H.7.1

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a) $\mathcal{L}_{GKH} = (D_\mu \Phi)^\dagger D^\mu \Phi$

$$= \frac{1}{2}\left[\left(\partial_\mu - \frac{i}{2} g W_\mu^a \sigma^a - \frac{i}{2} g' \overset{=1}{Y_\Phi} B_\mu\right)\begin{pmatrix}0\\ v+h\end{pmatrix}\right]^\dagger$$
$$\times \left[\left(\partial^\mu - \frac{i}{2} g W^{a\mu} \sigma^a - \frac{i}{2} g' \underset{=1}{Y_\Phi} B^\mu\right)\begin{pmatrix}0\\ v+h\end{pmatrix}\right]$$

mass term of field $\phi \sim \phi^2$, so no derivatives and other fields

$$W_\mu^a \sigma^a = \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix}$$

$$W_\mu^a \sigma^a \Phi = \begin{pmatrix} (W_\mu^1 - iW_\mu^2)(v+h) \\ -W_\mu^3 (v+h) \end{pmatrix},$$

$\Rightarrow$ mass terms in $\mathcal{L}$

$$\mathcal{L} \subset \frac{1}{2}\left|\frac{i}{2}\right|^2 v^2 \left[(g'B_\mu - gW_\mu^3)(g'B^\mu - gW^{3\mu}) + g^2|W_\mu^1 - iW_\mu^2|^2\right]$$

$$= \frac{v^2}{8}\left[(g'B_\mu - gW_\mu^3)(g'B^\mu - gW^{3\mu}) + g^2(W^1)^2 + g^2(W^2)^2\right]$$

b) $Q = T_3 + \frac{1}{2} Y$

$$[\underset{\frac{1}{2}\sigma^3}{T_3}, W_\mu^1 \sigma^1] = \frac{1}{2}\left[\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right] W_\mu^1$$
$$= \frac{1}{2}\left[\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} - \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\right] W_\mu^1$$
$$= W_\mu^1 \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

$$[T_3, W_\mu^2 \sigma^2] = \frac{1}{2} W_\mu^2 \left[\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\right]$$
$$= \frac{1}{2} W_\mu^2 \left[\begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix} - \begin{pmatrix}0 & i\\ i & 0\end{pmatrix}\right]$$
$$= -iW_\mu^2 \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$[T_3, W_\mu^3 \sigma^3] = 0$$

Define $\quad W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$

$$\Rightarrow [T_3, \frac{1}{\sqrt{2}}(W_\mu^1\sigma^1 - iW_\mu^2\sigma^2)]$$
$$= \frac{1}{\sqrt{2}}\left[W_\mu^1 \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix} - i W_\mu^2 \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}\right]$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^1 - iW_\mu^2 \\ -W_\mu^1 + iW_\mu^2 & 0\end{pmatrix} = W_\mu^+ \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix} \longrightarrow Q = \pm 1$$

$$\Rightarrow [T_3, \frac{1}{\sqrt{2}} W_\mu^1\sigma^1 + iW_\mu^2\sigma^2]$$
$$= \frac{1}{\sqrt{2}}\left[W_\mu^1 \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix} + i W_\mu^2 \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}\right]$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & W_\mu^1 + iW_\mu^2 \\ -W_\mu^1 - iW_\mu^2 & 0\end{pmatrix} = -W_\mu^- \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix} \longrightarrow Q = -1$$

c) $\quad [T^3, W_\mu^3\sigma^3] = [T^3, B_\mu] = 0 \longrightarrow T_3(W_\mu) = T_3(B_\mu) = 0$

$Y(B_\mu) = Y(W_\mu^3) = 0$? $\quad$ Is it definition or ?

