H8.1 Chenhuan Wang

a) $\int_{-\infty}^{\infty} dk^0 \underbrace{\delta(k^2-m^2)} \Theta(k^0) = \frac{1}{2E_k}$

$= \delta\left((k^0)^2 - |\vec{k}|^2 - m^2\right)$

$= \frac{1}{2E_k}\left[\delta(k^0-E_k) + \delta(k^0+E_k)\right]$

$\Rightarrow \int_{-\infty}^{\infty} \frac{dk^0}{2E_k} \Theta(k^0) \left[\delta(k^0-E_k)+\delta(k^0+E_k)\right] = \frac{1}{2E_k}$ ✓

b) $\int d^4k \underbrace{\delta(k^2-m^2)}_{L.-inv.} \Theta(k^0) \overset{L}{=} \int d^4k' \underbrace{|\det\Lambda|}_{1} \delta(k'^2-m^2)\Theta(k'^0)$

$\Lambda \in SO(3,1)^{\uparrow} \to \Lambda^0{}_0 > 1$

$\to \Theta(k^0)$ L. inv.

$= \int d^4k' \delta(k'^2-m^2)\Theta(k'^0)$

c) P: $\vec{k} \to -\vec{k}$ but it doesn't change the object

since $d^3k = d|\vec{k}|\,|\vec{k}|^2$

T: $k^0 \to -k^0$, $dk^0\,\Theta(k^0) \to dk^0\,\Theta(k^0)$

Two minus signs cancel out

T: $k^0 \to -k^0$

$\int_{\infty}^{-\infty} d(-k^0) \int d^3k\, \delta[(-k^0)^2 - |\vec{k}|^2 - m^2)\Theta(-k^0)$

$u = -k^0$

$= \int_{-\infty}^{\infty} du \int d^3k\, \delta[u^2 - |\vec{k}|^2 - m^2]\Theta(u) \quad \to$ inv.

$R_2 = \int \underbrace{\frac{d^3p_1}{(2\pi)^3 2E_1}} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^{(4)}(p-p_1-p_2)$

$= \int_{-\infty}^{\infty} dp_1^0\, d^3p_1\, \delta(p_1^2-m^2)\Theta(p_1^0)\frac{1}{(2\pi)^3}$ from a)

$\to$ L. inv.

$$R_2 = \int \frac{d^3p_1}{(2\pi)^3} \int dp_1^0\, \delta(p_1^2 - m_1^2)\, \Theta(p_1^0) \int \frac{d^3p_2}{(2\pi)^3} \int dp_2^0\, \delta(p_2^2 - m_2^2)\, \Theta(p_2^0) \longrightarrow L.\ inv.$$

$$\times (2\pi)^4 \underbrace{\delta^{(4)}(p - p_1 - p_2)}$$

$$\longrightarrow \delta^{(4)}(p - p_1 - p_2)\, |\det(\Lambda)|$$

d) $$R_2 = (2\pi)^{-2} \int_{-\infty}^{\infty} dp_1^0 \underbrace{\int_{-\infty}^{\infty} dp_2^0} \int_{\mathbb{R}^3} d^3p_1 \underbrace{\int_{\mathbb{R}^3} d^3p_2}\, \delta(p_1^2 - m_1^2)\, \Theta(p_1^0)$$

$$\times\, \delta(p_2^2 - m_2^2)\, \Theta(p_2^0)\, \underbrace{\delta^{(4)}(p - p_1 - p_2)}$$

$$= (2\pi)^{-2} \underbrace{\int_{-\infty}^{\infty} dp_1^0} \int_{\mathbb{R}^3} d^3p_1\, \underbrace{\delta(p_1^2 - m_1^2)\, \Theta(p_1^0)}\, \delta((p-p_1)^2 - m_2^2)\, \Theta((p-p_1)^0)$$

