Nuclear γ-γ Angular Correlations

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1 Aim

Electromagnetic multipole radiation has a non-isotropic angular distribution. Considering a gamma decay of a nucleus, the orientation of the emission distribution arises from the spin orientation of the parent nucleus.
In thermal equilibrium, the net orientation is zero as all spin states are populated with an equal density. All spin configurations are equally probable and in this case, the gamma ray distribution of an ensemble of nuclei is isotropic.

A non-equilibrium spin state can be achieved in a cascaded gamma decay of a nucleus. Detecting the fist photon of the cascade yields constraints on the spin probability of the resulting meta stable state of the nucleus. Therefore, a non isotropic angular distribution of the second photon can be measured if the first one was detected in a specific direction.

The aim of the experiment is to prepare the experiment and to measure the angular correlation of the $\gamma - \gamma$ cascade of ^{60}Co.

2 Knowledge

- Definition of the angular correlation of a γ-γ cascade. Explanation of the angular correlation of a hypothetical $0 \to 1 \to 0$ γ-γ cascade. Which quantities enter into the angular correlation coefficients? Perturbation of the angular correlation by hyperfine interaction.
- What information can one obtain by the measurement of angular correlation - with and without extranuclear perturbation?
- Design and operation of a scintillation spectrometer; fast-slow coincidence technique; components of the setup (SCA, CFD, ...); time resolution of the detector and of the coincidence unit; expected spectrum of ^{60}Co;
- Analysis of the experimental data: corrections for solid angle, accidental coincidences and deadjustment; Determination of the angular correlation coefficients by means of least-squares-fit to the data, determination of experimental errors.
- Decay-scheme of ^{60}Co (Fig. 2)

3 Literature

- *Siegbahn Vol. 2: α-,β- and γ-Ray Spectroscopy*, pp. 1029-1035, 1101-1104, 1190-1195, 1695
- Melissinos: *Experiments in Modern Physics*, pp. 412-429, 461-476
- Riezler/ Kopitzki: *Kernphysikalisches Praktikum*
- Schatz/ Weidinger: *Nuclear Condensed Matter Physics: Nuclear Methods and Applications*, Wiley, 1^{st} edition (1996), pp. 14-20, 63-68, 80-85
- Leo: *Techniques for Nuclear and Particle Physics Experiments*, Springer, 2^{nd} Rev. edition (1994)

4 Tasks for preparation

This experiment requires performing a few tasks before the experiment day, in addition to the preparation of the theory. These tasks are listed in the following.

The text requires knowledge about the experiment, so it makes sense to read the theory and the description of the experiment first.

4.1 Which distances to choose?

The angular correlation function is given for point like detectors. In addition to being impossible to build, those come with the problem that they cover zero solid angle resulting in zero count rate. The bigger a detector is, the bigger is the covered solid angle, therefore the count rate and therefore the time is shorter to achieve a given statistical precision.
Alternatively to increasing the size of the detector, a given detector can be moved closer to the source.

However, the increased count rate comes with a drawback: The increased covered solid angle smears out the angular correlation. This influence can be accounted for by correction factors from numerical calculations. Two factors limit the profit of these factors:

1. The factors are given without an error estimate.
2. The benefit of higher statistical precision resulting from a higher count rate comes with the drawback of an increasing angular coverage of the detector. This reduces the measurable asymmetry and therefore requires lower statistical errors to extract the same information from the experiment.

Use the following hints to examine this problem quantitatively:

- How does the count rate of one detector change with the distance?
- How does the coincidence rate change with the distance? Give a formula!
- How does the asymmetry seen in the experiment differ from the theoretical prediction? Again, give a formula.

- How do the correction factors influence the error of the corrected correlation coefficients?
- Using the values provided in Siegbahn for 1.5"x1" crystals: which distance is likely to yield most precise results?
- If you cannot get an answer with mathematical certainty, give an argument what seems reasonable.

4.2 Which angles to pick?

Among many compromises that have to be taken in this experiment, one is between measurement time per angle on the one side and number of angles on the other side. The total time available is limited, so you can either increase the statistical precision for few measurements or make many measurements which have a larger statistical error.

Deciding what is the optimal compromise requires a careful definition of the goal of the measurement.

