H9.1

a) $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{m^2}{2} V_\mu V^\mu \quad , \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$

$$\frac{\partial \mathcal{L}}{\partial V_\sigma} = -\frac{m^2}{2} V^\sigma$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\lambda V_\sigma)} = -\frac{1}{4} \frac{\partial}{\partial(\partial_\lambda V_\sigma)} \left[(\partial_\mu V_\nu - \partial_\nu V_\mu) g^{\alpha\mu} g^{\beta\nu} (\partial_\alpha V_\beta - \partial_\beta V_\alpha) \right]$$

$$= -\frac{1}{4} g^{\alpha\mu} g^{\beta\nu} \left[(\delta_{\mu\lambda} \delta_{\sigma\nu} - \delta_{\lambda\nu} \delta_{\sigma\mu})(\partial_\alpha V_\beta - \partial_\beta V_\alpha) + (\partial_\mu V_\nu - \partial_\nu V_\mu)(\delta_{\alpha\lambda} \delta_{\sigma\beta} - \delta_{\lambda\beta} \delta_{\sigma\alpha}) \right]$$

$$= -\frac{1}{4} \left[(g^{\alpha\lambda} g^{\beta\sigma} - g^{\alpha\sigma} g^{\beta\lambda})(\partial_\alpha V_\beta - \partial_\beta V_\alpha) + (\partial_\mu V_\nu - \partial_\nu V_\mu)(g^{\lambda\mu} g^{\sigma\nu} - g^{\sigma\mu} g^{\lambda\nu}) \right]$$

$$= -\frac{1}{4} \left[\partial^\lambda V^\sigma - \partial^\sigma V^\lambda - \partial^\sigma V^\lambda + \partial^\lambda V^\sigma + \partial^\lambda V^\sigma - \partial^\sigma V^\lambda - \partial^\sigma V^\lambda + \partial^\lambda V^\sigma \right]$$

$$= \partial^\sigma V^\lambda - \partial^\lambda V^\sigma$$

$\Rightarrow$ ELE : $\quad \partial_\lambda (\partial^\sigma V^\lambda - \partial^\lambda V^\sigma) - \frac{m^2}{2} V^\sigma = 0$

$$\Rightarrow \quad \left[(\partial^2 + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] V_\nu = 0$$

b) $$\partial_\mu \left\{ \left[(\partial^2 + m^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] V_\nu \right\} = 0$$

$$\partial_\mu \partial_\alpha \partial^\alpha g^{\mu\nu} V_\nu + m^2 g^{\mu\nu} \partial_\mu V_\nu - \partial_\mu \partial^\mu \partial^\nu V_\nu = 0$$

$\rightarrow$ if $m \neq 0$

$$\partial^\nu V_\nu = 0$$

c) $V_\mu = \epsilon_\mu(p)\, e^{-ip\cdot x} \quad \longrightarrow \quad \partial_\mu \left[\epsilon^\mu(p)\, e^{-ipx}\right] = \epsilon^\mu(p)\,(-ip_\mu)\, e^{-ipx} = 0$

$$\longrightarrow \quad p_\mu \epsilon^\mu(p) = 0$$

d) $P = (E, 0, 0, |\vec{p}|)^t$.

$$\longrightarrow \quad p_\mu \epsilon^\mu(p) = \begin{pmatrix} E \\ 0 \\ 0 \\ |\vec{p}| \end{pmatrix}^t \cdot \begin{cases} \frac{1}{m}\begin{pmatrix} |\vec{p}| \\ 0 \\ 0 \\ E \end{pmatrix} \\ \mp \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} \end{cases} = 0$$

$$\sum_\lambda \epsilon^*_\mu(\lambda)\,\epsilon_\nu(\lambda) = \frac{1}{m^2}\begin{pmatrix} |\vec{p}| \\ 0 \\ 0 \\ E \end{pmatrix}^* (|\vec{p}|, 0, 0, E)_{\mu\nu}$$

$$+ \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}^* (0 \ \ 1 \ \ i \ \ 0)_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}^* (0 \ \ 1 \ \ -i \ \ 0)_{\mu\nu}$$

$$= \frac{1}{m^2}\begin{pmatrix} |\vec{p}|^2 & 0 & 0 & E|\vec{p}| \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ E|\vec{p}| & 0 & 0 & E^2 \end{pmatrix}_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 & & & \\ & 1 & +i & \\ & -i & +1 & \\ & & & 0 \end{pmatrix}_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 & & & \\ & 1 & -i & \\ & +i & +1 & \\ & & & 0 \end{pmatrix}_{\mu\nu}$$

$$= \begin{pmatrix} |\vec{p}|^2/m^2 & & & E|\vec{p}|/m^2 \\ & 1 & & \\ & & +1 & \\ E|\vec{p}|/m^2 & & & E^2/m^2 \end{pmatrix}_{\mu\nu}$$

$$-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}_{\mu\nu} + \frac{1}{m^2}\begin{pmatrix} E^2 & & & E|\vec{p}| \\ & 0 & & \\ & & 0 & \\ E|\vec{p}| & & & |\vec{p}|^2 \end{pmatrix}_{\mu\nu}$$

