3. a)

$$= D_F^{(0)}(\vec{p}) + D_F^{(0)}(-i\Sigma(\not p))D_F^{(0)}$$

$$= D_F^{(0)} \frac{1}{1+i\Sigma(\not p)D_F^{(0)}}$$

$$= \frac{i}{\not p - m_0} \frac{1}{1-\frac{1}{\not p - m_0}\Sigma(\not p)} = \frac{i}{\not p - m_0 - \Sigma(\not p)}$$

$$\Sigma(\not p) = \Sigma(m_0) + (\not p - m_0)\Sigma'(m_0) + (\not p - m_0)\tilde{\Sigma}(\not p)$$

$$m = m_0 + \Sigma(m)$$

$$\to m_0 + \Sigma(\not p) = m_0 + \Sigma(m_0) + \cdots$$

$$= m + (\not p - m)\Sigma'(m_0) + (\not p - m)\tilde{\Sigma}(\not p)$$

$$= \frac{i}{\not p - m - (\not p - m)(\Sigma'(m_0) + \tilde{\Sigma}(\not p))}$$

$$= \frac{i}{\not p - m} \frac{1}{1 - \Sigma'(m_0) - \tilde{\Sigma}(\not p)}$$

no spinor

b) $$-i\Sigma_2(\not p) = \int \frac{d^4k}{(2\pi)^4} (-ie\gamma^\mu) \frac{-ig_{\mu\nu}}{(p-k)^2+i\varepsilon} \frac{(\not k + m)}{k^2-m^2+i\varepsilon} (-ie\gamma^\nu)$$

$$= (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu \frac{-i}{(p-k)^2} \frac{(\not k+m)}{k^2-m^2} \gamma_\mu$$

$$= +ie^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu (\not k + m)\gamma_\mu \int_0^1 dx \frac{1}{\underbrace{[x(p-k)^2 + (1-x)(k^2-m^2)]}^2}$$

$$= x p^2 - 2xpk + xk^2 + (1-x)k^2 - (1-x)m^2$$

$$= k^2 - 2xp\cdot k + x^2p^2 - x^2p^2 + xp^2$$

$$- (1-x)m^2$$

$$= (k - xp)^2 + p^2 x(-x+1) + m^2(1-x)$$

$$= \underbrace{(k-xp)}_{q}{}^2 + \underbrace{(1-x)(p^2 x + m^2)}_{\Delta}$$

$$= ie^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dx\, \gamma^\mu (\not{q} + x\not{p} + m)\gamma_\mu \frac{1}{(q^2+\Delta)^2}$$

$\not{q} \longrightarrow 0$, b.c. parity

c) $\gamma^\mu(\not{p}x + m_0\gamma_\mu)$

$$\gamma^\mu \gamma_\mu = \gamma^\mu \gamma^\nu g_{\nu\mu}$$
$$= \tfrac{1}{2} g_{\nu\mu}(\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu)$$
$$= \tfrac{1}{2} g_{\nu\mu} \cdot 2 g^{\mu\nu}$$
$$= d\, \mathbb{1}_d$$

$$\gamma^\mu\gamma^\nu\gamma_\mu = -\gamma^\mu\gamma_\mu\gamma^\nu + \gamma^\mu \cdot 2 g^\nu_\mu$$
$$= -d\mathbb{1}_d\gamma^\nu + 2\gamma^\nu$$
$$= (2-d)\gamma^\nu$$

$$\Rightarrow \quad \gamma^\mu(\not{p}x + m_0)\gamma_\mu = (2-d)\not{p}x + m_0 d$$

d) $$\mathcal{L}_{QED} = -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} + \bar\psi(i\not{D} - m)\psi$$
$$= \mathcal{L}_{EM} + \mathcal{L}_D - e\bar\psi\gamma^\mu\psi A_\mu$$

$[F_{\mu\nu}F^{\mu\nu}] = d$, $[F] = d/2$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$\rightarrow [A] = d/2 - 1$

$$[\psi] = [\bar\psi] = \frac{d-1}{2}$$

$$\Rightarrow \quad [e] + \cancel{d} - 1 + \frac{d}{2} - 1 = \cancel{d}$$

$$\Rightarrow \quad [e] = 2 - d/2$$

$$\underbrace{e = \mu^{\alpha} e_0}_{[e_0] = 0}, \qquad [\mu^{\alpha}] = \alpha = 2 - d/2$$

e) $$-i\Sigma = +ie^2 \int_0^1 dx \int \frac{d^d q}{(2\pi)^d} \left[(2-d)x\cancel{p} + d m_0\right] \frac{1}{(q^2-\Delta)^2}$$