Without any theoretical assumptions, measurements over the whole angular range are necessary, performed in small angular steps, in order to get a full picture of the angular correlation.

However, the picture changes if one assumes a certain angular distribution that is predicted by theory. Data points at certain angles give a stronger constraint on the fit parameters than those at other angles.

Imagine you want to measure the slope of a street. If you can assume that the street is a line (i.e. can be described by a linear function), you directly know that two points are enough to determine the slope. Even more, you know that it makes sense to put these two points as far apart as possible to achieve a good precision.

As the function describing the angular correlation is more complex, also it is more involved to find the optimal points to perform measurements.

To get into this, consider the theoretical prediction for the angular correlation of the 4-2-0-cascade which is given by

$$f(\theta) = A \cdot \left(1 + B \cdot \cos^2\theta + C \cdot \cos^4\theta\right) \tag{1}$$

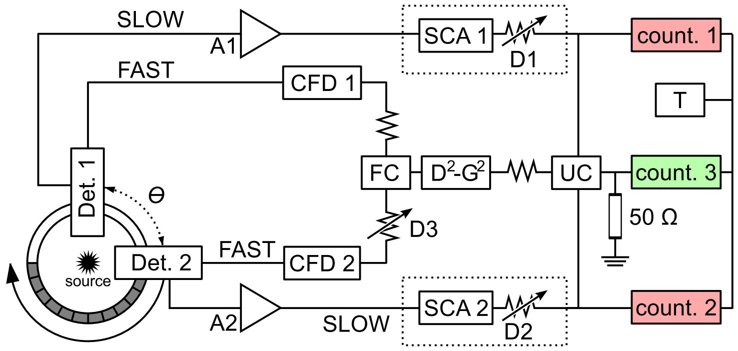
A is a scaling factor which is proportional to measurement time and intensity of the source. The information about the angular correlation is contained in parameters B and C, defining the intensity of the two contributions.
This can also be expressed differently by defining other coefficients:

$$\alpha = B + C$$
$$\beta = B - C.$$

Assuming that errors can be propagated in the Gaussian way, determining α and β precisely yields precise values for B and C.

To find out which spots are best suited to determine α and β, do the following steps:

- Express the angular correlation using the coefficients A, α, and β.
- Plot the function using the predicted values for α and β. Use $A = 1$.



A:	**a**mplifier	D²-G²:	**g**ate & **d**elay generator
SCA:	**s**ingle **c**hannel **a**nalyzer	FC:	**f**ast **c**oincidence
CFD:	**c**onstant **f**raction **d**iscriminator	UC:	**u**niversal **c**oincidence
D:	**d**elay	T:	**t**imer

Figure 1: Final setup of the coincidence measurement

- Plot the function again, but once with slightly varied α and once with slightly varied β
- How does the function change? At which points do you see the largest change?

4.3 How to correct for de-adjustment?

source not in the

In a practical setup, the alignment of the source can only be done to a certain precision. **How will a misalignment manifest in the data? How can you correct it?** Give formulas for the the count rate in each detector $f_{\text{movable}}(\theta)$ and $f_{\text{fixed}}(\theta)$ as well as the coincidence rate $f_{\text{coinc.}}(\theta)$.
To simplify the calculation, you can assume that there is no angular correlation (i. e. $B = C = 0$) and that there are no random coincidences.

5 Experimental Procedure

In this experiment, the angular correlation in the γ-γ cascade of ^{60}Co is investigated. Get familiar with theory using e. g. reference Siegbahn. Also, using external literature, get familiar with the principle of fast-slow coincidences, as well with the electronic modules used in the setup.

5.1 Preparation of the setup

Fig. 1 shows a schematic of the setup. Be careful to only connect inputs and outputs with matching polarities. Some of the devices have positive logic levels, other negative levels.

The tasks to prepare the setup are described in the following.

5.1.1 Energy Branch - Adjusting the Gain

slow circuit

Larger signals are easier to process. For example, a pick-up noise signal of 10mV will yield a more pronounced disturbance on 100 mV signal than on a 1 V signal. Therefore, turn up the gain of the main amplifier (A1, A2) as much as possible. Be aware that the upper limit for the gain is given by the signal range of the electronics used in the setup.

