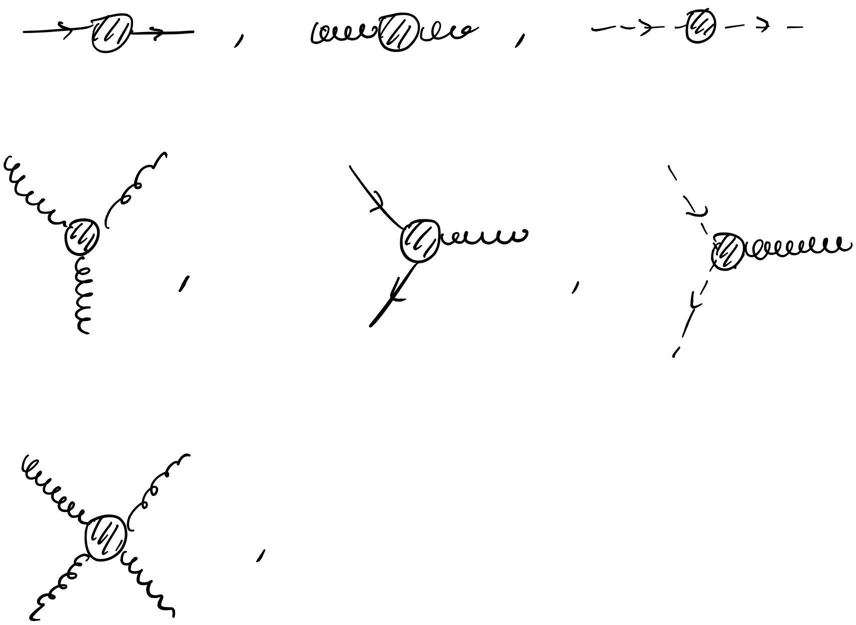
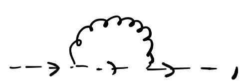
P.2

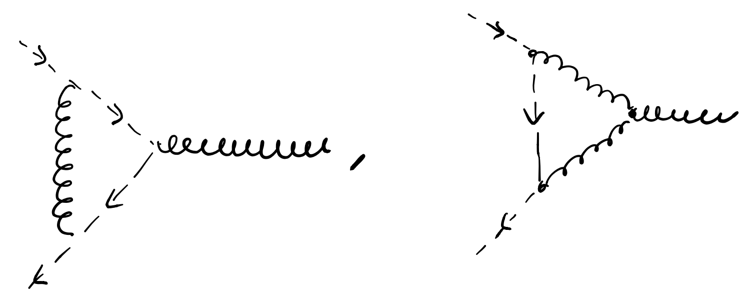
a)



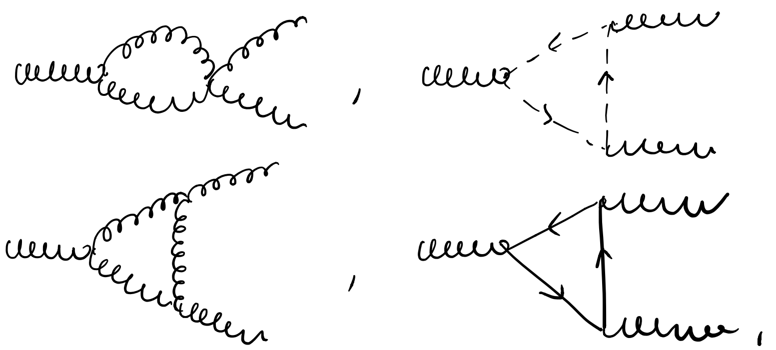
b) i)



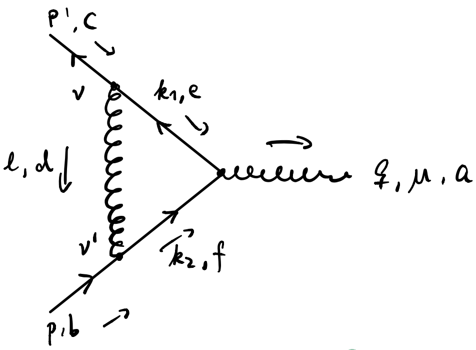
ii)



iii)



c)




