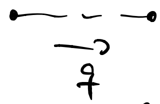
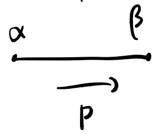
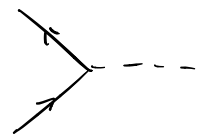
a) 1) $\overbrace{\phi(x)\,\phi(y)}$  $= \dfrac{i}{q^2 - m_\phi^2 + i\varepsilon}$

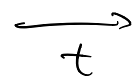
$\overbrace{\psi(x)_\alpha \bar{\psi}_\beta(x)}$  $= \dfrac{i(\not{p} + m)_{\alpha\beta}}{p^2 - m^2 + i\varepsilon}$ → Polarization sum, sum over all possible states

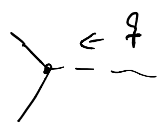
(photon: $\dfrac{-ig^{\mu\nu}}{q^2 + i\varepsilon}$)

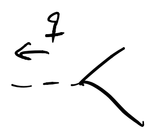
$\sum_s u\bar{u} = \not{p} + m$

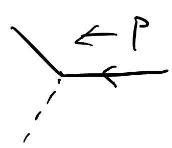
2) vertices  $= -ig$.

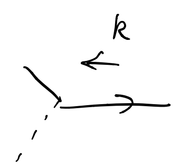
3) external leg contractions

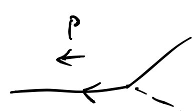


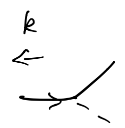
$\langle 0 | \overbrace{\phi | q} \rangle =$  $= 1$

$\langle \overbrace{q | \phi} | 0 \rangle =$  $= 1$

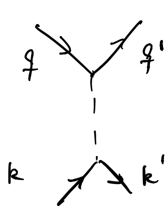
$\langle 0 | \psi | p, s \rangle =$  $= u^s(p)$

$\langle 0 | \overbrace{\bar{\psi} | k, s} \rangle =$  $= \bar{v}^s(k)$

$\langle p, s | \bar{\psi} | 0 \rangle =$  $= \bar{u}^s(p)$

$\langle k, s | \psi | 0 \rangle =$  $= v^s(k)$

$p^s(q) + n^\nu(k) \longrightarrow p^{s'}(p') + n^{\nu'}(k')$

$i\mathcal{M} =$ 

$$= \bar{u}^{s'}(q')(-ig)u^{s}(q)\,\frac{i}{(q'-q)^2-m_\phi^2+i\varepsilon}\,\bar{u}^{v'}(k')(-ig)u^{v}(k)$$

$$= -ig^2\,\bar{u}^{s'}(q')u^{s}(q)\,\frac{1}{(q'-q)^2-m_\phi^2+i\varepsilon}\,\bar{u}^{v'}(k')u^{v}(k)$$

No integral ← No loops!

b) $\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$,

$$u^s(q) = \sqrt{E_p+m}\begin{pmatrix} \xi^s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\xi^s \end{pmatrix} \xrightarrow{\vec{p}/m\to 0} \sqrt{2m}\begin{pmatrix} \xi^s \\ 0 \end{pmatrix}$$

$$\bar{u}^{s'}(q')u^{s}(q) \simeq 2m(\xi^{s'})^{\dagger}(\xi^{s}) = 2m\,\delta^{ss'}$$

$$(q'-q)^2 = ((q')^0-q^0)^2 - |\vec{q}\,'-\vec{q}\,|^2 \simeq -|\vec{q}\,'-\vec{q}\,|^2$$

$$i\mathcal{M} = \frac{ig^2}{|\vec{q}\,'-\vec{q}\,|^2+m_\phi^2-i\varepsilon}\,2m\,\delta^{ss'}\,2m\,\delta^{vv'}$$ (spin is seperately conserved)

c) $\langle p_f|\,S\,|p_i\rangle$

$$= \langle p_f|\,\mathbb{1}\,|p_i\rangle - \langle p_f|\,(2\pi)\delta(E_i-E_f)\,iV\,|p_i\rangle$$

