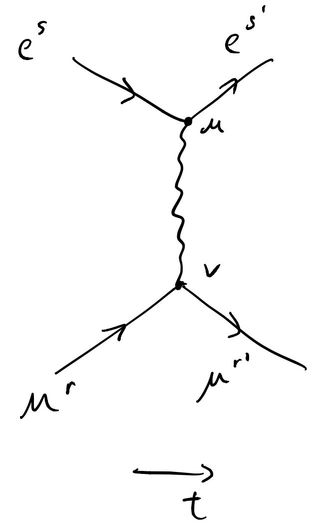
H.10 Chenhuan Wang

a) $e^s(p) + \mu^r(k) \longrightarrow e^{s'}(p') + \mu^{r'}(k')$

translated into feynman diagram:



$$i\mathcal{M} = \underset{\text{out}}{\bar{u}^{s'}(p')}(-ie\gamma^\mu)\underset{\text{in}}{u^s(p)}\frac{-ig_{\mu\nu}}{(p'-p)^2+i\varepsilon}\underset{\text{out}}{\bar{u}^{r'}(k')}(-ie\gamma^\nu)\underset{\text{in}}{u^r(k)}$$

$$= ie^2\bar{u}^{s'}(p')\gamma^\mu u^s(p)\frac{g_{\mu\nu}}{(p'-p)^2+i\varepsilon}\bar{u}^{r'}(k')\gamma^\nu u^r(k)$$

$$u^s(p) = \begin{pmatrix}\sqrt{p\cdot\sigma}\,\xi^s\\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s\end{pmatrix} = \begin{pmatrix}\sqrt{E-p^3}\,\xi^s\\ \sqrt{E+p^3}\,\xi^s\end{pmatrix} \xrightarrow[m\approx E]{\eta\to\infty} \sqrt{2m}\begin{pmatrix}\xi^s\\ 0\end{pmatrix}$$

$\xi = \begin{pmatrix}1\\0\end{pmatrix}$ or $\begin{pmatrix}0\\1\end{pmatrix}$

$$\bar{u}^{s'}(p')\gamma^\mu u^s(p) = 2m\,(\xi^{\dagger s'}, 0)\,\gamma^0\gamma^\mu\begin{pmatrix}\xi^s\\ 0\end{pmatrix}$$

$\mu = 0$,
$$= 2m\,(\xi^{\dagger s'}, 0)\begin{pmatrix}\xi^s\\ 0\end{pmatrix}$$
$$= 2m(\xi^{\dagger s'}\xi^s)$$

$$\mu = i, \qquad = 2m(\xi^{\dagger s'}, 0)\begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$= 2m\,(\xi^{s'\dagger}, 0)\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}\begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$= 0$$

$$= ie^2\, u^{s'\dagger}(p')\, u^s(p)\, \frac{1}{(p'-p)^2 + i\varepsilon}\, u^{r'\dagger}(k')\, u^r(k)$$

$$= ie^2\, 2m_\mu \delta^{s's}\, 2m_e \delta^{r'r}\, \underbrace{\frac{1}{-|\vec{p}\,'-\vec{p}|^2 + i\varepsilon}}_{} = -\frac{1}{|\vec{p}\,'-\vec{p}| + i\varepsilon}$$

$$\left[\begin{aligned} (p'-p)^2 &= ((q')^0 - q^0)^2 - |\vec{q}\,' - \vec{q}|^2 \\ &= (m-m)^2 - |\vec{q}\,'-\vec{q}|^2 \\ &= -|\vec{q}\,'-\vec{q}|^2 \end{aligned}\right]$$

b) $\quad i\mathcal{M} \overset{!}{=} -i\tilde{V}(\vec{p}\,'-\vec{p})\, 2m_\mu \delta^{rr'}\, 2m_e\, \delta^{ss'}$

$$\Rightarrow \quad \tilde{V}(\vec{p}\,'-\vec{p}) = \frac{e^2}{|\vec{p}\,'-\vec{p}|^2 + i\varepsilon}$$

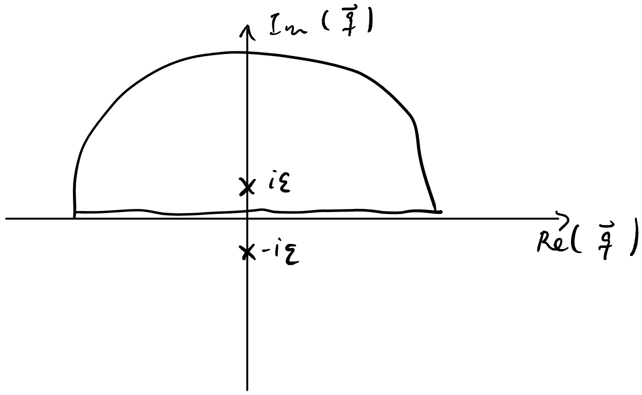
$$V(\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{x}}\, \tilde{V}(\vec{q}) = \int \frac{d^3q}{(2\pi)^3}\, e^{-i\vec{q}\cdot\vec{x}}\, \frac{e^2}{|\vec{q}|^2 + i\varepsilon}$$

