C3.1

a) $$\int_0^\infty d\theta \cdot \theta J_0(x\theta) J_0(x'\theta) = \frac{1}{x}\delta(x-x')$$

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \int d\Omega \, |f(\vec{k}', \vec{k})|^2$$

$$= \int d\Omega \cdot (-ik)^* \int_0^\infty db\, b\, J_0(kb\theta)[e^{2i\Delta(b)} - 1]^* \cdot (-ik) \cdot$$
$$\int_0^\infty db' b' J_0(kb'\theta)[e^{2i\Delta(b')} - 1] \quad , \quad \Delta(b) \in \mathbb{R}$$

$$= 2\pi k^2 \int d\theta \sin\theta \int_0^\infty db\, b\, J_0(kb\theta)[e^{-2i\Delta(b)} - 1] \int_0^\infty db' b' J_0(kb'\theta)[e^{2i\Delta(b')} - 1]$$

$$= 2\pi k^2 \int_0^\infty db\, b [e^{-2i\Delta(b)} - 1] \int_0^\infty db' b' [e^{2i\Delta(b)} - 1] \int d\theta \underbrace{\sin\theta}_{\simeq \theta} J_0(kb\theta) J_0(kb'\theta)$$

$$\xi = k\theta$$

why ∞

$$\int_0^\pi d\theta\, \theta\, J_0(kb\theta) J_0(kb'\theta)$$
$$= \frac{1}{k^2} \int_0^\infty d\xi\, \xi\, J_0(\xi b) J_0(\xi b')$$
$$= \frac{1}{k^2} \cdot \frac{1}{b} \delta(b-b')$$

since we assume the trajectory remains a straight line

$$= 2\pi k^2 \cdot \frac{1}{k^2} \int_0^\infty db [e^{-2i\Delta(b)} - 1] \int_0^\infty db' b' [e^{2i\Delta(b')} - 1] \cdot \frac{1}{b}\delta(b-b')$$

$$= 2\pi \int_0^\infty db\, b [e^{-2i\Delta(b)} - 1][e^{2i\Delta(b)} - 1]$$

$$= 2\pi \int_0^\infty db\, b\, (2 - 2\cos(2\Delta(b)))$$

$$1 - \cos 2x = 1 - (1 - 2\sin^2 x) = 2\sin^2 x$$

$$= 8\pi \int_0^\infty db\, b \sin^2 \Delta(b)$$

b) $$\mathrm{Im}[f(\vec{k}, \vec{k})]\big|_{\theta=0} = \mathrm{Im}[-ik \int_0^\infty db\, b\, J_0(kb\theta)[e^{2i\Delta(b)} - 1]]\big|_{\theta=0}$$
$$= \mathrm{Im}\{-k \int_0^\infty db\, b\, (i\cos(2\Delta(b)) - \sin(2\Delta(b)) - i)\}$$
$$= -k \int_0^\infty db\, b\, (\cos(2\Delta(b)) - 1)$$

$$= 2k \int_0^{\infty} db\ b\ \sin^2(\Delta(b))$$

c) $\text{Im}(f(\vec{k}', \vec{k}, \theta = 0)) = k\sigma / 4\pi$