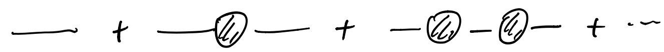
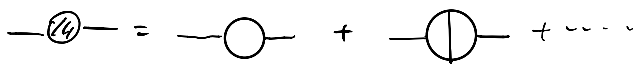
3. a) $\int d^4x\, e^{ipx} \langle \Omega | T\phi(x)\phi(0)|\Omega\rangle$

$=$ 

(with  $= -i\Sigma(p^2)$

$$= D_F^{(0)}(p^2) + D_F^{(0)}(p^2)\left(-i\Sigma(p^2)\right)D_F^{(0)}(p^2) + \cdots$$

$$= D_F^{(0)}(p^2)\frac{1}{1+i\Sigma(p^2)D_F^{(0)}(p^2)} = \frac{i}{p^2-m_0^2}\frac{1}{1+i\Sigma(p^2)\frac{i}{p^2-m_0^2}}$$

$$= \frac{i}{p^2-m_0^2-\Sigma(p^2)}$$

Expand $\Sigma(p^2) = \Sigma(m^2) + (p^2-m^2)\Sigma'(m^2) + (p^2-m^2)\tilde{\Sigma}(p^2)$

with $m_0^2 + \Sigma(m^2) = m^2$, $\tilde{\Sigma}(m^2) = 0$

$$\Rightarrow \quad m_0^2 + \Sigma(p^2) = m_0^2 + \Sigma(m^2) + \cdots$$

$$= m^2 + (p^2-m^2)\Sigma'(m^2) + (p^2-m^2)\tilde{\Sigma}(p^2)$$

$$\frac{i}{p^2-m_0^2-\Sigma(p^2)} = \frac{i}{p^2-m^2-(p^2-m^2)\left(\Sigma'(m^2)+\tilde{\Sigma}(p^2)\right)}$$

$$= \frac{i}{p^2-m^2}\frac{1}{1-\Sigma'(m^2)-\tilde{\Sigma}(p^2)}$$

$$= \frac{iZ}{p^2-m^2}\frac{1}{1-Z\tilde{\Sigma}(p^2)} = \frac{iZ}{p^2-m^2} + \text{Regular}$$

$$Z = \left(1-\Sigma'(m^2)\right)^{-1}$$

b) $\mathcal{L}_{int} = -\frac{\lambda}{3!}\phi^3 \quad \rightarrow$ vertex: $-i\lambda$

$$-i\Sigma_2(p^2) = \frac{(-i\lambda)^2}{2}\int\frac{d^dk}{(2\pi)^d}\frac{i}{(p+k)^2-m_0^2+i\varepsilon}\frac{i}{(-k)^2-m_0^2+i\varepsilon}$$

$$= \frac{+\lambda^2}{2(2\pi)^d}\int_0^1 dx\int d^dk\,\frac{1}{\underbrace{\left\{x[(p+k)^2-m_0^2]+(1-x)[k^2-m_0^2]\right\}^2}}$$

$$= k^2 - m_0^2 + x(p^2 + 2kp)$$
$$= (k+px)^2 - p^2x^2 - m_0^2 + xp^2$$
$$= (k+px)^2 - p^2 x(x-1) - m_0^2$$
$$=: q^2 - \Delta$$

$$= \frac{\lambda^2}{2(2\pi)^d} \int_0^1 dx \int d^d q \; \frac{1}{(q^2-\Delta)^2}$$

Wick rotation : $q_0 \longrightarrow i q_{0,E}$, $\vec{q} = \vec{q}_E$ (only w.r.t q !)

$$q^2 = q_0^2 - \vec{q}_E^{\,2} = -q_{0,E}^2 - \vec{q}_E^{\,2} = -\vec{q}_E^{\,2}$$

$$= \frac{i\lambda}{2(2\pi)^d} \int_0^1 dx \int d^d q_E \, \frac{1}{(q_E^2-\Delta)^2}$$

c)

$d \to 4$, $\quad \Sigma_2(p^2) \propto \int \frac{d^4 q_E}{q_E^4} \quad \to$ logarithmically divergent

$\frac{d}{dp^2} \Sigma_2(p^2) \propto \int \frac{d^4 q_E}{q_E^6} \quad \to$ convergent

d) $$-i\Sigma_2(p^2) = \frac{\lambda^2}{2(2\pi)^d} \int_0^1 dx \; \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-d/2}$$

$d = 4-\varepsilon$

$$= \frac{\lambda^2}{2(2\pi)^4} \frac{1}{(4\pi)^2} \frac{\Gamma(\frac{\varepsilon}{2})}{1} \underbrace{\int_0^1 dx \; \Delta^{\varepsilon/2}}_{=\underline{1}}$$

$$= \frac{\lambda^2}{2^8 \pi^6} \frac{1}{\varepsilon} + \cdots$$

$$Z = \left(1 - \frac{\partial}{\partial p^2} \Sigma(p^2)\Big|_{p^2 = m^2}\right)^{-1}$$

$$\frac{\partial}{\partial p^2} \Sigma(p^2) = \frac{i\lambda^2}{2(2\pi)^d} \int_0^1 dx \int d^d q_E \, \frac{1}{(q_E^2 - \Delta)^2} \quad , \quad \Delta = p^2 x(x-1) + m_0^2$$

$$= \frac{i\lambda^2}{2(2\pi)^d} \int_0^1 dx \int d^d q_E \ -2 \frac{1}{(q_E^2 - \Delta)^3} \cdot x(x-1)$$

$$= \frac{-i\lambda^2}{(2\pi)^d} \int_0^1 dx \ x(x-1) \cdot \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(3-d/2)}{\Gamma(3)} \Delta^{d/2-3}$$

$$= \frac{-i\lambda^2}{(2\pi)^d} \frac{1}{2(4\pi)^{d/2}} \int_0^1 dx \ x(x-1) \left[p^2 x(x-1) + m_0^2\right]^{d/2-3}$$

