worked on 7/9

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1) If $Q = I_{3L} + I_{3R} + X$

$Q(u_R) = I_{3R}(u_R) + X(u_R)$

$Q(u_L) = I_{3L}(u_L) + X(u_L)$

since $I_{3L}(u_L) \equiv \frac{1}{2} = I_{3R}(u_R)$ ✓

$Q(u_R) = Q(u_L)$

For left-handed fields

$X = Q - I_{3L}$

$$\left(\begin{array}{ll} \text{up quarks,} & \frac{1}{6} \\ \text{down quarks} & \frac{1}{6} \\ e_L \ldots & -\frac{1}{2} \\ \nu_L \ldots & -\frac{1}{2} \end{array} \right.$$

$\rightarrow X = \frac{1}{2}B - \frac{1}{2}L$ ✓

What about Y ?

$Q = I_{3L} + Y = I_{3L} + I_{3R} + X$

$\Rightarrow Y = I_{3R} + X$

For right-handed field

$X = Q - I_{3R}$

$$\left(\begin{array}{ll} \text{up } q: & \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \\ \text{down } q: & -\frac{1}{3} + \frac{1}{2} = \frac{1}{6} \\ e_R, \ldots: & -1 + \frac{1}{2} = -\frac{1}{2} \\ \nu_R, \ldots: & 0 - \frac{1}{2} = -\frac{1}{2} \end{array} \right.$$

$\rightarrow \underline{X = \frac{1}{2}B - \frac{1}{2}L}$ ✓

The hypercharges for lefthanded and righthanded fields are the same

2. $\mathcal{L}_{Yuk} = \sum y_\psi \bar{\psi}_L \phi \cdot \psi_R$

No colors, $SU(2)_{L/R}$ singlet $\rightarrow$ only consider $U(1)_X$

$\rightarrow U(\phi) = 0$ to be invariant under G_{LR}

✓

3. $\phi \cdot \psi_R$

$= \sum_{i,j} \epsilon_{ij} \phi_i \psi_{Rj}$

$= \phi_1 \psi_{R2} - \phi_2 \psi_{R1}$

$\rightarrow \bar{\psi}_L(\phi \cdot \psi_R)$

$= \bar{\psi}_L (\phi_1 \psi_{R2} - \phi_2 \psi_{R1})$ → what about SU(2) invariance?

$= \bar{\psi}_{L1} \phi_{11} \psi_{R2} + \bar{\psi}_{L2} \phi_{12} \psi_{R2} - \bar{\psi}_{L1} \phi_{21} \psi_{R1} - \bar{\psi}_{L2} \phi_{22} \psi_{R1}$

$\bar{\psi}_L$ is a $\bar{2}$, shouldn't it be $\bar{\psi}_{L1} \phi_{12} - \bar{\psi}_{L2} \phi_{11}$?

of course you can always rename the fields consistently, But!

since $\psi_{L/R\,1}$ and $\psi_{L/R\,2}$ have different charges

→ ϕ_{12} and ϕ_{21} don't violate charge conservation

→ $\mathcal{L} \supset \langle \phi_{12} \rangle \bar{\psi}_{L2} \psi_{R2} - \langle \phi_{21} \rangle \bar{\psi}_{L1} \psi_{R1}$

Acquiring these vev's doesn't break G_{LR} into G_{SM} or $SU(3) \times U(1)_{em}$

- there is no complete $SU(2)_L$ doublet (not invariant under $SU(2)_L$)
- there is still $U(1) \times$ unbroken ✓

