worked on $\frac{5}{10}$

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1)

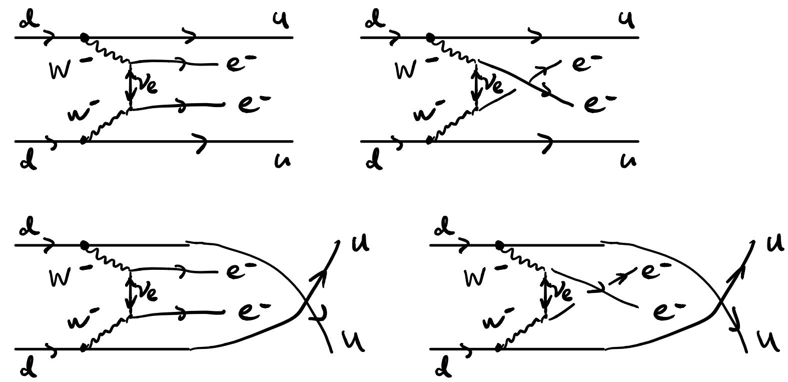
1. $(A, Z) \longrightarrow (A, Z+2) + e^- + e^-$ ✓

$\Longleftrightarrow \quad \underset{(udd)}{n} + n \longrightarrow \underset{(uud)}{p} + p + e^- + e^-$ ✓

$\Longleftrightarrow \quad d + d \rightarrow u + u + e^- + e^-$ ✓

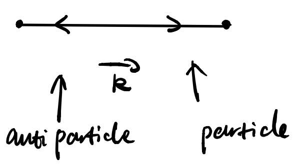
2.

$\overrightarrow{t}$



Good. Note that a charge assignment of W^- to intermediate state is not well-defined!

3.



$$= \frac{i \sum_s v \bar{u}}{k^2 - m^2 + i\varepsilon}$$

$$= \frac{-i(\not{k} + m)C}{k^2 - m^2 + i\varepsilon}$$

$$= -\frac{i}{\not{k} - m} C$$

$$\sum_s v \bar{u} = \sum C \bar{u}^T \bar{u}$$

Majorana condition

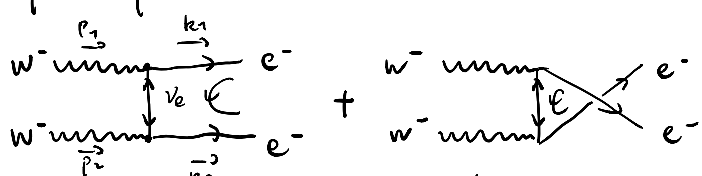
$\psi^c = \psi = C \bar{\psi}^T$

$v \cancel{e^{ipx}} = C \cancel{e^{ipx}} \bar{u}^T$

...

~ ✓

4. leptonic part $\longrightarrow$ take W^-'s as external states

$i\mathcal{M} =$ 

Schematically $\quad \mathcal{M}_a \propto \sum_k \underbrace{\frac{\not{p}_1 - \not{k}_1 + m_k}{t - m_k^2}}_{\text{propagator}} (U_{ek})^2$

two interaction vertices, interaction eigenstates $\hat{=}$ flavour eigenstates

$m_k \ll \text{MeV}$ $\qquad \simeq \sum_k (U_{ek})^2 \frac{m_k}{t}$

$$\mathcal{M}_b \propto \sum_k (U_{ek})^2 \frac{m_k}{u}$$

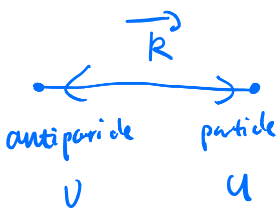
$$\Rightarrow \sum_i \mathcal{M}_i \propto \sum_k (U_{ek})^2 m_k$$

ignore $(\not p_1 - \not k_1)$ in denom. because of the gamma matrice bring extra minus sign commuting the γ_5 in P_L and P_R. So in the end one has $P_L P_R$ for $(\not p_1 - \not k_1)$, thus zero.

5. if $m_k \geq a$ a lower bound exists,

$\mathcal{M}_i \propto \sum_k (U_{ek})^2 m_k$ should have a lower bound, if $U^\dagger U = \mathbb{1}$.

↳ How's that U_{ek}^2 is unknown!?

