Lab report
K223 Nuclear γ-γ Angular Correlations

Chenhuan Wang and Harilal Bhattarai

August 13, 2020

In this experiment, we investigate nuclear properties of cobalt nuclei via angular correlation.

1 Introduction

2 Theory

Anisotropy of photon distribution would not be present, if there is thermal equilibrium among states. But equilibrium is not achieved in cascaded γ-γ , since the state of firstly emitted (firstly detected) photon constraints the angular momentum distribution of intermediate state because of angular momentum conservation[1].

Angular correlation of gamma rays of multipole moments $L_{1,2}$ from γ-γ cascade $I_i \to I \to I_f$ is defined as

$$W(\theta) = 1 + \sum_{k=2,\text{ even}}^{k_{\max}} A_{kk} P_k(\cos\theta) \tag{1}$$

with A_{kk} (known given the information of nucleus) coefficients, $P_k(\cos\theta)$ the Legendre polynomials, and $k_{\max} = \min(2I, 2L_1, 2L_2)$[2].

Coefficient A_{kk} is determined, generally with mixed multipole components L'_n and L_n ($n = 1, 2$), by

$$A_{kk} = A_k(L_1 L'_1 I_i I) A_k(L_2 L'_2 I_f I) \tag{2}$$

$$A_k(L_n L'_n I_{i,f} I) = \frac{F_k(L_n L_n I_{i,f} I) + 2\delta_1(\gamma) F_k(L_n L'_n I_{i,f} I) + \delta_1^2(\gamma) F_k(L'_n L'_n I_{i,f} I)}{1 + \delta_1^2(\gamma)} \tag{3}$$

$$\begin{aligned} F_k(LL'I'I) = {} & (-1)^{I'+I-1} \left[(2L+1)(2L'+1)(2I+1)(2k+1)\right]^{1/2} \\ & \times \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I & I & I' \end{Bmatrix} \end{aligned} \tag{4}$$

$$\delta_1(\gamma) = \frac{\langle I | L'_1 \pi'_1 | I_{i,f} \rangle}{\langle I | L_1 \pi_1 | I_{i,f} \rangle} \tag{5}$$

with round brackets being $3j$-symbols and curly brackets $6j$-symbols[2]. Their value can be easily found tabulated, e.g. in [3] and [4]. $\delta_n(\gamma)$ quantifies the mixing of two multipole moments and should be determined by some other methods. If we assume $L'_n = L_n + 1$ (this is reasonable because of selection rules), then there are 7 quantum numbers to nail down the coefficients: $I_i, I, I_f, \delta_{1,2}, L_{1,2}$[2].

Example with $0 \to 1 \to 0$ γ-γ cascade. Since the first and last states are of spin 0, the multipolarities of emitted photon must be 1 (whether it is electrical or magnetic depends on the parity of the quantum states). Thus $k_{\max} = 2$.

$$W(\theta) = 1 + A_{22} P_2(\cos\theta) \tag{6}$$

Since the photon can only have these multipole moments, there is no mixing, i.e. $L = L'$. The angular correlation coefficient can be calculated, it is given in [2]

$$W(\theta) = 1 + 0.1020 \cdot \frac{1}{2}\left(3\cos^2\theta - 1\right) = 0.949 + 0.153\cos^2\theta \tag{7}$$

Plot of this angular correlation can be found in figure 1.