$$\mathcal{L} \subset \frac{v^2}{8}(g'B_\mu - gW_\mu^3)(g'B^\mu - gW^{3\mu})$$
$$= \frac{v^2}{8}(W_\mu^3, B_\mu)\underbrace{\begin{pmatrix} g'^2 & gg' \\ gg' & g^2\end{pmatrix}}_{=A}\begin{pmatrix} W_\mu^3 \\ B_\mu\end{pmatrix}$$

$$D = U^{-1}AU, \qquad \begin{vmatrix} g'^2-\lambda & gg' \\ gg' & g^2-\lambda\end{vmatrix} = (g'^2-\lambda)(g^2-\lambda) - g^2g'^2$$
$$= \lambda^2 - (g^2+g'^2)\lambda \overset{!}{=} 0$$

$$\Rightarrow \lambda = 0 \quad \text{or} \quad \lambda = g^2 + g'^2$$

$$\rightarrow (A - 0\cdot\mathbb{1}) = A = \begin{pmatrix} g'^2 & gg' \\ gg' & g^2\end{pmatrix}, \rightarrow \begin{pmatrix} g \\ -g'\end{pmatrix}$$
$$(A - (g^2+g'^2)\mathbb{1}) = \begin{pmatrix} -g^2 & gg' \\ gg' & -g'^2\end{pmatrix}, \rightarrow \begin{pmatrix} g' \\ g\end{pmatrix}$$
$$\rightarrow U = \begin{pmatrix} g & g' \\ -g' & g\end{pmatrix}, \quad U^{-1} = \frac{1}{g\cdot g + g'g'}\begin{pmatrix} g & -g' \\ g' & g\end{pmatrix}$$

$$= \frac{v^2}{8} \underbrace{\left(W_\mu^3 \quad B_\mu\right) \begin{pmatrix} g & g' \\ -g' & g \end{pmatrix}}_{= \left(g W_\mu^3 - g' B_\mu \quad g' W_\mu^3 + g B_\mu\right)} \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}}_{\begin{pmatrix} g W_\mu^3 - g' B_\mu \\ g' W_\mu^3 + g B_\mu \end{pmatrix}} \cdot \frac{1}{g^2+g'^2}$$

$$= \frac{v^2}{8} \frac{1}{\sqrt{g^2+g'^2}} \left(g W_\mu^3 - g' B_\mu \quad g' W_\mu^3 + g B_\mu\right) \begin{pmatrix} g^2+g'^2 & \\ & 0 \end{pmatrix} \frac{1}{\sqrt{g^2+g'^2}} \begin{pmatrix} g W_\mu^3 - g' B_\mu \\ g' W_\mu^3 + g B_\mu \end{pmatrix}$$

(Define $\tan\theta_W = \frac{g'}{g}$

$\rightarrow \sin\theta_W = \frac{g'}{\sqrt{g'^2+g^2}}$, $\cos\theta_W = \frac{g}{\sqrt{g'^2+g^2}}$

$$= \frac{v^2}{8} \left(\cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \quad \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu\right) \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix}$$
$$\times \begin{pmatrix} \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu \\ \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu \end{pmatrix}$$

(Define
$$Z_\mu = -\cos\theta_W W_\mu^3 + \sin\theta_W B_\mu$$
$$A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$$
$\leftarrow$ The overall sign is chosen according to lecture (It is quadratic in field anyway)

$$= \frac{v^2}{8} \left(Z_\mu \quad A_\mu\right) \begin{pmatrix} g^2+g'^2 & \\ & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

$\rightarrow$ $m_A = 0$ (photon !)

$m_Z = \frac{v}{2}\sqrt{g^2+g'^2}$

d) Invert $Z_\mu(W_\mu^3, B_\mu)$
$A_\mu(W_\mu^3, B_\mu)$

$$-\cos\theta_W Z_\mu + \sin\theta_W A_\mu = W_\mu^3 \underbrace{- \sin\theta_W \cos\theta_W B_\mu + \sin\theta_W \cos\theta_W B_\mu}_{=0}$$

$$+\sin\theta_W Z_\mu + \cos\theta_W A_\mu = B_\mu + \ldots$$

$$\mathcal{L}_{GKL} = i\bar{L}_j D_\mu \gamma^\mu L_j + i\bar{R}_j D_\mu \gamma^\mu R_j$$

$$= i\bar{L}_j\left(\partial_\mu - \frac{i}{2} g W_\mu^a \sigma^a + \frac{i}{2} g' B_\mu\right)\gamma^\mu L_j$$

