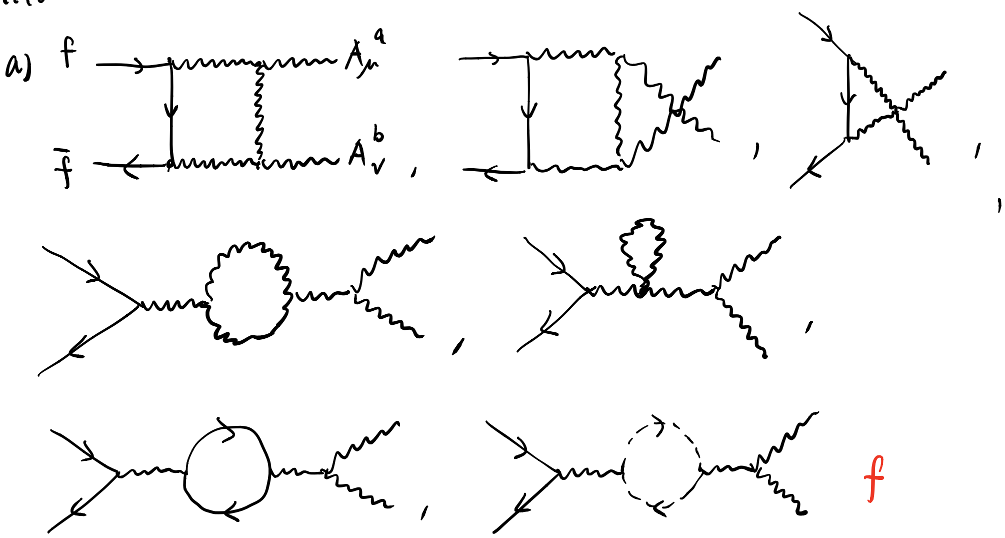
H.18

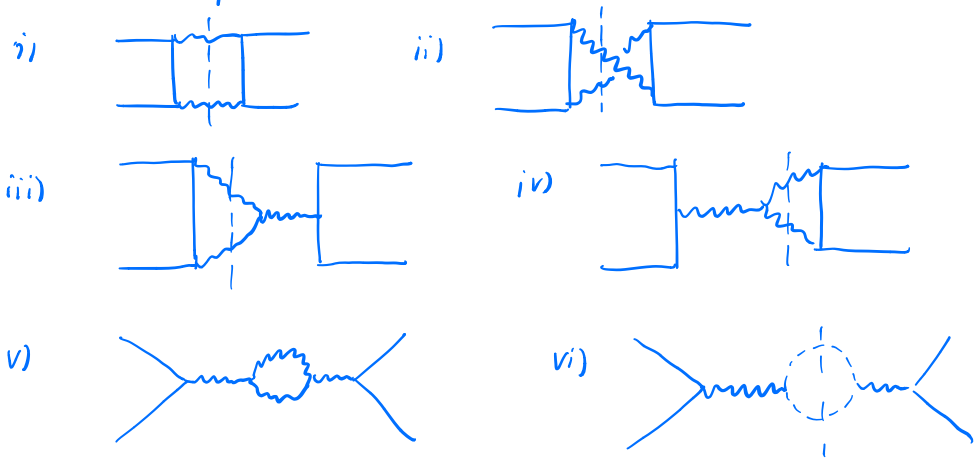
Chenhuan Wang

a)



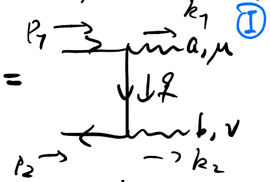
f

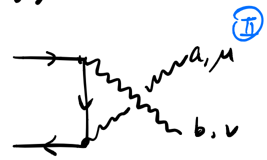
a) at one-loop order

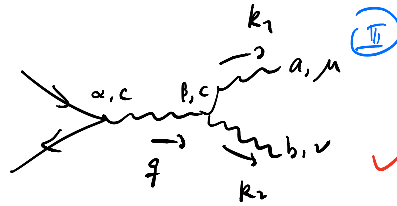


iii), iv), v), vi) are only in a non-ablian theory.

