worked on 5/8

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1) $$\Phi_i(x,\theta,\bar\theta) = \phi_i(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\phi_i(x) - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\partial^\mu\partial_\mu\phi_i(x) + \sqrt{2}\theta\xi_i(x) + \tfrac{i}{\sqrt{2}}\theta\theta\partial_\mu\xi_i(x)\sigma^\mu\bar\theta + \theta\theta F_i(x)$$

$\Phi_i^\dagger\Phi_k \supset$? (sloppy notation: $\phi^\dagger=\phi^*$, $\xi^\dagger=\bar\xi$)

①: $$\phi_i^\dagger\Phi_k = \phi_i^\dagger\phi_k - i\theta\sigma^\mu\bar\theta\phi_i^\dagger\partial_\mu\phi_k - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\phi_i^\dagger\partial^\mu\partial_\mu\phi_k + \sqrt{2}\theta\xi_k\phi_i^\dagger + \underbrace{\tfrac{i}{\sqrt{2}}\theta\theta\partial_\mu\xi_k\sigma^\mu\bar\theta}\phi_i^\dagger + \theta\theta\phi_i^\dagger F_k$$
$$= -\theta\theta\bar\theta\bar\sigma^\mu\partial_\mu\xi_k$$

missing Terms, e.g.: $(\sqrt{2}\theta\xi_i)^\dagger(\tfrac{i}{\sqrt{2}}\theta\theta\partial_\mu\xi\sigma^\mu\bar\theta)$ is of $\bar\theta\bar\theta\theta\theta$ type, right? ✓ oh; but you've included them further below.

②: $$(-i\theta\sigma^\mu\bar\theta\partial_\mu\phi_i)^\dagger\Phi_k = i\theta\sigma^\mu\bar\theta\partial_\mu\phi_i^\dagger\Phi_k$$
$$= i\theta\sigma^\mu\bar\theta\partial_\mu\phi_i^\dagger\phi_k + \underbrace{\theta\sigma^\mu\bar\theta\cdot\theta\sigma^\nu\bar\theta}_{\overset{(2.7k)}{=}\frac{1}{2}g^{\mu\nu}\theta\theta\bar\theta\bar\theta}\partial_\mu\phi_i^\dagger\partial_\nu\phi_k$$
$$-\tfrac{i}{4}\underbrace{\theta\sigma^\mu\bar\theta\,\theta\theta\bar\theta\bar\theta}_{=0,\text{ since }\theta\theta\bar\theta\bar\theta=2g_{\alpha\beta}\theta\sigma^\alpha\bar\theta\,\theta\sigma^\beta\bar\theta}\partial_\mu\phi_i^\dagger\partial^\nu\partial_\nu\phi_k + i\sqrt{2}\underbrace{\theta\sigma^\mu\bar\theta\,\theta\xi_k}_{=\theta^A(\sigma^\mu\bar\theta)_A\theta\xi_k=-\frac{1}{2}\xi_k\sigma^\mu\bar\theta\,\theta\theta=\frac{1}{2}\theta\theta\bar\theta\bar\sigma^\mu\xi_k}\partial_\mu\phi_i^\dagger$$
$$-\tfrac{1}{\sqrt{2}}\underbrace{\theta\sigma^\mu\bar\theta\,\theta\theta\partial_\mu\bar\xi_i\sigma^\nu\bar\theta}_{=0}\partial_\nu\phi_k + i\underbrace{\theta\sigma^\mu\bar\theta\,\theta\theta}_{\sim\theta^3=0}\partial_\mu\phi_i^\dagger F_k$$

③: $$(-\tfrac{1}{4}\theta\theta\bar\theta\bar\theta\partial_\mu\partial^\mu\phi_i^\dagger)\Phi_k$$
$$= -\tfrac{1}{4}\theta\theta\bar\theta\bar\theta\partial^2\phi_i^\dagger\phi_k + \tfrac{i}{4}\underbrace{\theta\theta\bar\theta\bar\theta\,\theta\sigma^\mu\bar\theta}_{=0}\partial^2\phi_i^\dagger\partial_\mu\phi_k + \tfrac{1}{16}\underbrace{(\theta\theta\bar\theta\bar\theta)^2}_{=0}\partial^2\phi_i^\dagger\partial^2\phi_k$$
$$-\tfrac{\sqrt{2}}{4}\underbrace{\theta\theta\bar\theta\bar\theta\,\theta\xi_k}_{=0}\partial^2\phi_i^\dagger - \tfrac{i}{4\sqrt{2}}\underbrace{\theta\theta\bar\theta\bar\theta\,\theta\theta}_{=0}\partial_\mu\xi_k\sigma^\mu\bar\theta\partial^2\phi_i^\dagger - \tfrac{1}{4}\underbrace{\theta\theta\bar\theta\bar\theta\,\theta\theta}_{=0}\partial^2\phi_i^\dagger F_k$$

