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1)

$$\sigma^\mu p_\mu = (\mathbb{1}, \vec{\sigma})^\mu (m, 0, 0, 0)_\mu = m \cdot \mathbb{1}$$

Not \bar{Q} should be
2-spinors!

$$\{Q_1, Q_2\} = \{\bar{Q}_i, \bar{Q}_j\} = 0$$

$$\{Q_1, \bar{Q}_i\} = 2\sigma^{\mu}_{1i} p_\mu = 2m(\mathbb{1})_{1i} = 2m$$

$$\{Q_1, \bar{Q}_j\} = 2\sigma^{\mu}_{1j} p_\mu = 2m(\mathbb{1})_{1j} = 0$$

$$\{Q_2, \bar{Q}_i\} = 2\sigma^{\mu}_{2i} p_\mu = 2m(\mathbb{1})_{2i} = 0$$

$$\{Q_2, \bar{Q}_j\} = 2\sigma^{\mu}_{2j} p_\mu = 2m(\mathbb{1})_{2j} = 2m$$

follow the calculation
in lecture, in which
we seem to use here
 $i=1$.

I guess for real, symm.
they are the same
(conjugate = original)

Looks like it

$$2. [J^3, \bar{Q}^i] = \frac{1}{2} \epsilon^{vrs} [M_{vs}, \bar{Q}^i]$$

$$= \frac{1}{2} \epsilon^{vrs} (-\bar{\sigma}_{vs})^i_{\dot{B}} \bar{Q}^{\dot{B}}$$

$$= -\frac{i}{8} \epsilon^{vrs} (\bar{\sigma}_v \sigma_s - \bar{\sigma}_s \sigma_v)^i_{\dot{B}} \bar{Q}^{\dot{B}}$$

here in principle
we could also
turn $\bar{\sigma} \rightarrow \sigma$,
but since we have
dotted indices,
we shouldn't.

Is this correct?

I guess ...

$$= -\frac{1}{4} \underbrace{\epsilon^{vrs} \epsilon_{vsp}}_{= \epsilon^{vrs} \epsilon_{pvs}} (\bar{\sigma}^p)^i_{\dot{B}} \bar{Q}^{\dot{B}}$$

$$\begin{cases} = 2 \delta_p^3 \\ = 2 \delta_p^3 \end{cases}$$

$$= -\frac{1}{2} (\bar{\sigma}^3)^i_{\dot{B}} \bar{Q}^{\dot{B}}$$

$$\begin{cases} \dot{B} \text{ has } + = i \\ \text{otherwise zero} \end{cases}$$

$$= -\frac{1}{2} (\bar{\sigma}^3)^i_{\dot{B}} \bar{Q}^{\dot{B}}$$

$$= +\frac{1}{2} \bar{Q}^i$$

$$\Rightarrow J^3 \bar{Q}^i = \bar{Q}^i (J^3 + \frac{1}{2})$$

$$\begin{cases} \sigma^\mu = (\mathbb{1}, \vec{\sigma}) \\ \bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma}) \\ \text{(latin letter } \epsilon \\ \{1, 2, 3\} !) \end{cases}$$

$$[\vec{J}^3, \vec{Q}^2] = -\frac{1}{2} (\vec{\sigma}^3)^2 \hat{z} \vec{Q}^2 = -\frac{1}{2} \vec{Q}^2$$

(same as before)

$$\Rightarrow \vec{J}^3 \vec{Q}^2 = \vec{Q}^2 (\vec{J}^3 - \frac{1}{2})$$

3,

$$(\vec{Q}^2 |m, j, \lambda\rangle) \quad m \quad J_3 \quad \vec{J}^2 \\ m \quad \lambda + \frac{1}{2} \quad j(j+1)$$

$$(\vec{Q}^2 |m, j, \lambda\rangle) \quad m \quad \lambda - \frac{1}{2} \quad j(j+1)$$

$$|m, j, \lambda\rangle \\ \uparrow \hat{p}^2 \quad \uparrow \vec{J}^3 \quad \uparrow \vec{J}^2$$

