BASIC ELECTRICAL ENGINEERING- 21ELE13

DC CIRCUITS :

<u>OHM'S LAW:</u>

The current flowing through the electric circuit is directly proportional to the potential difference across the circuit, inversely proportional to the resistance of the circuit, provided temperature and other parameters remained constant.

i.e, $I \propto V$

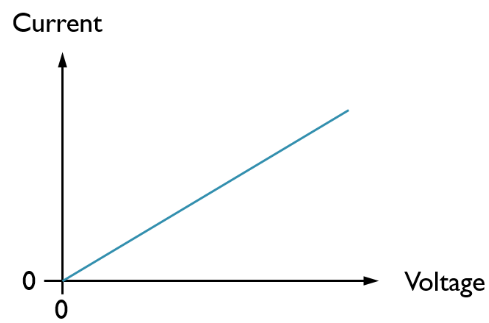
$I = V/R$

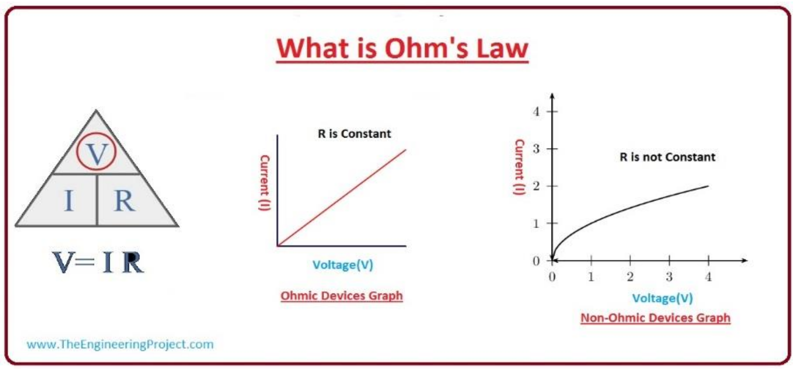
$V = RI$

limitations of the Ohm‟s law are,

(i) It is not applicable to the nonlinear devices such as diodes, Zener diodes, voltage regulators etc.

(ii) It does not hold good for non-metallic conductors such as silicon carbide.





<u>Kirchhoff‟s Laws:</u>

Kirchhoff‟s Current law: The law can be stated as, The total current flowing towards a junction point is equal to the total current flowing y from that junction point.

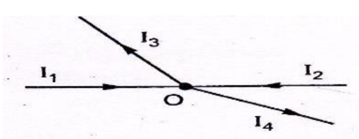


Fig. 1 Junction point

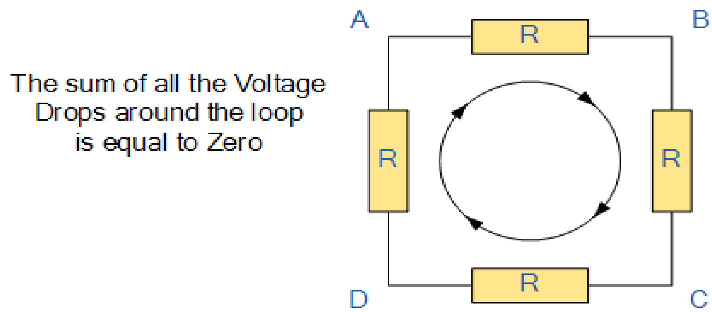
Sign convention : Currents flowing towards a junction point are assumed to be positive, while currents flowing away from a junction point assumed to be negative.

e.g. Refer to Fig. 1, currents I_1 and I_2 are positive while I_3 and I_4 are negative.

Applying KCL,

$I_1 + I_2 - I_3 - I_4 = 0$ i.e. $I_1 + I_2 = I_3 + I_4$

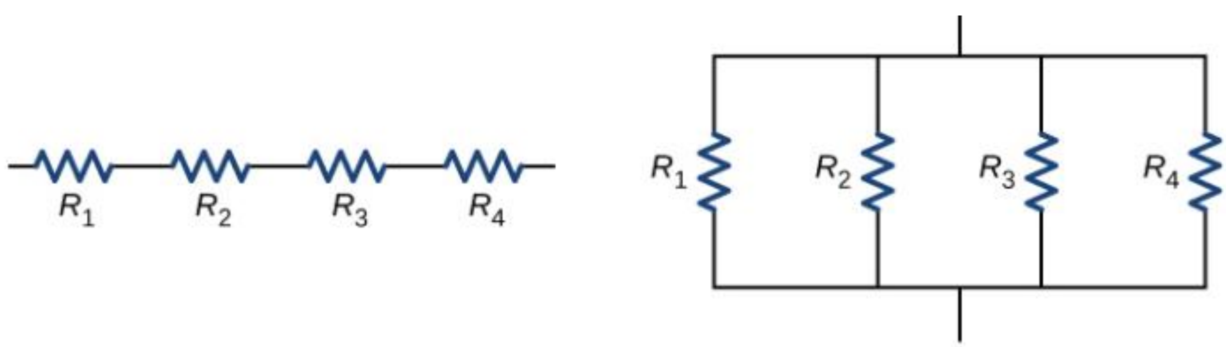
Kirchhoff‟s Voltage law: Kirchhoff's voltage law states that the sum of all voltages around any closed loop in a circuit must equal zero. Or In any closed electrical circuit algebraic sum of products of currents and resistances(Voltage drops) Plus algebraic sum of all the emfs in that circuit is Zero.



$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$

Analysis of series, parallel and series-parallel circuits excited by independent voltage sources :

Basically, a resistor limits the flow of charge in a circuit and is an ohmic device where V=IR. Most circuits have more than one resistor. If several resistors are connected together and connected to a battery, the current supplied by the battery depends on the **equivalent resistance** of the circuit.
The equivalent resistance of a combination of resistors depends on both their individual values and how they are connected. The simplest combinations of resistors are series and parallel connections.



