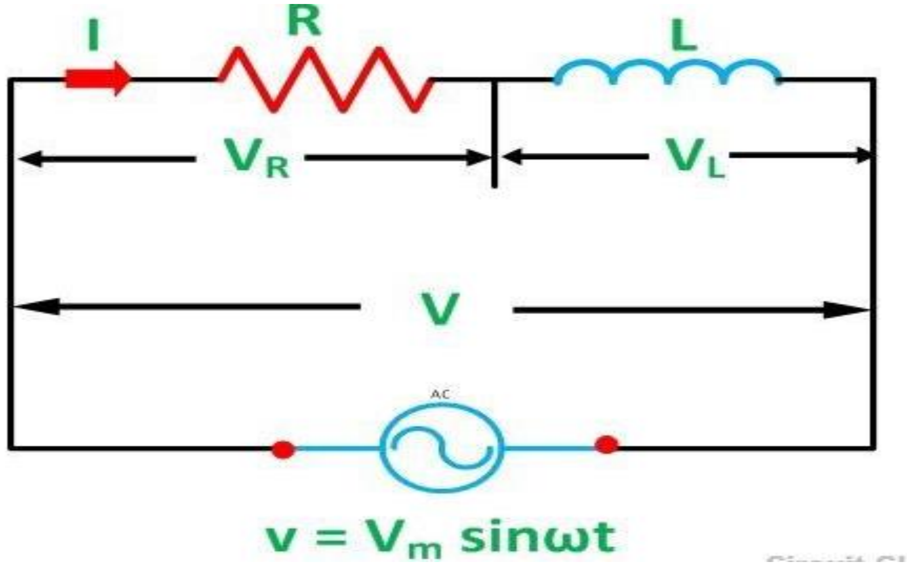
Module 2

Single-phase circuits

R-L Series circuit:

A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance of L (Henry) is known as **RL Series Circuit**. When an AC supply voltage V is applied, the current, I flows in the circuit.

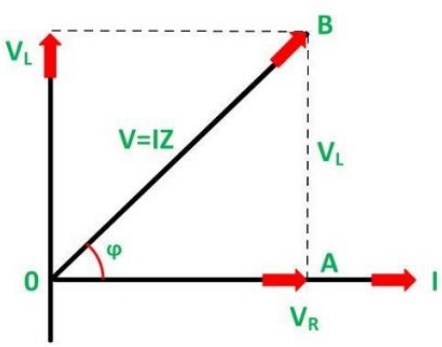
So, I_R and I_L will be the current flowing in the resistor and inductor respectively, but the amount of current flowing through both the elements will be same as they are connected in series with each other. The circuit diagram of RL Series Circuit is shown below:



Where,

- V_R – voltage across the resistor R
- V_L – voltage across the inductor L
- V – Total voltage of the circuit

The phasor diagram of the RL Series circuit is shown



The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.
- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.

- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.
  - Now,In right-angle triangle OAB
  - $V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$
  - $V = I\sqrt{R^2 + X_L^2}$ or
  - $I = L = \frac{V}{Z}$
  - $V_R = I_R$ and $V_L = IX_L$ where $X_L = 2\pi fL$
  - $Z = \sqrt{R^2 + X_L^2}$
  - Where, Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

In RL Series circuit the current lags the voltage by 90 degrees angle known as phase angle. It is given by the equation:

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

Power in R L Series Circuit

If the alternating voltage applied across the circuit is given by the equation:

$$v = V_m \operatorname{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

The equation of current I is given as: $i = I_m \sin(\omega t - \varphi) \ldots\ldots\ldots\ldots(2)$

Then the instantaneous power is given by the equation: $p = v\,i \ldots\ldots\ldots(3)$

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m \mathrm{Sin}\omega t) x \ I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below:

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\,I\cos\varphi$$

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots\ldots\ldots\ldots (4)$$

Where cosϕ is called the power factor of the circuit**.**

The power factor is defined as the ratio of resistance to the impedance of an AC Circuit.

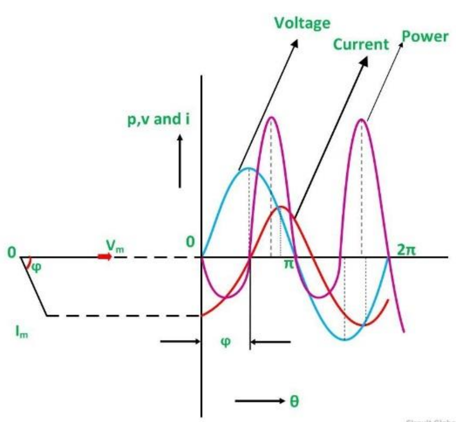
Putting the value of V and cosϕ from the equation (4) the value of power will be:

$$P = (IZ)(I)(R/Z) = I^2 R \ldots\ldots\ldots (5)$$

From equation (5) it can be concluded that the inductor does not consume any power in the circuit.