4. SU(2) triplet

I_3 charges defined as $[\sigma^3, \Delta^a \sigma^a]$

$$\vec{H} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \rightarrow \Delta = \vec{H} \cdot \vec{\tau} = \begin{pmatrix} H_3 & H_1 - iH_2 \\ H_1 + i_2 & -H_3 \end{pmatrix}$$

similar to W^μ in SM

→ $I_3(H_1 - iH_2) = 1$, $I_3(H_1 + iH_2) = -1$, $I_3(H_3) = 0$

→ $Q(\Delta_{12}) = 2$, $Q(\Delta_{21}) = 0$, $Q(\Delta_{11}) = Q(\Delta_{22}) = 1$ ✓

same assignments for Δ_L and Δ_R

5. Check for $SU(2)_L$,

$$\underbrace{\overset{\substack{\text{SU(2) doublet} \\ \text{(bar and c)}}}{\overline{\ell_L^c}}\ i\sigma_2 \Delta_L\ \overset{\text{SU(2) doublet}}{\ell_L}}_{\text{SU(2) doublet} \,\cdot\, \text{anti-doublet}}$$

→ invariant under $SU(2)_L$

→ same for $SU(2)_R$

Gauge invarince:

$$\overline{\ell_L^c}\ (i\sigma_2 \Delta_L)\ \ell_L \longrightarrow \overline{\ell_L^c}\ (i\sigma_2 \Delta_L)\ \ell_L$$

$\overline{\ell_L^c}\, U_L^\dagger \quad U_L (\cdots) U_L^\dagger \quad U_L \ell_L$

$X(\overline{\ell_L^c}) = X(\ell_L) = -\frac{1}{2}$, $X(\Delta) = 1$ $\Rightarrow$ $\sum X = 0$

$\langle \Delta_{L,R} \rangle$ leaves $U(1)_{em}$ unbroken $\rightarrow$ only neutral components aquire vev

$$\mathcal{L} \supset g_\ell (\overline{\ell_L^c}\; i\sigma_2)_2 \langle \Delta_L \rangle_{21}\, \ell_{L1}$$

$$\sim \left((\overline{\nu_L^c} \;\; \overline{e_L^c}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right)_2 \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_1$$

$$= (-\overline{e_L^c} \;\; \overline{\nu_L^c})_2 \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}_1$$

$$= \overline{\nu_L^c}\, \nu_L$$ ✓

$\langle \Delta_L \rangle \rightarrow$ left-handed neutrino

$\langle \Delta_R \rangle \rightarrow$ right-handed neutrino ✓

6. mass of $SU(2)_L$ gauge bosons $\ll$ of $SU(2)_R$ $\emptyset$

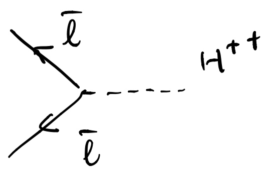
$\Rightarrow$ $|\langle \Delta_L \rangle| \ll |\langle \Delta_R \rangle|$ $\longrightarrow$ But you just showed $\langle \Delta_L \rangle$ contributes to $\overline{\nu_L^c}\,\nu_L$ and not $SU(2)_L$ Breaking !!!

(also implies massive right-handed neutrinos)

$|\langle \phi \rangle| \sim \mathcal{O}(200\ \text{GeV})$ like SM Higgs

$\rightarrow$ $|\langle \phi \rangle| < |\langle \Delta_L \rangle| < |\langle \Delta_R \rangle|$

7. highest charged comp. of Δ_L : $(\Delta_L)_{12} \sim H^{++}$



for decay (right to left)

for production if mass is low enough

On the right track! But what happened to e^c?

6. $\langle \Delta_L \rangle = \begin{pmatrix} 0 & 0 \\ \delta_L & 0 \end{pmatrix}$, $\langle \Delta_R \rangle = \begin{pmatrix} 0 & 0 \\ \delta_R & 0 \end{pmatrix}$

$M_{\nu_R} \sim g\, \delta_R \sim 4000\ \text{GeV}$

$M_{\nu_L} \propto g\, \delta_L \sim 0\ \text{GeV}$

$M_{W/Z} \propto g_Y \langle \phi \rangle \sim 80\ \text{GeV}$

$\Rightarrow$ $|\langle \Delta_R \rangle| > |\langle \phi \rangle| > |\langle \Delta_L \rangle|$