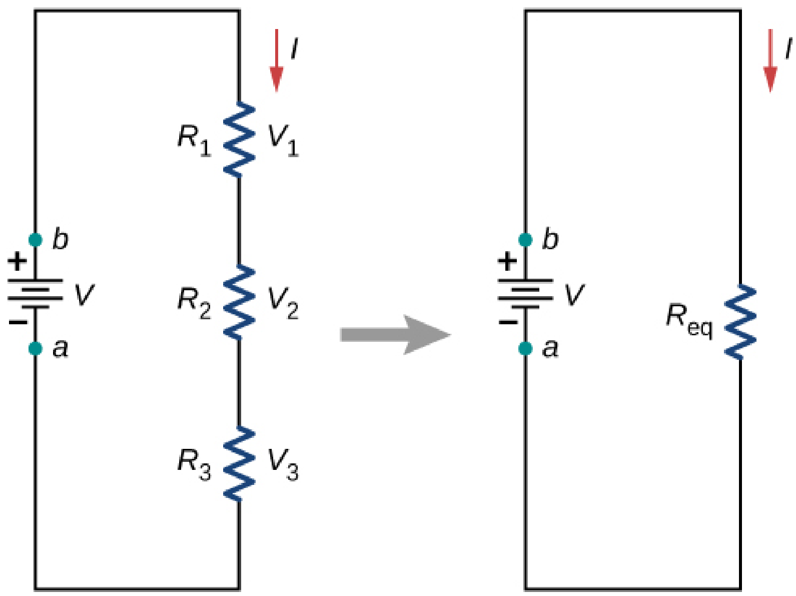
(a) Resistors connected in series (b) Resistors connected in parallel

In a **series circuit**, the output current of the first resistor flows into the input of the second resistor; therefore, the current is the same in each resistor. In a **parallel circuit**, all of the resistor leads on one side of the resistors are connected together and all the leads on the other side are connected together. In the case of a parallel configuration, each resistor has the same potential drop across it, and the currents through each resistor may be different, depending on the resistor.

The sum of the individual currents equals the current that flows into the parallel connections.

Resistors in Series

Resistors are said to be in series whenever the current flows through the resistors sequentially. Consider the below Figure , which shows three resistors in series with an applied voltage equal to V_{ab} . Since there is only one path for the charges to flow through, the current is the same through each resistor. The equivalent resistance of a set of resistors in a series connection is equal to the algebraic sum of the individual resistances.



(a) Original circuit (b) Equivalent circuit

In the above figure the current coming from the voltage source flows through each resistor, so the current through each resistor is the same. The current through the circuit depends on the voltage supplied by the voltage source and the resistance of the resistors. For each resistor, a potential drop occurs that is equal to the loss of electric potential energy as a current travels through each resistor. According to Ohm's law, the potential drop V across a resistor when a current flows through it is calculated using the equation V=IR , where I is the current in amps (A) and R is the resistance in ohms . Since energy is conserved, and the voltage is equal to the potential energy per charge, the sum of the voltage applied to the circuit by the source and the potential drops across the individual resistors around a loop should be equal to zero.

The sum of the potential drop of each resistor and the voltage supplied by the voltage source should equal zero:

$$V - V_1 - \quad \sum_{i=1}^{N} V_i = 0. \quad),$$

$$\begin{aligned} & V_1 + V_2 + V_3 \\ = \ & IR_1 + IR_2 + IR_3, \\ \Rightarrow I = \ & \frac{V}{R_1 + R_2 + R_3} = \frac{V}{R_{eq}}. \end{aligned}$$

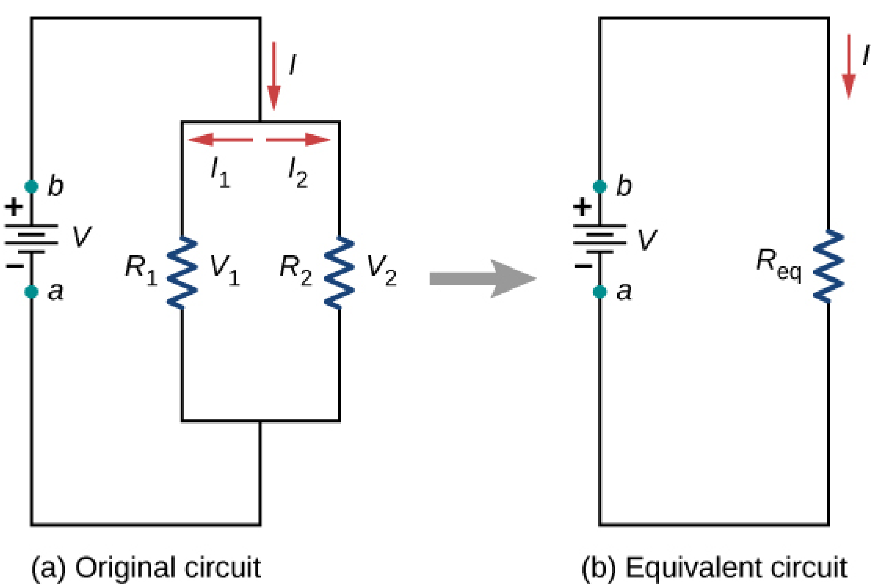
Since the current through each component is the same, the equality can be simplified to an equivalent resistance, which is just the sum of the resistances of the individual resistors.

Any number of resistors can be connected in series. If N resistors are connected in series, the equivalent resistance is

$$R_{\mathrm{eq}} = R_1 + R_2 + R_3 + \ldots + R_{N-1} + R_N = \sum_{i=1}^{N} R_i.$$

Resistors in Parallel:

Below fig shows resistors in parallel, wired to a voltage source. Resistors are in parallel when one end of all the resistors are connected by a continuous wire of negligible resistance and the other end of all the resistors are also connected to one another through a continuous wire of negligible resistance. The potential drop across each resistor is the same. Current through each resistor can be found using Ohm's law I=V/R, where the voltage is constant across each resistor. For example, an automobile's headlights, radio, and other systems are wired in parallel, so that each subsystem utilizes the full voltage of the source and can operate completely independently. The same is true of the wiring in your house or any building.