$$\left(E^2/m^2 = (|\vec{p}|^2 + m^2)/m^2 = 1 + |\vec{p}|^2/m^2 \right. \qquad \Longrightarrow (3)$$

$$= \begin{pmatrix} -1 + 1 + |\vec{p}|^2/m^2 & & & E|\vec{p}|/m^2 \\ & 1 & & \\ & & 1 & \\ E|\vec{p}|/m^2 & & & E^2/m^2 \end{pmatrix}_{\mu\nu}$$

e) $$[(-p^2+m^2)g^{\mu\nu}+p^\mu p^\nu]\Delta_{\nu\alpha}=\delta^\mu_\alpha$$

Plug in ansatz $\Delta_{\nu\alpha}=A\,g_{\nu\alpha}+\frac{B}{p^2}p_\nu p_\alpha$

$$[(-p^2+m^2)g^{\mu\nu}+p^\mu p^\nu][A g_{\nu\alpha}+\frac{B}{p^2}p_\nu p_\alpha]$$

$$=A(-p^2+m^2)\delta^\mu_\alpha+(-p^2+m^2)\frac{B}{p^2}p^\mu p_\alpha+A p^\mu p_\alpha+\frac{B}{\cancel{p^2}}p^\mu \cancel{p^\nu p_\nu} p_\alpha$$

$$\stackrel{!}{=}\delta^\mu_\alpha$$

$$\longrightarrow A=\frac{1}{-p^2+m^2}\ ,\qquad (-p^2+m^2)\frac{B}{p^2}p^\mu p_\alpha+\frac{1}{-p^2+m^2}p^\mu p_\alpha+B p^\mu p_\alpha=0$$

$$\longrightarrow (-p^2+m^2)\frac{B}{p^2}+\frac{1}{-p^2+m^2}+B=0$$

$$(-p^2+m^2)^2 B+p^2+Bp^2(-p^2+m^2)=0$$

$$\longrightarrow B=\frac{-p^2}{(-p^2+m^2)^2+(-p^2+m^2)p^2}$$

$$=\frac{-p^2}{-2m^2p^2+m^4+m^2p^2}$$

$$=\frac{-p^2}{-p^2+m^2}\frac{1}{m^2}$$

$$\longrightarrow \Delta_{\nu\alpha}=\frac{1}{-p^2+m^2}g_{\nu\alpha}+\frac{1}{p^2-m^2}\frac{1}{m^2}p_\nu p_\alpha$$

$$=\frac{1}{p^2-m^2}\underbrace{\left(-g_{\nu\alpha}+\frac{1}{m^2}p_\nu p_\alpha\right)}_{=\sum_\lambda \epsilon^*_\nu(\lambda)\epsilon_\alpha(\lambda)}$$

f) During propagating, all polarization states are possible and need to be summed over.

H9.2

a) $B \rightarrow f_1 f_2$

$m_W \sim 80\,GeV$

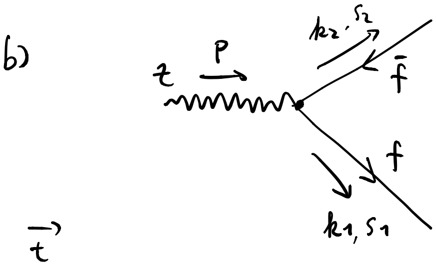
$W^+ \rightarrow e^+ \nu_e,\ \mu^+ \nu_\mu,\ \tau^+ \nu_\tau,\ u\bar{d},\ u\bar{s},\ ub,\ c\bar{d},\ c\bar{s},\ c\bar{b}$

$W^- \rightarrow e^- \bar{\nu}_e,\ \mu^- \bar{\nu}_\mu,\ \tau^- \bar{\nu}_\tau,\ \bar{u}d,\ \bar{u}s,\ \bar{u}b,\ \bar{c}d,\ \bar{c}s,\ \bar{c}b$

$m_Z \sim 90\,GeV$

$Z \rightarrow e^-e^+,\ \mu^-\mu^+,\ \tau^-\tau^+,\ \bar{u}u,\ \bar{d}d,\ \bar{c}c,\ \bar{s}s,\ \bar{b}b,\ \bar{u}c,\ u\bar{c},\ \bar{d}s,\ d\bar{s},\ \bar{d}b,\ d\bar{b}$
$\bar{s}b,\ s\bar{b}$

b)



c) $$i\mathcal{M} = \epsilon^\mu(\vec{p},\lambda)\,\bar{u}(k_1)\left[-i\frac{g}{\cos\theta_W}\gamma_\mu\left(\frac{c_V\mathbb{1} - c_A\gamma^5}{2}\right)\right]v(k_2)$$

$$= \frac{-ig}{2\cos\theta_W}\epsilon^\mu(\vec{p},\lambda)\,\bar{u}(k_1)\gamma_\mu(c_V\mathbb{1} - c_A\gamma^5)\,v(k_2)$$

d) $$\sum_{s_1,s_2}\frac{-ig}{2\cos\theta_W}\,\epsilon^\mu(\vec{p},\lambda)\,\bar{u}^{s_1}(k_1)\gamma_\mu(c_V\mathbb{1} - c_A\gamma^5)\,v^{s_2}(k_2)$$