$$+ i\bar{R}_j\left(\partial_\mu + i g' B_\mu\right)\gamma^\mu R_j$$

interaction between Leptons and gauge bosons

$$W_\mu^a \sigma^a = \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} = \begin{pmatrix} \sin\theta_W A_\mu - \cos\theta_W Z_\mu & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -\sin\theta_W A_\mu + \cos\theta_W Z_\mu \end{pmatrix}$$

$$\frac{1}{2}\bar{L}_j g W_\mu^a \sigma^a \gamma^\mu L_j - \frac{1}{2}\bar{L}_j g' B_\mu \gamma^\mu L_j - \bar{R}_j g' B_\mu \gamma^\mu R_j$$

$$= \frac{g}{2}\bar{L}_j \begin{pmatrix} \sin\theta_W A_\mu - \cos\theta_W Z_\mu & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -\sin\theta_W A_\mu + \cos\theta_W Z_\mu \end{pmatrix}\gamma^\mu L_j$$

$$- \frac{g'}{2}\bar{L}_j(\sin\theta_W Z_\mu + \cos\theta_W A_\mu)\gamma^\mu L_j - g'\bar{R}_j(\sin\theta_W Z_\mu + \sin\theta_W A_\mu)\gamma^\mu R_j$$

e) $e_L = U_e \hat{e}_L \qquad e_R = K_L \hat{e}_R$

flavour ES $\qquad$ mass ES

$\nu_L = U_L \hat{\nu}_L$

$$\rightarrow \mathcal{L}_{GKL} = \frac{g}{2}\left((\bar{\hat{\nu}}_L)_j U_e^\dagger \quad (\bar{\hat{e}}_L)_j U_L^\dagger\right)\begin{pmatrix} \sin\theta_W A_\mu - \cos\theta_W Z_\mu & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -\sin\theta_W A_\mu + \cos\theta_W Z_\mu \end{pmatrix}\gamma^\mu \begin{pmatrix} U_L(\hat{\nu}_L)_j \\ U_e(\hat{e}_L)_j \end{pmatrix}$$

$$- \frac{g'}{2}\left((\bar{\hat{\nu}}_L)_j \cancel{U_L^\dagger} \quad (\bar{\hat{e}}_L)_j \cancel{U_e^\dagger}\right)(\sin\theta_W Z_\mu + \cos\theta_W A_\mu)\gamma^\mu \begin{pmatrix} \cancel{U_e}(\hat{\nu}_L)_j \\ \cancel{U_e}(\hat{e}_L)_j \end{pmatrix}$$

$$- g'(\bar{\hat{e}}_R)_j \cancel{K_L^\dagger}(\sin\theta_W Z_\mu + \sin\theta_W A_\mu)\gamma^\mu \cancel{K_L}(\hat{e}_R)_j$$

U_e and K_e are unitary ($\bar{\hat{e}}_{L/R}\hat{e}_{L/R} = \bar{e}_{L/R} e_{L/R}$)

Focus on $W^\pm$ interactions

$$\mathcal{L} \supset \frac{g}{2}\left((\bar{\hat{\nu}}_L)_j U_e^\dagger \quad (\bar{\hat{e}}_L)_j U_e^\dagger\right)\begin{pmatrix} W^+\gamma^\mu U_e(\hat{e}_L)_j \\ W^-\gamma^\mu U_e(\hat{\nu}_L)_j \end{pmatrix}$$

$$= \frac{g}{2} (\hat{\bar{\nu}}_L)_j U_\ell^\dagger W^+ \gamma^\mu U_\ell (\hat{e}_L)_j + \frac{g}{2} (\hat{\bar{e}}_L)_j U_\ell^\dagger W^- \gamma^\mu U_\ell (\hat{\nu}_L)_j$$

$$= \frac{g}{2} (\hat{\bar{\nu}}_L)_j W^+ \gamma^\mu (\hat{e}_L)_j + \frac{g}{2} (\hat{\bar{e}}_L)_j W^- \gamma^\mu (\hat{\nu}_L)_j$$

$$= \frac{g}{2} \hat{\bar{\nu}}_j P_R W^+ \gamma^\mu P_L \hat{e}_j + \frac{g}{2} (\hat{\bar{e}})_j P_R W^- \gamma^\mu P_L \hat{\nu}_j$$