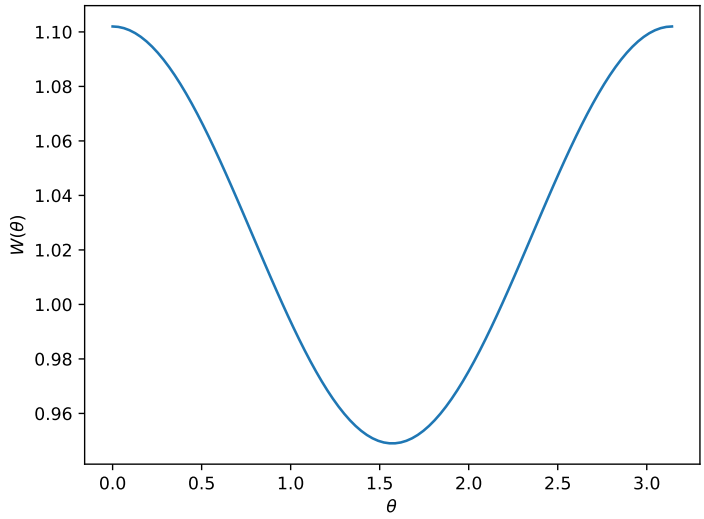

Figure 1: Angular correlation of hypothetical 010 cascade

Hyperfine structure can have influence on the angular correlation, since with quantization axis along the direction of first photon the first photon will cause transitions among the m-states. Thus in the end, the direction of second photon is altered. The perturbed angular correlation is given as

$$W(k_1, k_2, t) = \sum_{m_i, m_f, m_a, m'_a} \langle m_f|H_2\Lambda(t)|m_a\rangle \langle m_a|H_1|m_i\rangle \langle m_f|H_2\Lambda(t)|m'_a\rangle^* \langle m'_a|H_1|m_i\rangle^* \tag{8}$$

where $H_{1,2}$ represents the interaction between nucleus and radiation field and $\Lambda(t)$ is an unitary operator describing influence of extranuclear perturbation. k_1 and k_2 are wave vector of photons. [2].

Information can be obtained from measurement of γ-γ angular correlations (without extranuclear perturbation): spin angular momenta of excited states, the multipole orders, and the relative multipole composition of radiative transitions[5]. With extranuclear perturbation, we can in addition extract g-factor and quadrupole momentum of intermediate state. Internal fields of solids, liquids, and metal crystals can be investigated. And some changes in atomic shell is possible to study [2].

3 Experimental setups

3.1 Key components

Scintillation detector is used to detect ionizing radiation in general. Here we have gamma radiation. The purpose of scintillator is to lower photon energy via photoelectric effect, Compton scattering, and pair production [6]. It is then connected to photomultiplier tube (PMT) to generate signals.

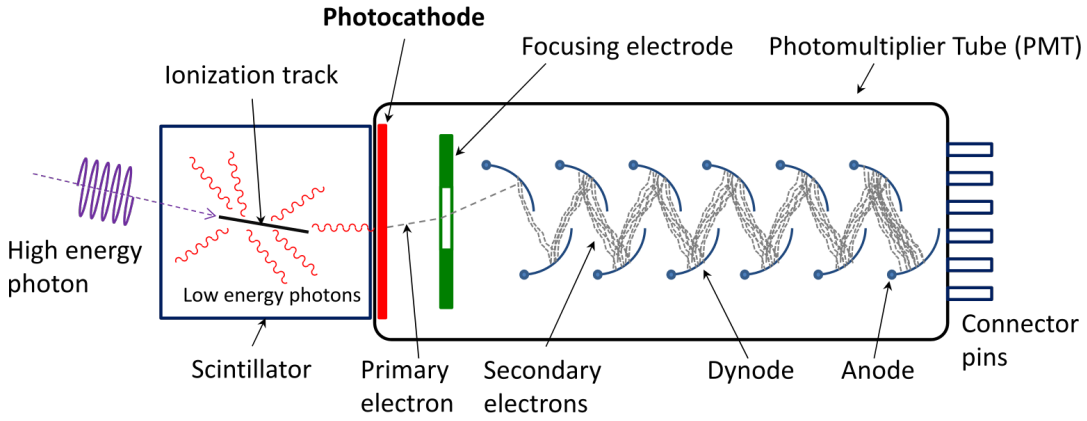

Figure 2: Scintillator with PMT [7]

Fast-slow coincidence is the technique to measure the ionizing radiation separately. The "slow" part will determine the energy of incoming radiation. And the "fast" part is used to measure the time as precisely as possible, since the photomultiplier will be brought to saturation and the height of the pulse is not proportional to radiation energy any more [8]. How exactly configure the fast and slow part? By adjusting the voltage?