$|m, o, o\rangle$ not annihilated by \vec{Q}_i or \vec{Q}_i

$$\Rightarrow \vec{J}^3 |m, o\rangle = o |m, o\rangle$$

$$\vec{J}^3 \vec{Q}_i |m, o\rangle = \frac{1}{2} |m, \frac{1}{2}\rangle$$

$$\vec{J}^3 \vec{Q}_i |m, o\rangle = -\frac{1}{2} |m, -\frac{1}{2}\rangle$$

$$\vec{J}^2 \rightarrow \text{tot. angular momentum } j = \frac{1}{2}$$

$$\Rightarrow j(j+1) = \frac{3}{4}$$

$$2) 1. \quad \mathcal{F}(x, \theta, \bar{\theta}) = f(x) + \sqrt{2} \theta g(x) + \sqrt{2} \bar{\theta} \bar{x}(x) + \theta \bar{\theta} M(x) + \bar{\theta} \bar{\theta} N(x) \\ + \theta \sigma^{\mu} \bar{\theta} A_{\mu}(x) + \theta \bar{\theta} \bar{\lambda}(x) + \bar{\theta} \bar{\theta} \theta \xi(x) + \frac{1}{2} \theta \bar{\theta} \bar{\theta} D(x)$$

(Why not include $h_{\mu\nu}$ field with something like $\theta \sigma^{\mu} \bar{\theta} \theta \sigma^{\nu} \bar{\theta}$?
 $\rightarrow \theta \sigma^{\mu} \bar{\theta} \theta \sigma^{\nu} \bar{\theta} = \frac{1}{2} g^{\mu\nu} \theta \theta \bar{\theta} \bar{\theta}$

Coordinate transp:

$$\left\{ \begin{array}{l} x^{\mu} \rightarrow x'^{\mu} = x^{\mu} - i\theta \sigma^{\mu} \bar{\epsilon} + i\epsilon \sigma^{\mu} \bar{\theta} =: x^{\mu} - G^{\mu}(\theta, \bar{\theta}, \epsilon, \bar{\epsilon}) \\ \theta \rightarrow \theta' = \theta + \epsilon \\ \bar{\theta} \rightarrow \bar{\theta}' = \bar{\theta} + \bar{\epsilon} \end{array} \right.$$

$$f(x) \rightarrow f(x') = f(x - i\theta \sigma^{\mu} \bar{\epsilon} + i\epsilon \sigma^{\mu} \bar{\theta}) \\ = f(x) + \partial_m f(x)(x^i - x'^i) + O((x')^2) \\ \approx f(x) + \partial_m f(x) G^{\mu}$$

$$\sqrt{2} \theta g(x) \rightarrow \sqrt{2} \theta' g(x') \\ = \sqrt{2} (\theta + \epsilon) (g(x) + \partial_m g(x) G^{\mu})$$

$$\sqrt{2} \bar{\theta} \bar{x}(x) \rightarrow \sqrt{2} \bar{\theta}' \bar{x}(x') \\ = \sqrt{2} (\bar{\theta} + \bar{\epsilon}) (\bar{x}(x) + \partial_m \bar{x}(x) G^{\mu})$$

$$\theta \bar{\theta} M(x) \rightarrow \theta' \bar{\theta}' M(x) \\ = (\theta + \epsilon)(\bar{\theta} + \bar{\epsilon})(M(x) + \partial_m M(x) G^{\mu}) \\ = (\theta \bar{\theta} + \theta \bar{\epsilon} + \epsilon \bar{\theta} + \cancel{\epsilon \bar{\epsilon}})(M(x) + \partial_m M(x) G^{\mu}) \quad \begin{array}{l} \text{(as always only} \\ \text{up to } O(\epsilon^2), \\ \text{also } G_m \sim O(\epsilon) \end{array}$$

$$\bar{\theta} \bar{\theta} N(x) \rightarrow (\bar{\theta} + \bar{\epsilon})(\bar{\theta} + \bar{\epsilon})(N(x) + \partial_m N(x) G^{\mu}) \\ = (\bar{\theta} \bar{\theta} + \bar{\theta} \bar{\epsilon} + \bar{\epsilon} \bar{\theta} + \cancel{\bar{\epsilon} \bar{\epsilon}})(N(x) + \partial_m N(x) G^{\mu})$$