b) $iT^{ab}_{\mu\nu} = i\mathcal{M}^{ab}_{\mu\nu}(f\bar{f} \to A^a_\mu A^b_\nu)$

= 

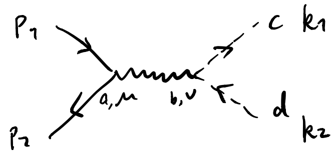
+ 

+ 

$$= \bar{v}\,\frac{ig}{2}\sigma^b\gamma_\nu\,\frac{i(\slashed{q}+m)}{q^2-m^2+i\varepsilon}\,\frac{ig}{2}\sigma^a\gamma_\mu u$$
$$+\bar{v}\,\frac{ig}{2}\sigma^a\gamma_\mu\,\frac{i(\slashed{q}+m)}{q^2-m^2+i\varepsilon}\,\frac{ig}{2}\sigma^b\gamma_\nu u$$
$$+\bar{v}\,\frac{ig}{2}\sigma^c\gamma_\alpha u\left(-i\frac{g_{\alpha\beta}}{q^2+i\varepsilon}\right)\cdot g f_{abc}\left[g^{\mu\nu}(k_2-k_1)^\beta + g^{\mu\beta}(q+k_1)^\nu + g^{\nu\beta}(-q-k_2)^\mu\right]$$

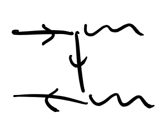
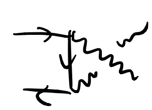
$$= -\frac{ig^2}{4}\left[\bar{v}\sigma^b\gamma^\nu\frac{(\slashed{q}+m)_{\mu\nu}}{q^2-m^2}\sigma^a\gamma^\mu u + \bar{v}\sigma^a\gamma^\mu\frac{(\slashed{q}+m)_{\mu\nu}}{q^2-m^2}\sigma^b\gamma^\nu u\right]$$
$$+\frac{g^2}{2}\bar{v}\sigma^c\gamma^\beta u\frac{1}{q^2}f_{abc}(g^{\mu\nu}(k_2-k_1)^\beta + g^{\mu\beta}(q+k_1)^\nu + g^{\nu\beta}(-q-k_2)^\mu))]$$

$$\Big(\ \bar{v}(p_2)\,ig\frac{\sigma^b}{2}\gamma_\nu\frac{i}{\slashed{p}_1-\slashed{k}-m}\,ig\frac{\sigma^a}{2}\gamma_\mu u(p_1) + \bar{v}(p_2)\,ig\frac{\sigma^a}{2}\gamma_\mu\frac{i}{\slashed{k}_1-\slashed{p}_1-m}\,ig\frac{\sigma^b}{2}\gamma_\nu u(p_1)$$
$$-g\varepsilon_{abc}[(k_1-k_2)_\lambda g_{\mu\nu} + (k_1+2k_2)_\mu g_{\nu\lambda} - (2k_1+k_2)_\nu g_{\mu\lambda}]\frac{i}{(k_1+k_2)^2}\bar{v}(p_2)\,ig\frac{\sigma^c}{2}\gamma^\lambda u(p_1)$$

iS^{cd} = 

$$= \bar{v}\,\frac{ig}{2}\sigma^a\gamma^\mu u\,(-i\delta_{ab})\left[\frac{g_{\mu\nu}}{q^2} - \underbrace{(1-\xi)\frac{q_\mu q_\nu}{q^4}}_{=0,\ \text{if } \xi=1}\right](-g)\epsilon^{cdb}k_1^\nu$$

$$= \frac{+g^2}{2q^2}\bar{v}\sigma^a\gamma^\mu u\,\epsilon^{cda}k_{1\mu}$$

in abelian case, only  , 

c) $k_1^\mu (t-\text{channel})_{\mu\nu}$

$$= k_1^\mu \bar{v} \frac{ig}{2}\sigma^b \gamma_\nu \frac{i(\not{q}+m)}{q^2-m^2+i\varepsilon} \frac{ig}{2}\sigma^a \gamma_\mu u$$

$\Big(q = p_1 - k_1$

$$\propto (\not{q}+m)(\not{p}_1 - \not{q})\, u(p_1)$$

$$= (\underbrace{\not{q}\not{p}_1}_{-\not{q}m} - \not{q}\cdot\not{q} + m\not{p}_1 - \cancel{m\not{q}})\, u(p_1)$$

$$= (-m^2 + m^2)\, u(p_1) \quad = 0 \quad = 2(p_1\cdot k_1)u(p_1)$$

$$q^2 = (p_1-k_1)^2 = \underbrace{p_1^2}_{m^2} - 2p_1\cdot k_1 - \underbrace{k_1^2}_{=0}$$

$\longrightarrow$ u-channel amplitude contracted with k^μ vanishes for the same reason.

