

Quantum Field Theory

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Contents

1 Klein-Gordon theory

(Real) Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \quad (2.0.1)$$

Quantization

$$\begin{aligned} [\phi(\mathbf{x}), \phi(\mathbf{x}')] &= [\pi(\mathbf{x}), \pi(\mathbf{x}')] = 0 \\ [\phi(\mathbf{x}), \pi(\mathbf{x}')] &= i\delta^{(3)}(\mathbf{x} - \mathbf{x}') \end{aligned} \quad (2.0.2)$$

Decomposition into Fourier modes

$$\phi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} + a_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\cdot\mathbf{x}} \right) \quad (2.0.3)$$

$$\pi(\mathbf{x}) = \int \frac{d^3p}{(2\pi)^3} (-i) \sqrt{\frac{E_p}{2}} \left(a_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{x}} - a_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\cdot\mathbf{x}} \right) \quad (2.0.4)$$

thus the commutation relations for ladder operators:

$$[a_{\mathbf{p}}, a_{\mathbf{p}'}] = [a_{\mathbf{p}}^\dagger, a_{\mathbf{p}'}^\dagger] = 0 \quad (2.0.5)$$

$$[a_{\mathbf{p}}, a_{\mathbf{p}'}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{p}') \quad (2.0.6)$$

Hamiltonian in terms of ladder operator

$$H = \int \frac{d^3p}{(2\pi)^3} E_p \left(a_{\mathbf{p}} a_{\mathbf{p}}^\dagger + \frac{1}{2} [a_{\mathbf{p}}, a_{\mathbf{p}}^\dagger] \right) \quad (2.0.7)$$

Normalisation it's also lorentz-invariante

$$\langle p|q \rangle = 2E_p (2\pi)^3 \delta^{(3)}(\mathbf{p} - \mathbf{q}) \quad (2.0.8)$$

2.1 Heisenberg-picture fields

Heisenberg-picture

$$|\psi_H\rangle = e^{iHt} |\psi_S(t)\rangle \quad (2.1.1)$$

$$O_H(t) = e^{iHt} O_S e^{-iHt} \quad (2.1.2)$$

Field operator

$$\phi(x) = \phi(\mathbf{x}, t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_{\mathbf{p}} e^{ipx} + a_{\mathbf{p}}^\dagger e^{-ipx} \right) \quad (2.1.3)$$

2.2 Commutations and propagators

Commutations

$$[\phi(x), \phi(y)] = D(x-y) - D(y-x) \begin{cases} = 0 & \text{if } (x-y) \text{ is space-like} \\ \neq 0 & \text{otherwise} \end{cases} \quad (2.2.1)$$

$$D(x-y) = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip(x-y)} \quad (2.2.2)$$

Propogator

$$\langle 0 | \phi(x) \phi(y) | 0 \rangle = D(x-y) \quad (2.2.3)$$

Feynman propagator

$$\begin{aligned} D_F(x-y) &= \langle 0 | T \phi(x) \phi(y) | 0 \rangle \\ &= \Theta(x^0 - y^0) D(x-y) + \Theta(y^0 - x^0) D(y-x) \end{aligned} \quad (2.2.4)$$

$$D_F(x-y) = \int \frac{d^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)} \quad (2.2.5)$$