$$= \frac{g}{2} \hat{\bar{\nu}}_j W^+ \gamma^\mu \hat{e}_j + \frac{g}{2} \hat{\bar{e}}_j W^- \gamma^\mu \hat{\nu}_j$$

f) Focus on (W^3_μ, B_μ) (Z_μ, A_μ) terms

$$\mathcal{L} \subset \frac{g}{2} \left((\hat{\bar{\nu}}_L)_j \cancel{U_\ell^\dagger} \quad (\hat{\bar{e}}_L)_j \cancel{U_L^\dagger} \right)^{?} \gamma^\mu \begin{pmatrix} W^3_\mu \cancel{U_\ell} (\hat{\nu}_L)_j \\ -W^3_\mu \cancel{U_\ell} (\hat{e}_L)_j \end{pmatrix} - \frac{g'}{2} \left((\hat{\bar{\nu}}_L)_j \ (\hat{\bar{e}}_L)_j \right) B_\mu \gamma^\mu \begin{pmatrix} (\hat{\nu}_L)_j \\ (\hat{e}_L)_j \end{pmatrix}$$

$$- g' (\hat{\bar{e}}_R)_j B_\mu \gamma^\mu (\hat{e}_R)_j$$

$$= \frac{g}{2} \left((\hat{\bar{\nu}}_L)_j \gamma^\mu W^3_\mu (\hat{\nu}_L)_j - (\hat{\bar{e}}_L)_j \gamma^\mu W^3_\mu (\hat{e}_L)_j \right) - \frac{g'}{2} \Big((\hat{\bar{\nu}}_L)_j B_\mu \gamma^\mu (\hat{\nu}_L)_j$$
$$+ (\hat{\bar{e}}_L)_j B_\mu \gamma^\mu (\hat{e}_L)_j \Big) - g' (\hat{\bar{e}}_R)_j B_\mu \gamma^\mu (\hat{e}_R)_j$$

$$= \frac{g}{2} \left[\hat{\bar{\nu}}_{Lj} \gamma^\mu (\sin\theta_W A_\mu - \cos\theta_W Z_\mu) \hat{\nu}_{Lj} + \underline{\hat{\bar{e}}_{Lj} \gamma^\mu (-\sin\theta_W A_\mu + \cos\theta_W Z_\mu) \hat{e}_{Lj}} \right]$$
$$- \frac{g'}{2} \left[\hat{\bar{\nu}}_{Lj} (\sin\theta_W A_\mu + \cos\theta_W Z_\mu) \gamma^\mu \hat{\nu}_{Lj} + \underline{\hat{\bar{e}}_{Lj} (\sin\theta_W A_\mu + \cos\theta_W Z_\mu)} \gamma^\mu \hat{e}_{Lj} \right]$$
$$- g' \underline{\hat{\bar{e}}_{Rj} (\sin\theta_W A_\mu + \cos\theta_W Z_\mu)} \gamma^\mu \hat{e}_{Rj}$$

$$= \hat{\bar{\nu}}_{Lj} \left(\frac{g}{2} - \frac{g'}{2} \right) \sin\theta_W A_\mu \gamma^\mu \hat{\nu}_{Lj} + \hat{\bar{\nu}}_{Lj} \left(-\frac{g}{2} - \frac{g'}{2} \right) \cos\theta_W Z_\mu \gamma^\mu \hat{\nu}_{Lj}$$

$$+ \hat{\bar{e}}_{Lj} \left[\left(-\frac{g}{2} - \frac{g'}{2} \right) \sin\theta_W A_\mu + \left(\frac{g}{2} - \frac{g'}{2} \right) \cos\theta_W Z_\mu \right] \gamma^\mu \hat{e}_{Lj} - g' \hat{\bar{e}}_{Rj} (\sin\theta_W A_\mu + \cos\theta_W Z_\mu) \gamma^\mu \hat{e}_{Rj}$$

IT WENT WRONG SOMEWHERE!

