1. a)

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} F_{\mu\nu}$$

$$= \bar{\psi}(i\not{\partial} - e\not{A} - m)\psi - \frac{1}{4}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha)(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$\frac{\partial}{\partial(\partial_\mu \psi)} = \bar{\psi} i \gamma^\mu \qquad \frac{\partial}{\partial \psi} = \bar{\psi}(-e\not{A} - m)$$

$$\Rightarrow \bar{\psi}(i\overleftarrow{\not{\partial}} + e\not{A} + m) = 0$$

$$\frac{\partial}{\partial(\partial_\sigma A_\lambda)} = -\frac{1}{4}\epsilon^{\mu\nu\alpha\beta}\Big[\big(\delta_{\alpha\sigma}\delta_{\beta\lambda} - \delta_{\beta\sigma}\delta_{\alpha\lambda}\big)F_{\mu\nu}$$

$$+ F_{\alpha\beta}\big(\delta_{\mu\sigma}\delta_{\nu\lambda} - \delta_{\nu\sigma}\delta_{\mu\lambda}\big)\Big]$$

$$= -\frac{1}{4}\Big[\underbrace{\epsilon^{\mu\nu\sigma\lambda}F_{\mu\nu} - \epsilon^{\mu\nu\lambda\sigma}F_{\mu\nu}}_{2F_{\mu\nu}\epsilon^{\mu\nu\sigma\lambda}} + \underbrace{\epsilon^{\sigma\lambda\alpha\beta}F_{\alpha\beta} - \epsilon^{\lambda\sigma\alpha\beta}F_{\alpha\beta}}_{2F_{\alpha\beta}\epsilon^{\sigma\lambda\alpha\beta}}\Big]$$

$$= -F_{\mu\nu}\epsilon^{\mu\nu\sigma\lambda}$$

$$\frac{\partial}{\partial A_\sigma} = -e\bar{\psi}\gamma^\mu\psi$$

$$\Rightarrow \partial_\sigma \epsilon^{\mu\nu\sigma\lambda} F_{\mu\nu} = e\bar{\psi}\gamma^\mu\psi$$

b)

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi + \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - M^2\phi^2) - g\bar{\psi}i\gamma^5\psi\phi$$

$d = 6$

$$2[\psi] + 1 = d$$

$$[\psi] = \frac{d-1}{2}$$

$$([\phi] + 1)\cdot 2 = d$$

$$[\phi] = \frac{d}{2} - 1$$

$$\Rightarrow [g] + 2[\psi] + [\phi] \overset{!}{=} d$$

$$[g] + (\cancel{d} - 1) + \left(\frac{d}{2} - 1\right) = \cancel{d}$$

$$\Rightarrow [g] = 2 - d/2 = 2 - 3 = -1 < 0$$

non-renormalizable

c) $\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s \left(a_p^s u_s(p) e^{-ipx} + b_p^{s+} v_s(p) e^{+ipx} \right)$

$\langle 0 | \psi(x) | \vec{p}, r \rangle$

$= \langle 0 | \psi(x) \sqrt{2E_p}\; a_p^{(r)+} | 0 \rangle$

$= \langle 0 | \int \frac{d^3p}{(2\pi)^3} u(p) e^{-ipx} \underbrace{a_p a_p^{(r)+}} | 0 \rangle$

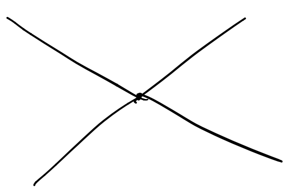
$= \{ a_p^s , a_p^{r+} \} = (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{q}) \delta_{rs}$

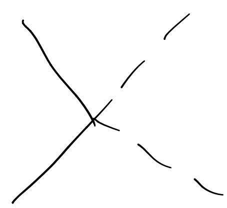
$= u(p)\, e^{-ipx}$

d) $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi + \frac{1}{2} \partial_\mu \pi \partial^\mu \pi - \sqrt{\lambda} (\phi^* \phi)^2 + \lambda (\phi^* \phi) \pi^2$
$- g (\phi^* \phi) \pi$