SCA stands for single channel analyzer. Basically it is advanced version of simple discriminator, as one can set both upper-level (ULD) and lower-level discriminator (LLD) for SCA. It reads the input pulse and check whether it is within the preset limits or not. If it is, SCA will produce a uniform digital signal. When applied to PMT, the height of pulse corresponds to energy of radiation. If SCA is built in after PMT, we are essentially picking out photons within the SCA window. Thus the name [9].

Simple discriminator outputs logic signal right after input goes higher than threshold. Since SCA also needs to check the ULD, it will produce output after maximal amplitude. There are two modes: non-timing and timing SCA. Non-timing SCA generates output right after the input comes back to the LLD, i.e. after its peak. It will create the so-called "time walk"

meaning that logic output will differ in time, even thought the inputs are simultaneous but of different height. Timing SCA, on the other hand, outputs logic signal as soon as the input hits its maximum [9].

CFD stands for constant fraction discriminator. As its name suggests, it gets triggered at some preset fraction of maximal amplitude, in order to reduce "walk". In simplest form, CFD works by splitting input signals, inverting one of them, and adding delay to the other. In the end, by combining these two together, we get logic signal with minimal walk [10].

TAC stands for time-to-amplitude converter, which outputs pulse with amplitude proportional to time difference between start and end signals.

Time resolution of fast coincidence unit is $< 100\,\text{ns}$ on any single input and $< 1\,\mu\text{s}$ on the coincidence output [11].

Expected Spectrum of ^{60}Co would predominantly consists of 0.31 MeV β-line and 1.1732 MeV, 1.3325 MeV γ-line [12]. Its γ-spectrum can be found in 3, where one can see two clear peaks corresponding to the γ-radiations and some background because of various effects, like pair production (higher E), Compton scattering (mid E), and photoelectric effect (low E) [6].

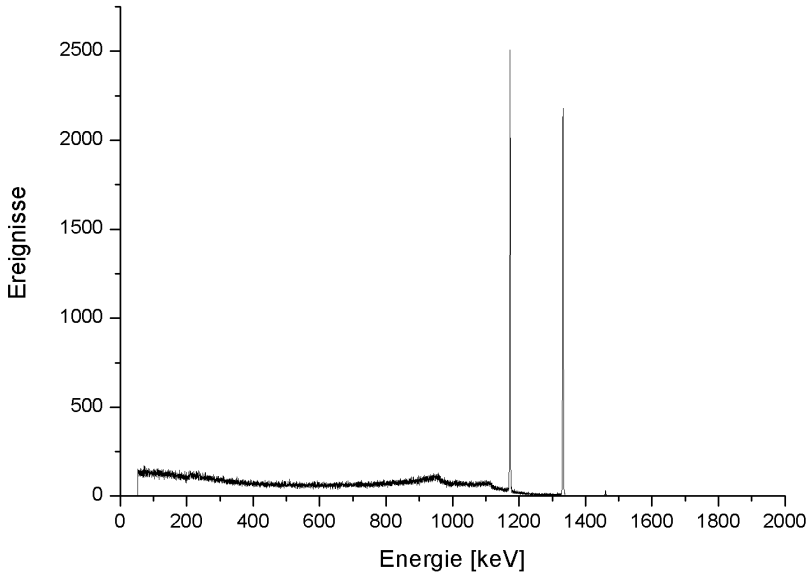

Figure 3: ^{60}Co γ-specturm [13]

