H.15

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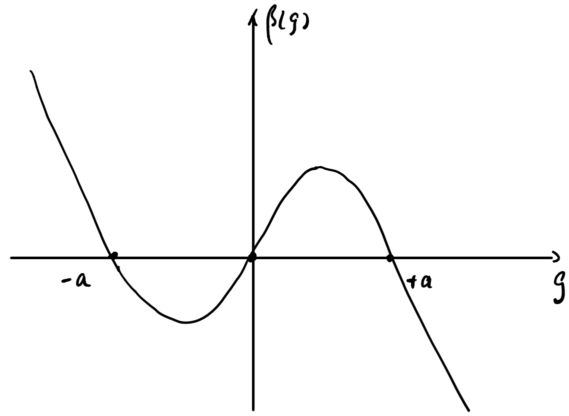
a) $\beta(g) = g(a^2-g^2) = g(a+g)(a-g)$



$\lim\limits_{g\to\infty} \beta(g) \hat{=} g\cdot g\cdot(-g) = -g^3 < 0$

$\longrightarrow -\infty$

$\lim\limits_{g\to-\infty} \beta(g) \hat{=} g\cdot g\cdot(-g) = -g^3 > 0$

$\longrightarrow \infty$

with $a>0$.

$$\underbrace{\frac{d}{dt}\bar{g} = \beta(\bar{g})} = \bar{g}(a^2-\bar{g}^2)$$

if $\beta\to\infty$, $\frac{d}{dt}\bar{g}\to\infty$.

b) $d\bar{g} = dt\ \bar{g}(a^2-\bar{g}^2)$

$$\int_{g_0}^{\bar{g}} \frac{dg'}{g'(a-g')(a+g')} = \int_0^t dt'$$

$$\frac{1}{g(a-g)(a+g)} = \frac{A}{g} + \frac{B}{a^2-g^2}$$

$$\left(\begin{array}{l} A(a^2-g^2)+Bg \overset{!}{=} 1 \\ \to A = 1/a^2,\ B = g/a^2 \end{array} \right.$$

$$= \frac{1}{a^2}\frac{1}{g} + \frac{1}{a^2}\underbrace{\frac{g}{a^2-g^2}}$$

$$\left(\begin{array}{l} = \frac{A}{a+g} + \frac{B}{a-g} \\ A(a-g)+B(a+g) \overset{!}{=} g \\ A = -B,\quad A-B = 1 \end{array} \right.$$

$$| \Rightarrow A = \frac{1}{2}, \quad B = -\frac{1}{2}$$

$$= \frac{1}{a^2}\left(\frac{1}{g} + \frac{1}{2}\frac{1}{a+g} - \frac{1}{2}\frac{1}{a-g} \right)$$

$$\Rightarrow \int_{g_0}^{\bar{g}} \frac{dg'}{g'(a-g')(a+g')} = \frac{1}{a^2}\Bigg[\underbrace{\int_{g_0}^{\bar{g}} \frac{dg'}{g'}}_{=[\ln g']_{g_0}^{\bar{g}}} + \frac{1}{2}\Bigg(\underbrace{\int_{g_0}^{\bar{g}} \frac{dg'}{a+g}}_{=[\ln g']_{g_0+a}^{\bar{g}+a}} - \underbrace{\int_{g_0}^{\bar{g}} \frac{dg'}{a-g}}_{=[\ln g']_{g_0-a}^{\bar{g}-a}} \Bigg)\Bigg]$$

$$= \frac{1}{a^2}\left[\ln(\bar{g}/g_0) + \frac{1}{2}\ln\left(\frac{|\bar{g}+a|}{|g_0+a|}\right) + \frac{1}{2}\ln\left(\frac{|\bar{g}-a|}{|g_0-a|}\right) \right]$$

RHS $= t$

$$\Rightarrow \ln\frac{\bar{g}}{g_0} + \ln\sqrt{\frac{|\bar{g}+a|}{|g_0+a|}} + \ln\sqrt{\frac{|\bar{g}-a|}{|g_0-a|}} = a^2 t$$

$$\ln\left(\bar{g}\sqrt{|\bar{g}^2-a^2|}\right) = a^2 t + \ln\left(g_0\sqrt{|g_0^2-a^2|}\right)$$

