3. a)

$$= D_F^{(0)}(\vec{p}) + D_F^{(0)}(-i\Sigma(\not p))D_F^{(0)}$$

$$= D_F^{(0)} \frac{1}{1+i\Sigma(\not p)D_F^{(0)}}$$

$$= \frac{i}{\not p - m_0} \frac{1}{1-\frac{1}{\not p - m_0}\Sigma(\not p)} = \frac{i}{\not p - m_0 - \Sigma(\not p)}$$

$$\left(\begin{aligned} &\Sigma(\not p) = \Sigma(m_0) + (\not p - m_0)\Sigma'(m_0) + (\not p - m_0)\breve{\Sigma}(\not p) \\ &m = m_0 + \Sigma(m) \\ &\to m_0 + \Sigma(\not p) = m_0 + \Sigma(m_0) + \cdots \\ &\qquad = m + (\not p - m)\Sigma'(m_0) + (\not p - m)\breve{\Sigma}(\not p) \end{aligned} \right.$$

$$= \frac{i}{\not p - m - (\not p - m)(\Sigma'(m_0) + \breve{\Sigma}(\not p))}$$

$$= \frac{i}{\not p - m} \frac{1}{1 - \Sigma'(m_0) - \breve{\Sigma}(\not p)}$$

no spinor

b) $$-i\Sigma_2(\not p) = \int \frac{d^4k}{(2\pi)^4} (-ie\gamma^\mu) \frac{-ig^{\mu\nu}}{(p-k)^2+i\varepsilon} \frac{(\not k + m)}{k^2-m^2+i\varepsilon} (-ie\gamma^\nu)$$

$$= (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{-i}{(p-k)^2} \frac{(\not k + m)}{k^2-m^2} \gamma_\mu$$

$$= +ie^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu (\not k + m)\gamma_\mu \int_0^1 dx \frac{1}{\underbrace{[x(p-k)^2 + (1-x)(k^2-m^2)]}^2}$$

$$= xp^2 - 2xpk + xk^2 + (1-x)k^2 - (1-x)m^2$$

$$= k^2 - 2xp\cdot k + x^2p^2 - x^2p^2 + xp^2$$

$$- (1-x)m^2$$

$$= (k - xp)^2 + p^2 x(-x+1) + m^2(1-x)$$

$$= \underbrace{(k-xp)}_{q}{}^2 + \underbrace{(1-x)(p^2x + m^2)}_{\Delta}$$

$$= ie^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx\, \gamma^\mu (\underbrace{\not{q}}_{\to 0,\ \text{b.c. parity}} + x\not{p} + m)\gamma_\mu \frac{1}{(q^2+\Delta)^2}$$

c) $\gamma^\mu(\not{p}x + m_0\gamma_\mu)$

$$\gamma^\mu \gamma_\mu = \gamma^\mu \gamma^\nu g_{\nu\mu}$$
$$= \frac{1}{2} g_{\nu\mu} (\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)$$
$$= \frac{1}{2} g_{\nu\mu} \cdot 2 g^{\mu\nu}$$
$$= d\, \mathbb{1}_d$$

$$\gamma^\mu\gamma^\nu\gamma_\mu = -\gamma^\mu\gamma_\mu\gamma^\nu + \gamma^\mu \cdot 2 g^\nu_\mu$$
$$= -d\mathbb{1}_d\gamma^\nu + 2\gamma^\nu$$
$$= (2-d)\gamma^\nu$$

$$\Rightarrow \quad \gamma^\mu(\not{p}x + m_0)\gamma_\mu = (2-d)\not{p}x + m_0 d$$

d) $$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi$$
$$= \mathcal{L}_{EM} + \mathcal{L}_D - e\bar{\psi}\gamma^\mu\psi A_\mu$$

$[F_{\mu\nu}F^{\mu\nu}] = d$, $[F] = d/2$ $\qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\to [A] = d/2 - 1$$

$$[\psi] = [\bar{\psi}] = \frac{d-1}{2}$$

$$\Rightarrow \quad [e] + \cancel{d} - 1 + \frac{d}{2} - 1 = \cancel{d}$$

$$\Rightarrow \quad [e] = 2 - d/2$$

$$\underbrace{e = \mu^\alpha e_0}_{[e_0]=0}, \qquad [\mu^\alpha] = \alpha = 2 - d/2$$

e) $$-i\Sigma = +ie^2 \int_0^1 dx \int \frac{d^d q}{(2\pi)^d} \left[(2-d)x\not{p} + d m_0\right] \frac{1}{(q^2-\Delta)^2}$$

$$\left(\begin{array}{l} q_0 \to i q_{0,E}, \quad \vec{q} \to \vec{q}_E \\ q^2 = q_0^2 - \vec{q}^2 = -q_E^2 \end{array} \right.$$