4 Task for preparation

4.1 Which distances to pick?

Obviously, the count (rate) should be proportional to the solid angle ignoring the anisotropy of radiation. This assumption should not bring too much influence to the results, as long as the size of detection is accounted for in final analysis as (systematic) error. In [2], true count rate is given by

$$N_i^t(\theta) = Mp_i\Omega_i\epsilon_i \tag{9}$$

with M the number of nuclear integrations per uni time, p_i the probability that this integration is selected, Ω_i is solid angle in unit 4π, and ϵ_i the detector efficiency. According to formula of solid angle, it means that

$$N_i^t(\theta) \propto \frac{1}{r_i^2} \tag{10}$$

Follow the same principle, true number of coincidence can be written as [2]

$$C^t(\theta) = Mp_1p_2\Omega_1\epsilon_1\Omega_2\epsilon_2\epsilon_c K(\theta) \tag{11}$$

where ϵ_c is the efficiency of coincidence unit and $K(\theta)$ is the directional correlation function. $K(\theta)$ is basically the "measured" version of $W(\theta)$, i.e. what we have in the real world. The coincidence rate is then $C^t(\theta)/N_1^t(\theta)$ thus

$$\frac{C^t(\theta)}{N_1^t(\theta)} \propto \frac{1}{r_2^2} \tag{12}$$

The angular "asymmetry" is represented as the coefficients A_{kk}. With correction factor, we write

$$A_{kk} = \frac{A_{kk}^{\exp}}{Q_{kk}} \tag{13}$$

And it can be calculated by [2]

$$Q_{kk} = Q_k(1)Q_k(2) \tag{14}$$

$$Q_k(i) = \frac{J_k(i)}{J_0(i)} \tag{15}$$

$$J_k(i) = \int_0^{\pi/2} \epsilon_i(E,\alpha)P_k(\cos\alpha)\sin\alpha\,\mathrm{d}\alpha \tag{16}$$

$$\epsilon(E,\alpha) = 1 - \exp\{-\tau(E)X(\alpha)\} \tag{17}$$

with $\tau(E)$ the total absorption coefficient and $X(\alpha)$ the distance traversed in the crystal.

The correction factor will certainly affect the error of angular correlation function, e.g. for (6)

$$\delta W(\theta) = \left|\frac{P_2(\cos\theta)}{Q_{kk}}\right| \delta(A_{22}^{\exp}) \tag{18}$$

According to table in [2], $h = 10\,\mathrm{cm}$ would provide the most precise measurement, since the Q_i's are closer to 1. So in the actual experiment, one can try to measure the event rate in a short time period. Then the distance should be chosen, so that enough data will be taken in the given time but still have maximal precision.

4.2 Which angles to pick?

The angular correlation function is given in the form of [1]

$$f(\theta) = A(1 + B\cos^2\theta + C\cos^4\theta) \tag{19}$$

It can be rewritten with $\alpha = B + C$ and $\beta = B - C$,

$$f(\theta) = A\left(1 + \frac{\alpha+\beta}{2}\cos^2\theta + \frac{\alpha-\beta}{2}\cos^4\theta\right) \tag{20}$$

Predicted values for A_{22} and A_{44} without mixing are given in [2]. Then the predicted correlation function is

$$\begin{aligned} W(\theta) &= 1 - \frac{A_{22}}{2} + \frac{3}{8}A_{44} + \left(\frac{3}{2}A_{22} - \frac{15}{4}A_{44}\right)\cos^2\theta + \frac{35}{8}A_{44}\cos^4\theta \\ &= 0.952412 + 0.118875\cos^2\theta + 0.039813\cos^4\theta \end{aligned} \tag{21}$$

Need to "scale" it, so that 0.9524 gets absorbed in A. Then we have

$$B = 0.124815, C = 0.041802 \tag{22}$$

Plot correlation with these two coefficients with slight variation, we have figure. 4 From it,

