Theoretical Astroparticle Physik
Homework 6

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1 Quickies

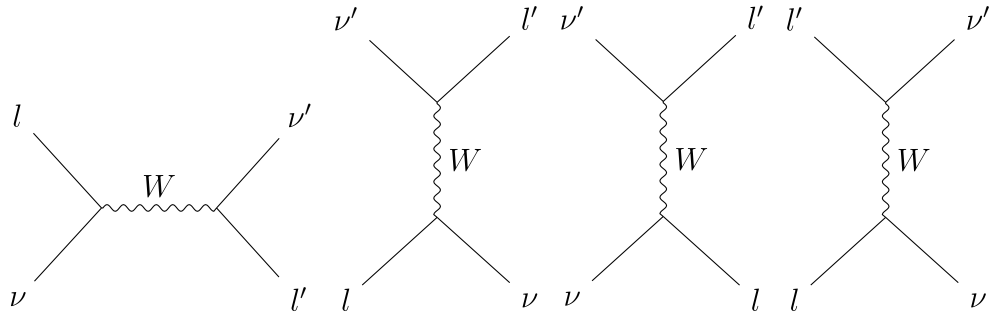
Well, One can consider two points each on opposite edge of the observable unierse. At some time in the early universe, both points would emit their last photon and it would travel to earth where we detect it as CMB. They should not be in causal contact since they are both outside of each other's lightcone. Yet, when we look at the CMB data, it shows that our observable universe is nearly isotropic and homogeneous. This implies that the entire observable universe must have been causally connected long enough to come into thermal equilibrium. This give rise to the horizon problem.

Although different regions are causally disconnected, they obey the same laws of physics. So why would we still expect them to be different? Our best explanation would be because of probabilistic property of thermodynamics. There might be fluctuations generated and they get disconnected causally.

2 Decoupling of electron- and muon-neutrino

Here we have neglected the existence of the Z-boson, neutrino oscillations and loop process.

a) At temperature above the muon mass ($T \gtrsim m_\mu \approx 105\,\mathrm{MeV}$) and below QCD phase transition ($T \lesssim T_{QCD} \approx 170\,\mathrm{MeV}$),
The only relevant processes are weak interacting process with exchange of W bosons. As we are at temperature above the muon mass, muon and anti-muon play a role. As shown below are the 'simplified' feynman diagrams. (Note that no fermion arrow were drawn, because it implies both possible directions. And also l and l' can be muon or electron, while ν and ν' can be muon- or electron-neutrinos.)



Other processes to think about:

(a) Since it stated that we are working at $T \lesssim T_{QCD} \approx 170\,\text{MeV}$, one may think that pion decays to leptons and leptons neutrinos should contribute, since pion mass is around 134 to 139 MeV. But again. since it is stated that ”hadronic degrees of freedom in the plasma is already decayed away”, we are not considering it here.

(b) There is also $\nu\nu \to WW$ and $\nu W \to \nu W$ process. But since the m_W is around $80\,\text{GeV}$ (in the GeV range, way higher than T), it is highly surpressed.

b) At temperature below the muon mass, the production of muons will be surpressed while existing muons will decay into electrons, electron-neutrinos and muon-neutrinos. The process possible left for the existing muon-neutrino is only $\nu_\mu - \mu$ annihilation which produce $\nu_e - e$. This is not the process that keeps muon-neutrinos in thermal equilibrium because the inverse of this process (muon-production) is highly surpressed. So, at this point, muon-neutrinos will be decoupled from the plasma. Note that this is true only if the existence of Z-boson, neutrino oscillation and loop process are neglected. If plasma is all the possible particles that interact either directly/indirectly to the photon, and the relativistic degree of freedom is the degrees of freedom of all these particles, then:

$$g_*(T > m_\mu) = 2_\gamma + \frac{7}{8}(4_e + 4_\mu + 2_{\nu_\mu} + 2_{\nu_e}) = 12.5 \tag{2.1}$$

$$g_*(T \ll m_\mu) = 2_\gamma + \frac{7}{8}(4_e + 2_{\nu_e}) = 7.25 \tag{2.2}$$

We had a discussion on what is the proper defintion of the relativistic degree of freedom of the plasma and the defintion of a plasma. We weren't really sure. So, here are some other possible answer if we were wrong above.