$$\left(\begin{array}{l} q_0 \to i q_{0,E}, \quad \vec{q} \to \vec{q}_E \\ q^2 = q_0^2 - \vec{q}^{\,2} = -q_E^2 \end{array} \right.$$

$$= -e^2 \int_0^1 dx \int \frac{d^d q_E}{(2\pi)^d} \frac{((2-d)x\cancel{p} + d m_0)}{(q^2+\Delta)^2}$$

$$= -e^2((2-d)x\cancel{p} + d m_0) \int_0^1 dx \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2-d/2)}{\underbrace{\Gamma(2)}_{=1}} \left(\frac{1}{\Delta}\right)^{2-d/2}$$

$$= \frac{-e^2((2-d)x\cancel{p} + d m_0)}{(4\pi)^{d/2}} \underbrace{\Gamma(2-d/2)} \underbrace{\int_0^1 dx\, \Delta^{d/2-2}}$$

$$= \int_0^1 dx\, \Delta^{\varepsilon}$$

$$\to \int_0^1 dx \left(1 + \varepsilon \ln \Delta\right)$$

$$= \int_0^1 dx \left\{1 + \varepsilon \ln(1-x) + \varepsilon \ln(m_0^2 - xp^2)\right\}$$

$$= 1 - \varepsilon\left((1-x)\ln(1-x) - (1-x)\right)\Big|_0^1 - \frac{\varepsilon}{p^2}\left((m_0^2 - xp^2)\ln(m_0^2-xp^2) - (m_0^2 - xp^2)\right)\Big|_0^1$$

$$= 1 - \varepsilon\Big\{-1 - \frac{1}{p^2}\Big[(m_0^2-p^2)\ln(m_0^2-p^2) - (\cancel{m_0^2} - p^2)$$
$$- \left(m_0^2 \ln m_0^2 - \cancel{m_0^2}\right)\Big]\Big\}$$

$$= 1 + \varepsilon + \frac{\varepsilon}{p^2}\left((m_0^2 - p^2)\ln(m_0^2 - p^2) + p^2 - m_0^2 \ln m_0^2 \right)$$

$$\longrightarrow 1$$

$$\Gamma(2 - d/2) = \Gamma\left(\frac{\varepsilon}{2}\right) = \frac{2}{\varepsilon} - \gamma_E + \mathcal{O}(\varepsilon)$$

2. $\mathcal{L}_{int} = -e\bar{\psi}\gamma^\mu A_\mu \psi - e\,\bar{\xi}\gamma^\mu A_\mu \xi$

a) $p = (E_p, \vec{p})$, $\qquad p^2 = E_p{}^2 - |\vec{p}|^2 \simeq 0 \qquad \Leftrightarrow \quad E_p = |\vec{p}|$

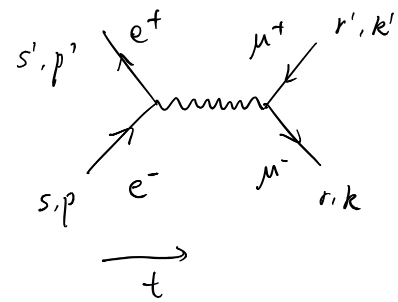
$p' = (E_p, -\vec{p})$, $\qquad p'^2 = E_p{}^2 - |\vec{p}|'^2 \simeq 0$

$k = (E_k, \vec{k})$,

$k' = (E_{k'}, -\vec{k})$,

$$s = (p+p')^2 = 2pp' = (k+k')^2 = 2m_\mu{}^2 + 2kk'$$
$$t = (p-k)^2 = m_\mu{}^2 - 2p\cdot k = (-p'+k')^2 = m_\mu{}^2 - 2p'\cdot k'$$
$$u = (p-k')^2 = m_\mu{}^2 - 2p'k$$
$$s + t + u = \sum m_i^2 = 2m_\mu^2$$

b) $\psi^{(s)}(p) + \bar{\psi}^{(s')}(p') \longrightarrow \xi^{(r)}(k) + \bar{\xi}^{(r')}(k')$



only s-channel possible!

