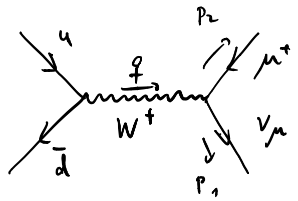
H10.1

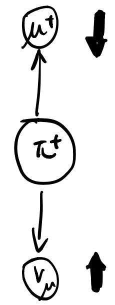
Chenluan Wang

a) $\underbrace{\pi^+}_{=u\bar{d}}(q) \to \nu_\mu(p_1) + \mu^+(p_2)$

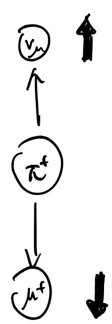


$\rightarrow$

In CMS of π^+



b) $\xrightarrow{P}$



P doesn't change direction of spin.

There is no right-handed neutrinos in SM.

c) d) Only axial part

Dirac eq. $\quad (i\not{\partial} - m)\psi = 0$

ν_μ : $\quad \gamma_\mu p_1^\mu u(p_1) = 0 \quad , \quad \bar{u}(p_1)\gamma_\mu p_1^\mu = 0$

μ^+ : $\quad (\gamma_\mu p_2^\mu - m_\mu) v(p_2) = 0$

$\Rightarrow \bar{u}(p_1) p_1^\mu \gamma_\mu = 0$.

$p_2^\mu \gamma_\mu v(p_2) = -i m_\mu v(p_2) \rightarrow i\mathcal{M} \sim m_\mu \bar{u}(p_1) v(p_2)$

$\cdots \quad \Rightarrow (p_1+p_2)^\alpha \gamma_\alpha$ (as factor) vanishes

$$\Rightarrow \mathcal{M} = i\frac{G_F}{\sqrt{2}} V_{ud}^{+} f_\pi \, m_\mu \, \bar{u}(p_1)(\mathbb{1}_4 - \gamma_5) v(p_2)$$

$$\rightarrow \mathcal{M}^* = \mathcal{M}^+ \propto [\bar{u}(\mathbb{1} - \gamma_5) v]^+$$
$$= v^+ (\mathbb{1} - \gamma_5)^+ \gamma_0^+ u$$
$$= \bar{v}(\mathbb{1} + \gamma_5) u$$

$$\Rightarrow \overline{|\mathcal{M}|^2} = \sum_{\text{spins}} \mathcal{M}\mathcal{M}^+$$
$$= \sum_{\text{spins}} \frac{G_F^2}{2} |V_{ud}|^2 f_\pi^2 m_\mu^2 \, \bar{u}(p_1)(\mathbb{1} - \gamma_5) v(p_2) \bar{v}(p_2)(\mathbb{1} + \gamma_5) u(p_1)$$
$$= \frac{G_F^2}{2} |V_{ud}|^2 f_\pi^2 m_\mu^2 \underbrace{\sum_{\text{spins}} \mathrm{tr}[\bar{u}(p_1)(\mathbb{1} - \gamma_5) v(p_2) \bar{v}(p_2)(\mathbb{1} + \gamma_5) u(p_1)]}$$

$$= \mathrm{tr}[\not{p}_1 (\mathbb{1} - \gamma_5)(\not{p}_2 + m_\mu)(\mathbb{1} + \gamma_5)]$$
$$= \mathrm{tr}[\underbrace{(\mathbb{1} + \gamma_5)} \not{p}_1 (\not{p}_2 + m_\mu) \underbrace{(\mathbb{1} + \gamma_5)}]$$
$$= 2\,\mathrm{tr}[\not{p}_1 (\not{p}_2 + m_\mu)(\mathbb{1} + \gamma_5)]$$
$$= 2\,\mathrm{tr}[\not{p}_1 (\not{p}_2 + \underbrace{m_\mu}_{=0} + \underbrace{\not{p}_2 \gamma_5}_{=0} + \underbrace{m_\mu \gamma_5}_{=0})]$$
$$= 8\,(p_1 \cdot p_2)$$

$$= 4 G_F^2 |V_{ud}|^2 f_\pi^2 m_\mu^2 (p_1 \cdot p_2)$$

e) Plug in the CMS momenta:

$$|\mathcal{M}|^2 = 4 G_F^2 |V_{ud}|^2 f_\pi^2 m_\mu^2 (E|\vec{p}_1| + |\vec{p}_1|^2)$$