- If plasma is only photons and existing charged particles, and the relativistic degree of freedom is the degrees of freedom of all these particles, then:

$$g_*(T > m_\mu) = 2_\gamma + \frac{7}{8}(4_e + 4_\mu) = 9 \tag{2.3}$$

$$g_*(T \ll m_\mu) = 2_\gamma + \frac{7}{8}(4_e) = 5.5 \tag{2.4}$$

- If plasma is only photons and existing charged particles, but the relativistic degree of freedom is the degrees of freedom of all relativistic particles, then:

$$g_*(T > m_\mu) = 2_\gamma + \frac{7}{8}(4_e + 4_\mu + 2_{\nu_\mu} + 2_{\nu_e} + 2_{\nu_\tau}) = 14.25 \tag{2.5}$$

$$g_*(T \ll m_\mu) = 2_\gamma + \frac{7}{8}(4_e + 2_{\nu_\mu} + 2_{\nu_e} + 2_{\nu_\tau}) = 10.75 \tag{2.6}$$

c) When $\frac{\Gamma}{H} > 1$ the expansion rate of the universe is slower than that of the scattering rate, and as such, it is still possible for the particle to interact/scatter and as such still at thermal equilibrium with the plasma.

When $\frac{\Gamma}{H} < 1$, the expansion rate of the universe is faster than that of the scattering rate, and as such, the particles are far apart and are unable to interact/scatter effectively anymore.

The particles are decoupled when

$$\begin{aligned}
\frac{\Gamma}{H} &= 1 \\
G_F^2 T_{dec}^5 &= \frac{T_{dec}^2}{M_{Pl}} \\
T_{dec} &= \left(\frac{1}{G_F^2 M_{Pl}}\right)^{\frac{1}{3}} \\
&= \left(\frac{1}{(1.166 \times 10^{-5}\,\text{GeV})^2 (1.22 \times 10^{19}\,\text{GeV})}\right)^{\frac{1}{3}} \\
&= 0.000\,844\,7\,\text{GeV} \\
&= 0.8447\,\text{MeV}
\end{aligned}$$

d) Knowing that,

$$sa^3 \propto g_* T^3 a^3 = \text{const} \tag{2.7}$$

One can compare the ratio of the degrees of freedom and temperature at different times

$$g_{*,\alpha} T_\alpha^3 = g_{*,\beta} T_\beta^3 \tag{2.8}$$

and one would end up with the following relation

$$T_\beta = \left(\frac{g_{*,\alpha}}{g_{*,\beta}}\right)^{1/3} T_\alpha \tag{2.9}$$

After neutrino decoupling, the neutrino temperature remains the same despite the expanding of the universe. However, when the temperature goes below electron mass, electrons and positrons annihilate leading to a higher photon temperature. So to find the temperature of the decoupled electron-neutrinos, one simply has to compare when $T > m_e$ and $T < m_e$.

At $T > m_e$, $T = T_{\nu_e}$, $g_{*,\nu_e} = 2_\gamma + \frac{7}{8} 4_e = 5.5$
At $T < m_e$, $T = T_\gamma$, $g_{*,\gamma} = 2_\gamma$

Now,

$$T_{\nu_e} = \left(\frac{g_{*,\gamma}}{g_{*,\nu_e}}\right)^{1/3} T_\gamma \tag{2.10}$$

$$= \left(\frac{4}{11}\right)^{1/3} (2.725\,\mathrm{K}) \tag{2.11}$$

$$\approx 1.95\,\mathrm{K} \tag{2.12}$$

e) To find the decoupled muon-neutrinos temperature, one simply follow the same procedure again with different relativistic effective numbers of degree of freedom. Taking $g_*(T > m_\mu) = 12.25$

$$T_{\nu_\mu} = \left(\frac{2}{12.25}\right)^{1/3} 2.725\,\mathrm{K} \tag{2.13}$$

$$\approx 1.479\,\mathrm{K} \tag{2.14}$$

Recall that for realtivistic particles that obey the Fermi-Dirac statistics,

$$n \propto T^3 \tag{2.15}$$

and since $T_{\nu_e} > T_{\nu_\mu}$, one would expect the number density of electron-neutrinos to be higher.