$$\theta \sigma^{\mu} \bar{\theta} A_{\mu}(x) \rightarrow (\theta + \epsilon) \sigma^{\mu} (\bar{\theta} + \bar{\epsilon})(A_{\mu}(x) + \partial_{\nu} A_{\mu}(x) G^{\nu}) \\ = (\theta \sigma^{\mu} \bar{\theta} + \theta \sigma^{\mu} \bar{\epsilon} + \epsilon \sigma^{\mu} \bar{\theta} + \cancel{\epsilon \sigma^{\mu} \bar{\epsilon}})(A_{\mu}(x) + \partial_{\nu} A_{\mu}(x) G^{\nu})$$

$$\Theta\Theta\bar{\Theta}\bar{\lambda}(x) \rightarrow (\Theta + \epsilon)(\Theta + \epsilon)(\bar{\Theta} + \bar{\epsilon})(\bar{\lambda}(x) + \partial_\mu \bar{\lambda}(x) A^\mu)$$

$$= (\Theta\Theta\bar{\Theta} + \Theta\Theta\bar{\epsilon} + \Theta\epsilon\bar{\Theta} + \cancel{\Theta\epsilon\bar{\epsilon}} + \cancel{\epsilon\Theta\bar{\Theta}} + \cancel{\epsilon\Theta\bar{\epsilon}})$$

$$+ \cancel{\epsilon\epsilon\bar{\Theta}} + \cancel{\epsilon\epsilon\bar{\epsilon}})(\bar{\lambda}(x) + \partial_\mu \bar{\lambda}(x) A^\mu)$$

$$\bar{\Theta}\bar{\Theta}\Theta\zeta(x) \rightarrow (\bar{\Theta} + \bar{\epsilon})(\bar{\Theta} + \bar{\epsilon})(\Theta + \epsilon)(\zeta(x) + \partial_\mu \zeta(x) A^\mu)$$

$$= (\bar{\Theta}\bar{\Theta}\Theta + \bar{\Theta}\bar{\Theta}\epsilon + \bar{\Theta}\bar{\epsilon}\Theta + \cancel{\bar{\Theta}\bar{\epsilon}\bar{\epsilon}} + \cancel{\bar{\epsilon}\bar{\Theta}\Theta} + \cancel{\bar{\epsilon}\bar{\Theta}\bar{\epsilon}})$$

$$+ \zeta(x) + \partial_\mu \zeta(x) A^\mu)$$

$$\frac{1}{2} \Theta\Theta\bar{\Theta}\bar{\Theta}D(x) \rightarrow \frac{1}{2}(\Theta + \epsilon)(\Theta + \epsilon)(\bar{\Theta} + \bar{\epsilon})(\bar{\Theta} + \bar{\epsilon})(D(x) + \partial_\mu D(x) A^\mu)$$

$$= \frac{1}{2} (\Theta\Theta\bar{\Theta}\bar{\Theta} + \Theta\Theta\bar{\Theta}\bar{\epsilon} + \Theta\Theta\bar{\epsilon}\bar{\Theta} + \cancel{\Theta\Theta\bar{\epsilon}\bar{\epsilon}})$$

$$+ \Theta\epsilon\bar{\Theta}\bar{\Theta} + \Theta\epsilon\bar{\Theta}\bar{\epsilon} + \cancel{\Theta\epsilon\bar{\epsilon}\bar{\Theta}} + \cancel{\Theta\epsilon\bar{\epsilon}\bar{\epsilon}}$$

$$+ \epsilon\Theta\bar{\Theta}\bar{\Theta} + \epsilon\Theta\bar{\Theta}\bar{\epsilon} + \cancel{\epsilon\Theta\bar{\epsilon}\bar{\Theta}} + \cancel{\epsilon\Theta\bar{\epsilon}\bar{\epsilon}}$$

$$+ \cancel{\epsilon\epsilon\bar{\Theta}\bar{\Theta}})(D(x) + \partial_\mu D(x) A^\mu)$$