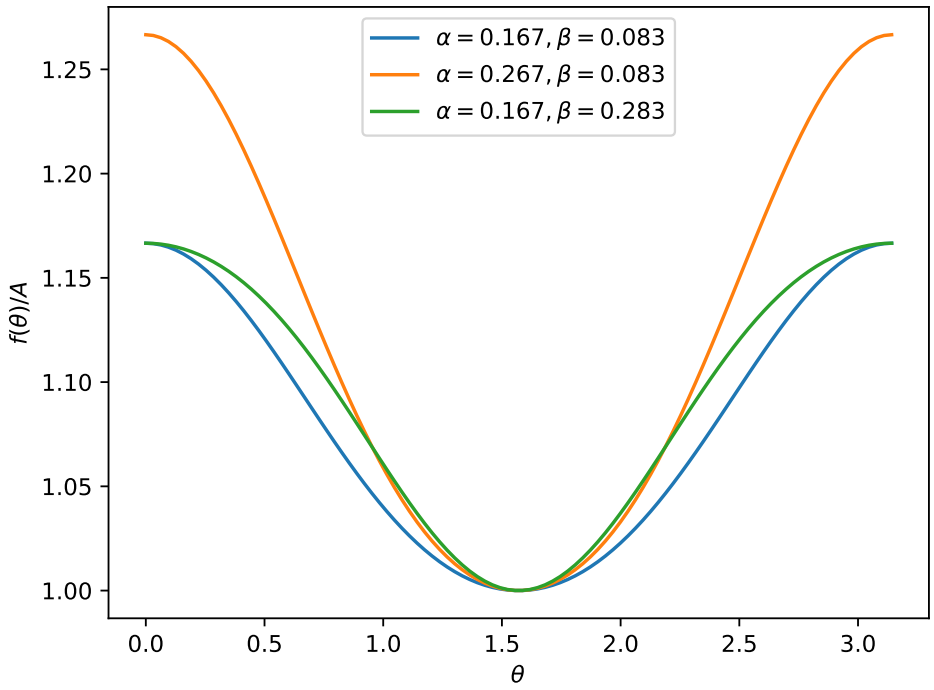

Figure 4: Correlation with different α and β

it is clear that to determine α, we need in principle only one point ($\theta = 0$ or $\theta = \pi$). But determination of β requires small increments in angle between 0 and $\pi/2$.

5 Analysis

Firstly the data needs to be corrected because of random coincidences and misalignment of the setup. Random coincidence rate $\dot{R}$ is simply subtracted from the coincidence rate. In calculation of error of count rate, error of random coincidence is considered as well.

$$\dot{R} = (2.218 \pm 0.061)\,\mathrm{s}^{-1} \tag{23}$$

It is thought to be acceptable to just use one random coincidence rate for all angles, since it has no angular dependence. Effects of misalignment on random coincidence need to be considered. That is why the random coincidence is to be subtracted first.

In figure. 5, one can see the trend in count rate of the mobile detector. This can be easily explained by source not being in the center of the setup. True data should be anti-proportional to count rates in figure. 5. Since measured angular correlation function is determined up to a proportional constant anyway, the normalization factor κ is calculated by the fraction of count rate at smallest angle (this choice is arbitrary) and count rate at respective angle.

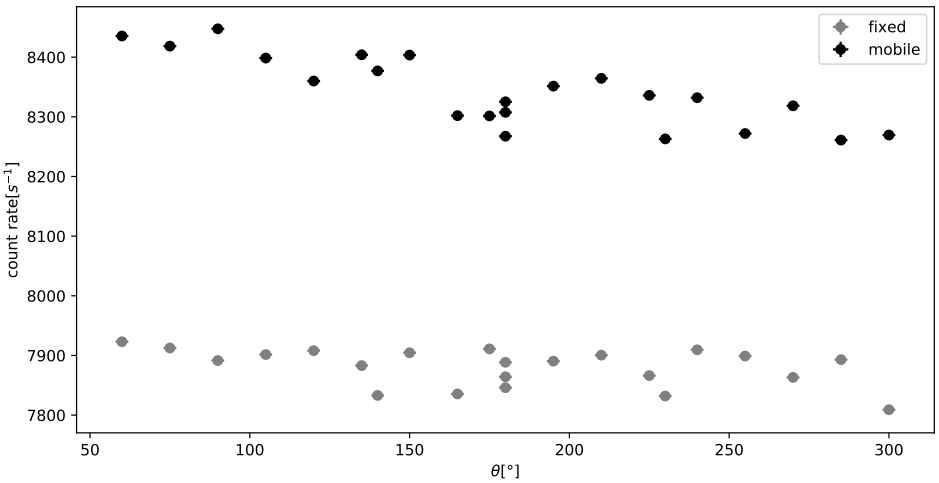

Figure 5: Raw count rate

Coincidence rates are normalized against the raw count rate with κ, in order to counter effects of misalignment. True coincidence rate $\dot{C}_{\mathrm{true}}$ is calculated via

$$\dot{C}_{\mathrm{true}} = \kappa \cdot (\dot{C}_{\mathrm{measured}} - \dot{R}) \tag{24}$$