$\phi^{(*)}$  $\frac{i}{p^2 - m^2 + i\varepsilon}$

π $\frac{i}{p^2 + i\varepsilon}$

 $-i4\sqrt{\lambda}$

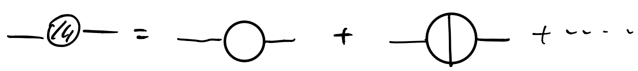
 $+i2\lambda$

$\underrightarrow{t}$ $\underrightarrow{t}$

 $-ig$

3. a) $\int d^4x\, e^{ipx} \langle \Omega | T\,\phi(x)\phi(0) | \Omega \rangle$

(with



$= -i\Sigma(p^2)$

$$= D_F^{(0)}(p^2) + D_F^{(0)}(p^2)\left(-i\Sigma(p^2)\right) D_F^{(0)}(p^2) + \cdots$$

$$= D_F^{(0)}(p^2)\, \frac{1}{1 + i\Sigma(p^2) D_F^{(0)}(p^2)} = \frac{i}{p^2 - m_0^2}\, \frac{1}{1 + i\Sigma(p^2)\frac{i}{p^2 - m_0^2}}$$

$$= \frac{i}{p^2 - m_0^2 - \Sigma(p^2)}$$

Expand $\Sigma(p^2) = \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) + (p^2 - m^2)\hat{\Sigma}(p^2)$

with $m_0^2 + \Sigma(m^2) = m^2$, $\hat{\Sigma}(m^2) = 0$

$$\Rightarrow \quad m_0^2 + \Sigma(p^2) = m_0^2 + \Sigma(m^2) + \cdots$$

$$= m^2 + (p^2 - m^2)\Sigma'(m^2) + (p^2 - m^2)\hat{\Sigma}(p^2)$$

$$\frac{i}{p^2 - m_0^2 - \Sigma(p^2)} = \frac{i}{p^2 - m^2 - (p^2 - m^2)\left(\Sigma'(m^2) + \hat{\Sigma}(p^2)\right)}$$

$$= \frac{i}{p^2 - m^2}\, \frac{1}{1 - \Sigma'(m^2) - \hat{\Sigma}(p^2)}$$

$$= \frac{iZ}{p^2 - m^2}\, \frac{1}{1 - Z\hat{\Sigma}(p^2)} = \frac{iZ}{p^2 - m^2} + \text{Regular}$$

$$Z = \left(1 - \Sigma'(m^2)\right)^{-1}$$

b) $\mathcal{L}_{int} = -\frac{\lambda}{3!}\phi^3 \quad \rightarrow \quad$ vertex: $-i\lambda$

$$-i\Sigma_2(p^2) = \frac{(-i\lambda)^2}{2} \int \frac{d^dk}{(2\pi)^d}\, \frac{i}{(p+k)^2 - m_0^2 + i\varepsilon}\, \frac{i}{(-k)^2 - m_0^2 + i\varepsilon}$$

$$= \frac{+\lambda^2}{2(2\pi)^d} \int_0^1 dx \int d^dk\, \frac{1}{\underbrace{\left\{ x\left[(p+k)^2 - m_0^2\right] + (1-x)\left[k^2 - m_0^2\right] \right\}^2}_{}}$$

$$= k^2 - m_0^2 + x(p^2 + 2kp)$$
$$= (k + px)^2 - p^2x^2 - m_0^2 + xp^2$$
$$= (k+px)^2 - p^2 x(x-1) - m_0^2$$
$$=: q^2 - \Delta$$

$$= \frac{\lambda^2}{2(2\pi)^d} \int_0^1 dx \int d^d q \, \frac{1}{(q^2 - \Delta)^2}$$

Wick rotation: $q_0 \to i q_{0,E}$, $\vec{q} = \vec{q}_E$ (only w.r.t q !)

$$q^2 = q_0^2 - \vec{q}_E^{\,2} = -q_{0,E}^2 - \vec{q}_E^{\,2} = -\vec{q}_E^{\,2}$$

$$= \frac{i\lambda}{2(2\pi)^d} \int_0^1 dx \int d^d q_E \, \frac{1}{(q_E^2 - \Delta)^2}$$

c)

$d \to 4$, $\Sigma_2(p^2) \propto \int \frac{d^4 q_E}{q_E^4}$ $\to$ logarithmically divergent

$\frac{d}{dp^2} \Sigma_2(p^2) \propto \int \frac{d^4 q_E}{q_E^6}$ $\to$ convergent

d) $$-i\Sigma_2(p^2) = \frac{\lambda^2}{2(2\pi)^d} \int_0^1 dx \, \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2 - d/2)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2 - d/2}$$

