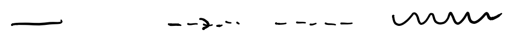
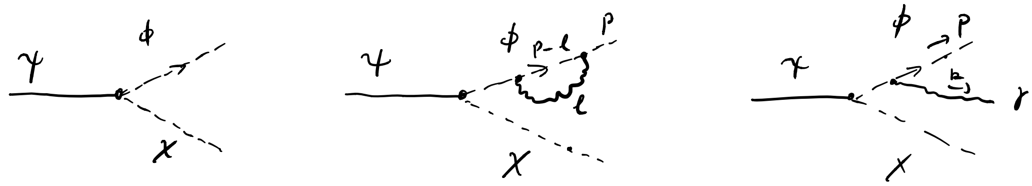
H.11

a) $\psi(P) \to \phi(p)\, \chi(q)\, [\gamma(k)]$

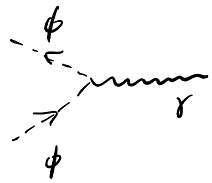




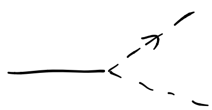
Feynman rules:

Propagator (ψ, ϕ, χ) : $\dfrac{i}{p^2 - m_i^2 + i\varepsilon}$

photon propagator : $\dfrac{i}{p^2 + i\varepsilon}$



$\sim -ie$



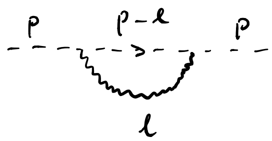
$\sim -i\lambda$

$\to$ $i\mathcal{M}_a = -i\lambda$

$$i\mathcal{M}_b = -i\lambda\,(-ie)^2 \int \frac{d^d l}{(2\pi)^d}\, \frac{i}{(p-l)^2 - m^2}\, \frac{i}{l^2}$$

$$i\mathcal{M}_c = -i\lambda\,(-ie)\, \frac{i}{(p-k)^2 - m^2}$$

b)



$$-i\Sigma(p^2) = (-ie)^2 \int \frac{d^d l}{(2\pi)^d}\, \frac{i}{(p-l)^2 - m^2}\, \frac{i}{l^2 - \mu^2}$$

$$= e^2 \int \frac{d^d l}{(2\pi)^d}\, \underbrace{\frac{1}{(p-l)^2 - m^2}\, \frac{1}{l^2 - \mu^2}}$$

$$= \int_0^1 dx \quad \{x[(p-\ell)^2 - m^2] + (1-x)(\ell^2 - \mu^2)\}^{-2}$$

$$= \int_0^1 dx \; \{ xp^2 - \underbrace{2xp\ell} + \cancel{x\ell^2} - xm^2 + \underline{\ell^2} - \cancel{x\ell^2} - \mu^2 + x\mu^2 \}^{-2}$$

$$= \int_0^1 dx \quad \{(\ell - xp)^2 - x^2p^2 + xp^2 - xm^2 + (x-1)\mu^2\}^{-2}$$

$$= \int_0^1 dx \; \{ \underbrace{(\ell - xp)^2}_{=: q^2} + \underbrace{x(-x+1)p^2 - xm^2 + (x-1)\mu^2}_{=: -\Delta(x)} \}^{-2}$$

$$= +e^2 \int \frac{d^dq}{(2\pi)^d} \int_0^1 dx \; \frac{1}{(q^2 - \Delta(x))^2}$$

Wick rotation: $q^0 \to i q^0_E$

$$= ie^2 \int \frac{d^dq_E}{(2\pi)^d} \int_0^1 dx \; \frac{1}{(q_E^2 + \Delta(x))^2}$$

$\longleftarrow$ $\ell \to 0$, $q \to -xp$, $p^2 = m^2$

$\sim \int_0^1 dx \; \frac{1}{[(x-1)\mu^2]^2}$

is IR-finite!

$$= \frac{+ie^2}{(4\pi)^2} \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi + \int_0^1 dx \; \ln\Delta(x) + \mathcal{O}(\epsilon) \right)$$

However, $\quad \Sigma'(m^2) = \left.\frac{d\Sigma(p^2)}{dp^2}\right|_{p^2 = m^2}$

$$= ie \int \frac{d^dq_E}{(2\pi)^d} \int_0^1 dx \; \underbrace{\frac{d}{dp^2} \frac{1}{(q_E^2 + \Delta)^2}}_{= -2\frac{1}{(q_E^2+\Delta)^3}\frac{d\Delta}{dp^2}} \Bigg|_{p^2 = m^2}$$

$$\Delta(x) = x(-x+1)p^2 - xm^2 + (x-1)\mu^2$$

$$\simeq xp^2 - xm^2 - \mu^2$$

$$\to \frac{d\Delta}{dp^2} = x \quad .$$

$$= ie \int \frac{d^d q_E}{(2\pi)^d} \int_0^1 dx \; \frac{-2}{(q_E^2+\Delta)^3} \; x \Big|_{p^2=m^2}$$

$$\frac{d}{dp^2} \int_0^1 dx \ln\Delta(x) = \int_0^1 dx \; \frac{1}{\Delta(x)} \cdot x(-x+1) = \int_0^1 dx \; \frac{x(-x+1)}{x(1-x)p^2 - xm^2 + (x-1)\mu^2}$$

$$= \int_0^1 dx \frac{x(-x+1)}{(x-1)(-xp^2+\mu^2) - xm^2}$$

$1-x \simeq 1$

$$\simeq \int_0^1 dx \; \frac{x}{+xp^2 - \mu^2 - xm^2}$$

$$= \int_0^1 dx \; \frac{x}{x(p^2-m^2) - \mu^2}$$

if $\mu \to 0$ $\quad \sim \int_0^1 dx \frac{1}{p^2-m^2}$.

$\longrightarrow \quad \Sigma'(m^2) \sim \infty$

c) $$d\Gamma_A = \frac{1}{2M} \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3q}{(2\pi)^3 2E_q} |\mathcal{M}_A|^2 (2\pi)^4 \delta^{(4)}(P-q-q)$$

$$d\Gamma_B = \frac{1}{2M} \frac{d^3p}{(2\pi)^3 2E_p} \frac{d^3q}{(2\pi)^3 2E_q} \frac{d^3k}{(2\pi)^3 2E_q} |\mathcal{M}_B|^2 (2\pi)^4 \delta^{(4)}(P-q-q)$$

$\longrightarrow$ equation (3)

d) $$|\tilde{\mathcal{M}}_B|^2 = \int \frac{d^3k}{(2\pi)^3 2E_k} |\mathcal{M}_B|^2$$

$$= \int \frac{d^3k}{(2\pi)^3 2E_k} \quad \lambda^2 e^2 \left(\frac{1}{(p-k)^2 - m^2} \right)^2$$

$$= \frac{4\pi}{8\pi^3} \lambda^2 e^2 \int_{E_{min}}^{\infty} \frac{dk \quad k^2}{2\sqrt{k^2+\mu^2}} \left[(p-k)^2 - m^2 \right]^{-2}$$