H.16

a) $\beta(g) = -b(g-a)$, $b>0$

$$\frac{d}{dt}\bar{g} = \beta(\bar{g}) = -b(\bar{g}-a)$$

$$\int_{g_0}^{\bar{g}} \frac{d\bar{g}'}{\bar{g}'-a} = \int_0^t -b\, dt'$$

$$\ln\left|\frac{\bar{g}-a}{g_0-a}\right| = -bt$$

$$\ln|\bar{g}-a| = -bt + \ln|g_0-a|$$

$$|\bar{g}-a| = e^{-bt+\ln|g_0-a|}$$

$$= t^{-b}\cdot|g_0-a|$$

$\rightarrow$ $t\rightarrow\infty$. RHS = 0. $\Rightarrow$ $\bar{g}\rightarrow a$ (exponentially)

b) $\beta(g) = -b(g-a)^n$, $b>0$, $n>1$

$$\frac{d}{dt}g' = \beta(g')$$

$$\frac{dg'}{dt} = -b(g'-a)^n$$

$$\frac{dg'}{(g'-a)^n} = -b\,dt$$

$$\rightarrow \frac{1}{-n+1}\left[(\bar{g}-a)^{-n+1} - (g_0-a)^{-n+1}\right] = -bt$$

$$(\bar{g}-a)^{-n+1} = -b(-n+1)t + (g_0-a)^{-n+1}$$

$$(\bar{g}-a)^{n-1} = \left[+b(n-1)t + C\right]^{-1}$$

$\rightarrow$ $t\rightarrow\infty$. RHS $\rightarrow 0$ $\Rightarrow$ $\bar{g}-a\rightarrow 0$ (power in t)

$\bar{g}\rightarrow a$

H.17 $\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \bar{q}_f \left[i\gamma^\mu (\partial_\mu - ig A_\mu^b \frac{\lambda_b}{2}) - m_f \right] q_f$

a) term(s) in $\mathcal{L}$ responsible for quark-gluon interaction:

color triplet, contains spinors with three colors

$$-\sum_f \bar{q}_f (i\gamma^\mu) \, ig A_\mu^b \frac{\lambda_b}{2} q_f = +\frac{g}{2} \sum_f \bar{q}_f \gamma^\mu A_\mu^b \lambda_b q_f$$

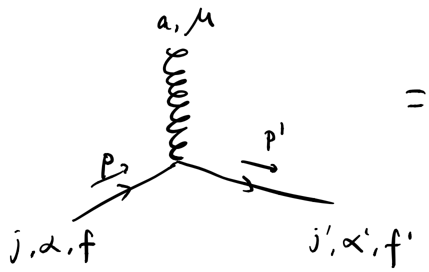
$$\frac{\delta \mathcal{L}}{\delta (\bar{q}_f)_{j,\alpha}} = -\frac{g}{2} \left[\gamma^\mu A_\mu^b (\lambda_b q_f)_j \right]_\alpha$$

$$\frac{\delta^2 \mathcal{L}}{\delta (q_{f'})_{j',\alpha'} \delta (\bar{q}_f)_{j,\alpha}} = -\frac{g}{2} (\gamma^\mu)_{\alpha\alpha'} A_\mu^b (\lambda_b)_{jj'} \delta_{ff'}$$

$$\frac{\delta^3 \mathcal{L}}{\delta A^a_\mu \delta q_{f'} \delta \bar{q}_f} = -\frac{g}{2} (\gamma^\mu)_{\alpha\alpha'} \delta^{ab} (\lambda_b)_{jj'} \delta_{ff'}$$

$$= -\frac{g}{2} (\gamma^\mu)_{\alpha\alpha'} (\lambda^a)_{jj'} \delta_{ff'}$$

$(2\pi)^4 \delta^{(4)}(p+q-p')$ enters in S-matrix

$\Rightarrow$ 

$$= ig \left(\frac{\lambda^a}{2} \right)_{jj'} (\gamma^\mu)_{\alpha'\alpha} \delta_{f'f} (2\pi)^4 \delta^{(4)}(p+q-p')$$

b) $\mathcal{L} \supset -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$

$$\supset -\frac{1}{4} \left[\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \right] \left[\cdots \right]_a^{\mu\nu}$$

