a) $\mathcal{L} = \bar\psi(i\not\partial - e\not A - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2$,

$[\mathcal{L}] = d$, $\quad [\partial_\mu] = 1$

$\Rightarrow [\psi] = \frac{d-1}{2}$, $\quad [A_\mu] = \frac{d}{2} - 1$, $\quad [\bar\psi A \psi] = \frac{3}{2}d - 2$,

$[e] = d - (\frac{3}{2}d - 2) = 2 - \frac{d}{2}$

Mass scale μ, $[\mu] = 1$, $\quad e = \mu^{(2-\frac{d}{2})}\tilde{e}$

$$\mathcal{L}_{int} = -e\bar\psi\not A\psi = -\mu^{2-\frac{d}{2}}\tilde{e}\bar\psi\not A\psi$$

b) $$i\mathcal{M} = (-i\tilde{e}\mu^{2-d/2})^3\int\frac{d^dk}{(2\pi)^d}\bar{u}^{(s')}(p')\gamma^\nu\frac{i(\not p' - \not k + m)}{(p'-k)^2 - m^2 + i\varepsilon}\gamma^\mu\frac{i(\not p - \not k + m)}{(p-k)^2 - m^2 + i\varepsilon}$$

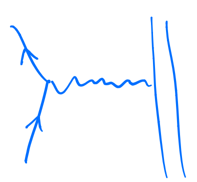
$$\times\gamma^\rho u^{(s)}(p)\frac{-ig_{\mu\nu}}{k^2 + i\varepsilon}\varepsilon_\mu^{(\bar{s})}(q)$$

(Position of term representing photon doesn't matter.

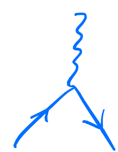
$$= \bar{u}^{(s')}(p')\Gamma^\mu(p',p)u^{(s)}(p)\varepsilon_\mu^{(\bar{s})}(q)(-i\tilde{e}\mu^{2-\frac{d}{2}})$$

$$\Gamma^\mu(p',p) = -i(\tilde{e}\mu^{2-d/2})^2\int\frac{d^dk}{(2\pi)^d}\frac{\gamma^\nu(\not p' - \not k + m)\gamma^\mu(\not p - \not k + m)\gamma_\nu}{[(p-k')^2 - m^2 + i\varepsilon][(p-k)^2 - m^2 + i\varepsilon][k^2 + i\varepsilon]}$$

leave $(-i\tilde{e}\mu^{2-d/2})$ outside to match the first order diagram



some source



c) $q_\mu F^\mu(p,p',q) = 0 \Rightarrow \bar{u}^{(s')}(p')q_\mu\Gamma^\mu(p',p)u^{(s)}(p) = 0$

$$A\,\bar{u}^{(s')}(p')q_\mu\gamma^\mu u^{(s)}(p) = A(\underbrace{\bar{u}^{(s')}(p')\not p'}_{\bar{u}^{(s')}(p')m}u^{(s)}(p) - \bar{u}^{(s')}(p')\underbrace{\not p u^{(s)}(p)}_{m u^{(s)}(p)}) = 0$$

$$B\,\bar{u}^{(s)}(p')q_\mu(p'+p)^\mu u^{(s)}(p) = B\,\bar{u}^{(s')}(p')((p')^2 - (p)^2)u^{(s)}(p) = 0$$

$$C\,\bar{u}^{(s)}(p')q_\mu p^\mu u^{(s')}(p') = C q^2\bar{u}^{(s')}(p')u^{(s)}(p) \neq 0$$