$$= (2\pi)^{-2} \int_{\mathbb{R}^3} d^3p_1\, \frac{1}{2E_1}\, \delta((p-p_1)^2 - m_2^2)\, \Theta((p-p_1)^0) \checkmark$$

$$= (2\pi)^{-2} \int_{\mathbb{R}^3} \frac{d^3p_1}{2E_1}\, \delta((p-p_1)^2 - m_2^2)\, \Theta((p-p_1)^0) \checkmark$$

e) $$p = p_1 + p_2 \longrightarrow E_1 + E_2 = \sqrt{s}$$

$$E_1 = \sqrt{|\vec{p}_{cm}|^2 + m_1^2} = \sqrt{s} - E_2$$

$$(p_1 + p_2)^2 - (p_1 - p_2)(p_1 + p_2)$$

$$= (p_1 + p_2)(p_1 + p_2 - p_1 + p_2)$$

$$p_1^2 = m_1^2 = E_1^2 - |\vec{p}_{cM}|^2$$

$$s + m_1^2 - m_2^2 = p^2 + p_1^2 - p_2^2 = [p + (p_1 - p_2)]^2 \underbrace{- 2p(p_1 - p_2) + 2p_1 p_2}$$

$$\propto \begin{pmatrix} E_1 + E_2 \\ 0 \end{pmatrix} \begin{pmatrix} E_1 - E_2 \\ 2\vec{p}_{cm} \end{pmatrix} - \begin{pmatrix} E_1 \\ \vec{p}_{cm} \end{pmatrix} \begin{pmatrix} E_2 \\ \vec{p}_{cm} \end{pmatrix}$$

$$= E_1^2 + E_2^2 - E_1 E_2 + |\vec{p}_{cm}|^2$$

$$2\sqrt{s}\, E_1 = 2(E_1 + E_2) E_1 = 2(p_1 + p_2) \cdot p_1 = (p_1 + p_2)(p_1 + p_2 + p_1 - p_2)$$

$$= (p_1 + p_2)^2 + (p_1 + p_2)(p_1 - p_2)$$

$$= (p_1 + p_2)^2 + p_1^2 - p_2^2$$

$$= s + m_1^2 - m_2^2$$

$$\rightarrow \quad E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}$$

Analogously $$E_2 = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}}$$

$$p^2 = s = (p_1 + p_2)^2 = m_1^2 + m_2^2 - 2 p_1 \cdot p_2 = m_1^2 + m_2^2 - 2\left[E_1 E_2 + |\vec{p}_{cm}|^2\right]$$

$$\rightarrow \quad s = m_1^2 + m_2^2 - 2 E_1 E_2 - 2 |\vec{p}_{cm}|^2$$

$$s^2 = (m_1^2 + m_2^2)\, s - 2 \underbrace{\sqrt{s}\, E_1}\, \underbrace{\sqrt{s}\, E_2} - 2 s\, |\vec{p}_{cm}|^2$$

$$2s\, |\vec{p}_{cm}|^2 = -s^2 + (m_1^2 + m_2^2)\, s - \frac{1}{2}(s + m_1^2 - m_2^2)(s + m_2^2 - m_1^2)$$

$$4s\, |\vec{p}_{cm}|^2 = \overset{+2\,?}{-2}\, s^2 + 2 s m_1^2 + 2 s m_2^2 - s^2 - \cancel{s m_2^2} + \cancel{s m_1^2} - \cancel{s m_1^2} - m_1^2 m_2^2$$
$$+ m_1^4 + \cancel{s m_2^2} + m_2^4 - m_1^2 m_2^2$$

$$= \lambda (s, m_1^2, m_2^2)$$

$$\Rightarrow \quad |\vec{p}_{cm}|^2 = \frac{\lambda^{\frac{1}{2}}(\ldots)}{2\sqrt{s}}$$