$$\to \mathcal{L}_{3g} = -\frac{1}{4} g f^{abc} A_\mu^b A_\nu^c (\partial^\mu A_a^\nu - \partial^\nu A_a^\mu)$$
$$- \frac{1}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) g f_{abc} A_b^\mu A_c^\nu$$
$$= -\frac{g}{2} \underbrace{f^{abc}}_{\text{anti-symm.}} A_\mu^b A_\nu^c (\partial^\mu A_a^\nu - \partial^\nu A_a^\mu)$$

$$= g f^{abc} A_\mu^b A_\nu^c (\partial^\nu A_a^\mu)$$
$$= g f^{lmn} A_\alpha^m A_\beta^n (\partial^\beta A_l^\alpha)$$

$$\frac{\delta \mathcal{L}}{\delta A^a_\mu} = g f^{lmn} (\delta^{ma} \delta_{\mu\alpha} A_\beta{}^n + A^m_\alpha \delta^{an} \delta_{\mu\beta}) (\partial^\beta A_l{}^\alpha)$$

$$+ g f^{lmn} A^m_\alpha A^n_\beta (-i p^\beta) \delta^a_l g^{\mu\alpha}$$

$$= g f^{lan} A^n_\beta (\partial^\beta A_l^\mu) + g f^{lma} A^m_\alpha (\partial^\mu A_l{}^\alpha)$$

$$+ g f^{amn} (A^m)^\mu A^n_\beta \; (-i p^\beta)$$

$$\frac{\delta^2 \mathcal{L}}{\delta A^b_\nu \; \delta A^a_\mu} = g f^{lan} \; \delta^{bn} \; \delta_{\nu\beta} (\partial^\beta A_l^\mu) + g f^{lan} A^n_\beta (-i q^\beta) \delta^b_l g^{\mu\nu}$$

$$+ g f^{lma} \delta^{bm} \delta_{\alpha\nu} (\partial^\mu A^\alpha_l) + g f^{lma} A^m_\alpha \; (-i q^\mu) \delta^b_l g^{\alpha\nu}$$

$$+ g f^{amn} \delta^{mb} g^{\mu\nu} A^n_\beta \; (-i p^\beta) + g f^{amn} (A^m)^\mu \delta^{bn} \delta^{\nu\beta} (-i p^\beta)$$

$$= g f^{lab} (\partial^\nu A_l^\mu) + g f^{ban} A^n_\beta \; (-i q^\beta) g^{\mu\nu}$$

$$+ g f^{lba} (\partial^\mu A^\nu_l) + g f^{bma} (A^m)^\nu \; (-i q^\mu)$$

$$+ g f^{abn} g^{\mu\nu} A_\beta{}^n (-i p^\beta) + g f^{amb} \; (A^m)^\mu (-i p^\nu)$$

$$\frac{\delta^3 \mathcal{L}}{\delta A^c_\rho \, \delta A^b_\nu \, \delta^a_\mu} = g f^{lab} \; (-i r^\nu) \delta^c_l g^{\mu\rho} + g f^{ban} \; \delta^c_n \; \delta^\rho_\beta \; (-i q^\beta) g^{\mu\nu}$$

$$+ g f^{lba} \; (-i r^\mu) \delta^c_l g^{\rho\nu} + g f^{bma} \; \delta^c_m \; g^{\rho\nu} (-i q^\mu)$$

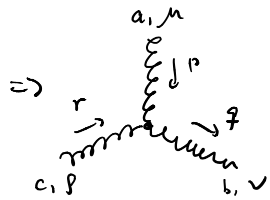
$$+ g f^{abn} g^{\mu\nu} \; \delta^n_c \; \delta^\rho_\beta (-i p^\beta) + g f^{amb} \; \delta^m_c \; g^{\mu\rho} \; (-i p^\nu)$$

$$= -ig \Big\{ f^{cab} \; r^\nu g^{\mu\rho} + f^{bac} q^\rho g^{\mu\nu} + f^{cba} \; r^\mu g^{\rho\nu} + g f^{bca} \; g^{\rho\nu} q^\mu$$