$$e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

g) $\mathcal{L}_Y^Q = -(Y^d)_{ij}\, \bar{Q}_i \Phi D_j - (Y^u)_{ij}\, \bar{Q}_i \tilde{\Phi} U_j + h.c.$

$$Q_i = \begin{pmatrix} (u_L)_i \\ (d_L)_i \end{pmatrix}, \quad D_i = (d_R)_i, \quad U_i = (u_R)_i, \quad \tilde{\Phi} = i\tau_2 \Phi$$

$$u_{L/R} = U_{L/R}\, \hat{u}_{L/R} \qquad d_{L/R} = D_{L/R}\, \hat{d}_{L/R}$$

$$Y^u_{diag} = U_L^\dagger Y^u U_R$$

$$\mathcal{L}_Y^Q = -(Y^d)_{ij} \begin{pmatrix} (\bar{u}_L)_i \\ (\bar{d}_L)_i \end{pmatrix}^T \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} (d_R)_j - (Y^u)_{ij} \begin{pmatrix} (\bar{u}_L)_i \\ (\bar{d}_L)_i \end{pmatrix}^T \frac{i\tau_2}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} (u_R)_j + h.c.$$

$$= -\frac{1}{\sqrt{2}} (Y^d)_{ij}\, (\bar{d}_L)_i (v+h) (d_R)_j - \frac{1}{\sqrt{2}} (Y^u)_{ij}\, (\bar{u}_L)_i (v+h) (u_R)_j + h.c.$$

$$\left[\begin{pmatrix} 0 & -i \\ +i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v+h \end{pmatrix} = -i \begin{pmatrix} v+h \\ 0 \end{pmatrix} \right.$$

$$\supset -\frac{v}{\sqrt{2}}\, \bar{d}_L Y^d d_R - \frac{v}{\sqrt{2}}\, \bar{u}_L Y^u u_R$$

$$= -\frac{v}{\sqrt{2}}\, \hat{\bar{d}}_L \underbrace{D_L^\dagger Y^d D_R}_{= Y^d_{diag}} \hat{d}_R - \frac{v}{\sqrt{2}}\, \hat{\bar{u}}_L \underbrace{U_L^\dagger Y^u U_R}_{= Y^u_{diag}} \hat{u}_R$$

$$\longrightarrow m_d = \frac{v}{\sqrt{2}} Y^d_{diag}, \quad m_u = \frac{v}{\sqrt{2}} Y^u_{diag}$$

h) $\mathcal{L}_{GKQ} = i \bar{Q}_j D_\mu \gamma^\mu Q_j + i \bar{U}_j D_\mu \gamma^\mu U_j + i \bar{D}_j D_\mu \gamma^\mu D_j$

$\left[Y(Q) = \frac{1}{3}, \; Y(U) = \frac{4}{3}, \; Y(D) = -\frac{2}{3} \right.$

$$= i \bar{Q}_j \left(\partial_\mu - \frac{i}{2} g W_\mu^a \sigma^a - \frac{i}{2} g' \frac{1}{3} B_\mu \right) \gamma^\mu Q_j$$

$$+ i \bar{U}_j \left(\partial_\mu - \frac{i}{2} g' \frac{4}{3} B_\mu \right) \gamma^\mu U_j$$

$$+ i \bar{D}_j \left(\partial_\mu + \frac{i}{2} g' \frac{2}{3} B_\mu \right) \gamma^\mu D_j$$

Only interaction involve B_μ, W^3_μ

$$\supset i \bar{Q}_j \left(-\frac{i}{2} g W_\mu^3 \sigma^3 - \frac{i}{2} g' \frac{1}{3} B_\mu \right) \gamma^\mu Q_j$$

$$+ \frac{2}{3} g' \bar{U}_j B_\mu \gamma^\mu U_j - \frac{1}{3} g' \bar{D}_j B_\mu \gamma^\mu D_j$$

$$= \frac{g}{2} \begin{pmatrix} (\bar{u}_L)_j \\ (\bar{d}_L)_j \end{pmatrix}^T \begin{pmatrix} W_\mu^3 & W_\mu^+ \\ W_\mu^- & -W_\mu^3 \end{pmatrix} \gamma^\mu \begin{pmatrix} (u_L)_j \\ +(d_L)_j \end{pmatrix} + \begin{pmatrix} (\bar{u}_L)_j \\ (\bar{d}_L)_j \end{pmatrix}^T \frac{g'}{6} B_\mu \gamma^\mu \begin{pmatrix} (u_L)_j \\ (d_L)_j \end{pmatrix}$$

