohibited march in steps.

Theory of resonance:

Condition for resonance:

In case of forced vibrations amplitude of the forced oscillation is given by

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2+(\omega^2-p^2)^2}} \quad \text{-------------------------------(1)}$$

Equation (1) shows that the amplitude varies with the frequency p of the applied force. For a particular value of p, the amplitude becomes maximum. This phenomenon is known as amplitude resonance. Amplitude is maximum when

$\sqrt{4b^2p^2 + (\omega^2 - p^2)^2}$ is minimum

i.e. $\frac{d}{dp}(4b^2p^2 + (\omega^2 - p^2)^2) = 0$

$4b^2(2p) + 2(\omega^2 - p^2)(-2p) = 0$

$2b^2 - (\omega^2 - p^2) = 0$

$p^2 = \omega^2 - 2b^2$ ---(2)

It is clear from the above that, the denominator reaches its minimum when equation (3) is satisfied. further condition reduces to $p^2 = \omega^2$ when b is negligible.

When $p = \omega$ amplitude is

$$a_{max} = \frac{\frac{F}{m}}{\sqrt{4b^2\omega^2}} = \frac{\frac{F}{m}}{2b\omega}$$

$a_{max} \rightarrow \infty \; as \; b \rightarrow 0$

Conditions for resonance are

1. Frequency of the applied force must be equal to natural frequency of the vibrating body i.e. $p = \omega$
2. Damping factor b must be minimum.

Sharpness of resonance:

We have seen that the amplitude of the forced oscillation is maximum at resonance. When the frequency of the driving force is increased or decreased from resonant frequency the amplitude falls off from the maximum value.

Thus the term sharpness of the resonance means the rate of fall in amplitude with change of frequency of applied force on each side of resonance frequency.

sharpness of resonance = $\frac{\textit{Change in amplitude}}{\textit{Change in frequency}}$

We know that amplitude of vibration at resonance is

$$a_{max} = \frac{\frac{F}{m}}{2b\omega}$$

When there is no resonance amplitude of vibration is given by

$$a = \frac{\left(\frac{F}{m}\right)}{\sqrt{4b^2p^2+(\omega^2-p^2)^2}}$$

Near resonance $p = \omega$ and $(\omega^2\text{-}p^2) = 0$

Therefore $a = \frac{\frac{F}{m}}{2bp}$

Change in amplitude $= a_{\max} - a = \frac{\frac{F}{m}}{2b\omega} - \frac{\frac{F}{m}}{2bp} = \frac{F/m}{2b}\left(\frac{1}{\omega} - \frac{1}{p}\right) = \frac{F/m(p-\omega)}{2b\omega p}$

Change in frequency = $p\text{-}\omega$

But the sharpness of resonance = $\frac{\textit{Change in amplitude}}{\textit{Change in frequency}}$

Therefore sharpness of resonance = $\frac{\frac{F}{m}}{2b\omega p}$

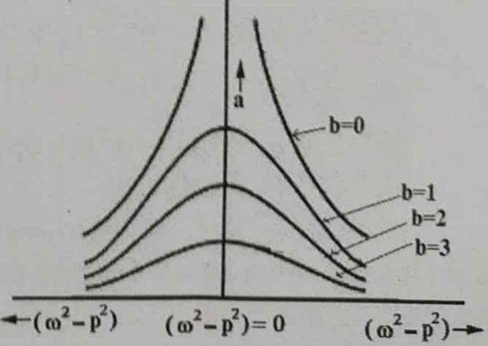
Therefore sharpness of resonance depends inversely as b

Effect of damping on sharpness of resonance

Figure shows graphically the response of amplitude to various degrees of damping. One can notice that curves are rather flat for larger values of b. On the other hand curve for smaller value of b exhibits pronounced peak and it refer to sharp resonance. For b = 0 amplitude will be infinity which never exists in reality.

Significance of sharpness of resonance:

The amplitude of oscillations of an oscillating body or system rises to maximum when the frequency of the external force is equal to the natural frequency of the oscillating system. However the rise of the amplitude will be very sharp when the damping is very small.



SHOCK WAVES

Definition of Mach number:

Mach number (M) is the ratio of speed of an object moving through a fluid to the speed of sound in the fluid

$$Mach\ Number = \frac{Object\ speed}{Speed\ of\ sound\ in\ medium}$$

$$M = \frac{v}{a}$$

Where M is the Mach number, v is the speed of the object relative to the medium, and a is the speed of sound in the medium.