④: $(\sqrt{2}\,\bar\theta\xi_i^\dagger)\Phi_k$

$$= \sqrt{2}\,\bar\theta\xi_i^\dagger\phi_k - i\sqrt{2}\,\underbrace{\bar\theta\xi_i^\dagger\theta\sigma^\mu\bar\theta}_{= -\bar\theta\xi_i^\dagger\bar\theta(\bar\sigma^\mu\theta) = \frac{1}{2}\xi_i^\dagger\bar\sigma^\mu\theta\bar\theta\bar\theta = -\frac{1}{2}\bar\theta\bar\theta\theta\sigma^\mu\xi_i^\dagger}\partial_\mu\phi_k - \frac{\sqrt{2}}{4}\overbrace{\bar\theta\xi_i^\dagger\theta\theta\bar\theta\bar\theta}^{=0}\partial^2\phi_k + 2\bar\theta\xi_i^\dagger\theta\xi_k$$

$$+\underbrace{i\bar\theta\xi_i^\dagger\theta\theta\partial_\mu\xi_k\sigma^\mu\bar\theta}_{= i\theta\theta\bar\theta\xi_i^\dagger\partial_\mu\xi_k\sigma^\mu\bar\theta} + \underbrace{\sqrt{2}\,\bar\theta\xi_i^\dagger\theta\theta F_k}_{= \sqrt{2}\,\theta\theta\bar\theta\xi_i^\dagger F_k}$$

$$= i\theta\theta\,\bar\theta\xi_i^\dagger(\partial_\mu\xi_k\sigma^\mu)_{\dot A}\bar\theta^{\dot A}$$

$$= i\theta\theta\,\bar\theta\xi_i^\dagger\,\bar\theta_{\dot A}(\partial_\mu\xi_k\sigma^\mu)^{\dot A}$$

$$= -\frac{i}{2}\theta\theta\,\xi_i^\dagger\partial_\mu\xi_k\sigma^\mu\,\bar\theta\bar\theta$$

$$= -\frac{i}{2}\theta\theta\bar\theta\bar\theta\,\partial_\mu\xi_k\sigma^\mu\xi_i^\dagger$$

⑤: $\left(\frac{-i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu\partial_\mu\xi_i^\dagger\right)\Phi_k$

$$= -\frac{i}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\xi_i^\dagger\phi_k \underbrace{- i\bar\theta\bar\theta\theta\sigma^\mu\partial_\mu\xi_i^\dagger\theta\xi_k}_{} + \left(\begin{array}{l}\text{terms with higher power of } (\theta,\bar\theta)\\ \text{will vanish eventually}\end{array}\right)$$

$$= -i\bar\theta\bar\theta\,\theta^A(\sigma^\mu\partial_\mu\xi_i^\dagger)_A\theta\xi_k$$

$$= \frac{i}{2}\bar\theta\bar\theta\,\xi_k\sigma^\mu\partial_\mu\xi_i^\dagger\,\theta\theta$$

$$= \frac{i}{2}\theta\theta\bar\theta\bar\theta\,\xi_k\sigma^\mu\partial_\mu\xi_i^\dagger$$

⑥: $(\bar\theta\bar\theta F_i^\dagger)\Phi_k$

$$= \bar\theta\bar\theta F_i^\dagger\phi_k + \sqrt{2}\,\bar\theta\bar\theta\theta\xi_k F_i^\dagger + \theta\theta\bar\theta\bar\theta F_i^\dagger F_k \quad (+ \longrightarrow 0)$$

$$\theta\theta(\theta)(\sigma^\mu\bar\theta)$$

$$= -\frac{1}{2}(\theta\sigma^\mu\bar\theta)\theta\theta$$

$$= -\frac{1}{2}\theta\theta\,\theta\sigma^\mu\bar\theta = 0$$

Together

$\Phi_i^\dagger \Phi_k$

$$= \phi_i^\dagger \phi_k - i\theta\sigma^\mu\bar\theta\, \phi_i^\dagger \partial_\mu \phi_k - \frac{1}{4}\theta\theta\bar\theta\bar\theta\, \phi_i^\dagger \partial^\mu \partial_\mu \phi_k + \sqrt{2}\,\theta\xi_k \phi_i^\dagger$$

$$- \frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu \partial_\mu \xi_k \phi_i^\dagger + \theta\theta\phi_i^\dagger F_k$$

$$+ i\theta\sigma^\mu\bar\theta\, \partial_\mu \phi_i^\dagger \phi_k + \frac{1}{2}\theta\theta\bar\theta\bar\theta\, \partial_\mu \phi_i^\dagger \partial^\mu \phi_k + \frac{i}{\sqrt{2}}\theta\theta\bar\theta\bar\sigma^\mu \xi_k \partial_\mu \phi_i^\dagger$$

