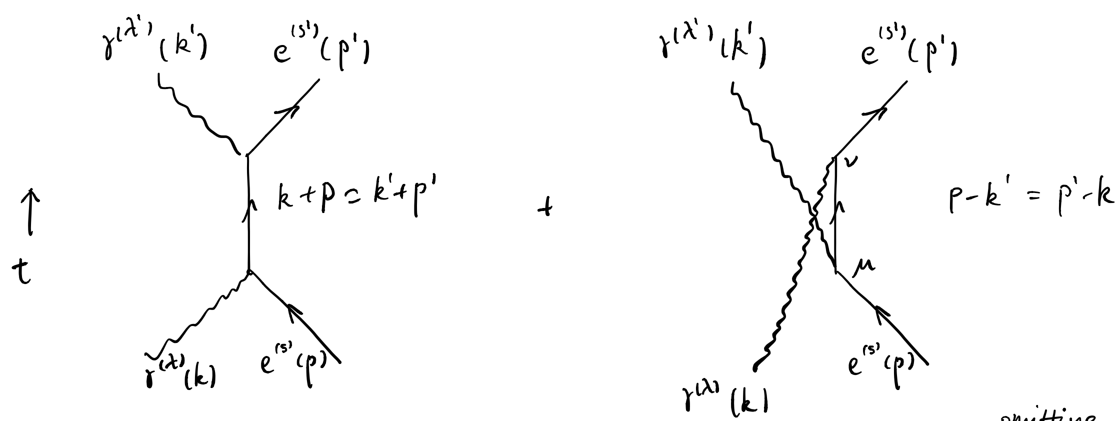
H.11

a) Two diagrams for $e^{(s)}(p) + \gamma^{(\lambda)}(k) \longrightarrow e^{(s')}(p') + \gamma^{(\lambda')}(k')$



omitting the spin index afterward

$$i\mathcal{M} = \bar{u}^{(s')}(p')(-ie\gamma^{\mu})\,\epsilon^{(\lambda')*}_{\mu}(k')\frac{i(\not{k}+\not{p}+m)}{(p+k)^2 - m_e^2 + i\varepsilon}\epsilon^{(\lambda)}_{\nu}(k)(-ie\gamma^{\nu})u^{(s)}(p)$$

$$+\ \bar{u}(p')(-ie\gamma^{\nu})\,\epsilon_{\nu}(k)\frac{i(\not{p}-\not{k}'+m)}{(p-k')^2 - m_e^2 + i\varepsilon}\epsilon^{*}_{\mu}(k')(-ie\gamma^{\mu})u(p)$$

for some reason(s) the order of ϵ and ϵ^* $\rightarrow$ doesn't matter

$$= -ie^2\,\epsilon_{\nu}(k)\epsilon^{*}_{\mu}(k')\,\bar{u}(p')\left[\gamma^{\mu}\frac{\not{k}+\not{p}+m}{(p+k)^2-m^2}\gamma^{\nu} + \gamma^{\nu}\frac{\not{p}-\not{k}+m}{(p-k')^2-m^2}\gamma^{\mu}\right]u(p)$$

$$\left[\begin{array}{lcl} p^2 = m^2, \quad k^2 = 0 & \Rightarrow & (p+k)^2 = p^2 + 2k\cdot p + k^2 = m^2 + 2k\cdot p \\ \qquad\qquad\quad k'^2 = 0 & & (p-k')^2 = p^2 - 2k'\cdot p + k'^2 = m^2 - 2k'\cdot p \end{array}\right]$$

$$= -ie^2\,\epsilon_{\mu}(k)\epsilon^{*}_{\nu}(k')\,\bar{u}(p')\left[\frac{\gamma^{\mu}(\not{k}+\not{p}+m)\gamma^{\nu}}{2k\cdot p} - \frac{\gamma^{\nu}(\not{p}-\not{k}+m)\gamma^{\mu}}{2k'\cdot p}\right]u(p)$$

