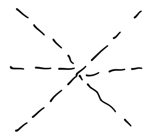
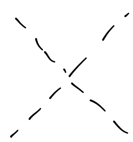
A.1 ϕ^6 : $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2 - \frac{g}{6!}\phi^6 - \frac{\lambda}{4!}\phi^4$

a)  $= \dfrac{i}{p^2 - m^2 + i\varepsilon}$

 $= -ig$

 $= -i\lambda$

b) $[\mathcal{L}] = d \Rightarrow [\phi] = \frac{d}{2} - 1$

$\Rightarrow [g] = d - 6\left(\frac{d-2}{2}\right) = 6 - 2d$

$[\lambda] = d - 4\left(\frac{d-2}{2}\right) = 4 - d$

if a coupling c, $[c] > 0 \Rightarrow$ super-renormalizable

$[c] = 0 \Rightarrow$ renormalizable

$[c] < 0 \Rightarrow$ non-renormalizable

- $d > 3$, $[g] < 0$, $[\lambda] < 1$, $\Rightarrow$ non-renormalizable
- $d = 3$, renormalizable
- $d < 3$, super renormalizable

c) $D = \underbrace{d\,L}_{\#\text{ of Loops}} - 2\underbrace{I}_{\#\text{ of internal propagators}}$ by observing the loop integral

$L = I - (V_g + V_\lambda) + 1$, $d = 3$

$\Rightarrow D = 3(I - V_g - V_\lambda + 1) - 2I$

$= I - 3(V_g + V_\lambda - 1)$

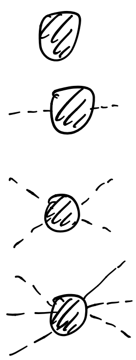
$2I + E = 6V_g + 4V_\lambda$

$I = \dfrac{6V_g + 4V_\lambda - E}{2}$

$$\Rightarrow \quad D = 3V_g + 2V_\lambda - \tfrac{1}{2}E - 3(V_g + V_\lambda - 1)$$
$$= 3 - \tfrac{1}{2}E - V_\lambda$$

d) $\mathcal{L}$ is $\phi \to -\phi$ invariant. Only even # of external legs

We will only look at diagrams with $V_\lambda = 0$, since $D \propto -V_\lambda$



$D=3$

$D=2$

$D=1$

$D=0$

e)

4-point one-loop

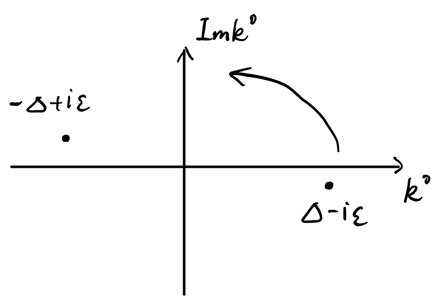
convergent

f)

$$= \frac{1}{2}(-ig)\int \frac{d^3k}{(2\pi)^3}\,\frac{i}{k^2 - m^2 + i\varepsilon}$$

symm. factor

$$k^2 - m^2 + i\varepsilon = (k^0)^2 - |\vec{k}|^2 - m^2 + i\varepsilon$$
$$= (k^0)^2 - \Delta^2 + i\varepsilon$$
$$= (k^0 - \Delta + i\varepsilon)(k^0 + \Delta - i\varepsilon)$$



with $\Delta > 0$

Without hitting the poles one could rotate the integration contour clockwise

$$k^0 \longrightarrow i k_E^0$$

$$i\mathcal{M} = -\frac{ig}{2} \int \frac{d^3 k_E}{(2\pi)^3} \frac{1}{k_E^2 + m^2}$$

introducing cutoff Λ

$$= -\frac{ig}{2} \lim_{\Lambda \to \infty} \frac{4\pi}{(2\pi)^2} \int_0^\Lambda d|\vec{k}| \frac{|\vec{k}|^2}{|\vec{k}|^2 + m^2}$$

$$= \frac{ig}{4\pi^2} \lim_{\Lambda \to \infty} \left[m \arctan\left(\frac{\Lambda}{m}\right) - \Lambda \right]$$