$$- \frac{1}{4}\theta\theta\bar\theta\bar\theta\, \partial^2 \phi_i^\dagger \phi_k$$

$$+ \sqrt{2}\,\bar\theta\xi_i^\dagger \phi_k + \frac{i}{\sqrt{2}}\bar\theta\bar\theta\theta\sigma^\mu \xi_i^\dagger \partial_\mu \phi_k + 2\bar\theta\xi_i^\dagger \theta\xi_k - \frac{i}{2}\theta\theta\bar\theta\bar\theta\, \partial_\mu \xi_k \sigma^\mu \xi_i^\dagger + \sqrt{2}\,\theta\theta\bar\theta\,\xi_i^\dagger F_k$$

$$- \frac{i}{\sqrt{2}}\bar\theta\bar\theta\,\theta\sigma^\mu \partial_\mu \xi_i^\dagger \phi_k + \frac{i}{2}\theta\theta\bar\theta\bar\theta\, \xi_k\sigma^\mu \partial_\mu \xi_i^\dagger$$

$$+ \bar\theta\bar\theta F_i^\dagger \phi_k + \sqrt{2}\,\bar\theta\bar\theta\theta\xi_k F_i^\dagger + \theta\theta\bar\theta\bar\theta F_i^\dagger F_k$$

$$= \phi_i^* \phi_k + 2i\theta\sigma^\mu\bar\theta\ \phi_i^\dagger [\partial_\mu] \phi_k + \theta\theta\bar\theta\bar\theta\Big[+\frac{1}{2}\partial_\mu\phi_i^*\partial^\mu\phi_k - \frac{1}{4}(\phi_i^*\partial^2\phi_k + \partial^2\phi_i^*\phi_k)$$

$$+ i\,\xi_k\sigma^\mu[\partial_\mu]\bar\xi_i + F_i^* F_k\Big]$$

$$+ \sqrt{2}\,\theta\xi_k\phi_i^* + \sqrt{2}\,\theta\theta\bar\theta\big(\bar\xi_i F_k + \bar\sigma^\mu \xi_k[\partial_\mu]\phi_i^\dagger\big) + \theta\theta\phi_i^* F_k + \sqrt{2}\,\bar\theta\bar\xi_i\phi_k$$

$$+ \sqrt{2}\,\bar\theta\bar\theta\theta\big(\xi_k F_i^* + \sigma^\mu\bar\xi_i[\partial_\mu]\phi_k\big) + 2\bar\theta\bar\xi_i\theta\xi_k + \bar\theta\bar\theta F_i^*\phi_k$$

somehow this corresponds to $\partial_\mu\phi_k^*\partial^\mu\phi_i$

otherwise $\checkmark \to$ (3.119)

? By part integration?

2) 1. $V_{WZ}(x,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta A_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x)$

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta \iff x^\mu = y^\mu + i\theta\sigma^\mu\bar\theta$$

$$\bar y^\mu = x^\mu + i\theta\sigma^\mu\bar\theta \iff x^\mu = \bar y^\mu - i\theta\sigma^\mu\bar\theta$$

$$A_\mu(x) = A_\mu(y) + \partial_\nu^{(y)} A_\mu(x)\Big|_{x=y}(x-y)^\nu + \mathcal{O}((y-x)^2)$$

$$\simeq A_\mu(y) + \partial_\nu^{(y)} A_\mu(y)\cdot i\theta\sigma^\nu\bar\theta$$

$$= A_\mu(\bar y) - \partial_\nu^{(\bar y)} A_\mu(\bar y)\cdot i\theta\sigma^\nu\bar\theta$$

$$\Rightarrow V_{WZ}(y,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta\left(A_\mu(y) + \partial_\nu^{(y)}A_\mu(y)\cdot i\theta\sigma^\nu\bar\theta\right) + \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\lambda(y) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(y)$$

additional terms out of expansion contain higher power of $\theta,\bar\theta$
$\to 0$ ✓

$$= \theta\sigma^\mu\bar\theta A_\mu(y) + \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\lambda(y) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(y) + i\underbrace{\theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\theta}_{=\frac{1}{2}g^{\mu\nu}\theta\theta\bar\theta\bar\theta}\partial_\nu^{(y)}A_\mu(y)$$

✓

$$\Rightarrow V_{WZ}(y,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta A_\mu(y) + \theta\theta\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\theta\lambda(y) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left(D(y) + i\partial_\mu^{(y)}A^\mu(y)\right)$$

No "i" from lecture...

