C9.1

a) $P_B \psi(\vec{r}_1, \vec{r}_2) = \frac{1}{2!} \sum_P \xi^P \psi(\vec{r}_{P_1}, \vec{r}_{P_2})$

$= \frac{1}{2}\left(\psi(\vec{r}_1, \vec{r}_2) + \psi(\vec{r}_2, \vec{r}_1)\right)$

$P_F \psi(\vec{r}_1, \vec{r}_2) = \frac{1}{2}\left(\psi(\vec{r}_1, \vec{r}_2) - \psi(\vec{r}_2, \vec{r}_1)\right)$

b) $P^2_{\{B \atop F\}} \psi(\vec{r}_1, \ldots, \vec{r}_N)$

$= \frac{1}{N!} \sum_P \xi^P \frac{1}{N!} \sum_{P'} \xi^{P'} \psi(\vec{r}_{P'P_1}, \ldots, \vec{r}_{P'P_N})$

$= \frac{1}{N!} \sum_P P_{Q\{B \atop F\}} \psi(\ldots) = P_{Q\{B \atop F\}} \psi(\ldots)$

Even + Even = Even	even · even = even
odd + odd = Even	odd · even = even
odd + Even = odd	odd · odd = odd

c) (1) $\longrightarrow$ $|\alpha_1 \ldots \alpha_N\} = \sqrt{N!}\, P_F |\alpha_1 \ldots \alpha_N)$

$= \frac{1}{\sqrt{N!}} \sum_P (-1)^P |\alpha_1, \ldots, \alpha_N)$

if $\alpha_i = \alpha_j = \alpha$

$\Rightarrow$ exchange α_i, α_j states.

$(-1)^{P'} = -(-1)^P$

the total wave function does not change $\Rightarrow 0$

d) Bosons:
$$\{\alpha_1' \ldots \alpha_N' \mid \alpha_1 \ldots \alpha_N\}$$
$$= N! \left(\alpha_1 \ldots \alpha_N \right| \hat{P}_B \; \hat{P}_B \left| \alpha_1, \ldots, \alpha_N \right)$$
$\uparrow$
$$P_{\{B \atop F\}}^{\dagger} = P_{\{B \atop F\}}$$
$$= \frac{1}{N!} \sum_P \langle \alpha_{P_1} | \otimes \langle \alpha_{P_2} | \otimes \cdots \otimes \langle \alpha_{P_N} | \sum_{P'} | \alpha_{P_1'} \rangle \otimes \cdots \otimes | \alpha_{P_N'} \rangle$$
$$= \frac{1}{N!} \sum_P n_{\alpha_1} \cdots n_{\alpha_{max}} = \prod_\alpha (n_\alpha !)$$

Obviously $\langle \alpha | \beta \rangle = \delta_{\alpha\beta}$ even if $\langle \alpha | \in \mathcal{H}_i$, $|\beta\rangle \in \mathcal{H}_j$, $i \neq j$?

Fermions:
$$\{\alpha_1' \ldots \alpha_N' \mid \alpha_1 \ldots \alpha_N\}$$
$$= \frac{1}{N!} \sum_P \langle \alpha_{P_1} | \otimes \cdots \otimes \langle \alpha_{P_N} | (-1)^P (-1)^{P'} | \alpha_{P_1'} \rangle \otimes \cdots \otimes | \alpha_{P_N'} \rangle$$
$$= (-1)^P \qquad n_\alpha ! \equiv 1$$

e) $$\Rightarrow |\alpha_1 \cdots \alpha_N\rangle = \frac{1}{\sqrt{\prod_\alpha n_\alpha !}} \{\alpha_1 \ldots \alpha_N\}$$

$$\Rightarrow \langle \alpha_1 \ldots \alpha_N | \alpha_1 \ldots \alpha_N \rangle = 1$$

f)