Theoretical Astroparticle Physik
Homework 7

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1 Quickies

a) For heavier elements, BBN are unable to explain the abundance of it. Stellar and supernova nucleosynthesis are the two ways to explain this. Stellar nucleosynthesis explain how new heavy nuclei are produced in stars during stellar evolution. It explains the abundances of elements from carbon to iron that we see today. Stars like our sun act as thermonuclear furnaces in which H and He are fused into heavier nuclei by increasingly high temperatures as the composition of the core evolves.[Wiki]

   Supernova nucleosynthesis occurs in the energetic environment in supernovae, in which the elements between silicon and nickel are synthesized in quasiequilibrium[13] established during fast fusion that attaches by reciprocating balanced nuclear reactions to 28Si. [Wiki]

b) The reactions crucial in the early universe can be categorized as follows:

   i) $p(n,\gamma)D$, production of deuterium, initial stage.
   ii) $D(p,\gamma)^3He$, $D(D,n)^3He$,$D(D,p)T$,$^3He(n,p)T$, preliminary reactions preparing material for ^4He production.
   iii) $T(D,n)^4He$,$^3He(D,p)^4He$, production of 4 He.
   iv) $T(\alpha,\gamma)^7Li$, $^3He(\alpha,\gamma)^7Be$,$^7Be(n,p)^7Li$, production of the heaviest elements. $^7Li(p,\alpha)^4He$, burning of ^7Li.

   The simplest nucleus that can be produced in the early universe is Deuterium (^2H) via $p(n,\gamma)D$. This reaction would not take place if the temperature are high enough to overcome its binding energy and break D back to its constituents. Most of the light nuclei needs D to be produced. Hence, it is the production of deuterium that determines whether the nucleosynthesis will take place as well as the nucleosynthesis temperature. Thus, deuterium bottleneck.

However, since the binding energy of ^4He is greater than that of deuterium, it is possible to to produce ^4He without going through Deuterium. But, this does not happen because of something. I think not enough neutron or proton? or something bout temperature not high enough to produce 4He, but D...then since D abundant, D becomes dominating process?

2 Dark Radiation during BBN

a) We know the temperature of deuterium production

$$T_{NS} \approx 80\,\mathrm{keV}$$

In order to compute the age of the Universe, need g_* first. Neutrinos decoupled at $\sim 2\,\mathrm{MeV}$ and their temeratures are $(4/11)^{1/3}$. Then

$$g_* = 2 + \frac{7}{8} \cdot 3 \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} = 3.36 \tag{2.1}$$

As a result,

$$t_{NS} = \frac{M_{Pl}}{1.66\sqrt{g_*}} \frac{1}{2T_{NS}^2} = 203.3\,\mathrm{s} \tag{2.2}$$

b) Neutron freeze-out must happened before the T_{NS}, otherwise no way to burn the deuterium to produce Helium-4. The g_* is then the same as last part.

$$\begin{aligned} \Gamma_n &\stackrel{!}{=} H \\ C_n G_F^2 T^5 &\stackrel{!}{=} T^2/M_{Pl}^* \\ T_n^3 &= \left(M_{Pl}^* C_n G_F^2\right)^{-1} \\ T_n &= \left(M_{Pl}^* C_n G_F^2\right)^{-1/3} = 1.16\,\mathrm{MeV} \end{aligned} \tag{2.3}$$

c) If there were yet-to-be-discovered light particle, which could be in thermal equilirum with photons in present era, the freeze-out temperature will also be different, since we have explicitely used g_* in the previous calculation.

Specifically, g_* would be larger and consequently $T_n \propto g_*^{1/6}$ will be higher and the neutron decoupling would have happened slightly earlier. Since at this temperature neutrons and protons are non-relativistic, they obey Boltzmann distribution $n_{n,p}(T) \propto T_n^{3/2} e^{-m/T}$. For the sake of simplicity, we just ignore the mass difference between protons and neutrons here.