Two-body decay must have discrete spectrum!

$$q = p_1 + p_2 \quad \Rightarrow \quad m_\pi = |\vec{p}_1| + E$$
$$\rightarrow \quad E^2 = |\vec{p}_1|^2 + m_\mu^2$$
$$\Leftrightarrow \quad (m_\pi - |\vec{p}_1|)^2 = |\vec{p}_1|^2 + m_\mu^2$$
$$\Rightarrow \quad m_\pi^2 - 2m_\pi |\vec{p}_1| = m_\mu^2$$
$$\rightarrow \quad |\vec{p}_1| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi},$$

f) $\lambda(m_\pi^2, 0, m_\mu^2) = m_\pi^4 + m_\mu^4 - 2 m_\pi^2 m_\mu^2$

$$\Rightarrow \Gamma(\pi^+ \to \nu_\mu \mu^+) = \frac{1}{16\pi m_\pi^3} (m_\pi^4 + m_\mu^4 - 2 m_\pi^2 m_\mu^2)^{\frac{1}{2}} \cdot 4 G_F^2 |V_{ud}|^2 f_\pi^2 m_\mu^2$$

$$\times \frac{m_\pi^2 - m_\mu^2}{2 \cancel{m_\pi}} \cdot \cancel{m_\pi}$$

$$= \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} \cdot \frac{m_\mu^2}{m_\pi^3} \underbrace{(m_\pi^2 - m_\mu^2)^2}_{= m_\pi^4 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2}$$

$$= \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\mu^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

g) $\Gamma_\mu / \Gamma_e \sim 10^4$

h) Dirac - Basis

General free solution with $m=0$

$$u(p) = N \begin{pmatrix} \chi^{(s)} \\ \frac{\vec{\sigma} \cdot \hat{\vec{p}}}{E} \chi^{(s)} \end{pmatrix}$$

$$\gamma^5 u(p) = N \begin{pmatrix} \frac{\vec{\sigma} \cdot \vec{p}}{E} \chi^{(s)} \\ \chi^{(s)} \end{pmatrix} = N \begin{pmatrix} \vec{\sigma} \cdot \hat{\vec{p}} \chi^{(s)} \\ \chi^{(s)} \end{pmatrix}$$

(since $(\vec{\sigma} \cdot \hat{\vec{p}})^2 = \hat{\vec{p}}^2 \cdot \mathbb{1} = \mathbb{1}$

$$= \vec{\sigma} \cdot \hat{p}\; u(p)$$

vertex in (1) $\quad (\mathbb{1} - \gamma_5) \propto P_L \to$ left-handed particle

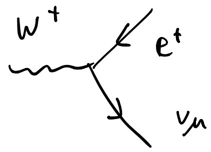
($\gamma_\mu P_L = P_R \gamma_\mu$

and $\bar{u} P_R$ is a left-handed particle)

i) $P_L \nu \rightarrow$ left-handed (massive) particle

There is no right-handed neutrino.

(j)



(in this kind of vertex)

From last part, $W^{\pm}$ couples to left-handed particles only.

? They're indeed left-handed as in the vertex.

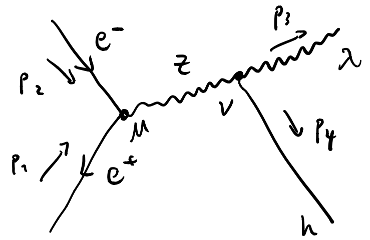
k) $P_R = \frac{1}{2}(\mathbb{1} + \gamma_5)$

$$P_R \, v_L(p) = 0$$

H 10.2

Why no $e^+e^- \to Zh$ in LHC? LHC: pp-collider!

a)



b)

$$i\mathcal{M} = -\frac{ig}{2\cos(\theta_W)}\, \bar{v}(p_1)\,\gamma^\mu (C_V^e \mathbb{1} - C_A^e \gamma^5)\, u(p_2)\, \frac{1}{s - M_Z^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2}\right)$$

$$\times \frac{i g m_Z}{\cos(\theta_W)}\, g^{\nu\lambda}\, \epsilon_\lambda(p_3)$$

$$= \frac{g^2 m_Z}{2\cos^2\theta_W}\frac{1}{s - M_W^2}\, \bar{v}(p_1)\gamma^\mu (C_V^e \mathbb{1} - C_A^e \gamma^5) u(p_2)\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2}\right)\epsilon^\nu(p_3)$$

c)

