

Quantum Field Theory

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October 4, 2019

Contents

1 Classical field theory

1.1 Field theory in continuum

Euler-Lagrange-equation

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \quad (1.1.1)$$

momentum density

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)} \quad (1.1.2)$$

Hamiltonian density

$$\mathcal{H}(\phi(x), \pi(x)) = \pi(x) \dot{\phi}(x) - \mathcal{L}(\phi, \partial_\mu \phi) \quad (1.1.3)$$

1.2 Noether Theorem

If a Lagrangian field theory has an infinitesimal symmetry, then there is an associated current j^μ , which is conserved.

$$\partial_\mu j^\mu = 0 \quad (1.2.1)$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta \phi - X^\mu \quad (1.2.2)$$

Energy-momentum tensor (stress-energy tensor)

Asymmetric version

$$\Theta^\mu_\nu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \delta^\mu_\nu \mathcal{L} \quad (1.2.3)$$

General version

$$T^{\mu\nu} = \Theta^{\mu\nu} + \partial_\lambda f^{\mu\nu\lambda} \quad (1.2.4)$$

with $f^{\lambda\mu\nu} = -f^{\mu\lambda\nu}$ or $\partial_\mu \partial_\nu f^{\lambda\mu\nu} = 0$