C 8.1

a) $\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu} \mathbb{1}$; $(\gamma^\mu)^+ = \gamma^0 \gamma^\mu \gamma^0$

$$i \gamma^\mu \partial_\mu \psi = m \frac{c}{\hbar} \psi$$

$$\xrightarrow{L} \quad i \gamma^\mu \partial_\mu' S \psi = \frac{mc}{\hbar} S \psi \qquad \text{with} \quad S^{-1} \gamma^\mu S = \Lambda^\mu{}_\nu \gamma^\nu$$

Take dagger of both side of DE:

$$(i \gamma^\mu \partial_\mu \psi)^+ = \frac{mc}{\hbar} \psi^+$$

$$-i \underbrace{\partial_\mu \psi^+}_{\psi^+ \partial_\mu} \gamma^0 \gamma^\mu \gamma^0 = \frac{mc}{\hbar} \psi^+ \quad , \qquad \psi^+ \longrightarrow \psi^+ \tilde{S}$$

$$\xrightarrow{L} \quad -i \Lambda_\mu{}^\nu \partial_\nu \psi^+ \tilde{S} \gamma^0 \gamma^\mu \gamma^0 = \frac{mc}{\hbar} \psi^+ \tilde{S}$$

$$\Rightarrow \quad \tilde{S} \gamma^0 \gamma^\mu \gamma^0 \tilde{S}^{-1} = (\Lambda^\mu{}_\nu \gamma^\nu)^+ = \Lambda^\mu{}_\nu \gamma^0 \gamma^\nu \gamma^0$$

$(\gamma^0)^{-1} = \gamma^0$ from anticommutation

$$\Rightarrow \quad \underline{\gamma^0 \tilde{S} \gamma^0} \gamma^\mu \gamma^0 \tilde{S}^{-1} \gamma^0 = \Lambda^\mu{}_\nu \gamma^\nu$$
$$= S^{-1} \gamma^\mu S$$

$$\Rightarrow \quad S^{-1} = \gamma^0 \tilde{S} \gamma^0 \qquad \Rightarrow \quad \tilde{S} = \gamma^0 S^{-1} \gamma^0$$

or $\quad S = \gamma^0 \tilde{S}^{-1} \gamma^0$

b) $\psi \xrightarrow{L} S\psi$ show that $\bar\psi = \psi^\dagger\gamma^0 \longrightarrow \bar\psi S^{-1}$

$$\overline{(i\gamma^\mu\partial_\mu\psi)} = \overline{\left(\frac{mc}{\hbar}\psi\right)} \quad , \quad \bar\psi' = \bar\psi S'$$

$$i\partial_\mu\bar\psi\,\gamma^0\gamma^\mu\gamma^0 = \frac{mc}{\hbar}\bar\psi$$

$$\xrightarrow{L} \quad i\partial'_\mu\bar\psi\,S'\gamma^0\gamma^\mu\gamma^0 = \frac{mc}{\hbar}\bar\psi S'$$

$$i\partial'_\mu\bar\psi\quad S'\gamma^0\gamma^\mu\gamma^0\,S'^{-1} = \frac{mc}{\hbar}\bar\psi$$

$$i\partial'_\mu\psi^\dagger\underbrace{\gamma^0 S'\gamma^0\gamma^\mu\gamma^0\,S'^{-1}\gamma^0}\, = \frac{mc}{\hbar}\psi^\dagger$$

$$\gamma^0 S'\gamma^0\gamma^\mu\gamma^0 S'^{-1}\gamma^0 \overset{!}{=} (\Lambda^\mu{}_\nu\gamma^\nu)^\dagger = \gamma^0 S^{-1}\gamma^\nu S\gamma^0$$

$$\Rightarrow \quad S'\gamma^0 = S^{-1}$$

$$\Rightarrow \quad S' = S^{-1}\gamma^0$$

$$S'\gamma^0\gamma^\mu\gamma^0 = \Lambda^\mu{}_\nu\,\gamma^\nu = S^{-1}\gamma^\mu S$$

$\times$

b) $\bar\psi = \psi^\dagger\gamma^0 \longrightarrow \underbrace{\psi^\dagger\gamma^0}_{\bar\psi} S^{-1}\gamma^0\gamma^0 = \bar\psi S^{-1}$

c) $\bar{\psi}\psi = \psi^{+}\gamma^{0}\psi$

$$\bar{\psi}'\psi' = \bar{\psi}\, S^{-1}\, S\psi = \bar{\psi}\psi$$

d) $\bar{\psi}'\gamma^{\mu}\psi' = \bar{\psi}\underbrace{S^{-1}\gamma^{\mu}S}\psi = \Lambda^{\mu}{}_{\nu}\,\bar{\psi}\gamma^{\nu}\psi$

e) $j^{\mu} = \bar{\psi}\gamma^{\mu}\psi$

$$\left[\left(i\gamma^{\mu}\partial_{\mu} - \frac{m\hbar}{c}\right)\psi\right]^{+} = 0$$

$$\psi^{+}\left(-i\gamma^{0}\gamma^{\mu}\gamma^{0} - \frac{m\hbar}{c}\gamma^{0}\gamma^{0}\right) = 0$$

$$\psi^{+}\left(-i\gamma^{0}\gamma^{\mu} - \frac{m\hbar}{c}\gamma^{0}\right) = 0 \qquad \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} + \frac{m\hbar}{c}\right) = 0$$

$$\partial_{\mu}J^{\mu} = \partial_{\mu}\bar{\psi}\gamma^{\mu}\psi + \bar{\psi}\gamma^{\mu}\partial_{\mu}\psi = 0$$