worked on 4/4

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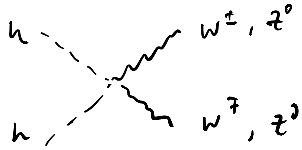
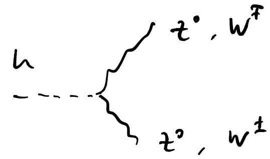
1) 1.

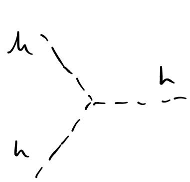
Possible vertices

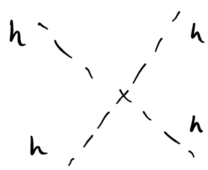


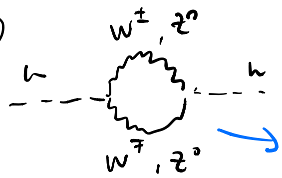


So by h you mean any member of the Higgs sector?

yes





① 

→ log. div.

② 

→ And may I ask why there is no w loop with quartic coupling?

③ 

Only the second kind of diagram leads to quad. divergence, since

$$\Pi^h_{②}(0) = \frac{i}{2}\frac{g^2}{\cos^2\theta_W}\int \frac{d^4k}{(2\pi)^4}\frac{-i}{k^2-m_Z^2} \times 2 \quad \frac{1}{2}$$

↑ symm. factor

$$\Pi^h_{③}(0) = \int d^4k \, \frac{ig^2}{2} g_{\mu\nu} \frac{-ig^{\mu\nu}}{k^2-m_W^2} \propto \frac{4g^2}{2}\int \frac{d^4k}{k^2-m_W^2}$$

why are we multiplying by symm factor?! $\times 1/2$?

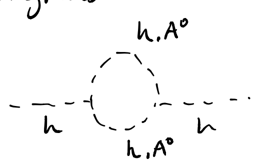
2. spin-0 propagators $\hat{=}$ higgs and A^0

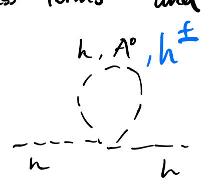
In Feynman gauge: $\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v+h+iA^0)\end{pmatrix}$

From $\sim \Phi^+\Phi$, $\sim(\Phi^+\Phi)^2$ in $\mathcal{L}$

$\Rightarrow$ A^0 mass terms and interaction ($\sim A^{0\,4}$)

Diagrams:

① 

② 

→ wait! there is also the $h, \phi^\pm$ coupling. right?

Again second kind of diagrams leads to quad. divergence. → and you will to specify the symmetry factors!

$$\Pi^h_{②}(0) = -\frac{3}{4} i g^2 \frac{m_h^2}{m_W^2}\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_h^2} - \frac{i}{4}g^2\frac{m_h^2}{m_W^2}\int\frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m_Z^2}$$

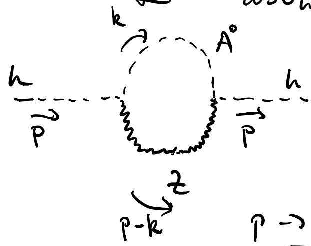
$-2\times 2 > 0$ ✓

$$② = \int d^4k \frac{i}{k^2-m_h^2}\left(-ig^2\frac{m_h^2}{m_W^2}\right)\Big[\underbrace{\frac{1}{4}\times\frac{1}{2}+\frac{3}{4}\times\frac{1}{2}}_{\text{symm. fact.}}+\frac{1}{4}\times 1\Big] \propto g^2\frac{m_h^2}{m_W^2}\frac{3}{4}\int d^4k\frac{1}{k^2-m_h^2}$$

3. Diagrams with one gauge, one scalar

Expand $(\partial_\mu \Phi)^\dagger \partial^\mu \Phi \quad \longrightarrow \quad Z - A^0 - h$ vertex

$$\mathcal{L} \supset \frac{-ig}{\cos\theta_W} Z_\mu \left[(\partial^\mu h) A^0 - h (\partial^\mu A^0) \right]$$

① 

$$= \int \frac{d^4k}{(2\pi)^4} (p-k)^\mu \frac{g}{\cos\theta_W} \cdot (-p+k)^\nu \frac{g}{\cos\theta_W} \frac{i}{k^2 - m_Z^2} \frac{-i g_{\mu\nu}}{(p-k)^2 - m_Z^2}$$

$$\xrightarrow{p \to 0} -\int \frac{d^4k}{(2\pi)^4} k^2 \left(\frac{g}{\cos\theta_W} \right)^2 \frac{1}{(k^2 - m_Z^2)^2}$$

$\rightarrow$ quad. divergent

4. The masses in the integral are not important (?)

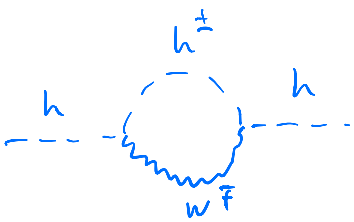
Σ coefficients of quad. divergent terms

$$= \frac{g^2}{\cancel{\cos^2\theta_W}} + g^2 \frac{m_h^2}{m_W^2} - \frac{g^2}{\cancel{\cos^2\theta_W}}$$

on the right track though
you are missing diagrams
and factors.

$\rightarrow$ if m_h arbitrary. yes. can cancel divergence in $t\bar{t}$ diagram

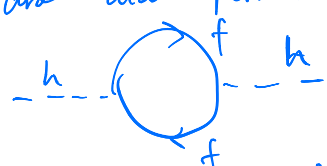
$\left(m_h^2 \sim \frac{m_W^2}{g^2} f^{(t)2} \right)$

3. ② 

$$= \int d^4k \, \frac{g^2}{4} (-k^2) \frac{i}{k^2 - m_{h^\pm}^2} \frac{-i}{(k^2 - m_W^2)}$$

$$\propto -\frac{g^2}{4} \int d^4k \frac{k^2}{(k^2 - m_{h^\pm}^2)(k^2 - m_W^2)}$$

4. There are also fermion loops



$$= \underbrace{-}_{\text{ferm. loop}} \underbrace{3}_{\text{color}} \int d^4k \left[-\frac{ig}{2} \frac{m_f}{m_W} \frac{i}{\not{k} - m_t} \frac{-ig}{2} \frac{m_f}{m_W} \frac{i}{\not{k} - m_t} \right]$$

All together

$$\propto \Big\{ \underbrace{+2g^2 + \frac{g^2}{\cos^2\theta_W}}_{1.} + \underbrace{\frac{3}{4} g^2 \frac{m_h^2}{m_W^2}}_{2.} \underbrace{-2\frac{g^2}{4} - \frac{g^2}{4\cos^2\theta_W}}_{3.} \underbrace{-3g^2\frac{m_f}{m_W^2}}_{4.} \Big\} \Lambda^2 \stackrel{!}{=} 0$$

$$\Rightarrow \quad 2m_W^2 + m_Z^2 + m_h^2 - 4m_f^2 = 0$$