$$P = (p_1 + p_2) \quad \rightarrow \quad p_2^2 = (P - p_1)^2$$

$$\Rightarrow m_2^2 = s + m_1^2 - 2 E_1 \sqrt{s}$$

$$\Rightarrow E_1 = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}}$$

$$|p_{cm}^2| = E_1^2 - m_1^2$$

$$= \frac{(s + m_1^2 - m_2^2)^2}{4s} - m_1^2 = \frac{\lambda(s, m_1^2, m_2^2)}{4s}$$

f) $$R_2 = \frac{1}{(2\pi)^2} \int \frac{d^3 p_1}{2E_2} \delta((p-p_1)^2 - m_2^2)\, \Theta((p-p_1)^0)$$

$$= \frac{1}{(2\pi)^2} \frac{1}{2} \cdot \int d\Omega \int d|\vec{p}_1| \cdot |\vec{p}_1|^2 \frac{2\sqrt{s}}{s+m_1^2-m_2^2} \underbrace{\delta((p-p_1)^2 - m_2^2)}_{\text{ensures on-shell}} \underbrace{\Theta((p-p_1)^0)}_{\text{ensures positive energy}}$$

$$= \frac{1}{(2\pi)^2} \int d\Omega \cdot \frac{|\vec{p}_1|^2}{2E_1} \frac{1}{2|\vec{p}_1|}$$

from $\delta(\ldots)$

$$= \frac{1}{16\pi^2} \frac{\lambda^{\frac{1}{2}}(\ldots)}{s+m_1^2-m_2^2} \int d\Omega$$

?

$$R_2 = \frac{1}{(2\pi)^2} \int \frac{d^3 p_1}{2E_1} \delta[(p-p_1)^2 - m_2^2]\, \Theta(p^0 - p_1^0)$$

$$= \frac{1}{(2\pi)^2} \int \frac{d^3 p_1}{2E_2} \delta(s+m_1^2 - 2E\sqrt{s} - m_2^2)\, \Theta(p^0 - p_1^0)$$

$$d^3 p_1 = d|\vec{p}_1|\, |\vec{p}_1|^2\, d\Omega$$

$$E_1^2 = |\vec{p}_1|^2 + m_1^2 \rightarrow E\, dE_1 = |\vec{p}_1|\, d|\vec{p}_1|$$

$$= \frac{1}{(2\pi)^2} \int dE_1 \frac{\sqrt{E_1^2 - m_1^2}}{2} \delta(s+m_1^2 - m_2^2 - 2\sqrt{s}\, E_1)\, \Theta(\sqrt{s} - E_1) \int d\Omega_1$$

$$= \frac{1}{(2\pi)^2} \frac{1}{2} \int \frac{\sqrt{E_1^2 - m_1^2}}{2\sqrt{s}} \delta\left[\frac{s+m_1^2-m_2^2}{2\sqrt{s}} - E_1\right] dE_1$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4\sqrt{s}} \left[\left(\frac{s+m_1^2-m_2^2}{2\sqrt{s}}\right)^2 - m_1^2\right]^{\frac{1}{2}} \int d\Omega = \frac{1}{32\pi^2 s} \lambda^{1/2}(\ldots) \int d\Omega$$

$$\gamma = \frac{E}{m}, \quad \beta = \frac{E}{p}$$

g) $$|v_A - v_B|\, 2E_A\, 2E_B \stackrel{?}{=} 4\sqrt{(p_A \cdot p_B)^2 - m_A^2 \cdot m_B^2}$$

$$p_A = (E_A, \vec{p}_A)^T, \quad p_B = (E_B, \vec{p}_B)^T,$$

$$|\vec{p}_A| = \gamma m_A v_A, \quad |\vec{p}_B| = \gamma' m_B v_B, \quad \gamma = (1-v^2)^{-\frac{1}{2}}$$

$$E_A^2 - \gamma^2 m_A^2 v_A^2 = m_A^2 \Leftrightarrow \frac{E_A^2}{m_A^2} = 1 + \gamma^2 v_A^2 = 1 + \frac{v_A^2}{1-v_A^2} = \frac{1}{1-v_A^2} \Leftrightarrow \frac{m_A}{E_A} = \sqrt{1-v_A^2} = \frac{1}{\gamma}$$