Dirac equation with $m_e \simeq 0$

$$\not{p}_2\, u(p_2) = 0, \quad \bar{v}(p_1)\not{p}_1 = 0$$

$$\rightarrow \quad i\mathcal{M} = \frac{-g^2 m_Z}{2\cos^2\theta_W}\frac{1}{s - M_W^2}\, \bar{v}(p_1)\gamma^\mu (C_V^e \mathbb{1} - C_A^e \gamma^5) u(p_2)\, g_{\mu\nu}\epsilon^\nu(p_3)$$

d)

$$\overline{|\mathcal{M}|^2} = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2$$

$$= \frac{1}{4}\sum_{\text{spins}} \frac{g^4 m_Z^2}{4\cos^4\theta_W}\frac{1}{(s - M_W^2)^2}\cdot \bar{v}\gamma^\mu (C_V^e \mathbb{1} - C_A^e\gamma^5) u\, \epsilon_\mu \cdot \epsilon_\mu^* \,\bar{u}\gamma^\nu (C_V^e \mathbb{1} - C_A^e \gamma^5) v$$

$$= \frac{g^4 m_Z^2}{16\cos^4\theta_W (s - M_W^2)^2}\, tr\Big[\sum_{\text{spins}} \bar{v}\gamma^\mu (C_V^e \mathbb{1} - C_A^e\gamma^5) u\, \epsilon_\mu \epsilon_\nu^*\, \bar{u}\gamma^\nu (C_V^e \mathbb{1} - C_A^e\gamma^5) v\Big]$$

$$= \frac{g^4 m_Z^2}{16\cos^4\theta_W (s - M_W^2)^2}\sum_{\text{Polarisation}} \epsilon_\nu^* \epsilon_\mu\, tr\Big[\sum_{\text{spins}} v\bar{v}\,\gamma^\mu (C_V^e \mathbb{1} - C_A^e\gamma^5)\, u\bar{u}\,\gamma^\nu (C_V^e \mathbb{1} - C_A^e\gamma^5)\Big]$$

$$= \frac{g^4 m_Z^2}{16\cos^4\theta_W (s - M_W^2)^2}\sum_{\text{Polarisation}} \epsilon_\nu^* \epsilon_\mu\, tr\Big[\not{p}_1\gamma^\mu \underbrace{(C_V^e \mathbb{1} - C_A^e\gamma^5)}_{\cdot(-1)^2} \not{p}_2\gamma^\nu (C_V^e \mathbb{1} - C_A^e\gamma^5)\Big]$$

Focus on the trace

$$= \mathrm{tr}\{\not p_1 \gamma^\mu \not p_2 \gamma^\nu [(C_V^e)^2 - 2C_V^e C_A^e \gamma^5 + (C_A^e)^2]\}$$

$$= 4[(C_V^e)^2 + (C_A^e)^2](p_1)_\alpha (p_2)_\beta (g^{\alpha\mu} g^{\beta\nu} - g^{\alpha\beta} g^{\mu\nu} + g^{\alpha\nu} g^{\mu\beta})$$
$$-8iC_V^e C_A^e (p_1)_\alpha (p_2)_\beta \, \epsilon^{\alpha\mu\beta\nu}$$

e)

For $\theta \neq 0$:

