H9.1

simplifies to

$$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = -\frac{1}{4}[(\partial_\mu V_\nu - \partial_\nu V_\mu)(\partial^\mu V^\nu - \partial^\nu V^\mu)] = -\frac{1}{2}[\partial_\mu V_\nu \cdot \partial^\mu V^\nu - \partial_\mu V_\nu \cdot \partial^\nu V^\mu]$$

a) $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}V_\mu V^\mu \quad , \quad F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$

$$\frac{\partial \mathcal{L}}{\partial V_\sigma} = -\frac{m^2}{2}V^\sigma$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\lambda V_\sigma)} = -\frac{1}{4}\frac{\partial}{\partial(\partial_\lambda V_\sigma)}\left[(\partial_\mu V_\nu - \partial_\nu V_\mu) g^{\alpha\mu} g^{\beta\nu}(\partial_\alpha V_\beta - \partial_\beta V_\alpha)\right]$$

$$= -\frac{1}{4} g^{\alpha\mu} g^{\beta\nu}\left[(\delta_{\mu\lambda}\delta_{\sigma\nu} - \delta_{\lambda\nu}\delta_{\sigma\mu})(\partial_\alpha V_\beta - \partial_\beta V_\alpha) + (\partial_\mu V_\nu - \partial_\nu V_\mu)(\delta_{\alpha\lambda}\delta_{\sigma\beta} - \delta_{\lambda\beta}\delta_{\sigma\alpha})\right]$$

$$= -\frac{1}{4}\left[(g^{\alpha\lambda}g^{\beta\sigma} - g^{\alpha\sigma}g^{\beta\lambda})(\partial_\alpha V_\beta - \partial_\beta V_\alpha) + (\partial_\mu V_\nu - \partial_\nu V_\mu)(g^{\lambda\mu}g^{\sigma\nu} - g^{\sigma\mu}g^{\lambda\nu})\right]$$

$$= -\frac{1}{4}[\underline{\partial^\lambda V^\sigma} - \underline{\underline{\partial^\sigma V^\lambda}} - \underline{\underline{\partial^\sigma V^\lambda}} + \underline{\partial^\lambda V^\sigma} + \underline{\partial^\lambda V^\sigma} - \underline{\underline{\partial^\sigma V^\lambda}} - \underline{\underline{\partial^\sigma V^\lambda}} + \underline{\partial^\lambda V^\sigma}]$$

$$= \partial^\sigma V^\lambda - \partial^\lambda V^\sigma$$

$\Rightarrow$ ELE : $\quad \partial_\lambda(\partial^\sigma V^\lambda - \partial^\lambda V^\sigma) - \frac{m^2}{2}V^\sigma = 0$

$$\Rightarrow [(\partial^2 + m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]V_\nu = 0$$

b) $$\partial_\mu\{[(\partial^2 + m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]V_\nu\} = 0$$

$$\cancel{\partial_\mu\partial_\alpha\partial^\alpha g^{\mu\nu}V_\nu} + m^2 g^{\mu\nu}\partial_\mu V_\nu - \cancel{\partial_\mu\partial^\mu\partial^\nu V_\nu} = 0$$

$\rightarrow$ if $m \neq 0$

$$\partial^\nu V_\nu = 0$$

c) $V_\mu = \epsilon_\mu(p)\, e^{-ip\cdot x} \quad \longrightarrow \quad \partial_\mu\left[\epsilon^\mu(p)\, e^{-ipx}\right] = \epsilon^\mu(p)\,(-ip_\mu)\,e^{-ipx} = 0$

$\longrightarrow \quad p_\mu \epsilon^\mu(p) = 0$

$\longrightarrow$ 3 DOF

d) $p = (E, 0, 0, |\vec{p}|)^t$.