The amplifiers output is linear up to $V_{\max} = 9\,\text{V}$. Using the oscilloscope, verify that the relevant energies of this experiment are in the linear range.

Add plots of the signals to your report.

5.1.2 Timing Branch - Adjustment of the Constant-Fraction-Discriminator (CFD)

Additionally to a zero crossing detector, a CFD has a threshold-discriminator which allows to filter out zero crossings that belong to electronic noise and not to a genuine scintillation signal. This threshold has to be adjusted such, that most noise events are filtered out while genuine events are still detected. Removing noise requires a threshold as high as possible, while detecting even small signals requires a threshold as low as possible.

Performing a threshold scan allows to find the proper compromise.

To do so, connect the negative outputs of the CFDs to the counters. Measure the count rate in dependence of the threshold of the CFD, once with and once without ^{60}Co-Source installed. Chose an appropriate gate duration for this measurement. Remember: $\Delta N \approx \sqrt{N}$. A relative accuracy of 10% is sufficient to determine the proper threshold.

Add plots of both channels to your report, mark the threshold settings you chose for later measurement.

5.1.3 Setting up the Fast Coincidence

Any electronics need some time to process signals. Also cables introduce a certain delay due to the propagation speed of signals. Consider that different components may have a different delays, even two modules of the same time.

To properly identify coincidences, it is necessary to add additional delays in such a way, that true coincidences arrive at the same time at the input of the fast coincidence unit. To determine the proper delay, one can measure the coincidence rate while varying the delay.

Connect the outputs of both CFDs to the oscilloscope. Trigger on one of the signals. On the other channel, pulses concentrate in a small time window. These are true coincidences. Events appearing at random times are uncorrelated and will contribute to random coincidences.

Insert a fixed delay into one of the fast branches and a variable delay into the other one. While the variable delay is set to minimum, the fixed delay should be long enough that real coincidences never arrive within the resolving time of the fast coincidence.

You need to pick a certain resolving time for the first measurements (e.g. 25 ns). Later in this measurement, you will verify if this choice is reasonable.

Connect the outputs of the CFDs to the fast coincidence unit. Measure the count rate for different delay settings of the variable delay unit. Pick an appropriate time for the gate generator, considering the same arguments like in the previous measurement.

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The resulting count rate in dependence of the delay should look like a box with smeared edges. Determine the with and the center of the distribution.

To do so, fit the function

leading edge importanted

$$f(t) = A_1 \cdot \left(\frac{1}{2} + \text{erf}\left(\frac{t - t_0}{\sigma} \right) \cdot \text{erf}\left(\frac{t_1 - t}{\sigma} \right) \right) + A_0 \tag{2}$$

to the data. **Explain what the parameters represent.** Which property of the setup corresponds to the width of the function? Which property to the width of the slopes?

The resolving time should be as short as possible to reduce random coincidences. Also it should be as long as possible to ensure that true coincidences are certainly detected.

How will the shape of the prompt curve change if the resolving time is chosen too short or too long? Was the choice reasonable?

5.1.4 Setting up the Slow Coincidence

To allow the universal coincidence to properly process all incoming signals, the signals from both SCAs and the output of the fast coincidence need to be aligned in timing.

The SCAs have a built-in delay that can be adjusted. To adjust the timing of the fast coincidence unit, connect a gate and delay generator module to the output of the fast coincidence.

Use the oscilloscope to align the timing of these three signals. **Do you need to align the leading edges of the signals or do you need maximum overlap of the pulses?** Add a oscilloscope screenshot of the correctly adjusted pulses.

leading edg

5.1.5 Calibrating the Single Channel Analyzer, selecting the gamma events.

The SCAs are used to select events in which gamma rays of ^{60}Co deposited their whole energy in the corresponding detector. To be able to adjust the thresholds of the SCAs accordingly, you need to perform an energy calibration. To do so, record an energy spectrum using the SCAs.

Therefore, set the SCA to the window mode (set toggle switch to WIN). In this setting, the window size is only set by the second knob, allowing to easily keep the window size constant while varying its position. Having a constant window width is a prerequisite for directly comparing measurements from different window positions.

Normalization!

