Lab report
K223 Nuclear γ-γ Angular Correlations

Chenhuan Wang and Harilal Bhattarai

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1 Introduction

2 Theory

Anisotropy of photon distribution would not be present, if there is thermal equilibrium among states. But equilibrium is not achieved in cascaded γ-γ , since the state of firstly emitted (firstly detected) photon constraints the angular momentum distribution of intermediate state because of angular momentum conservation[1].

Angular correlation of gamma rays of multipole moments $L_{1,2}$ from γ-γ cascade $I_i \to I \to I_f$ is defined as

$$W(\theta) = 1 + \sum_{k=2,\text{ even}}^{k_{\max}} A_{kk} P_k(\cos\theta) \tag{2.1}$$

with A_{kk} (known given the information of nucleus) coefficients, $P_k(\cos\theta)$ the Legendre polynomials, and $k_{\max} = \min(2I, 2L_1, 2L_2)$[2].

Coefficient A_{kk} is determined, generally with mixed multipole components L'_n and L_n $(n = 1, 2)$, by

$$A_{kk} = A_k(L_1 L'_1 I_i I) A_k(L_2 L'_2 I_f I) \tag{2.2}$$

$$A_k(L_n L'_n I_{i,f} I) = \frac{F_k(L_n L_n I_{i,f} I) + 2\delta_1(\gamma) F_k(L_n L'_n I_{i,f} I) + \delta_1^2(\gamma) F_k(L'_n L'_n I_{i,f} I)}{1 + \delta_1^2(\gamma)} \tag{2.3}$$

$$\begin{aligned} F_k(LL'I'I) = {} & (-1)^{I'+I-1} \left[(2L+1)(2L'+1)(2I+1)(2k+1)\right]^{1/2} \\ & \times \begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} L & L' & k \\ I & I & I' \end{Bmatrix} \end{aligned} \tag{2.4}$$

$$\delta_1(\gamma) = \frac{\langle I | L'_1 \pi'_1 | I_{i,f} \rangle}{\langle I | L_1 \pi_1 | I_{i,f} \rangle} \tag{2.5}$$

with round brackets being $3j$-symbols and curly brackets $6j$-symbols[2]. Their value can be easily found tabulated, e.g. in [3] and [4]. $\delta_n(\gamma)$ quantifies the mixing of two multipole moments and should be determined by some other methods. If we assume $L'_n = L_n + 1$ (this is reasonable because of selection rules), then there are 7 quantum numbers to nail down the coefficients: $I_i, I, I_f, \delta_{1,2}, L_{1,2}$[2].

Example with $0 \to 1 \to 0$ γ-γ cascade. Since the first and last states are of spin 0, the multipolarities of emitted photon must be 1 (whether it is electrical or magnetic depends on the parity of the quantum states). Thus $k_{\max} = 2$.

$$W(\theta) = 1 + A_{22} P_2(\cos\theta) \tag{2.6}$$

Since the photon can only have these multipole moments, there is no mixing, i.e. $L = L'$. The angular correlation coefficient can be calculated, it is given in [2]

$$W(\theta) = 1 + 0.1020 \cdot \frac{1}{2}\left(3\cos^2\theta - 1\right) = 0.949 + 0.153\cos^2\theta \tag{2.7}$$

Plot of this angular correlation can be found in figure 1.

Hyperfine structure can have influence on the angular correlation, since with quantization axis along the direction of first photon the first photon will cause transitions among the m-states. Thus in the end, the direction of second photon is altered [2].