$$+ f^{abc} \; g^{\mu\nu} p^\rho + g f^{acb} \; g^{\mu\rho} \; p^\nu \Big\}$$

$$= -ig f^{abc} \Big\{ \underline{\underline{g^{\mu\rho} r^\nu}} - \underline{g^{\mu\nu} q^\rho} - g^{\rho\nu} r^\mu + g^{\rho\nu} q^\mu + \underline{g^{\mu\nu} p^\rho} - \underline{\underline{g^{\mu\rho} p^\nu}} \Big\}$$

$$= -ig f^{abc} \Big\{ g^{\mu\rho} (r-p)^\nu + g^{\mu\nu} (p-q)^\rho + g^{\rho\nu} (q-r)^\mu \Big\}$$

$\Rightarrow$ 

$$= g f^{abc} \left\{ g^{\mu\rho} (r-p)^{\nu} + g^{\mu\nu} (p-q)^{\rho} + g^{\rho\nu} (q-r)^{\mu} \right\}$$
$$\times (2\pi)^4 \delta^{(4)} (p+q+r)$$

c) $$\mathcal{L}_{4g} = -\frac{1}{4} g f^{abc} A^b_\mu A^c_\nu \, g f_{amn} A^\mu_m A^\nu_n$$

$$= -\frac{g^2}{4} f^{abc} f_{amn} A^b_\mu A^c_\nu A^\mu_m A^\nu_n$$

$$= -\frac{g^2}{4} f^{abc} f_{amn} g^{\mu\alpha} g^{\nu\beta} A^b_\mu A^c_\nu A^m_\alpha A^n_\beta$$

$$= \underbrace{-\frac{g^2}{4} f^{jkl} f_{jmn} g^{\sigma\alpha} g^{\tau\beta}}_{=: B} A^k_\sigma A^l_\tau A^m_\alpha A^n_\beta$$

$$\frac{\delta \mathcal{L}_{4g}}{\delta A^a_\mu}$$
$$= B \left(\delta^{ak} \delta^\mu_\sigma A^l_\tau A^m_\alpha A^n_\beta + A^k_\sigma \delta^{la} \delta^\mu_\tau A^m_\alpha A^n_\beta + A^k_\sigma A^l_\tau \delta^{am} \delta^\mu_\alpha A^n_\beta \right.$$
$$\left. + A^k_\sigma A^l_\tau A^m_\alpha \delta^{an} \delta^\mu_\beta \right)$$
$$= -\frac{g^2}{4} f^{jal} f_{jmn} g^{\mu\alpha} g^{\tau\beta} A^l_\tau A^m_\alpha A^n_\beta - \frac{g^2}{4} f^{jka} f_{jmn} g^{\sigma\alpha} g^{\mu\beta} A^k_\sigma A^m_\alpha A^n_\beta$$
$$- \frac{g^2}{4} f^{jkl} f_{jan} g^{\sigma\mu} g^{\tau\beta} A^k_\sigma A^l_\tau A^n_\beta - \frac{g^2}{4} f^{jkl} f_{jma} g^{\sigma\alpha} g^{\tau\mu} A^k_\sigma A^l_\tau A^m_\alpha$$

$$\frac{\delta^2 \mathcal{L}_{4g}}{\delta A^b_\nu \, \delta A^a_\mu}$$
$$= -\frac{g^2}{4} f^{jal} f_{jmn} g^{\mu\alpha} g^{\tau\beta} \left(\delta^{bl} \delta^\nu_\tau \underline{A^m_\alpha A^n_\beta} + \delta^{bm} \delta^\nu_\alpha \underline{A^l_\tau A^n_\beta} + \delta^{bn} \delta^\nu_\beta \underline{A^l_\tau A^m_\alpha} \right)$$
$$- \frac{g^2}{4} f^{jka} f_{jmn} g^{\sigma\alpha} g^{\mu\beta} \left(\delta^{bk} \delta^\nu_\sigma \underline{A^m_\alpha A^\mu_\beta} + \delta^{bm} \delta^\nu_\alpha \boxed{A^k_\sigma A^n_\beta} + \delta^{bn} \delta^\nu_\beta A^k_\sigma A^m_\alpha \right)$$
$$- \frac{g^2}{4} f^{jkl} f_{jan} g^{\sigma\mu} g^{\tau\beta} \left(\delta^{bk} \delta^\nu_\sigma \underline{A^l_\tau A^n_\beta} + \delta^{bl} \delta^\nu_\tau \boxed{A^k_\sigma A^n_\beta} + \delta^{bn} \delta^\nu_\beta \underline{A^k_\sigma A^l_\tau} \right)$$
$$- \frac{g^2}{4} f^{jkl} f_{jma} g^{\sigma\alpha} g^{\tau\mu} \left(\delta^{kb} \delta^\nu_\sigma \underline{A^l_\tau A^m_\alpha} + \delta^{lb} \delta^\nu_\tau A^k_\sigma A^m_\alpha + \delta^{mb} \delta^\nu_\alpha \underline{A^k_\sigma A^l_\tau} \right)$$