$\Rightarrow C \equiv 0$

d) $\bar{u}^{(s')}(p')\, i\sigma^{\mu\nu}(p'_\nu - p_\nu)\, u^{(s)}(p) = \bar{u}^{(s')}(p')\, \frac{1}{2}[\gamma^\nu, \gamma^\mu]\, (p'_\nu - p_\nu)\, u^{(s)}(p)$

$$\left(\quad [\gamma^\nu, \gamma^\mu] = \gamma^\nu\gamma^\mu - \gamma^\mu\gamma^\nu = 2\gamma^\nu\gamma^\mu - 2g^{\mu\nu}\mathbb{1} = -[\gamma^\mu, \gamma^\nu] \right)$$

$$= \bar{u}^{(s')}(p')\left[(\gamma^\nu\gamma^\mu - g^{\mu\nu})p'_\nu - (g^{\mu\nu} - \gamma^\mu\gamma^\nu]p_\nu\right]u^{(s)}(p)$$

$$= \bar{u}^{(s')}(p')\left[\not{p}'\gamma^\mu - (p'+p)^\mu + \gamma^\nu\not{p}\right]u^{(s)}(p) = \bar{u}^{(s')}(p')[2m\gamma^\mu - (p'+p)^\mu]u^{(s)}(p)$$

$$\bar{u}^{(s')}(p')\,\Gamma^\mu(p',p)\,u^{(s)}(p) = A\bar{u}^{(s')}(p')\gamma^\mu u^{(s)}(p) + B(p'+p)^\mu\, \bar{u}^{(s')}(p')u^{(s)}(p)$$

$$= \underbrace{(A+2mB)}_{F_1(q^2)}\ \bar{u}^{(s')}(p')\gamma^\mu u^{(s)}(p) + \underbrace{(-2mB)}_{F_2(q^2)}\ \bar{u}^{(s')}(p')\,\frac{i\sigma^{\mu\nu}(p'-p)_\nu}{2m}\,u^{(s)}(p)$$

e) $$\Gamma^\mu = -i(\tilde{e}\mu^{2-d/2})^2 \int \frac{d^dk}{(2\pi)^d}\ \frac{\gamma^\nu(\not{p}'-\not{k}+m)\gamma^\mu(\not{p}-\not{k}+m)\gamma_\nu}{[(p'-k)^2-m^2+i\varepsilon][(p-k)^2-m^2+i\varepsilon][k^2+i\varepsilon]}$$

$$= -i(\tilde{e}\mu^{2-d/2})^2 \int \frac{d^dk}{(2\pi)^d}\cdot 2\int_0^1 dx\,dy\,dz$$

$$\times\ \frac{\delta(x+y+z-1)\,\gamma^\nu(\not{p}'-\not{k}+m)\gamma^\mu(\not{p}-\not{k}+m)\gamma_\nu}{[y(\not{p}-\not{k})^2 - ym^2 + x(p-k)^2 - xm^2 + zk^2]^3}$$

$$= -2i(\tilde{e}\mu^{2-d/2})^2 \int \frac{d^dk}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy\ \frac{\gamma^\nu(\not{p}'-\not{k}-m)\gamma^\mu(\not{p}-\not{k}+m)\gamma_\nu}{[p'^2y + p^2x - m^2(x+y) + k^2 - 2k(px+p'y)]^3}$$

Denominator $= \left[k^2 - 2k(px+p'y) - m^2(x+y) + p'^2y + p^2x\right]^3$

$$= \left[(k-(px+p'y))^2 - ((px+p'y)^2 + m^2(x+y) - p'^2y - p^2x)\right]^3$$

$$= \left[(k-(px+p'y)^2) - (m^2(x+y) - p^2x(1-x) - p'^2y(1-y) + 2pp'xy)\right]^3$$

$$= \left[(k-(px+p'y))^2 - \Delta\right]^3$$

$\tilde{k} = k - (px+p'y)$,

$$\Gamma^\mu = -2i\frac{\tilde{e}^2\mu^{4-d}}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy\int d^d\tilde{k}$$

$$\frac{\gamma^\nu(\not{p}' - \not{k} - \not{p}x - \not{p}'y + m)\gamma^\mu(\not{p} - \not{k} - \not{p}x - \not{p}'y + m)\gamma_\nu}{(\hat{k}^2 - \Delta)^3}$$

$$= -2i\,\frac{\breve{e}^{\mu}\mu^{4-d}}{(2\pi)^d}\int_0^1 dx\int_0^{1-x} dy\int d^d\hat{k}\,\frac{I_1}{I_2^3}$$