1. Renormalisation and UV cutoffs

1.1 Introduction

- Renormalisation: absorb UV divergences in Feynman diags by
  1. redefining bare parameters (m, λ, . . .)
     $\longrightarrow$ "bare perturbation theory"
  2. adding counterterms (δ_m, δ_λ, . . .)
     $\longrightarrow$ "renormalised perturbation theory"
- example: ϕ^4 theory $\longrightarrow$ works in practice

1.2 Formal and physical cutoffs

- UV divergences: dependence on large momenta / small distances
- QED contains effective parameters m_e, α_{QED} $\longrightarrow$ not calculable; maybe from more fundamental theory: compare fluid dynamics, magnetism $\longleftrightarrow$ atomic physics ["fundamental"]

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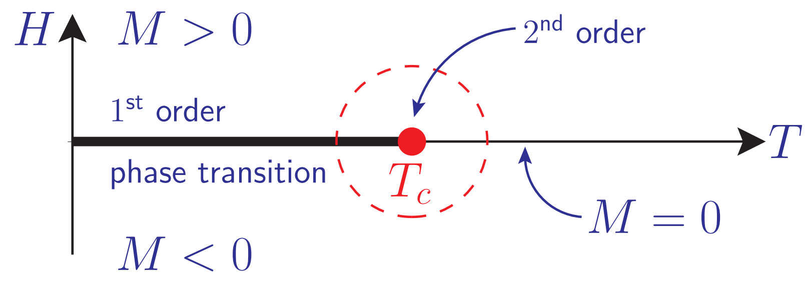
1.3 Landau theory of phase transitions

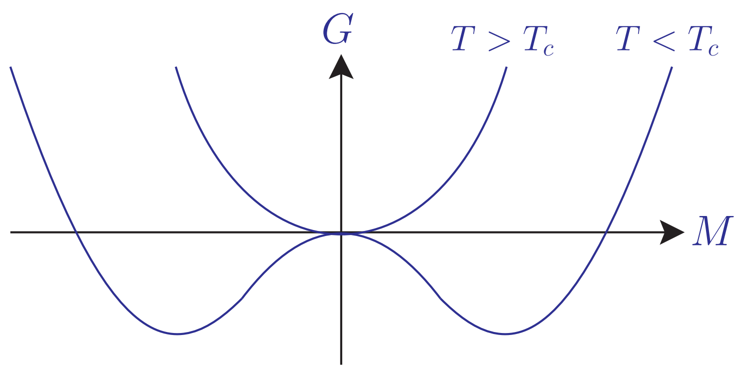
- example: uniaxial ferromagnet
- Gibbs free energy in the critical region $T \approx T_c$, $M \approx 0$:



$$G(M,T) = A(T) + \underbrace{B(T)}_{b(T-T_c)+\ldots} M^2 + \underbrace{C(T)}_{c+\ldots} M^4 + \ldots$$

- for $H = 0$: from $\frac{\partial G}{\partial M}\Big|_T = 0$

$$M = \begin{cases} 0 & T \geq T_c \\ \pm\sqrt{\frac{b}{2c}(T_c - T)} & T \leq T_c \end{cases}$$



$\longrightarrow$ universal prediction: $M \propto (T - T_c)^\beta$, $\beta = \frac{1}{2}$ critical exponent

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- microscopic model with local spin density $s(\mathbf{x})$, $M = \int d^3x\, s(\mathbf{x})$:

$$G[s(\mathbf{x})] = \int d^3x \Big[\underbrace{\frac{1}{2}(\nabla s(x))^2}_{\text{spin alignment}} + b(T-T_c)s(\mathbf{x})^2 + cs(\mathbf{x})^4 - \underbrace{H(\mathbf{x})s(\mathbf{x})}_{\text{local external field}}\Big]$$

- spin response to $H(\mathbf{x}) = H_0\delta^{(3)}(\mathbf{x})$:

$$\frac{\delta G}{\delta s(\mathbf{x})} = 0 \quad \Rightarrow \quad -\nabla^2 + 2b(T-T_c)s(\mathbf{x}) = H_0\delta^{(3)}(\mathbf{x})$$

$$s(\mathbf{x}) = D(\mathbf{x}) = \frac{H_0}{4\pi}\frac{e^{-x/\xi}}{x}\,, \quad \text{correlation length } \xi = [2b(T-T_c)]^{-1/2}$$

$\longrightarrow$ Yukawa potential with "mass" $m = 1/\xi$

for $T \to T_c$: $\xi \to \infty$ / $m \to 0$, correlation length diverges

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quantum fluctuations in Quantum Field Theory	$\Leftrightarrow$	thermal fluctuations in critical phenomena
indep. of details in short-range / high-energy physics	$\Leftrightarrow$	universality (critical exponents)