$$RHS = 4\left[(E_A \cdot E_B - \vec{p}_A \cdot \vec{p}_B)^2 - m_A^2 \cdot m_B^2\right]^{1/2}$$

$$= 4 E_A E_B \left[\left(1 - \frac{\vec{p}_A \cdot \vec{p}_B}{E_A E_B}\right)^2 - \frac{m_A^2}{E_A^2} \cdot \frac{m_B^2}{E_B^2}\right]^{\frac{1}{2}}$$

$$\theta = \pi \hookrightarrow \quad = \left(1 - \frac{\gamma m_A v_A \gamma' m_B v_B \cdot \cos\theta}{E_A E_B}\right)^2 - \left(\frac{m_A m_B}{E_A E_B}\right)^2$$

$$= \left[1 - \frac{v_A}{\sqrt{1-v_A^2}} \frac{v_B}{\sqrt{1-v_B^2}} \sqrt{1-v_A^2}\sqrt{1-v_B^2}\right]^2 - (1-v_A^2)(1-v_B^2)$$

$$= (1 - v_A v_B)^2 - (1-v_A)(1-v_B)$$

$$= \cancel{1} - 2 v_A v_B + \cancel{v_A^2 v_B^2} - \cancel{1} + v_A^2 + v_B^2 - \cancel{v_A^2 v_B^2}$$

$$= (v_A - v_B)^2$$

$$= 4 E_A E_B \, |v_A - v_B|$$ ✓

$$(p_A \cdot p_B)^2 - m_A^2 m_B^2$$

$$= \left[\frac{1}{2} \underbrace{(p_A + p_B)^2}_{=s} - \frac{1}{2} m_A^2 - \frac{1}{2} m_B^2\right]^2 - m_A^2 m_B^2$$

$$= \frac{1}{4}\left(s^2 + m_A^4 + m_B^4 - 2 s m_A^2 - 2 s m_B^2 + 2 m_A^2 m_B^2\right) - m_A^2 m_B^2$$

$$= \frac{1}{4}\left(s^2 + m_A^4 + m_B^4 - 2 s m_A^2 - 2 s m_B^2 - 2 m_A^2 m_B^2\right)$$

$$= \frac{1}{4} \lambda \left(s, m_A^2, m_B^2\right)$$

$$\rightarrow 4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2} = 2 \lambda^{\frac{1}{2}} (s, m_A^2, m_B^2)$$ ✓

h) $$d\sigma = \frac{1}{2 \lambda^{\frac{1}{2}}(s, m_A^2, m_B^2)} |\mathcal{M}|^2 \, dR_2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2\lambda^{\frac{1}{2}}(s, m_A^2, m_B^2)} \frac{\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)}{32\pi^2 \, s} |\mathcal{M}|^2$$

$$= \frac{1}{64\pi^2 s} \underbrace{\frac{\lambda^{\frac{1}{2}}(\cdots)}{\lambda^{\frac{1}{2}}(\cdots)}}_{} |\mathcal{M}|^2$$

$$= \frac{|\vec{p}_f|}{|\vec{p}_i|} \, , \quad \text{since} \quad \lambda^{\frac{1}{2}}(s, m_1^2, m_2^2) = 2\sqrt{s}\, |\vec{p}_{cm}|$$ ✓

$$\underset{m_A = m_B}{\text{if}} \quad = 1$$

i) $t = (p_A - p_1)^2 = m_A^2 + m_1^2 - 2\vec{p}_A \cdot \vec{p}_1$

$$= m_A^2 + m_1^2 - 2|\vec{p}_A||\vec{p}_1| \cdot \cos\theta$$

$$= m_A^2 + m_1^2 - \cancel{2}\, \frac{\lambda^{\frac{1}{2}}(s, m_1^2, m_2^2)}{\cancel{2}\sqrt{s}}\, \frac{\lambda^{\frac{1}{2}}(s, m_A^2, m_B^2)}{2\sqrt{s}} \cos\theta$$