$$+\frac{2}{3}g'(\bar{u}_R)_j B_\mu \gamma^\mu (u_R)_j - \frac{g'}{3}(\bar{d}_R)_j B_\mu \gamma^\mu (d_R)_j$$

$$= \frac{g}{2}\begin{pmatrix}(\bar{u}_L)_j \\ (\bar{d}_L)_j\end{pmatrix}^T \gamma^\mu \begin{pmatrix} W^3_\mu u_{Lj} + W^+_\mu d_{Lj} \\ W^-_\mu u_{Lj} - W^3_\mu d_{Lj}\end{pmatrix}$$
$$+\frac{g'}{6}B_\mu(\bar{u}_{Lj}\gamma^\mu u_{Lj} + \bar{d}_{Lj}\gamma^\mu d_{Lj})$$
$$+\frac{2}{3}g' B_\mu (\bar{u}_R)_j \gamma^\mu (u_R)_j - \frac{g'}{3}(\bar{d}_R)_j B_\mu \gamma^\mu (d_R)_j$$

$$= \frac{g}{2}\Big[\bar{u}_{Lj}\gamma^\mu W^3_\mu u_{Lj} + \underbrace{\bar{u}_{Lj}\gamma^\mu W^+_\mu d_{Lj}} + \underbrace{\bar{d}_{Lj}\gamma^\mu W^-_\mu u_{Lj}} - \bar{d}_{Lj}\gamma^\mu W^3_\mu d_{Lj}\Big]$$
$$+\frac{g'}{6}B_\mu(\bar{u}_{Lj}\gamma^\mu u_{Lj} + \bar{d}_{Lj}\gamma^\mu d_{Lj})$$
$$+\frac{2}{3}g' B_\mu (\bar{u}_R)_j \gamma^\mu (u_R)_j - \frac{g'}{3}(\bar{d}_R)_j B_\mu \gamma^\mu (d_R)_j$$

$$= \bar{u}_{Lj}\gamma^\mu\left(\frac{g}{2}W^3_\mu + \frac{g'}{6}B_\mu\right)u_{Lj} + \frac{2}{3}g' B_\mu \bar{u}_{Rj}\gamma^\mu u_{Rj}$$
$$\bar{d}_{Lj}\gamma^\mu\left(-\frac{g}{2}W^3_\mu + \frac{g'}{6}B_\mu\right)d_{Lj} - \frac{g'}{3}B_\mu \bar{d}_{Rj}\gamma^\mu d_{Rj}$$
$$+\bar{u}_{Lj}\gamma^\mu W^+_\mu d_{Lj} + \bar{d}_{Lj}\gamma^\mu W^-_\mu u_{Lj}$$

$$= \bar{u}_{Lj}\gamma^\mu\left[\frac{g}{2}(\sin\theta_W A_\mu - \cos\theta_W Z_\mu) + \frac{g'}{6}(\cos\theta_W A_\mu + \sin\theta_W Z_\mu)\right]u_{Lj}$$
$$+\frac{2}{3}g'(\sin\theta_W Z_\mu + \cos\theta_W A_\mu)\bar{u}_{Rj}\gamma^\mu u_{Rj}$$
$$+\bar{d}_{Lj}\gamma^\mu\left[-\frac{g}{2}(-\cos\theta_W Z_\mu + \sin\theta_W A_\mu) + \frac{g'}{6}(\sin\theta_W Z_\mu + \cos\theta_W A_\mu)\right]d_{Lj}$$
$$-\frac{g'}{3}(\sin\theta_W Z_\mu + \cos\theta_W A_\mu)\bar{d}_{Rj}\gamma^\mu d_{Rj}$$
$$+\bar{u}_{Lj}\gamma^\mu W^+_\mu d_{Lj} + \bar{d}_{Lj}\gamma^\mu W^-_\mu u_{Lj}$$