$\Rightarrow$ $k^\mu T^{ab}_{\mu\nu}$ vanishes for abelian case

$\longrightarrow$ RHS of (2) $= \frac{1}{2}\int d\rho_2\, T^{ab}_{\mu\nu} T^{*ab}_{\mu'\nu'} \left(-g^{\mu\mu'} + \cancel{\frac{k^\mu \eta^{\mu'} + k^{\mu'}\eta^{\mu}}{k\cdot\eta}}\right) \times \left(-g^{\nu\nu'} + \cancel{\frac{k^\nu \eta^{\nu'} + k^{\nu'}\eta^{\nu}}{k\cdot\eta}}\right)$

$$= \frac{1}{2}\int d\rho_2\, T^{ab}_{\mu\nu} T^{*ab}_{\mu'\nu'}\, g^{\mu\mu'} g^{\nu\nu'}$$

$\longrightarrow$ $\int d\rho_2\, S^{ab} S^{*ab} = 0$

no ghost-contribution required! ✓

c) ii)

$$k_1^\mu T_{\mu\nu}^{ab}$$

$$= k_1^\mu T_{3\,\mu\nu}^{ab} \quad f$$

$$= k_1^\mu \frac{g^2}{2} \bar{v}\, \sigma^c \gamma^\beta u \frac{1}{q^2} \underbrace{f^{abc}}_{=\epsilon^{abc}} [g_{\mu\nu}(k_2-k_1)_\beta + g_{\mu\beta}(q+k_1)_\nu + g_{\nu\beta}(-q-k_2)_\mu]$$

$$= \frac{g^2}{2q^2} \epsilon^{abc}\, \bar{v}(p_2)\, \sigma^c \gamma^\beta u(p_1) k_1^\mu [g_{\mu\nu}(k_2-k_1)_\beta + g_{\mu\beta}(q+k_1)_\nu + g_{\nu\beta}(-q-k_2)_\mu]$$

$$k_1^\mu[--] = k_{1\nu}(k_2-k_1)_\beta + k_{1\beta}(q+k_1)_\nu + g_{\nu\beta}(\underbrace{(-q-k_2)\cdot k_1}_{\substack{=(-k_1-2k_2)\cdot k_1 \\ = -2(k_1\cdot k_2)}})]$$

$$-S^{ab} k_{2\nu} = \frac{-g^2}{2q^2} \bar{v}\, \sigma^c \gamma^\mu u\, \epsilon^{abc} k_{1\mu} k_{2\nu}$$

$$= \frac{-g^2}{2q^2} \epsilon^{abc} \bar{v}(p_2)\, \sigma^c \gamma^\mu u(p_1)\, k_{1\mu} k_{2\nu}$$

$$\rightarrow k_1^\mu T_{\mu\nu}^{ab} = -S^{ab} k_{2\nu}$$

similarly $\quad T_{\mu\nu}^{ab} k_2^\nu = -S^{ab} k_{1\mu}$

$$\Rightarrow k_1^\mu T_{\mu\nu}^{ab} k_2^\nu = -S^{ab} k_{2\nu} k_2^\nu = -S^{ab} \cdot (k_2)^2 = 0$$

c) Look at Ⅰ and Ⅱ

$$k_1^\mu T_{\mathrm{I}\,\mu\nu}^{ab} = -g^2 \bar{v}(p_2) \left\{ \frac{\sigma^b \sigma^a}{4} \gamma_\nu \frac{\overbrace{\cancel{p_1} - \cancel{k_1} + m}^{2p_1\cdot k_1}}{\underbrace{(p_1-k_1)^2 - m^2}_{-2p_1\cdot k_1}} \cancel{k_1} + \frac{\sigma^a \sigma^b}{4} \cancel{k_1} \frac{\overbrace{\cancel{p_2} - \cancel{k_1} + m}^{-2p_2\cdot k_1}}{\underbrace{(p_2-k_1)^2 - m^2}_{-2p_2\cdot k_1}} \gamma_\nu \right\} u(p_1)$$

$$= -g^2 \bar{v}(p_2) \left[\frac{\sigma^a}{2}, \frac{\sigma^b}{2} \right] \gamma_\nu u(p)$$

$$= -ig^2 \varepsilon_{abc} \bar{v}(p_2) \frac{\sigma^c}{2} \gamma_\nu u(p_1)$$

(=0 , if ablian theory)

$$k_1^\mu T_{\mathrm{III}\,\mu\nu}^{ab} = ig^2 \varepsilon^{abc} [\overset{(i)}{(k_1-k_2)_\lambda k_{1\nu}} + \overset{(ii)}{2k_1\cdot k_2\, g_{\nu\lambda}} - \overset{(iii)}{(2k_1+k_2)_\nu k_{1\lambda}}]$$

$$\times \frac{1}{(k_1+k_2)^2}\bar{v}(p_2)\frac{\sigma^c}{2}\gamma^\lambda u(p)$$