Waveform and Power Curve of the RL Series Circuit

The **waveform** and **power curve** of the RL series circuit is shown below:

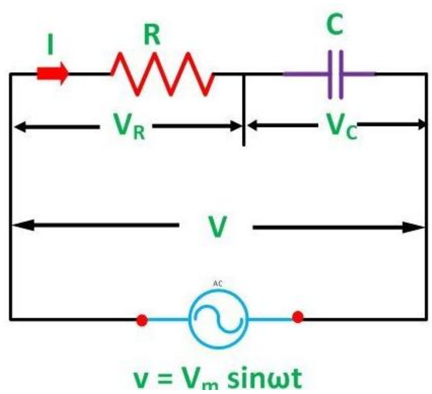


The various points on the power curve are obtained by the product of voltage and current.

If you analyze the curve carefully, it is seen that the power is negative between angle 0 and ϕ and between 180 degrees and (180 + ϕ) and during the rest of the cycle the power is positive. The current lags the voltage and thus they are not in phase with each other.

RC Series Circuit:

A circuit that contains pure resistance R ohms connected in series with a pure capacitor of capacitance C farads is known as **RC Series Circuit.** A sinusoidal voltage is applied and current I flows through the resistance (R) and the capacitance (C) of the circuit.

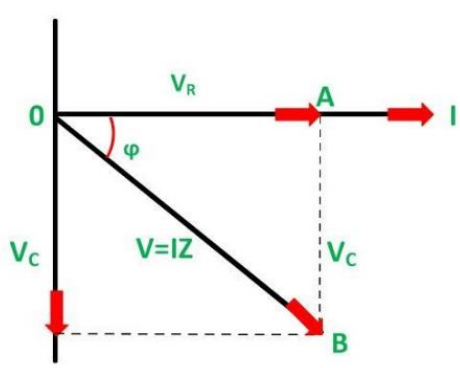


Where,

- V_R – voltage across the resistance R
- V_C – voltage across capacitor C
- V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit

The phasor diagram of the RC series circuit is shown below:



Steps to draw a Phasor Diagram

The following steps are used to draw the phasor diagram of RC Series circuit

- Take the current I (r.m.s value) as a reference vector
- Voltage drop in resistance VR = IR is taken in phase with the current vector
- Voltage drop in capacitive reactance VC = IXC is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
- The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now,

Where $X_C = I/2\pi fC$

In right triangle OAB,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).

Phase angle

From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Power in RC Series Circuit

If the alternating voltage applied across the circuit is given by the equation

$$v = V_m Sin\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

Then, $$i = I_m sin(\omega t + \varphi) \ldots\ldots\ldots..(2)$$

Therefore, the instantaneous power is given by p = vi

Putting the value of v and i from the equation (1) and (2) in p = vi

$$P = (V_m Sin\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete cycle is given by:

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

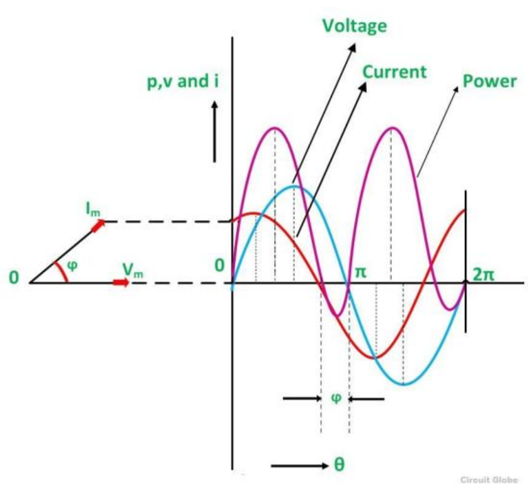
Where cosϕ is called the **power factor** of the circuit.

$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots\ldots\ldots\ldots.(3)$$

Putting the value of V and cosϕ from the equation (3) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \ldots\ldots\ldots..(4)$$

From the equation (4) it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.



Waveform and Power Curve of the RC Series Circuit

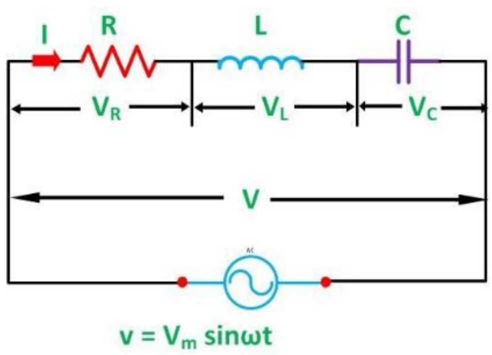
The waveform and power curve of the RC circuit is shown below: The various points on the power curve are obtained from the product of the instantaneous value of voltage and current.

The power is negative between the angle (180° – ϕ) and 180° and between (360° -ϕ) and 360° and in the rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is **positive**.

RLC Series Circuit

When a pure resistance of R ohms, a pure inductance of L Henry and a pure capacitance of C farads are connected together in series combination with each other then **RLC Series Circuit** is formed. As all the three elements are connected in series so, the current flowing through each element of the circuit will be the same as the total current I flowing in the circuit.