↳ I got the same; will look into it!

same for $\bar y$: $$V_{WZ}(\bar y,\theta,\bar\theta) = \theta\sigma^\mu\bar\theta A_\mu(\bar y) + \theta\theta\bar\theta\bar\lambda(\bar y) + \bar\theta\bar\theta\theta\lambda(\bar y) + \frac{1}{2}\theta\theta\bar\theta\bar\theta\left(D(\bar y) - i\partial_\mu^{(\bar y)}A^\mu(\bar y)\right)$$

✓

2、 $D_A^{(y)} = \partial_A - 2i\sigma^\mu_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_\mu^{(y)}$

$D_A^{(y)} V_{WZ}(y,\theta,\bar{\theta})$

$= (\partial_A - 2i\sigma^\mu_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_\mu^{(y)})[\theta\sigma^\mu\bar{\theta}A_\mu(y) + \theta\theta\bar{\theta}\bar{\lambda}(y) + \bar{\theta}\bar{\theta}\theta\lambda(y)$
$+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D(y) + i\partial_\mu^{(y)}A^\mu(y))]$

$=$ ① $+$ ②

① $= \partial_A(\theta\sigma^\mu\bar{\theta})A_\mu(y) + \partial_A(\theta\theta)\bar{\theta}\bar{\lambda}(y) + \bar{\theta}\bar{\theta}\,\partial_A(\theta\lambda(y))$
$+ \frac{1}{2}\partial_A(\theta\theta)\bar{\theta}\bar{\theta}(D(y) + i\partial_\mu^{(y)}A^\mu(y))$

$$\left\{\begin{aligned}
\partial_A(\theta\sigma^\mu\bar{\theta}) &= \partial_A\theta^B\sigma^\mu_{B\dot{C}}\bar{\theta}^{\dot{C}} \overset{(I16a)}{=} \delta_A^B\sigma^\mu_{B\dot{C}}\bar{\theta}^{\dot{C}} = \sigma^\mu_{A\dot{C}}\bar{\theta}^{\dot{C}} \\
\partial_A(\theta\theta) &= \partial_A(\theta^B\theta_B) = \partial_A\theta^B\cdot\theta_B + \theta^B\cdot\partial_A\theta_B \\
&= \delta_A^B\theta_B - \theta^B(-\epsilon_{AB}) \\
&= \theta_A + \epsilon_{AB}\theta^B \\
&= 2\theta_A \quad ✓\\
\partial_A(\theta\lambda(y)) &= \partial_A(\theta^B\lambda_B(y)) = \delta_A^B\lambda_B(y) = \lambda_A(y)
\end{aligned}\right.$$

$= (\sigma^\mu\bar{\theta})_A A_\mu(y) + 2\theta_A\bar{\theta}\bar{\lambda}(y) + \bar{\theta}\bar{\theta}\lambda_A(y) + \theta_A\bar{\theta}\bar{\theta}(D(y) + i\partial_\mu^{(y)}A^\mu(y))$ ✓

② $= -2i\sigma^\mu_{A\dot{B}}\bar{\theta}^{\dot{B}}\partial_\mu^{(y)}[\theta\sigma^\nu\bar{\theta}A_\nu(y) + \theta\theta\bar{\theta}\bar{\lambda}(y) + \bar{\theta}\bar{\theta}\theta\lambda(y)$
$+ \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}(D(y) + i\partial_\nu^{(y)}A^\nu(y))]$

$$= -2i\Big[\underbrace{(\sigma^\mu\bar\theta)_A\,\theta\sigma^\nu\bar\theta}_{=\bar\theta\bar\theta[\frac{1}{2}g^{\mu\nu}\theta_A - i(\sigma^{\mu\nu}\theta)_A]}\,\partial_\mu^{(y)}A_\nu(y) + \underbrace{(\sigma^\mu\bar\theta)_A\,\theta\theta\bar\theta\,\partial_\mu^{(y)}\bar\lambda(y)}\Big]$$

$$= \theta\theta\,\sigma^\mu_{A\dot B}\,\bar\theta^{\dot B}\,\underbrace{\bar\theta\,\partial_\mu^{(y)}\bar\lambda(y)}$$

$$= \bar\theta^{\dot B}\,\bar\theta_{\dot C}\left[\partial_\mu^{(y)}\bar\lambda(y)\right]^{\dot C}$$

$$= \epsilon_{\dot C\dot D}\,\bar\theta^{\dot B}\,\bar\theta^{\dot D}\left[\partial_\mu^{(y)}\bar\lambda(y)\right]^{\dot C}$$

$$= \epsilon_{\dot C\dot D}\,\tfrac{1}{2}\epsilon^{\dot B\dot D}\,\bar\theta\bar\theta\left[\partial_\mu^{(y)}\bar\lambda(y)\right]^{\dot C}$$