Why is it important to have the same window size during all measurement points? What would be the effect of a changed size?

Pick a window width. A high width gives a high count rate, but smears out all structures of the energy spectrum in the selected energy range. A window width of 10 ... 20 channels (corresponding to a potentiometer setting of 100...200) might be a reasonable start. **How will you see that you need to change the window size? Write in your report why you conclude that the setting was appropriate to gather all required information.**

Increase size, if everywhere the

Pick a gate time, considering the same argument as in the measurements before.

Perform the measurement for different window positions. Pick enough positions to resolve both photo peaks. You can pick the positions more sparse in the region of the compton continuum.

Add a plot containing both spectra to your report. Insert the chosen SCA windows. Using the positions of the photopeaks and assuming that 0 MeV corresponds to 0 channels, perform a coarse calibration to verify that you identified the photopeaks correctly.

5.2 Main Measurement

5.2.1 Measurement of the angular correlation

Adjust the SCA windows to cover both photopeaks each. Set both detectors to the measurement distance. Perform three types of measurements in total:

1. A measurement over a large angular range with a fine stepping (e.g. 20°), but with comparably short time per position.
2. A measurement at a few certain positions to achieve a precise data.
3. A measurement at one specific position, once as early as possible, once in the middle of the other measurements, and one at the end.

The first measurement allows to get an idea of the overall angular dependence. The second one allows to pin down the correlation coefficients as precisely as possible using the assumption that the correlation can be described by the predicted distribution. The third one allows to get an idea of the stability of the setup. If a repeated measurement yields different results, a similar variation can be expected on other positions, introducing a systematic error. Comment: It is not in question that a systematic error of this kind is present but only how large it is. It is particularly important to find out if the systematic or the statistic error dominates.

5.2.2 Measurement of random coincidences

The measured coincidence rate is the sum of true and random coincidences. To get the true coincidence rate from the measurements, the random coincidence rate needs to be determined. In order to do so, set the variable delay in the fast circuit to such a value that true coincidences will never arrive within the resolving time. **Does an error arise from not changing the delay in the**
No
slow circuit? Argue under which conditions it is negligible. Are these conditions fulfilled? Measure random coincidences until a sufficient statistical precision is achieved. This can be considered the case when the statistical error on the random coincidence rate does not significantly increase the error of the overall result.

Compare the measured value to the value that is expected from single detector count rate and resolving time of the coincidence unit.

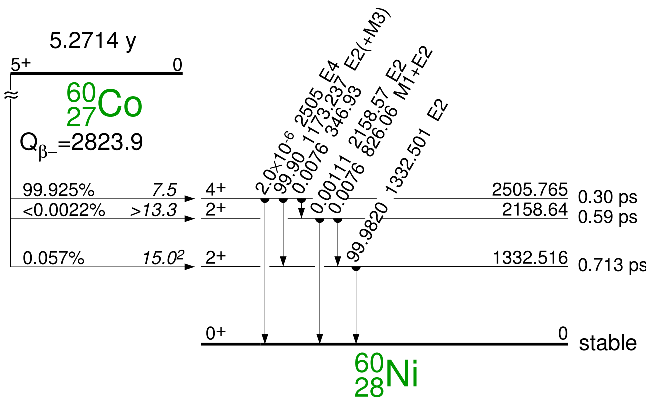

Figure 2: Decay scheme of ^{60}Co, taken from [1]

6 Analysis of the Data

1. Subtract random coincidences from the measured rates and compensate the misalignment of the setup. Can you use the same random coincidence rate for all angles? Alternatively, explain quantitatively why such a correction is not necessary.

2. Apply a least squares fit of the predicted function to the resulting data and apply solid angle corrections. Correction factors can be found in *Siegbahn Vol.2 pp. 1695.*

3. Try to fit angular correlation functions for cascades with different spins e. g. 0-1-0. To which degree can those be excluded?

4. Plot the angular distribution including data, fit, and prediction.

References

[1] R. B. Firestone, Table of Isotopes 8th edition, (Wiley, New York, 1996)

[2] K. Siegbahn, ALPHA-, BETA-, AND GAMMA-RAY SPECTROSCOPY, Vol. 2, North Holland Publishing Company, Amsterdam (1965), p 1695