The **RLC Circuit** is shown below:



In the RLC Series circuit

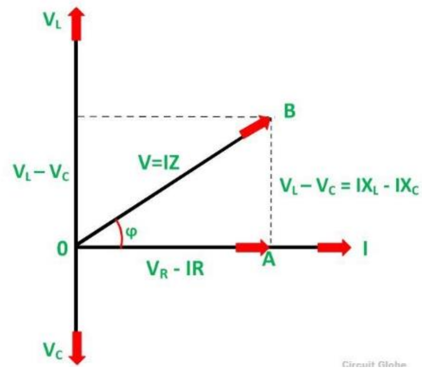
$\mathbf{X_L = 2\pi fL}$ and $\mathbf{X_C = 1/2\pi fC}$

When the AC voltage is applied through the RLC Series circuit the resulting current I flows through the circuit, and thus the voltage across each element will be:

- $V_R = IR$ that is the voltage across the resistance R and is in phase with the current I.
- $V_L = IX_L$ that is the voltage across the inductance L and it leads the current I by an angle of 90 degrees.
- $V_C = IX_C$ that is the voltage across capacitor C and it lags the current I by an angle of 90 degrees.

Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC series circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.



Steps to draw the Phasor Diagram of the RLC Series Circuit

- Take current I as the reference as shown in the figure above
- The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle.
- The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90-degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees.
- The two vector V_L and V_C are opposite to each other.

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Where, Z is the total opposition offered to the flow of current by an RLC Circuit and is known as **Impedance** of the circuit.

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

$$P = VI\cos\varphi = I^2R$$

The product of voltage and current is defined as power. Where cosϕ is the power factor of the circuit and is expressed as:

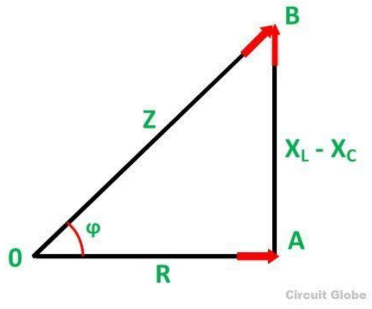
$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is **unity**.

Impedance Triangle of RLC Series Circuit

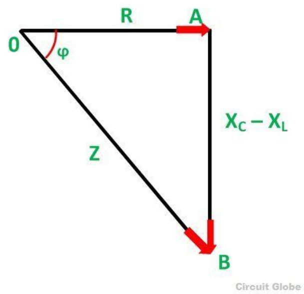
When the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle. The impedance triangle of the RL series circuit, when ($X_L > X_C$) is shown below:



If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a **lagging phase angle**.

Impedance triangle is shown below when the circuit acts as an RC series circuit ($X_L < X_C$)

When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as capacitive and the phase angle will be leading.



Applications of RLC Series Circuit

The following are the application of the RLC circuit:

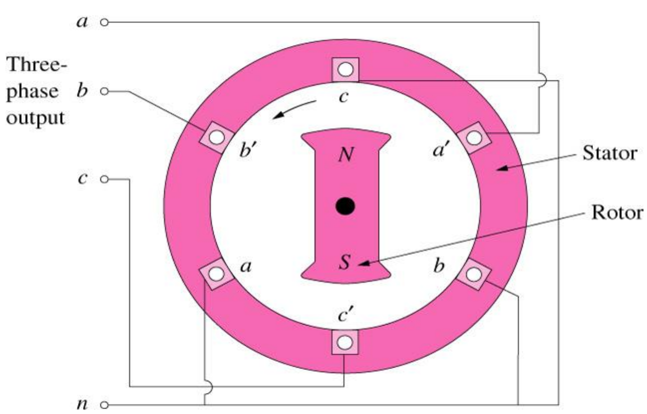
- It acts as a variable tuned circuit
- It acts as a low pass, high pass, bandpass, bandstop filters depending upon the type of frequency.
- The circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