Minus from order of dotted indices

$$= -\tfrac{1}{2}\,\delta^{\dot B}_{\dot C}\,\bar\theta\bar\theta\left[\partial_\mu^{(y)}\bar\lambda\right]^{\dot C}$$

$$= -\tfrac{1}{2}\theta\theta\bar\theta\bar\theta\left[\sigma^\mu\partial_\mu^{(y)}\bar\lambda(y)\right]_A$$

$$\Rightarrow D_A^{(y)}V_{WZ}(y,\theta,\bar\theta)$$

$$= (\sigma^\mu\bar\theta)_A A_\mu(y) + 2\theta_A\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\lambda_A(y) + \bar\theta\bar\theta\theta_B\delta^B_A D(y)$$

$$+ \cancel{i\theta_A\bar\theta\bar\theta\,\partial^{\mu(y)}_{\mu}A_\mu(y)} - \cancel{i\bar\theta\bar\theta\theta_A\,\partial_\mu^{(y)}A_\mu(y)} - \bar\theta\bar\theta\cdot\underbrace{2(\sigma^{\mu\nu}\theta)_A\,\partial_\mu^{(y)}A_\nu(y)}$$

$$+ i\theta\theta\bar\theta\bar\theta\left[\sigma^\mu\partial_\mu^{(y)}\bar\lambda(y)\right]_A$$

$$= (\sigma^{\mu\nu}\theta)_A\,\partial_\mu^{(y)}A_\nu(y) - (\sigma^{\nu\mu}\theta)_A\,\partial_\mu^{(y)}A_\nu(y)$$

$$= (\sigma^{\mu\nu}\theta)_A\left(\partial_\mu^{(y)}A_\nu(y) - \partial_\nu^{(y)}A_\mu(y)\right)$$

$$= (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y)$$

$$= (\sigma^\mu\bar\theta)_A A_\mu(y) + 2\theta_A\bar\theta\bar\lambda(y) + \bar\theta\bar\theta\lambda_A(y) + \bar\theta\bar\theta\left(\theta_A D(y) - (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y)\right)$$

$$+ i\theta\theta\bar\theta\bar\theta\left[\sigma^\mu\partial_\mu^{(y)}\bar\lambda(y)\right]_A$$

Similar calculation for $D_{\dot A}^{(\bar y)}V_{WZ}(\bar y,\theta,\bar\theta)$

$(\circledast)$ I think you're mixing things up here; if working in (y), then $\bar{D}^{(y)}_{\dot{A}} = -\bar{\partial}_{\dot{A}}$ and there is no extra term of ∂_μ!

3. $$\bar{D}\bar{D} = \bar{D}^{(y)}_{\dot{A}}\bar{D}^{(y)\dot{A}} = (-\bar{\partial}_{\dot{A}} + \cancel{i\theta^B\sigma^\mu_{B\dot{A}}\partial^{(y)}_\mu})\epsilon^{\dot{A}\dot{B}}(-\bar{\partial}_{\dot{B}} + \cancel{i\theta^C\sigma^\nu_{C\dot{B}}\partial^{(y)}_\nu})$$

$$(\circledast)? \ominus = \epsilon^{\dot{A}\dot{B}}\partial_{\dot{A}}\partial_{\dot{B}} - \epsilon^{\dot{A}\dot{B}}\theta^B\sigma^\mu_{B\dot{A}}\partial^{(y)}_\mu\theta^C\sigma^\nu_{C\dot{B}}\partial^{(y)}_\nu$$

$$= -\bar{\partial}_{\dot{A}}\bar{\partial}^{\dot{A}} - \epsilon^{\dot{A}\dot{B}}(\theta\sigma^\mu)_{\dot{A}}(\theta\sigma^\nu)_{\dot{B}}\partial^{(y)}_\mu\partial^{(y)}_\nu$$

$$= -\bar{\partial}_{\dot{A}}\bar{\partial}^{\dot{A}} - \cancel{(\theta\sigma^\mu)_{\dot{A}}(\theta\sigma^\nu)^{\dot{A}}\partial^{(y)}_\mu\partial^{(y)}_\nu}$$

$D_A V$ is to be understood as $D^{(y)}_A V_{WZ}(y,\theta,\bar{\theta})$, i.e. result from last part

$$\Rightarrow -4W_A = [-\bar{\partial}\bar{\partial} - (\theta\sigma^\mu)_{\dot{A}}(\theta\sigma^\nu)^{\dot{A}}\partial^{(y)}_\mu\partial^{(y)}_\nu]D^{(y)}_A V_{WZ}(y,\theta,\bar{\theta})$$

$$\left[\ \bar{\partial}\bar{\partial}(\bar{\theta}\bar{\theta}) = \bar{\partial}_{\dot{A}}\bar{\partial}^{\dot{A}}\bar{\theta}_{\dot{B}}\bar{\theta}^{\dot{B}} = \bar{\partial}_{\dot{A}}\delta^{\dot{A}}_{\dot{B}}\bar{\theta}^{\dot{B}} = \delta^{\dot{A}}_{\dot{B}}\delta^{\dot{B}}_{\dot{A}} = 4\right.$$

$$-4W_A = -4\left(\lambda_A(y) + \theta_A D(y) - (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y) + i\theta\theta[\sigma^\mu\partial^{(y)}_\mu\bar{\lambda}(y)]_A\right)$$