$$
\begin{aligned}
= -\frac{g^2}{4}\Big[& \big(f^{jab} f_{jmn} g^{\mu\alpha} g^{\nu\beta} + f^{jba} f_{jmn} g^{\nu\alpha} g^{\mu\beta}\big)\, \underline{A^m_\alpha A^n_\beta} \\
& + \big(f^{jal} f_{jbn} g^{\mu\nu} g^{\tau\beta} + f^{jbl} f_{jan} g^{\mu\nu} g^{\tau\beta}\big)\, \underline{A^l_\tau A^n_\beta} \\
& + \big(f^{jal} f_{jmb} g^{\mu\alpha} g^{\tau\nu} + f^{jbl} f_{jma} g^{\nu\alpha} g^{\tau\mu}\big)\, \underline{\underline{A^l_\tau A^m_\alpha}} \\
& + \big(f^{jka} f_{jbn} g^{\sigma\nu} g^{\mu\beta} + f^{jkb} f_{jan} g^{\sigma\mu} g^{\nu\beta}\big)\, \boxed{A^k_\sigma A^n_\beta} \\
& + \big(f^{jka} f_{jmb} g^{\sigma\alpha} g^{\mu\nu} + f^{jkb} f_{jma} g^{\sigma\alpha} g^{\nu\mu}\big)\, A^k_\sigma A^m_\alpha \\
& + \big(f^{jkl} f_{jab} g^{\sigma\mu} g^{\tau\nu} + f^{jkl} f_{jba} g^{\sigma\nu} g^{\tau\mu}\big)\, \underline{\underline{A^k_\sigma A^l_\tau}} \Big]
\end{aligned}
$$

$$
\frac{\delta^3 \mathcal{L}_{4g}}{\delta A^c_\rho\, \delta A^b_\nu\, \delta A^a_\mu}
$$

$$
\begin{aligned}
= -\frac{g^2}{4}\Big[& \big(f^{jab} f_{jmn} g^{\mu\alpha} g^{\nu\beta} + f^{jba} f_{jmn} g^{\nu\alpha} g^{\mu\beta}\big) \cdot \big(\delta^{cm} \delta^\rho_\alpha A^n_\beta + \delta^{cn} \delta^\rho_\beta A^m_\alpha\big) \\
& + \big(f^{jal} f_{jbn} g^{\mu\nu} g^{\tau\beta} + f^{jbl} f_{jan} g^{\mu\nu} g^{\tau\beta}\big) \cdot \big(\delta^{lc} \delta^\rho_\tau A^n_\beta + \delta^{nc} \delta^\rho_\beta A^l_\tau\big) \\
& + \big(f^{jal} f_{jmb} g^{\mu\alpha} g^{\tau\nu} + f^{jbl} f_{jma} g^{\nu\alpha} g^{\tau\mu}\big) \cdot \big(\delta^{lc} \delta^\rho_\tau A^m_\alpha + \delta^{mc} \delta^\rho_\alpha A^l_\tau\big) \\
& + \big(f^{jka} f_{jbn} g^{\sigma\nu} g^{\mu\beta} + f^{jkb} f_{jan} g^{\sigma\mu} g^{\nu\beta}\big) \cdot \big(\delta^{kc} \delta^\rho_\sigma A^n_\beta + \delta^{nc} \delta^\rho_\beta A^k_\sigma\big) \\
& + \big(f^{jka} f_{jmb} g^{\sigma\alpha} g^{\mu\nu} + f^{jkb} f_{jma} g^{\sigma\alpha} g^{\nu\mu}\big) \cdot \big(\delta^{kc} \delta^\rho_\sigma A^m_\alpha + \delta^{mc} \delta^\rho_\alpha A^k_\sigma\big) \\
& + \big(f^{jkl} f_{jab} g^{\sigma\mu} g^{\tau\nu} + f^{jkl} f_{jba} g^{\sigma\nu} g^{\tau\mu}\big) \cdot \big(\delta^{kc} \delta^\rho_\sigma A^l_\tau + \delta^{lc} \delta^\rho_\tau A^k_\sigma\big) \Big]
\end{aligned}
$$