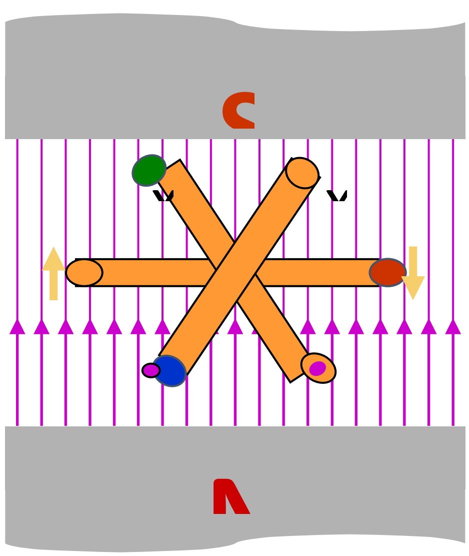
Three Phase Circuits

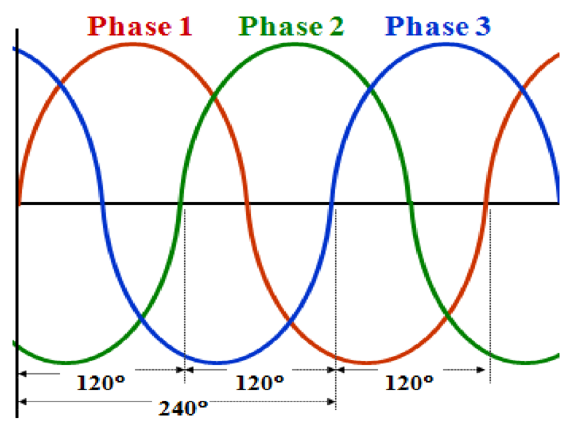
ADVANTAGES OF THREE PHASE SYSTEM

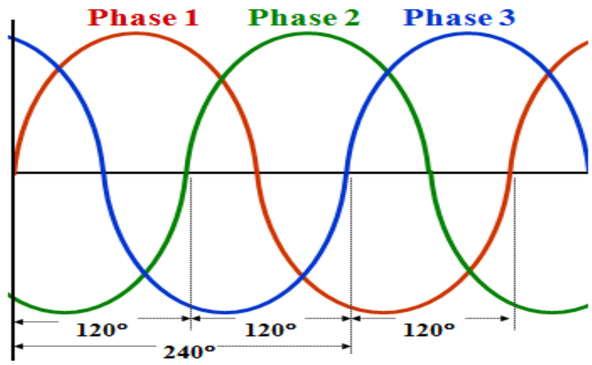
- 3 phase apparatus is more efficient than 1phase apparatus
- For same capacity, a three phase apparatus costs less than a single phase apparatus.
- The size of 3 phase apparatus is smaller in size of a single phase apparatus of the same capacity and hence requires less material for construction.
- Three phase motors are self starting whereas single phase motors are not self starting.
- Three phase motors produce uniform torque whereas, the torque produced by single phase motors is pulsating.