Rearrange everything according to order in $\Theta, \bar{\Theta}$

$$\sim \sigma(\theta^0, \bar{\theta}^0) = \delta f(x) = \sqrt{2} \epsilon \xi(x) + \sqrt{2} \bar{\epsilon} \bar{\chi}(x)$$

$$\sim \sigma(\theta^1, \bar{\theta}^0) = \delta(\sqrt{2} \theta \xi(x)) = (\Theta\epsilon + \epsilon\Theta) M(x) - i \partial_\mu f(x) \Theta \tau^\mu \bar{\epsilon} + \Theta \tau^\mu \bar{\epsilon} A_\mu$$

$$\Rightarrow \sqrt{2} \theta^A \delta \xi_A(x) = (\underbrace{\Theta^A \epsilon_A + \epsilon^A \Theta_A}_{= -\epsilon_A \Theta^A}) M(x) + \Theta \tau^\mu \bar{\epsilon} (-i \partial_\mu f(x) + A_\mu)$$

$$= \Theta^A \epsilon_A$$

$$\Rightarrow \sqrt{2} \theta^A \delta(\xi_A(x)) = 2 \Theta^A \epsilon_A M(x) + \Theta^A (\sigma^\mu \bar{\epsilon})_A (-i \partial_\mu f(x) + A_\mu)$$

$$\Rightarrow \delta \xi_A(x) = \sqrt{2} \epsilon_A M(x) + \frac{1}{\sqrt{2}} (\sigma^\mu \bar{\epsilon})_A (-i \partial_\mu f(x) + A_\mu)$$

$$\begin{aligned}\sim \theta(\theta^0, \bar{\theta}^1) &= \sqrt{2} \bar{\theta} \delta \bar{x}(x) = (\bar{\theta} \bar{E} + \bar{E} \bar{\theta}) N(x) + \underbrace{E \sigma^\mu \bar{\theta}}_{\text{PDF}} A_\mu(x) + i \underbrace{E \sigma^\mu \bar{\theta}}_{\partial_\mu f(x)} \partial_\mu f(x) \\ \Rightarrow \delta(\sqrt{2} \bar{\theta}_A \dot{\bar{x}}^A(x)) &= 2 \bar{\theta}_A \bar{E} \dot{\bar{x}}^A N(x) + \underbrace{E \sigma^\mu \bar{\theta}}_{\text{Identities}} (A_\mu + i \partial_\mu f(x)) \\ &\quad \stackrel{\text{PDF}}{\hookrightarrow} = -\bar{\theta} \bar{\tau}^\mu E \\ &= -\bar{\theta}_A (\bar{\tau}^\mu E)^A \\ \Rightarrow \delta \dot{\bar{x}}^A(x) &= \sqrt{2} \bar{E} \dot{\bar{x}}^A N(x) - \frac{i}{\sqrt{2}} (\bar{\sigma}^\mu E)^A (A_\mu + i \partial_\mu f(x)) \\ &\quad \uparrow \\ &\quad \text{Should be a mistake in lecture}\end{aligned}$$

$$\begin{aligned}\sim \theta(\theta^2, \bar{\theta}^0) &= \theta \bar{\theta} \delta M(x) = \underbrace{-i \sqrt{2} \theta \partial_\mu \xi(x) \theta \sigma^\mu \bar{E}}_{\text{product of four z-comp. spinors}} + \theta \bar{\theta} \bar{E} \bar{\lambda}(x) \\ &\quad (\sigma^\mu \bar{E} \text{ as whole})\end{aligned}$$

similar to (17g)

$$\begin{aligned}\text{LHS} = \theta^A \xi_A \theta^B \chi_B &= \theta^A \epsilon_{AC} \xi^C \epsilon^{BD} \theta_D \chi_B \\ &= -\epsilon_{AC} \epsilon^{BD} \theta^A \theta_D \xi^C \chi_B \\ &= -(\delta_C^B \delta_A^D - \delta_A^B \delta_C^D) \theta^A \theta_D \xi^C \chi_B \\ &= -\theta^A \theta_A \xi^B \chi_B + \theta^A \theta_C \xi^C \chi_A \\ \Rightarrow \theta \xi \theta \chi &= -\frac{1}{2} \theta \theta \xi \chi\end{aligned}$$

