a) 1)

$$\phi(x)\,\phi(y) \quad \bullet\!-\!-\!-\!\bullet \quad \underset{q}{\longrightarrow} \quad = \quad \frac{i}{q^2 - m_\phi^2 + i\varepsilon}$$

$$\psi(x)_\alpha\,\bar\psi_\beta(x) \quad \underset{p}{\overset{\alpha \qquad \beta}{\longrightarrow}} \quad = \quad \frac{i\,(\not p + m)_{\alpha\beta}}{p^2 - m^2 + i\varepsilon} \quad \text{Polarization summ.}$$

sum over all possible states

$$\left(\text{photon:}\ \frac{-ig^{\mu\nu}}{q^2 + i\varepsilon}\right) \qquad\qquad \sum_s u\bar u = \not p + m$$

2) vertices

 $= -ig$

3) external leg contractions

$$\langle 0 |\, \phi\, | q\rangle = \quad \overset{\leftarrow\ q}{\text{---}} \quad = 1 \qquad\qquad \overset{\longrightarrow}{t}$$

$$\langle q |\, \phi\, | 0\rangle = \quad \overset{\leftarrow\ q}{\text{---}} \quad = 1$$

$$\langle 0 |\, \psi\, | p,s\rangle = \quad \overset{\leftarrow\ p}{} \quad = u^s(p) \qquad \langle 0|\,\bar\psi\,| k,s\rangle = \quad \overset{\leftarrow\ k}{} \quad = \bar v^s(k)$$

$$\langle p,s |\, \bar\psi\, | 0\rangle = \quad \overset{\leftarrow\ p}{} \quad = \bar u^s(p) \qquad \langle k,s |\,\psi\,|0\rangle = \quad \overset{\leftarrow\ k}{}$$

$$= v^s(k)$$

$$p^s(q) + n^v(k) \longrightarrow p^{s'}(p') + n^{v'}(k')$$

$$iM = \quad$$

$$= \bar{u}^{s'}(q')(-ig)u^s(q) \frac{i}{(q'-q)^2 - m_\phi^2 + i\varepsilon} \bar{u}^{v'}(k')(-ig)u^v(k)$$

$$= -ig^2 \bar{u}^{s'}(q')u^s(q) \frac{1}{(q'-q)^2 - m_\phi^2 + i\varepsilon} \bar{u}^{v'}(k')u^v(k)$$

No integral ⟵ No loops!

b) $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$,

$$u^s(q) = \sqrt{E_p + m} \begin{pmatrix} \xi^s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} \xi^s \end{pmatrix} \xrightarrow{\vec{p}/m \to 0} \sqrt{2m} \begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$\bar{u}^{s'}(q')u^s(q) \simeq 2m(\xi^{s'})^\dagger(\xi^s) = 2m\,\delta^{ss'}$$

$$(q'-q)^2 = ((q')^0 - q^0)^2 - |\vec{q}' - \vec{q}|^2 \simeq -(\vec{q}' - \vec{q})^2$$

$$i\mathcal{M} = \frac{ig^2}{|\vec{q}' - \vec{q}|^2 + m_\phi^2 + i\varepsilon} 2m\,\delta^{ss'} 2m\,\delta^{vv'} \qquad \text{(spin is separately conserved)}$$

c) $\langle P_f | S | P_i \rangle$

$$= \langle P_f | \mathbb{1} | P_i \rangle - \langle P_f | (2\pi)\delta(E_i - E_f) i V | P_i \rangle$$

$$V = \int d^3x\, d^3y\; V(|\vec{x} - \vec{y}|)\, \bar{p}(\vec{x})\, p(\vec{x})\, \bar{n}(\vec{y})\, n(\vec{y})$$
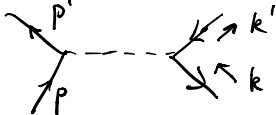
$$\langle q', s'; k', v' | V | q, s; k, v \rangle$$

$$= \int d^3x\, d^3y\; V(|\vec{x} - \vec{y}|) \langle q', s'; k', v' | \bar{p}(x) p(x) \bar{n}(y) n(y) | q, s; k, v \rangle$$

$$= \int d^3x\, d^3y\; V(|\vec{x} - \vec{y}|)\, \bar{u}^{s'}(q')\, u^s(q)\, \bar{u}^{v'}(k')\, u^v(k)\, e^{-ix(q-q')} x$$

$$e^{-i\vec{y}(\vec{k}-\vec{k}')}$$

$$\vec{x}=\tfrac{1}{2}(\vec{w}+\vec{z})$$
$$\vec{y}=\tfrac{1}{2}(\vec{w}-\vec{z}) \longrightarrow$$

$$\approx \int d^3x\, d^3y\; V(|\vec{x}-\vec{y}|)\; 2m\,\delta^{ss'}\, 2m\,\delta^{vv'}\, e^{i\vec{x}(\vec{q}-\vec{q}')}\, e^{i\vec{y}(\vec{k}-\vec{k}')}$$

$$= 2m\,\delta_{ss'}\, 2m\,\delta_{vv'}\,\delta^{(3)}(\vec{q}+\vec{k}-\vec{q}'-\vec{k}')\int d^3z\; \underbrace{V(|\vec{z}|)\, e^{iz(\vec{q}-\vec{q}')}}_{\displaystyle \breve{V}(|\vec{q}-\vec{q}'|)}$$

$$V(\vec{x}) = \int \frac{d^3q}{(2\pi)^3}\,\frac{-g^2}{|\vec{q}|^2+m_\phi^2}\, e^{-i\vec{q}\cdot\vec{x}} = -\frac{g^2}{4\pi^2}\int_{-1}^{+1}dz\int_0^\infty d|\vec{q}|\,|\vec{q}|^2$$
$$\times \frac{1}{|\vec{q}|^2+m_\phi^2}\, e^{-i|\vec{q}|rz}$$

$$= -\frac{g^2}{4\pi^2}\int_0^\infty d|\vec{q}|\,|\vec{q}|^2\,\frac{e^{i|\vec{q}|r}-e^{-i|\vec{q}|r}}{i\,|\vec{q}|\,r}\,\frac{1}{|\vec{q}|^2+m_\phi^2}$$

$$= \frac{-g^2}{4\pi^2 i\,r}\int_{-\infty}^{+\infty} d|\vec{q}|\,\frac{|\vec{q}|\,e^{i|\vec{q}|r}}{|\vec{q}|^2+m_\phi^2}$$

$$= -\frac{g^2}{4\pi^2 i r}\,(2\pi i)\,\mathrm{Res}\left(\frac{q\,e^{iqr}}{q^2+m_\phi^2}, q=+im_\phi\right) = -\frac{g^2}{4\pi r}\,e^{-m_\phi r}$$

d) $\quad iM =$

$$= -ig^2\,\bar{u}^{s'}(p')\,u^s(p)\,\frac{1}{(p'-p)^2-m_\phi^2}\,\bar{v}^v(k)\,v^{v'}(k')\,(-1)$$

$$\langle p',k'|\,\overline{\psi}(x)\,\psi(x)\,\overline{\psi}(y)\,\psi(y)\,|p,k\rangle = -1\cdot(---)$$

$$\langle p',k'|\,\overline{\psi}(x)\,\psi(x)\,\overline{\psi}(y)\,\psi(y)\,|p,k\rangle$$

$$= \langle p', k' | \psi(x) \psi(y) \bar{\psi}(x) \bar{\psi}(y) | p, k \rangle$$

$T ($