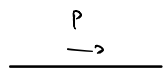
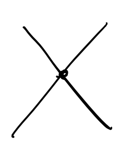
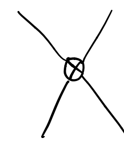
A.14

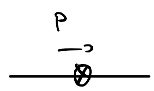
a) Renormalized P.T $\qquad \phi = \sqrt{Z}\,\phi_r$

$$\mathcal{L} = \frac{Z}{2}(\partial_\mu \phi_r)^2 - \frac{Z m_0^2}{2}\phi_r^2 - \frac{Z^2\lambda_0}{4!}\phi_r^4$$

$$= \frac{1}{2}(\partial_\mu \phi_r)^2 - \frac{m^2}{2}\phi_r^2 - \frac{\lambda}{4!}\phi_r^4 + \frac{\delta Z}{2}(\partial_\mu \phi_r)^2 - \frac{\delta m}{2}\phi_r^2 - \frac{\delta\lambda}{4!}\phi_r^4$$

$$\Rightarrow \begin{cases} \delta Z = Z - 1 \\ \delta m = Z m_0^2 - m^2 \\ \delta\lambda = \lambda_0 Z^2 - \lambda \end{cases}$$

b)  $= \dfrac{i}{p^2 - m^2 + i\varepsilon}$  $= -i\lambda$  $= -i\,\delta\lambda$

 $= i(p^2\delta Z - \delta m)$

c) E $\longrightarrow$ external lines

V $\longrightarrow$ vertices

I $\longrightarrow$ internal lines

L $\longrightarrow$ Loops

$$L = I - V + 1$$ $\longleftarrow$ valid for all theories

I: # of undetermined integral $\hat{=} \int d^4p$

V: $\hat{=}$ # of delta functions

1: overall delta functions

$$4V = E + 2I$$

1 external line $\longrightarrow$ 1 vertex

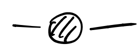
1 internal line $\longrightarrow$ 2 vertices

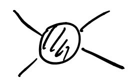
1 vertex $\longrightarrow$ 4 lines

$$D = 4L - 2I \longleftarrow \int \frac{d^4 p}{p^2 - m^2}$$
$$= 4(I - V + 1) - 2I$$
$$= 4 - E$$

d) $\phi \to -\phi$, amplitudes with odd # of external lines vanish

$D = 4$  $\longrightarrow$ 4^{th}

$D = 2$  $\longrightarrow$ quadratically divergent

$D = 0$  $\longrightarrow$ logarithmetically

e) $[\mathcal{L}] = d$, $[\partial^\mu] = 1$

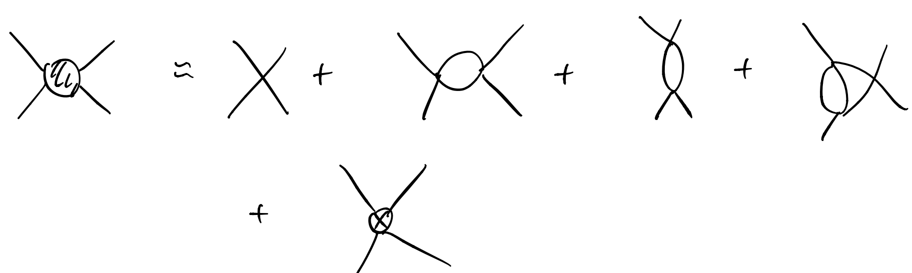
$$d = [(\partial_\mu \phi)(\partial^\mu \phi)] = 2(1 + [\phi]) \iff [\phi] = \frac{d}{2} - 1$$

$$d = [\lambda_0 \phi^4] = 4[\phi] + [\lambda_0] = 2d - 4 + [\lambda_0]$$
$$\Rightarrow [\lambda_0] = 4 - d$$
$$\Rightarrow \lambda = \mu^{4-4} \tilde{\lambda}, \quad [\mu] = 1$$

$\tilde{\lambda}$ is dimensionless

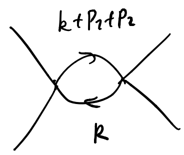
f) divergences $\Rightarrow$ poles in $\frac{1}{\varepsilon}$

MS scheme ; only absorb poles

$i\mathcal{M} =$ 

(must be finite)

s-channel diagram with incoming momenta p_1, p_2



$$= (-i\lambda)^2 \underset{\substack{\uparrow \\ \text{symm. factor}}}{\frac{1}{2}} \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2-m^2+i\varepsilon} \frac{i}{(k+p_1+p_2)^2-m^2+i\varepsilon} =: A(s)$$

$$i\mathcal{M} = -i\lambda + A(s) + A(t) + A(u) - i\delta\lambda$$

expand $A(s)$ arond $A(0)$

$$A(s) = \underset{\substack{\uparrow \\ \text{divergent} \\ \text{part}}}{A(0)} + \underbrace{\frac{\partial^2}{\partial^2 p_1^2} A(p_1^2)\Big|_{p_1^2=0} p_1^2}_{\text{finite}}$$

$$A(s) \sim A(0) = \tilde{\lambda}^2 \mu^{8-2d} \frac{1}{2} \int \frac{d^dk}{(2\pi)^d} \frac{1}{(k^2-m^2+i\varepsilon)^2}$$

$$= \tilde{\lambda}^2 \mu^{8-2d} \frac{1}{2} \frac{i}{(4\pi)^{d/2}} \frac{\Gamma\left(\frac{4-d}{2}\right)}{\Gamma(2)} m^{d-4}$$

$$d = 4-\varepsilon \rightarrow = \tilde{\lambda}^2 \mu^{2\varepsilon} \frac{1}{2} \frac{i}{(4\pi)^{2-\varepsilon/2}} \Gamma(\varepsilon/2)\, m^{-\varepsilon}$$

$$A(s) \sim \lambda^2 \frac{i}{(4\pi)^2} \frac{1}{\varepsilon}$$

$$\mathcal{M} \sim \frac{\lambda^2}{(4\pi)^2} \frac{1}{\varepsilon} \cdot 3 - \delta\lambda$$ ← ignore tree level contribution

in order to keep $\mathcal{M}$ finite $\rightarrow \delta\lambda = \lambda^2 \frac{3}{16\pi^2} \frac{1}{\varepsilon}$