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1) 1.

$$\sigma^\mu P_\mu = (\mathbb{1}, \vec{\sigma})^\mu (m, 0,0,0)_\mu = m \cdot \mathbb{1}$$

$$\{Q_1, Q_2\} = \{\bar{Q}_{\dot 1}, \bar{Q}_{\dot 2}\} = 0$$

$$\{Q_1, \bar{Q}_{\dot 1}\} = 2\sigma^\mu{}_{1\dot 1} P_\mu = 2m\,(\mathbb{1})_{1\dot 1} = 2m$$

$$\{Q_1, \bar{Q}_{\dot 2}\} = 2\sigma^\mu{}_{1\dot 2} P_\mu = 2m\,(\mathbb{1})_{1\dot 2} = 0$$

$$\{Q_2, \bar{Q}_{\dot 1}\} = 2\sigma^\mu{}_{2\dot 1} P_\mu = 2m\,(\mathbb{1})_{2\dot 1} = 0$$

$$\{Q_2, \bar{Q}_{\dot 2}\} = 2\sigma^\mu{}_{2\dot 2} P_\mu = 2m\,(\mathbb{1})_{2\dot 2} = 2m$$

follow the calculation in lecture, in which we seem to use here $\dot 1 = 1$. I guess for real, symm. they are the same (conjugate = original)

2.
$$[J^3, \bar{Q}^{\dot 1}] = \frac{1}{2}\epsilon^{3\nu s}\,[M_{\nu s}, \bar{Q}^{\dot 1}]$$
$$= \frac{1}{2}\epsilon^{3\nu s}(-\bar{\sigma}_{\nu s})^{\dot 1}{}_{\dot B}\,\bar{Q}^{\dot B}$$
$$= -\frac{i}{8}\epsilon^{3\nu s}(\bar{\sigma}_\nu \sigma_s - \bar{\sigma}_s \sigma_\nu)^{\dot 1}{}_{\dot B}\,\bar{Q}^{\dot B}$$

here in principle we could also turn $\bar{\sigma} \to \sigma$, but since we have dotted indices, we shouldn't. Is this correct?

$$= \frac{i}{8}\epsilon^{3\nu s}[\bar{\sigma}_\nu, \bar{\sigma}_s]^{\dot 1}{}_{\dot B}\,\bar{Q}^{\dot B}$$
$$= \frac{i}{8}\epsilon^{3\nu s}\cdot 2i\,\epsilon_{\nu s p}(\bar{\sigma}^p)^{\dot 1}{}_{\dot B}\,\bar{Q}^{\dot B}$$
$$= -\frac{1}{4}\underbrace{\epsilon^{3\nu s}\epsilon_{\nu s p}}\,(\bar{\sigma}^p)^{\dot 1}{}_{\dot B}\,\bar{Q}^{\dot B}$$

($\sigma^\mu = (\mathbb{1}, \vec{\sigma})$, $\bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$, Latin letter $\in \{1,2,3\}$!)

$$\left(\begin{aligned} &= \epsilon^{3\nu s}\epsilon_{p\nu s} \\ &= 2\delta^3_p \end{aligned} \right.$$

$$= -\frac{1}{2}(\bar{\sigma}^3)^{\dot 1}{}_{\dot B}\,\bar{Q}^{\dot B}$$

($\dot B$ has to $= \dot 1$, otherwise zero

$$= -\frac{1}{2}(\bar{\sigma}^3)^{\dot 1}{}_{\dot 1}\,\bar{Q}^{\dot 1}$$
$$= +\frac{1}{2}\bar{Q}^{\dot 1} \qquad \Rightarrow J^3\bar{Q}^{\dot 1} = \bar{Q}^{\dot 1}(J^3 + \frac{1}{2})$$

$$[J^3, \bar{Q}^{\dot{2}}] = -\frac{1}{2}(\bar{\sigma}^3)^{\dot{2}}{}_{\dot{2}}\bar{Q}^{\dot{2}} = -\frac{1}{2}\bar{Q}^{\dot{2}}$$

(same as before)

$$\Rightarrow \quad J^3 \bar{Q}^{\dot{2}} = \bar{Q}^{\dot{2}}\left(J^3 - \frac{1}{2}\right)$$

3.