$$V = \int d^3x\,d^3y\;V(|\vec{x}-\vec{y}|)\,\bar{p}(\vec{x})\,p(\vec{x})\,\bar{n}(\vec{y})\,n(\vec{y})$$

$$\langle q',s';k',v'|\,V\,|q,s;k,v\rangle$$

$$= \int d^3x\,d^3y\;V(|\vec{x}-\vec{y}|)\langle q',s';k',v'|\,\bar{p}(x)p(x)\bar{n}(y)n(y)\,|q,s;k,v\rangle$$

$$= \int d^3x\,d^3y\;V(|\vec{x}-\vec{y}|)\,\bar{u}^{s'}(q')u^{s}(q)\,\bar{u}^{v'}(k')u^{v}(k)\,e^{-ix(q-q')}x$$

$e^{-iy(k-k')}$

$$\approx \int d^3x\, d^3y\; V(|\vec{x}-\vec{y}|)\, 2m\, \delta^{ss'}\, 2m\, \delta^{vv'}\, e^{i\vec{x}(\vec{q}-\vec{q}')}\, e^{i\vec{y}(\vec{k}-\vec{k}')}$$

$\vec{x} = \frac{1}{2}(\vec{w}+\vec{z})$

$\vec{y} = \frac{1}{2}(\vec{w}-\vec{z})$ $\longrightarrow$

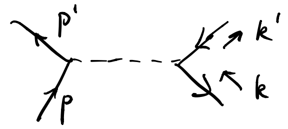
$$= 2m\, \delta_{ss'}\, 2m\, \delta_{vv'}\, \delta^{(3)}(\vec{q}+\vec{k}-\vec{q}'-\vec{k}') \int d^3z\, \underbrace{V(|\vec{z}|)\, e^{iz(\vec{q}-\vec{q}')}}_{\tilde{V}(|\vec{q}-\vec{q}'|)}$$

$$V(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} \frac{-g^2}{|\vec{q}|^2+m_\phi^2}\, e^{-i\vec{q}\cdot\vec{x}} = -\frac{g^2}{4\pi^2} \int_{-1}^{+1} dz \int_0^\infty d|\vec{q}|\, |\vec{q}|^2 \times \frac{1}{|\vec{q}|^2+m_\phi^2}\, e^{-i|\vec{q}|rz}$$

$$= -\frac{g^2}{4\pi^2} \int_0^\infty d|\vec{q}|\, |\vec{q}|^2\, \frac{e^{i|\vec{q}|r} - e^{-i|\vec{q}|r}}{i|\vec{q}|r}\, \frac{1}{|\vec{q}|^2+m_\phi^2}$$

$$= \frac{-g^2}{4\pi^2 i r} \int_{-\infty}^{+\infty} \frac{|\vec{q}|\, e^{i|\vec{q}|r}}{|\vec{q}|^2+m_\phi^2}$$

$$= -\frac{g^2}{4\pi^2 i r}\, (2\pi i)\, \mathrm{Res}\left(\frac{q\, e^{iqr}}{q^2+m_\phi^2},\ q = +im_\phi \right) = -\frac{g^2}{4\pi r}\, e^{-m_\phi r}$$

d) $i\mathcal{M} =$ 

$$= -ig^2\, \bar{u}^{s'}(p')\, u^s(p)\, \frac{1}{(p'-p)^2 - m_\phi^2}\, \bar{v}^{v}(k)\, v^{v'}(k')\, (-1)$$

$$\langle p', k' | \bar{\psi}(x)\psi(x)\bar{\psi}(y)\psi(y) | p, k \rangle = -1 \cdot (- - -)$$

$$\langle p', k' | \bar{\psi}(x)\psi(x)\bar{\psi}(y)\psi(y) | p, k \rangle$$

$$= \langle p', k' | \psi(x)\, \psi(y)\, \bar{\psi}(x)\, \bar{\psi}(y)\, | p, k \rangle$$

$T\,($