The current flowing from the voltage source in the figure depends on the voltage supplied by the voltage source and the equivalent resistance of the circuit. In this case, the current flows from the voltage source and enters a junction, or node, where the circuit splits flowing through resistors R1 and R2. As the charges flow from the battery, some go through resistor R1 and some flow through resistor R2. The sum of the currents flowing into a junction must be equal to the sum of the currents flowing out of the junction:

$$\sum I_{\mathrm{in}} = \sum I_{\mathrm{out}}.$$

There are two loops in this circuit, and the voltage across the resistors in parallel are the same .

$$\begin{aligned} I &= I_1 + I_2 \\ &= \frac{V_1}{R_1} + \frac{V_2}{R_2} \\ &= \frac{V}{R_1} + \frac{V}{R_2} \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{\mathrm{eq}}} \\ \Rightarrow R_{\mathrm{eq}} &= \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}. \end{aligned}$$

Generalizing to any number of N resistors, the equivalent resistance R_{eq} of a parallel connection is related to the individual resistances by

$$R_{\mathrm{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_{N-1}} + \frac{1}{R_N}\right)^{-1} = \left(\sum_{i=1}^{N} \frac{1}{R_i}\right)^{-1}.$$

Power and Energy:

Power: The rate at which the work is being done in an electrical circuit is called an electric power. In other words, the electric power is defined as the rate of the transfer of energy. The electric power is produced by the generator and can also be supplied by the electrical batteries.
$P=V\,I=I^2\,R$, The unit of power is Watt.

$$Electrical\ Power = \frac{Work\ done\ in\ an\ electrical\ current}{time}$$

$$P = \frac{VIt}{t} = VI = IR^2 = \frac{V^2}{R}$$

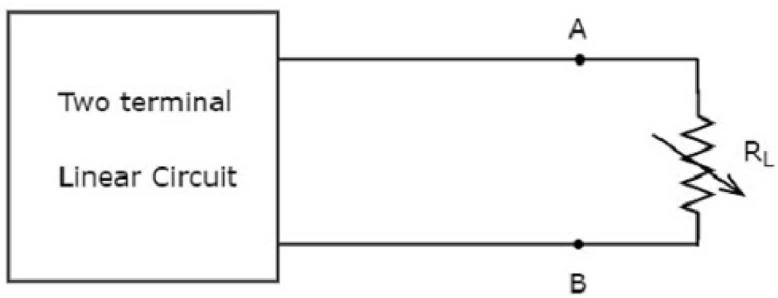
Energy : Electrical energy is the work done by electric charge. If current i ampere flows through a conductor or through any other conductive element of potential difference v volts across it, for time t second, the electric energy is, E=VIt Jouls
If one watt power is being consumed for 1 hour time, the energy consumed is one watt-hour

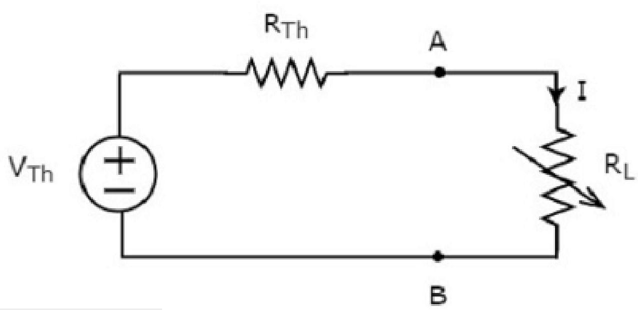
Maximum Power Transfer Theorem applied to series circuit:

The **Maximum Power Transfer Theorem** can be defined as, a resistive load is connected to a DC-network, when the load resistance (R_L) is equivalent to the internal resistance then it receives the highest power is known as Thevenin's equivalent resistance of the source network. The theorem defines how to select the load resistance (RL) when the source resistance is given once. It is a general misunderstanding for applying the theorem in the reverse situation.

Proof of Maximum Power Transfer Theorem

Replace any two terminal linear network or circuit to the left side of variable load resistor having resistance of R_L ohms with a Thevenin's equivalent circuit. We know that Thevenin's equivalent circuit resembles a practical voltage source.





$$P_L = I^2 R_L$$

Substitute in the $I = \frac{V_{Th}}{R_{Th}+R_L}$ above equation.

$$P_L = \left(\frac{V_{Th}}{(R_{Th}+R_L)}\right)^2 R_L$$

$$\Rightarrow P_L = {V_{Th}}^2 \left\{\frac{R_L}{(R_{Th}+R_L)^2}\right\}$$ **Equation 1**

Condition for Maximum Power Transfer

For maximum or minimum, first derivative will be zero. So, differentiate Equation 1 with respect to R_L and make it equal to zero.

$$\frac{dP_L}{dR_L} = {V_{Th}}^2\{\frac{(R_{Th}+R_L)^2 \times 1 - R_L \times 2(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\} = 0$$

$$\Rightarrow (R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L) = 0$$

$$\Rightarrow (R_{Th}+R_L)(R_{Th}+R_L-2R_L) = 0$$

$$\Rightarrow (R_{Th}-R_L) = 0$$

$$\Rightarrow R_{Th} = R_L \; or \; R_L = R_{Th}$$

Therefore, the **condition for maximum power** dissipation across the load is RL=RTh That means, if the value of load resistance is equal to the value of source resistance i.e., Thevenin's resistance, then the power dissipated across the load will be of maximum value.

