# Quantum Field Theory

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# Contents

# 2 Klein-Gordon theory

**(Real) Lagrangian density**

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 \tag{2.0.1}$$

**Quantization**

$$[\phi(\boldsymbol{x}), \phi(\boldsymbol{x'})] = [\pi(\boldsymbol{x}), \pi(\boldsymbol{x'})] = 0$$
$$[\phi(\boldsymbol{x}), \pi(\boldsymbol{x'})] = i\delta^{(3)}(\boldsymbol{x} - \boldsymbol{x'}) \tag{2.0.2}$$

**Decomposition into Fourier modes**

$$\phi(\boldsymbol{x}) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left( a_{\boldsymbol{p}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} + a_{\boldsymbol{p}}^\dagger e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \right) \tag{2.0.3}$$

$$\pi(\boldsymbol{x}) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} (-i) \sqrt{\frac{E_p}{2}} \left( a_{\boldsymbol{p}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} - a_{\boldsymbol{p}}^\dagger e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} \right) \tag{2.0.4}$$

thus the commutation relations for ladder operators:

$$\left[ a_{\boldsymbol{p}}, a_{\boldsymbol{p'}} \right] = \left[ a_{\boldsymbol{p}}^\dagger, a_{\boldsymbol{p'}}^\dagger \right] = 0 \tag{2.0.5}$$

$$\left[ a_{\boldsymbol{p}}, a_{\boldsymbol{p'}}^\dagger \right] = (2\pi)^3 \delta^{(3)}(\boldsymbol{p} - \boldsymbol{p'}) \tag{2.0.6}$$

**Hamiltonian in terms of ladder operator**

$$H = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} E_p \left( a_{\boldsymbol{p}} a_{\boldsymbol{p}}^\dagger + \frac{1}{2} \left[ a_{\boldsymbol{p}}, a_{\boldsymbol{p}}^\dagger \right] \right) \tag{2.0.7}$$

**Normlisation**    it's also lorentz-invariante

$$\langle p | q \rangle = 2E_p (2\pi)^3 \delta^{(3)}(\boldsymbol{p} - \boldsymbol{q}) \tag{2.0.8}$$

## 2.1 Heisenberg-picture fields

**Heisenberg-picture**

$$|\psi_H\rangle = e^{iHt} |\psi_s(t)\rangle \tag{2.1.1}$$

$$O_H(t) = e^{iHt} O_S e^{-iHt} \tag{2.1.2}$$

**Field operator**

$$\phi(x) = \phi(\boldsymbol{x}, t) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left( a_p e^{ipx} + a_p^\dagger e^{-ipx} \right) \tag{2.1.3}$$

## 2.2 Commutations and propogators

**Commutations**

$$[\phi(x), \phi(y)] = D(x - y) - D(y - x) \begin{cases} = 0 & \text{if } (x - y) \text{ is space-like} \\ \neq 0 & \text{otherwise} \end{cases} \tag{2.2.1}$$

$$D(x - y) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-ip(x-y)} \tag{2.2.2}$$

**Propogator**

$$\langle 0|\phi(x)\phi(y)|0\rangle = D(x - y) \tag{2.2.3}$$

**Feynman propagator**

$$\begin{aligned} D_F(x - y) &= \langle 0|T\phi(x)\phi(y)|0\rangle \\ &= \Theta(x^0 - y^0)D(x - y) + \Theta(y^0 - x^0)D(y - x) \end{aligned} \tag{2.2.4}$$

$$D_F(x - y) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{i}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)} \tag{2.2.5}$$