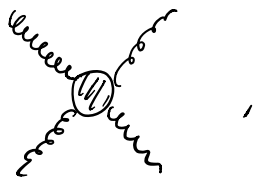
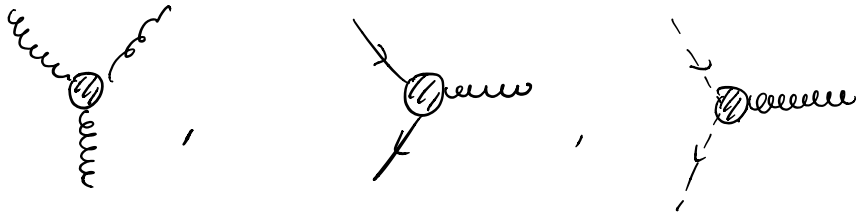


P.2

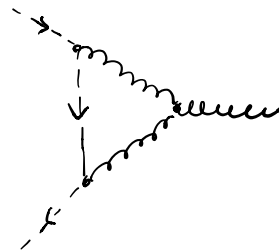
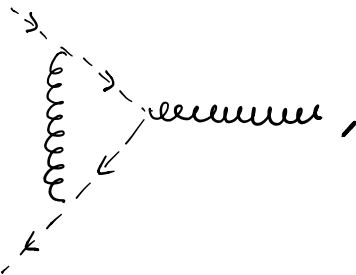
a)



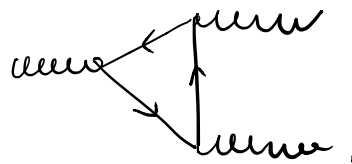
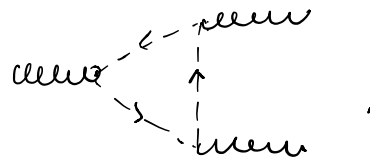
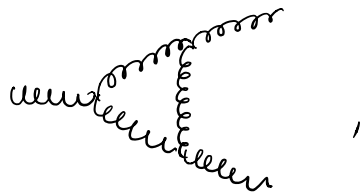
b) i)



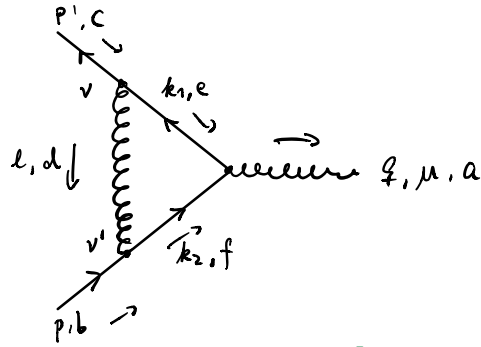
ii)



iii)



c)



$$k_1 = p' - l, \quad k_2 = p + l$$

$$i\mathcal{M}^\mu = \int \frac{d^4 l}{(2\pi)^4} i g \gamma^\nu (t^d)_{ce} \frac{i}{\not{k}_1 - m} i g \gamma^\mu (t^a)_{ef} \frac{i}{\not{k}_2 - m} i g \gamma^\nu (t^d)_{fb} \frac{-i g_{\nu\mu}}{l^2}$$

$$= -i g^3 \int \frac{d^4 l}{(2\pi)^4} \gamma^\nu \frac{1}{\not{k}_1 - m} \gamma^\mu \frac{1}{\not{k}_2 - m} \gamma_\nu \frac{1}{l^2} \cdot (GTF)$$

$$(GFF) = (t^d)_{ce} (t^a)_{ef} (t^d)_{fb}$$

$$= (t^d t^a t^d)_{cb}$$

$$t^d t^a t^d = t^d t^d t^a + t^d \underbrace{[t^a, t^d]}_{= i f^{adc} t^c}$$

$$= C_2(F) t^a + \underbrace{i f^{adc} t^d t^c}_{= \frac{1}{2} f^{adc} t^d t^c + \frac{1}{2} f^{adc} t^d t^c}$$

$$= \frac{1}{2} f^{adc} t^d t^c + \frac{1}{2} f^{adc} t^d t^c$$

$$= \frac{1}{2} f^{acd} t^c t^d + \frac{1}{2} f^{adc} t^d t^c$$

$$= \frac{1}{2} f^{adc} [t^d, t^c]$$

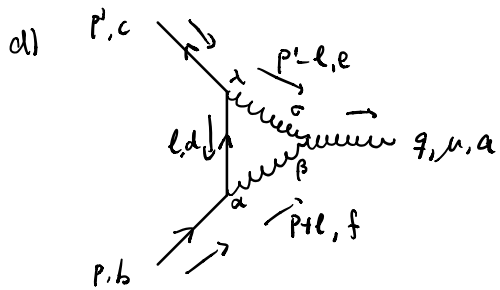
$$= \frac{1}{2} f^{adc} \cdot i f^{dce} t^e$$

$$= \underbrace{f^{ae} C_2(G)}_{= g^{ae} C_2(G)} t^e$$

$$= C_2(F) t^a + i \cdot \frac{i}{2} C_2(G) t^a$$

$$= \left[-\frac{1}{2} C_2(G) + C_2(F) \right] t^a$$

$$\rightarrow i\mathcal{M}^\mu = i g \Lambda_{\text{QED}}^\mu(p, q, p'; e \rightarrow g) \left[-\frac{1}{2} C_2(G) + C_2(F) \right] (t^a)_{cb}$$



