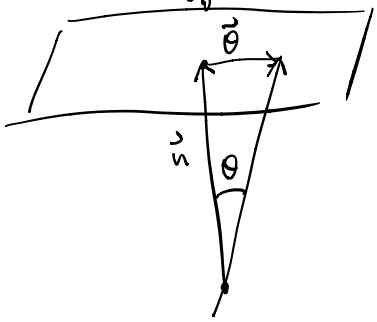
P3.1

SIS : $\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$; $\Sigma(\xi) = \frac{\sigma_v^2}{2G\xi}$

Convergence : $\kappa(\theta) = \frac{\Sigma(D_d\theta)}{\Sigma_{cr}}$

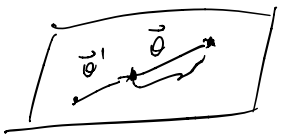
$$\psi(\theta) = \frac{1}{\pi}\int_{\mathbb{R}^2} d^2\theta' \, \kappa(\vec{\theta}') \ln|\vec{\theta} - \vec{\theta}'|$$

$$= \frac{1}{\pi}\int_0^{\theta} d\theta' \cdot 2\pi\theta' \, \kappa(\dots) \ln|\theta - \theta'| \qquad (\theta' = |\vec{\theta}'|$$



Pg. 12 lens eq. $\beta = \theta - \alpha(\theta)$ (β: true pos., θ: apparent angular position, α: scaled deflection angle)

$$\vec{\alpha}(\theta) = \frac{1}{\pi}\int d^2\theta'\ \kappa(\theta')\,\frac{\vec{\theta}-\vec{\theta}'}{|\vec{\theta}-\vec{\theta}'|^2}$$



$$\overset{\text{a.symm}}{=} \frac{\vec{\theta}}{|\vec{\theta}|^2}\,2\int_0^{|\vec{\theta}|} d\theta'\,\theta'\,\kappa(\theta')$$

$\vec{\theta} \to (\theta, r)$

$$\kappa(\theta) = \frac{\Sigma(D_d\theta)}{\Sigma_{cr}} = \frac{\sigma_v^2}{2G(D_d\theta)\,\frac{c^2}{4\pi G}\frac{D_s}{D_d D_{ds}}}$$

$$= \frac{\sigma_v^2}{2\theta}\,\frac{4\pi}{c^2}\,\frac{D_{ds}}{D_s}$$

$$= \frac{\theta_E}{2\theta}$$

$$= \frac{\vec{\theta}}{|\vec{\theta}|^2}\,2\int_0^{|\theta|} d\theta'\,\theta'\,\frac{\theta_E}{2\theta'}$$

$$\alpha(\theta) = \frac{\theta}{\theta^2}\cdot\theta_E\,\theta = \theta_E\frac{\theta}{|\theta|}$$

$$\Rightarrow \quad \beta = \theta - \theta_E\frac{\theta}{|\theta|}$$

$$\frac{\beta}{\theta_E} = \frac{\theta}{\theta_E} - \frac{\theta}{|\theta|}$$

$$\uparrow \quad \uparrow \quad \uparrow_{\frac{x}{|x|}}$$
$$y \quad x$$
$$\geq 0$$

$$\Rightarrow \quad y<1 \quad : \quad x_{\pm} = y \pm 1$$

$$\frac{\theta}{\theta_E} = \frac{\beta}{\theta_E} \pm 1$$

$$\Rightarrow \quad \theta_{A,B} = \beta \pm \theta_E$$

$$\Delta\theta = 2\theta_E$$

P3.3

$$\det A = \frac{\beta}{\theta}\frac{d\beta}{d\theta}$$

$$= \frac{\beta}{\theta}\frac{d\left(\theta - \theta_E \frac{\theta}{|\theta|}\right)}{d\theta}$$

$$= \underbrace{\frac{\beta}{\theta}}_{\frac{y}{x}}\left(1 - \frac{d}{d\theta}\left(\frac{\theta}{|\theta|}\right)\right)$$

$$= \frac{x-1}{x}$$

$$\Rightarrow (\det A)^{-1} = \mu = \frac{x}{x-1}$$

P3.4

$$c\Delta t = (1+z_d)\frac{D_d D_s}{D_{ds}}\left[\underbrace{\tau(\theta_A;\beta) - \tau(\theta_B;\beta)}\right]$$

$$= \frac{1}{2}(\beta - \theta_A)^2 - \psi(\theta_A) - \frac{1}{2}(\beta - \theta_B)^2 + \psi(\theta_B)$$

$$= \frac{1}{2}\left(-2\beta\theta_A + \theta_A^2 + 2\beta\theta_B - \theta_B^2\right) + \psi(\theta_B) - \psi(\theta_A)$$

$$= \beta(\theta_B - \theta_A) + \theta_A^2 - \theta_B^2 + \underbrace{\psi(\theta_B) - \psi(\theta_A)}_{?}$$

P3.5

Shape of images ?

Magnitification ?

P3.6

$$D^2(\lambda) = \sum_k \left(A(t_k) - B(t_k + \lambda)\right)^2$$

$$D^2(\underbrace{\hat{\lambda}}_{\text{true}} + \delta\lambda) = \sum_k \left(A(t_k) - B(t_k + \hat{\lambda} + \delta\lambda)\right)^2$$

$$\approx \sum_k \left(A(t_k) - B(t_k + \hat{\lambda}) - \frac{d}{d\lambda} B(t_k + \lambda)\,\delta\lambda\right)^2$$

$$\approx \sum_k \left(B'(t_k + \lambda)\,\delta\lambda\right)^2$$