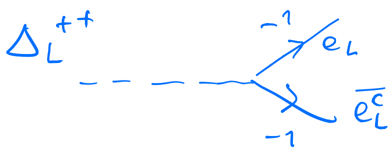
7. $\longrightarrow \Delta_L^{12} = \Delta_L^{++}$.

interaction term in (5):

$$g_L \left(\overline{\nu_L^c} \quad \overline{e_L^c} \right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} & \Delta_L^{++} \\ & \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= g_L \left(\overline{\nu_L^c} \quad \overline{e_L^c} \right) \begin{pmatrix} & \\ & -\Delta_L^{++} \end{pmatrix} \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$= -g_L \overline{e_L^c} \Delta_L^{++} e_L$$

$\Rightarrow$ 

U(1)em broken?

2) $\psi_3 = \begin{pmatrix} b^c \\ \tau \\ \nu_\tau \end{pmatrix} \qquad \chi_3 = \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix}, H^5 \qquad F = 0$

$\psi_2 = \begin{pmatrix} s^c \\ \mu \\ \nu_\mu \end{pmatrix} \qquad \chi_2 = \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} \qquad F = -1$

$\psi_1 = \begin{pmatrix} d^c \\ e \\ \nu_e \end{pmatrix} \qquad \chi_1 = \begin{pmatrix} u \\ d \\ e^c \end{pmatrix} \qquad F = -2$

Sorry, I really can't follow your solution!
Not the field assignments, not the chiralities?
and why H_5?!

$F(f) = 1$

$$\mathcal{L}^{eff}_{Yuk} = \sum_{ij} [\, \lambda^d_{ij}\, \varepsilon^{|F(\psi_i)+F(\chi_j)|}\, \bar{\psi}^c_i \chi_j H_5^+ - \frac{\lambda^u_{ij}}{4} \varepsilon^{|F(\chi_i)+F(\chi_j)|}\, \bar{\chi}^c_i \chi_j H_5 + h.c.]$$

$i=1$;

$$\mathcal{L}_1 = \sum_j [\, \lambda^d_{1j}\, \varepsilon^{|-2+F(\chi_j)|}\, \bar{\psi}^c_1 \chi_j H_5^+ - \frac{\lambda^u_{1j}}{4} \varepsilon^{|-2+F(\chi_j)|}\, \bar{\chi}^c_1 \chi_j H_5 + h.c.]$$

$$= \lambda^d_{11} \varepsilon^4 (\bar{d} \;\; \bar{e^c} \;\; \bar{\nu_e^c}) \begin{pmatrix} u \\ d \\ e^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{11}}{4} \varepsilon^4 (\bar{u^c} \;\; \bar{d^c} \;\; \bar{e}) \begin{pmatrix} u \\ d \\ e^c \end{pmatrix} H_5 + h.c.$$

$$+ \lambda^d_{12} \varepsilon^3 (\bar{d} \;\; \bar{e^c} \;\; \bar{\nu_e^c}) \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{12}}{4} \varepsilon^3 (\bar{u^c} \;\; \bar{d^c} \;\; \bar{e}) \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} H_5 + h.c.$$

$$+ \lambda^d_{13} \varepsilon^2 (\bar{d} \;\; \bar{e^c} \;\; \bar{\nu_e^c}) \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{13}}{4} \varepsilon^2 (\bar{u^c} \;\; \bar{d^c} \;\; \bar{e}) \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix} H_5 + h.c.$$

$$\mathcal{L}_2 = \lambda^d_{21} \varepsilon^3 (\bar{s} \;\; \bar{\mu^c} \;\; \bar{\nu_\mu^c}) \begin{pmatrix} u \\ d \\ e^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{21}}{4} \varepsilon^3 (\bar{c^c} \;\; \bar{s^c} \;\; \bar{\mu}) \begin{pmatrix} u \\ d \\ e^c \end{pmatrix}$$

$$+ \lambda^d_{22} \varepsilon^2 (\bar{s} \;\; \bar{\mu^c} \;\; \bar{\nu_\mu^c}) \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{22}}{4} \varepsilon^2 (\bar{c^c} \;\; \bar{s^c} \;\; \bar{\mu}) \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} \qquad + h.c.$$

