Question Bank for First internal

Part1

1. Prove that $\tan\phi = r\frac{d\theta}{dr}$. 2. Prove that $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.

Find the angle between the following two curves.

3. $r = a(1-\sin\theta)$, $r = b(1+\sin\theta)$
4. $r^n = a^n\cos n\theta$, $r^n = b^n \sin n\theta$.
5. $r = \frac{2a}{(1-\cos\theta)}$, $r = \frac{2b}{(1+\cos\theta)}$
6. $r = \frac{a\theta}{1+\theta}$ and $r = \frac{a}{1+\theta^2}$.
7. $r = \sin\theta + \cos\theta$, $r = 2\sin\theta$.

Find the pedal equation of the following curves.

8. $r = a(1-\sin\theta)$
9. $r^n = a^n\cos n\theta$
10. $r(1-\cos\theta) = 2a$
11. $r^2 = a^2\sec 2\theta$
12. $r^m = a^m(\cos m\theta + \sin m\theta)$
13. $r^n = a^n\sin n\theta + b^n\cos n\theta$
14. Derive the radius of curvature in Cartesian form.
15. Derive the radius of curvature in polar form.

Find the radius of curvature for the following curves.

16. $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$
17. $xy^3 = a^4$ at the point (a, a)
18. $r^n = a^n\sin n\theta$ at any point (x, y)
19. For the cardioid $r = a(1+\cos\theta)$,
20. Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 4$ at the point where it cuts the line $y = x$.

Part2

Using Maclaurin's series expand the following functions

1. $y = \log\sec x$
2. $\log(1+\sin x)$
3. $\log(1+e^x)$
4. $\tan^{-1}x$
5. $\sqrt{(1+\sin 2x)}$

Evaluate the following limits.

6. $\underset{x\to a}{Lt}\left(2-\frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$
7. $\underset{x\to 0}{Lt}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$
8. $\underset{x\to\frac{\pi}{2}}{Lt}(\sec x)^{\cot x}$
9. $\underset{x\to 0}{Lt}(\cot x)^{\frac{1}{\log x}}$
10. $\underset{x\to\frac{\pi}{2}}{Lt}(\tan x)^{\tan 2x}$

11. If x increases at the rate of 2 cm/sec at the instant when $x = 3\ cm.$ and $y = 1\ cm.$, at what rate must y changing in order that the function $2xy - 3x^2y$ shell be neither increasing nor decreasing?
12. At a given instant the sides of a rectangle are 4ft and 3 ft, and they are increasing at the rate of 1.5ft/sec and 0.5ft/sec respectively. Find the rate at which the area is increasing at that instant.
13. If $u = x\log xy$ where $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$.
14. If $u = f(y-z, z-x, x-y), then\ prove\ that\ u_x + u_y + u_z = 0.$

15. If $x = u(1+v),\ \ y = v(1+u), show\ that\ \frac{\partial(x,\ y)}{\partial(u,\ v)} = 1 + \mathrm{u} + \mathrm{v}$.

16. If $u = x + y + z,\ v = x^2 + y^2 + z^2\ and\ w = xy + yz + zx, then\ find\ \ \frac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)}$.

17. If $ux = yz,\ vy = zx,\ \ wz = xy,$ then show that $\frac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)} = 4$.

18. If $u = x + y + z,\ uv = y + z\ and\ uvw = z, then\ find\ \ \frac{\partial(x,\ y,\ z)}{\partial(u,\ v,\ w)}$.

Question paper pattern

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