$$\delta M = \frac{i}{\sqrt{2}} \partial_\mu \xi \bar{E} + \bar{E} \bar{\lambda}$$

$$\begin{aligned}\sim \theta(\theta^0, \bar{\theta}^2) &= \bar{\theta} \bar{\theta} \delta N(x) = \sqrt{2} \bar{\theta} \partial_\mu \bar{x} \underbrace{i E \sigma^\mu \bar{\theta}}_{\text{PDF}} + \bar{\theta} \bar{\theta} E \xi \\ &= -\bar{\theta} \bar{\sigma}^\mu E\end{aligned}$$

$$\Rightarrow \delta N(x) = \frac{i}{\sqrt{2}} \partial_\mu \bar{x} \bar{\sigma}^\mu E + E \xi$$

$$\delta N = -\frac{i}{\sqrt{2}} E \sigma^\mu \partial_\mu \bar{x} + E \xi$$

$$\sim \partial(\theta\sigma^\mu\bar{\theta}) = i\sqrt{2} \underbrace{\theta\partial_\mu \bar{\epsilon}\sigma^\mu\bar{\theta}}_{=①} - i\sqrt{2} \underbrace{\bar{\theta}\partial_\mu \bar{x}\theta\sigma^\mu\bar{\epsilon}}_{=②} + \underbrace{(\theta\epsilon\bar{\theta} + \epsilon\theta\bar{\theta})}_③\lambda \\ + \underbrace{(\bar{\theta}\bar{\epsilon}\theta + \bar{\epsilon}\bar{\theta}\theta)\xi(x)}_④$$

$$\begin{aligned} ① &= \theta\partial_\mu \bar{\epsilon}\sigma^\mu\bar{\theta} = \partial_\mu \bar{\epsilon}\theta\epsilon\sigma^\mu\bar{\theta} \\ (17e) &= -\frac{1}{2}\partial_\mu \bar{\epsilon}\epsilon\theta\sigma^\mu\bar{\theta} + \partial_\mu \bar{\epsilon}\sigma^{\mu\nu}\epsilon\theta\sigma_\nu\bar{\theta} \\ &= -\frac{1}{2}\theta\sigma^\mu\bar{\theta}\partial_\mu \bar{\epsilon}\epsilon - \theta\sigma^\mu\bar{\theta}\underbrace{\partial^\nu \bar{\epsilon}\sigma_{\nu\mu}\epsilon}_{= -\epsilon\sigma_{\mu\nu}\partial^\nu \bar{\epsilon}} \end{aligned}$$

$$\begin{aligned} ② &= \bar{\theta}\partial_\mu \bar{x}\theta\sigma^\mu\bar{\epsilon} = \theta\sigma^\mu\bar{\epsilon}\bar{\theta}\partial_\mu \bar{x} \\ &= (\partial_\mu x\theta\epsilon\sigma^\mu\bar{\theta})^+ \\ &= (-\frac{1}{2}\partial_\mu x\epsilon\theta\sigma^\mu\bar{\theta} + \partial_\mu x\sigma^{\mu\nu}\epsilon\theta\sigma_\nu\bar{\theta})^+ \\ &= -\frac{1}{2}\theta\sigma^\mu\bar{\theta}\bar{\epsilon}\partial_\mu \bar{x} + \theta\sigma_\nu\bar{\theta}\bar{\epsilon}\bar{\sigma}^{\mu\nu}\partial_\mu \bar{x} \end{aligned}$$

$$\begin{aligned} ③ &= \theta\epsilon\bar{\theta}\bar{\lambda} + \epsilon\theta\bar{\theta}\bar{\lambda} = 2\theta\epsilon\bar{\theta}\bar{\lambda} \\ (17b) &\stackrel{=} \theta\sigma^\mu\bar{\theta}\epsilon\sigma_\mu\bar{\lambda} \end{aligned}$$

