Theoretical Astroparticle Physik
Homework 7

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1 Dark Radiation during BBN

a) We know the temperature of deuterium production

$$T_{NS} \approx 80\,\text{keV}$$

In order to compute the age of the Universe, need g_* first. Neutrinos decoupled at $\sim 2\,\text{MeV}$ and their temeratures are $(4/11)^{1/3}$. Then

$$g_* = 2 + \frac{7}{8} \cdot 3 \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} = 3.36 \tag{1.1}$$

As a result,

$$t_{NS} = \frac{M_{Pl}}{1.66\sqrt{g_*}} \frac{1}{2T_{NS}^2} = 203.3\,\text{s} \tag{1.2}$$

b) Neutron freeze-out must happened before the T_{NS}, otherwise no way to burn the deuterium to produce Helium-4. The g_* is then the same as last part.

$$\begin{aligned} \Gamma_n &\overset{!}{=} H \\ C_n G_F^2 T^5 &\overset{!}{=} T^2/M_{Pl}^* \\ T_n^3 &= \left(M_{Pl}^* C_n G_F^2\right)^{-1} \\ T_n &= \left(M_{Pl}^* C_n G_F^2\right)^{-1/3} = 1.16\,\text{MeV} \end{aligned} \tag{1.3}$$

c) If there were yet-to-be-discovered light particle, which could be in thermal equilirum with photons in present era, the freeze-out temperature will also be different, since we have explicitely used g_* in the previous calculation.

Specifically, g_* would be larger and consequently $T_n \propto g_*^{1/6}$ will be higher and the neutron decoupling would have happened slightly earlier. Since at this temperature neutrons and protons are non-relativistic, they obey Boltzmann distribution $n_{n,p}(T) \propto T_n^{3/2} e^{-m/T}$. For the sake of simplicity, we just ignore the mass difference between protons and neutrons here.

In the expression of $\frac{n_n(T_{NS})}{n_p(T_{NS})}$, we see t_{NS} and it depends on g_* in the sense that g_* grows, t_{NS} decreases. Then the $\frac{n_n(T_{NS})}{n_p(T_{NS})}$ will gets larger and in the end, Helium-4 will become more abundant.

d) First express effective degrees of freedom in terms of ΔN_{eff}

$$g_* = 2 + \frac{7}{8} \cdot (3 + \Delta N_{\text{eff}}) \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} \tag{1.4}$$

g_* dependence enters not only T_n, but also t_{NS}. Thus

$$\begin{aligned}\frac{\mathrm{d}X_{^4\text{He}}}{\mathrm{d}N_{\text{eff}}} &= -\frac{2}{\left(1 + \frac{n_p(T_{NS})}{n_n(T_{NS})}\right)^2} \frac{\mathrm{d}\frac{n_p(T_{NS})}{n_n(T_{NS})}}{\mathrm{d}N_{\text{eff}}} \\ &= -\frac{X_{^4\text{He}}^2}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\partial T_n}{\partial N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\partial t_{NS}}{\partial N_{\text{eff}}} \right)\end{aligned}$$

Write d into Δ and rearrange the equation, we have

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} = -\frac{X_{^4\text{He}}}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\partial T_n}{\partial N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\partial t_{NS}}{\partial N_{\text{eff}}} \right) \Delta N_{\text{eff}} \tag{1.5}$$

It contains four derivatives, thus it is more economical to calculate it with Mathematica. With $T_n \approx 1.4\,\text{MeV}$, $t_{NS} \approx 200\,\text{s}$, $X_{^4\text{He}} \approx 45\%$, $\Delta N_e f f = 1$, and $\mu_n = \mu_p$, we get

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} \approx 2.8\% \tag{1.6}$$

2 Neutron burning

a) Given $\langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D} \approx 6 \times 10^{-20}\mathrm{cm}^3\mathrm{s}^{-1}$, $\eta_B = 6.2 \times 10^{-10}$

$$\begin{aligned}\Gamma_{p(n,\gamma)D} &= n_p \cdot \langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D} \\ &= \eta_B n_\gamma \langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D} \\ &= \eta_B (g_\gamma \frac{\zeta(3)}{\pi^2} T^3) \langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D}\end{aligned} \tag{2.1}$$

Take $T_{NS} = 80\,\mathrm{keV} = 80 \times 10^3 \times 8065\mathrm{cm}^{-1}$, $g_\gamma = 2$, $\zeta(3) \approx 1.2025$

$$\Gamma_{p(n,\gamma)D} = 0.002\,435\,\mathrm{s}^{-1} \tag{2.2}$$

something felt wrong, I use the same equation with the values given in Gorubnov(pg191), I cant get the $0.5s^{-1}$ given in the book

For a radiation dominated universe, taking $M_{pl}^* \approx M_{pl} \approx 1.2209 \times 10^{19}\,\mathrm{GeV}$. I dont know what M_{Pl} to use.

$$\begin{aligned}H(T_{NS}) &= \frac{T^2}{M_{pl}^*} \\ &= \frac{(80 \times 10^{-6}\,\mathrm{GeV})^2}{1.2209 \times 10^{19}\,\mathrm{GeV}} \\ &= 5.242 \times 10^{-30}\,\mathrm{GeV}\end{aligned} \tag{2.3}$$

$\Gamma_{p(n,\gamma)D} \gg H(T_{NS})$? dafuq with the units