$$\longrightarrow \quad p_\mu \epsilon^\mu(p) = \begin{pmatrix} E \\ 0 \\ 0 \\ |\vec p| \end{pmatrix}^t \cdot \begin{cases} \frac{1}{m}\begin{pmatrix} |\vec p| \\ 0 \\ 0 \\ E \end{pmatrix} \\ \mp \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{pmatrix} \end{cases} = 0$$

$$\sum_\lambda \epsilon^*_\mu(\lambda)\,\epsilon_\nu(\lambda) = \frac{1}{m^2}\begin{pmatrix} |\vec p| \\ 0 \\ 0 \\ E \end{pmatrix}^* (|\vec p|, 0, 0, E)_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}^* (0 \;\; 1 \;\; i \;\; 0)_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}^* (0 \;\; 1 \;\; -i \;\; 0)_{\mu\nu}$$

$$= \frac{1}{m^2}\begin{pmatrix} |\vec p|^2 & 0 & 0 & E|\vec p| \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ E|\vec p| & 0 & 0 & E^2 \end{pmatrix}_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 & & & \\ & 1 & +i & \\ & -i & +1 & \\ & & & 0 \end{pmatrix}_{\mu\nu} + \frac{1}{2}\begin{pmatrix} 0 & & & \\ & 1 & -i & \\ & +i & +1 & \\ & & & 0 \end{pmatrix}_{\mu\nu}$$

$$= \begin{pmatrix} |\vec p|^2/m^2 & & & E|\vec p|/m^2 \\ & 1 & & \\ & & +1 & \\ E|\vec p|/m^2 & & & E^2/m^2 \end{pmatrix}_{\mu\nu}$$

$$-g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}_{\mu\nu} + \frac{1}{m^2}\begin{pmatrix} E^2 & & & E|\vec p| \\ & 0 & & \\ & & 0 & \\ E|\vec p| & & & |\vec p|^2 \end{pmatrix}_{\mu\nu}$$

$\left(E^2/m^2 = (|\vec p|^2 + m^2)/m^2 = 1 + |\vec p|^2/m^2 \right. \quad \Rightarrow (3)$

$$= \begin{pmatrix} -1+1+|\vec p|^2/m^2 & & & E|\vec p|/m^2 \\ & 1 & & \\ & & 1 & \\ E|\vec p|/m^2 & & & E^2/m^2 \end{pmatrix}_{\mu\nu}$$

e) $[(-p^2+m^2)g^{\mu\nu} + p^\mu p^\nu]\Delta_{\nu\alpha} = \delta^\mu_\alpha$

Plug in ansatz $\Delta_{\nu\alpha} = A\, g_{\nu\alpha} + \frac{B}{p^2} p_\nu p_\alpha$

$$[(-p^2+m^2)g^{\mu\nu} + p^\mu p^\nu][A g_{\nu\alpha} + \frac{B}{p^2} p_\nu p_\alpha]$$

$$= A(-p^2+m^2)\delta^\mu_\alpha + (-p^2+m^2)\frac{B}{p^2} p^\mu p_\alpha + A p^\mu p_\alpha + \frac{B}{\cancel{p^2}} p^\mu \cancel{p^\nu p_\nu} p_\alpha$$

$$\doteq \delta^\mu_\alpha$$

$$\to A = \frac{1}{-p^2+m^2}, \qquad (-p^2+m^2)\frac{B}{p^2} p^\mu p_\alpha + \frac{1}{-p^2+m^2} p^\mu p_\alpha + B p^\mu p_\alpha = 0$$

$$\to (-p^2+m^2)\frac{B}{p^2} + \frac{1}{-p^2+m^2} + B = 0$$

$$(-p^2+m^2)^2 B + p^2 + B p^2 (-p^2+m^2) = 0$$

$$\to B = \frac{-p^2}{(-p^2+m^2)^2 + (-p^2+m^2)p^2}$$

$$= \frac{-p^2}{-2m^2p^2 + m^4 + m^2p^2}$$

$$= \frac{-p^2}{-p^2+m^2}\frac{1}{m^2}$$

$$\to \Delta_{\nu\alpha} = \frac{1}{-p^2+m^2} g_{\nu\alpha} + \frac{1}{p^2-m^2}\frac{1}{m^2} p_\nu p_\alpha$$

$$= \frac{1}{p^2-m^2}\underbrace{\left(-g_{\nu\alpha} + \frac{1}{m^2} p_\nu p_\alpha\right)}_{= \sum_\lambda \epsilon^*_\nu(\lambda)\epsilon_\alpha(\lambda)}$$

f)

During propagating, all polarization states are possible and need to be summed over.