(+ $\partial^{(y)}_\mu\partial^{(y)}_\nu$ terms for some reason vanish) $\to \circledast$

$$\Rightarrow W_A = \lambda_A(y) + \theta_A D(y) - (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y) + i\theta\theta[\sigma^\mu\partial^{(y)}_\mu\bar{\lambda}(y)]_A$$

$$\bar{W}_{\dot{A}} = \bar{\lambda}_{\dot{A}}(\bar{y}) + \bar{\theta}_{\dot{A}}D(\bar{y}) - (\bar{\sigma}^{\mu\nu}\bar{\theta})_{\dot{A}}F_{\mu\nu}(\bar{y}) - i\bar{\theta}\bar{\theta}[\sigma^\mu\partial^{(\bar{y})}_\mu\lambda(\bar{y})]_{\dot{A}}$$

4. $$W^A W_A = \epsilon^{AB}W_B W_A + \cdots$$

$$= \epsilon^{AB}(\theta_B\theta_A D^2(y) + \underbrace{\theta_B D(y)(\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y) + (A\leftrightarrow B)}_{\to 0}$$

$$+ i\theta\theta\lambda_B[\sigma^\mu\partial^{(y)}_\mu\bar{\lambda}(y)]_A + (A\leftrightarrow B)$$

$$+ (\sigma^{\mu\nu}\theta)_B F_{\mu\nu}(\sigma^{\alpha\beta}\theta)_A F_{\alpha\beta}) + \cdots$$

3. $\bar{D}\bar{D} = \bar{D}^{(y)}_{\dot{A}}\, \bar{D}^{(y)\dot{A}} = \bar{\partial}_{\dot{A}}\, \bar{\partial}^{\dot{A}}$

$$\Rightarrow \quad W_A = -\frac{1}{4}\, \bar{\partial}_{\dot{A}}\, \bar{\partial}^{\dot{A}}\, D^{(y)}_A\, V_{WZ}(y,\theta,\bar{\theta})$$

$$= -\frac{1}{4}\bar{\partial}\bar{\partial}\Big[\bar{\theta}\bar{\theta}\lambda_A(y) + \bar{\theta}\bar{\theta}\big(\theta_A D(y) - (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y)\big) + i\theta\theta\bar{\theta}\bar{\theta}\,[\sigma^{\mu}\partial^{(y)}_{\mu}\bar{\lambda}(y)]_A\Big]$$

$$\left(\begin{aligned} \bar{\partial}\bar{\partial}(\bar{\theta}\bar{\theta}) &= \bar{\partial}_{\dot{A}}\big[\bar{\partial}^{\dot{A}}\,\bar{\theta}_{\dot{B}}\,\bar{\theta}^{\dot{B}}\big] = \bar{\partial}_{\dot{A}}\big(\delta^{\dot{A}}_{\dot{B}}\,\bar{\theta}^{\dot{B}} - \bar{\theta}_{\dot{B}}(-\epsilon^{\dot{A}\dot{B}})\big) \\ &= \bar{\partial}_{\dot{A}}\big(\bar{\theta}^{\dot{A}} + \bar{\theta}^{\dot{A}}\big) \\ &= 2\,\bar{\partial}_{\dot{A}}\,\bar{\theta}^{\dot{A}} \\ &= 4 \end{aligned} \right.$$

$$= -\Big(\lambda_A(y) + \theta_A D(y) - (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y) + i\theta\theta[\sigma^{\mu}\partial^{(y)}_{\mu}\bar{\lambda}(y)]_A\Big)$$

$$= -\lambda_A(y) - \theta_A D(y) + (\sigma^{\mu\nu}\theta)_A F_{\mu\nu}(y) - i\theta\theta[\sigma^{\mu}\partial^{(y)}_{\mu}\bar{\lambda}(y)]_A$$

4. $$W^A W_A\Big|_{\theta\theta} = \theta\theta D^2(y) + (\sigma^{\mu\nu}\theta)(\sigma^{\alpha\beta}\theta)F_{\mu\nu}F_{\alpha\beta} + 2i\theta\theta\,\lambda(y)\sigma^{\mu}\partial^{(y)}_{\mu}\bar{\lambda}(y)$$