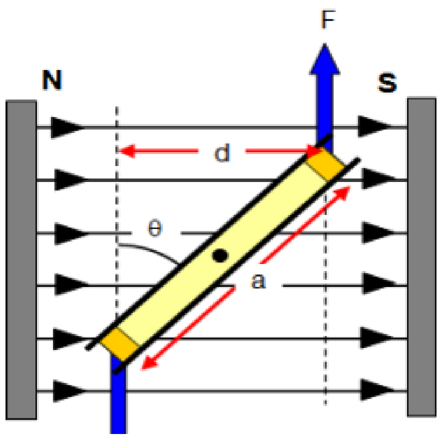
Applications of Maximum Power Transfer

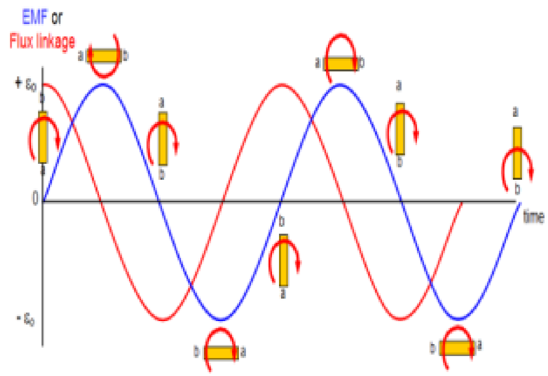
- In a public address system, the circuit is adjusted for maximum power transfer by making load resistance (speaker) equal to the source resistance (amplifier). When source and load have the same resistance, they are said to be matched.
- In car engines, the power delivered to the starter motor of the car will depend upon the effective resistance of the motor and the internal resistance of the battery. If the two resistances are equal, maximum power will be transferred to the motor to turn to the engine.

Single Phase Circuits:

Generation of sinusoidal voltage:

Consider a rectangular coil of N turns placed in a uniform magnetic field as shown in the figure.





The coil is rotating in the anticlockwise direction at a uniform angular velocity of ω rad/sec. When the coil is in the vertical position, the flux linking the coil is zero because the plane of the coil is parallel to the direction of the magnetic field. Hence at this position, the emf induced in the coil is zero. When the coil moves by some angle in the anticlockwise direction, there is a rate of change of flux linking the coil and hence an emf is induced in the coil. When the coil reaches the horizontal position, the flux linking the coil is maximum, and hence the emf induced is also maximum. When the coil further moves in the anticlockwise direction, the emf induced in the coil reduces. Next when the coil comes to the vertical position, the emf induced becomes zero. After that the same cycle repeats and the emf is induced in the opposite direction. When the coil completes one complete revolution, one cycle of AC voltage is generated. An alternating quantity changes continuously in magnitude and alternates in direction at regular intervals of time.

Alternating quantity: An alternating quantity is one which acts in alternate positive and negative directions, whose magnitude undergoes a definite series of changes in definite intervals of time and in which the sequence of changes of negative is identical with the sequence of changes of positive.

Waveform: “The graph between an alternating quantity (voltage or current) and time is called waveform”, generally, alternating quantity is depicted along the Y-axis and time along the X-axis.

Instantaneous value: The value of an alternating quantity at any instant is called instantaneous value.The instantaneous values of alternating voltages and current are represented by “e‟ and “i‟

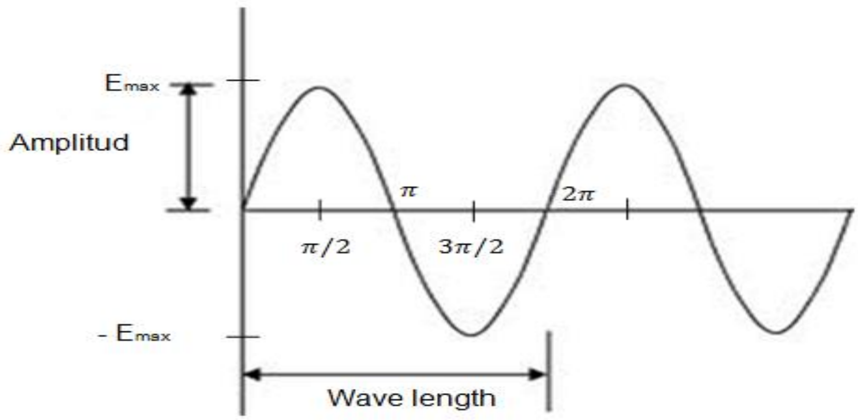
Alternation and cycle: When an alternating quantity goes through one half cycle (complete set of +ve or –ve values) it completes an alternation, and when it goes through a complete set of +ve and –ve values, it is said to have completed one cycle.

Periodic Time and Frequency: The time taken in seconds by an alternating quantity to complete one cycle is known as periodic time and is denoted by T.

Frequency: The number of cycles completed per second by an alternating quantity is known as frequency and is denoted by "f". in the SI system, the frequency is expressed in hertz.
The number of cycles completed per second = f.

Periodic Time T = Time taken in completing one cycle.
In India, the standard frequency for power supply is 50 Hz. It means that alternating voltage or current completes 50 cycles in one second.

Amplitude: The maximum value, positive or negative, which an alternating quantity attains during one complete cycle, is called amplitude or peak value or maximum value. The amplitude of alternating voltage and current is represented by E_m and I_m respectively.



Flux linking the coil at any instant = No.Of turns x Flux linking= N φ_{max} Cosωt

But as per Faradays Law, instantaneous emf induced in the coil is

$$e = -d/dt(\text{ Flux Linkage})$$

$$e = -d/dt(N\,\varphi_{max} \cos\omega t)$$

$$e = -N\,\varphi_{max}\omega\,(-\sin\omega t)$$

$e = N\omega\varphi_{max} \sin\omega t$ Volts. --------------(i)

E will be maximum when $\omega t=90^o$

Therefore $E_m = N\omega\varphi_{max}$ Volts. -------------(ii)

Put (ii) in (i) will get **$e = E_m \sin \omega t$** where $\omega = 2\pi f$, $\omega t = \theta$

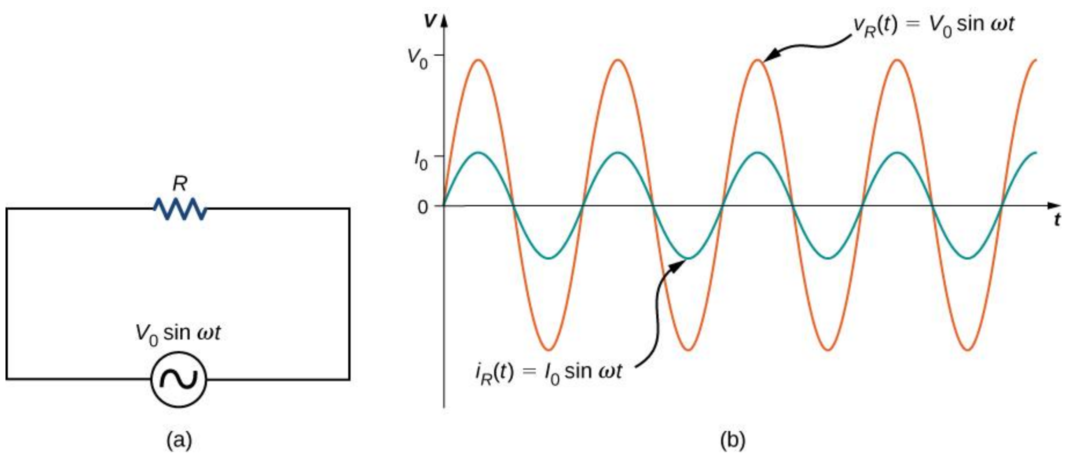
$e = E_m \sin 2\pi f\, t$

if T is the time period of A.C voltage then $e = E_m \sin(2\pi/T)t$ volts.