→ proof in Schwartz

H 9.2

a) $B \to f_1 f_2$

$m_W \sim 80\,GeV$

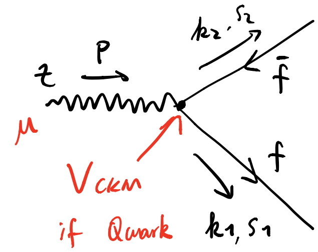
$W^+ \to e^+ \nu_e,\ \mu^+ \nu_\mu,\ \tau^+ \nu_\tau,\ u\bar{d},\ u\bar{s},\ ub,\ c\bar{d},\ c\bar{s},\ c\bar{b}$

$W^- \to e^- \bar{\nu}_e,\ \mu^- \bar{\nu}_\mu,\ \tau^- \bar{\nu}_\tau,\ \bar{u}d,\ \bar{u}s,\ \bar{u}b,\ \bar{c}d,\ \bar{c}s,\ \bar{c}b$

$m_Z \sim 90\,GeV$

$Z \to e^- e^+,\ \mu^- \mu^+,\ \tau^- \tau^+,\ \bar{u}u,\ \bar{d}d,\ \bar{c}c,\ \bar{s}s,\ \bar{b}b,\ \bar{u}c,\ u\bar{c},\ \bar{d}s,\ d\bar{s},\ \bar{d}b,\ d\bar{b}$
$\bar{s}b,\ s\bar{b}$

b)



$\vec{t}$

c) $$i\mathcal{M} = \epsilon^\mu(\vec{p},\lambda)\,\bar{u}(k_1)\left[-i\frac{g}{\cos\theta_W}\gamma_\mu\left(\frac{C_V\mathbb{1} - C_A\gamma^5}{2}\right)\right]v(k_2)$$

$$= \frac{-ig}{2\cos\theta_W}\epsilon^\mu(\vec{p},\lambda)\,\bar{u}(k_1)\gamma_\mu(C_V\mathbb{1} - C_A\gamma^5)\,v(k_2)$$

d) $$\sum_{s_1,s_2}\frac{-ig}{2\cos\theta_W}\epsilon^\mu(\vec{p},\lambda)\,\bar{u}^{s_1}(k_1)\gamma_\mu(C_V\mathbb{1} - C_A\gamma^5)\,v^{s_2}(k_2)$$

e) $$\overline{|\mathcal{M}|^2} = \frac{1}{3}\sum_\lambda\sum_{s_1,s_2}\left|\frac{-ig}{2\cos\theta_W}\right|^2\epsilon^\mu(\vec{p},\lambda)\,\epsilon^{*\nu}(\vec{p},\lambda)\,\bar{u}^{s_1}(k_1)\gamma_\mu(C_V\mathbb{1} - C_A\gamma^5)\,v^{s_2}(k_2)$$

$$\times\,\bar{v}^{s_2}(k_2)\,\gamma_\nu\,(C_V\mathbb{1} - C_A\gamma^5)\,u^{s_1}(k_1)$$

$$= \frac{g^2}{12\cos^2\theta_W}\left(-g^{\mu\nu} + \frac{p^\mu p^\nu}{M_Z^2}\right)$$

$$\times \sum_{S_1,S_2} tr[\bar{u}^{S_1}(k_1)\gamma_\mu(C_V\mathbb{1}-C_A\gamma^5)v^{S_2}(k_2)\bar{v}^{S_2}(k_2)\gamma_\nu(C_V\mathbb{1}-C_A\gamma^5)u^{S_1}(k_1)]$$

$$(m_f=0) \quad \simeq \frac{g^2}{12\cos^2\theta_W}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{M_Z^2}\right) tr[\not{k}_1\gamma_\mu(C_V\mathbb{1}-C_A\gamma^5)\not{k}_2\gamma_\nu(C_V\mathbb{1}-C_A\gamma^5)]$$

$-2C_VC_A\, tr[\not{k}_1\gamma_\mu\gamma^5\not{k}_2\gamma_\nu]$

$\propto \epsilon^{\alpha\mu\beta\gamma}k_{1\alpha}k_{2\beta}$

$$(C_V\mathbb{1}-C_A\gamma^5)^2 = C_V^2+C_A^2-2C_VC_A\gamma^5$$

$\to 0$?

fully anti-symm. in $\mu\nu$.