$$\left[\begin{array}{l} (\not{p}+m)\gamma^{\nu}u(p) = (p_{\mu}\gamma^{\mu}+m)\gamma^{\nu}u(p) \\ \{\gamma^{\mu},\gamma^{\nu}\} = 2g^{\mu\nu} \rightarrow \quad = (-p_{\mu}\gamma^{\nu}\gamma^{\mu} + 2g^{\mu\nu}p_{\mu} + m\gamma^{\nu})u(p) \\ \qquad\qquad\qquad = (2p^{\nu} - \gamma^{\nu}\underbrace{(\not{p}-m)\;)\,u(p)}_{=0,\ \text{using } \Psi = u(p)e^{\pm ip\cdot x} \text{ and dirac eq.}} \\ \qquad\qquad\qquad = 2p^{\nu}u(p) \end{array}\right]$$

$$= -ie^2 \epsilon_\mu(k)\epsilon_\nu^*(k')\bar{u}(p')\left[\frac{\gamma^\mu \not{k}\gamma^\nu + 2p^\nu\gamma^\mu}{2k\cdot p} - \frac{-\gamma^\nu \not{k}'\gamma^\mu + 2p^\mu\gamma^\nu}{2p\cdot k'}\right]u(p)$$

$$\Rightarrow \quad \mathcal{M} = -e^2(\epsilon_\mu^{(\lambda')}(k'))^* \epsilon_\nu^{(\lambda)}(k)\mathcal{M}^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = \bar{u}^{(s')}(p')\left[\frac{\gamma^\mu \not{k}\gamma^\nu + 2p^\nu\gamma^\mu}{2k\cdot p} - \frac{-\gamma^\nu \not{k}'\gamma^\mu + 2p^\mu\gamma^\nu}{2p\cdot k'}\right]u^{(s)}(p)$$

$$\int d^4x \; j_\mu A^\mu \longrightarrow \epsilon^{(\lambda)}$$

$$\Rightarrow \quad \mathcal{M}^{\mu\nu} \propto \int d^4x \; e^{ikx} \langle f | j^\mu j^\nu | i \rangle$$

using $\quad \partial_\mu j^\mu = 0$

$$\Rightarrow \quad k'_\mu \mathcal{M}^{\mu\nu} = 0$$

c) $k'_\mu \mathcal{M}^{\mu\nu} = k_\mu \mathcal{M}^{\mu\nu} = 0$ and choose $k^\mu = (k,0,0,k)^\mu$,

$k'^\mu = (k',0,0,k')^\mu$

$$\Rightarrow \quad k'_\mu \mathcal{M}^{\mu\nu} = k'\mathcal{M}^{0\nu} - k'\mathcal{M}^{3\nu} = k\mathcal{M}^{0\nu} - k'\mathcal{M}^{3\nu} = 0$$

$$\Rightarrow \quad \mathcal{M}^{0\nu} = \mathcal{M}^{3\nu}$$

$$e^4 \sum_{\lambda,\lambda'=0}^{3} (-g_{\lambda\lambda})(-g_{\lambda'\lambda'}) \mathcal{M}^{\mu\nu}(\mathcal{M}^{\mu'\nu'})^* (\epsilon_\mu^{(\lambda')}(k'))^* \epsilon_{\mu'}^{(\lambda')}(k')\epsilon_\nu^{(\lambda)}(k)(\epsilon_{\nu'}^{(\lambda)}(k))^*$$

$$= e^4 \mathcal{M}^{\mu\nu}(\mathcal{M}^{\mu'\nu'})^* \cdot \sum_{\lambda'=0}^{3} g_{\lambda'\lambda'}(\epsilon_{\mu'}^{(\lambda')}(k'))^*\epsilon_\mu^{(\lambda')}(k) \cdot \sum_{\lambda=0}^{3} g_{\lambda\lambda}\epsilon_\nu^{(\lambda)}(k)(\epsilon_{\nu'}^{(\lambda)}(k))^*$$

$$= e^4 \mathcal{M}^{\mu\nu}(\mathcal{M}^{\mu'\nu'})^* \left[\sum_{\lambda'=1}^{2} -(\epsilon_{\mu'}^{(\lambda')}(k'))^*\epsilon_\mu^{(\lambda')}(k) + \underbrace{\epsilon_{\mu'}^{(0)}(k')\epsilon_\mu^{(0)}(k) - \epsilon_{\mu'}^{(3)}(k')\epsilon_\mu^{(3)}(k)}\right]$$

$$\times \left[\quad \cdots \quad \right]$$

$$= \epsilon^{(0)}\epsilon^{(0)} - \left(\epsilon^{(0)} - \frac{k}{|k|}\right)^2$$