$d = 4 - \varepsilon$

$$= \frac{\lambda^2}{2(2\pi)^4} \frac{1}{(4\pi)^2} \frac{\Gamma(\frac{\varepsilon}{2})}{1} \underbrace{\int_0^1 dx \, \Delta^{\varepsilon/2}}_{=1}$$

$$= \frac{\lambda^2}{2^8 \pi^6} \frac{1}{\varepsilon} + \cdots$$

$$Z = \left(1 - \frac{\partial}{\partial p^2} \Sigma(p^2)\Big|_{p^2 = m^2}\right)^{-1}$$

$$\frac{\partial}{\partial p^2} \Sigma(p^2) = \frac{i\lambda^2}{2(2\pi)^d} \frac{\partial}{\partial p^2} \int_0^1 dx \int d^d q_E \, \frac{1}{(q_E^2 - \Delta)^2} \qquad \Delta = p^2 x(x-1) + m_0^2$$

$$= \frac{i\lambda^2}{2(2\pi)^d} \int_0^1 dx \int d^d q_E \; -2 \, \frac{1}{(q_E^2 - \Delta)^3} \cdot x(x-1)$$

$$= \frac{-i\lambda^2}{(2\pi)^d} \int_0^1 dx \; x(x-1) \cdot \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(3-d/2)}{\Gamma(3)} \Delta^{d/2-3}$$

$$= \frac{-i\lambda^2}{(2\pi)^d} \frac{1}{2(4\pi)^{d/2}} \int_0^1 dx \; x(x-1) \left[p^2 x(x-1) + m_0^2\right]^{d/2-3}$$

At $p^2 = m^2$, $m^2 - m_0^2 = O(\lambda^2)$

$$\propto \int_0^1 dx \; \frac{x(x-1)}{x^2 - x + 1} \; \frac{1}{m_0^2}$$

2. $\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi + \bar{\chi}(i\not{\partial} - m)\chi + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - M^2\phi^2)$

$\qquad - g\bar{\psi}i\gamma^5\psi\phi - \bar{g}\bar{\chi}i\gamma^5\chi\phi$

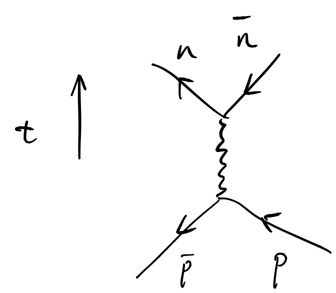
$\mu > m$

$$\left(\begin{array}{l} \psi^{(s)}(p) + \bar{\psi}^{(\bar{s})}(\bar{p}) \longrightarrow \chi^{(r)}(k) + \bar{\chi}^{(\bar{r})}(\bar{k}) \\ \qquad p\,p \longrightarrow n\,n \end{array} \right.$$

a) $\qquad -ig\, i\gamma^5 = g\gamma^5 \qquad ; \qquad -i\bar{g}\, i\gamma^5 = \bar{g}\gamma^5$

b)

$p\bar{p} \longrightarrow n\bar{n}$



$$i\mathcal{M} = \bar{v}_p^{(r)}(\bar{p})\,(g\gamma^5)\,u_p^{(r')}(p)\,\frac{i}{(p_1+p_2)^2 - M^2 + i\varepsilon}\,\bar{u}_n^{(s)}(k)\,(\bar{g}\gamma^5)\,v_n^{(s')}(\bar{k})$$

$+ \cdots (?)$

c) $$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{spin}(g\bar{g})^2\, \bar{v}_p^{(r)}(\bar{p})\gamma^5 u_p^{(r')}(p)\frac{1}{s - M^2}\bar{u}_n^{(s)}(k)\gamma^5 v_n^{(s')}(\bar{k})$$

$$\bar{v}_n^{(s')}(\bar{k})\underbrace{\gamma^0\gamma^5\gamma^0}_{\gamma^5} u_n^{(s)}(k)\frac{1}{s - M^2}\bar{u}_p^{(r')}(p)\gamma^5 v_p^{(r)}(\bar{p})$$

to be aware of γ^5

$$= \frac{(g\bar{g})^2}{4(s-M^2)^2}\sum_{\substack{r,r'\\ s,s'}} \underbrace{tr\left[(\bar{\not{p}} - m)(-\not{p} + m)\right]}_{\begin{array}{l} = tr(\bar{\not{p}}\not{p} + m^2) \\ = 4(\bar{p}\cdot p + m^2) \end{array}}\ \underbrace{tr\left[(\bar{\not{k}} - \mu)(-\not{k} + \mu)\right]}_{\begin{array}{l} = tr(\bar{\not{k}}\not{k} + \mu^2) \\ = 4(\bar{k}\cdot k + \mu^2) \end{array}}$$