	m	J_3	$\vec{J}^2$
$(\bar{Q}^{\dot{1}} \vert m, j, \lambda\rangle)$	m	$\lambda + \frac{1}{2}$	$j(j+1)$
$(\bar{Q}^{\dot{2}} \vert m, j, \lambda\rangle)$	m	$\lambda - \frac{1}{2}$	$j(j+1)$

2) 1. $\mathcal{F}(x,\theta,\bar\theta) = f(x) + \sqrt{2}\,\theta\xi(x) + \sqrt{2}\,\bar\theta\bar\chi(x) + \theta\theta M(x) + \bar\theta\bar\theta N(x)$

$+ \theta\sigma^\mu\bar\theta A_\mu(x) + \theta\theta\bar\theta\bar\lambda(x) + \bar\theta\bar\theta\theta\varphi(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x)$

(Why not include $h_{\mu\nu}$ field with something like $\theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\theta$?
$\rightarrow \theta\sigma^\mu\bar\theta\theta\sigma^\nu\bar\theta = \frac{1}{2}g^{\mu\nu}\theta\theta\bar\theta\bar\theta$

Coordinate trafo:

$$\begin{cases} x^\mu \rightarrow x'^\mu = x^\mu - i\theta\sigma^\mu\bar\epsilon + i\epsilon\sigma^\mu\bar\theta =: x^\mu - G^\mu(\theta,\bar\theta,\epsilon,\bar\epsilon) \\ \theta \rightarrow \theta' = \theta + \epsilon \\ \bar\theta \rightarrow \bar\theta' = \bar\theta + \bar\epsilon \end{cases}$$

$$f(x) \rightarrow f(x') = f(x - i\theta\sigma^\mu\bar\epsilon + i\epsilon\sigma^\mu\bar\theta)$$
$$= f(x) + \partial_\mu f(x)(x' - x)^\mu + \mathcal{O}((x')^2)$$
$$\simeq f(x) + \partial_\mu f(x)\,G^\mu$$

$$\sqrt{2}\,\theta\xi(x) \rightarrow \sqrt{2}\,\theta'\xi(x')$$
$$= \sqrt{2}(\theta+\epsilon)(\xi(x) + \partial_\mu\xi(x)G^\mu)$$

$$\sqrt{2}\,\bar\theta\bar\chi(x) \rightarrow \sqrt{2}\,\bar\theta'\bar\chi(x')$$
$$= \sqrt{2}(\bar\theta+\bar\epsilon)(\bar\chi(x) + \partial_\mu\bar\chi(x)G^\mu)$$

$$\theta\theta M(x) \rightarrow \theta'\theta' M(x')$$
$$= (\theta+\epsilon)(\theta+\epsilon)(M(x) + \partial_\mu M(x)G^\mu)$$
$$= (\theta\theta + \theta\epsilon + \epsilon\theta + \cancel{\epsilon\epsilon})(M(x) + \partial_\mu M(x)G^\mu)$$

(as always only up to $\mathcal{O}(\epsilon^2)$, also $G_\mu \sim \mathcal{O}(\epsilon)$)

$$\bar\theta\bar\theta N(x) \rightarrow (\bar\theta+\bar\epsilon)(\bar\theta+\bar\epsilon)(N(x) + \partial_\mu N(x)G^\mu)$$
$$= (\bar\theta\bar\theta + \bar\theta\bar\epsilon + \bar\epsilon\bar\theta + \cancel{\bar\epsilon\bar\epsilon})(N(x) + \partial_\mu N(x)G^\mu)$$

$$\theta\sigma^\mu\bar\theta A_\mu(x) \rightarrow (\theta+\epsilon)\sigma^\mu(\bar\theta+\bar\epsilon)(A_\mu(x) + \partial_\nu A_\mu(x)G^\nu)$$
$$= (\theta\sigma^\mu\bar\theta + \theta\sigma^\mu\bar\epsilon + \epsilon\sigma^\mu\bar\theta + \cancel{\epsilon\sigma^\mu\bar\epsilon})(A_\mu(x) + \partial_\nu A_\mu(x)G^\nu)$$

