Theoretical Astroparticle Physik
Homework 7

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1 Dark Radiation during BBN

a) We know the temperature of deuterium production

$$T_{NS} \approx 80\,\mathrm{keV}$$

In order to compute the age of the Universe, need g_* first. Neutrinos decoupled at $\sim 2\,\mathrm{MeV}$ and their temeratures are $(4/11)^{1/3}$. Then

$$g_* = 2 + \frac{7}{8} \cdot 3 \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} = 3.36 \tag{1.1}$$

As a result,

$$t_{NS} = \frac{M_{Pl}}{1.66\sqrt{g_*}} \frac{1}{2T_{NS}^2} = 203.3\,\mathrm{s} \tag{1.2}$$

b) Neutron freeze-out must happened before the T_{NS}, otherwise no way to burn the deuterium to produce Helium-4. The g_* is then the same as last part.

$$\begin{aligned}
\Gamma_n &\overset{!}{=} H \\
C_n G_F^2 T^5 &\overset{!}{=} T^2/M_{Pl}^* \\
T_n^3 &= \left(M_{Pl}^* C_n G_F^2\right)^{-1} \\
T_n &= \left(M_{Pl}^* C_n G_F^2\right)^{-1/3} = 1.16\,\mathrm{MeV}
\end{aligned} \tag{1.3}$$

c) If there were yet-to-be-discovered light particle, which could be in thermal equilirum with photons in present era, the freeze-out temperature will also be different, since we have explicitely used g_* in the previous calculation.

Specifically, g_* would be larger and consequently $T_n \propto g_*^{1/6}$ will be higher and the neutron decoupling would have happened slightly earlier. Since at this temperature neutrons and protons are non-relativistic, they obey Boltzmann distribution $n_{n,p}(T) \propto T_n^{3/2} e^{-m/T}$. For the sake of simplicity, we just ignore the mass difference between protons and neutrons here.

In the expression of $\frac{n_n(T_{NS})}{n_p(T_{NS})}$, we see t_{NS} and it depends on g_* in the sense that g_* grows, t_{NS} decreases. Then the $\frac{n_n(T_{NS})}{n_p(T_{NS})}$ will gets larger and in the end, Helium-4 will become more abundant.

d) First express effective degrees of freedom in terms of ΔN_{eff}

$$g_* = 2 + \frac{7}{8} \cdot (3 + \Delta N_{\text{eff}}) \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} \tag{1.4}$$

g_* dependence enters not only T_n, but also t_{NS}. Thus

$$\begin{aligned}
\frac{\mathrm{d}X_{^4\text{He}}}{\mathrm{d}N_{\text{eff}}} &= -\frac{2}{\left(1 + \frac{n_p(T_{NS})}{n_n(T_{NS})}\right)^2} \frac{\mathrm{d}\frac{n_p(T_{NS})}{n_n(T_{NS})}}{\mathrm{d}N_{\text{eff}}} \\
&= -\frac{X_{^4\text{He}}^2}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\partial T_n}{\partial N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\partial t_{NS}}{\partial N_{\text{eff}}} \right)
\end{aligned}$$

Write d into Δ and rearrange the equation, we have

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} = -\frac{X_{^4\text{He}}}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\partial T_n}{\partial N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\partial t_{NS}}{\partial N_{\text{eff}}} \right) \Delta N_{\text{eff}} \tag{1.5}$$

It contains four derivatives, thus it is more economical to calculate it with Mathematica. With $T_n \approx 1.4\,\text{MeV}$, $t_{NS} \approx 200\,\text{s}$, $X_{^4\text{He}} \approx 45\%$, $\Delta N_e f f = 1$, and $\mu_n = \mu_p$, we get

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} \approx 2.8\% \tag{1.6}$$