3. 

$$= \frac{i\sum v\, u^T}{k^2 - m^2} = \frac{i\sum v\,(-u^T C^{-1})\cdot C}{k^2 - m^2} = \frac{-i\sum v\bar{v}\cdot C}{k^2 - m^2} = \frac{-i(\not k + m)C}{k^2 - m^2}$$

5. total amp. : $\mathcal{M} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3$

$$\propto (U_{e1})^2 m_1 + (U_{e2})^2 m_2 + (U_{e3})^2 m_3$$

$$\Rightarrow \mathcal{M} > A \min(m_1, m_2, m_3)\left((U_{e1})^2 + (U_{e2})^2 + (U_{e3})^2\right)$$

$$\mathcal{M} < A \max(m_1, m_2, m_3)\left[(U_{e1})^2 + (U_{e2})^2 + (U_{e3})^2\right]$$

$$< \underset{\uparrow\ \text{prop. const.}}{A} \max(\cdots)\ \underbrace{\left[|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2\right]}_{=1 \ \leftarrow\ \text{unitarity}}$$

$$U U^\dagger = \mathbb{1}$$

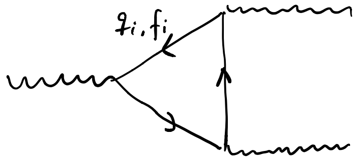
$$\Rightarrow \mathcal{M} < \max(m_1, m_2, m_3)$$

2) 1. External gauge boson : B, W^i $i=1,2,3$

Sorry; first need to fix what combination of gauge groups you're checking.
and need to show why these constraints follow from anomaly cancellation.

Relevant interaction is

$$\mathcal{L}_{int} \supset \bar{L}\, i\gamma^\mu \left(\frac{i}{2} g' B_\mu - i g \frac{\tau^i}{2} W^i_\mu\right) L$$



$$q_i = \begin{array}{llll} u_L\, c_L\, t_L & Y = \frac{1}{3} & I_3 = \frac{1}{2}; & u_R^c\, c_R^c\, t_R^c \quad Y = -\frac{4}{3} \quad I_3 = 0 \\ d_L\, s_L\, b_L & Y = \frac{1}{3} & I_3 = -\frac{1}{2}; & d_R^c\, s_R^c\, b_R^c \quad Y = +\frac{2}{3} \quad I_3 = 0 \end{array}$$

$$f_i = \begin{array}{llll} \nu_{e_L}\, \nu_{\mu_L}\, \nu_{\tau_L} & Y = -1 & I_3 = \frac{1}{2}; & \\ e_L\, \mu_L\, \tau_L & Y = -1 & I_3 = -\frac{1}{2}; & e_R^c\, \mu_R^c\, \tau_R^c \quad Y = 2 \quad I_3 = 0 \end{array}$$

$$\rightarrow \sum_{q_i} Y(q_i) = \underset{\substack{\uparrow \\ \text{color}}}{3} \cdot \underset{\substack{\uparrow \\ \text{3 generations}}}{3} \cdot \left(\frac{1}{3} + \frac{1}{3} - \frac{4}{3} + \frac{2}{3}\right) = 0$$

$$\sum_{f_i} I_3(f_i)^2\, Y(f_i) = \left(\cancel{3}\cdot\left(\frac{1}{2}\right)^2 \cdot \cancel{\frac{1}{3}} + \cancel{3}\cdot\left(-\frac{1}{2}\right)^2 \cdot \cancel{\frac{1}{3}}\right) \overset{\substack{\text{color} \\ \downarrow}}{3} + 3\cdot\left(\frac{1}{2}\right)^2 \cdot (-1) + 3\cdot\left(-\frac{1}{2}\right)^2 \cdot (-1)$$

$$= 3 \cdot \frac{1}{2} - \frac{3}{4} - \frac{3}{4} = 0$$

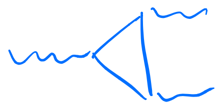
$$\sum_{f_i} Y(f_i)^3 = 3\cdot\left[3\cdot\left(\frac{1}{3}\right)^3 + 3\cdot\left(\frac{1}{3}\right)^3 + 3\cdot\left(\frac{-4}{3}\right)^3 + 3\cdot\left(\frac{2}{3}\right)^3\right] + 3\cdot(-1)^3 + 3\cdot(-1)^3 + 3\cdot 2^3$$

$$= \frac{1}{3} + \frac{1}{3} + \frac{-4^3 + 2^3}{3} - 3 - 3 + 24$$

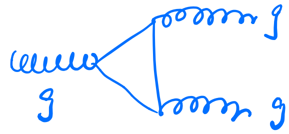
$$= -18 - 6 + 24$$

$$= 0$$

2. Diagrams with two gravitons and one gauge boson are harmless,

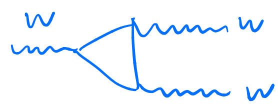
2. 1) In general:  $\propto \mathrm{Tr}[T^a\{T^b, T^c\}]$