$$\theta\theta\bar\theta\,\bar\lambda(x) \to (\theta+\epsilon)(\theta+\epsilon)(\bar\theta+\bar\epsilon)(\bar\lambda(x)+\partial_\mu\bar\lambda(x)\,a^\mu)$$
$$= (\theta\theta\bar\theta + \theta\theta\bar\epsilon + \theta\epsilon\bar\theta + \cancel{\theta\epsilon\bar\epsilon} + \epsilon\theta\bar\theta + \cancel{\epsilon\theta\bar\epsilon}$$
$$+ \cancel{\epsilon\epsilon\bar\theta} + \cancel{\epsilon\epsilon\bar\epsilon})(\bar\lambda(x)+\partial_\mu\bar\lambda(x)a^\mu)$$

$$\bar\theta\bar\theta\theta\zeta(x) \to (\bar\theta+\bar\epsilon)(\bar\theta+\bar\epsilon)(\theta+\epsilon)(\zeta(x)+\partial_\mu\zeta(x)\,a^\mu)$$
$$=(\bar\theta\bar\theta\theta + \bar\theta\bar\theta\epsilon + \bar\theta\bar\epsilon\theta + \cancel{\bar\theta\bar\epsilon\epsilon} + \bar\epsilon\bar\theta\theta + \cancel{\bar\epsilon\bar\theta\epsilon} + \cancel{\bar\epsilon\bar\epsilon\theta} + \cancel{\bar\epsilon\bar\epsilon\epsilon})$$
$$\times(\zeta(x)+\partial_\mu\zeta(x)\,a^\mu)$$

$$\tfrac{1}{2}\theta\theta\bar\theta\bar\theta D(x) \to \tfrac{1}{2}(\theta+\epsilon)(\theta+\epsilon)(\bar\theta+\bar\epsilon)(\bar\theta+\bar\epsilon)(D(x)+\partial_\mu D(x)\,a^\mu)$$
$$= \tfrac{1}{2}(\theta\theta\bar\theta\bar\theta + \theta\theta\bar\theta\bar\epsilon + \theta\theta\bar\epsilon\bar\theta + \cancel{\theta\theta\bar\epsilon\bar\epsilon}$$
$$+\theta\epsilon\bar\theta\bar\theta + \cancel{\theta\epsilon\bar\theta\bar\epsilon} + \cancel{\theta\epsilon\bar\epsilon\bar\theta} + \cancel{\theta\epsilon\bar\epsilon\bar\epsilon}$$
$$+\epsilon\theta\bar\theta\bar\theta + \cancel{\epsilon\theta\bar\theta\bar\epsilon} + \cancel{\epsilon\theta\bar\epsilon\bar\theta} + \cancel{\epsilon\theta\bar\epsilon\bar\epsilon}$$
$$+ \cancel{\cdots})(D(x)+\partial_\mu D(x)\,a^\mu)$$

Rearrange everything according to order in $\theta, \bar\theta$

$$\sim \sigma(\theta^0, \bar\theta^0) = \delta f(x) = \sqrt{2}\,\epsilon\xi(x) + \sqrt{2}\,\bar\epsilon\bar\chi(x)$$
$$\sim \sigma(\theta^1, \bar\theta^0) = \delta(\sqrt{2}\,\theta\xi(x)) = (\theta\epsilon+\epsilon\theta)M(x) - i\partial_\mu f(x)\,\theta\sigma^\mu\bar\epsilon + \theta\sigma^\mu\bar\epsilon A_\mu$$
$$\Rightarrow \sqrt{2}\,\theta^A\,\delta\xi_A(x) = (\theta^A\epsilon_A + \underbrace{\epsilon^A\theta_A}_{\substack{= -\epsilon_A\theta^A \\ = \theta^A\epsilon_A}})M(x) + \theta\sigma^\mu\bar\epsilon(-i\partial_\mu f(x) + A_\mu)$$

$$\Rightarrow \sqrt{2}\,\theta^A\,\delta(\xi_A(x)) = 2\theta^A\epsilon_A M(x) + \theta^A(\sigma^\mu\bar\epsilon)_A(-i\partial_\mu f(x) + A_\mu)$$
$$\Rightarrow \delta\xi_A(x) = \sqrt{2}\,\epsilon_A M(x) + \frac{1}{\sqrt{2}}(\sigma^\mu\bar\epsilon)_A(-i\partial_\mu f(x) + A_\mu)$$

