Lab report
K223 Nuclear γ-γ Angular Correlations

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1 Introduction

2 Theory

Anisotropy of photon distribution would not be present, if there is thermal equilibrium among states. But equilibrium is not achieved in cascaded γ-γ , since the state of firstly emitted (firstly detected) photon constraints the angular momentum distribution of intermediate state because of angular momentum conservation[1].

Angular correlation of gamma rays of multipole moments $L_{1,2}$ from γ-γ cascade $I_i \to I \to I_f$ is defined as

$$W(\theta) = 1 + \sum_{k=2,\text{ even}}^{k_{\max}} A_{kk} P_k(\cos\theta) \tag{2.1}$$

with A_{kk} (known given the information of nucleus) coefficients, $P_k(\cos\theta)$ the Legendre polynomials, and $k_{\max} = \min(2I, 2L_1, 2L_2)$[2].

Coefficient A_{kk} is determined, generally with mixed multipole components L'_n and L_n $(n = 1, 2)$, by

$$A_{kk} = A_k(L_1 L'_1 I_i I) A_k(L_2 L'_2 I_f I) \tag{2.2}$$

$$A_k(L_n L'_n I_{i,f} I) = \frac{F_k(L_n L_n I_{i,f} I) + 2\delta_1(\gamma) F_k(L_n L'_n I_{i,f} I) + \delta_1^2(\gamma) F_k(L'_n L'_n I_{i,f} I)}{1 + \delta_1^2(\gamma)} \tag{2.3}$$

$$\begin{aligned} F_k(LL'I'I) = (-1)^{I'+I-1} \left[(2L+1)(2L'+1)(2I+1)(2k+1)\right]^{1/2} \\ \times \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I & I & I' \end{Bmatrix} \end{aligned} \tag{2.4}$$

$$\delta_1(\gamma) = \frac{\langle I | L'_1 \pi'_1 | I_{i,f} \rangle}{\langle I | L_1 \pi_1 | I_{i,f} \rangle} \tag{2.5}$$

with round brackets being $3j$-symbols and curly brackets $6j$-symbols[2]. Their value can be easily found tabulated, e.g. in [3] and [4]. $\delta_n(\gamma)$ quantifies the mixing of two multipole moments and should be determined by some other methods. If we assume $L'_n = L_n + 1$ (this is reasonable because of selection rules), then there are 7 quantum numbers to nail down the coefficients: $I_i, I, I_f, \delta_{1,2}, L_{1,2}$[2].

Example with $0 \to 1 \to 0$ γ-γ cascade. Since the first and last states are of spin 0, the multipolarities of emitted photon must be 1 (whether it is electrical or magnetic depends on the parity of the quantum states). Thus $k_{\max} = 2$.

$$W(\theta) = 1 + A_{22} P_2(\cos\theta) \tag{2.6}$$

Since the photon can only have these multipole moments, there is no mixing, i.e. $L = L'$. The angular correlation coefficient can be calculated, it is given in [2]

$$W(\theta) = 1 + 0.1020 \cdot \frac{1}{2}\left(3\cos^2\theta - 1\right) = 0.949 + 0.153\cos^2\theta \tag{2.7}$$

Hyperfine structure can have influence on the angular correlation, since with quantization axis along the direction of first photon the first photon will cause transitions among the m-states. Thus in the end, the direction of second photon is altered [2].

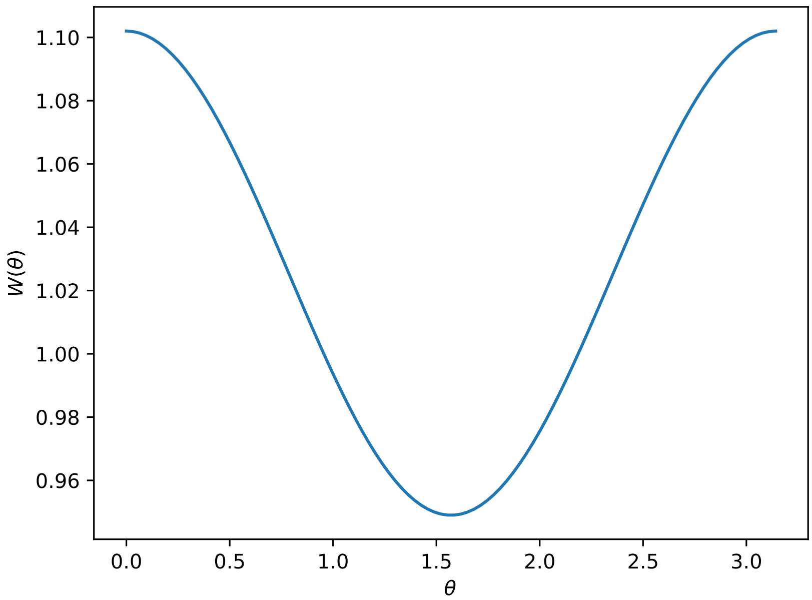

Figure 1: Angular correlation of hypothetical 010 cascade

Information can be obtained from measurement of γ-γ angular correlations (without extranuclear perturbation): spin angular momenta of excited states, the multipole orders, and the relative multipole composition of radiative transitions[5]. With extranuclear perturbation, we can in addition extract g-factor and quadrupole momentum of intermediate state. Internal fields of solids, liquids, and metal crystals can be investigated. And some changes in atomic shell is possible to study [2].

3 Experimental setups

3.1 Key components

Scintillation detector is used to detect ionizing radiation in general. Here we have gamma radiation. The purpose of scintillator is to lower photon energy via photoelectric effect, Compton scattering, and pair production [6]. It is then connected to photomultiplier tube to generate signals.

Fast-slow coincidence is the technique to measure the ionizing radiation separately. The "slow" part will determine the energy of incoming radiation. And the "fast" part is used to measure the time as precisely as possible, since the photomultiplier will be brought to saturation and the height of the pulse

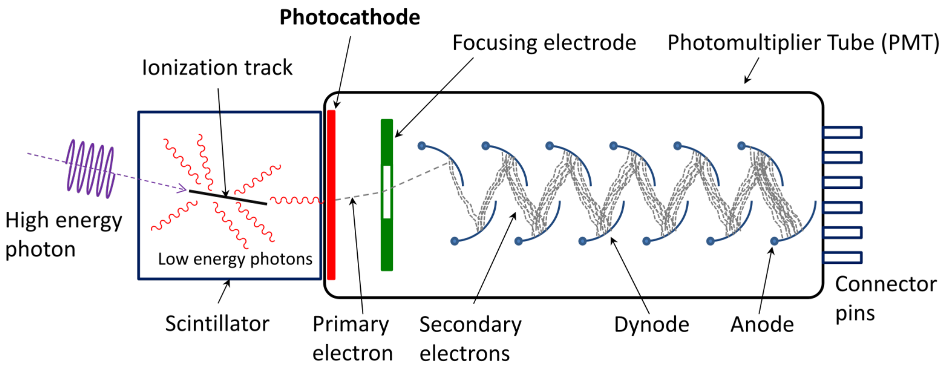

Figure 2: Scintillator with PMT [7]

is not proportional to radiation energy any more. How exactly configure the fast and slow part? By adjusting the voltage?

SCA stands for single channel analyzer

CFD stands for constant fraction discriminator

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