The equation for polarisation vectors

$$p_\mu \epsilon^\mu(p) = 0 , \qquad p^\mu = (E_Z, \vec{p})^\mu = (E_Z, 0, |\vec{p}|\sin\theta, |\vec{p}|\cos\theta)$$
$$p_\mu = (E_Z, -\vec{p})^\mu = (E_Z, 0, -|\vec{p}|\sin\theta, -|\vec{p}|\cos\theta)$$

$$(E_Z, 0, -|\vec{p}|\sin\theta, -|\vec{p}|\cos\theta) \cdot \epsilon^\mu(\lambda) \overset{!}{=} 0$$

$$\rightsquigarrow \quad \epsilon^\mu(\lambda=0) = \frac{1}{m_Z}\begin{pmatrix} |\vec{p}| \\ 0 \\ E_Z\sin\theta \\ E_Z\cos\theta \end{pmatrix} \xrightarrow{\theta=0} \frac{1}{m_Z}\begin{pmatrix} |\vec{p}| \\ 0 \\ 0 \\ E_Z \end{pmatrix}$$

$$\rightarrow \epsilon^\mu(\lambda=\pm 1) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix}$$

$$\sum_\lambda \epsilon_\mu^* \epsilon_\nu$$

$$= \frac{1}{m_Z^2}\left[\begin{pmatrix} |\vec{p}_3| \\ 0 \\ -E_Z\sin\theta \\ -E_Z\cos\theta \end{pmatrix}^* (|\vec{p}_3|, 0, -E_Z\sin\theta, -E_Z\cos\theta)\right]_{\mu\nu} + \frac{1}{2}\left[\begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}^* (0, 1, i, 0)\right]_{\mu\nu}$$

$$+ \frac{1}{2}\left[\begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}^* (0, 1, -i, 0)\right]_{\mu\nu}$$

$$= \frac{1}{m_Z^2}\begin{pmatrix} |\vec{p}_3|^2 & 0 & -|\vec{p}_3|E_Z\sin\theta & -|\vec{p}_3|E_Z\cos\theta \\ 0 & 0 & 0 & 0 \\ -|\vec{p}_3|E_Z\sin\theta & 0 & E_Z^2\sin^2\theta & E_Z^2\sin\theta\cos\theta \\ -|\vec{p}_3|E_Z\cos\theta & |\vec{p}_3|E_Z\sin\theta & E_Z^2\sin\theta & E_Z^2\cos^2\theta \end{pmatrix}_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & i & 0 \\ 0 & -i & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -i & 0 \\ 0 & i & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}_{\mu\nu}$$

$$
= \frac{1}{m_Z^2} \begin{pmatrix} |\vec{p}_3|^2 & 0 & -|\vec{p}_3| E_Z \sin\theta & -|\vec{p}_3| E_Z \cos\theta \\ 0 & m_Z^2 & 0 & 0 \\ -|\vec{p}_3| E_Z \sin & 0 & E_Z^2 \sin^2\theta + m_Z^2 & E_Z^2 \sin\theta \cos\theta \\ -|\vec{p}_3| E_Z \cos\theta & |\vec{p}_3| E_Z \sin\theta & E_Z^2 \sin\theta \cos\theta & E_Z^2 \cos^2\theta \end{pmatrix}_{\mu\nu}
$$

f) $\sum_\lambda \epsilon(\lambda)^*_\mu \, \epsilon(\lambda)_\nu = -g_{\mu\nu} + \frac{1}{m_Z^2} (p_3)_\mu (p_3)_\nu$

$$
= -\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} + \frac{1}{m_Z^2} \left[\begin{pmatrix} E_Z \\ 0 \\ -|\vec{p}_3| \sin\theta \\ -|\vec{p}_3| \cos\theta \end{pmatrix} \left(E_Z, \; 0, \; -|\vec{p}_3| \sin\theta, \; -|\vec{p}_3| \cos\theta \right) \right]_{\mu\nu}
$$

$$
= \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} + \frac{1}{m_Z^2} \begin{pmatrix} E_Z^2 & 0 & -E_Z |\vec{p}_3| \sin\theta & -E_Z |\vec{p}_3| \cos\theta \\ 0 & 0 & 0 & 0 \\ -E_Z |\vec{p}_3| \sin\theta & 0 & |\vec{p}_3|^2 \sin^2\theta & |\vec{p}_3|^2 \sin\theta \cos\theta \\ -E_Z |\vec{p}_3| \cos\theta & 0 & |\vec{p}_3|^2 \sin\theta \cos\theta & |\vec{p}_3|^2 \cos^2\theta \end{pmatrix}_{\mu}
$$

$$
= \frac{1}{m_Z^2} \left(\begin{matrix} |\vec{p}_3|^2 & 0 & -E_Z |\vec{p}_3| \sin\theta & -E_Z |\vec{p}_3| \cos\theta \\ 0 & m_Z^2 & 0 & 0 \\ -E_Z |\vec{p}_3| \sin\theta & 0 & |\vec{p}_3|^2 \sin^2\theta + m_Z^2 & \\ & & & \end{matrix} \right.
$$