Since Mach number is a ratio of speeds, it is a dimension less quantity. As name indicates it is a pure number.

Speed of sound:

The speed of sound a in air or any medium at temperature T is given by

$a = \sqrt{\gamma RT}$ Where γ the ratio of specific heats and R is is the specific gas constant.

Distinctions between acoustic, ultrasonic, subsonic, supersonic waves, Transonic waves and Hypersonic waves:

Acoustic waves:

Acoustic waves are longitudinal waves that travel in the medium with the speed of sound in the medium. These waves can propagate in solids liquids and gases.

Ultrasonic waves:

Ultrasonic waves are acoustic waves having frequencies beyond 20K Hz. The human ear is not sensitive to these waves.

Subsonic Waves:

If the speed of the mechanical wave or body moving in the fluid is less than the speed of the sound then such a speed is referred to as subsonic speed and the wave is subsonic wave. All subsonic waves have Mach number $M<1$.

The speeds of the almost all the vehicles such as motor cars or trains and speeds of the birds are subsonic.

For body moving with subsonic speed, the sound emitted by the body manages to move ahead and away from the body.

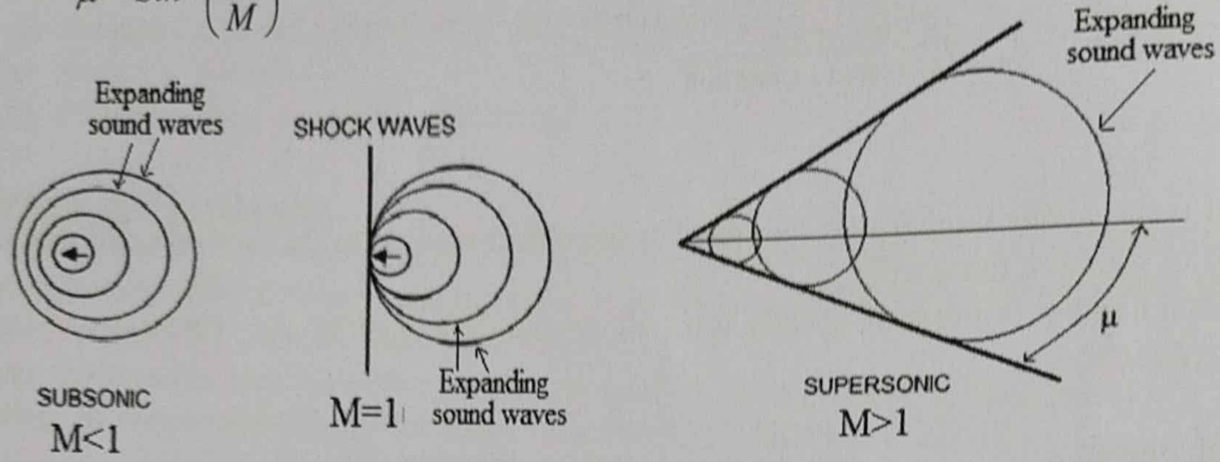
Supersonic waves:

Supersonic waves are mechanical waves which travel with speed greater than that of the sound i.e. Mach number $M>1$.

A body with supersonic speed zooms ahead by piercing its own sound curtains, leaving behind a series of expanding sound waves with their centres displaced continuously along its trajectory. The amplitudes of supersonic waves are very high and they affect the medium through which they move. Fighter planes fly with supersonic speed.

A common tangent drawn to the expanding sound waves emitted from the body at supersonic speed formulates a cone called **Mach cone**. The angle made by the tangent with the axis of the mach cone is called **Mach angle** μ. M is related to Mach number by the relation

$$\mu = Sin^{-1}\left(\frac{1}{M}\right)$$



Transonic waves:
Transonic waves have speed such that its Mach number is between 0.8 and 1.2. This is the region where the phase changes from subsonic to supersonic takes place. This is the grey area where there is overlapping of some of the characteristics of both the subsonic and the supersonic speeds.

Hypersonic waves:
Supersonic waves which are moving with speed such that Mach number $M > 5$ are called **hypersonic waves.**

Description of a shock wave
A Shock wave is a narrow surface that manifests as a discontinuity in a fluid medium in which it is propagating with supersonic speed. The disturbance is characterised by sudden increase in pressure temperature and density of the gas through which it propagates.