$$
\begin{aligned}
= -\frac{g^2}{4}\Big[& A^n_\beta \big(f^{jab} f_{jcn} g^{\mu\rho} g^{\nu\beta} + f^{jba} f_{jcn} g^{\nu\rho} g^{\mu\beta} \\
& \qquad + f^{jac} f_{jbn} g^{\mu\nu} g^{\rho\beta} + f^{jbc} f_{jan} g^{\mu\nu} g^{\rho\beta} \\
& \qquad + f^{jca} f_{jbn} g^{\rho\nu} g^{\mu\beta} + f^{jcb} f_{jan} g^{\rho\mu} g^{\nu\beta}\big) \\
& + A^m_\alpha \big(f^{jab} f_{jmc} g^{\mu\alpha} g^{\nu\rho} + f^{jba} f_{jmc} g^{\nu\alpha} g^{\mu\rho} \\
& \qquad + f^{jac} f_{jmb} g^{\mu\alpha} g^{\rho\nu} + f^{jbc} f_{jma} g^{\nu\sigma} g^{\rho\mu} \\
& \qquad + f^{jca} f_{jmb} g^{\rho\alpha} g^{\mu\nu} + f^{jcb} f_{jma} g^{\rho\alpha} g^{\nu\mu}\big) \\
& + A^l_\tau \big(f^{jal} f_{jbc} g^{\mu\nu} g^{\tau\rho} + f^{jbl} f_{jac} g^{\mu\nu} g^{\tau\rho} \\
& \qquad + f^{jal} f_{jcb} g^{\mu\rho} g^{\tau\nu} + f^{jbl} f_{jca} g^{\nu\rho} g^{\tau\mu} \\
& \qquad + f^{jcl} f_{jab} g^{\rho\mu} g^{\tau\nu} + f^{jcl} f_{jba} g^{\rho\nu} g^{\tau\mu}\big) \\
& + A^k_\sigma \big(f^{jka} f_{jbc} g^{\sigma\nu} g^{\mu\rho} + f^{jkb} f_{jac} g^{\sigma\mu} g^{\nu\rho}
\end{aligned}
$$

$$+ f^{jka} f_{jcb}\, g^{\sigma\rho} g^{\mu\nu} + f^{jkb} f_{jca}\, g^{\sigma\rho} g^{\nu\mu}$$

$$+ f^{jkc} f_{jab}\, g^{\sigma\mu} g^{\rho\nu} + f^{jkc} f_{jba}\, g^{\sigma\nu} g^{\rho\mu})]$$

$$= -\frac{g^2}{4} \Big\{ A^n_\beta \Big[f^{jab} f_{jcn} (g^{\mu\rho} g^{\nu\beta} - g^{\nu\rho} g^{\mu\beta}) + f^{jac} f_{jbn} (g^{\mu\nu} g^{\rho\beta} - g^{\rho\nu} g^{\mu\beta})$$

$$+ f^{jbc} f_{jan} (g^{\mu\nu} g^{\rho\beta} - g^{\rho\mu} g^{\nu\beta}) \Big]$$

$$+ A^m_\alpha \Big[f^{jab} f_{jmc} (g^{\mu\alpha} g^{\nu\rho} - g^{\nu\alpha} g^{\mu\rho}) + f^{jac} f_{jmb} (g^{\mu\alpha} g^{\rho\nu} - g^{\rho\alpha} g^{\mu\nu})$$

$$+ f^{jbc} f_{jma} (g^{\nu\alpha} g^{\rho\mu} - g^{\rho\alpha} g^{\nu\mu}) \Big]$$

