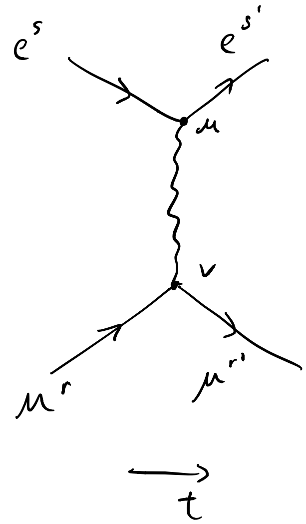
H.10 Chenhuan Wang 18/20

a) $e^{s}(p) + \mu^{r}(k) \longrightarrow e^{s'}(p') + \mu^{r'}(k')$

translated into feynman diagram:



$$i\mathcal{M} = \underset{\text{out}}{\bar{u}^{s'}(p')}(-ie\gamma^{\mu})\underset{\text{in}}{u^{s}(p)}\frac{-ig_{\mu\nu}}{(p'-p)^2+i\varepsilon}\underset{\text{out}}{\bar{u}^{r'}(k')}(-ie\gamma^{\nu})\underset{\text{in}}{u^{r}(k)}$$

$$= ie^2\bar{u}^{s'}(p')\gamma^{\mu}u^{s}(p)\frac{g_{\mu\nu}}{(p'-p)^2+i\varepsilon}\bar{u}^{r'}(k')\gamma^{\nu}u^{r}(k)$$

$$u^{s}(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^{s}\\ \sqrt{p\cdot\bar{\sigma}}\,\xi^{s}\end{pmatrix} = \begin{pmatrix}\sqrt{E-p^3}\,\xi^{s}\\ \sqrt{E+p^3}\,\xi^{s}\end{pmatrix} \xrightarrow[m\approx E]{\eta\to\infty} \sqrt{2m}\begin{pmatrix}\xi^{s}\\ 0\end{pmatrix}$$

$$\xi = \begin{pmatrix}1\\0\end{pmatrix} \text{ or } \begin{pmatrix}0\\1\end{pmatrix}$$

$$\bar{u}^{s'}(p')\gamma^{\mu}u^{s}(p) = 2m\,(\xi^{\dagger s'}, 0)\,\gamma^{0}\gamma^{\mu}\begin{pmatrix}\xi^{s}\\ 0\end{pmatrix}$$

$\mu = 0$,
$$= 2m\,(\xi^{\dagger s'}, 0)\begin{pmatrix}\xi^{s}\\ 0\end{pmatrix}$$
$$= 2m(\xi^{\dagger s'}\xi^{s})$$

$$\mu = i, \quad = 2m(\xi^{\dagger s'}, 0)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$= 2m(\xi^{s'\dagger}, 0)\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}\begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$= 0$$

$$= ie^2 u^{s'\dagger}(p') u^s(p) \frac{1}{(p'-p)^2 + i\varepsilon} u^{r'\dagger}(k') u^r(k)$$

$$= ie^2 2m_\mu \delta^{s's} 2m_e \delta^{r'r} \underbrace{\frac{1}{-|\vec{p}'-\vec{p}|^2 + i\varepsilon}}_{= -\frac{1}{|\vec{p}'-\vec{p}| + i\varepsilon}}$$

$$\left[\begin{aligned} (p'-p)^2 &= ((q')^0 - q^0)^2 - |\vec{q}' - \vec{q}|^2 \\ &= (m-m)^2 - |\vec{q}' - \vec{q}|^2 \\ &= -|\vec{q}' - \vec{q}|^2 \end{aligned} \right]$$

b) $$i\mathcal{M} \overset{!}{=} -i\tilde{V}(\vec{p}'-\vec{p})\, 2m_\mu \delta^{rr'}\, 2m_e \delta^{ss'}$$

$$\Rightarrow \tilde{V}(\vec{p}'-\vec{p}) = \frac{e^2}{|\vec{p}'-\vec{p}|^2 + i\varepsilon}$$

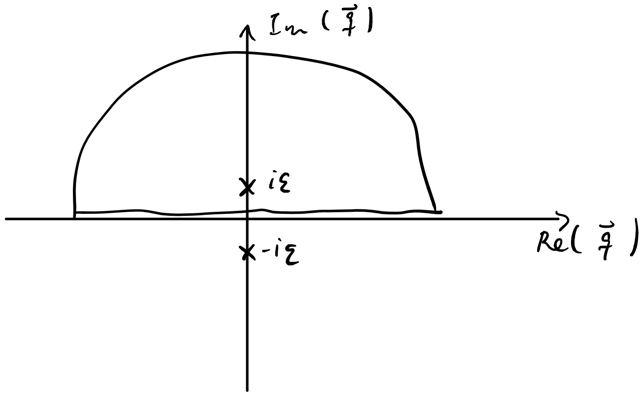
$$V(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{x}} \tilde{V}(\vec{q}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}\cdot\vec{x}} \frac{e^2}{|\vec{q}|^2 + i\varepsilon}$$