(total symm. in a, b)

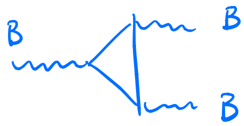
A) $SU(3)^3 \equiv$ 

No anomalies

SU(3) is vector theory, coupled identically to both chiralities

B) $SU(2)^3 \equiv$ 

$$\propto \mathrm{Tr}[\tau^a, \{\tau^b, \tau^c\}] = \tfrac{1}{2}\delta^{ab}\,\mathrm{Tr}[\tau^a] = 0$$

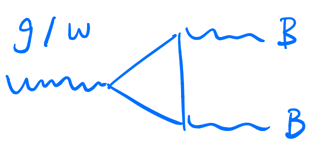
C) $U(1)^3 \equiv$ 

$$\propto \mathrm{Tr}[Y^a, \{Y^b, Y^c\}] \propto Y^3 \quad \text{for a fermion}$$

$\Rightarrow \sum_{\text{fermion}} Y^3(f) = 0$ as anomaly cancellation condition

(only use left chiral fields)

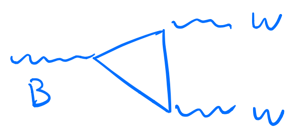
Note that $U(1)_Y$ is chiral, $Y(\psi_L) \neq Y(\psi_R)$!

D) $SU(N \in \{2,3\})\, U^2(1) \equiv$ 

$$\mathrm{Tr}[\tau^a\{Y, Y\}] \propto \mathrm{Tr}[\tau^a] = 0$$

$\tau \to T$ for SU(3)

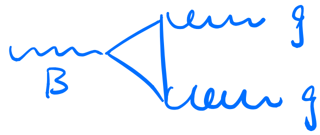
The same for SU(2)SU(3)U(1)

E) $SU(2)^2\, U(1) \equiv$ 

$$\propto \mathrm{Tr}[Y\{\tau^a, \tau^b\}] \propto$$

Fermion in loop must be left-chiral, $\leftarrow$ coupled to $SU(2)$

$\Rightarrow \sum_{L\text{-fermions}} Y(f) = 0 \quad \Leftarrow \sum I_3^2(f)\, Y(f) = 0$ on sheet

F) $SU^2(3)\, U(1) \equiv$ 

$$\propto Tr[Y\{T^a, T^b\}] \propto \delta^{ab} Y$$

$\Rightarrow \sum_{quarks} Y(q) = 0$

2. graviton $\to SO(4)$

$SO(4)^2\, SU(2 \text{ or } 3) \to$ no anomalies

$Tr[T^a] = Tr[\tau^a] = 0$

$\to SO(4)^2\, U(1)_Y \to Tr[Y] = Y \to \sum_{\substack{\text{all}\\ \text{fermions}}} Y = 0$

Now we have 4 constraints,

There are but $Y_L, Y_{e_R}, Y_{Q_L}, Y_{u_R}, Y_{d_R}$

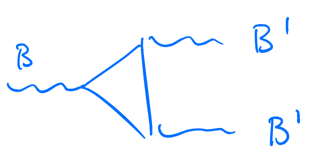
5 assignments of Y, 4 I_3's

$\to$ cannot det. $Y(f)$ uniquely. Ratios?

3. $U(1)_X \to B'$

The same constraints from 1. and 2. but replace Y with X.

There could also be



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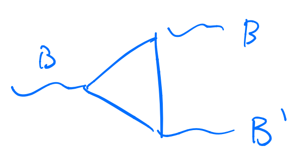

$\Rightarrow$ in total we have $4+4+2=10$ constraints

5+5 ratios of Y and X

$\Rightarrow$ single solution for rations of Y

If one say $Y_X = k Y_Y$

$\Rightarrow$ 6 equations : 4 for Y (= X), 2 for mixed(X, Y)

6 parameters : 5 ratios, 1 k

$\Rightarrow$ $Y_X = k Y_Y$

4. $U(1)_X$ with ν_R

$\hookrightarrow$ one more parameter

$\Rightarrow$ as before if $Y_X = k Y_Y$

6 eq., 7 parameters $\rightarrow$ no soluti.

$\Rightarrow$ $Y_X \neq k Y_Y$

($U(1)_X$ is $U(1)_{B-L}$!)