$\propto k_{\mu^{(\prime)}}$ or $k_\mu k_{\mu'}$

with ward identities $\longrightarrow 0$

$$= e^4 \mathcal{M}^{\mu\nu} (\mathcal{M}^{\mu'\nu'})^* \sum_{\lambda'=1}^{2} (\epsilon_{\mu'}^{(\lambda')}(k'))^* \epsilon_{\mu}^{(\lambda')}(k) \sum_{\lambda=1}^{2} \epsilon_{\nu}^{(\lambda)}(k') \epsilon_{\nu}^{(\lambda)}(k)$$

$$= \sum_{\lambda,\lambda'=1,2} |\mathcal{M}|^2$$

d) $$|\overline{\mathcal{M}^2}| = \frac{1}{4} \sum_{\lambda,\lambda'=1}^{2} \sum_{s,s'} |\mathcal{M}|^2 = \frac{e^4}{4} \sum_{\lambda,\lambda'=1}^{2} \sum_{s,s'} (-g_{\lambda\lambda})(-g_{\lambda'\lambda'}) \mathcal{M}^{\mu\nu} (\mathcal{M}^{\mu'\nu'})^*$$

$$\times (\epsilon_{\mu}^{(\lambda')}(k'))^* \epsilon_{\mu'}^{(\lambda')}(k') \epsilon_{\nu}^{(\lambda)}(k) (\epsilon_{\nu'}^{(\lambda)}(k))^*$$

$$= \frac{e^4}{4} \sum_{s,s'} g_{\mu\mu'} g_{\nu\nu'} \mathcal{M}^{\mu\nu} (\mathcal{M}^{\mu'\nu'})^*$$

$$= \frac{e^4}{4} g_{\mu\mu'} g_{\nu\nu'} \sum_{s,s'} \bar{u}^{(s')}(p') A^{\mu\nu} u^{(s)}(p) \left(\bar{u}^{(s')}(p') A^{\mu'\nu'} u^{(s)}(p)\right)^*$$

with $$A^{\mu\nu} = \frac{\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu}{2k\cdot p} - \frac{-\gamma^\nu \not{k}' \gamma^\mu + 2\gamma^\nu p^\mu}{2k'\cdot p}$$

$$(A^{\mu'\nu'})^* = (A^{\mu'\nu'})^\dagger$$

$$= \frac{\gamma^0 \gamma^\nu \cancel{\gamma^0} k_\alpha \cancel{\gamma^0} \gamma^\alpha \cancel{\gamma^0 \gamma^0} \gamma^\mu \gamma^0 + 2p^\nu \gamma^0 \gamma^\mu \gamma^0}{2k\cdot p}$$

$$- \frac{-\gamma^0 \gamma^\mu \cancel{\gamma^0} k'_\alpha \cancel{\gamma^0} \gamma^\alpha \cancel{\gamma^0 \gamma^0} \gamma^\nu \gamma^0 + 2p^\mu \gamma^0 \gamma^\nu \gamma^0}{2k'\cdot p}$$

note the order of μ,ν is not exactly the same as $A^{\mu\nu}$

$$= \gamma^0 \left(\frac{\gamma^\nu \not{k} \gamma^\mu + 2p^\nu \gamma^\mu}{2k\cdot p} - \frac{-\gamma^\mu \not{k}' \gamma^\nu + 2p^\mu \gamma^\nu}{2k'\cdot p} \right) \gamma^0 := \gamma^0 \hat{A}^{\mu'\nu'} \gamma^0$$

$$(\mathcal{M}^{\mu'\nu'})^* = \left(u^{(s)\dagger}(p') \gamma^0 A^{\mu'\nu'} u^{(s)}(p)\right)^\dagger$$