$$= -e^2 \int_0^1 dx \int \frac{d^d q_E}{(2\pi)^d} \frac{((2-d)x\not{p} + d m_0)}{(q^2+\Delta)^2}$$

$$= -e^2((2-d)x\not{p} + d m_0) \int_0^1 dx \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\underbrace{\Gamma(2)}_{=1}} \left(\frac{1}{\Delta}\right)^{2-d/2}$$

$$= \frac{-e^2((2-d)x\not{p} + d m_0)}{(4\pi)^{d/2}} \underbrace{\Gamma(2-d/2)} \underbrace{\int_0^1 dx \, \Delta^{d/2-2}}$$

$$= \int_0^1 dx \, \Delta^\varepsilon$$

$$\to \int_0^1 dx \, (1 + \varepsilon \ln \Delta)$$

$$= \int_0^1 dx \left\{1 + \varepsilon \ln(1-x) + \varepsilon \ln(m_0^2 - xp^2)\right\}$$

$$= 1 - \varepsilon\left((1-x)\ln(1-x) - (1-x)\right)\Big|_0^1 - \frac{\varepsilon}{p^2}\left((m_0^2 - xp^2)\ln(m_0^2 - xp^2) - (m_0^2 - xp^2)\right)\Big|_0^1$$

$$= 1 - \varepsilon\Big\{-1 - \frac{1}{p^2}\Big[(m_0^2 - p^2)\ln(m_0^2 - p^2) - (\cancel{m_0^2} - p^2)$$
$$- (m_0^2 \ln m_0^2 - \cancel{m_0^2})\Big]\Big\}$$

$$= 1 + \varepsilon + \frac{\varepsilon}{p^2}\left((m_0^2 - p^2)\ln(m_0^2 - p^2) + p^2 - m_0^2 \ln m_0^2 \right)$$

$$\longrightarrow 1$$

$$\Gamma(2 - d/2) = \Gamma\left(\frac{\varepsilon}{2}\right) = \frac{2}{\varepsilon} - \gamma_E + \mathcal{O}(\varepsilon)$$

2. $\mathcal{L}_{int} = -e\bar{\psi}\gamma^\mu A_\mu \psi - e\,\bar{\xi}\gamma^\mu A_\mu \xi$

a) $p = (E_p, \vec{p})$, $\quad p^2 = E_p^2 - |\vec{p}|^2 \simeq 0 \quad \Leftrightarrow \quad E_p = |\vec{p}|$

$p' = (E_p, -\vec{p})$, $\quad p'^2 = E_p^2 - |\vec{p}|'^2 \simeq 0$

$k = (E_k, \vec{k})$,

$k' = (E_{k'}, -\vec{k})$,

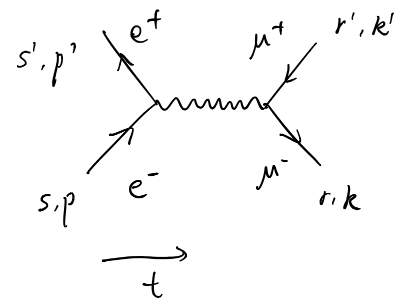
$$S = (p+p')^2 = 2pp' = (k+k')^2 = 2m_\mu^2 + 2kk'$$

$$t = (p-k)^2 = m_\mu^2 - 2p\cdot k = (-p'+k')^2 = m_\mu^2 - 2p'\cdot k'$$

$$u = (p-k')^2 = m_\mu^2 - 2p'k$$

$$S + t + u = \sum m_i^2 = 2m_\mu^2$$

b) $\psi^{(s)}(p) + \bar{\psi}^{(s')}(p') \longrightarrow \xi^{(r)}(k) + \bar{\xi}^{(r')}(k')$



only s-channel possible!

$$i\mathcal{M} = \bar{v}_e^{(s')}(p')(-ie\gamma^\mu)u_e^{(s)}(p)\frac{-ig_{\mu\nu}}{(p+p')^2+i\varepsilon}\bar{u}_\mu^{(r)}(k)(-ie\gamma^\nu)v_\mu^{(r')}(k')$$

$$= ie^2\,\bar{v}_e^{(s')}(p')\gamma^\mu u_e^{(s)}(p)\frac{1}{(p+p')^2}u_\mu^{(r)}(k)\gamma_\mu v_\mu^{(r')}(k')$$

c) $\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{spins}\mathcal{M}\mathcal{M}^+$

$$= \frac{e^4}{4s^2}\sum \bar{v}_e^{(s')}(p')\gamma^\mu u_e^{(s)}(p)\,u_\mu^{(r)}(k)\gamma_\mu v_\mu^{(r')}(k') \;\vdots\; \bar{v}_\mu^{(r')}(k')\gamma_\mu u_\mu^{(1)}$$