As current follows the voltage , Alternating current is given by,

$i = i_m \sin \omega t$

A.C Voltage and Current Waveforms



Rms(root mean square) value of an alternating quantity: RMS value of an alternating quantity is defined as, that steady state current when flowing through a given resistance for a given time produces same amount of heat as produced by an alternating current when flowing through the same resistance for the same time.

$P = i^2R$ and

$P_{ave} = I_{eff}^2 R$

$$P_{ave} = \frac{\int_0^T P\,dt}{T}$$

$$I_{eff}^2 R = \frac{\int_0^T i^2 R\,dt}{T}$$

$$I_{eff} = \sqrt{\frac{\int_0^T i^2\,dt}{T}}$$

If wave is not symmetrical then $T = 2\pi$

$$I_{eff} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(I_m sin\omega t)^2 d\omega t}$$

$$I_{eff} = \sqrt{\frac{Im^2}{2\pi}\int_0^{2\pi}\frac{(1-cos2\omega\, t)}{2}d\omega t}$$

$$I_{eff} = \sqrt{\frac{Im^2}{2\pi}\int_0^{2\pi}\frac{1}{2}\omega t}$$

$$I_{eff} = \sqrt{\frac{Im^2}{4\pi}2\pi}$$

$$I_{eff} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = I_{eff} = \frac{I_m}{\sqrt{2}}$$

Similarly $V_{rms} = \frac{V_m}{\sqrt{2}}$

Average value: The Arithmetical average of all the values of an alternating quantity over one cycle is called average value

We have $e = E_m \sin \omega t$

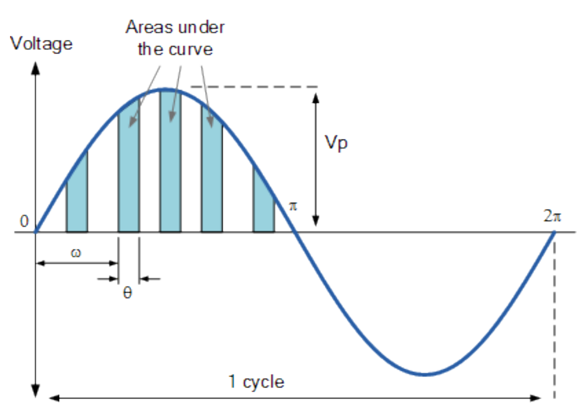
0 and π are the limits of integration since we are determining the average value of voltage over one half a cycle.Since we now know the area under the positive (or negative) half cycle, we can easily determine the average value of the positive (or negative) region of a sinusoidal waveform by integrating the sinusoidal quantity overhalf a cycle and dividing by half the period.

$$\text{Area} = \int_0^{\pi} V_P \sin(\omega t)\, dt$$

$$V_{AVE} = \frac{1}{\pi}\int_0^{\pi} V_P \sin\theta\, d\theta$$

$$V_{AVE} = \frac{V_P}{\pi}(-\cos\theta)_0^{\pi}$$

$$= \frac{2V_P}{\pi} = \frac{2}{\pi}V_P = 0.637V_P$$



Peak Factor(Ka) : It is the ratio between maximum value and RMS value of an alternating wave
For a sinusoidal alternating voltage:

$$\textbf{Peak Factor} = \frac{\text{Maximum Value}}{\text{R.M.S Value}} = \frac{E_M}{0.707\,E_M} = \mathbf{1.414}$$

Maximum value is 1.414 times the RMS Value.
Peak Factor is also known as Crest Factor or Amplitude Factor.

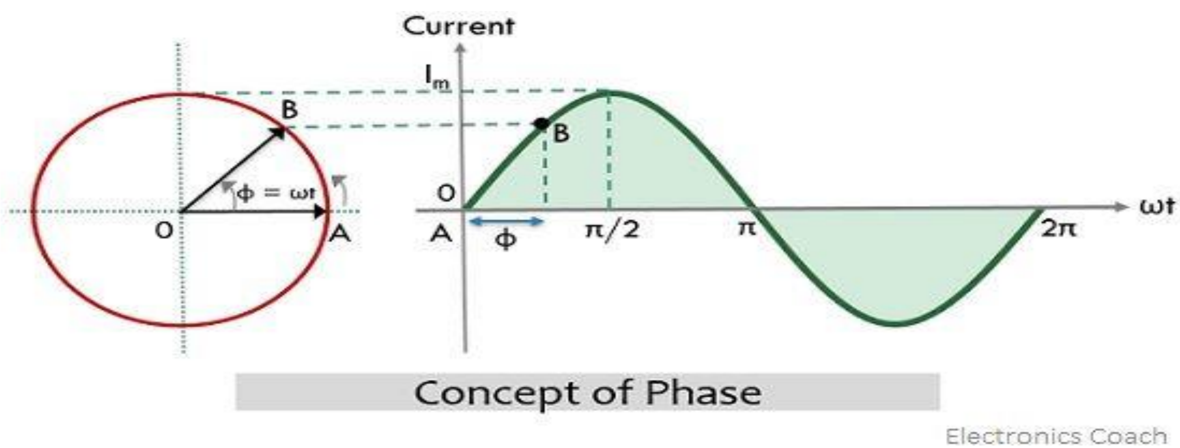
Form Factor(Kf) : The ratio between RMS value and Average value of an alternating quantity (Current or Voltage) is known as Form Factor.

