Theoretical Astroparticle Physik
Homework 6

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1 Decoupling of electron- and muon-neutrino

Here we have neglected the existence of the Z-boson, neutrino oscillations and loop process.

a) At temperature above the muon mass ($T \gtrsim m_\mu \approx 105\,\text{MeV}$) and below QCD phase transition ($T \lesssim T_{QCD} \approx 170\,\text{MeV}$),
The only relevant process are weak interacting process with exchange of W bosons. As we are at temperaure above the muon mass, muon and anti-muon plays a role. As shown below are the simplified feynman diagrams.

Refer to the FD.png. I can use tik to draw the FD again if you want to. But of course Im lazy. lol. And also I have not included any fermion lines here because it represent both possibilities.

Other process to think about:

(a) There is also pion decays to leptons and leptons neutrinos which I'm not sure whether or not to include here. As pion mass is arounf 134 to 139 MeV. it is stated that we are working at temperature "below the QCD phase transition" I understand in the sense that we need to ignore perturbative QCD process. I would assume treating pions as a boson rather than two quarks would make sense. But again. it is stated that "hadronic degrees of freedom in the plasma is already decayed away". Im not sure what does that means. I guess we should ignore processes with hadrons?

(b) and there is also neutrino neutrino to W W and/or neutrino W to neutrino W process, and I think it make sense to ignore this, because m_W is too high so probably fucking surpressed.

b) At temperature below the muon mass, the production of muons will be surpressed while existing muons will decay into electrons, electron-neutrinos and muon-neutrinos. In the absence of muons, neutrino are unable to interact with anything else, since the only process we considering here is weak interaction via W(in which muon neutrino is coupled to muons via W bosons.) So, at this point muon-neutrinos will be decoupled from the plasma. Note that this is true only if the existence of Z-boson, neutrino oscillation and loop process are neglected.

$$g_*(T > m_\mu) = 2_\gamma + \frac{7}{8}(4_e + 4_\mu + 2_{\nu_\mu} + 2_{\nu_e}) = 12.5 \tag{1.1}$$

$$g_*(T < m_\mu) = 2_\gamma + \frac{7}{8}(4_e + +2_{\nu_e}) = 7.25 \tag{1.2}$$

- When a degree of freedom is not a degree of freedom anymore, it means that the particle is decoupled right? In some text I found online (eg:https://ethz.ch/content/dam/ethz/special-interest/phys/particle-physics/cosmologygro dam/Courses/TheoreticalCosmology/2017/solution%203.pdf), however in their case they didnt ignore the existence of Z boson, as such muon- and tau-neutrinos are still able to be coupled despite not having any muons or taus left. I still dont have a good idea of thermal equilibrium in the context of particle physics :poop:
- 3 points for this, do I need to calculate some shit? If so, any idea how?

c) when $\frac{\Gamma}{H} > 1$ the expansion rate of the universe is sloower than that of the scaterring rate, and as such, it is still posible for the particle to interact/scatter and as such still at thermal equilibirum with the plasma.

when $\frac{\Gamma}{H} < 1$, the expansion rate of the universe is faster than that of the scattering rate, and as such, the particle are far apart and are unable to interact/scatter effectively anymore.

The particles are decoupled when

$$\begin{aligned}
\frac{\Gamma}{H} &= 1 \\
G_F^2 T_{dec}^5 &= \frac{T_{dec}^2}{M_{Pl}} \\
T_{dec} &= \left(\frac{1}{G_F^2 M_{Pl}}\right)^{\frac{1}{3}} \\
&= \left(\frac{1}{(1.166\times10^{-5}\,\text{GeV})^2(1.22\times10^{19}\,\text{GeV})}\right)^{\frac{1}{3}} \\
&= 0.000\,844\,7\,\text{GeV} \\
&= 0.8447\,\text{MeV}
\end{aligned}$$

This is lower than the value given in the Gorbunov text ($2-3$MeV), probably due to the approximation of M_{Pl}^* to M_{Pl}. (Recall that $H = \frac{T^2}{M_{Pl}^*}$ where $M_{Pl}^* = \frac{1}{1.66\sqrt{g_*}} M_{Pl}$). In doing so, one has ignored the dependence of the relativistic degree of freedom. In wiki its around 1MeV which is true for us

d) Knowing that,

$$sa^3 \propto g_* T^3 a^3 = \text{const} \tag{1.3}$$

One can compare the ratio of the degrees of freedom and temperature at different times

$$g_{*,\alpha} T_\alpha^3 = g_{*,\beta} T_\beta^3 \tag{1.4}$$

and one would end up with the following relation

$$T_\beta = \left(\frac{g_{*,\alpha}}{g_{*,\beta}}\right)^{1/3} T_\alpha \tag{1.5}$$

After neutrino decoupling, the neutrino temperature remains the same despite the expanding of the universe. However, when the temperature goes below electron mass, electrons and positrons annihilate leading to a higher photon temperature. So to find the temperature of the decoupled electron-neutrinos, one simply has to compare when $T > m_e$ and $T < m_e$.

