Theoretical Astroparticle Physik
Homework 7

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1 Quickies

a) For heavier elements, BBN are unable to explain the abundance of it. Stellar and supernova nucleosynthesis are the two ways to explain this. Stellar nucleosynthesis explain how new heavy nuclei are produced in stars during stellar evolution. It explains the abundances of elements from carbon to iron that we see today. Stars like our sun act as thermonuclear furnaces in which H and He are fused into heavier nuclei by increasingly high temperatures as the composition of the core evolves. Similarly, supernova nucleosynthesis occurs in supernovae. Elements between silicon and nickel are synthesized in this energetic environment.
(Reference: https://en.wikipedia.org/wiki/Nucleosynthesis#Stellar_nucleosynthesis)

b) The reactions crucial in the early universe can be categorized as follows:

   i) p(n, γ)D, production of deuterium, initial stage.
   ii) D(p,γ)^{3}He, D(D, n)^{3}He, D(D, p)T,^{3}He(n, p)T, preliminary reactions preparing material for ^{4}He production.
   iii) T(D, n)^{4}He,^{3}He(D, p)^{4}He, production of ^{4}He.
   iv) T(α,γ)^{7}Li, ^{3}He(α, γ)^{7}Be, ^{7}Be(n, p)^{7}Li, production of the heaviest elements. ^{7}Li(p, α)^{4}He, burning of ^{7}Li.

   The simplest nucleus that can be produced in the early universe is Deuterium (^{2}H) via p(n, γ)D. This reaction would not take place if the temperature are high enough to overcome its binding energy and break D back to its constituents. Most of the light nuclei needs D to be produced. Hence, it is the production of deuterium that determines whether the nucleosynthesis will take place as well as the nucleosynthesis temperature. Thus, deuterium bottleneck.

2 Dark Radiation during BBN

a) We know the temperature of deuterium production

$$T_{NS} \approx 80\,\mathrm{keV}$$

In order to compute the age of the Universe, need g_* first. Neutrinos decoupled at $\sim 2\,\mathrm{MeV}$ and their temeratures are $(4/11)^{1/3}$. Then

$$g_* = 2 + \frac{7}{8} \cdot 3 \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} = 3.36 \tag{2.1}$$

As a result,

$$t_{NS} = \frac{M_{Pl}}{1.66\sqrt{g_*}} \frac{1}{2T_{NS}^2} = 203.3\,\mathrm{s} \tag{2.2}$$

b) For some reason, we assume the electrons are still in thermal contact with the plasma (at least this assumption is self-consistent). Then

$$g_* = 2 + \frac{7}{8} \cdot 4 + \frac{7}{8} \cdot 3 \cdot 2 \cdot = 10.75 \tag{2.3}$$

Thus

$$\begin{aligned} \Gamma_n &\stackrel{!}{=} H \\ C_n G_F^2 T^5 &\stackrel{!}{=} T^2 / M_{Pl}^* \\ T_n^3 &= \left(M_{Pl}^* C_n G_F^2\right)^{-1} \\ T_n &= \left(M_{Pl}^* C_n G_F^2\right)^{-1/3} = 1.4\,\mathrm{MeV} \end{aligned} \tag{2.4}$$

c) If there were yet-to-be-discovered light particle, which could be in thermal equilirum with photons in present era, the freeze-out temperature of neutron will also be different, since we have explicitely used g_* in the previous calculation.

Specifically, g_* would be larger and consequently $T_n \propto g_*^{1/6}$ will be higher and the neutron decoupling would have happened slightly earlier. Since at this temperature neutrons and protons are non-relativistic, they obey Boltzmann distribution $n_{n,p}(T) \propto T_n^{3/2} e^{-m/T}$. For the sake of simplicity, we just ignore the mass difference between protons and neutrons here.