② $$= (\sigma^{\mu\nu}\theta)^A(\sigma^{\alpha\beta}\theta)_A F_{\mu\nu}F_{\alpha\beta}$$

$$= \epsilon^{AB}(\sigma^{\mu\nu}\theta)_B(\sigma^{\alpha\beta}\theta)_A F_{\mu\nu}F_{\alpha\beta}$$

$$= \epsilon^{AB}\,\sigma^{\mu\nu}{}_B{}^C\,\underline{\theta_C}\,\sigma^{\alpha\beta}{}_A{}^D\,\underline{\theta_D}\,F_{\mu\nu}F_{\alpha\beta}$$

$$= \frac{1}{2}\underbrace{\epsilon^{AB}\epsilon_{CD}}_{=\delta^A_D\delta^B_C - \delta^A_C\delta^B_D}\theta\theta\,\sigma^{\mu\nu}{}_B{}^C\,\sigma^{\alpha\beta}{}_A{}^D\,F_{\mu\nu}F_{\alpha\beta}$$

$$= \frac{1}{2}\theta\theta\Big[\underbrace{\sigma^{\mu\nu}{}_B{}^B\,\sigma^{\alpha\beta}{}_A{}^A}_{=0} - \underbrace{\sigma^{\mu\nu}{}_B{}^A\,\sigma^{\alpha\beta}{}_A{}^B}_{=\mathrm{tr}[\sigma^{\mu\nu}\sigma^{\alpha\beta}]}\Big]F_{\mu\nu}F_{\alpha\beta}$$

$$= -\frac{1}{2}\theta\theta\cdot\frac{1}{2}\big(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha} + i\epsilon^{\mu\nu\alpha\beta}\big)F_{\mu\nu}F_{\alpha\beta}$$

$$= -\frac{1}{4}\theta\theta\big[F^{\alpha\beta}F_{\alpha\beta} - F^{\beta\alpha}F_{\alpha\beta} + 2i\tilde{F}^{\mu\nu}F_{\mu\nu}\big]$$

$$= \Theta\Theta D^2(y) + 2i\Theta\Theta\, \lambda^A(y)\,[\sigma^\mu \partial_\mu^{(y)} \bar\lambda(y)]_A$$
$$+ \underbrace{(\sigma^{\mu\nu}\theta)^A (\sigma^{\alpha\beta}\theta)_A}\, F_{\mu\nu} F_{\alpha\beta} \quad + \quad \dots$$

wait a second! this has Lorentz structure TrIG1!

$= tr[\dots]$ (trace in $\mu\nu\alpha\beta$, it is lorentz scalar even before trace)

~✓

$$\overset{[I4j]}{=} \frac{1}{2}\theta\theta\,(g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha} + i\epsilon^{\mu\nu\alpha\beta})\, F_{\mu\nu}F_{\alpha\beta}$$
$$= \frac{1}{2}\Theta\Theta\,(F_{\mu\nu}F^{\mu\nu} - F_{\mu\nu}F^{\nu\mu} + 2i\, F_{\mu\nu}\hat{F}^{\mu\nu})$$
$$= \Theta\Theta\,(F_{\mu\nu}F^{\mu\nu} + i\, F_{\mu\nu}\hat{F}^{\mu\nu})$$

$$= \Theta\Theta\,(D^2(y) + 2i\,\lambda(y)\sigma^\mu\partial_\mu^{(y)}\bar\lambda(y) + F_{\mu\nu}F^{\mu\nu} + i\, F_{\mu\nu}\hat{F}^{\mu\nu})$$

coefficients don't match with lecture.

3) 1. $$\mathcal{L}_{SMT} = -\frac{1}{2}(\varphi_i, \varphi_i^*)\underbrace{m_{ij}^{(s)\,2}}_{\text{matrix}}\begin{pmatrix}\varphi_j \\ \varphi_j^*\end{pmatrix}$$

potentially problematic, only transpose, not "dagger"

this I think was a typo on the sheet.

So you'll get the point :)

$$= -\frac{1}{2}(\varphi_i, \varphi_i^*)\begin{pmatrix} V_1 & V_2 \\ V_3 & V_4\end{pmatrix}\begin{pmatrix}\varphi_j \\ \varphi_j^*\end{pmatrix}$$

$$\Rightarrow\quad -2\mathcal{L}_{SMT} = (\varphi_i V_1 + \varphi_i^* V_3,\ \varphi_i V_2 + \varphi_i^* V_4)\begin{pmatrix}\varphi_j \\ \varphi_j^*\end{pmatrix}$$
$$= \varphi_i\varphi_j V_1 + \varphi_i^*\varphi_j V_3 + \varphi_i\varphi_j^* V_2 + \varphi_i^*\varphi_j^* V_4$$

$$\Rightarrow\quad V_1 = \frac{\partial^2 V}{\partial\varphi_i\partial\varphi_j} =: V_{ij}\ ,\quad V_2 = \frac{\partial^2 V}{\partial\varphi_i\partial\varphi_j^*} =: V_i^{\ j}$$

$$V_3 = \frac{\partial^2 V}{\partial\varphi_i^*\partial\varphi_j} =: V^i_{\ j}\ ,\quad V_4 = \frac{\partial^2 V}{\partial\varphi_i^*\partial\varphi_j^*} =: V^{ij}$$