$$i\mathcal{M} = \bar{v}_e^{(s')}(p')(-ie\gamma^\mu)u_e^{(s)}(p)\frac{-ig_{\mu\nu}}{(p+p')^2 + i\varepsilon}\bar{u}_\mu^{(r)}(k)(-ie\gamma^\nu)v_\mu^{(r')}(k')$$
$$= ie^2\,\bar{v}_e^{(s')}(p')\gamma^\mu u_e^{(s)}(p)\frac{1}{(p+p')^2}\bar{u}_\mu^{(r)}(k)\gamma_\mu v_\mu^{(r')}(k')$$

c) $\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{spins}\mathcal{M}\mathcal{M}^\dagger$

$$= \frac{e^4}{4s^2}\sum \bar{v}_e^{(s')}(p')\gamma^\mu u_e^{(s)}(p)\, u_\mu^{(r)}(k)\gamma_\mu v_\mu^{(r')}(k')\;\bar{v}_\mu^{(r')}(k')\gamma_\nu u_\mu^{(r)}(k)\bar{u}_e^{(s)}(p)\gamma^\nu v_e^{(s')}(p')$$

$$= \frac{e^4}{4s^2}\,\mathrm{tr}\left[(\not{p}' - m_e)\gamma^\mu(\not{p} + m_e)\gamma^\nu\right]\mathrm{tr}\left[(\not{k} + m_\mu)\gamma_\mu(\not{k}' - m_\mu)\gamma_\nu\right]$$

$$\approx \frac{e^4}{4s^2}\, \mathrm{tr}[\not{p}'_\alpha \gamma^\mu \not{p}_\beta \gamma^\nu]\, \mathrm{tr}[\not{k}_\alpha \gamma_\mu \not{k}'_\beta \gamma_\nu - m_\mu^2 \gamma_\mu \gamma_\nu]$$

$$= \frac{e^4}{4s^2}\, 4\cdot 4 \left\{ p'_\alpha p_\beta (g^{\alpha\mu} g^{\beta\nu} - g^{\alpha\beta} g^{\mu\nu} + g^{\alpha\nu} g^{\mu\beta}) \right\}$$

$$\times \left\{ k^\alpha k'^\beta (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\mu\beta}) - m_\mu^2 g_{\mu\nu} \right\}$$

$$= \frac{4e^4}{s^2} (p'^\mu p^\nu - \underbrace{p'\cdot p}_{\frac{1}{2}s} g^{\mu\nu} + p'^\nu p^\mu)(k_\mu k'_\nu - \underbrace{k\cdot k'}_{= \underbrace{pp'}_{\frac{1}{2}s} - \cancel{m_\mu^2}} g_{\mu\nu} + k_\nu k'_\mu - \cancel{m_\mu^2 g_{\mu\nu}})$$

$$= \frac{4e^4}{s^2} (p'^\mu p^\nu + p'^\nu p^\mu - \frac{s}{2} g^{\mu\nu})(k_\mu k'_\nu + k_\nu k'_\mu - \frac{s}{2} g_{\mu\nu})$$

$$= \frac{4e^4}{s^2} \Big\{ (p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k) - \frac{s}{2} \underbrace{(p\cdot p')}_{\frac{s}{2}}$$

$$+ (p'\cdot k')(p\cdot k) + (p'\cdot k)(p\cdot k') - \frac{s}{2} \underbrace{(p\cdot p')}_{\frac{s}{2}}$$

$$- \frac{s}{2}(k\cdot k') - \frac{s}{2}(k\cdot k') + \frac{s^2}{4} 4 \Big\}$$

$$= \frac{4e^4}{s^2} \Big\{ 2\cdot \left[\tfrac{1}{2}(m_\mu^2 - u)\right]^2 + 2\cdot \left[\tfrac{1}{2}(m_\mu^2 - t)\right]^2 \underline{- 2\cdot \frac{s^2}{4}}$$

$$- s\cdot (\tfrac{1}{2}s - m_\mu^2) \underline{+ s^2} \Big\}$$

$$= \frac{4e^4}{s^2} \cdot \frac{1}{2} \left(m_\mu^4 - 2m_\mu^2 u + u^2 + m_\mu^4 - 2m_\mu^2 t + t^2 + \cancel{s^2} - \cancel{s^2} + 2s m_\mu^2 \right)$$

$$= \frac{2e^4}{s^2} \left(2m_\mu^4 + 2s m_\mu^2 - 2m_\mu^2 \underbrace{(u+t)}_{= 2m_\mu^2 - s} + u^2 + t^2 \right)$$

$$= \frac{2e^4}{s^2} \left(-2m_\mu^4 + 4s m_\mu^2 + u^2 + t^2 \right)$$

d) $m_\mu \longrightarrow 0 \qquad \overline{|\mathcal{M}|^2} = \frac{2e^4}{s^2}(u^2 + t^2)$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{2e^4}{s^2} (u^2 + t^2)$$

$$\angle(\vec{p},\vec{k}) = \cos\theta \quad \Rightarrow \quad p = (1,\ 0,\ 0,\ 1)\,p$$
$$k = (1,\ \sin\theta,\ 0,\ \cos\theta)\,k$$
$$p = k$$

$$\rightarrow \quad t^2 = (p-k)^4 = (-2\,p\cdot k)^2 = 4p^4\left[(1,0,0,1)^T\cdot(1,\sin\theta,0,\cos\theta)\right]^2$$
$$= 4p^4(1-\cos\theta)^2$$