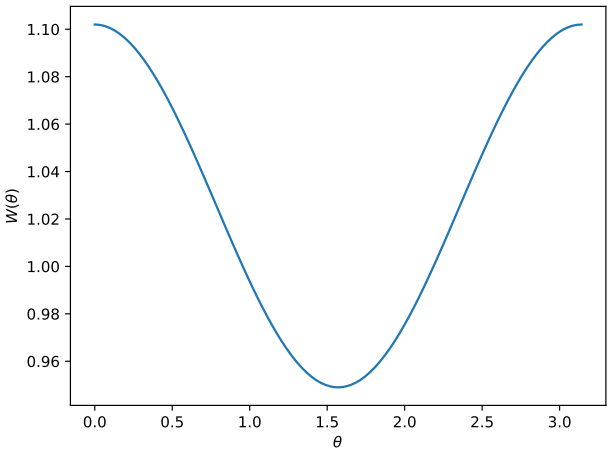

Figure 1: Angular correlation of hypothetical 010 cascade

Information can be obtained from measurement of γ-γ angular correlations (without extranuclear perturbation): spin angular momenta of excited states, the multipole orders, and the relative multipole composition of radiative transitions[5]. With extranuclear perturbation, we can in addition extract g-factor and quadrupole momentum of intermediate state. Internal fields of solids, liquids, and metal crystals can be investigated. And some changes in atomic shell is possible to study [2].

3 Experimental setups

3.1 Key components

Scintillation detector is used to detect ionizing radiation in general. Here we have gamma radiation. The purpose of scintillator is to lower photon energy via photoelectric effect, Compton scattering, and pair production [6]. It is then connected to photomultiplier tube (PMT) to generate signals.

Fast-slow coincidence is the technique to measure the ionizing radiation separately. The ”slow” part will determine the energy of incoming radiation. And the ”fast” part is used to measure the time as precisely as possible, since the photomultiplier will be brought to saturation and the height of the pulse is not proportional to radiation energy any more [8]. How exactly configure the fast and slow part? By adjusting the voltage?

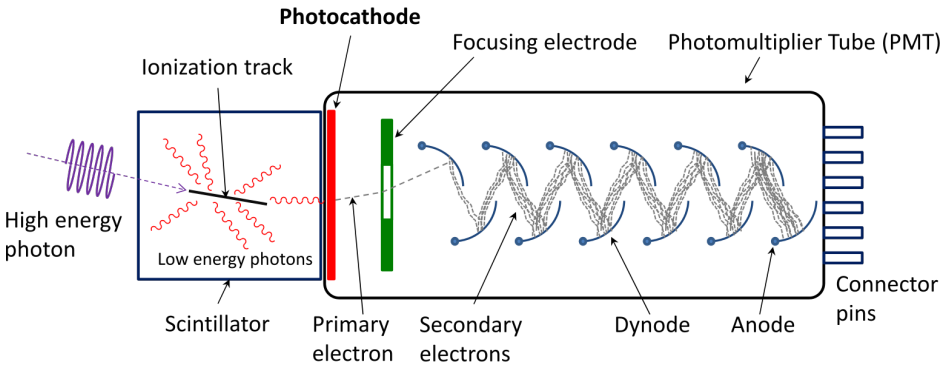

Figure 2: Scintillator with PMT [7]

SCA stands for single channel analyzer. Basically it is advanced version of simple discriminator, as one can set both upper-level (ULD) and lower-level discriminator (LLD) for SCA. It reads the input pulse and check whether it is within the preset limits or not. If it is, SCA will produce a uniform digital signal. When applied to PMT, the height of pulse corresponds to energy of radiation. If SCA is built in after PMT, we are essentially picking out photons within the SCA window. Thus the name [9].

Simple discriminator outputs logic signal right after input goes higher than threshold. Since SCA also needs to check the ULD, it will produce output after maximal amplitude. There are two modes: non-timing and timing SCA. Non-timing SCA generates output right after the input comes back to the LLD, i.e. after its peak. It will create the so-called "time walk" meaning that logic output will differ in time, even thought the inputs are simultaneous but of different height. Timing SCA, on the other hand, outputs logic signal as soon as the input hits its maximum [9].

CFD stands for constant fraction discriminator. As its name suggests, it gets triggered at some preset fraction of maximal amplitude, in order to reduce "walk". In simplest form, CFD works by splitting input signals, inverting one of them, and adding delay to the other. In the end, by combining these two together, we get logic signal with minimal walk [10].

TAC stands for time-to-amplitude converter, which outputs pulse with amplitude proportional to time difference between start and end signals.

Time resolution of fast coincidence unit is $< 100\,\text{ns}$ on any single input and $< 1\,\mu\text{s}$ on the coincidence output [11].