It has a compression wavefront with large changes in pressure, density and temperature across it. The thickness of the shockwave front is very small of the order of 100nm.
Shock waves can be produced by sudden dissipation of mechanical energy in a medium enclosed in a small space.
Explosions, lightning, or other phenomena that create violent changes in pressure can also produce shock waves.
Shock waves are also produced by the object moving with supersonic speed.
Shock waves are strong or weak depending on the magnitude of the instantaneous change in pressure and temperature in the medium

- For strong shock waves Mach no. is high

Example: nuclear explosion, shock waves created during thunder or lightning.

- For weaker shock waves the Mach no. is low (close to 1)

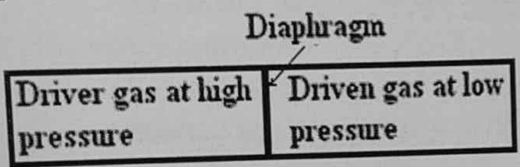
Example: explosion of cracker, bursting of tyre.

roperties of shock waves:

1. They always travel in the medium with Mach number exceeding 1.
2. Shock waves obey the laws of fluid dynamics.
3. The effect caused by the shock waves results in increase in entropy.
4. When a shock wave is formed, there is a distinct surface created by the medium itself called shock front. Typical thickness of a shock front is few micrometers.
5. Shock waves are characterized by sudden increase in pressure, temperature and density of the gas through which it propagates.
6. Shock waves propagate in a manner different from that of ordinary acoustic waves.
7. Shock waves travel faster than sound and their speed increases as the amplitude is increased.
8. Intensity of a shock wave decreases faster than that of a sound wave because some of the energy of the shock wave is expended to heat the medium in which it travels.
9. While they turn around a convex corner, they breakup into very large number of expanding supersonic waves diverging from a central spot and the process is called supersonic expansion fan.

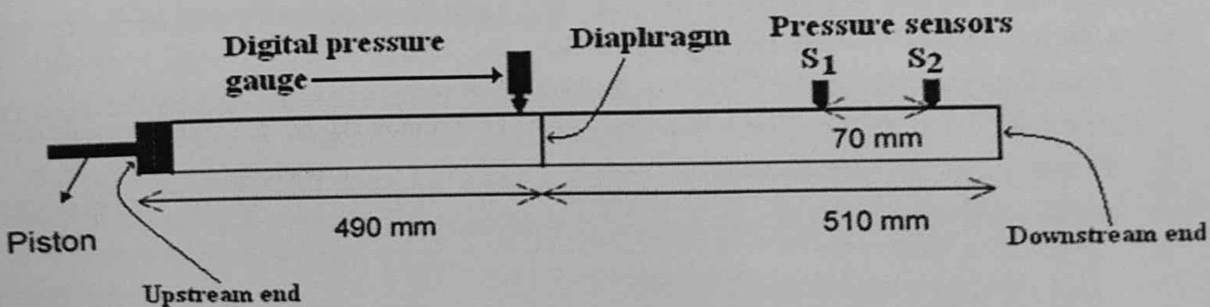
Shock tubes:

Shock tube is a simple device that is used to generate the moving shock waves in the controlled environment. The shock tube is a cylindrical tube of metal. It has two sections driver section and driven sections separated by a metal diaphragm. The diver section contains gas at high pressure called driver gas and driven section contains gas of low pressure called driven gas. When the diaphragm ruptures due to the high pressure gas in the driver section, it generates a shock wave that moves along the length of the driven side, there by increasing the pressure, temperature and density of the driven gas.



Reddy's Tube:

Reddy's tube is a hand operated shock tube capable of producing shock waves by using human energy. It is a long cylindrical tube with two sections separated by diaphragm. Its one end is fitted with a piston and the other end is closed or open to the surroundings.



Construction:

- Reddy tube is a hand operated shock tube used to produce shock waves by using human energy.
- It consists of a cylindrical stainless-steel tube of about 30 mm in diameter and of length about 1 meter.
- It is divided into two sections each of length about 50 cm. One is called driver section and the other one is driven section. The two sections are separated by 0.1 mm thick aluminium or Mylar or paper diaphragm.
- Far end of driver section is fitted with a piston whereas the far end of the driven section is closed.
- A digital pressure gauge is fixed in the driver section next to the diaphragm.
- Two piezoelectric sensors S1 and S2 separated by 70 mm are mounted towards the closed end of the shock tube.
- The driver section is filled with a gas called as the driver gas which is held at a relatively high pressure due to the compressing action of the piston.
- A port is provided at the closed end of the driven section for filling the test gas to the required pressure.
- The gas in the driven section is termed as driven gas or test gas