$$\textbf{Form Factor} = \frac{\text{RMS Value}}{\text{Average Value}}$$

$$= \frac{0.707\,E_M}{0.637\,E_M} \text{ Or } \frac{0.707\,I_M}{0.637\,I_M} = \mathbf{1.11}$$

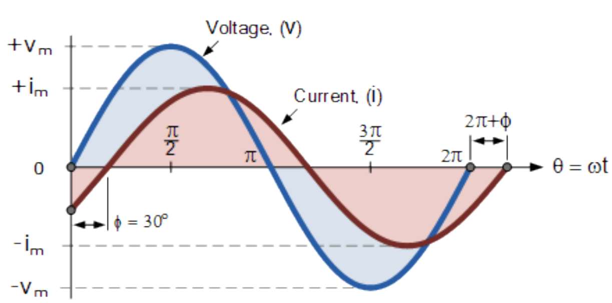
RMS value is 1.11 times the Average Value.

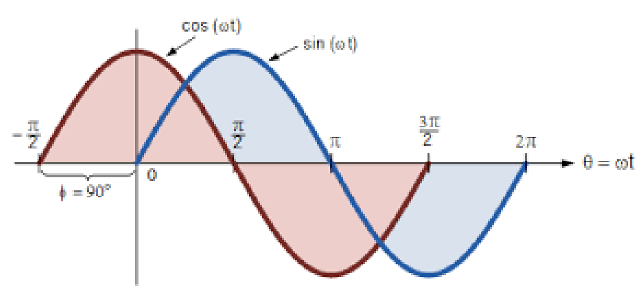
Phase of an alternating quantity: Phase of an alternating quantity at any instant is the fractional part of period or cycle through which the quantity has advanced from its selected origin.

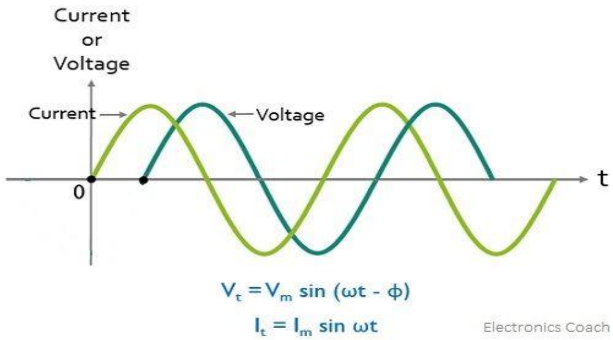


Concept of Phase

Phase difference : Here quantity ahead in phase is said to be lead the other quantity, where the second quantity is said to be lag behind the first one.



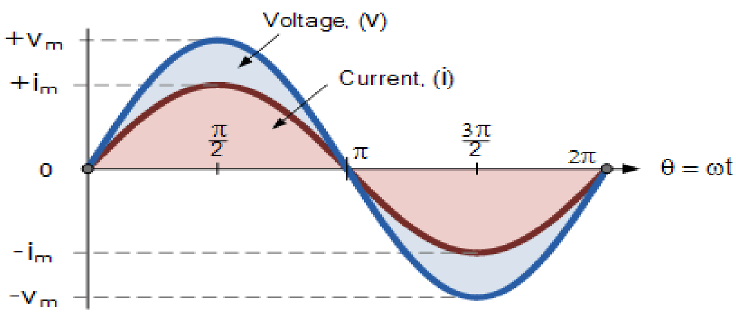




Here the the voltage waveform goes through its zero and Max.value first then the current goes through its zero and Max.Value, i.e after the Time lag angle φ.

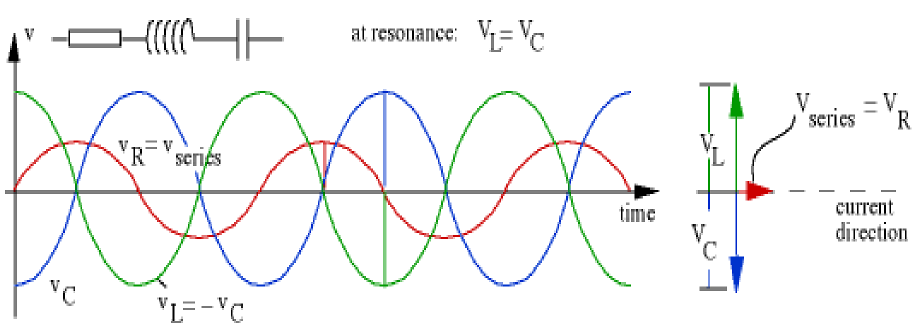
$$\mathbf{v = V_m\ Sin\ \omega t}$$
$$i = i_m (\mathrm{Sin}\ \omega t - \varphi)$$

In phase: when two quantities are said to be in phase with each other , if they pass through zero values at the same instances and raise in the same direction.

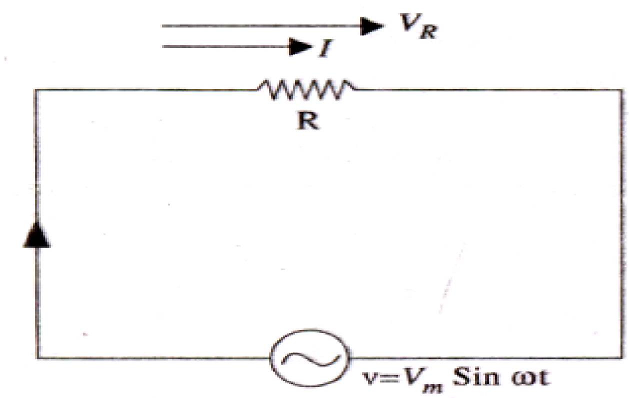


$$\mathbf{v = V_m\ Sin\ \omega t}$$
$$i = i_m\ \mathrm{Sin}\ \omega t$$

Out of Phase: If two quantities pass through zero values at the same instants but raise in opposite directions are said to be in phase opposition.(I.e phase difference of 180 degree).