$$+ A^l_\tau \Big[f^{jal} f_{jbc} (g^{\mu\nu} g^{\tau\rho} - g^{\mu\rho} g^{\tau\nu}) + f^{jbl} f_{jac} (g^{\mu\nu} g^{\tau\rho} - g^{\nu\rho} g^{\tau\mu})$$

$$+ f^{jcl} f_{jab} (g^{\rho\mu} g^{\tau\nu} - g^{\rho\nu} g^{\tau\mu}) \Big]$$

$$+ A^k_\sigma \Big[f^{jka} f_{jbc} (g^{\sigma\nu} g^{\mu\rho} - g^{\sigma\rho} g^{\mu\nu}) + f^{jkb} f_{jac} (g^{\sigma\mu} g^{\nu\rho} - g^{\sigma\rho} g^{\nu\mu})$$

$$+ f^{jkc} f_{jab} (g^{\sigma\mu} g^{\rho\nu} - g^{\sigma\nu} g^{\rho\mu}) \Big] \Big\}$$

$$\frac{\delta^4 \mathcal{L}}{\delta A^d_\kappa\, \delta A^c_\rho\, \delta A^b_\nu\, \delta A^a_\mu}$$

$$= -\frac{g^2}{4} \Big\{ \delta^{nd} \delta^\kappa_\beta \Big[f^{jab} f_{jcn} (g^{\mu\rho} g^{\nu\beta} - g^{\nu\rho} g^{\mu\beta}) + f^{jac} f_{jbn} (g^{\mu\nu} g^{\rho\beta} - g^{\rho\nu} g^{\mu\beta})$$

$$+ f^{jbc} f_{jan} (g^{\mu\nu} g^{\rho\beta} - g^{\rho\mu} g^{\nu\beta}) \Big]$$

$$+ \delta^{md} \delta^\kappa_\alpha \Big[f^{jab} f_{jmc} (g^{\mu\alpha} g^{\nu\rho} - g^{\nu\alpha} g^{\mu\rho}) + f^{jac} f_{jmb} (g^{\mu\alpha} g^{\rho\nu} - g^{\rho\alpha} g^{\mu\nu})$$

$$+ f^{jbc} f_{jma} (g^{\nu\alpha} g^{\rho\mu} - g^{\rho\alpha} g^{\nu\mu}) \Big]$$

$$+ \delta^{ld} \delta^\kappa_\tau \Big[f^{jal} f_{jbc} (g^{\mu\nu} g^{\tau\rho} - g^{\mu\rho} g^{\tau\nu}) + f^{jbl} f_{jac} (g^{\mu\nu} g^{\tau\rho} - g^{\nu\rho} g^{\tau\mu})$$

$$+ f^{jcl} f_{jab} (g^{\rho\mu} g^{\tau\nu} - g^{\rho\nu} g^{\tau\mu}) \Big]$$

$$+ \delta^{kd} \delta^\kappa_\sigma \Big[f^{jka} f_{jbc} (g^{\sigma\nu} g^{\mu\rho} - g^{\sigma\rho} g^{\mu\nu}) + f^{jkb} f_{jac} (g^{\sigma\mu} g^{\nu\rho} - g^{\sigma\rho} g^{\nu\mu})$$

$$+ f^{jkc} f_{jab} (g^{\sigma\mu} g^{\rho\nu} - g^{\sigma\nu} g^{\rho\mu}) \Big] \Big\}$$

$$= -\frac{g^2}{4} \Big\{ f^{jab} f_{jcd} (g^{\mu\rho} g^{\nu\kappa} - g^{\nu\rho} g^{\mu\kappa}) + f^{jac} f_{jbd} (g^{\mu\nu} g^{\rho\kappa} - g^{\rho\nu} g^{\mu\kappa}) + f^{jbc} f_{jad} (g^{\mu\nu} g^{\rho\kappa} - g^{\rho\mu} g^{\nu\kappa})$$