$$= u^{(s)\dagger}(p) (A^{\mu'\nu'})^\dagger \gamma^0 u^{(s)}(p')$$

$$= \bar{u}^{(s)}(p) \gamma^0 (A^{\mu'\nu'})^\dagger \gamma^0 u^{(s)}(p')$$

$$= \bar{u}^{(s)}(p) A^{\mu'\nu'} u^{(s')}(p')$$

$$= \frac{e^4}{4} g_{\mu\mu'} g_{\nu\nu'} \sum_{s,s'} \underbrace{\bar{u}^{(s')}(p') \underset{\substack{\uparrow \\ \text{a matrix!}}}{A^{\mu\nu}} u^{(s)}(p) \bar{u}^{(s)}(p) \underset{\substack{\uparrow \\ \text{a matrix!}}}{\tilde{A}^{\mu'\nu'}} u^{(s')}(p')}$$

$$= \bar{u}_\alpha(p')(A^{\mu\nu})_{\alpha\beta} u_\beta(p) \bar{u}_\gamma(p) (\tilde{A}^{\mu'\nu'})_{\gamma\delta} u_\delta(p')$$

$$= u_\beta(p) \bar{u}_\gamma(p) \, (A^{\mu\nu})_{\alpha\beta} \, u_\delta(p') \, \bar{u}_\alpha(p')(\tilde{A}^{\mu'\nu'})_{\gamma\delta}$$

$$= \frac{e^4}{4} g_{\mu\mu'} g_{\nu\nu'} \sum_{\alpha,\beta,\gamma,\delta} (\not{p}+m)_{\beta\gamma} (A^{\mu\nu})_{\alpha\beta} (\not{p}'+m)_{\delta\alpha} (\tilde{A}^{\mu'\nu'})_{\gamma\delta}$$

$$= \frac{e^4}{4} g_{\mu\mu'} g_{\nu\nu'} \sum (\not{p}'+m)_{\delta\alpha} (A^{\mu\nu})_{\alpha\beta} (\not{p}+m)_{\beta\gamma} (\tilde{A}^{\mu'\nu'})_{\gamma\delta}$$

$$= \frac{e^4}{4} \underbrace{g_{\mu\mu'} g_{\nu\nu'}}_{\text{numbers!}} \, \mathrm{tr} \left\{ (\not{p}'+m) \left[\frac{\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu}{2k\cdot p} - \frac{-\gamma^\nu \not{k}' \gamma^\mu + 2\gamma^\nu p^\mu}{2k'\cdot p} \right] \right.$$

$$\left. \times (\not{p}+m) \left[\frac{\gamma^{\nu'} \not{k} \gamma^{\mu'} + 2\gamma^{\mu'} p^{\nu'}}{2k\cdot p} - \frac{-\gamma^{\mu'} \not{k}' \gamma^{\nu'} + 2\gamma^{\nu'} p^{\mu'}}{2k'\cdot p} \right] \right\}$$

$$= \frac{e^4}{4} \mathrm{tr} \left\{ (\not{p}'+m) \left[\frac{\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu}{2k\cdot p} - \frac{-\gamma^\nu \not{k}' \gamma^\mu + 2\gamma^\nu p^\mu}{2k'\cdot p} \right] (\not{p}+m) \right.$$

$$\left. \times \left[\frac{\gamma_\nu \not{k} \gamma_\mu + 2\gamma_\mu p_\nu}{2k\cdot p} - \frac{-\gamma_\mu \not{k}' \gamma_\nu + 2\gamma_\nu p_\mu}{2k'\cdot p} \right] \right\}$$

$$= \frac{e^4}{4} \left[\frac{I_1}{(2k\cdot p)^2} + \frac{I_2}{(2k'\cdot p)^2} + \frac{I_3 + I_4}{(2k\cdot p)(2k'\cdot p)} \right]$$

with $I_1 = \mathrm{tr}\left[(\not{p}'+m)(\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu)(\not{p}+m)(\gamma_\nu \not{k} \gamma_\mu + 2\gamma_\mu p_\nu) \right]$

$I_2 = I_1$ with $k \leftrightarrow -k'$

$I_3 = \mathrm{tr}\left[(\not{p}'+m)(\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu)(\not{p}+m)(\gamma_\mu \not{k}' \gamma_\nu - 2\gamma_\nu p_\mu) \right]$