Prop is symm in $\mu\nu$

$$= \frac{g^2}{12\cos^2\theta_W}\left(-g^{\mu\nu}+\frac{p^\mu p^\nu}{M_Z^2}\right)(C_V^2+C_A^2)\, tr[\not{k}_1\gamma_\mu\not{k}_2\gamma_\nu]$$

$\to 0$, in cms

?

f) $tr[\not{k}_1\gamma_\mu\not{k}_2\gamma_\nu]$

$$= 4k_1^\alpha k_2^\beta(g_{\alpha\mu}g_{\beta\nu}-g_{\alpha\beta}g_{\mu\nu}+g_{\alpha\nu}g_{\mu\beta})$$

$$= 4[k_{1\mu}k_{2\nu}-(k_1\cdot k_2)g_{\mu\nu}+k_{1\nu}k_{2\mu}]$$

$$\to \overline{|\mathcal{M}|^2} = \frac{-4g^2}{12\cos^2\theta_W}(C_V^2+C_A^2)[(k_1\cdot k_2)-4(k_1\cdot k_2)+(k_1\cdot k_2)]$$

$$= \frac{2g^2}{3\cos^2\theta_W}(C_V^2+C_A^2)(E^2+|\vec{p}_{cm}|^2)$$

$$2E = M_Z = 2P_{cm}$$

$$\overline{|\mathcal{M}|^2} = \frac{g^2}{12\cos^2\theta_W}(C_V^2+C_A^2)\left[-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_Z^2}\right]$$

$$\times[4(k_1^\mu k_2^\nu-(k_1\cdot k_2)g^{\mu\nu}+k_1^\nu k_2^\mu)]$$

$$\simeq \frac{4g^2}{3\cos^2\theta_W}(C_V^2+C_A^2)|\vec{P}_{cm}|^2$$

$$= \frac{g^2(C_V^2+C_A^2)}{3\cos^2\theta_W}\left[(k_1\cdot k_2)+\frac{2(p\cdot k_1)(p\cdot k_2)}{m_Z^2}\right]$$

$$\to \frac{d\Gamma}{d\Omega} = \frac{P_{cm}}{32\pi^2M_Z^2}\cdot\frac{4g^2}{3\cos^2\theta_W}(C_V^2+C_A^2)P_{cm}^2$$

$$= \frac{g^2}{8\pi^2M_Z^2}\frac{1}{3\cos^2\theta_W}(C_V^2+C_A^2)\cdot\frac{1}{8}M_Z^3$$

$$\to \Gamma = 4\pi\frac{d\Gamma}{d\Omega} \qquad \text{(no } \theta,\varphi \text{ dependence)}$$

$$= \frac{g^2}{48\pi\cos^2\theta_W}(C_V^2+C_A^2)M_Z$$

$\to \Gamma_f$ is same for all $f \in$ lepton

$C_V = T_3 - 2Q\sin^2\theta_W$, $C_A = T_3$

	C_V	C_A
ℓ	0	$-1/2$
q_u	$1/6$	$1/2$
q_d	$-1/3$	$-1/2$
ν	$1/2$	$1/2$

g) $g^2=0.4$, $\sin^2\theta_W=\frac{1}{4} \Leftrightarrow \cos^2\theta_W=\frac{3}{4}$, $M_Z=90\,GeV$

$$\to \Gamma = 0.32\,GeV\cdot(C_V^2+C_A^2)$$

$\Gamma(Z\to ee) = 79.9\,MeV$, $\Gamma(Z\to u\bar{u}) = 3\cdot 91\,MeV = 273\,MeV$

$\Gamma(Z\to d\bar{d}) = 349\,MeV$

$\rightarrow \Gamma_Z^{visible} = 3 \cdot (79.9 + 273 + 349)\, MeV = \underline{2.1\ GeV}$

$\Gamma_Z^{visible} = 3\Gamma(Z\to ee) + 3\Gamma(Z\to d\bar{d}) + 2\Gamma(Z\to u\bar{u}) \simeq 1.8\,GeV$

h) Discrepancy : decay into neutrinos

For ν: $(C_A^2 + C_V^2) = \frac{1}{2}$

$\rightarrow \Gamma(Z\to \nu\nu) = 0.16\ GeV$

$\rightarrow \Delta\Gamma \overset{\sim}{=} 300\ GeV \approx 160\ GeV \cdot 3$

$\Gamma_{invisible} = N\,\Gamma(Z\to\nu\bar{\nu}) = N\,(160\ MeV)$

$N = \Gamma_{invis} / \Gamma(Z\to\nu\bar{\nu}) \approx 3. \ldots$

(In SM)