$$+ \lambda^d_{23} \varepsilon (\bar{s} \;\; \bar{\mu^c} \;\; \bar{\nu_\mu^c}) \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{23}}{4} \varepsilon (\bar{c^c} \;\; \bar{s^c} \;\; \bar{\mu}) \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix}$$

$$\mathcal{L}_3 = \lambda^d_{31} \varepsilon^2 (\bar{b} \;\; \bar{\tau^c} \;\; \bar{\nu_\tau^c}) \begin{pmatrix} u \\ d \\ e^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{31}}{4} \varepsilon^2 (\bar{b^c} \;\; \bar{t^c} \;\; \bar{\tau}) \begin{pmatrix} u \\ d \\ e^c \end{pmatrix}$$

$$+ \lambda^d_{32} \varepsilon (\bar{b} \;\; \bar{\tau^c} \;\; \bar{\nu_\tau^c}) \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{32}}{4} \varepsilon (\bar{b^c} \;\; \bar{t^c} \;\; \bar{\tau}) \begin{pmatrix} c \\ s \\ \mu^c \end{pmatrix} \qquad + h.c.$$

$$+ \lambda^d_{33} (\bar{b} \;\; \bar{\tau^c} \;\; \bar{\nu_\tau^c}) \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix} H_5^+ - \frac{\lambda^u_{33}}{4} (\bar{b^c} \;\; \bar{t^c} \;\; \bar{\tau}) \begin{pmatrix} b \\ t \\ \tau^c \end{pmatrix}$$

ignore mixing of leptons and quarks for simplicity

$$\mathcal{M}_q = \underset{\substack{\uparrow \\ \text{from} \\ \text{h.c.}}}{2}\langle H_5\rangle
\begin{array}{c} \\ u \\ d \\ c \\ s \\ b \\ t \end{array}
\begin{array}{c}
\begin{array}{cccccc} \quad u \quad & \quad d \quad & \quad c \quad & \quad s \quad & \quad b \quad & \quad t \quad \end{array} \\
\begin{pmatrix}
-\frac{\lambda^u_{11}}{4}\varepsilon^4 & 0 & -\frac{\lambda^u_{12}}{4}\varepsilon^3 & 0 & -\frac{\lambda^u_{13}}{4}\varepsilon^2 & 0 \\
\lambda^d_{11}\varepsilon^4 & -\frac{\lambda^u_{11}}{4}\varepsilon^4 & \lambda^d_{12}\varepsilon^3 & -\frac{\lambda^u_{12}}{4}\varepsilon^3 & \lambda^d_{13}\varepsilon^2 & -\frac{\lambda^u_{13}}{4}\varepsilon^2 \\
-\frac{\lambda^u_{21}}{4}\varepsilon^3 & 0 & -\frac{\lambda^u_{22}}{4}\varepsilon^2 & 0 & -\frac{\lambda^u_{23}}{4}\varepsilon & 0 \\
\lambda^d_{21}\varepsilon^3 & -\frac{\lambda^u_{21}}{4}\varepsilon^3 & \lambda^d_{22}\varepsilon^2 & -\frac{\lambda^u_{22}}{4}\varepsilon^2 & \lambda^d_{23}\varepsilon & -\frac{\lambda^u_{23}}{4}\varepsilon \\
-\frac{\lambda^u_{31}}{4}\varepsilon^2 & 0 & -\frac{\lambda^u_{32}}{4}\varepsilon^2 & 0 & -\frac{\lambda^u_{33}}{4} & 0 \\
\lambda^d_{31}\varepsilon^2 & -\frac{\lambda^u_{31}}{4}\varepsilon^2 & \lambda^d_{32}\varepsilon & -\frac{\lambda^u_{32}}{4}\varepsilon^2 & \lambda^d_{13}\varepsilon^2 & -\frac{\lambda^u_{33}}{4}
\end{pmatrix}
\end{array}$$

similar for leptons...