$I_4 = I_3$ with $k' \leftrightarrow -k$

e) In CMS: $\vec{p}+\vec{k}=\vec{p}'+\vec{k}'=0$

$$\underline{p\cdot k} = p\cdot(-p) = -p^2 = -m^2 = \underline{p'\cdot k'}$$

mom. cons: $p+k=p'+k'$

$$p'\cdot k' = p'(p+k-p') = p'\cdot p+p'\cdot k-p'^2$$

$$p\cdot k = p(p'+k'-p) = p\cdot p'+p\cdot k'-p^2$$

$$\Rightarrow \quad \underline{p'\cdot k = p\cdot k'}$$

$$\underline{k\cdot k'} = k(p+k-p') = kp-kp' = k(p-p') = -p(p-p') = \underline{p\cdot p'+m^2}$$

$$\underline{p\cdot p'} = p\cdot(p+k-k') = \underline{m^2+p\cdot k-p\cdot k'}$$

$$I_1 = tr[(\not{p}'+m)(\gamma^\mu \not{k}\gamma^\nu+2\gamma^\mu p^\nu)(\not{p}+m)(\gamma_\nu\not{k}\gamma_\mu+2\gamma_\mu p_\nu)]$$

$$= tr[(\not{p}'\gamma^\mu\not{k}\gamma^\nu+2\not{p}'\gamma^\mu p^\nu+m\gamma^\mu\not{k}\gamma^\nu+2m\gamma^\mu p^\nu)$$

$$\times(\underset{\substack{\uparrow\\ 4\gamma}}{\not{p}\gamma_\nu\not{k}\gamma_\mu}+\underset{\substack{\uparrow\\ 2\gamma}}{2\not{p}\gamma_\mu p_\nu}+\underset{\substack{\uparrow\\ 3\gamma}}{m\gamma_\nu\not{k}\gamma_\mu}+\underset{\substack{\uparrow\\ 1\gamma}}{2m\gamma_\mu p_\nu})]$$

$$= tr[\not{p}'\gamma^\mu\not{k}\underbrace{\gamma^\nu\not{p}\gamma_\nu}_{=-2\not{p}}\not{k}\gamma_\mu+2\not{p}'\gamma^\mu\not{k}\gamma^\nu\not{p}\gamma_\mu p_\nu$$

$$+2\not{p}'\gamma^\mu p^\nu\not{p}\gamma_\nu\not{k}\gamma_\mu+4\not{p}'\gamma^\mu p^\nu\not{p}\gamma_\mu p_\nu$$

$$+m^2\gamma^\mu\not{k}\underbrace{\gamma^\nu\gamma_\nu}\not{k}\gamma_\mu+2m^2\gamma^\mu\not{k}\gamma^\nu\gamma_\mu p_\nu$$

$$+2m^2\gamma^\mu p^\nu\gamma_\nu\not{k}\gamma_\mu+4m^2\gamma^\mu p^\nu\gamma_\mu p_\nu]$$

(all traces with odd numbers of gamma matrices vanish!)

$$= tr[4\not{p}'\not{k}\not{p}\not{k}+2\not{p}'\underbrace{\gamma^\mu\not{k}\not{p}\not{p}\gamma_\mu}+2\not{p}'\underbrace{\gamma^\mu\not{p}\not{p}\not{k}\gamma_\mu}$$

$$+4m^2\not{p}'\underbrace{\gamma^\mu\not{p}\gamma_\mu}+4m^2\underbrace{\gamma^\mu\not{k}\not{k}\gamma_\mu}+2m^2\underbrace{\gamma^\mu\not{k}\not{p}\gamma_\mu}$$

$$+ 2m^2 \underbrace{\gamma^\mu \not{p} \not{k} \gamma_\mu} + 4m^2 \cdot 4 \cdot m^2]$$

$$= \mathrm{tr}[4\not{p}' \not{k} \not{p} \not{k} - 4 \not{p}' \not{p} \not{p} \not{k} - 4 \not{p}' \not{k} \not{p} \not{p} - 8m^2 \not{p}' \not{p} + \underbrace{16 m^2 (k\cdot k) \mathbb{1}}_{=0}$$

$$+ 8m^2 (k\cdot p) \mathbb{1} + 8m^2 (p\cdot k) \mathbb{1} + 16 m^4]$$

$$= 16 p'_\alpha k_\beta p_\mu k_\nu (g^{\alpha\beta} g^{\mu\nu} - g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu})$$