Alternating Current Through Pure Ohmic Resistance :



Consider a circuit as shown in figure for which applied voltage is given by

$V = V_m \sin\omega t$

R = ohmic resistance,

i = instantaneous current

Due to applied alternating voltage,an alternating current 'i' flows through it.The applied voltage has to overcome ohmic voltage drop only.Hence, for equilibrium

$V = V_m \sin\omega t = iR$ ----Eq(1)

$$i = \frac{Vm}{R}\sin\omega t ----Eq(2)$$

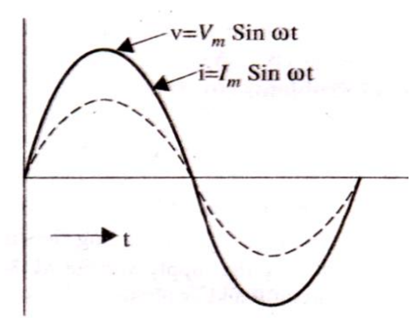
Current 'i' will be maximun when $\sin\omega t = 1$

$I_m = V_m/R$

Hence, Eq(2) becomes, $i = I_m \sin\omega t$ ----Eq(3)

Comparing Eq(1) and Eq(3)

We find that the alternating voltage and current are in phase with each other



Instantaneous power P = V * I

$$= V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t$$

Power consists of constant part $V_m I_m/2$ and fluctuating part

$$\frac{V_m I_m}{2}\cos 2\omega t$$

For the 2nd part, the frequency is double that of voltage and current waves. For a complete cycle, the average value of

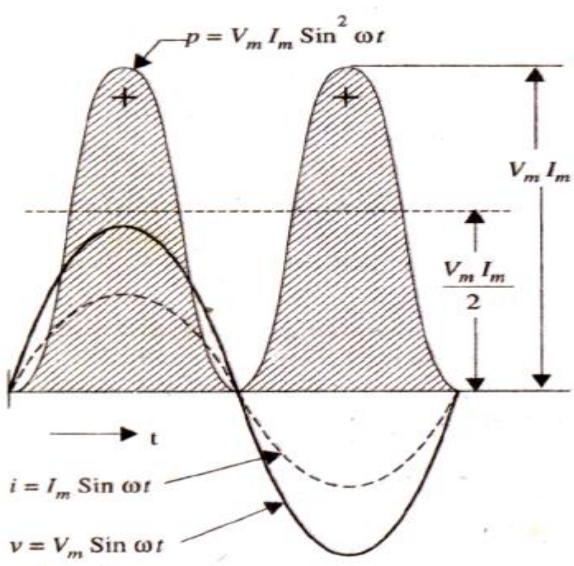
$$\frac{V_m I_m}{2}\cos 2\omega t = 0$$

So we have P = V * I watts
where V = r.m.s. value of applied voltage
I = r.m.s. value of the current

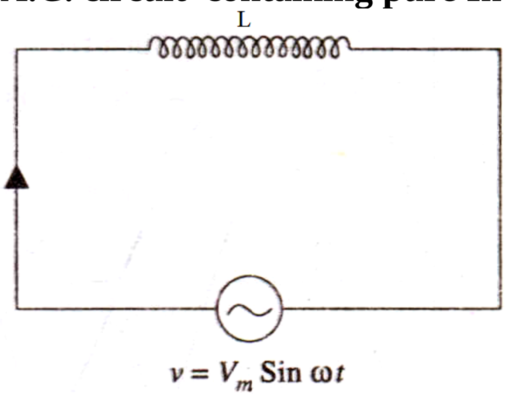
Hence, power for whole cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

Power curves: figure shows in a purely resistive circuit, power is never zero. This is so because the instantaneous values of voltage and current are always either both positive or negative and hence the product is always positive.



A.C. circuit containing pure Inductance :



Whenever an alternating voltage is applied to a pure inductive coil,a back e.m.f is produced ,The induced E.M.F. will oppose the applied Voltage in opposite direction. It is produced due to the self-inductance of the coil. This back e.m.f. at every step, opposes the rise or fall of current through the coil. As there is no ohmic voltage drop, the applied voltage has to overcome this self-induced e.m.f. only.

$$v = L \frac{di}{dt}$$

$$Now \quad v = V_m \sin \omega t$$

$$V_m \sin \omega t = L \frac{di}{dt}$$

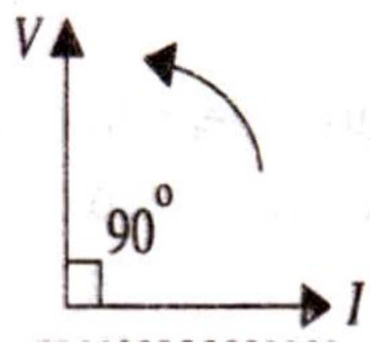
$$di = \frac{Vm}{L} \sin \omega t \; dt$$

$$i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$= \frac{V_m}{\omega L} (- \cos \omega t)$$

$$= - \frac{V_m}{\omega L} (\cos \omega t)$$

$$i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$



Maximum value of ‘i’ is $I_m = V_m/\omega L$ when $\sin(\omega t - \pi/2)$ is unity.

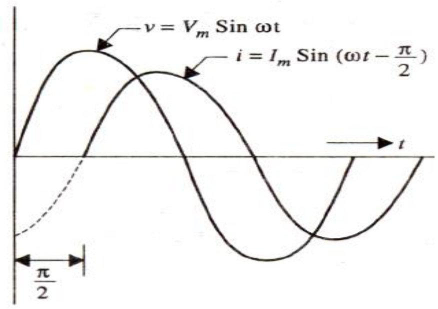
Hence equation of the current becomes $i = I_m \sin(\omega t - \pi/2)$

Here the current lags behind the applied voltage by a quarter cycle. Or the phase difference between the two is ‘$\pi/2$’ with voltage leading.

We have seen that $I_m = V_m/\omega L$

Here, 'ωL', plays the part of resistance. It is called the inductive reactance of the coil and is given in ohms if ‘L’ is in henrys and ‘ω’ in radian/second. It is denoted by ‘X_L’.