$$i\mathcal{M} = \bar{v}(p') ig(t^e)_{cd} \gamma^\lambda \frac{i}{\not{k} - m} ig(t^f)_{db} \gamma^\alpha u(p) \frac{-ig_{\lambda\sigma}}{(p' - l)^2} \frac{-ig_{\alpha\beta}}{(p + l)^2} \\ \times g f^{efa} [g^{\sigma\beta} (p' - l - p - l)^\mu + g^{\beta\mu} (p + l + q)^\sigma + g^{\mu\sigma} (-q - p' + l)^\beta] \in^{*M}_a(q)$$

$$\begin{aligned} (GTF) &= (t^e)_{cd} (t^f)_{db} f^{efa} \\ &= (t^e t^f)_{cb} f^{efa} \\ &= \underbrace{(t^e t^f f^{efa})}_{= \frac{i}{2} f^{efa} f^{efa} t^f} \\ &= \frac{i}{2} G_2(G)(t^a)_{cb} \end{aligned}$$

P.3 $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \bar{\psi} (i \not{D} - m) \psi + \frac{g}{2} (B^a)^2 + B^a \partial^\mu A_\mu^a - \bar{\eta}^a \partial^\mu D_\mu^{ab} \eta^b$

a) EOM w.r.t B^a

$$\frac{\partial \mathcal{L}}{\partial B^a} = \frac{g}{2} B^a + \partial^\mu A_\mu^a = 0$$

$$\mathcal{L}_B = \frac{g}{2} (B^a)^2 + B^a \partial^\mu A_\mu^a = -B_a \partial^\mu A_\mu^a + B^a \partial^\mu A_\mu^a = 0$$

b) $S\psi = ig\lambda \eta^a t^a \psi$

$$S(S\psi) = ig\lambda \underbrace{[(S\eta^a) t^a \psi + \eta^a t^a S\psi]}_{= -\frac{g}{2} \lambda f^{abc} \eta^b \eta^c t^a \psi + \eta^a t^a \cdot ig\lambda \eta^b t^b \psi}$$

$$= ig^2 \lambda^2 \underbrace{\left[-\frac{1}{2} f^{abc} \eta^b \eta^c t^a + i \eta^a t^a \eta^b t^b \right] \psi}$$

$$= -\frac{1}{2} \eta^b \eta^c f^{abc} t^a + i \underbrace{\eta^a \eta^b t^a t^b}$$

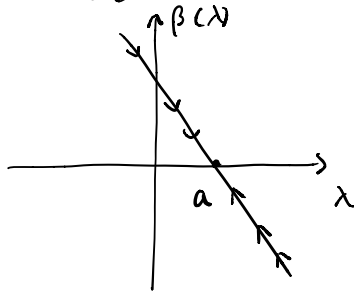
$$= \frac{1}{2} \eta^a \eta^b [t^a, t^b]$$

$$= \frac{1}{2} \eta^a \eta^b i f^{abc} t^c$$

$$= -\frac{1}{2} \eta^b \eta^c \cancel{f^{abc}} t^a - \frac{1}{2} \eta^a \eta^b \cancel{f^{abc}} t^c$$

P.4

$$a) \quad \beta(\lambda) = \frac{\partial}{\partial t} \lambda = -b(\lambda - a), \quad b > 0, \quad t = \ln \mu$$



$$\int_{\lambda_0}^{\bar{\lambda}} d\lambda = -b(\lambda - a) dt$$

$$\int_{\lambda_0}^{\bar{\lambda}} \frac{d\lambda}{b(a - \lambda)} = \int_0^t dt'$$

$$\Rightarrow \frac{1}{b} [\ln(a - \lambda)]_{\lambda=\lambda_0}^{\lambda=\bar{\lambda}} = t$$

$$\rightarrow \ln(a - \lambda) = b t$$

$$\rightarrow \lambda = \begin{cases} a - e^{bt} & , \quad a - \lambda \geq 0 \\ e^{-bt} + a & , \quad \lambda - a > 0 \end{cases}$$

$$t \rightarrow \infty, \quad \lambda \rightarrow a$$

$$b) \quad \beta(\lambda) = -b(\lambda - a)^n = \frac{\partial}{\partial t} \lambda$$

$$\rightarrow d\lambda = -b(\lambda - a)^n dt$$

$$-\frac{1}{b} \int_{\lambda_0}^{\bar{\lambda}} \frac{d\lambda}{(\lambda - a)^n} = \int_{t=0}^{\bar{t}=t} dt'$$

$$-\frac{1}{b} \frac{1}{-n+1} [(\lambda - a)^{-n+1}]_{\lambda_0}^{\bar{\lambda}} = t$$

$$\rightarrow (\bar{\lambda} - a)^{-n+1} - (\lambda_0 - a)^{-n+1} = b(n-1)t$$

$$(\bar{\lambda} - a)^{-n+1} = b(n-1)t + (\lambda_0 - a)^{-n+1}$$

$$\bar{\lambda} = [b(n-1)t + (\lambda_0 - a)^{-n+1}]^{\frac{1}{-n+1}} + a$$