$$\sim \mathcal{O}(\theta^0, \bar\theta^1) = \sqrt{2}\,\bar\theta\ \delta\bar\chi(x) = (\bar\theta\bar\epsilon + \bar\epsilon\bar\theta)N(x) + \epsilon\sigma^\mu\bar\theta A_\mu(x) + i\epsilon\sigma^\mu\bar\theta\partial_\mu f(x)$$

$$\Rightarrow \delta(\sqrt{2}\,\bar\theta_{\dot A}\ \bar\chi^{\dot A}(x)) = 2\bar\theta_{\dot A}\bar\epsilon^{\dot A}N(x) + \underbrace{\epsilon\sigma^\mu\bar\theta}(A_\mu + i\partial_\mu f(x))$$

Identities PDF (7a) $\hookrightarrow$

$$= -\bar\theta\bar\sigma^\mu\epsilon$$

$$= -\bar\theta_{\dot A}(\bar\sigma^\mu\epsilon)^{\dot A}$$

$$\Rightarrow \delta\bar\chi^{\dot A}(x) = \sqrt{2}\,\bar\epsilon^{\dot A}N(x) - \frac{1}{\sqrt{2}}(\bar\sigma^\mu\epsilon)^{\dot A}(A_\mu + i\partial_\mu f(x))$$

$\uparrow$ Should be a mistake in lecture

$$\sim \mathcal{O}(\theta^2, \bar\theta^0) = \theta\theta\,\delta M(x) = -i\sqrt{2}\,\underbrace{\theta\partial_\mu\xi(x)\theta\sigma^\mu\bar\epsilon}_{\substack{\text{product of four 2-comp. spinor} \\ (\sigma^\mu\bar\epsilon \text{ as whole})}} + \theta\theta\bar\epsilon\bar\lambda(x)$$

similar to (17g)

$$LHS = \theta^A\xi_A\theta^B\chi_B = \theta^A\epsilon_{AC}\xi^C\epsilon^{BD}\theta_D\chi_B$$

$$= -\epsilon_{AC}\epsilon^{BD}\theta^A\theta_D\xi^C\chi_B$$

$$= -(\delta^B_C\delta^D_A - \delta^B_A\delta^D_C)\theta^A\theta_D\xi^C\chi_B$$

$$= -\theta^A\theta_A\xi^B\chi_B + \theta^A\theta_C\xi^C\chi_A$$

$$\Rightarrow \theta\xi\theta\chi = -\frac{1}{2}\theta\theta\,\xi\chi$$

$$\delta M = \frac{i}{\sqrt{2}}\partial_\mu\xi\bar\epsilon + \bar\epsilon\bar\lambda$$

$$\sim \mathcal{O}(\theta^0, \bar\theta^2) = \bar\theta\bar\theta\,\delta N(x) = \sqrt{2}\,\bar\theta\partial_\mu\bar\chi\, i\underbrace{\epsilon\sigma^\mu\bar\theta}_{= -\bar\theta\bar\sigma^\mu\epsilon} + \bar\theta\bar\theta\,\epsilon\zeta$$

$$\Rightarrow \delta N(x) = +\frac{i}{\sqrt{2}}\partial_\mu\bar\chi\bar\sigma^\mu\epsilon + \epsilon\zeta$$

$$\delta N = -\frac{i}{\sqrt{2}}\epsilon\sigma^\mu\partial_\mu\bar\chi + \epsilon\zeta$$

$$\sim \mathcal{O}(\theta\sigma^\mu\bar\theta) = \underbrace{i\sqrt{2}\,\theta\partial_\mu\xi\,\epsilon\sigma^\mu\bar\theta}_{=①} \underbrace{-i\sqrt{2}\,\bar\theta\partial_\mu\bar\chi\,\theta\sigma^\mu\bar\epsilon}_{=②} + \underbrace{(\theta\epsilon\bar\theta + \epsilon\theta\bar\theta)\bar\lambda}_{③}$$

$$+ \underbrace{(\bar\theta\bar\epsilon\theta + \bar\epsilon\bar\theta\theta)\xi(x)}_{④}$$