In the expression of $\frac{n_n(T_{NS})}{n_p(T_{NS})}$, we see t_{NS} and it depends on g_* in the sense that g_* grows, t_{NS} decreases. We could not make further statement how exactly the fraction will change since the function expression is rather complicated. We need to calculate in next part anyway.

d) First express effective degrees of freedom in terms of ΔN_{eff}

$$g_* = 2 + \frac{7}{8} \cdot (3 + \Delta N_{\text{eff}}) \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} \tag{2.5}$$

g_* dependence enters not only T_n, but also t_{NS}. Thus

$$\begin{aligned}\frac{\mathrm{d}X_{^4\text{He}}}{\mathrm{d}N_{\text{eff}}} &= -\frac{2}{\left(1 + \frac{n_p(T_{NS})}{n_n(T_{NS})}\right)^2} \frac{\mathrm{d}\frac{n_p(T_{NS})}{n_n(T_{NS})}}{\mathrm{d}N_{\text{eff}}} \\ &= -\frac{X^2_{^4\text{He}}}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\mathrm{d}T_n}{\mathrm{d}N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\mathrm{d}t_{NS}}{\mathrm{d}N_{\text{eff}}} \right)\end{aligned}$$

Write d into Δ and rearrange the equation, we have

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} = -\frac{X_{^4\text{He}}}{2} \left(\frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial T_n} \frac{\mathrm{d}T_n}{\mathrm{d}N_{\text{eff}}} + \frac{\partial \frac{n_p(T_{NS})}{n_n(T_{NS})}}{\partial t_{NS}} \frac{\mathrm{d}t_{NS}}{\mathrm{d}N_{\text{eff}}} \right) \Delta N_{\text{eff}} \tag{2.6}$$

It contains four derivatives, thus it is more economical to calculate it with Mathematica. With $T_n \approx 1.4\,\text{MeV}$, $t_{NS} \approx 200\,\text{s}$, $X_{^4\text{He}} \approx 45\%$, $\Delta N_{\text{eff}} = 1$, and $\mu_n = \mu_p$, we get

$$\frac{\Delta X_{^4\text{He}}}{X_{^4\text{He}}} \approx 2.8\% \tag{2.7}$$

3 Neutron burning

a) Given $\langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D} \approx 6 \times 10^{-20}\text{cm}^3\text{s}^{-1}$, $\eta_B = 6.2 \times 10^{-10}$

$$\begin{aligned}\Gamma_{p(n,\gamma)D} &= n_p \cdot \langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D} \\ &= \eta_B n_\gamma \langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D} \\ &= \eta_B (g_\gamma \frac{\zeta(3)}{\pi^2} T^3) \langle\sigma|\vec{v}|\rangle_{p(n,\gamma)D}\end{aligned} \tag{3.1}$$

During freeze out, $T = T_p$,

$$\begin{aligned} H(T_p) &= \Gamma_{p(n,\gamma)D} \qquad (3.2) \\ \frac{T_p^2}{M_{pl}^*} &= \eta_B (g_\gamma \frac{\zeta(3)}{\pi^2} T_p^3) \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D} \\ T_p &= \frac{\pi^2}{M_{pl}^* \eta_B (g_\gamma \zeta(3)) \langle \sigma |\vec{v}| \rangle_{p(n,\gamma)D}} \qquad (3.3) \end{aligned}$$

Assuming that the freeze out temperature is in the order of $\mathcal{O}(10keV)$ or smaller(which is well below the mass of electrons), there should be no relic electrons and positrons because all of them would annihilate to produce photons, as we learned from the our last assignment.

$$g_* = 2 + \frac{7}{8} \cdot 4 + \frac{7}{8} \cdot 3 \cdot 2 \cdot \left(\frac{4}{11}\right)^{4/3} = 3.36 \qquad (3.4)$$

And with $g_\gamma = 2$, $\zeta(3) \approx 1.2025$, $M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}}$, we have

$$T_p = 0.316\,\mathrm{keV} \qquad (3.5)$$

We can see that $T_p \ll T_{NS}$, which make sense because if the protons were to freeze out before T_{NS}, the relic protons wouldnt be in thermal equilibrium to produce Deuterium and consequently allowing BBN.

b) Knowing that the temperature during the matter-radiation equality is about $3\,\mathrm{eV}$, and from what we calcaluted, $T_p \gg 3\,\mathrm{eV}$. the reaction freeze out is during the radiation dominated era. This is also why it make sense to use this as an assumption in the first part.

$$\begin{aligned} H(T_p) &= \frac{1}{2t_p} \qquad (3.6) \\ \frac{T_p^2}{M_{pl}^*} &= \frac{1}{2t_p} \\ t_p &= \frac{M_{pl}^*}{2T_p^2} \\ &= 1.299 \times 10^7\,\mathrm{s} \end{aligned}$$