$$- 16 p'_\alpha p_\beta p_\mu k_\nu (g^{\alpha\beta} g^{\mu\nu} - g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu})$$

$$- 16 p'_\alpha k_\beta p_\mu p_\nu (g^{\alpha\beta} g^{\mu\nu} - g^{\alpha\mu} g^{\beta\nu} + g^{\alpha\nu} g^{\beta\mu})$$

$$- 32 m^2 p' \cdot p + 64 m^2 k\cdot p + 72 m^4$$

$$= 16 \Big((p'\cdot k)(p\cdot k) - (p'\cdot p)\underbrace{(k\cdot k)}_{=0} + (p'\cdot k)(k\cdot p) - \cancel{(p'\cdot p)}(p\cdot k) + \cancel{(p'\cdot p)}(p\cdot k)$$

$$- (p'\cdot k)\underbrace{(p\cdot p)}_{=m^2} - (p'\cdot k) m^2 + \cancel{(p'\cdot p)(k\cdot p)} - \cancel{(p'\cdot p)(k\cdot p)} \Big)$$

$$- 32 m^2 p'\cdot p + 64 m^2 k\cdot p + 72 m^4$$

$$= 16 [2(p\cdot k')(p\cdot k) - 2m^2 (p'\cdot k)] - 32 m^2 (p\cdot k - p\cdot k' + m^2)$$

$$+ 64 m^2 k\cdot p + 72 m^2$$

$$= 16 [2(p\cdot k')(p\cdot k) - 2m^2 \underbrace{(p'\cdot k)}_{= p\cdot k'} - 2m^2 (p\cdot k - p\cdot k' + m^2) + 4 m^2 k\cdot p + 4 m^4]$$

$$= 16 [2(p\cdot k')(p\cdot k) - \cancel{2m^2 (p\cdot k')} - 2m^2 (p\cdot k) + \cancel{2m^2 (p\cdot k')} - 2m^4 + 4m^2 k\cdot p + 4m^4]$$

$$= 32 [m^4 + m^2 (p\cdot k) + (p\cdot k')(p\cdot k)]$$

$$I_3 = \mathrm{tr}[(\not{p}' + m)(\gamma^\mu \not{k} \gamma^\nu + 2\gamma^\mu p^\nu)(\not{p} + m)(\gamma_\mu \not{k}' \gamma_\nu - 2\gamma_\nu p_\mu)]$$

$$= \mathrm{tr}\Big[(\not{p}' \gamma^\mu \not{k} \gamma^\nu + 2 \not{p}' \gamma^\mu p^\nu + m \gamma^\mu \not{k} \gamma^\nu + 2 m \gamma^\mu p^\nu)$$

$$\times (\underbrace{\not{p} \gamma_\mu \not{k}' \gamma_\nu}_{4\gamma} - \underbrace{2 \not{p} \gamma_\nu p_\mu}_{2\gamma} + \underbrace{m \gamma_\mu \not{k}' \gamma_\nu}_{3\gamma} - \underbrace{2 m \gamma_\nu p_\mu}_{1\gamma}) \Big]$$

$$
\begin{aligned}
= \mathrm{tr}\big[& \not{p}'\gamma^\mu \not{k}\gamma^\nu \not{p}\gamma_\mu \not{k}'\gamma_\nu - 2\not{p}'\gamma^\mu \not{k}\gamma^\nu \not{p}\gamma_\nu p_\mu \\
& + 2\not{p}'\gamma^\mu p^\nu \not{p}\gamma_\mu \not{k}'\gamma_\nu - 4\not{p}'\gamma^\mu p^\nu \not{p}\gamma_\nu p_\mu \\
& + m^2\gamma^\mu \not{k}\gamma^\nu \gamma_\mu \not{k}'\gamma_\nu - 2m^2\gamma^\mu \not{k}\gamma^\nu \gamma_\nu p_\mu \\
& + 2m^2\gamma^\mu p^\nu \gamma_\mu \not{k}'\gamma_\nu - 4m^2\gamma^\mu p^\nu \gamma_\nu p_\mu \big]
\end{aligned}
$$