2. $$\operatorname{tr} m^{(s)2} = \operatorname{tr}\begin{pmatrix} V_{ij} & V_i{}^j \\ V^i{}_j & V^{ij} \end{pmatrix}$$
$$= V_{ij} + V^{ij}$$
$$= \frac{\partial}{\partial \varphi_i}\frac{\partial V}{\partial \varphi_j} + \frac{\partial}{\partial \varphi_i^*}\frac{\partial V}{\partial \varphi_j^*} \qquad V = \sum_i W_i \overline{W}^i$$
$$= \sum_k \left[\frac{\partial}{\partial \varphi_i}\left(W_{jk}\overline{W}^k + W_k \overline{W}_j{}^k\right) + \frac{\partial}{\partial \varphi_i^*}\left(W^j{}_k \overline{W}^k + W_k \overline{W}^{jk}\right)\right]$$
$$= \sum_k \Big[W_{ijk}\overline{W}^k + W_{jk}\overline{W}_i{}^k + W_{ik}\overline{W}_j{}^k + W_k \overline{W}_{ij}{}^k + W^{ij}{}_k \overline{W}^k + W^j{}_k \overline{W}^{ik} + W^i{}_k \overline{W}^{jk} + W_k \overline{W}^{ijk}\Big]$$
...

3) 1. $$\mathcal{L}_{SMT} = -\frac{1}{2}(\varphi_i, \varphi^{*j})\, m_{ij}^{(s)2} \begin{pmatrix} \varphi^{*j} \\ \varphi_j \end{pmatrix}$$

$$V = \sum_i W_i \overline{W}^i$$
$$W_i = \frac{\partial W}{\partial \varphi_i} = h_i + m_{ij}\varphi_j + \frac{1}{2} f_{ijk}\varphi_j \varphi_k$$
$$\overline{W}_i = h_i^+ + m_{ij}^* \varphi^{+j} + \frac{1}{2} f_{ijk}^* \varphi^{+j}\varphi^{+k}$$
$$V_{ij} = \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j} = f_{ijk}\left(h_k^* + m_{kl}^* \varphi^{+l} + \frac{1}{2} f_{klm}^* \varphi^{+l}\varphi^{+m}\right)$$
$$V_i{}^j = \frac{\partial^2 V}{\partial \varphi_i \partial \varphi^{*j}} = (m_{ik} + f_{ikm}\varphi_m)(m_{jk}^* + f_{jkl}^* \varphi^{+l})$$
$$V^i{}_j = \frac{\partial^2 V}{\partial \varphi^{*i}\partial \varphi_j} = (m_{ik}^* + f_{ikm}^* \varphi^{+m})(m_{jk} + f_{jkl}\varphi^l)$$
$$V^{ij} = \frac{\partial^2 V}{\partial \varphi^{*i}\partial \varphi^{*j}} = f_{ijk}^+ \left(h_k + m_{kl}\varphi_l + \frac{1}{2} f_{klm}\varphi_l \varphi_m\right)$$

$$m^{(s)\,2}{}_{ij} = \begin{pmatrix} V^i{}_j & V^{ij} \\ V_{ij} & V_i{}^j \end{pmatrix}$$

2. $$W_{ij} = \frac{\partial^2 W}{\partial\varphi_i \partial\varphi_j} = m_{ij} + f_{ijk}\varphi_k$$

$$W_{ip}\overline{W}^{pj} = \dots = V_i{}^j$$

$$\Rightarrow \operatorname{tr}\left(m^{(s)2}\right) = V^i{}_i + V_i{}^i$$

$$= W_{ip}\overline{W}^{pi} + W^{ip}\overline{W}_{pi}$$

$$= \operatorname{tr}\left(m^{(f)} m^{(f)+} + m^{(f)} m^{(f)+}\right)$$

$$= 2\operatorname{tr}\left(m^{(f)} m^{(f)+}\right)$$

3. $$\langle F_i^* \rangle = -\langle W_i \rangle = 0 \Rightarrow \underbrace{V_{ij} = V_{ji}}_{\text{off-diagonal}} = 0$$

$$\Rightarrow V^i{}_j \text{ and } V^j{}_i \text{ are EV!}$$