$$① = \theta\partial_\mu\xi\,\epsilon\sigma^\mu\bar\theta = \partial_\mu\xi\,\theta\epsilon\sigma^\mu\bar\theta$$

$$\overset{(17\ell)}{=} -\frac{1}{2}\partial_\mu\xi\,\epsilon\theta\sigma^\mu\bar\theta + \partial_\mu\xi\,\sigma^{\mu\nu}\epsilon\theta\sigma_\nu\bar\theta$$

$$= -\frac{1}{2}\theta\sigma^\mu\bar\theta\,\partial_\mu\xi\,\epsilon - \theta\sigma^\mu\bar\theta\,\underbrace{\partial^\nu\xi\,\sigma_{\nu\mu}\epsilon}_{= -\epsilon\sigma_{\mu\nu}\partial^\nu\xi}$$

$$② = \bar\theta\partial_\mu\bar\chi\,\theta\sigma^\mu\bar\epsilon = \theta\sigma^\mu\bar\epsilon\,\bar\theta\partial_\mu\bar\chi$$

$$= (\partial_\mu\chi\,\theta\epsilon\sigma^\mu\bar\theta)^+$$

$$= \left(-\frac{1}{2}\partial_\mu\chi\,\epsilon\theta\sigma^\mu\bar\theta + \partial_\mu\chi\,\sigma^{\mu\nu}\epsilon\theta\sigma_\nu\bar\theta\right)^+$$

$$= -\frac{1}{2}\theta\sigma^\mu\bar\theta\,\bar\epsilon\partial_\mu\bar\chi + \theta\sigma_\nu\bar\theta\,\bar\epsilon\bar\sigma^{\mu\nu}\partial_\mu\bar\chi$$

$$③ = \theta\epsilon\bar\theta\bar\lambda + \epsilon\theta\bar\theta\bar\lambda = 2\theta\epsilon\bar\theta\bar\lambda$$

$$\overset{(17k)}{=} \theta\sigma^\mu\bar\theta\,\epsilon\sigma_\mu\bar\lambda$$

$$④ = 2\bar\theta\bar\epsilon\theta\xi = 2\theta\xi\bar\theta\bar\epsilon$$

$$= \theta\sigma^\mu\bar\theta\,\xi\sigma_\mu\bar\epsilon$$

$$\Rightarrow \delta A_\mu = \xi\sigma_\mu\bar\epsilon + \epsilon\sigma_\mu\bar\lambda + \frac{i}{\sqrt{2}}\bar\epsilon\partial_\mu\bar\chi - i\sqrt{2}\,\bar\epsilon\bar\sigma^{\mu\nu}\partial_\mu\bar\chi - \frac{i}{\sqrt{2}}\partial_\mu\xi\epsilon + i\sqrt{2}\,\epsilon\sigma_{\mu\nu}\partial^\nu\xi$$

$$\sim \mathcal{O}(\theta^2, \bar\theta^1) = \theta\theta\bar\theta\,\delta\lambda(x)$$

$$= i\,\theta\theta\partial_\mu M\,\epsilon\sigma^\mu\bar\theta + \frac{1}{2}(\theta\theta\bar\epsilon\bar\theta + \theta\theta\bar\theta\bar\epsilon)D$$

$$+ -i\,\theta\sigma^\mu\bar\theta\,\partial_\nu A_\mu\,\theta\sigma^\nu\bar\epsilon$$

$$= \theta\theta\bar\theta\left(-i\partial_\mu M\bar\sigma^\mu\epsilon + \bar\epsilon D\right) - i\underbrace{\theta\sigma^\mu\bar\theta\,\partial_\nu A_\mu\,\theta\sigma^\nu\bar\epsilon}$$

Why can't use (17k)? $\theta \neq \epsilon$!!!