cancels out the contribution of I, II

$$\Rightarrow k_1^\mu T^{ab}_{\mu\nu} = -ig^2\varepsilon_{abc}\frac{k_{1\nu}}{(k_1+k_2)^2}\underbrace{\bar{v}(p_2)[\underbrace{\not{k}_1+\not{k}_2}_{=\not{p}_1+\not{p}_2}]\frac{\sigma^c}{2}u(p_1)}_{=0} - ig^2\varepsilon_{abc}\frac{k_{2\nu}}{(k_1+k_2)^2}\bar{v}(p_2)\not{k}\frac{\sigma^c}{2}u(p_1)$$

$$= -S^{ab}k_{2\nu}$$

Similarly $\quad T^{ab}_{\mu\nu}k_2^\nu = -S^{ab}k_{1\mu}$

$$k_1^\mu T^{ab}_{\mu\nu}k_2^\nu = -S^{ab}k_{2\nu}k_2^\nu = 0$$

d) $$\mathrm{RHS} = \frac{1}{2}\int d\rho_2\, T^{ab}_{\mu\nu} T^{*\,ab}_{\mu'\nu'}\, P^{\mu\mu'}(k_1)\, P^{\nu\nu'}(k_2)$$

$$= \frac{1}{2}\int d\rho_2\, T^{ab}_{\mu\nu} T^{*\,ab}_{\mu'\nu'} \left(-g^{\mu\mu'} + \frac{k_1^\mu \eta^{\mu'} + k_1^{\mu'}\eta^\mu}{k\cdot\eta}\right)\left(-g^{\nu\nu'} + \frac{k_2^\nu \eta^{\nu'} + k_2^{\nu'}\eta^\nu}{k\cdot\eta}\right)$$

$$= \frac{1}{2}\int d\rho_2\, T^{ab}_{\mu\nu} T^{*\,ab}_{\mu'\nu'} \Big[+ g^{\mu\mu'} g_{\nu\nu'} - g^{\mu\mu'} \frac{k_2^\nu \eta^{\nu'} + k_2^{\nu'}\eta^\nu}{k_2\cdot\eta} - g^{\nu\nu'} \frac{k_1^\mu \eta^{\mu'} + k_1^{\mu'}\eta^\mu}{k_1\cdot\eta}$$

$$+ \cancel{\frac{(k_1^\mu \eta^{\mu'} + k_1^{\mu'}\eta^\mu)(k_2^\nu \eta^{\nu'} + k_2^{\nu'}\eta^\nu)}{(k_2\cdot\eta)(k_1\cdot\eta)}} \Big]$$

combined with $T^{ab}_{\mu\nu}$ or $T^{*\,ab}_{\mu'\nu'}$

$$= \frac{1}{2}\int d\rho_2 \Big[T^{ab}_{\mu\nu} T^{*\,ab}_{\mu'\nu'} g^{\mu\mu'} g_{\nu\nu'} - \frac{g^{\mu\mu'}}{k_2\cdot\eta}(-S^{ab} k_{1\mu}) T^{*ab}_{\mu'\nu'} \eta^{\nu'} - \frac{g^{\mu\mu'}}{k_2\cdot\eta}(-S^{*ab} k_{1\mu'}) T^{ab}_{\mu\nu} \eta^{\nu}$$

$$- \frac{g^{\nu\nu'}}{k_1\cdot\eta}(-S^{ab} k_{2\nu}) T^{*\,ab}_{\mu'\nu'} \eta^{\mu'} - \frac{g^{\nu\nu'}}{k_1\cdot\eta}(-S^{*ab} k_{2\nu'}) T^{ab}_{\mu\nu} \eta^{\mu} \Big]$$

$$= \frac{1}{2}\int d\rho_2 \Big[T^{ab}_{\mu\nu} T^{*\,ab}_{\mu'\nu'} g^{\mu\mu'} g_{\nu\nu'} - \frac{1}{\cancel{k_2\cdot\eta}} S^{ab} S^{*ab} \cancel{k_{2\nu'}\eta^{\nu'}} - \frac{1}{\cancel{k_2\cdot\eta}} S^{*ab} S^{ab} \cancel{k_{2\nu}\eta^{\nu}}$$

$$- \frac{1}{\cancel{k_1\cdot\eta}} S^{ab} S^{*\,ab} \cancel{k_{1\mu'}\eta^{\mu'}} - \frac{1}{\cancel{k_1\cdot\eta}} S^{*ab} S^{ab} \cancel{k_{1\mu}\eta^{\mu}}$$

$$= \int d\rho_2 \Big[\frac{1}{2} T^{ab}_{\mu\nu} T^{*\,ab}_{\mu'\nu'} g^{\mu\mu'} g_{\nu\nu'} - S^{*ab} S^{ab} \Big] = \mathrm{LHS}$$