During experiment, the setup and its environment might (inevitably) change, i.e. temperature. Measurements at 180 are repeated multiple times. Some variations are seen. This introduce (one source of) systematic error and will be included in the further analysis.

$$\Delta\dot{C}_{\mathrm{sys}} = 0.564 \tag{25}$$

Data after these corrections are plotted in figure. 6. Vertical error bars include statistical error and systematic error. Error of angle is estimated to be about 2 degrees.

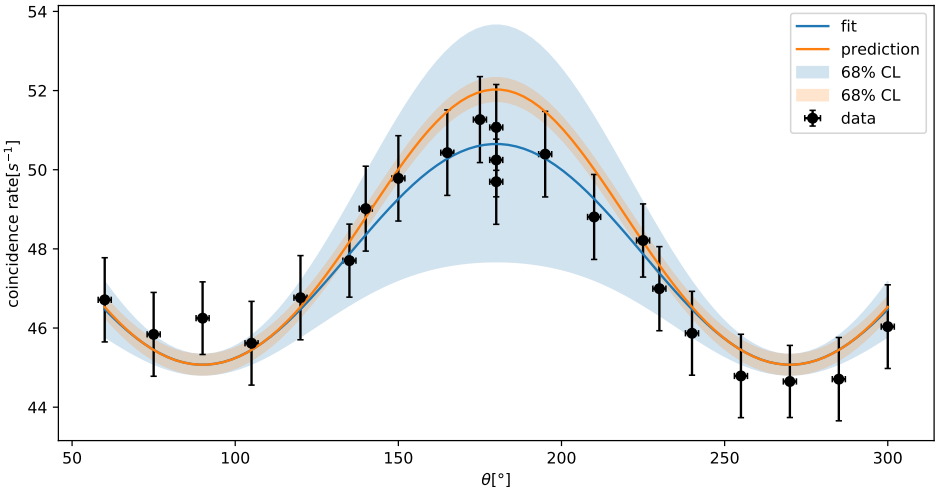

Figure 6: Angular correlation with fit and prediction

A least squares fit of data using function in the form of (19) is carried out. Fitted function is drawn in blue in figure. 6 alongside with its confidence intervals. Parameters are

$$\begin{aligned} A &= 45.072 \pm 0.258 \\ B &= 0.125 \pm 0.031 \\ C &= -0.001 \pm 0.029 \end{aligned} \tag{26}$$

Based on this value of A, we can plot the predicted function equation. (21). Because of finite size of detector, prediction curve is "corrected" by factor Q_k to correspond measured correlation curve. Values of Q_k are taken from [2] with distance to source $h = 5\,\text{cm}$. Energy of γ-radiation in experiment is between 1 MeV and 1.5 MeV and only photopeaks are relevant. For convenience, mean value of Q_k of these two energies is used in calculation.

$$Q_2 = 0.934,\ Q_4 = 0.792 \tag{27}$$

Value of A contains error, thus in figure. 6 CLs are drawn around the prediction curve.

Although there are some deviation between prediction and fit curves, they are well within each other's 1σ range. Around 90 and 270 degrees, they match perfectly as expected. We have used A value from fitting in drawing prediction curve. An independent determination of A might be meaningful, but it is certainly beyond the scope of this experiment.

There is quite obvious asymmetry (with respect to 180 degrees) present in the angular correlation. Ideally, this should not be in data, or at least after being corrected by considering misalignment. To better see the difference, data points are plotted in figure. 7. From this, we can see that the asymmetry can be well explained by the errors.