?

h) $\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v+h+iA^0) \end{pmatrix}$

$$\rightarrow \quad (D_\mu \Phi)^\dagger D^\mu \Phi = \left((D_\mu \phi^+)^*, \frac{1}{\sqrt{2}} D_\mu (v+h+iA^0) \right) \begin{pmatrix} D^\mu \phi^+ \\ \frac{1}{\sqrt{2}} D^\mu (v+h+iA^0) \end{pmatrix}$$

$$= (D_\mu \phi^+)^* (D^\mu \phi^+) + \frac{1}{2} D_\mu (v+h+iA^0) D^\mu (v+h+iA^0)$$

$$D_\mu = \partial_\mu + \frac{i}{2} g' B_\mu - \frac{ig}{2} \tau^i W_\mu^i$$

$$B_\mu = \cos\theta_W A_\mu + \sin\theta_W Z_\mu, \quad W_\mu^3 = \sin\theta_W A_\mu - \cos\theta_W Z_\mu$$

$$\rightarrow \quad D_\mu \supset \frac{i}{2} g' \sin\theta_W Z_\mu + \frac{ig}{2} g \, \tau^3 \cos\theta_W Z_\mu$$

H 10.3

a) $$\det[\Lambda] = \det[1 + iL + \mathcal{O}(L^2)] \overset{!}{=} 1$$

$$\Lambda^* \Lambda \equiv \mathbb{1}$$

$\Rightarrow$ L is a $N\times N$ matrix $\rightarrow N^2$

Because of $\det(\Lambda) \overset{!}{=} 1 \rightarrow N^2-1$

b) $$ad(T_i)\, T_j = [T_i, T_j] = T_i T_j - T_j T_i = i f_{ij}^{\ k}\, T_k$$

$\rightarrow \dim[\,ad(T_i)] = N$

$\rightarrow ad(T_i)_j^{\ k}\, T_k = i f_{ij}^{\ k}\, T_k \quad \rightarrow ad(T_i)_j^{\ k} = i f_{ij}^{\ k}$

c) $$ad(J_i)_{jk} = i\,\epsilon_{ijk}$$

$$\Rightarrow \quad ad(J_i) = i \begin{pmatrix} 0 & \delta_{i3} & -\delta_{i2} \\ -\delta_{i3} & 0 & \delta_{i1} \\ \delta_{i2} & -\delta_{i1} & 0 \end{pmatrix}$$

3 Generators. $[J_i, J_j] = i\epsilon_{ijk} J_k$

$$\Rightarrow \quad J_i = \frac{1}{\sqrt{2}}\sigma_i \quad \Rightarrow \quad \frac{1}{2}[\sigma_i, \sigma_j] = \frac{1}{2}\cdot 2i\,\epsilon_{ijk}\,\sigma_k = i\epsilon_{ijk}\sigma_k$$

One-dimensinal: $J_i \equiv 1$

d) $$3 \oplus 2 = 5$$

e) $$J_\pm = \frac{1}{\sqrt{2}}(J_1 \pm iJ_2)\,, \qquad J_3$$

$$\rightarrow [J_+, J_+] = [J_-, J_-] = 0$$

$$\begin{aligned}[J_+, J_-] = -[J_-, J_+] &= \frac{1}{2}[(J_1 + iJ_2), (J_1 - iJ_2)] \\ &= \frac{i}{2}\left(-[J_1, J_2] + [J_2, J_1]\right) \\ &= -i \cdot i J_3 \\ &= J_3\end{aligned}$$

$$[J_3, J_+] = \frac{1}{\sqrt{2}}\left([J_3, J_1] + i[J_3, J_2]\right) = \frac{1}{\sqrt{2}}\left(iJ_2 + i\cdot i\cdot(-J_1)\right) = \frac{1}{\sqrt{2}}(iJ_2 + J_1) = J_+$$

$$[J_3, J_-] = \dots = J_-$$