$$= \partial_\nu A_\mu\,\theta\sigma^\mu\bar\theta\,\theta\sigma^\nu\bar\epsilon$$

$$= \partial_\nu A_\mu\,\theta\sigma^\nu\bar\epsilon\,\theta\sigma^\mu\bar\theta$$

$$= \partial_\nu A_\mu\,\bar\epsilon^{\dot A}(\theta\sigma^\nu)_{\dot A}\,\bar\theta\bar\sigma^\mu\theta$$

$$= -\partial_\nu A_\mu\,\bar\epsilon^{\dot A}\theta\theta\left[\tfrac{1}{2}\bar\theta_{\dot A}\,g^{\mu\nu} + i(\bar\theta\bar\sigma^{\mu\nu})_{\dot A}\right]$$

$$= -\tfrac{1}{2}\partial_\mu A^\mu\,\theta\theta\bar\theta\bar\epsilon - i\,\theta\theta\bar\theta\bar\sigma^{\mu\nu}\bar\epsilon\,\partial_\nu A_\mu$$

$$\Rightarrow \delta\bar\lambda = -i\partial_\mu M\bar\sigma^\mu\epsilon + \bar\epsilon D - \tfrac{i}{2}\partial_\mu A^\mu\theta - \partial_\nu A_\mu\,\sigma^{\mu\nu}\theta$$

$$\sim \mathcal{O}(\theta^1,\bar\theta^2) = \bar\theta\bar\theta\theta\,\delta\varsigma(x)$$

$$= -i\,\bar\theta\bar\theta\,\partial_\mu N\,\theta\sigma^\mu\bar\epsilon + \tfrac{1}{2}\left(\theta\epsilon\bar\theta\bar\theta + \epsilon\theta\bar\theta\bar\theta\right)D(x)$$

$$+ i\,\theta\sigma^\mu\bar\theta\,\partial_\nu A_\mu\,\epsilon\sigma^\nu\bar\theta$$

$$= \bar\theta\bar\theta\theta\left(-i\partial_\mu N\sigma^\mu\bar\epsilon + \epsilon D\right) + i\partial_\nu A_\mu\,\theta\,\underbrace{\sigma^\mu\bar\theta\,\epsilon\sigma^\nu\bar\theta}$$

$$\Rightarrow \delta\varsigma(x) = -i\partial_\mu N\sigma^\mu\bar\epsilon + \epsilon D + \tfrac{i}{2}\epsilon\partial^\mu A_\mu - \sigma^{\mu\nu}\epsilon\,\partial_\mu A_\nu$$

$$\sim \mathcal{O}(\theta^2,\bar\theta^2) = \tfrac{1}{2}\theta\theta\bar\theta\bar\theta\,\delta D(x)$$

$$= i\,\theta\theta\underbrace{\bar\theta\,\partial_\mu\bar\lambda\,\epsilon\sigma^\mu\bar\theta} - i\,\bar\theta\bar\theta\underbrace{\theta\,\partial_\mu\varsigma\,\theta\sigma^\mu\bar\epsilon}$$

$$= -\bar\theta\,\partial_\mu\bar\lambda\,\bar\theta(\bar\sigma^\mu\epsilon) \qquad = \theta\,\partial_\mu\varsigma\,\theta(\sigma^\mu\bar\epsilon)$$

(17h)

$$= \tfrac{1}{2}\partial_\mu\bar\lambda\,\bar\sigma^\mu\epsilon\,\bar\theta\bar\theta \qquad = -\tfrac{1}{2}\partial_\mu\varsigma\,\sigma^\mu\bar\epsilon\,\theta\theta$$

$$= \tfrac{1}{2}\bar\theta\bar\theta\,\partial_\mu\bar\lambda\,\bar\sigma^\mu\epsilon \qquad = -\tfrac{1}{2}\theta\theta\,\partial_\mu\varsigma\,\sigma^\mu\bar\epsilon$$

$$= \tfrac{i}{2}\theta\theta\bar\theta\bar\theta\left(\partial_\mu\bar\lambda\,\bar\sigma^\mu\epsilon + \partial_\mu\varsigma\,\sigma^\mu\bar\epsilon\right)$$

$$\Rightarrow \delta D = i\left(\partial_\mu\bar\lambda\,\bar\sigma^\mu\epsilon + \partial_\mu\varsigma\,\sigma^\mu\bar\epsilon\right)$$

In this assignment, I have used (all being 2-comp. spinors)

$$\xi\zeta\chi\tau = (\xi\zeta)^A(\chi\tau)_A$$
$$= -(\chi\tau)_A(\xi\zeta)^A$$
$$= (\chi\tau)^A(\xi\zeta)_A$$
$$= \chi\tau\xi\zeta$$