$k_1 = p' - l, \quad k_2 = p + l$

$$i\mathcal{M}^\mu = \int \frac{d^4 l}{(2\pi)^4}\, ig\gamma^\nu (t^d)_{ce} \frac{i}{\not k_1 - m}\, ig\gamma^\mu (t^a)_{ef} \frac{i}{\not k_2 - m}\, ig\gamma^{\nu'} (t^d)_{fb}\, \frac{-ig_{\nu\nu'}}{l^2}$$

$$= -(i)^6 g^3 \int \frac{d^4 l}{(2\pi)^4} \gamma^\nu \frac{1}{\not k_1 - m} \gamma^\mu \frac{1}{\not k_2 - m} \gamma_\nu \frac{1}{l^2} \cdot (GTF)$$

$$(GFF) = (t^d)_{ce} (t^a)_{ef} (t^d)_{fb}$$

$$= (t^d t^a t^d)_{cb}$$

$$t^d t^a t^d = t^d t^d t^a + t^d \underbrace{[t^a, t^d]}_{= i f^{adc} t^c}$$

$$= C_2(F)\, t^a + \underbrace{i f^{adc}\, t^d t^c}$$

$$= \tfrac{i}{2} f^{adc} t^d t^c + \tfrac{i}{2} f^{adc} t^d t^c$$

$$= \tfrac{i}{2} f^{acd} t^c t^d + \tfrac{i}{2} f^{adc} t^d t^c$$

$$= \tfrac{i}{2} f^{adc} [t^d, t^c]$$

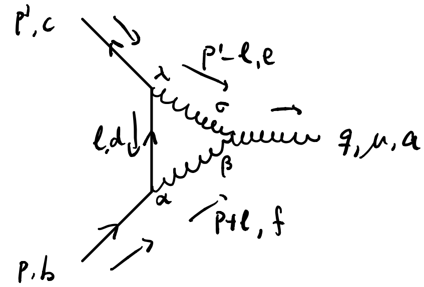
$$= \tfrac{i}{2} \underbrace{f^{adc} \cdot i f^{dce}}_{= \delta^{ae} C_2(G)} t^e$$

$$= C_2(F)\, t^a + i \cdot \tfrac{i}{2}\, C_2(G)\, t^a$$

$$= \left[-\tfrac{1}{2} C_2(G) + C_2(F)\right] t^a$$

$$\rightarrow \quad i\mathcal{M}^\mu = ig\, \Lambda^\mu_{QED}(p, q, p'; e \to g) \left[-\tfrac{1}{2} C_2(G) + C_2(F)\right] (t^a)_{cb}$$

d)




$$i\mathcal{M} = \bar{v}(p')\, ig(t^e)_{cd}\, \gamma^\lambda \frac{i}{\not{l}-m}\, ig(t^f)_{db}\gamma^\alpha u(p) \frac{-ig_{\lambda\sigma}}{(p'-l)^2}\frac{-ig_{\alpha\beta}}{(p+l)^2}$$
$$\times g f^{efa}\left[g^{\sigma\beta}(p'-l-p-l)^\mu + g^{\beta\mu}(p+l+q)^\sigma + g^{\mu\sigma}(-q-p'+l)^\beta\right] \epsilon^{*\mu}_a(q)$$

$$(GTF) = (t^e)_{cd}(t^f)_{db} f^{efa}$$
$$= (t^e t^f)_{cb} f^{efa}$$
$$= (\underbrace{t^e t^f f^{efa}}_{})_{cb}$$
$$= \frac{i}{2} f^{efg} f^{efa} t^g$$
$$= \frac{i}{2} C_2(G)(t^a)_{cb}$$