$$④ = 2\bar{\theta}\bar{\epsilon}\theta\xi = 2\theta\xi\bar{\theta}\bar{\epsilon}$$

$$= \theta\sigma^\mu\bar{\theta}\xi\sigma_\mu\bar{\epsilon}$$

$$\Rightarrow \delta A_\mu = \cancel{\xi\sigma_\mu\bar{\epsilon}} + \epsilon\sigma_\mu\bar{\lambda} + \frac{i}{\sqrt{2}}\bar{\epsilon}\partial_\mu \bar{x} - i\sqrt{2}\bar{\epsilon}\bar{\sigma}^{\mu\nu}\partial_\mu \bar{x} \\ - \frac{i}{\sqrt{2}}\partial_\mu \bar{\epsilon}\epsilon + i\sqrt{2}\epsilon\sigma_{\mu\nu}\partial^\nu \bar{\epsilon}$$

$$\sim \partial(\theta^2, \bar{\theta}^2) = \theta\theta\bar{\theta}\bar{\theta}\delta\lambda(x)$$

$$\begin{aligned} &= i\theta\theta\partial_\mu M\epsilon\sigma^\mu\bar{\theta} + \frac{1}{2}(\theta\theta\bar{\epsilon}\bar{\theta} + \theta\theta\bar{\theta}\bar{\epsilon})D \\ &+ -i\theta\sigma^\mu\bar{\theta}\partial_\nu A_\mu\theta\sigma^\nu\bar{\epsilon} \end{aligned}$$

$$= \theta\bar{\theta}(-i\partial_\mu M\bar{\sigma}^\mu E + \bar{E}D) - i\theta\sigma^\mu\bar{\theta}\partial_\nu A_\mu\theta\sigma^\nu\bar{E}$$

Why can't use

(I $\neq k$)?

$\theta \neq E$!!!



$$= \partial_\nu A_\mu \theta\sigma^\mu\bar{\theta}\theta\sigma^\nu\bar{E}$$

$$= \partial_\nu A_\mu \theta\sigma^\nu\bar{E}\theta\sigma^\mu\bar{\theta}$$

$$= \partial_\nu A_\mu \bar{E}^A (\theta\sigma^\nu)_A \bar{\theta}\bar{\sigma}^\mu\theta$$

$$= \partial_\nu A_\mu \bar{E}^A \theta\theta [\frac{1}{2} \bar{\theta}_A g^{\mu\nu} + i(\bar{\theta}\bar{\sigma}^{\mu\nu})_A]$$

$$= -\frac{1}{2} \partial_\mu A^\mu \theta\theta\bar{\theta}\bar{E} - i\theta\theta\bar{\theta}\sigma^{\mu\nu}E\partial_\mu A_\nu$$

(\neq)

$$\Rightarrow \delta\bar{\lambda} = -i\partial_\mu M\bar{\sigma}^\mu E + \bar{E}D - \frac{i}{2}\partial_\mu A^\mu \theta - \partial_\nu A_\mu \bar{\sigma}^{\mu\nu}\theta$$

\bar{E}

$(\bar{\sigma}^{\mu\nu}\theta)$

$$\sim \theta(\theta^1, \bar{\theta}^2) = \bar{\theta}\bar{\theta}\theta\delta S(x)$$

$$= -i\bar{\theta}\bar{\theta}\partial_\mu N\theta\sigma^\mu\bar{E} + \frac{1}{2}(\theta E\bar{\theta}\bar{\theta} + E\theta\bar{\theta}\bar{\theta})D(x)$$

$$+ i\theta\sigma^\mu\bar{\theta}\partial_\nu A_\mu E\sigma^\nu\bar{\theta}$$

$$= \bar{\theta}\bar{\theta}\theta(-i\partial_\mu N\bar{\sigma}^\mu\bar{E} + \bar{E}D) + i\partial_\nu A_\mu \theta\sigma^\mu\bar{\theta}E\sigma^\nu\bar{\theta}$$

$$\Rightarrow \delta S(x) = -i\partial_\mu N\bar{\sigma}^\mu\bar{E} + \bar{E}D + \frac{i}{2}E\partial^\mu A_\mu - \sigma^{\mu\nu}E\partial_\mu A_\nu$$