$$
\mathrm{tr}\left(\not{p}'\gamma^\mu \not{k}\gamma^\nu \not{p}\gamma_\mu \not{k}'\gamma_\nu\right) \longleftarrow \mathrm{tr}(ABC) = \mathrm{tr}(CBA)
$$

$$
\begin{aligned}
&= \not{p}\gamma_\mu \not{k}'\gamma_\nu \not{k}\gamma^\nu \not{p}'\gamma^\mu \\
&= -2\not{p}\gamma_\mu \not{k}'\not{k}\not{p}'\gamma^\mu \\
&= +4\not{p}\not{k}'\not{k}\not{p}'
\end{aligned}
$$

$$
\begin{aligned}
&\mathrm{tr}\left(\gamma^\mu \not{k}\gamma^\nu \gamma_\mu \not{k}'\gamma_\nu\right) \\
&= \mathrm{tr}\left(\not{k}'\gamma_\nu \gamma^\nu \gamma_\mu \gamma^\mu \not{k}\right) \\
&= \mathrm{tr}\left(16\not{k}'\not{k}\,\mathbb{1}\right)
\end{aligned}
$$

$$
\begin{aligned}
= \mathrm{tr}\big[& +4\not{p}\not{k}'\not{k}\not{p}' + 4\not{p}'\not{p}\not{k}\not{p} - 4\not{p}'\not{p}\not{k}'\not{p} - 4\not{p}'\not{p}\not{p}\not{p} \\
& + 16m^2\not{k}'\not{k} - 8m^2\not{p}\not{k} + 8m^2\not{k}'\not{p} - 4m^2\not{p}\not{p} \big]
\end{aligned}
$$

$$
\begin{aligned}
= 16\Big(& +(p\cdot k')(k\cdot p') - (p\cdot k)(k'\cdot p') + (p\cdot p')(k'\cdot k) \\
& + (p'\cdot p)(k\cdot p) - (p'\cdot k)(p\cdot p) + (p'\cdot p)(p\cdot k) \\
& - (p'\cdot p)(k'\cdot p) + (p'\cdot k')(p\cdot p) - (p'\cdot p)(p\cdot k') \\
& - (p'\cdot p)m^2 + 4m^2(k'\cdot k) - 2m^2(p\cdot k) + 2m^2(k'\cdot p) - m^2(p\cdot p) \Big)
\end{aligned}
$$

$$
\begin{aligned}
= 16\Big\{ & (p\cdot k')^2 - (p\cdot k)^2 + (p\cdot p' - m^2)(p\cdot p') + 2(p'\cdot p)(k\cdot p) - m^2(p\cdot k') - 2(p\cdot p')(p\cdot k') \\
& + m^2(p\cdot k) - m^2(p\cdot p') + 4m^2(p\cdot p' - m^2) - 2m^2(p\cdot k) + 2m^2(p\cdot k') - m^4 \Big\}
\end{aligned}
$$

$$
\begin{aligned}
= 16\Big\{ & (p\cdot k')^2 - (p\cdot k)^2 + (p\cdot p' - 2m^2)(p\cdot p') + 2(p'\cdot p)(p\cdot k - p\cdot k') \\
& + m^2(+p\cdot k' - p\cdot k - \underset{\substack{\uparrow \\ = p\cdot k - p\cdot k' + m^2}}{p\cdot p'}) + 4m^2\underbrace{(p\cdot p' - m^2)}_{p\cdot k - p\cdot k'} - m^4 \Big\}
\end{aligned}
$$

$$= 16 \{ (p\cdot k')^2 - (p\cdot k)^2 + ((p\cdot k - p\cdot k') - m^2)((p\cdot k - p\cdot k') + m^2)$$

$$+ 2((p\cdot k - p\cdot k') + m^2)(p\cdot k - p\cdot k') + m^2(2p\cdot k' - 2p\cdot k - m^2) + 4m^2(p\cdot k - p\cdot k') - m^4 \}$$

$$= 16 \{ (p\cdot k')^2 - \cancel{(p\cdot k)^2} + \cancel{(p\cdot k)^2} + (p\cdot k')^2 - 2(p\cdot k)(p\cdot k') - m^4 + 2(p\cdot k)^2 + 2(p\cdot k')^2$$

