Question bank for first internal

Part-1

A. Evaluate the following integrals.

1. $\int_0^1 \int_0^y e^{\frac{x}{y}}\, dx\, dy.$ 2. $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx.$

3. $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$.

4. $\iint xy(x+y)\, dxdy$ over the area between $y = x^2$ and $y = x.$

5. Evaluate by changing the order of integration

i) $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx.$ ii) $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\ dxdy$. iii) $\int_0^1 \int_x^{\sqrt{x}} xy\, dy\, dx$.

iv) $\int_0^a \int_y^a \frac{xdxdy}{x^2+y^2}$. v) $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx.$

6. Evaluate by changing in to polar form.

i) $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ ii) $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2) dx\, dy$ iii) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{xdydx}{x^2+y^2}$.

B

1. Evaluate the following triple integrals.

i. $\int_0^1 \int_{y^2}^1 \int_0^{1-x} xdzdxdy$. ii. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dxdydz$.

iii. $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2+y^2+z^2) dzdydx$. iv. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyzdzdydx$.

v. $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z) dxdydz$.

2. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ by double integration.

3. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

4. Find the area enclosed by the curve $r = a(1+\cos\theta)$ above the initial line.

5. Find the volume of the tetrahedron bounded by the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ and the coordinate planes, using double integration.

6. Calculate the volume of the solid bounded by the planes $x = 0,\ y = 0,\ x+y+z = 1,\ and\ z = 0.$

C

1. Prove that $\beta(m\,,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

2. Define Beta, gamma functions, and prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

3. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}} d\theta = \pi$.

4. Evaluate $\int_0^1 x^5(1-x^3)^{10} dx$ using Beta gamma functions.

5. Evaluate $\int_0^\infty e^{-4x} x^{3/2} dx$ using gamma function

Part-2

A

1. Prove that $curl grad \emptyset = 0$.

2. Prove that $div curl\, F = 0$.
3. Find the unit normal vector to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1,\ 2, -1)$.
4. Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at the point $(1, -2, -1)$ in the direction of $2i - j - 2k$.
5. Prove that $div(\emptyset F) = grad\emptyset \circ F + \emptyset div F$.
6. Calculate the angle between the normals to the surface $xy = z^2$ at the points $(4,\ 1,\ 2)$ and $(3,\ 3, -3)$.
7. Prove that $curl(\emptyset F) = grad\emptyset \times F + \emptyset curl F$.
8. Find the angle between the two surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1,\ 2)$.

B

1. If $F = grad\ [x^3 + y^3 + z^3 - 3xyz]$, find $divF$ and $curlF$.
2. Find $a,\ b,\ c,$ if $F = (x + by - z)i + (2x - y + cz)j + (ax + y - z)k$ is irrotational. And also find scalar potential $\emptyset$ such that $F = \nabla\emptyset$.
3. Find the value of a if $F = (ax^2y + yz)i + (xy^2 - xz^2)j + (2xyz - 2x^2y^2)k$ is solenoidal, And if $F = xy^2i + 2x^2yzj - 3yz^2k$, find $div(F)$.
4. Show that $\frac{xi+yj}{x^2+y^2}$ is both solenoidal and irrotational.
5. If $F = (x + y + 1)i + j - (x + y)k$, Show that $F \circ curlF = 0$.
6. Find the constants a and b if $F = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational, and also find the scalar potential.

Question paper pattern

<table>
<tr><td colspan="3">Part1:</td></tr>
<tr><td>1.
a)
b)
c)</td><td>Or</td><td>2.
a)
b)
c)</td></tr>
<tr><td colspan="3">Part2:</td></tr>
<tr><td>3.
a)
b)</td><td>Or</td><td>4.
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