

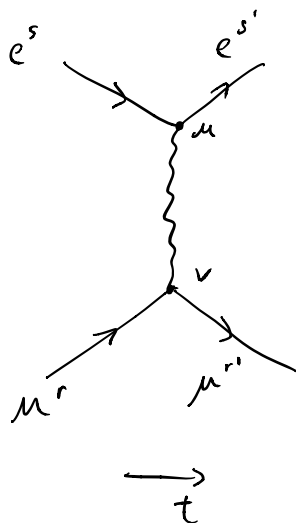
H.10

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18/20

$$a) \quad e^s(p) + \mu^r(k) \rightarrow e^{s'}(p') + \mu^{r'}(k')$$

translated into feynman diagram:



$$i\mathcal{M} = \bar{u}^{s'}(p')(-ie\gamma^\mu) u^s(p) \frac{-ig_{\mu\nu}}{(p'-p)^2 + i\epsilon} \bar{u}^{r'}(k')(-ie\gamma^\nu) u^r(k)$$

\uparrow out \uparrow in \uparrow out \uparrow in

$$= ie^2 \bar{u}^{s'}(p') \gamma^\mu u^s(p) \frac{g_{\mu\nu}}{(p'-p)^2 + i\epsilon} \bar{u}^{r'}(k') \gamma^\nu u^r(k)$$

$$u^s(p) = \begin{pmatrix} \sqrt{p \cdot \sigma} \xi^s \\ \sqrt{p \cdot \bar{\sigma}} \xi^s \end{pmatrix} = \begin{pmatrix} \sqrt{E - p^3} \xi^s \\ \sqrt{E + p^3} \xi^s \end{pmatrix} \xrightarrow{p \rightarrow \infty} \sqrt{2m} \begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$\xi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
 $m \approx E$

$$\bar{u}^{s'}(p') \gamma^\mu u^s(p) = 2m \begin{pmatrix} \xi^{s\dagger}, 0 \end{pmatrix} \gamma^0 \gamma^\mu \begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$\left[\begin{array}{l} \mu=0, \\ \mu=1, \end{array} \right. \quad \begin{aligned} &= 2m \begin{pmatrix} \xi^{s\dagger}, 0 \end{pmatrix} \begin{pmatrix} \xi^s \\ 0 \end{pmatrix} \\ &= 2m \begin{pmatrix} \xi^{s\dagger} \xi^s \end{pmatrix} \end{aligned}$$



$$\begin{aligned}
 \mu=i, \quad &= 2m (\xi^{s'}, 0) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \begin{pmatrix} \xi^s \\ 0 \end{pmatrix} \\
 &= 2m (\xi^{s'}, 0) \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix} \begin{pmatrix} \xi^s \\ 0 \end{pmatrix} \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 &= ie^2 u^{s'}(p') u^s(p) \frac{1}{(p'-p)^2 + i\varepsilon} u^{r'}(k') u^r(k) \\
 &= ie^2 2m \delta^{s's} 2m \delta^{r'r} \frac{1}{-|\vec{p}' - \vec{p}|^2 + i\varepsilon} = - \frac{1}{|\vec{p}' - \vec{p}|^2 + i\varepsilon} \\
 &\left[\begin{aligned} (p'-p)^2 &= (q')^0 - q^0)^2 - |\vec{q}' - \vec{q}|^2 \\ &= (m-m)^2 - |\vec{q}' - \vec{q}|^2 \\ &= -|\vec{q}' - \vec{q}|^2 \end{aligned} \right]
 \end{aligned}$$

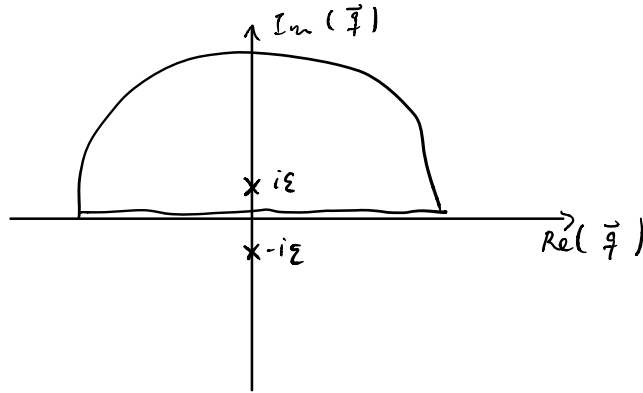
$$b) \quad i\mathcal{M} = -i \tilde{V}(\vec{p}' - \vec{p}) 2m \delta^{r'r} 2m \delta^{s's}$$

$$\Rightarrow \tilde{V}(\vec{p}' - \vec{p}) = \frac{e^2}{|\vec{p}' - \vec{p}|^2 + i\varepsilon}$$