2) 1. $U(1)_F$ Charge : Q_F

$$Q_F(f) = Q_F(\epsilon) = +1$$

$$Q_F(f_{1L}) = -2 \qquad Q_F(f_{1R}) = +2$$

$$Q_F(f_{2L}) = -1 \qquad Q_F(f_{2R}) = +1$$

$$Q_F(f_{3L}) = 0 \qquad Q_F(f_{3R}) = 0 \quad , \quad Q_F(H) = 0$$

so that tree-level mass term $\bar{f}_i f_i = \bar{f}_{iL} f_{iR} + \bar{f}_{iR} f_{iL}$

has nonvanishing ΣQ_F, $\rightarrow$ forbidden!

(To generate mass through loops !)

We can also add ϵ factors to the mass term by hand,

to make it $U(1)_F$ -inv.

$\Rightarrow$ e.g. $\epsilon^0 \bar{f}_3 f_3$, $\epsilon^4 \bar{f}_1 f_1$

In matrix form:

$$\begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array} \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^1 \\ \epsilon^2 & \epsilon^1 & \epsilon^0 \end{pmatrix}$$

2. Assume two generations of fermions

① Allow $M \bar{F}_L F_R^c$, $M \bar{F}_R F_L^c$

② Allow $h \bar{f}_{R,2} f_{L,2}$

③ Forbid $h \bar{f}_{R,1} f_{L,1}$, $h \bar{f}_{R,1} f_{L,2}$, $h \bar{f}_{R,2} f_{L,1}$

④ Allow $f \bar{f}_{R2} F_L^c$, $f^* \bar{f}_{L2} F_R^c$

⑤ Allow $h \bar{f}_{R,1} F_L$, $h \bar{F}_R f_{L1}$

$$① \Rightarrow -Q_F(F_L) - Q(F_R) = 0$$

$$Q_F(F_L) = -Q(F_R) \qquad (\text{as for } f)$$

$$② \rightarrow Q(f_{R2}) = -Q(f_{L2})$$

same as 1, but the value might differ

$$④ \Rightarrow Q_F(F_L) = Q_F(f) - Q_F(f_{R2}) = 1 - Q_F(f_{R2})$$

$$Q(F_R) = -Q_F(f) - Q_F(f_{L2}) = -1 - Q_F(f_{L2})$$

$$\Rightarrow \quad Q_F(F_L) = 1 + Q_F(f_{R2})$$

$$Q_F(F_R) = -Q_F(F_L) = -1 - Q_F(f_{L2})$$

$$\Rightarrow \quad Q_F(f_{R2}) = Q_F(f_{L2}) = 0$$

$$Q_F(F_L) = 1, \quad Q_F(F_R) = -1$$

$$⑤ \Rightarrow \quad Q_F(h) - Q_F(f_{R1}) + Q_F(F_L) \stackrel{!}{=} 0$$

$$\Rightarrow \quad 0 - Q_F(f_{R1}) + 1 = 0$$

$$\Rightarrow Q_F(f_{R1}) = 1, \quad Q_F(f_{L1}) = -1$$

Can show ③ is forbidden

(In reality, $f\,\bar{f}_{R1} f_{L2}$ is $U(1)_F$ conserving, but $SU(2)_L$ violating)

Mass matrix

$$\begin{array}{c|cccc} & F & F^c & f_1 & f_2 \\ \hline F & & & & \\ F^c & & & & \\ f_1 & & & & \\ f_2 & & & & \end{array} \quad \begin{pmatrix} 0 & M & v & 0 \\ M & 0 & 0 & \langle f \rangle \\ v & 0 & 0 & 0 \\ 0 & \langle f \rangle & 0 & v \end{pmatrix}$$

after SSB.
$\langle f \rangle = \langle f \rangle$
$\langle h \rangle = v$

Low energy, integrate out F

$$m_{f_1} \sim \frac{\langle f \rangle^2}{M^2} v \qquad \text{somehow} \ldots$$