Question bank for second internal

Part-1

1. If $F = 3xy\boldsymbol{i} - y^2\boldsymbol{j}$, find the value of $\int_C F \cdot dR$, where C is the curve $y = 2x^2$ from $(0,0)$ to $(1,2)$.
2. Find the work done in moving a particle by the force $F = 3x^2\boldsymbol{i} + (2xz - y)\boldsymbol{j} + z\boldsymbol{k}$, along a straight line from (0, 0, 0) to (2, 1, 3).
3. Using Green's theorem evaluate $\int_C [(xy + y^2)\boldsymbol{dx} + x^2\boldsymbol{dy}]$, where C is bounded by $y = x$ and $y = x^2$.
4. Using the Green's theorem, evaluate $\int_C [(y - \sin x)dx + \cos x\, dy]$ where C is the plane triangle enclosed by the lines $y = 0,\ x = \frac{\pi}{2}\ \ and\ \ y = \frac{2}{\pi}x$.
5. Apply Green's theorem to prove that the area enclosed by a plane curve C is $\frac{1}{2}\int_C (xdy - ydx)$. Hence find the area of an ellipse whose semi-major and semi-minor axes are of lengths a and b.
6. Using Stoke's theorem evaluate $\int_C F \cdot dR$, where $F = (x^2 + y^2)\boldsymbol{i} - 2xy\boldsymbol{j}$ taken around the rectangle bounded by the lines $x = \pm a,\ y = 0,\ y = b$.
7. Using Stoke's theorem evaluate $\int_C F \cdot dR$, where $F = (2x - y)\boldsymbol{i} - yz^2\boldsymbol{j} - y^2z\boldsymbol{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy-plane.
8. If $F = 3y\boldsymbol{i} - xz\boldsymbol{j} + yz^2\boldsymbol{k}$ and S is the surface of the paraboliod $2z = x^2 + y^2$ bounded by $z = 2$. Evaluate $\int\int_S (\nabla \times F) \cdot ds$ using Stoke's theorem.

Part-2

A

1. Form the PDE by eliminating the arbitrary constants of the equation $z = xy + y\sqrt{x^2 - a^2} + b$.]
2. Form the PDE by eliminating the arbitrary function of $xyz = \emptyset(x + y + z)$.
3. Form the PDE by eliminating the arbitrary functions of $z = xf(x + t) + g(x + t)$.
4. Form the PDE by eliminating the arbitrary functions of $z = f(2x + 3y) + g(x + 2y)$.
5. Form the PDE by eliminating the arbitrary function of $z = yf(x) + xg(y)$.
6. Form the PDE by eliminating the arbitrary function of $F(x + y + z,\ x^2 + y^2 + z^2) = 0$.

B Identify the equation by direct integration method for the following PDE.

1. $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + \sin(2x - y) = 0$.
2. $\frac{\partial^2 z}{\partial x \partial y} - \sin x \sin y = 0$. For which $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$, $z = 0$ when y is odd multiple of $\frac{\pi}{2}$.
3. $\frac{\partial^2 z}{\partial y^2} = z$, given that when $y = 0$, $z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.
4. $\frac{\partial^2 z}{\partial x^2} = xy$, given that $\frac{\partial z}{\partial x} = \log(1 + y)$ when $x = 1$, and $z = 0$ when $x = 0$.
5. $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial z}{\partial x} + 2z = 0$, given that when $x = 0$, $z = 0$ and $\frac{\partial z}{\partial x} = \cos y$.
6. $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + 2z = 0$, given that when $x = 0$, $z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

C Identify the equation of following Lagrange's linear PDE.

1. $(mz - ny)p + (nx - lz)q = ly - mx$.
2. $x(y - z)p + y(z - x)q = z(x - y)$.

3. $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$.
4. $y^2p - xyq = x(z-2y)$.
5. Explain the derivation of one dimensional wave equation in the form, $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$.
6. Explain the derivation of one dimensional Heat equation in the form, $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$.

Part-3(Similar problems)

1. Using Regula-falsi method compute the real root of the equation $xe^x - \sin x = 0$, in$(-3,\ -2.8)$.
2. Using Regula-falsi method compute the real root of the equation $xe^x - \cos x = 0$, in$(0.5,\ 0.6)$.
3. Using Regula-falsi method compute the real root of the equation $x^3 - 2x - 5 = 0$, in$(2,\ 2.3)$.
4. Using Newton-Raphson method compute the real root of the equation $x\sin x + \cos x = 0$, near $x = \pi$.
5. Using Newton-Raphson method compute the real root of the equation $x^3 + x^2 + 3x + 4 = 0$, near $x = -1$.
6. Using Newton-Raphson method compute the real root of the equation $3x = \cos x + 1$, near $x = 0.5$.

Question paper pattern

<table>
<tr><td colspan="3">Part1</td></tr>
<tr><td>1 (4 Marks)</td><td>Or</td><td>2 (4 Marks)</td></tr>
<tr><td colspan="3">Part2</td></tr>
<tr><td>3 a. (4 Marks)
b. (4 Marks)
c. (4 Marks)</td><td>Or</td><td>4 a. (4 Marks)
b. (4 Marks)
c. (4 Marks)</td></tr>
<tr><td colspan="3">Part3</td></tr>
<tr><td>5 (4 Marks)</td><td>Or</td><td>6 (4 Marks)</td></tr>
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