H.5 a) P is operator in Hilbert space acts only on a_s, $a_s^\dagger$ 18/18

$P\psi(\underline{x},t)P^{-1} \qquad (P = P^\dagger = P^{-1})$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s [P a_s(p) P\, u_s(p) e^{-ipx} + P b_s^\dagger(p) P v_s(p) e^{+ipx}]$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s [\eta_a a_s(-\underline{p}) \underbrace{u_s(\underline{p})}_{=\gamma^0 u_s(-\underline{p})} e^{-ipx} - \eta_b^* b_s^\dagger(-\underline{p}) \underbrace{v_s(\underline{p})}_{=\gamma_0 v_s(-\underline{p})} e^{+ipx}]$$

$$= \eta_a \gamma^0 \psi(t,-\underline{x}) |0\rangle, \text{ if } \eta_a = -\eta_b^*$$

$\eta_a^2 = \eta_b^2 = \pm 1$, since its fermionic field, only even number of a_p or a_q

$\Rightarrow \eta_a \eta_b = -\eta_a \eta_a^* = -1$, i.e. $\eta_a \eta_a^* = 1$

$\eta_a = x+iy$: $(x+iy)(x-iy) = x^2+y^2 = 1$

$(x+iy)(x+iy) = \pm 1, \quad 2ixy = 0, \quad x=0$ or $y=0$

$\Rightarrow \eta_a = \pm 1, \pm i$

$\eta_b = \mp 1, \pm i$

b) $T\,\psi(\underline{x},t)\,T^{-1} \qquad (T = T^\dagger = T^{-1})$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s [T a_s(p) u_s(p) e^{-ipx} T + T b_s^\dagger(p) v_s(p) e^{+ipx} T]$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s [(-)^{\frac{1}{2}+s} a_{-s}(-\underline{p}) u_s^*(p) e^{+ipx} + (-)^{\frac{1}{2}+s} b_{-s}^\dagger(-\underline{p}) v_s^*(p) e^{-ipx}]$$

$$i\gamma^5\gamma^2\gamma^0 u_s(\underline{p}) = (-)^{\frac{1}{2}-s} [u_{-s}(-\underline{p})]^*$$

$$LHS = -\gamma^0\gamma^1\gamma^2\gamma^3\gamma^2\gamma^0 = +\gamma^0\gamma^1(\gamma^2)^2\gamma^3\gamma^0 = -\gamma^0\gamma^1\gamma^3\gamma^0$$

$$= -(\gamma^0)^2\gamma^1\gamma^3 = -\gamma^1\gamma^3$$

$$\Rightarrow \quad -\gamma^1\gamma^3 u_s(\underline{p}) = (-)^{\frac{1}{2}-s}[u_{-s}(-\underline{p})]^*$$

$$\Leftrightarrow \quad -(-)^{s-\frac{1}{2}}\gamma^1\gamma^3 u_s(\underline{p}) = [u_{-s}(-\underline{p})]^*$$

$$\Leftrightarrow \quad -(-)^{-s-\frac{1}{2}}\gamma^1\gamma^3 u_{-s}(-\underline{p}) = [u_s(\underline{p})]^*$$

analogically $\quad -(-)^{-s-\frac{1}{2}}\gamma^1\gamma^3 v_{-s}(-\underline{p}) = [v_s(\underline{p})]^*$

$$= -\gamma^1\gamma^3 \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s [a_{-s}(-\underline{p})u_{-s}(-\underline{p})e^{+ipx} + b^+_{-s}(-\underline{p})v_{-s}(-\underline{p})e^{-ipx}]\,|0\rangle$$

inside integral $-\underline{p} \to \tilde{\underline{p}}$, $\quad p = (E_{\underline{p}}, \underline{p}) \longrightarrow \tilde{p} = (E_{\underline{p}}, -\underline{p})$

$$= -\gamma^1\gamma^3 \int \frac{d^3\tilde{p}}{(2\pi)^3}\frac{1}{\sqrt{2E_{\tilde{p}}}}\sum_s [a_{-s}(\tilde{p})u_{-s}(\tilde{p})e^{-i\tilde{p}(-t,\underline{x})} + b^+_{-s}(\tilde{p})v_s(\tilde{p})e^{i\tilde{p}(-t,\underline{x})}]\,|0\rangle$$

$$= -\gamma^1\gamma^3\,\psi(-t,\underline{x})\,|0\rangle$$

c) $\quad C\,\psi(\underline{x},t)\,C^{-1} \qquad\qquad (C^+ = C = C^{-1} = -i\gamma^2)$

$$= \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s [\underbrace{C\,a_s(\underline{p})\,C}_{b_s(\underline{p})}\underbrace{u_s(\underline{p})}_{=-i\gamma^2 v_s^*(\underline{p})}e^{-ipx} + \underbrace{C\,b^+_s(\underline{p})\,C}_{a^+_s(\underline{p})}\underbrace{v_s(\underline{p})}_{=-i\gamma^2 u_s^*(\underline{p})}e^{-ipx}]$$

$$= -i\gamma^2\,\psi^*(\underline{x},t)$$

H.6

How do we get expression (19) ?