GENERATION OF THREE PHASE POWER



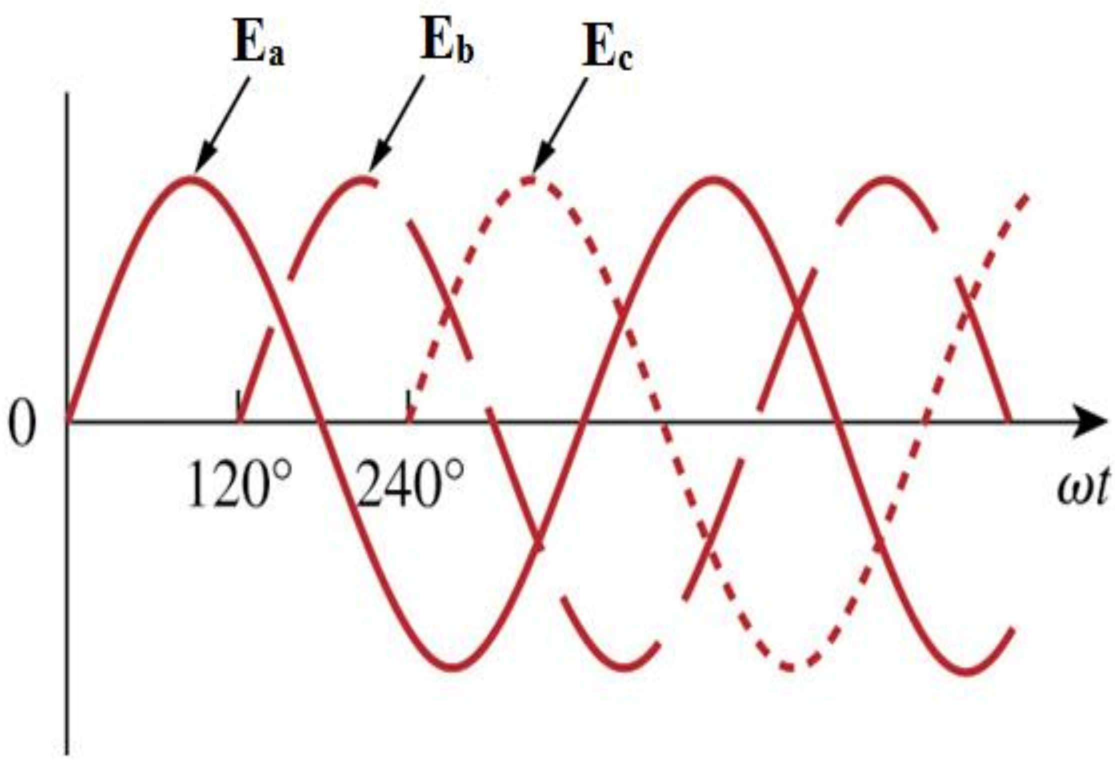




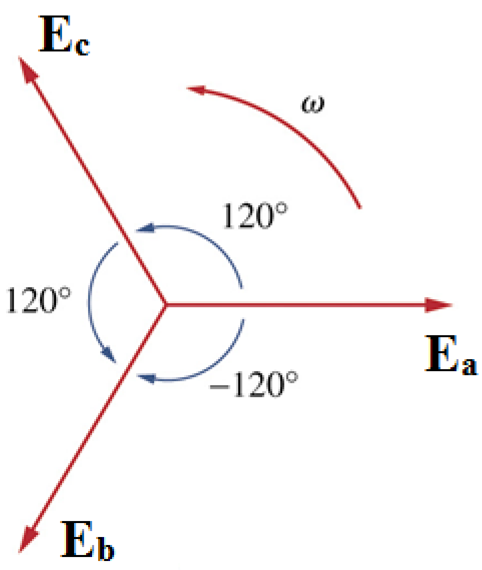


Phase 3

Phase 1 Phase 2



Start ¼ Cycle½ Cycle¾ Cycle



$$e_a = E_m \sin \omega t$$

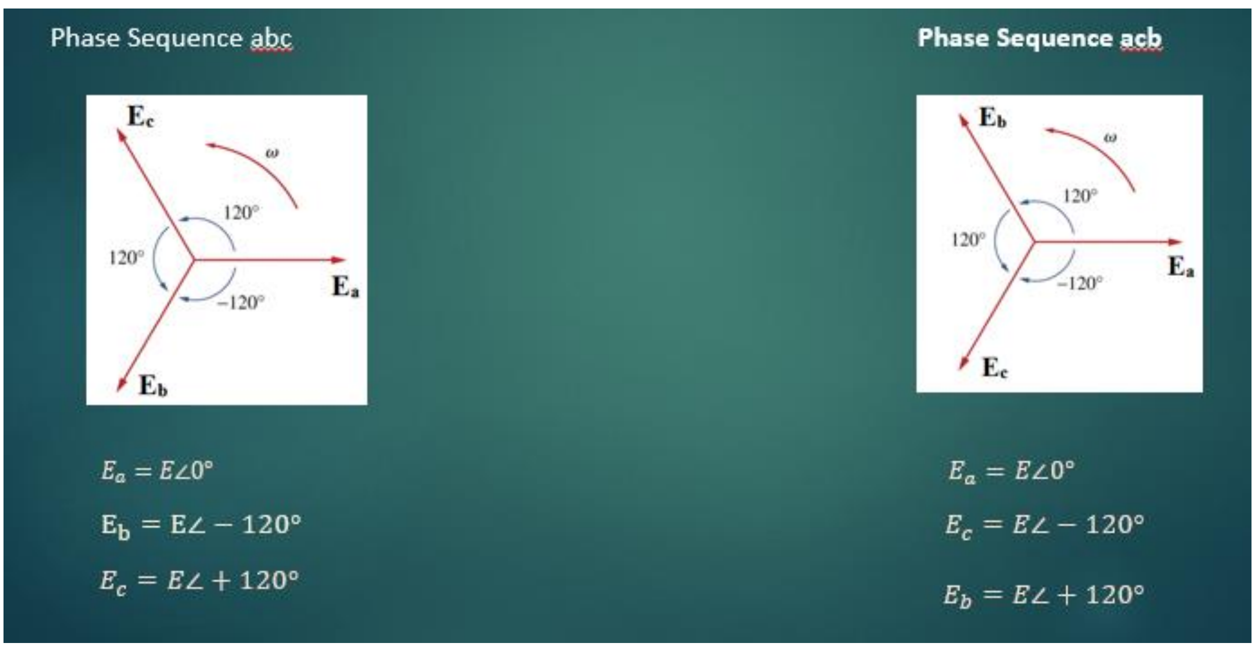
$$e_b = E_m \sin(\omega t - 120)$$

$$e_c = E_m \sin(\omega t - 240) = E_m \sin(\omega t + 120)$$

$$e_a + e_b + e_c = 0$$

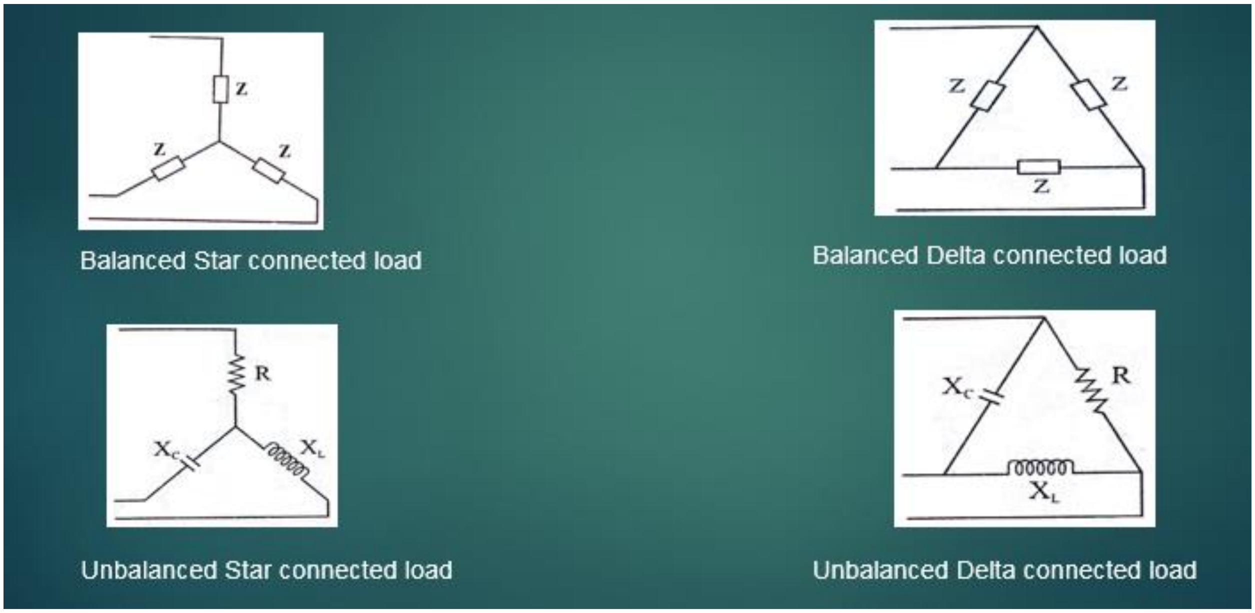
PHASE SEQUENCE:

The phase sequence is the order in which the maximum values of the three phase voltages occur.