$$= \frac{e^2}{(2\pi)^3} \int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\varphi \int_0^\infty d|\vec{q}|\, |\vec{q}|^2\, e^{-i|\vec{q}||\vec{x}|\cos\theta}\, \frac{1}{|\vec{q}|^2 + i\varepsilon}$$

$$= \frac{e^2}{(2\pi)^2} \int_{-1}^{1} d\cos\theta \int_0^\infty d|\vec{q}|\, e^{-i|\vec{q}||\vec{x}|\cos\theta}\, \frac{|\vec{q}|^2}{|\vec{q}|^2 - i\varepsilon}$$

$$= \left(\frac{e}{2\pi}\right)^2 \int_0^\infty d|\vec q|\, \frac{1}{+i|\vec q||\vec x|}\left(e^{i|\vec q||\vec x|\cos\theta} - e^{-i|\vec q||\vec x|\cos\theta}\right)\frac{|\vec q|^2}{|\vec q|^2+i\varepsilon}$$

$$= -\left(\frac{e}{2\pi}\right)^2 \frac{1}{|\vec x|}\, i \int_{-\infty}^{\infty} d|\vec q|\, e^{i|\vec q||\vec x|\cos\theta}\, \frac{|\vec q|}{|\vec q|^2+i\varepsilon}$$



Pole at $\vec q^{\,2} - i\varepsilon = 0$

$\Rightarrow \vec q^{\,2} = i\varepsilon$

$\vec q = \pm i\varepsilon$

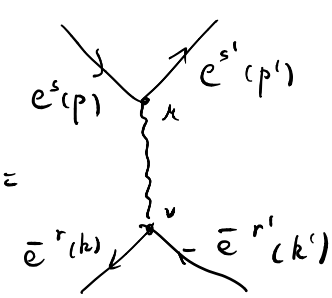
$$= -\left(\frac{e}{2\pi}\right)^2 \frac{1}{|\vec x|}\, i \cdot 2\pi i \;\mathrm{Res}\left(\frac{e^{iqx\cos\theta}\, q}{q^2+i\varepsilon}\, ,\; q = i\varepsilon\right)$$

$$= +\frac{e^2}{2\pi}\,\frac{1}{|\vec x|}\;\frac{e^{i\cdot i\varepsilon x\cos\theta}\,\cancel{i\varepsilon}}{2\cancel{i\varepsilon}}$$

$$\underset{\varepsilon\to 0}{=} +\frac{e^2}{4\pi|\vec x|} \underset{|\vec x|\equiv r}{=} +\frac{e^2}{4\pi r}$$

c) fermion - antifermion scattering

$e + \bar e \longrightarrow e + \bar e =$



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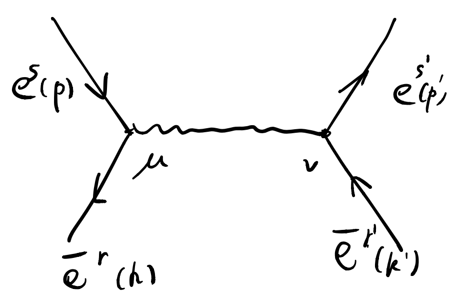


t - chanel

$$i\mathcal{M}_u = \bar{u}^s(p)(-ie\gamma^\mu)u^{s'}(p') \frac{-ig^{\mu\nu}}{(p-p')^2+i\varepsilon} \bar{v}^s(p) v^{s'}(p')$$

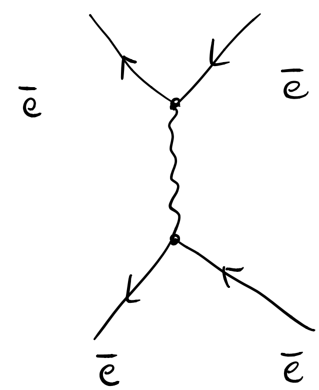
s - chanel

$$i\mathcal{M}_s = \bar{u}^s(p)(-ie\gamma^\mu)\bar{v}^r(p) \frac{-ig^{\mu\nu}}{(p-p')^2+i\varepsilon} u^{s'}(p')(-ie\gamma^\nu)v^{r'}(k')$$

since $\quad v^{s'}(p) v^s(p) = -2m\delta^{ss'}$

$$\longrightarrow \quad V(r) = -\frac{e^2}{4\pi r}$$

antifermion - antifermion:



$$(-1)^2 \longrightarrow V(r) = \frac{e^2}{4\pi r}$$