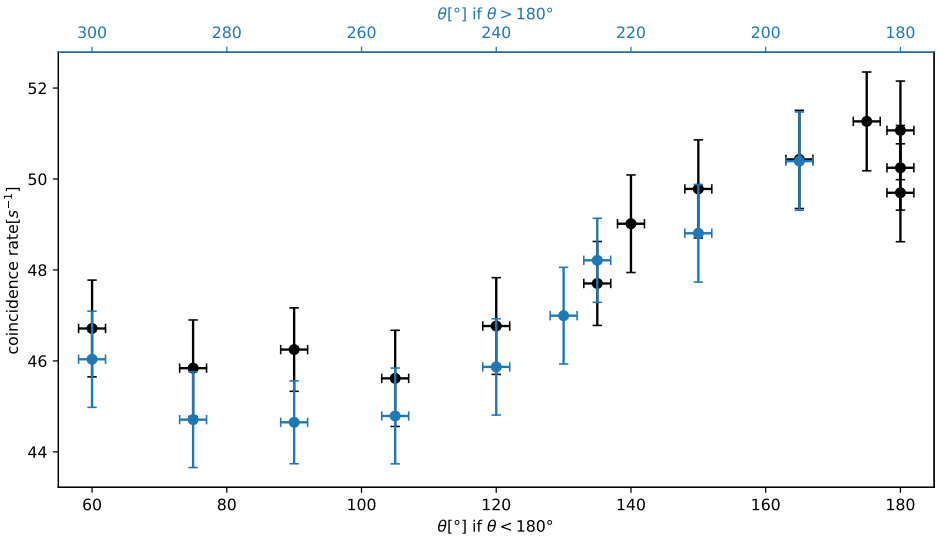

Figure 7: Angular correlation to see asymmetry. Color of data points correspond to the color of axis to use.

References

[1] Unknown. *Experiment description: Nuclear γ-γ Angular Correlations*. 2019.

[2] Kai Siegbahn, ed. *α-, β-, and γ-Ray Spectroscopy*. Vol. 2. North-Holland Publishing Company, 1965.

[3] P.D. Stevenson. "Analytic angular momentum coupling coefficient calculators". In: *Computer Physics Communications* 147.3 (2002), pp. 853–858. ISSN: 0010-4655. DOI: https://doi.org/10.1016/S0010-4655(02)00462-9. URL: http://www.sciencedirect.com/science/article/pii/S0010465502004629.

[4] Plasma Laboratory of Weizmann Institute of Science. *369j-symbol Calculator*. URL: https://plasma-gate.weizmann.ac.il/369j.html.

[5] Robert Allan Wilson. "Directional correlation of the 346-136 keV gamma-gamma cascade in Ta181". MA thesis. Portland State University. Department of Physics, 1969. URL: https://pdxscholar.library.pdx.edu/open_access_etds/75/.

[6] Hermann Kolanoski and Norbert Wermes. *Teilchendetektoren*. Springer Berlin Heidelberg, 2016.

[7] Qwerty123uiop. *File:PhotoMultiplierTubeAndScintillator.svg*. 2013. URL: https://commons.wikimedia.org/wiki/File:PhotoMultiplierTubeAndScintillator.svg.

[8] G. Iaci and M. Lo Savio. "A fast-slow coincidence system". In: *Nuclear Instruments and Methods* 65.1 (1968), pp. 103–109. ISSN: 0029-554X. DOI: https://doi.org/10.1016/0029-554X(68)90014-1. URL: http://www.sciencedirect.com/science/article/pii/0029554X68900141.

[9] Ortec. *Single-Channel Pulse-Height Analyzers*.

[10] Canberra Elektronik. *Constant Fraction Discriminator: Model 1326, 1428* Operating Manual*.

[11] Ortec. *Fast coincidence*.

[12] R.B.firestone. *Table of Isotopes*. 8th ed. Wiley, New York, 1996.

[13] Traitor. *File:60Co gamma spectrum energy.png*. 2007. URL: https://en.wikipedia.org/wiki/File:60Co_gamma_spectrum_energy.png.