At $T > m_e$, $T = T_{\nu_e}$, $g_{*,\nu_e} = 2_\gamma + \frac{7}{8}4_e = 5.5$
At $T < m_e$, $T = T_\gamma$, $g_{*,\gamma} = 2_\gamma$
Now,

$$T_{\nu_e} = \left(\frac{g_{*,\gamma}}{g_{*,\nu_e}}\right)^{1/3} T_\gamma \tag{1.6}$$

$$= \left(\frac{4}{11})^{1/3}(2.725\,\mathrm{K}\right) \tag{1.7}$$

$$\approx 1.95\,\mathrm{K} \tag{1.8}$$

e) To find the decoupled muon-neutrinos temperature, one simply follow the same procedure again with different relativistic effective numbers of degree of freedom. Taking $g_*(T < m_\mu) = 7.25$

$$T_{\nu_\mu} = \left(\frac{2}{7.25}\right)^{1/3} 2.725\,\mathrm{K} \tag{1.9}$$

$$\approx 1.77\,\mathrm{K} \tag{1.10}$$

Recall that for realtivistic particles that obey the Fermi-Dirac statistics,

$$n \propto T^3 \tag{1.11}$$

and since $T_{\nu_e} > T_{\nu_\mu}$, one would expect the number density of electron-neutrinos to be higher.

2 Graviton Decoupling

a) Since gravitational interaction is characterised by M_{pl}, through simple dimensional analysis, we estimate the interaction rate

$$\Gamma = \frac{T^5}{M_{pl}^4} \tag{2.1}$$

The Hubble time in radiation-dominated era is $H = T^2/M_{pl}$. The particle of interest decouples when $\Gamma \sim H$, so

$$\frac{\Gamma}{H} = \left(\frac{T}{M_{pl}}\right)^3 \sim 1$$

Thus we have

$$T_{G,\mathrm{dec}} \sim M_{pl} = 1.2 \times 10^{19}\,\mathrm{GeV} \tag{2.2}$$

It happened rather near the beginning of the Universe. It also justifies the assumption that it happended in the radiation-dominated era.

b) With this high temperature, all particles in SM were relativistic. They were all in thermal equilibrium, thus $(T_i/T) = 1$. Since this is before recombination and QCD phase transition, we only consider elementary particles.

Ths bosons are the Higg boson , gluons, $W^{\pm}$, Z^0 and γ. The fermion part includes quarks and leptons. Note that in SM there is no right-handed neutrinos.

$$
\begin{aligned}
g_{*s}(T_{G,\text{dec}}) &= \sum_{\text{bosons}} g_i + \frac{7}{8}\sum_{\text{fermions}} g_i && (2.3)\\
&= (1\cdot 1 + 1\cdot 8\cdot 2 + 2\cdot 3 + 1\cdot 3 + 1\cdot 2) + \frac{7}{8}2\cdot(3\cdot 2\cdot 6 + 3\cdot 2 + 3\cdot 1) && (2.4)\\
&= 106.75 && (2.5)
\end{aligned}
$$

Somehow considering the particles/fields before EWSB doesn't give the correct value. The total degrees of freedom are not the same, since the Higgs loses its one degree of freedom to make $W^{\pm}$ and Z^0 massive and thus give them in total 3 degrees of freedoms.

c) In the present Universe, there are only photons and neutrinos still relativitic. Although neutrinos are relativistic, they have already decoupled and not in thermal equilibrium with photons anymore. From the lecture, $T_{\gamma,0}/T_{\nu,0} \simeq (11/4)^{1/3}$

$$
g_{*s}(T_0) = 1\cdot 2 + \frac{7}{8}\cdot 6\cdot\frac{4}{11} = 3.91 \tag{2.6}
$$

Instead of considering only electron-photon component in determining neutrino temeperature, we consider all the particles

$$
g_{*s}(T)a^3T^3 = \text{const}
$$

Using this to compare the time of graviton decoupling and present Universe.

$$
\begin{aligned}
g_{*s}(T_{G,\text{dec}})a_{G,\text{dec}}^3 T_{G,\text{dec}}^3 &= g_{*s}(T_0)a_0^3T_0^3\\
\Rightarrow T_{G,0}^3 &= \left(T_{G,\text{dec}}\frac{a}{a_0}\right)^3 = T_0^3\frac{g_{*s}(T_0)}{g_{*s}(T_{G,\text{dec}})}\\
\Rightarrow T_{G,0} &= T_0\left(\frac{g_{*s}(T_0)}{g_{*s}(T_{G,\text{dec}})}\right)^{1/3} = 0.897\,\text{K} && (2.7)
\end{aligned}
$$

Since graviton is massless, it should be relativistic

$$n_{G,0} = 1 \cdot \frac{\zeta(3)}{\pi^2} T_{G,0}^3 = 7.25\,\mathrm{cm}^{-3} \tag{2.8}$$

As expected, we have far less gravitons floating around than neutrinos, since graviton decoupling happened before neutrino decoupling.

Shouldn't graviton always be massless?