In the expression of $\frac{n_n(T_{NS})}{n_p(T_{NS})}$, we see t_{NS} and it depends on g_* in the sense that g_* grows, t_{NS} decreases. Then the $\frac{n_n(T_{NS})}{n_p(T_{NS})}$ will gets larger and in the end, Helium-4 will become more abundant.

d) First express effective degrees of freedom in terms of ΔN_{eff}

$$g_* = 2 + \frac{7}{8} \cdot (3 + \Delta N_{\text{eff}}) \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} \tag{2.4}$$

g_* dependence enters not only T_n, but also t_{NS}. Thus

$$\begin{aligned} \frac{\mathrm{d}X_{^4\text{He}}}{\mathrm{d}N_{\text{eff}}} &= -\frac{2}{\left(1 + \frac{n_p(T_{NS})}{n_n(T_{NS})}\right)^2} \frac{\mathrm{d}\frac{n_p(T_{NS})}{n_n(T_{NS})}}{\mathrm{d}N_{\text{eff}}} \\ &= -\frac{X_{^4\text{He}}^2}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\partial T_n}{\partial N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\partial t_{NS}}{\partial N_{\text{eff}}} \right) \end{aligned}$$

Write d into Δ and rearrange the equation, we have

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} = -\frac{X_{^4\text{He}}}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\partial T_n}{\partial N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\partial t_{NS}}{\partial N_{\text{eff}}} \right) \Delta N_{\text{eff}} \tag{2.5}$$

It contains four derivatives, thus it is more economical to calculate it with Mathematica. With $T_n \approx 1.4\,\text{MeV}$, $t_{NS} \approx 200\,\text{s}$, $X_{^4\text{He}} \approx 45\%$, $\Delta N_e ff = 1$, and $\mu_n = \mu_p$, we get

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} \approx 2.8\% \tag{2.6}$$

3 Neutron burning

a) Given $\langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \approx 6 \times 10^{-20} \text{cm}^3\text{s}^{-1}$, $\eta_B = 6.2 \times 10^{-10}$

$$\begin{aligned} \Gamma_{p(n,\gamma)D} &= n_p \cdot \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \\ &= \eta_B n_\gamma \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \\ &= \eta_B (g_\gamma \frac{\zeta(3)}{\pi^2} T^3) \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \end{aligned} \tag{3.1}$$

Take $T_{NS} = 80\,\text{keV} = 80 \times 10^3 \times 8065\text{cm}^{-1}$, $g_\gamma = 2$, $\zeta(3) \approx 1.2025$

$$\Gamma_{p(n,\gamma)D} = 0.002\,435\,\text{s}^{-1} \tag{3.2}$$

something felt wrong, I use the same equation with the values given in Gorubnov(pg191), I cant get the $0.5s^{-1}$ given in the book

For a radiation dominated universe, taking $M^*_{pl} \approx M_{pl} \approx 1.2209 \times 10^{19}\,\text{GeV}$. I dont know what M_{Pl} to use.

$$\begin{aligned} H(T_{NS}) &= \frac{T^2}{M^*_{pl}} \\ &= \frac{(80 \times 10^{-6}\,\text{GeV})^2}{1.2209 \times 10^{19}\,\text{GeV}} \\ &= 5.242 \times 10^{-28}\,\text{GeV} \end{aligned} \tag{3.3}$$

$\Gamma_{p(n,\gamma)D} \gg H(T_{NS})$? dafuq with the units

b) Since we are taking the assumption that the freezeout temperature is when the universe is radiation dominated, we use

$$\begin{aligned} H(T_{NS}) &= \frac{1}{2t} \qquad (3.4) \\ 5.242 \times 10^{-28}\,\text{GeV} &= \frac{1}{2t} \\ t &= \frac{1}{2 \cdot (5.242 \times 10^{-28}\,\text{GeV})} \\ &= 9.538 \times 10^{26}\,\text{GeV} \end{aligned}$$

Something bout the the units . :|