Lab Report
S261: Optical Astronomy and Gravitational Lensing

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1 Theoretical Background

1.1 Cosmic Expansion

The general expression for a spatially homogeneous and isotropic universe can be written as [**manual**]

$$ds^2 = c^2dt^2 - a^2(t)[dw^2 + f_K^2(w)(d\theta + sin^2\theta d\psi^2)] \tag{1}$$

Where t is cosmic time and a(t) is the cosmic scale factor which describe the isotropic expansion. Finally, the 'K' denotes the space-time curvature is given by,

$$f_K(w) = \left\{ \begin{array}{cc} \frac{1}{\sqrt{K}} sin(\sqrt{K}w) & K > o \\ w & K = 0 \\ \frac{1}{\sqrt{-K} sinh(\sqrt{-K}w)} & K < 0 \end{array} \right\} \tag{2}$$

Where K= 0 represents a flat geometry.
The redshift of a source is given by,

$$z = \frac{\lambda_{obj} - \lambda_{em}}{\lambda_{em}} \tag{3}$$

Where, λ_{obs} and λ_{em} are, respectively, the wavelengths at time of observation and emission. Equation 3 is directly related to the scale factor as,

$$1 + z = \frac{1}{a(t_{em})} \tag{4}$$

This means that a source at redshift z = 1 is observed at a time when the Universe was half of its current size (a = 1/2).

Due to the expansion of the Universe, a set of comoving observers sees the recession of surrounding objects. The corresponding velocity is,

$$v = \dot{a}x = \frac{\dot{a}}{a}r = H(t)r \tag{5}$$

where r = ax, and $H(t) = \frac{\dot{a}}{a}$ is the Hubble parameter, a measure of the cosmic expansion rate. The local Hubble law according to today's result are given by the following formula,

$$v_{esc} = H_0 D \tag{6}$$

Where, H_0 is Hubble constant and D is the distance between object and observer.

1.2 Distances

Accordingly, one defines the angular diameter distance as exactly this ratio,

$$D_{ang}(z) = 2R/\delta = a(z) f_K(w) \tag{7}$$

Where, 'R' is the radius of the distant object, 'δ' is the angular diameter, and 'z' is the cosmological redshift. If we consider an observer at redshift z_1 gives the angular diameter of another object at redshift z_2, so equation 7 becomes

$$D_{ang}(z_1, z_2) = a(z_2) f_K[w(z_2) = w(z_1)] \tag{8}$$

Another distance measure relates the observed flux, S, of a source to its luminosity, L. For a known luminosity, the distance to the source can be determined as,

$$D_{lum}(z) = \sqrt{\frac{L}{4\pi S}} \tag{9}$$

1.3 Quasars

1.4 Gravitational lensing

1.4.1 Lens Equation

For a given source, the lens equation is given by,

$$\beta = \theta - \alpha(\theta) \tag{10}$$

Where, θ be the apparent angular position of the source in the sky, β is its true position, and α is the scaled deflection angle.

The angles can be related to the physical distances as,

$$\eta = \frac{D_s}{D_d}\xi - D_{ds}\hat{\alpha}(\xi) \tag{11}$$

With, $D_d\theta = \xi$, $\eta = D_s\beta$, and $\hat{\alpha}$ is the true deflection angle which is related to the scaled deflection angle via:

$$\hat{\alpha}(\xi) = \frac{4G}{c^2}\int d^2\xi' \sum(\xi') \frac{\xi - \xi'}{\mid \xi - \xi' \mid^2} \tag{12}$$

Where, $\sum(\xi')$ is the surface mass density and $\mid \xi - \xi' \mid$ is the impact parameter for $\sum(\xi')$. Now, we can define a dimensionless surface mas density with convergence as,

$$\kappa(\theta) = \frac{\sum(D_d\theta)}{\sum_{cr}} \tag{13}$$

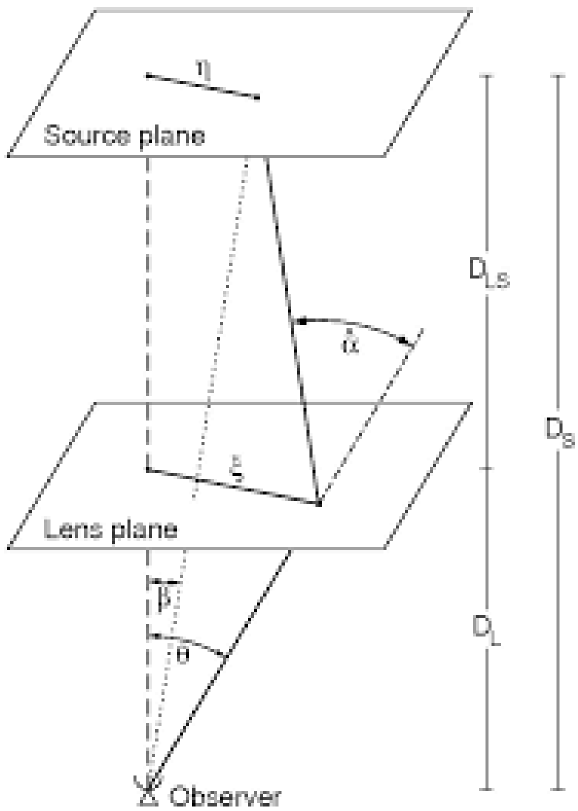

Figure 1: Schematic diagram of the gravitational lensing system [**wiki**].

with the critical surface mass density,

$$\sum_{cr} = \frac{c^2}{4\pi G}\frac{D_s}{D_d D_{ds}} \tag{14}$$

Now, on rewriting the expression for scaled deflection angle,

$$\alpha(\theta) = \frac{1}{\pi}\int d^2\theta'_\kappa(\theta')\frac{\theta-\theta'}{\mid \theta-\theta' \mid^2} \tag{15}$$

Along with this we also have to define a deflection potential which is to gives information about the mass distribution of the lens is as follow,

$$\psi(\theta) = \frac{1}{\pi}\int d^2\theta'_\kappa(\theta')In \mid \theta-\theta' \mid \tag{16}$$

The use of this quantity is motivated because it encloses all information of the mass distribution of the lens. By means of the deflection potential the following relations can be derived:

$$\alpha(\theta) = \bigtriangledown\psi(\theta) \tag{17}$$

For a better understanding further reading of the references is advised. From the deflection potential a further scalar function, the Fermat potential, can be derived:

$$\tau(\alpha;\beta) = \frac{1}{2}(\beta-\alpha)^2 - \psi(\theta) \tag{18}$$

Finally to the magnification of the images the ration of a lens to unlensed flux is given by,

$$\mu = (detA)^{-1} \tag{19}$$

Where A is the Jacobian matrix of lens mapping

$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} \tag{20}$$