$$= \frac{e^2}{(2\pi)^3} \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\varphi \int_0^\infty d|\vec{q}|\, |\vec{q}|^2 e^{-i|\vec{q}||\vec{x}|\cos\theta} \frac{1}{|\vec{q}|^2 + i\varepsilon}$$

$$= \frac{e^2}{(2\pi)^2} \int_{-1}^{1} d\cos\theta \int_0^\infty d|\vec{q}|\, e^{-i|\vec{q}||\vec{x}|\cos\theta} \frac{|\vec{q}|^2}{|\vec{q}|^2 - i\varepsilon}$$

$$= \left(\frac{e}{2\pi}\right)^2 \int_0^\infty d|\vec{q}|\, \frac{1}{+i|\vec{q}||\vec{x}|}\left(e^{i|\vec{q}||\vec{x}|} - e^{-i|\vec{q}||\vec{x}|}\right)\frac{|\vec{q}|^2}{|\vec{q}|^2+i\varepsilon}$$

$$= -\left(\frac{e}{2\pi}\right)^2 \frac{1}{|\vec{x}|}\, i \int_{-\infty}^{\infty} d|\vec{q}|\, e^{i|\vec{q}||\vec{x}|}\frac{|\vec{q}|}{|\vec{q}|^2+i\varepsilon}$$



Pole at $\vec{q}^{\,2} - i\varepsilon = 0$

$\Rightarrow \vec{q}^{\,2} = i\varepsilon$

$\vec{q} = \pm i\varepsilon$

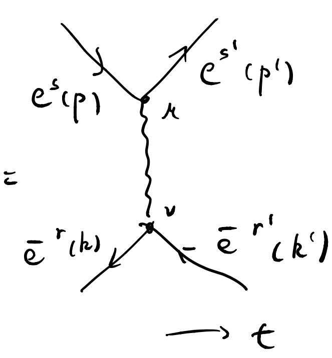
$$= -\left(\frac{e}{2\pi}\right)^2 \frac{1}{|\vec{x}|}\, i \cdot 2\pi i \;\mathrm{Res}\left(\frac{e^{iqx}\, q}{q^2+i\varepsilon},\ q = i\varepsilon\right)$$

$$= +\frac{e^2}{2\pi}\frac{1}{|\vec{x}|}\,\frac{e^{i\cdot i\varepsilon x}\,\cancel{i\varepsilon}}{2\cancel{i\varepsilon}}$$

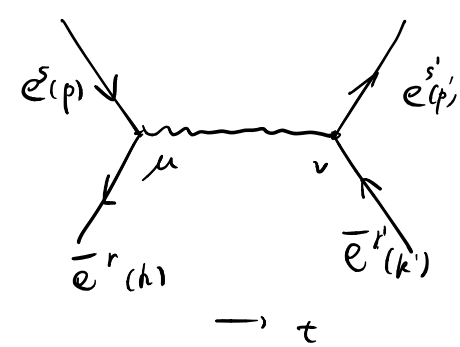
$$\underset{\varepsilon\to 0}{=} + \frac{e^2}{4\pi|\vec{x}|} \underset{|\vec{x}|\equiv r}{=} + \frac{e^2}{4\pi r}$$

c) fermion - antifermion scattering

$e + \bar{e} \longrightarrow e + \bar{e} =$



$+$



t - channel

$$i\mathcal{M}_u = \bar{u}^s(p)(-ie\gamma^\mu)u^{s'}(p') \frac{-ig^{\mu\nu}}{(p-p')^2+i\varepsilon} \bar{v}^s(p) v^{s'}(p')$$

s - channel

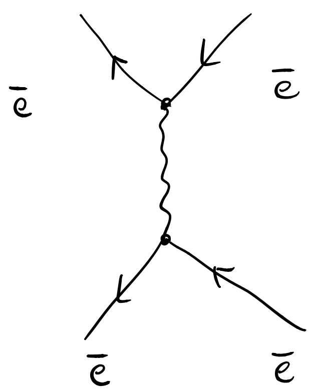
$$i\mathcal{M}_s = \bar{u}^s(p)(-ie\gamma^\mu)\bar{v}^r(p) \frac{-ig^{\mu\nu}}{(p-p')^2+i\varepsilon} u^{s'}(p')(-ie\gamma^\nu)v^{r'}(k')$$

has to show explicitely like before

since $\quad v^{s'}(p) v^{s}(p) = -2m\delta^{ss}$

$$\longrightarrow \quad V(r) = -\frac{e^2}{4\pi r}$$

antifermion - antifermion:



$$(-1)^2 \longrightarrow V(r) = \frac{e^2}{4\pi r}$$