$$u^2 = (p-k')^4 = (-2\,p'k)^2 = \left(-2\,p^2(1,0,0,-1)^T\cdot(1,\sin\theta,0,\cos\theta)\right)^2$$
$$= 4p^4\,(1+\cos\theta)^2$$

$$\rightarrow \quad u^2+t^2 = 8p^4(1+\cos^2\theta)\,,$$
$$s = 2pp' = 2\cdot p\cdot p(1,0,0,1)^T(1,0,0,-1)$$
$$= 2p^2(1+1) = 4p^2$$

$$\Rightarrow \quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{\cancel{4}e^4}{\cancel{s^2}}\cdot\frac{1}{\cancel{2}}\,\cancel{s^2}(1+\cos^2\theta)$$
$$= \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

1. a) $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$= \bar{\psi}(i\not{\partial} + e\not{A} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = \bar{\psi}\, i\gamma^\mu$, $\frac{\partial \mathcal{L}}{\partial \psi} = \bar{\psi}(e\not{A} - m)$

$\Rightarrow \bar{\psi}(i\gamma^\mu \overleftarrow{\partial_\mu} + e\not{A} - m) = 0$

$\psi \to e^{i\alpha\gamma^5}\psi, \quad \bar{\psi} \to \psi^+ e^{-i\alpha\gamma^5}\gamma^0$

$\to (1 + i\alpha\gamma^5)\psi \qquad \to \psi^+(1 - i\alpha\gamma^5)\gamma^0$

$\to \quad j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\Delta\psi = \bar{\psi}\cdot i\gamma^\mu \cdot i\alpha\gamma^5\psi$

$= -\alpha\bar{\psi}\gamma^\mu\gamma^5\psi$

b) $\mathcal{L}_{int} = c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(\bar{\psi}\, i\sigma^{\mu\nu}\psi)F_{\mu\nu} + c_3(\bar{\psi}\, i\gamma^5\not{A}\psi)\phi^2$

$+ c_4(\phi(\partial^\mu\phi)A_\mu)$

$\mathcal{L}_D = \bar{\psi}(i\not{\partial} - m)\psi \quad \to 2[\psi] + 1 = [\mathcal{L}] = d$

$\Rightarrow [\psi] = \frac{d-1}{2}$

$\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{m^2}{2}\phi^2 \quad \to 2[\phi] + 2 = d$

$\Rightarrow [\phi] = \frac{d}{2} - 1$

$\mathcal{L}_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad \to [F_{\mu\nu}] = \frac{d}{2}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \to [A] = \frac{d}{2} - 1$

$\Rightarrow [c_1] = 0$,

$[c_2] + 2[\psi] + [F] = [c_2] + \cancel{d} - 1 + \frac{d}{2} \overset{!}{=} \cancel{d}$

$\to [c_2] = 1 - d/2 \underset{d=4}{=} -1$

$[c_3] + 2[\psi] + 2[\phi] + [A] \overset{!}{=} d$

$\to [c_3] = \cancel{d} - (\cancel{d} - 1) - (d - 2) - (\frac{d}{2} - 1)$

$= 1 - d + 2 - \frac{d}{2} + 1$

$$= -\frac{3}{2}d+4$$

$$\underset{d=4}{=} -6+4 = -2$$

$$[c_4] + 2\left(\frac{d}{2}-1\right) + \cancel{1} + \frac{d}{2} - \cancel{1} \overset{!}{=} d$$

$$\Rightarrow [c_4] = \cancel{d} - (\cancel{d}-2) - \frac{d}{2}$$

$$= 2 - \frac{d}{2} \underset{d=4}{=} 0$$

$\rightarrow$ non-renormalisable

c) Fermi - statistics. Through ex. we found two electrons in one system cannot occupy same quantum level. $\longrightarrow$ anti-comm.

$$(a_p^{(s)})^+ (a_q^{(r)})^+ |0\rangle = |p,s;q,r\rangle$$

d) ϕ^4: For every loop: $L = I - V + 1$

For vertex: $V = \frac{1}{4}E + \frac{1}{2}I$

From Integral $D = dL - 2I$

$$\rightarrow D = d(I-V+1) - 2I$$

$$= dI - dV + d - 2I$$

$$= d(2V-\tfrac{1}{2}E) - dV + d - 2(2V-\tfrac{1}{2}E)$$

$$= 2dV - \tfrac{1}{2}dE - \underline{dV} + \underline{d} - \underline{4V} + E$$

$$= d + (d-4)V + E\left(1-\tfrac{1}{2}d\right)$$

e) /