$$\sim \theta(\theta^2, \bar{\theta}^2) = \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\delta D(x)$$

$$= i\theta\theta\bar{\theta}\partial_\mu \bar{\lambda} E\sigma^\mu\bar{\theta} - i\bar{\theta}\bar{\theta}\theta\partial_\mu S\theta\sigma^\mu\bar{E}$$

$$(I \neq k) \quad = -\bar{\theta}\partial_\mu \bar{\lambda} \bar{\theta}(\bar{\sigma}^\mu E)$$

$$= \frac{1}{2}\partial_\mu \bar{\lambda} \bar{\sigma}^\mu E \bar{\theta}\bar{\theta}$$

$$= \frac{1}{2}\bar{\theta}\bar{\theta}\partial_\mu \bar{\lambda} \bar{\sigma}^\mu E$$

$$= \theta\partial_\mu S\theta(\sigma^\mu\bar{E})$$

$$= \frac{1}{2}\partial_\mu S\sigma^\mu\bar{E}\theta\theta$$

$$= -\frac{1}{2}\theta\theta\partial_\mu S\sigma^\mu\bar{E}$$

$$= \frac{i}{2}\theta\theta\bar{\theta}\bar{\theta}(\partial_\mu \bar{\lambda} \bar{\sigma}^\mu E + \partial_\mu S\sigma^\mu\bar{E})$$

$$\Rightarrow \delta D = i(\partial_\mu \bar{\lambda} \bar{\sigma}^\mu E + \partial_\mu S\sigma^\mu\bar{E})$$

In this assignment, I have used (all being 2-comp. spins)

$$\S\S \chi\tau = (\S\S)^A (\chi\tau)_A \quad \times$$

$$\S\S \chi\tau = \chi\tau \S\S$$

$$= -(\chi\tau)_A (\S\S)^A$$

but for different reason

$$= (\chi\tau)^A (\S\S)_A$$

$$\S^A S_A \chi^B \tau_B = \chi^B \S^A S_A \tau_B$$

$$= \chi^B \tau_B \S^A S_A$$

$$\stackrel{\downarrow}{=} (-1)^2$$

(*)

$$\theta \sigma^\mu \bar{\theta} \sigma^\nu \bar{e}$$

$$= - \bar{\theta} \bar{\sigma}^\mu \theta (\theta \sigma^\nu)_{\dot{A}} \bar{e}^{\dot{A}}$$

$$= \bar{\theta} \bar{\sigma}^\mu \theta \bar{e}^{\dot{A}} (\theta \sigma^\nu)_{\dot{A}}$$

$$= + \bar{e}^{\dot{A}} (\theta \sigma^\nu)_{\dot{A}} \bar{\theta} \bar{\sigma}^\mu \theta$$

$$= + \bar{e}^{\dot{A}} (-\theta \theta) [\frac{1}{2} \bar{\theta}_{\dot{A}} g^{\nu\mu} + i (\bar{\theta} \bar{\sigma}^\mu)_{\dot{A}}]$$

$$= + \bar{e}^{\dot{A}} \theta \theta [\frac{1}{2} \bar{\theta}_{\dot{A}} g^{\nu\mu} + i (\bar{\theta} \bar{\sigma}^\mu)_{\dot{A}}]$$

$$= + \frac{1}{2} \theta \theta \bar{e} \bar{e} g^{\nu\mu} + i \theta \theta \bar{\theta} \bar{\sigma}^\mu \bar{e}$$

$$\Rightarrow \delta \bar{e} = -i \partial_\mu M \bar{\sigma}^\mu e + \bar{e} D - i \partial_\mu A_\mu (+ \frac{1}{2} \bar{e} g^{\nu\mu} + i \bar{\sigma}^\mu \bar{e})$$

$$= \bar{e} D - i \partial_\mu M \bar{\sigma}^\mu e - \frac{i}{2} \bar{e} \partial_\mu A^\mu + \partial_\nu A_\mu \bar{\sigma}^{\nu\mu} \bar{e}$$