Commutation from discrete to continuous

$[q_i, p_i] = i\delta_{ij}$, $\quad [q_i, q_j] = [p_i, p_j] = 0$

in order to $\quad [\phi(\underline{x}), \pi(\underline{y})] = i\delta^{(3)}(\underline{x}-\underline{y})$, ....

$\Rightarrow \quad q \to \phi, \quad p \to \pi\, d^3x \quad , \quad \delta_{ij}/d^3x \to \delta^{(3)}(\underline{x}-\underline{y})$

A.4)

a) i) $\gamma^0 L(\vec{p})\gamma^0 = \gamma^0 \exp(\frac{1}{2}\omega_{ij}S^{ij})\exp(\omega_{03}S^{03})\gamma^0$

$$= \gamma^0 \exp(\tfrac{1}{2}\omega_{ij}S^{ij})\gamma^0 \exp(-\omega_{03}S^{03})$$

$$\left[ S^{\mu\nu} = \tfrac{1}{4}[\gamma^\mu, \gamma^\nu] \quad , \quad \{\gamma^\mu, \gamma^\nu\} = 2\mathbb{1} g^{\mu\nu} \right]$$

$$= \gamma^0\gamma^0 \exp(\tfrac{1}{2}\omega_{ij}S^{ij})\exp(-\omega_{03}S^{03})$$

$\uparrow$ two anticommutations

ii) $i\gamma^2 L(\vec{p})\, i\gamma^2 = (L(\vec{p}))^*$ ; $\quad (\gamma^2)^* = -\gamma^2, \ (\gamma^\mu)^* = \gamma^\mu, \ \mu \neq 2$

$(\omega_{03}S^{03})^* = \left(\frac{\omega_{03}}{4}[\gamma^0, \gamma^3]\right)^* = \omega_{03}S^{03}$

$$(\omega_{ij}S^{ij})^* = \frac{\omega_{03}}{4}[(\gamma^i)^*, (\gamma^j)^*] = \begin{cases} -\omega_{ij}S^{ij}, & \text{if } i=2 \text{ or } j=2 \\ +\omega_{\mu j}S^{\mu j}, & \text{else} \end{cases}$$

$$(i\gamma^2)\,\omega_{\mu j}S^{\mu j}\,(i\gamma^2) = \begin{cases} -\omega_{\mu j}S^{\mu j}, & \text{if } \mu=2 \text{ or } j=2 \\ \omega_{\mu j}S^{\mu j}, & \text{else} \end{cases}$$

using $\{\gamma^\mu, \gamma^\nu\} = 2\mathbb{1} g^{\mu\nu}$

$$\Rightarrow i\gamma^2 L(\vec{p})\, i\gamma^2 = i\gamma^2 \exp(\tfrac{1}{2}\omega_{ij}S^{ij})\underbrace{i\gamma^2\ i\gamma^2}_{=\mathbb{1}}\exp(\omega_{03}S^{03})\, i\gamma^2$$

$$= \exp((\tfrac{1}{2}\omega_{ij}S^{ij})^*)\exp((\omega_{03}S^{03})^*) = (L(\vec{p}))^*$$

b) $\gamma^5 u_s(\vec{0}) = (-1)^{\frac{1}{2}-s} v_{-s}(0)$, $\quad \gamma^5 v_s(0) = -(-1)^{\frac{1}{2}-s} u_{-s}(0)$

$\gamma^0 u_s(\vec{0}) = u_s(\vec{0})$, $\quad \gamma^0 v_s(\vec{0}) = -v_s(\vec{0})$, $\quad$ anti-particle has negative parity !

c) i) $\gamma^0 u_s(\vec{p}) = \underbrace{\gamma^0 L(\vec{p})\gamma^0}_{L(-\vec{p})}\underbrace{\gamma^0 u_s(0)}_{u_s(0)} = L(-\vec{p})u_s(0) = u_s(-\vec{p})$

$$\gamma^0 v_s(\vec{p}) = \underbrace{\gamma^0 \; L(\vec{p})\,\gamma^0}_{L(-\vec{p})} \underbrace{\gamma^0 \, v_s(\vec{0})}_{v_s(\vec{0})} = -v_s(-\vec{p})$$

ii) $$i\gamma^2 u_s(\vec{p}) = \underbrace{i\gamma^2 L(\vec{p}) \;\; i\gamma^2}_{(L(\vec{p}))^*} \cdot \underbrace{i\gamma^2 u_s(\vec{0})}_{-v_s(\vec{0}) = -[v_s(\vec{0})]^*} = -[v_s(\vec{p})]^*$$

$$i\gamma^2 v_s(\vec{p}) = \underbrace{i\gamma^2 \, L(\vec{p}) \, i\gamma^2}_{(L(\vec{p}))^*} \underbrace{i\gamma^2 \, v_s(\vec{0})}_{-[u_s(\vec{0})]^*} = -[u_s(\vec{p})]^*$$

iii) $$\gamma^5 \, i\gamma^2 \gamma^0 u_s(\vec{p}) = \gamma^5 \, i\gamma^2 u_s(-\vec{p}) = -\gamma^5 [v_s(-\vec{p})]^* = -[L(-\vec{p})\gamma^5 v_s(\vec{0})]^*$$
$$= (-1)^{\frac{1}{2}-s} [u_{-s}(-\vec{p})]^*$$

$$\gamma^5 \, i\gamma^2 \gamma^0 v_s(\vec{p}) = \gamma^5 [u_s(-\vec{p})]^* = [L(-\vec{p})\gamma^5 u_s(\vec{0})]^* = (-1)^{\frac{1}{2}-s} [v_{-s}(-\vec{p})]^*$$

d) $$P(i\partial\!\!\!/ - m)P^{-1} = \gamma^0(i\gamma^\mu\partial_\mu - m)\gamma^0 = \gamma^0(i\gamma^0\partial_0 + i\gamma^i\partial_i - m)\gamma^0$$
$$= (i\gamma^0\partial_0 - i\gamma^i\partial_i - m)$$

$$(i\partial\!\!\!/ - m)\, C\psi = -(i\gamma^\mu\partial_\mu - m)\, i\gamma^2\psi = -i\gamma^2 i\gamma^2 \, (i\partial\!\!\!/ - m)\, i\gamma^2 \psi^*$$
$$= -i\gamma^2 ( i(\underbrace{i\gamma^2\gamma^\mu \, i\gamma^2}_{-(\gamma^\mu)^*})\partial_\mu - m)\psi^*$$
$$= -i\gamma^2(-i\partial\!\!\!/^* - m)\psi^* = -i\gamma^2[(-i\partial\!\!\!/ - m)\psi]^* = 0$$