BALANCED LOAD

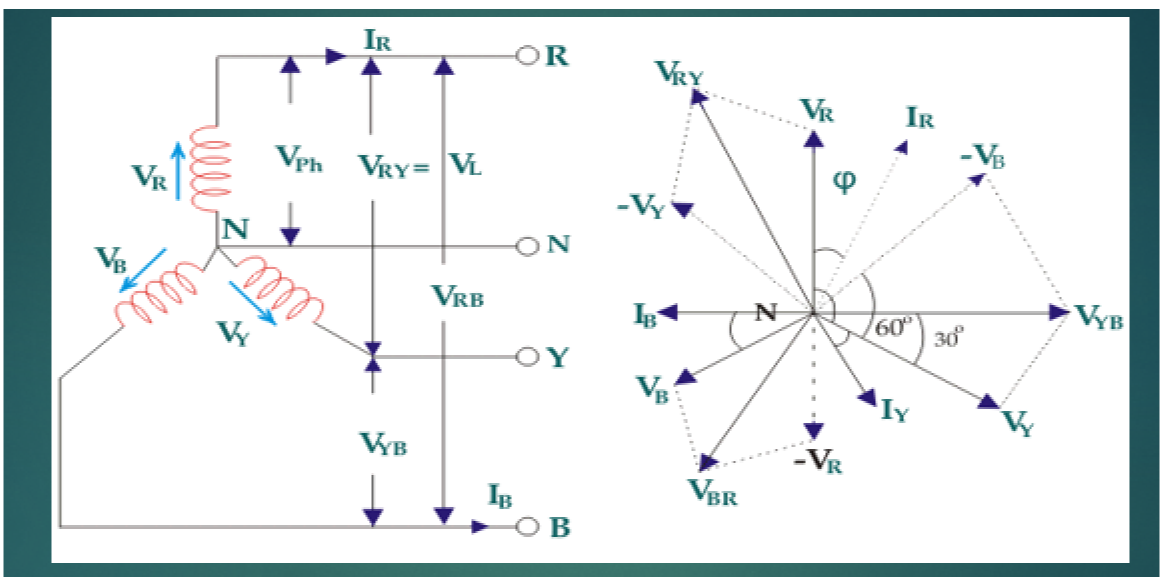
A three phase load is balanced, when the impedances of all the three phases are exactly the same.



To derive the relations between line and phase currents and voltages of a star connected system,

we have first to draw a balanced star connected system.

Let us take due to load impedance the current lags the applied voltage in each phase of the system by an angle ϕ.

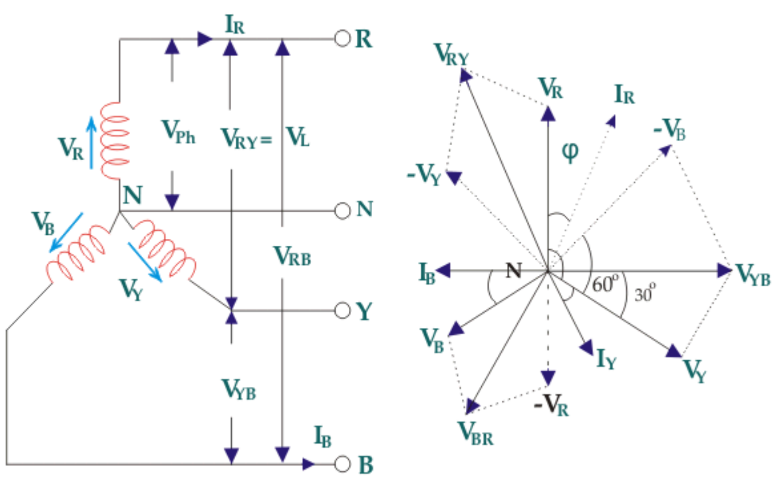


Let us say, the magnitude of the voltage across the red phase i.e. magnitude of the voltage between neutral point (N) and red phase terminal (R) is V_R.
Similarly, the magnitude of the voltage across yellow phase is V_Y and the magnitude of the

voltage across blue phase is V_B. In the balanced star system, magnitude of phase voltage in each phase is V_{ph}.
$\therefore V_{RN} = V_{YN} = V_{BN} = V_{ph}$

We know in the star connection, line current is same as phase current. The magnitude of this current is same in all three phases and say it is I_L.
$\therefore I_R = I_Y = I_B = I_L$, Where, I_R is line current of R phase, I_Y is line current of Y phase and I_B is line current of B phase. Again, phase current, I_{ph} of each phase is same as line current I_L in star connected system.
$\therefore I_R = I_Y = I_B = I_L = I_{ph}$.