$$= \frac{(g\hat{g})^2}{(s-M^2)^2}\, S^2$$

with $\quad S^2 = 2m^2 + p\cdot\bar{p} = 2\mu^2 + k\cdot\bar{k}$

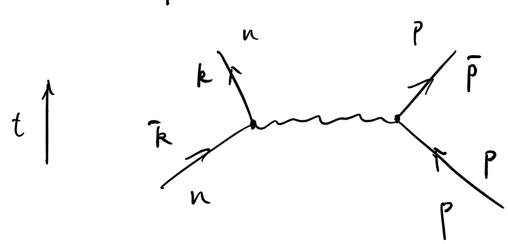
d) $$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\,\frac{|\vec{p}_f|}{|\vec{p}_i|}\,\overline{|\mathcal{M}|^2}$$

$$= \frac{1}{64\pi^2 s}\,\frac{\sqrt{s}}{\sqrt{s}}\left(\frac{g\bar{g}}{s-M^2}\right)^2 S^2$$

$$= \frac{1}{64\pi^2}\left(\frac{g\hat{g}}{s-M^2}\right)^2 S$$

$s < 4\mu^2$, not able to produce $n\bar{n}$-pair.

e) $$\psi^{(s)}(p) + \chi^{(\tilde{r})}(\bar{k}) \rightarrow \psi^{(\tilde{s})}(\bar{p}) + \chi^{(r)}(k)$$



$\breve{s} = (p+\bar{k})^2$

$\breve{t} = (p-\bar{p})^2$

$\breve{u} = (p-k)^2$

$$i\mathcal{M} = \bar{u}_n^{(\tilde{s})}(k)\,(\bar{g}\gamma^5)\,u_n^{(s)}(\bar{k})\,\frac{-ig_{\mu\nu}}{(p+\bar{k})^2 - M^2}\,\bar{u}_p^{(\tilde{s})}(\bar{p})\,(g\gamma^5)\,u_p^{(s)}(p)$$

$$= \frac{\bar{g}g}{(\breve{s}-M^2)}\,\bar{u}_n^{(\tilde{s})}(k)\gamma^5 u_n^{(s)}(\bar{k})\,\bar{u}_p^{(\tilde{s})}(\bar{p})\gamma^5 u_p^{(s)}(p)$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{spin}} \mathcal{M}^* \mathcal{M}$$

$$= \frac{1}{4}\left(\frac{g\bar{g}}{\tilde{s}-M^2}\right)^2 \sum \bar{u}_n^{(\bar{s})}(k)\gamma^5 u_n^{(s)}(\bar{k})\bar{u}_p^{(\bar{s})}(\bar{p})\gamma^5 u_p^{(s)}(p)\,|\,\bar{u}_p^{(s)}(p)\gamma^5 u_p^{(\bar{s})}(\bar{p})\bar{u}_n^{(s)}(\bar{k})\gamma^5 u_n^{(\bar{s})}(k)$$

$$= \frac{1}{4}\left(\frac{g\bar{g}}{\tilde{s}-M^2}\right)^2 tr\left[(\not{k}+\mu)\gamma^5(\not{\bar{k}}+\mu)\gamma^5\right] tr\left[(\not{\bar{p}}+m)\gamma^5(\not{p}+m)\gamma^5\right]$$

$$= \frac{1}{4}\left(\frac{g\bar{g}}{\tilde{s}-M^2}\right)^2 tr\left[(\not{k}+\mu)\underbrace{(\gamma^5)^2}_{=\mathbb{1}}(-\not{\bar{k}}+\mu)\right] tr\left[(\not{\bar{p}}+m)(-\not{p}+m)\right]$$

$$= \frac{1}{4}\left(\frac{g\bar{g}}{\tilde{s}-M^2}\right)^2 \underbrace{tr\left[-\not{k}\not{\bar{k}}+\mu^2\right]}_{=4(k\cdot\bar{k}-\mu^2)} \underbrace{tr\left[-\not{p}\not{\bar{p}}+m^2\right]}_{=4\cdot(p\cdot\bar{p}-m^2)}$$

$$= \frac{4(g\bar{g})^2}{(\tilde{s}-M^2)^2}(k\cdot\bar{k}-\mu^2)(p\cdot\bar{p}-m^2)$$

$$\Big(\quad \tilde{t} = (p-\bar{p})^2 = 2m^2 - 2p\bar{p}$$
$$= (k-\bar{k})^2 = 2\mu^2 - 2k\bar{k}$$

$$= \left(\frac{g\bar{g}}{\tilde{s}-M^2}\right)^2 \tilde{t}^2$$