Expected Spectrum of ^{60}Co would predominantly consists of 0.31 MeV β-line and 1.1732 MeV, 1.3325 MeV γ-line [12]. Its γ-spectrum can be found in 3, where one can see two clear peaks corresponding to the γ-radiations and some background because of various effects, like pair production (higher E), Compton scattering (mid E), and photoelectric effect (low E) [6].

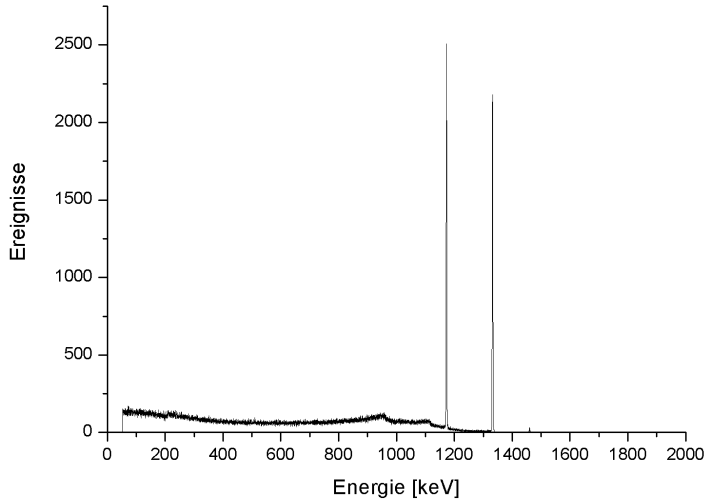

Figure 3: ^{60}Co γ-specturm [13]

4 Task for preparation

4.1 Which distances to pick?

Obviously, the count (rate) should be proportional to the solid angle ignoring the anisotropy of radiation. This assumption should not bring too much influence to the results, as long as the size of detection is accounted for in final analysis as (systematic) error. In [2], true count rate is given by

$$N_i^t(\theta) = M p_i \Omega_i \epsilon_i \tag{4.1}$$

with M the number of nuclear integrations per uni time, p_i the probability that this integration is selected, Ω_i is solid angle in unit 4π, and ϵ_i the detector efficiency. According to formula of solid angle, it means that

$$N_i^t(\theta) \propto \frac{1}{r_i^2} \tag{4.2}$$

Follow the same principle, true number of coincidence can be written as [2]

$$C^t(\theta) = Mp_1p_2\Omega_1\epsilon_1\Omega_2\epsilon_2\epsilon_c K(\theta) \tag{4.3}$$

where ϵ_c is the efficiency of coincidence unit and $K(\theta)$ is the directional correlation function. $K(\theta)$ is basically the "measured" version of $W(\theta)$, i.e. what we have in the real world. The coincidence rate is then $C^t(\theta)/N_1^t(\theta)$ thus

$$\frac{C^t(\theta)}{N_1^t(\theta)} \propto \frac{1}{r_2^2} \tag{4.4}$$

The angular "asymmetry" is represented as the coefficients A_{kk}. With correction factor, we write

$$A_{kk} = \frac{A_{kk}^{\exp}}{Q_{kk}} \tag{4.5}$$

And it can be calculated by [2]

$$Q_{kk} = Q_k(1)Q_k(2) \tag{4.6}$$

$$Q_k(i) = \frac{J_k(i)}{J_0(i)} \tag{4.7}$$

$$J_k(i) = \int_0^{\pi/2} \epsilon_i(E,\alpha)P_k(\cos\alpha)\sin\alpha\,\mathrm{d}\alpha \tag{4.8}$$

$$\epsilon(E,\alpha) = 1 - \exp\{-\tau(E)X(\alpha)\} \tag{4.9}$$

with $\tau(E)$ the total absorption coefficient and $X(\alpha)$ the distance traversed in the crystal.

The correction factor will certainly affect the error of angular correlation function, e.g. for (2.6)

$$\delta W(\theta) = \left|\frac{P_2(\cos\theta)}{Q_{kk}}\right| \delta(A_{22}^{\exp}) \tag{4.10}$$

According to table in [2], $h = 10\,\mathrm{cm}$ would provide the most precise measurement, since the Q_i's are closer to 1. So in the actual experiment, one can try to measure the event rate in a short time period. Then the distance should be chosen, so that enough data will be taken in the given time but still have maximal precision.

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