C 2.1

a) $\ell = 0$:
$$J_{0+\frac{1}{2}}(x) = J_{\frac{1}{2}}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+\frac{1}{2}+1)} \left(\frac{x}{2}\right)^{2n+\frac{1}{2}}$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \left(\sqrt{\pi}\, \frac{1\cdot 3 \cdot \dots \cdot (2n+2-1)}{2^{n+1}}\right)^{-1} \left(\frac{x}{2}\right)^{2n+\frac{1}{2}}$$
$$= \frac{1}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{2^{n+1}}{1\cdot 3\cdot 5\cdot \dots \cdot (2n+1)} \frac{x^{2n+\frac{1}{2}}}{2^{2n+\frac{1}{2}}}$$
$$= \frac{1}{\sqrt{\pi}} \cdot \sqrt{2x} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{x^{2n}}{2^n} \cdot \frac{2\cdot (n+1)!}{(2n+2)!}$$
$$= \sqrt{\frac{2x}{\pi}} \sum_{n=0}^{\infty} (-1)^n x^{2n+1} \frac{(n+1)!}{(2n+2)!\, n! \cdot 2^{n-1}} \cdot \frac{1}{x}$$
$$\hookrightarrow \frac{n+1}{(2n+2)!\, 2^{n-1}} = \frac{1}{(2n+1)!\, 2^n}$$

$\ell = 0$, $j_0(x) = \sin x / x$
$$\Gamma(n+\tfrac{3}{2}) = \frac{(2n+2)!\sqrt{\pi}}{4^{n+1}(n+1)!}$$
$$J_{1/2}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(n+\frac{3}{2})} \left(\frac{x}{2}\right)^{2n+\frac{1}{2}}$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n 4^{n+1} (n+1)!}{n!\,(2n+2)!\,\sqrt{\pi}} \left(\frac{x}{2}\right)^{2n+\frac{1}{2}}$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n 4 (n+1) \cdot x^{2n}}{(2n+2)!\,\sqrt{\pi}} \cdot \sqrt{\frac{x}{2}}$$
$$= \sqrt{\frac{x}{2\pi}}\, 4 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2\,(2n+1)!} = \sqrt{\frac{2x}{\pi}} \frac{\sin x}{x} = j_0$$

$$\Gamma(n+\tfrac{3}{2}) = \sqrt{\pi}\frac{(2n+2)!}{4^{n+1}(n+1)!} = \sqrt{\pi} \frac{(2n+2)!}{2^{2n+1}\, 2\cdot(n+1)!} = \sqrt{\pi} \frac{1\cdot 3 \cdot \dots (2n+1)}{2^{2n+1}}$$

C 2.2
$$(p-(k-i\varepsilon))(p-(-k+i\varepsilon))$$
$$= p^2 + (k-i\varepsilon)(-k+i\varepsilon) - p(-k+i\varepsilon) - p(k-i\varepsilon)$$
$$= p^2 - k^2 + \underline{2i\varepsilon k} + \cancel{\varepsilon^2} + \cancel{pk} - \cancel{ip\varepsilon} - \cancel{pk} + \cancel{ip\varepsilon}$$

$p = k - i\varepsilon = \alpha_1$ $\qquad (p^2 - k^2 - i\varepsilon) = (p-(k-i\varepsilon))(p-(-k+i\varepsilon))$

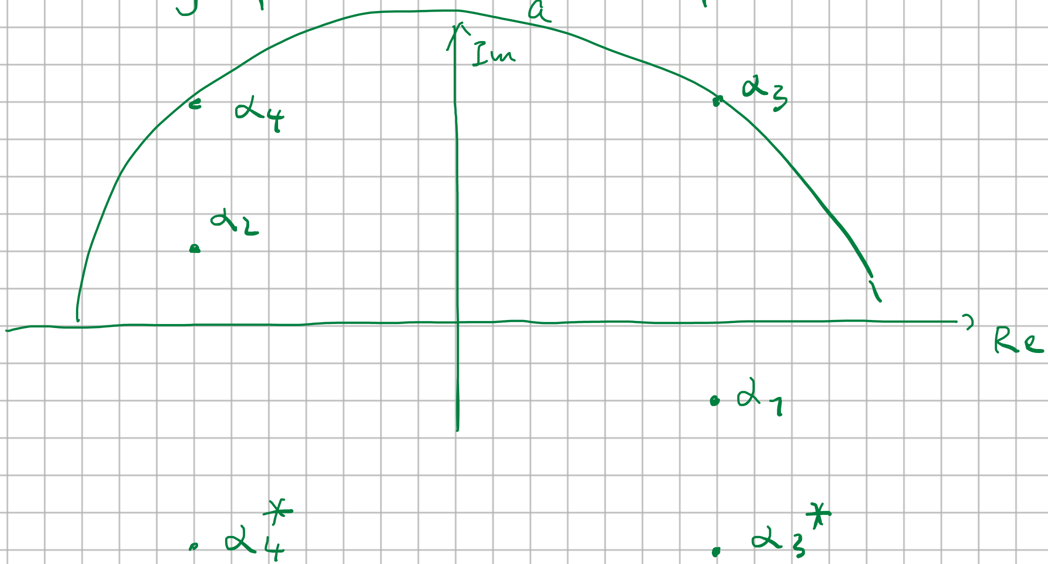
$p = -k + i\varepsilon = \alpha_2$ $\qquad (p-k)^2 + \mu^2 = (p-k+i\mu)(p-k-i\mu)$

$p = k \pm i\mu = \alpha_3, \alpha_3^*$ $\qquad (p+k)^2 + \mu^2 = (p+k+i\mu)(p+k+i\mu)$

$p = -k \pm i\mu = \alpha_4, \alpha_4^*$

Residue theorem:

$$\oint dz f(z) = 2\pi i \sum_a \mathrm{Res}(f, a)$$



Integrate over the positive half, Res: $\oint dz \frac{f(z)}{z-a} = 2\pi i$

1) $$\frac{\alpha_2^2}{(\alpha_2-\alpha_1)(\alpha_2-\alpha_3)(\alpha_2-\alpha_3^*)(\alpha_2-\alpha_1)(\alpha_2-\alpha_4^*)}$$

$$\varepsilon \to 0 \quad = \frac{(-k+i\varepsilon)^2}{(-2k+2i\varepsilon)(-2k-i\mu)(-2k+i\mu)(-i\mu)(+i\mu)} = \frac{k}{-2\mu^2(4k^2+\mu^2)}$$

2) $$\left.\frac{p^2}{(p-\alpha_1)(p-\alpha_2)(p-\alpha_3^*)(p-\alpha_4)(p-\alpha_1)}\right|_{p=\alpha_3} = \frac{k+i\mu}{8\mu^2 k(-2k-i\mu)}$$

$$\Rightarrow \oint dz f(z) = \frac{-\pi i}{2\mu^2(2k-i\mu)}$$