$$+ f^{jab} f_{jdc} (g^{\mu\kappa} g^{\nu\rho} - g^{\nu\kappa} g^{\mu\rho}) + f^{jac} f_{jdb} (g^{\mu\kappa} g^{\rho\nu} - g^{\rho\kappa} g^{\mu\nu}) + f^{jbc} f_{jda} (g^{\nu\kappa} g^{\rho\mu} - g^{\rho\kappa} g^{\nu\mu})$$

$$+ f^{jad} f_{jbc} (g^{\mu\nu} g^{\kappa\rho} - g^{\mu\rho} g^{\kappa\nu}) + f^{jbd} f_{jac} (g^{\mu\nu} g^{\kappa\rho} - g^{\nu\rho} g^{\kappa\mu}) + f^{jcd} f_{jab} (g^{\rho\mu} g^{\kappa\nu} - g^{\rho\nu} g^{\kappa\mu})$$

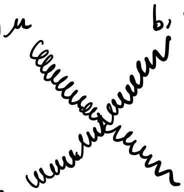
$$+ f^{jda} f_{jbc} (g^{\kappa\nu} g^{\mu\rho} - g^{\kappa\rho} g^{\mu\nu}) + f^{jdb} f_{jac} (g^{\kappa\mu} g^{\nu\rho} - g^{\kappa\rho} g^{\nu\mu}) + f^{jdc} f_{jab} (g^{\kappa\mu} g^{\rho\nu} - g^{\kappa\nu} g^{\rho\mu}) \Big\}$$

$$
\begin{aligned}
= -\frac{g^2}{4}\Big\{ & f^{jab} f_{jcd}\big(g^{\mu\rho}g^{\nu\kappa} - g^{\nu\rho}g^{\mu\kappa} - g^{\mu\kappa}g^{\nu\rho} + g^{\nu\kappa}g^{\mu\rho} + g^{\rho\mu}g^{\kappa\nu} - g^{\rho\nu}g^{\kappa\mu} \\
& \qquad - g^{\mu\kappa}g^{\nu\rho} + g^{\nu\kappa}g^{\mu\rho}\big) \\
& + f^{jac} f_{jbd}\big(g^{\mu\nu}g^{\rho\kappa} - g^{\rho\nu}g^{\mu\kappa} - g^{\mu\kappa}g^{\rho\nu} + g^{\rho\kappa}g^{\mu\nu} + g^{\mu\nu}g^{\kappa\rho} - g^{\nu\rho}g^{\kappa\mu} \\
& \qquad - g^{\mu\kappa}g^{\nu\rho} + g^{\rho\kappa}g^{\mu\nu}\big) \\
& + f^{jbc} f_{jad}\big(g^{\mu\nu}g^{\rho\kappa} - g^{\rho\mu}g^{\nu\kappa} - g^{\nu\kappa}g^{\mu\rho} + g^{\rho\kappa}g^{\mu\nu} + g^{\mu\nu}g^{\kappa\rho} - g^{\mu\rho}g^{\nu\kappa} \\
& \qquad - g^{\nu\kappa}g^{\mu\rho} + g^{\rho\kappa}g^{\mu\nu}\big)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
= -g^2\Big\{ & f^{jab} f_{jcd}\big(g^{\mu\rho}g^{\nu\kappa} - g^{\mu\kappa}g^{\nu\rho}\big) + f^{jac} f_{jbd}\big(g^{\mu\nu}g^{\rho\kappa} - g^{\mu\kappa}g^{\nu\rho}\big) \\
& + f^{jbc} f_{jad}\big(g^{\mu\nu}g^{\rho\kappa} - g^{\mu\rho}g^{\nu\kappa}\big)\Big\}
\end{aligned}
$$

(minus sign ?)

$\Rightarrow$



$$
\begin{aligned}
= ig^2\Big[& f^{jab} f^{jcd}\big(-g^{\mu\kappa}g^{\nu\rho} + g^{\mu\rho}g^{\nu\kappa}\big) \\
& + f^{jac} f^{jbd}\big(-g^{\mu\kappa}g^{\nu\rho} + g^{\mu\nu}g^{\rho\kappa}\big) \\
& + f^{jbc} f^{jad}\big(-g^{\mu\rho}g^{\nu\kappa} + g^{\mu\nu}g^{\rho\kappa}\big)\Big] (2\pi)^4 \delta^{(4)}(p+q+r+w)
\end{aligned}
$$