It is also clear from Figure that the average demand of power from the supply for a complete cycle is zero.
The maximum value of the instantaneous power is



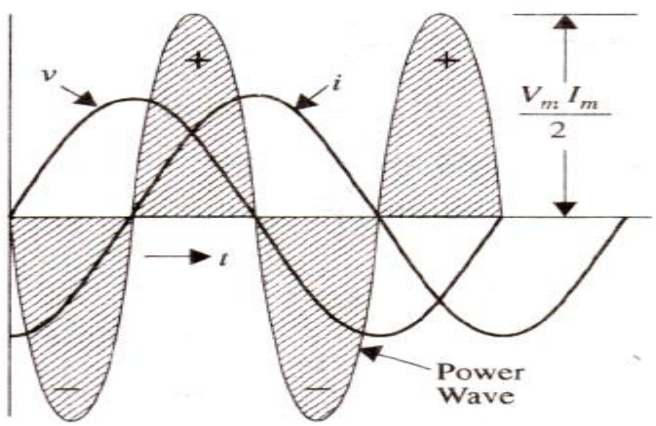
Instantaneous power P = V * I

$$= V_m I_m \sin \omega t \sin \left(\omega t - \frac{\pi}{2}\right) = -V_m I_m \sin \omega t \cos \omega t = -\frac{V_m I_m}{2} \sin 2\omega t$$

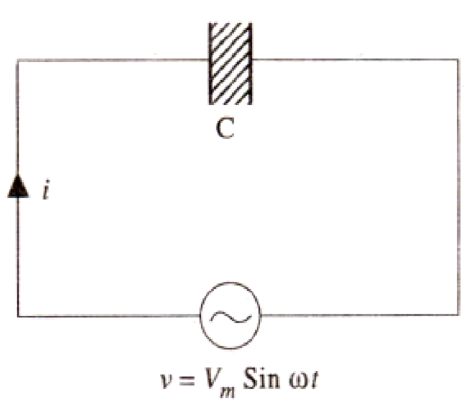
$$P = \frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \; dt = 0$$

It is also clear from Figure that the average demand of power from the supply for a complete cycle is zero.
The maximum value of the instantaneous power is

$$\frac{V_m I_m}{2}$$



AC Circuit Containing Pure Capacitance:



When an alternating voltage is applied to the plates of a capacitor, it is charged first in one direction and then in the opposite direction

v = p.d. between plates at any instant q = charge on plates at that instant

Then, $q = C\,V = C\,V_m \sin\omega t$

Now, current 'i' is given by the rate of flow of charge

$i = dq/dt = d/dt\,(C\,V_m \sin\omega t) = \omega C\,V_m \cos\omega t$

$$i = \frac{V_m}{1/\omega C}\cos\omega t = \frac{V_m}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

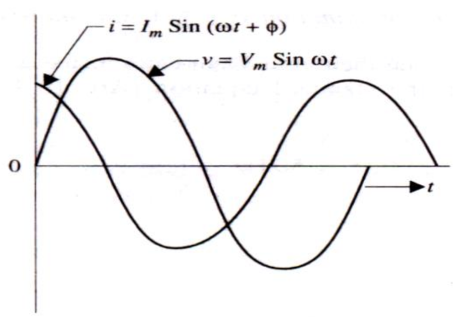
$$I_m = \frac{V_m}{1/\omega C} = \omega C V_m$$

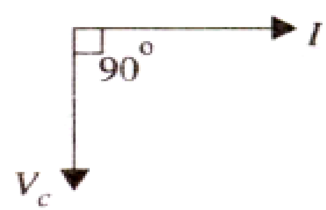
$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

The denominator '$1/\omega C$' is known as capacitive reactance and is in ohms if 'C' is in farads and 'ω' in radian/second. It is denoted by 'X_c'. It is seen that if the applied voltage is given by

$$v = V_m \sin\omega t$$

$$i = I_m \sin(\omega t + 90^o).$$





The vector representation shows phase difference between its voltage and current is 90^o with the current leading. Note that V_c is along the negative direction of Y-axis.

Instantaneous power $P = V * I$

$$= V_m \sin\omega t * I_m \sin(\omega t + 90^o)$$

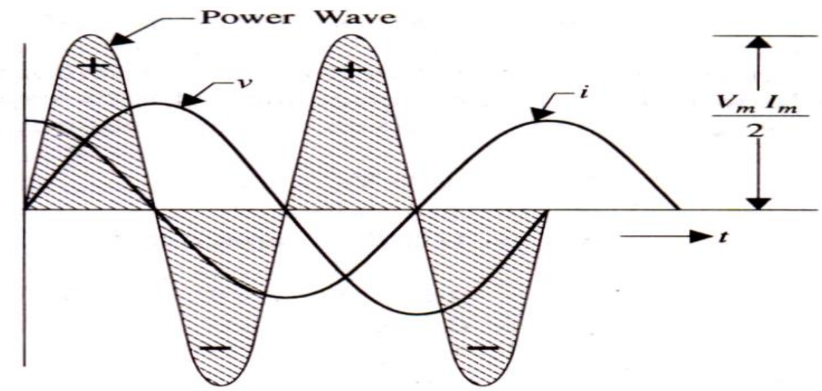
$$= V_m\, I_m \sin\omega t \cos\omega t$$

$$= 1/2\; V_m\, I_m \sin 2\omega t$$

Power for the whole cycle is

$$P = V_m I_m \int_0^{2\pi} \sin 2\omega t \; dt = 0$$

Power Curves:



In capacitive circuit, the average demand of power from supply is zero inductive circuit).
as Power wave is a sine wave of frequency double that of the voltage and current waves
The maximum value of the instantaneous power is

$$\frac{V_m I_m}{2}$$

Powerfactor: It is defined as the cosine of the angle of lead or lag. P.f=cosΦ

Real Power: It is the power multiplying apparent power by the power factor and is expressed in watts or Kwatts=VAcosΦ

Apparent power: The product of RMS values of current and voltage is called apparent power.

Reactive power: It is the power developed in the inductive reactance of the circuit. The quantity VIsinΦ is called the reactive power.