At $p^2 = m^2$, $m^2 - m_0^2 = O(\lambda^2)$

$$\propto \int_0^1 dx \ \frac{x(x-1)}{x^2 - x + 1} \frac{1}{m_0^2}$$

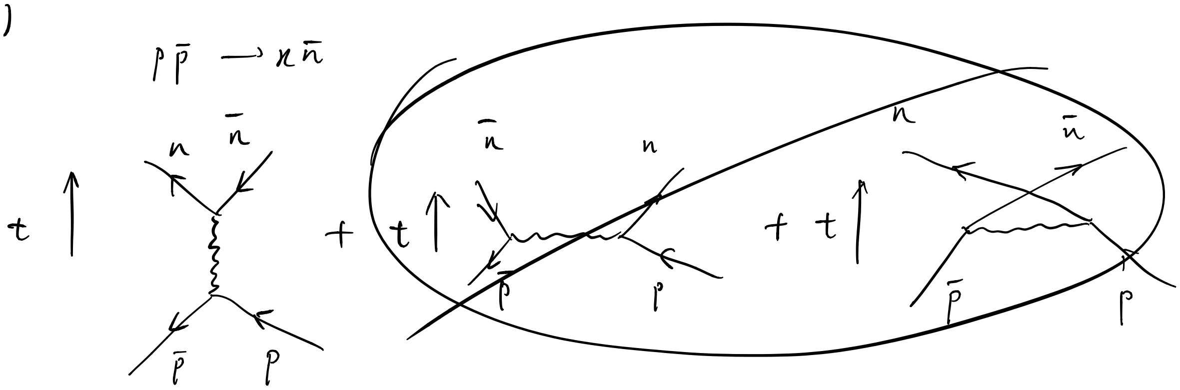
2. $\mathcal{L} = \bar\psi(i\partial\!\!\!/ - m)\psi + \bar\chi(i\partial\!\!\!/ - m)\chi + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - M^2\phi^2)$
$\quad - g\bar\psi i\gamma^5\psi\phi - \bar g\bar\chi i\gamma^5\chi\phi$

$M > m$

$$\left(\begin{array}{l} \psi^{(s)}(p) + \bar\psi^{(\bar s)}(\bar p) \longrightarrow \chi^{(r)}(k) + \bar\chi^{(\bar r)}(\bar k) \\ \quad pp \longrightarrow nn \end{array}\right.$$

a) $-ig\, i\gamma^5 = g\gamma^5$; $-i\bar g\, i\gamma^5 = \bar g\gamma^5$

b) $p\bar p \longrightarrow n\bar n$



$$i\mathcal{M} = \bar v_p^{(r)}(\bar p)\,(g\gamma^5)\,u_p^{(r')}(p)\,\frac{i}{(p_1+p_2)^2 - M^2 + i\epsilon}\,\bar u_n^{(s)}(k)\,(\bar g\gamma^5)\,v_n^{(s')}(\bar k)$$

$+ \cdots (?)$

c) $$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{spin}(g\bar g)^2\, \bar v_p^{(r)}(\bar p)\gamma^5 u_p^{(r')}(p)\frac{1}{s-M^2}\bar u_n^{(s)}(k)\gamma^5 v_n^{(s')}(\bar k)$$
$$\bar v_n^{(s')}(\bar k)\underbrace{\gamma^0\gamma^5\gamma^0}_{\gamma^5} u_n^{(s)}(k)\frac{1}{s-M^2}\bar u_p^{(r')}(p)\gamma^5 v_p^{(r)}(\bar p)$$
$$= \frac{(g\bar g)^2}{4(s-M^2)^2}\sum_{\substack{r,r'\\ s,s'}} \underbrace{tr[(\bar p\!\!\!/ - m)(p\!\!\!/ + m)]}\ \underbrace{tr[(\bar k\!\!\!/ - \mu)(k\!\!\!/ + \mu)]}$$
$$= tr(\bar p\!\!\!/ p\!\!\!/ - m^2) \qquad = tr(\bar k\!\!\!/ k\!\!\!/ - \mu^2)$$
$$= 4(\bar p\cdot p - m^2) \qquad = 4(\bar k\cdot k - \mu^2)$$

$$= \frac{(g\tilde{g})^2}{(s-M^2)^2} \; s^2$$

with $\quad s = (p+\bar{p})^2 = p^2 + 2p\cdot\bar{p} + \bar{p}^2$

$$= 2m^2 + 2(m, \vec{p})\cdot(m, -\vec{p})$$

$$= \vec{p}^{\,2} \; = \vec{k}^{\,2}$$

d) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_f|}{|\vec{p}_i|} \overline{|M|^2}$$

$$= \frac{1}{64\pi^2 s} \frac{\sqrt{s}}{\sqrt{s}} \left(\frac{g\tilde{g}}{s-M^2}\right)^2 s^2$$

$$= \frac{1}{64\pi^2} \left(\frac{g\tilde{g}}{s-M^2}\right)^2 s$$

$s < 4\mu^2$, not able to produce $n\bar{n}$-pair.

e)