Now, let us say, the voltage across R and Y terminal of the star connected circuit is V_{RY}.
The voltage across Y and B terminal of the star connected circuit is V_{YB}.
From the diagram, it is found that
$V_{RY} = V_R + (-V_Y)$
Similarly, $V_{YB} = V_Y + (-V_B)$
And, $V_{BR} = V_B + (-V_R)$
Now, as angle between V_R and V_Y is 120^o(electrical),
the angle between V_R and $-V_Y$ is $180^o - 120^o = 60^o$.



$$V_L = |V_{RY}| = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^o}$$
$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}V_{ph} \times \frac{1}{2}}$$
$$= \sqrt{3}V_{ph}$$
$$\therefore V_L = \sqrt{3}V_{ph}$$

Thus, for the star-connected system line voltage = √3 × phase voltage.
Line current = Phase current
As, the angle between voltage and current per phase is φ, the electric power per phase is

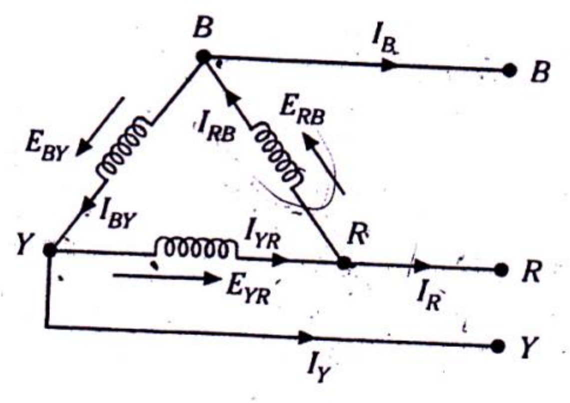
$$V_{ph} I_{ph} cos\phi = \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

So the total power of three phase system is

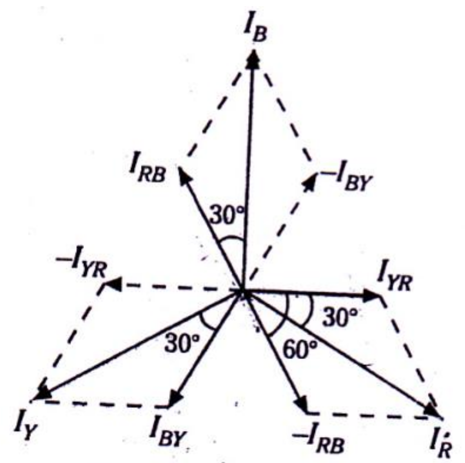
$$3 \times \frac{V_L}{\sqrt{3}} I_L \cos\phi = \sqrt{3} V_L I_L \cos\phi$$

Relationship between Line and Phase Values and Expression for Power for Balanced Delta Connection

When the starting end of one coil is connection to the finishing end of another coil, as shown in Fig. delta or mesh connection is obtained. The direction of the e.m.f.s is as shown in the diagram



(a) Connection Diagram



(b) Vector Diagram

Fig. 3.83

Line current, $I_R = I_{YR} - I_{RB}$ (vector difference)

$= I_{YR} + (-I_{RB})$ (vector sum)

As the phase angle between currents I_{YR} and $-I_{RB}$ is 60^0

$$\therefore \quad I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_{YR}I_{RB}\cos 60^0}$$

For a balanced load, the phase current in each winding is equal and let it be $= I_P$.

$\therefore$ Line current, $I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2I_P I_P x0.5} = \sqrt{3}\ I_P$

Similarly, line current, $I_Y = I_{BY} - I_{YR} = \sqrt{3}\ I_P$

And line current, $I_B = I_{RB} - I_{BY} = \sqrt{3}\ I_P$

In a delta network, there is only one phase between any pair of line outers, so the potential difference between the outers, called the line voltage, is equal to phase voltage.

i.e. Line voltage, E_L = phase voltage, E_P

Power output per phase = $E_P I_P \cos \phi$,

where $\cos \phi$ is the power factor of the load.

Total power output, $P = 3E_P I_P \cos \phi$

$$= 3E_L \frac{I_L}{\sqrt{3}} \cos \phi$$

$$= \sqrt{3}\, E_L I_L \cos \phi$$

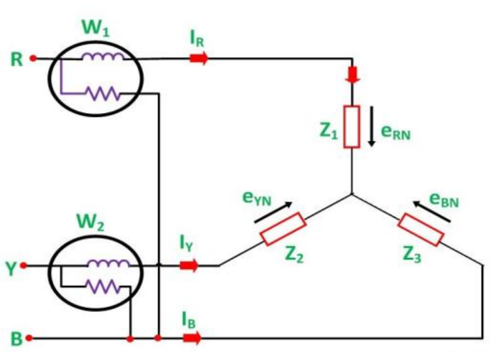
i.e. Total power output= $\sqrt{3}$ x Line voltage x Line current x p.f.

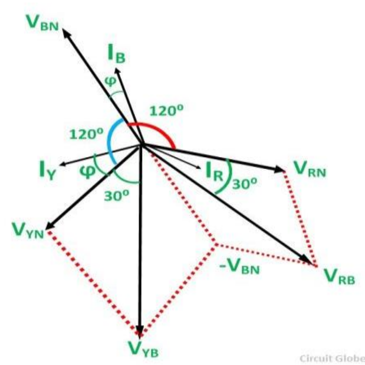
Show that in a three phase, balanced circuit, two wattmeters are sufficient to measure the total three phase power and power factor of the circuit.