$$= A_\mu \bar{u}_{Lj}\gamma^\mu\left(\frac{g}{2}\sin\theta_W + \frac{g'}{6}\cos\theta_W\right)u_{Lj} + A_\mu \bar{u}_{Rj}\gamma^\mu\left(\frac{2}{3}g'\cos\theta_W\right)u_{Rj}$$
$$+Z_\mu \bar{u}_{Lj}\gamma^\mu\left(-\frac{g}{2}\cos\theta_W + \frac{g'}{6}\sin\theta_W\right)u_{Lj} + Z_\mu \bar{u}_{Rj}\gamma^\mu\left(\frac{2}{3}g'\sin\theta_W\right)u_{Rj}$$
$$+A_\mu \bar{d}_{Lj}\gamma^\mu\left(-\frac{g}{2}\sin\theta_W + \frac{g'}{6}\cos\theta_W\right)d_{Lj} + A_\mu \bar{d}_{Rj}\gamma^\mu\left(-\frac{g'}{3}\cos\theta_W\right)d_{Rj}$$
$$+Z_\mu \bar{d}_{Lj}\gamma^\mu\left(\frac{g}{2}\cos\theta_W + \frac{g'}{6}\sin\theta_W\right)d_{Lj} + Z_\mu \bar{d}_{Rj}\gamma^\mu\left(-\frac{g'}{3}\sin\theta_W\right)d_{Rj}$$
$$+\bar{u}_{Lj}\gamma^\mu W^+_\mu d_{Lj} + \bar{d}_{Lj}\gamma^\mu W^-_\mu u_{Lj}$$

H.7.2

a) $(MM^\dagger)^\dagger = MM^\dagger \rightarrow$ hermitian

$\rightarrow MM^\dagger = S M_d^s S^\dagger$

Have a matrix $F = \text{diag}(e^{i\phi_1}, \ldots, e^{i\phi_N})$, ϕ_i N parameters

$S \rightarrow SF$, $S M_d^s S^\dagger \rightarrow SF M_d^s F^\dagger S^\dagger = S M_d^s S^\dagger$

$\rightarrow$ right-hand side has N more parameters

b) $V = H^{-1} M$, $H = S M_d S^\dagger$ hermitian

$$
\begin{aligned}
V V^\dagger &= H^{-1} M M^\dagger (H^{-1})^\dagger \\
&= H^{-1} S M_d^2 S^\dagger H^{-1} \\
&= H^{-1} S M_d S^\dagger S M_d S^\dagger H^{-1} \\
&= H^{-1} H H H^{-1} = \mathbb{1}_N
\end{aligned}
$$

c) $M = S M_d T^\dagger = S M_d (V^\dagger S)^\dagger = S M_d S^\dagger V$

$$
\begin{aligned}
\rightarrow MM^\dagger &= S M_d \underbrace{S^\dagger \underbrace{V V^\dagger}_{=\mathbb{1}} S}_{=\mathbb{1}} M_d^\dagger S^\dagger \\
&= S M_d M_d^\dagger S^\dagger \\
&= S M_d^2 S^\dagger \quad , \quad M_d \in \mathbb{R}^N \times \mathbb{R}^N
\end{aligned}
$$

# of parameter in $S = N^2$ # of p in $V = N^2$ $\rightarrow 2N^2$

d) $V_{CKM} = U_u^\dagger U_d$, U_i are unitary ($U(N)$)

$\rightarrow (N-1)^2$ parameters

e) $$\underbrace{2N^2/2}_{\text{unitary}} - \underbrace{(2N-1)}_{\substack{\text{redefinition} \\ \text{of quark} \\ \text{phases}}} - \underbrace{\frac{N(N-1)}{2}}_{\substack{\text{orthogonal} \\ \text{matrix} \\ \text{parameter}}} = N^2 - 2N + 1 - \frac{N^2}{2} + \frac{N}{2}$$

$$= \frac{1}{2} N^2 - \frac{3}{2} N + 1$$

$$= \frac{1}{2} (N^2 - 3N + 2)$$

$$= \frac{1}{2} (N-1)(N-2)$$

f) $N = 2$, $\#$ of $\delta = 0$

$N = 3$, $\#$ of $\delta = 1$ $\longrightarrow$ CP-violation