$$\begin{aligned}
 V(\vec{x}) &= \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{x}} \tilde{V}(\vec{q}) = \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{x}} \frac{e^2}{|\vec{q}|^2 + i\varepsilon} \\
 &= \frac{e^2}{(2\pi)^3} \int_{-1}^1 d\cos\theta \int_0^{2\pi} d\varphi \int_0^\infty d|\vec{q}| |\vec{q}|^2 e^{-i|\vec{q}| |\vec{x}| \cos\theta} \frac{1}{|\vec{q}|^2 + i\varepsilon} \\
 &= \frac{e^2}{(2\pi)^2} \int_{-1}^1 d\cos\theta \int_0^\infty d|\vec{q}| e^{-i|\vec{q}| |\vec{x}| \cos\theta} \frac{|\vec{q}|^2}{|\vec{q}|^2 + i\varepsilon}
 \end{aligned}$$

$$= \left(\frac{e}{2\pi}\right)^2 \int_0^\infty d|\vec{q}| \frac{1}{+i|\vec{q}||\vec{x}|} \left(e^{i|\vec{q}||\vec{x}|} - e^{-i|\vec{q}||\vec{x}|} \right) \frac{|\vec{q}|^2}{|\vec{q}|^2 + i\varepsilon}$$

$$= -\left(\frac{e}{2\pi}\right)^2 \frac{1}{|\vec{x}|} i \int_{-\infty}^\infty d|\vec{q}| e^{i|\vec{q}||\vec{x}|} \frac{|\vec{q}|}{|\vec{q}|^2 + i\varepsilon}$$



pole at $\vec{q}^2 - i\varepsilon = 0$

$$\Rightarrow \vec{q}^2 = i\varepsilon$$

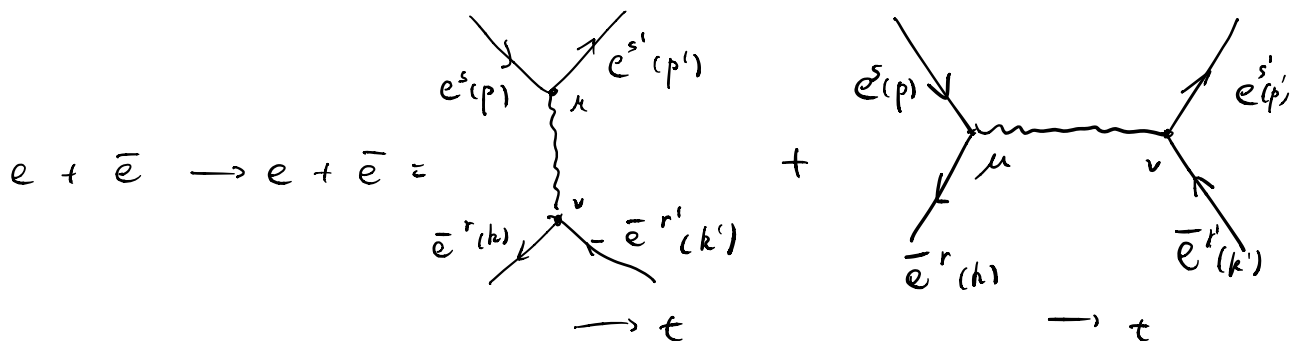
$$\vec{q} = \pm i\varepsilon$$

$$= -\left(\frac{e}{2\pi}\right)^2 \frac{1}{|\vec{x}|} i \cdot 2\pi i \operatorname{Res} \left(\frac{e^{i\vec{q}x}}{\vec{q}^2 + i\varepsilon}, \vec{q} = i\varepsilon \right)$$

$$= + \frac{e^2}{2\pi} \frac{1}{|\vec{x}|} \frac{e^{i \cdot i\varepsilon x}}{2i\varepsilon}$$

$$\varepsilon \rightarrow 0 \Rightarrow + \frac{e^2}{4\pi |\vec{x}|} \quad |\vec{x}| \equiv r \Rightarrow + \frac{e^2}{4\pi r}$$

c) fermion - antifermion scattering



t-channel

$$i\mathcal{M}_t = \bar{u}^s(p) (-ie\gamma^\mu) u^{s'}(p') \frac{-ig^{\mu\nu}}{(p-p')^2 + i\epsilon} \bar{v}^s(p) v^{s'}(p')$$

s-channel

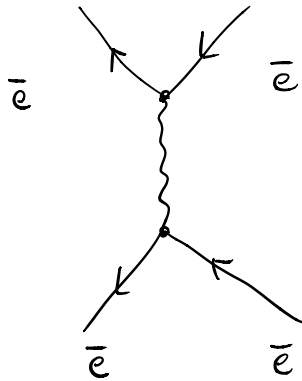
$$i\mathcal{M}_s = \bar{v}^r(k) (-ie\gamma^\mu) u^s(p) \frac{-ig^{\mu\nu}}{(p-p')^2 + i\epsilon} \bar{u}^{s'}(p') (-ie\gamma^\nu) v^{r'}(k')$$

since $v^{s'}(p') v^s(p) = -2m \delta^{ss'} \otimes$

$$\rightarrow V(r) = -\frac{e^2}{4\pi r}$$

has to show
explicitly like
before

antifermion - antifermion:



$$(-1)^2 \rightarrow V(r) = \frac{e^2}{4\pi r}$$