P.3 $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \bar{\Psi}(i\not{D} - m)\Psi + \frac{\xi}{2}(B^a)^2 + B^a \partial^\mu A^a_\mu - \bar{\eta}^a \partial^\mu D^{ab}_\mu \eta^b$

a) EOM w.r.t $B^a$

$$\frac{\partial \mathcal{L}}{\partial B^a} = \xi \cdot B^a + \partial^\mu A^a_\mu = 0$$

Since $B^a$ field is not physical we can plug EOM back in $\mathcal{L}$.

$$\mathcal{L}_B = \frac{\xi}{2}(B^a)^2 + B^a \partial^\mu A^a_\mu = -\frac{1}{2\xi}(\partial^\mu A^a_\mu)^2$$

b) $\delta\Psi = ig\lambda\eta^a t^a \Psi$

$$\delta(\delta\Psi) = ig\lambda\underbrace{\left[(\delta\eta^a)t^a\Psi + \eta^a t^a \delta\Psi\right]}$$

$\lambda$: Grassmann valued

$$= -\frac{g}{2}\lambda f^{abc}\eta^b\eta^c t^a\Psi + \eta^a t^a \cdot ig\lambda\eta^b t^b \Psi$$

$$= ig^2\lambda\underbrace{\left[-\frac{1}{2}\lambda f^{abc}\eta^b\eta^c t^a - i\lambda\eta^a t^a \eta^b t^b\right]}\Psi$$

$$= \lambda\left(-\frac{1}{2}\eta^b\eta^c f^{abc} t^a - i\underbrace{\eta^a\eta^b t^a t^b}\right)$$

$$= \frac{1}{2}\eta^a\eta^b[t^a, t^b]$$

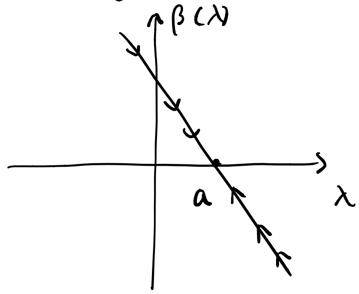
$$= \frac{1}{2}\eta^a\eta^b \, i f^{abc} t^c$$

$$= \lambda\left(\frac{1}{2}\eta^b\eta^c f^{abc} t^a + \frac{1}{2}\eta^a\eta^b f^{abc} t^c\right)$$

$= 0$

P.4

a) $\beta(\lambda) = \frac{\partial}{\partial t}\lambda = -b(\lambda - a)$ , $b > 0$ , $t = \ln\mu$




$$d\lambda = -b(\lambda - a)\, dt$$

$$\int_{\lambda_0}^{\lambda} \frac{d\lambda}{b(a-\lambda)} = \int_0^t dt'$$

$$\Rightarrow \frac{1}{b}\left[\ln|a-\lambda|\right]_{\lambda=\lambda_0}^{\lambda=\bar{\lambda}} = t$$

$$\rightarrow \ln|a-\lambda| = bt$$

$$\rightarrow \lambda = \begin{cases} a - e^{bt} , & a - \lambda \geq 0 \\ e^{-bt} + a , & \lambda - a > 0 \end{cases}$$

$$t \to \infty, \quad \lambda \to a$$

b) $\beta(\lambda) = -b(\lambda - a)^n = \frac{\partial}{\partial t}\lambda$

$$\rightarrow d\lambda = -b(\lambda - a)^n\, dt$$

$$-\frac{1}{b}\int_{\lambda_0}^{\bar{\lambda}} \frac{d\lambda}{(\lambda-a)^n} = \int_{t'=0}^{t'=t} dt'$$

$$-\frac{1}{b}\frac{1}{-n+1}\left[(\lambda-a)^{-n+1}\right]_{\lambda_0}^{\bar{\lambda}} = t$$

$$\rightarrow (\bar{\lambda}-a)^{-n+1} - (\lambda_0 - a)^{-n+1} = b(n-1)t$$

$$(\bar{\lambda}-a)^{-n+1} = b(n-1)t + (\lambda_0 - a)^{-n+1}$$

$$\bar{\lambda} = \left[b(n-1)t + (\lambda_0 - a)^{-n+1}\right]^{\frac{1}{-n+1}} + a$$