a) $P\,|\Psi, S=0,1\rangle$

wave function to bind two particles

one for singlett, three for triplett

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \phi_s(|\underline{p}|)\, P \left\{ a^\dagger_{\frac{1}{2}}(\underline{p}) \underbrace{b^\dagger_{-\frac{1}{2}}}_{PP}(\underline{p}) + (-)^{S-1} a^\dagger_{-\frac{1}{2}}(\underline{p}) \underbrace{b^\dagger_{\frac{1}{2}}}_{PP}(-\underline{p}) \underbrace{\}}_{PP} \right|0\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left\{ \eta_a^* \eta_b^*\, a^\dagger_{\frac{1}{2}}(-\underline{p})\, b^\dagger_{-\frac{1}{2}}(+\underline{p}) + (-)^{S-1} \eta_a^* \eta_b^*\, a^\dagger_{-\frac{1}{2}}(-\underline{p})\, b^\dagger_{\frac{1}{2}}(+\underline{p}) \right\}|0\rangle$$

$$= \eta_a^* \eta_b^* \int \frac{d^3\tilde{p}}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left\{ a^\dagger_{\frac{1}{2}}(\tilde{\underline{p}})\, b^\dagger_{-\frac{1}{2}}(-\tilde{\underline{p}}) + (-)^{S-1} a^\dagger_{-\frac{1}{2}}(\tilde{\underline{p}})\, b^\dagger_{\frac{1}{2}}(-\tilde{\underline{p}}) \right\}|0\rangle$$

$$= -|\eta_a|^2\, |\Psi, S=0,1\rangle$$

$$= -|\Psi, S=0,1\rangle$$

positive parity!

b) $C\,|\Psi, S=0,1\rangle$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \phi_s(|\underline{p}|) \left\{ C a^\dagger_{\frac{1}{2}}(\underline{p})\, C C\, b^\dagger_{-\frac{1}{2}}(-\underline{p}) + (-)^{S-1} C a^\dagger_{-\frac{1}{2}}(\underline{p})\, C C\, b^\dagger_{\frac{1}{2}}(-\underline{p}) \right\} \underbrace{C C |0\rangle}_{=|0\rangle}$$

$$= \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \phi_s(|\underline{p}|) \left\{ b^\dagger_{\frac{1}{2}}(\underline{p})\, a^\dagger_{-\frac{1}{2}}(-\underline{p}) + (-)^{S-1} b^\dagger_{-\frac{1}{2}}(\underline{p})\, a^\dagger_{\frac{1}{2}}(-\underline{p}) \right\}|0\rangle$$

$$= (-1)(-)^{S-1} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \phi_s(|\underline{p}|) \left\{ (-)^{S-1} a^\dagger_{-\frac{1}{2}}(-\underline{p})\, b^\dagger_{\frac{1}{2}}(\underline{p}) + a^\dagger_{\frac{1}{2}}(-\underline{p})\, b^\dagger_{-\frac{1}{2}}(\underline{p}) \right\}|0\rangle$$

$$\left[\left((-)^{S-1}\right)^2 = 1, \quad \{a_s, b_s\} = \{a_s^\dagger, b_s^\dagger\} = 0 \right]$$

changing $-\underline{p} \to \tilde{\underline{p}}$

$$= (-)^S\, |\Psi, S=0,1\rangle$$

c) $C A_\mu C^{-1} = -A_\mu \Rightarrow$ photo is its own antiparticle

eigenvalue -1

$$C|n\rangle = C\, \underbrace{|\gamma\rangle \otimes \cdots \otimes |\gamma\rangle}_{n}$$

$= (-1)^n |n\rangle$

$C|\Psi, S=0\rangle = |\Psi, S=0\rangle$ $\Rightarrow$ decay into even numbers of photons
2, 4, 6, ...

$C|\Psi, S=1\rangle = -|\Psi, S=1\rangle$ $\Rightarrow$ decay into odd numbers of photons
3, 5, 7, ...

For these two decays, same initial state, S=1 decay has more possibilities for final states.

$\Rightarrow \Gamma_{S=1} > \Gamma_{S=0}$

$\Rightarrow \tau_{S=1} < \tau_{S=0}$, not consistent...

2γ bigger phase space $\rightarrow$ decay easier
3γ