3 Graviton Decoupling

a) Since gravitational interaction is characterised by M_{pl}, through simple dimensional analysis, we estimate the interaction rate

$$\Gamma = \frac{T^5}{M_{pl}^4} \tag{3.1}$$

The Hubble parameter in radiation-dominated era is $H = T^2/M_{pl}$. The particle of interest decouples when $\Gamma \sim H$, so

$$\frac{\Gamma}{H} = \left(\frac{T}{M_{pl}}\right)^3 \sim 1$$

Thus we have

$$T_{G,\text{dec}} \sim M_{pl} = 1.2 \times 10^{19}\,\text{GeV} \tag{3.2}$$

It happened rather near the beginning of the Universe. It also justifies the assumption that it happended in the radiation-dominated era.

b) With this high temperature, all particles in SM were relativistic. They were all in thermal equilibrium, thus $(T_i/T) = 1$. Since this is before recombination and QCD phase transition, we only consider elementary particles.

Ths bosons are the Higg boson , gluons, $W^{\pm}$, Z^0 and γ. The fermion part includes quarks and leptons. Note that in SM there is no right-handed neutrinos.

$$g_{*s}(T_{G,\text{dec}}) = \sum_{\text{bosons}} g_i + \frac{7}{8} \sum_{\text{fermions}} g_i \tag{3.3}$$

$$= (1 \cdot 1 + 1 \cdot 8 \cdot 2 + 2 \cdot 3 + 1 \cdot 3 + 1 \cdot 2) + \frac{7}{8} 2 \cdot (3 \cdot 2 \cdot 6 + 3 \cdot 2 + 3 \cdot 1) \tag{3.4}$$

$$= 106.75 \tag{3.5}$$

c) In the present Universe, there are only photons and neutrinos still relativitic. Although neutrinos are relativistic, they have already decoupled and not in thermal equilibrium with photons anymore. From the lecture, $T_{\gamma,0}/T_{\nu,0} \simeq (11/4)^{1/3}$

$$g_{*s}(T_0) = 1 \cdot 2 + \frac{7}{8} \cdot 6 \cdot \frac{4}{11} = 3.91 \tag{3.6}$$

Instead of considering only electron-photon component in determining neutrino temeperature, we consider all the particles

$$g_{*s}(T)a^3T^3 = \text{const}$$

Using this to compare the time of graviton decoupling and present Universe.

$$\begin{aligned}
&g_{*s}(T_{G,\text{dec}})a^3_{G,\text{dec}}T^3_{G,\text{dec}} = g_{*s}(T_0)a_0^3T_0^3 \\
&\Rightarrow T^3_{G,0} = \left(T_{G,\text{dec}}\frac{a}{a_0}\right)^3 = T_0^3\frac{g_{*s}(T_0)}{g_{*s}(T_{G,\text{dec}})} \\
&\Rightarrow T_{G,0} = T_0\left(\frac{g_{*s}(T_0)}{g_{*s}(T_{G,\text{dec}})}\right)^{1/3} = 0.897\,\text{K} \qquad (3.7)
\end{aligned}$$

Since graviton is massless, it should be relativistic

$$n_{G,0} = 1\cdot\frac{\zeta(3)}{\pi^2}T^3_{G,0} = 7.25\,\text{cm}^{-3} \qquad (3.8)$$

As expected, we have far less gravitons floating around than neutrinos, since graviton decoupling happened before neutrino decoupling.