Working:

- The driver gas is compressed by pushing the piston towards the diaphragm until it ruptures.
- Due to rupture, the driver gas rushes into the driven section, and pushes the driven gas towards the far downstream end. This generates a moving shock wave that travels along length of the driven section. The shock wave instantaneously raises the temperature and pressure of the driven (test gas) gas when the shock moves over it.
- The propagating primary shock wave is reflected from the downstream end. After the reflection, the test gas undergoes further compression which increases temperature and pressure to still higher values.
- This state of high values of pressure and temperature is maintained at the downstream end until an expansion wave reflected from the upstream end of the driver tube arrives there and neutralizes the compression partially. Expansion waves are created at the instant the diaphragm is ruptured and they travel in a direction opposite to that of the shock wave.
- The period over which the extreme temperature and pressure conditions at the downstream end is sustained, is of the order of milliseconds.
- The pressure rise caused by the primary shock waves and also the reflected shock waves are sensed as signals by the sensors S1 and S2 respectively and are recorded in a digital cathode ray oscilloscope (CRO).
- From the recording in the CRO the shock arrival times are found out by the associated time base calculations. Using the data obtained, Mach number, pressure and temperatures can be measured.
- It is capable of producing Mach number exceeding 1.5.

Characteristics of Reddy tube:

1. The Reddy tube operates on the principle of free piston driven shockwaves
2. It is a hand operated shock producing device.
3. It is capable of producing Mach number exceeding 1.5.
4. The rupture pressure is a function of the thickness of the diaphragm.
5. The temperature exceeding 900 K can be easily obtained by the Reddy tube by using helium as the drive gas and argon as the driven gas. This temperature is useful in the chemical kinetic studies.

Application of shock waves:

1. ***Cell transformation***:
   Shock waves have been used for genetic transformation where a particular DNA of interest is introduced into the cell without damaging/ destroying the living cell. This has wide biological application.

2. ***Wood preservation***:
   By using shock waves chemical preservatives in the form of solution could be pushed into the interior of the wood samples such as bamboo. This method makes the process of introduction of the preservatives into the wood much faster and more efficient.

3. ***Pencil industry***:
   While manufacturing the pencils wood is used to cover the graphite rod. The wood is softened by socking in a polymer at 70°C for about three hours and then dried. In the modern process using shock waves, the wood is placed in the liquid and the shock wave is sent through it. The liquid gets into the wood instantaneously. This reduces the manufacturing time.

4. ***Kidney stone treatment***:
   Shock waves are used to break the kidney stones into small pieces. These small pieces can travel easily through the urinary tract. It is called extra corporal lithotripsy

5. ***Needleless drug delivery***:
   By using shock waves drugs can be injected into the body without using needles. The patient doesn't experience any pain.

6. ***Treatment of dry bore wells***:
   Sometimes the seepage routs of the bore wells are blocked by sand particles clogging the pores. It has been observed that, a shockwave sent through such a dry bore well, clears the blockages.

7. **Gas dynamic studies**
   The extreme conditions of pressure and temperature can be produced in the shock tubes enables the study of high temperature gas dynamics. This knowledge is crucial in the study of supersonic motion of the bodies and hypersonic re-entry of space vehicles into the atmosphere.

Engineering Physics
Question Bank
(CBCS Scheme-2021-22)
Module – 1

1. Define simple harmonic motion. Give four examples for SHM
2. Starting from Hooke's' law derive the differential equation for SHM. Mention its solution
3. Give the Characteristics of SHM.
4. Define force constant. Give the physical significance of the force constant.
5. Derive the expression for equivalent force constant for 2 springs in series and parallel.
6. Explain free, damped and forced Oscillation with examples for each.
7. What are damped oscillations? Derive the expression for decaying amplitude and hence discuss three cases in detail.
8. Discuss the theory of forced vibrations and hence obtain the expression for amplitude and Phase.
9. What is resonance? Obtain the condition for resonance. Give the example for resonance.
10. Write note on 1) sharpness of resonance, 2) Quality factor,
11. What is Mach number? Distinguish between subsonic, Supersonic waves. Transonic and Hypersonic waves.
12. What are shock waves? Mention the characteristics of shock waves
13. Classify the shock waves on the basis of Mach number and mention examples for each
14. With a neat diagram explain the construction and working of Reddy's tube.
15. Explain applications of Shock waves