$$- 4(p\cdot k)(p\cdot k') + 2m^2\cancel{(p\cdot k - p\cdot k')} + 2m^2\cancel{(p\cdot k' - p\cdot k)} + 4m^2(p\cdot k - p\cdot k') - 2m^4 \}$$

$$= -16 \left[2m^4 + m^2(p\cdot k - p\cdot k') \right]$$

f) $$\frac{d\sigma}{d\cos\theta} = \frac{1}{8\pi} \frac{1}{4m^2} \left(\frac{\omega'}{\omega}\right)^2 \cdot \frac{e^4}{4} \times$$

$$\left\{ 32 \frac{m^4 + m^2 p\cdot k + (p\cdot k')(p\cdot k)}{(2\, k\cdot p)^2} + 32 \frac{m^4 - m^2 p\cdot k' + (p\cdot k)(p\cdot k')}{(2k'\cdot p)^2} \right.$$

$$\left. - 16 \frac{2m^4 + m^2\cancel{(p\cdot k - p\cdot k')} + 2m^4 - m^2\cancel{(p\cdot k' - p\cdot k)}}{(2k\cdot p)(2k'\cdot p)} \right\}$$

$$= \frac{\pi\alpha^2}{\cancel{8}m^2} \left(\frac{\omega'}{\omega}\right)^2 \cdot \cancel{8} \left\{ m^4 \left(\frac{1}{(k\cdot p)^2} + \frac{1}{(k'\cdot p)^2} - \frac{2}{(k\cdot p)(k'\cdot p)} \right) \right.$$

$$\left. + 2m^2 \left(\frac{1}{k\cdot p} - \frac{1}{k'\cdot p} \right) + \frac{p\cdot k}{p\cdot k'} + \frac{p\cdot k'}{p\cdot k} \right\}$$

in lab frame $\quad p = (m, 0)$, $\; k = (\omega, \omega \hat{z}) \quad \Longrightarrow \quad p' = (E', \vec{p}\,')$, $\; k' = (\omega', \omega' \sin\theta, 0, \omega' \cos\theta)$

$(k\cdot p) = m\omega \;, \quad (p\cdot k') = m\omega'$

$$= \frac{\pi\alpha^2}{m^2} \left(\frac{\omega'}{\omega}\right)^2 \left\{ m^4 \cdot \left(\frac{1}{\omega m} - \frac{1}{\omega' m} \right)^2 + 2m^2 \left(\frac{1}{m\omega} - \frac{1}{m\omega'} \right) \right.$$

$$\left. + \frac{\omega}{\omega'} + \frac{\omega'}{\omega} \right\}$$

$$= \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2 \left\{ m^2\left(\frac{1}{\omega} - \frac{1}{\omega'}\right)^2 + 2m\left(\frac{1}{\omega} - \frac{1}{\omega'}\right) + \frac{\omega'}{\omega} + \frac{\omega}{\omega'} \right\}$$

$$\left[\begin{aligned}
p'^2 = m^2 &= (p+k-k')^2 = p^2 + 2p\cdot(k-k') - 2k\cdot k' \\
&= m^2 + 2(m,0)(\omega-\omega', -\omega'\sin\theta, 0, -\omega'\cos\theta + \omega) - 2\omega\omega'(1-\cos\theta) \\
&= m^2 + 2m(\omega-\omega') - 2\omega\omega'(1-\cos\theta) \\
&\Leftrightarrow \quad 2m(\omega-\omega') = 2\omega\omega'(1-\cos\theta) \\
&\qquad \frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1-\cos\theta)
\end{aligned}\right]$$

$$= \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2 \left\{ \cancel{m^2}\cdot\frac{1}{\cancel{m^2}}(1-\cos\theta)^2 - 2\cancel{m}\cdot\frac{1}{\cancel{m}}(1-\cos\theta) + \frac{\omega'}{\omega} + \frac{\omega}{\omega'} \right\}$$

$$= \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2 \left(1 - \cancel{2\cos\theta} + \cos^2\theta - 2 + \cancel{2\cos\theta} + \frac{\omega'}{\omega} + \frac{\omega}{\omega'} \right)$$

$$= \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2 \left(\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta \right)$$