Two wattmeter method: The current coils of the two wattmeters are connected in any two lines, while the voltage coil of each wattmeters is connected between its own current coil terminal and line without current coil. Consider star connected balanced load and two wattmeters connected as shown in figure. Let us consider the rms values of the currents and voltages to prove that sum of two wattmeter gives total power consumed by three phase load.

Two Wattmeter Method of Measurement of 3 phase of power

Two Wattmeter Method can be employed to measure the power in a 3 phase three wire star or delta connected the balanced or unbalanced load. In Two wattmeter method the current coils of the wattmeter are connected with any two lines, say R and Y and the potential coil of each wattmeter is joined on the same line, the third line i.e. B as shown below in figure (A)





Considering the above figure in which Two Wattmeters W_1 and W_2 are connected, the instantaneous current through the current coil of Wattmeter W_1 is given by the equation shown below.

Current through the wattmeter W1= i**R**
Instantaneous potential difference across the potential coil of Wattmeter, W_1 is given as

Instantaneous power measured by the Wattmeter, W_1 is

$$W_1 = e_{RN} - e_{BN} \qquad W_1 = i_R\,(e_{RN} - e_{BN})\ \ldots\ldots\ldots(1)$$

The instantaneous current through the current coil of Wattmeter, W2= Iy

Instantaneous potential difference across the potential coil of Wattmeter, W_2 is given as

$$\boxed{W_2 = e_{YN} - e_{BN}}$$

Instantaneous power measured by the Wattmeter, W_2 is

$$\boxed{W_2 = i_Y\,(e_{YN} - e_{BN})\ \ldots\ldots\ldots..(2)}$$

Therefore, the Total Power Measured by the Two Wattmeters W_1 and W_2 will be obtained by adding the equation (1) and (2).

$$W_1 + W_2 = i_R\,(e_{RN} - e_{BN}) + i_Y\,(e_{YN} - e_{BN})$$

$$W_1 + W_2 = i_R e_{RN} + i_Y e_{YN} - e_{BN}\,(i_R + i_Y) \quad \text{or}$$

$$W_1 + W_2 = i_R e_{RN} + i_Y e_{YN} + i_B e_{BN} \qquad (\text{i.e.}\ i_R + i_Y + i_B = 0)$$

$$W_1 + W_2 = P$$

Where P – the total power absorbed in the three loads at any instant.

<u>Effect of power factor on wattmeter readings:</u>

Wattmeter 1 can be expressed as : VRB * IR * angle between VRB and IR

Wattmeter 2 can be expressed as : VYB * IB * angle between VYB and IB

$$W_1 = V_{RB} I_R \cos\,(30^\circ - \varphi) \qquad W_2 = V_{YB} I_Y \cos\,(30^\circ + \varphi)$$

$$W_1 + W_2 = V_L I_L \cos(30^\circ - \varphi) + V_L I_L \cos(30^\circ + \varphi)$$

$$W_1 + W_2 = V_L I_L\,[\cos(30^\circ - \varphi) + \cos(30^\circ + \varphi)] \quad \text{or}$$

$$W_1 + W_2 = V_L I_L\,[\cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi + \cos 30^\circ \cos\varphi - \sin 30^\circ \sin\varphi\,] \text{ or}$$

$$W_1 + W_2 = V_L I_L (2 \cos 30^\circ \cos\varphi) \quad \text{or}$$

$$W_1 + W_2 = V_L I_L \left(2\, \frac{\sqrt{3}}{2} \cos\varphi\right)$$

$$W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi$$

$$W_1 + W_2 = P\ \ldots\ldots(1)$$

Effect of power factor on wattmeter readings:

As we know that, $W_1 + W_2 = \sqrt{3}\, V_L I_L \cos\varphi \;\ldots\ldots\ldots (2)$

and

$$W_1 = V_L I_L \cos(30^\circ - \varphi) \;\; \text{and}$$

$$W_2 = V_L I_L \cos(30^\circ + \varphi)$$

Now,

$$W_1 - W_2 = V_L I_L \left[\cos(30^\circ - \varphi) - \cos(30^\circ + \varphi)\right] \quad \text{or}$$

$$W_1 - W_2 = V_L I_L \left[\cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi - \cos 30^\circ \cos\varphi + \sin 30^\circ \sin\varphi\right] \text{ or}$$

$$W_1 - W_2 = 2\, V_L I_L \sin 30^\circ \sin\varphi$$

$$W_1 - W_2 = V_L I_L \sin\varphi \;\ldots\ldots\ldots (3)$$

Dividing equation (3) by equation (2) we get,

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\varphi}{\sqrt{3}\, V_L I_L \cos\varphi} \quad \text{or}$$

$$\tan\varphi = \sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}$$

Power factor of the load is given as

$$\cos\varphi = \cos \tan^{-1} \sqrt{3}\, \frac{W_1 - W_2}{W_1 + W_2}$$