e) $$\overline{|\mathcal{M}|^2} = \frac{1}{3}\sum_\lambda\sum_{s_1,s_2}\left|\frac{-ig}{2\cos\theta_W}\right|^2\epsilon^\mu(\vec{p},\lambda)\,\epsilon^{*\nu}(\vec{p},\lambda)\;\bar{u}^{s_1}(k_1)\gamma_\mu(c_V\mathbb{1} - c_A\gamma^5)\,v^{s_2}(k_2)$$

$$\times\ \bar{v}^{s_2}(k_2)\,\gamma_\nu\,(c_V\mathbb{1} - c_A\gamma^5)\,u^{s_1}(k_1)$$

$$= \frac{g^2}{12\cos^2\theta_W}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M_Z^2}\right)$$

$$\times \sum_{s_1,s_2} tr\left[\bar{u}^{s_1}(k_1)\gamma_\mu (c_V \mathbb{1} - c_A\gamma^5) v^{s_2}(k_2)\bar{v}^{s_2}(k_2)\gamma_\nu (c_V\mathbb{1} - c_A\gamma^5) u^{s_1}(k_1)\right]$$

$$(m_f = 0) \quad \hat{=} \frac{g^2}{12\cos^2\theta_W}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M_Z^2}\right) tr\left[\not{k}_1 \gamma_\mu (c_V\mathbb{1} - c_A\gamma^5)\not{k}_2\gamma_\nu \underbrace{(c_V\mathbb{1}-c_A\gamma^5)}\right]$$

$$(c_V\mathbb{1} - c_A\gamma^5)^2 = c_V^2 + c_A^2 \underbrace{-2c_Vc_A\gamma^5}_{\to 0\ ?}$$

$$= \frac{g^2}{12\cos^2\theta_W}\left(-g^{\mu\nu} + \underbrace{\frac{p^\mu p^\nu}{M_Z^2}}_{=0,\ \text{in cms}}\right)(c_V^2 + c_A^2)\, tr[\not{k}_1\gamma_\mu \not{k}_2\gamma_\nu]$$

f) $tr[\not{k}_1\gamma_\mu\not{k}_2\gamma_\nu]$

$$= 4k_1^\alpha k_2^\beta \left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu} + g_{\alpha\nu}g_{\mu\beta}\right)$$

$$= 4\left[k_{1\mu}k_{2\nu} - (k_1\cdot k_2)g_{\mu\nu} + k_{1\nu}k_{2\mu}\right]$$

$$\to \overline{|\mathcal{M}|^2} = \frac{-4g^2}{12\cos^2\theta_W}(c_V^2 + c_A^2)\left[(k_1\cdot k_2) - 4(k_1\cdot k_2) + (k_1\cdot k_2)\right]$$

$$= \frac{2g^2}{3\cos^2\theta_W}(c_V^2+c_A^2)\left(E^2 + |\vec{p}_{cm}|^2\right) \qquad 2E = M_Z = 2p_{cm}$$

$$\hat{=} \frac{4g^2}{3\cos^2\theta_W}(c_V^2 + c_A^2)|\vec{p}_{cm}|^2$$

$$\to \frac{d\Gamma}{d\Omega} = \frac{p_{cm}}{32\pi^2 M_Z^2}\cdot\frac{4g^2}{3\cos^2\theta_W}(c_V^2+c_A^2)\,p_{cm}^2$$

$$= \frac{g^2}{8\pi^2 M_Z^2}\,\frac{1}{3\cos^2\theta_W}(c_V^2+c_A^2)\cdot\frac{1}{8}M_Z^3$$

$$\to \Gamma = 4\pi\frac{d\Gamma}{d\Omega} \qquad (\text{no } \theta,\varphi \text{ dependence})$$

$$= \frac{g^2}{48\pi\cos^2\theta_W}(c_V^2 + c_A^2)M_Z$$

g) $g^2 = 0.4,\quad \sin^2\theta_W = \frac{1}{4} \Leftrightarrow \cos^2\theta_W = \frac{3}{4},\quad M_Z = 90\,GeV$

$$\to \Gamma = 0.32\ GeV\cdot(c_V^2 + c_A^2)$$

$\Gamma(Z\to ee) = 79.9\ MeV \quad , \quad \Gamma(Z\to u\bar{u}) = 3\cdot 91\ MeV = 273\ MeV$

$\Gamma(Z\to d\bar{d}) = 349\ MeV$

$\rightarrow \Gamma_Z^{visible} = 3 \cdot (79.9 + 273 + 349)\ \mathrm{MeV} = 2.1\ \mathrm{GeV}$

h) Discrepancy : decay into neutrinos

For ν: $(c_A^2 + c_V^2) = \frac{1}{2}$

$\rightarrow \Gamma(Z \rightarrow \nu\bar{\nu}) = 0.96\ \mathrm{GeV}$

$\rightarrow \Delta\Gamma \stackrel{\wedge}{=} 300\ \mathrm{GeV} \approx 160\ \mathrm{GeV} \cdot 3$