$$d\Omega = d\cos\theta \, d\varphi$$

$$\rightarrow \quad \frac{dt}{d\cos\theta} = \frac{1}{2s}\, \lambda^{\frac{1}{2}}(\cdot\cdot)\, \lambda^{\frac{1}{2}}(\cdot\cdot)$$

$$\frac{d\Omega}{dt} = d\varphi \cdot \frac{2s}{\lambda^{\frac{1}{2}}(\cdot\cdot)\,\lambda^{\frac{1}{2}}(\cdot\cdot)} \cdot d\cos\theta$$

$$\Rightarrow \quad \frac{d\sigma}{dt} = \frac{d\sigma}{d\Omega}\frac{d\Omega}{dt} = \frac{1}{64\pi^2 \cancel{s}}\, \frac{\cancel{\lambda^{\frac{1}{2}}(1,2)}}{\lambda^{\frac{1}{2}}(A,B)} \cdot \underbrace{d\varphi}_{=2\pi}\, \frac{2\cancel{s}}{\cancel{\lambda^{\frac{1}{2}}(1,2)}\,\lambda^{\frac{1}{2}}(A,B)}\, \underbrace{d\cos\theta}_{=1}\, |M|^2$$

after integration

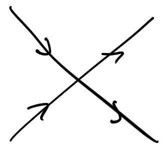
$$= \frac{1}{16\pi} \cdot \frac{1}{\lambda(A,B)}\, |M|^2$$

H8.2

a) $[J_\mu^\pm] = 2[\psi] = 3 \quad \rightarrow \quad [G_F] + 2[J_\mu^\pm] \overset{!}{=} 4$

$\rightarrow [G_F] = -2$

b)

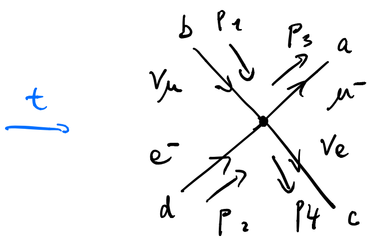


$\sim -i\frac{4}{\sqrt{2}} G_F (\gamma_\nu P_L)_{cd} (\gamma^\nu P_L)_{ab}$

$(1-\gamma_5)$ V - A process

parity max. violated

$\nu_\mu(p_1)\, e^-(p_2) \longrightarrow \mu^-(p_3)\, \nu_e(p_4)$



$$i\mathcal{M} = [u(p_1)]_b [u(p_2)]_d (-i)\frac{4G_F}{\sqrt{2}} (\gamma_\nu P_L)_{cd} (\gamma^\nu P_L)_{ab} [\bar{u}(p_3)]_a [\bar{u}(p_4)]_c$$

$$= \frac{-4i}{\sqrt{2}}\, \bar{u}(p_4)(\gamma_\nu P_L) u(p_2) \cdot \bar{u}(p_3)(\gamma^\nu P_L) u(p_1) \quad ✓$$

c) $[\bar{u}(p_i)\gamma_\nu(\mathbb{1}_4 - \gamma_5) u(p_j)]^*$

$= (\dots)^\dagger$

$= u^\dagger(p_j)\, (\mathbb{1}_4 - \gamma_5)^\dagger \gamma_\nu^\dagger \gamma^0 u(p_i) \quad , \quad \gamma_\nu^\dagger = \gamma^0 \gamma_\nu \gamma^0$

$= \bar{u}(p_j)\, (\mathbb{1}_4 + \gamma_5) \gamma_\nu u(p_i) \quad = \bar{u}(p_j) \gamma_\nu (\mathbb{1}_4 - \gamma_5) u(p_i)$

d)

$$\overline{|\mathcal{M}|^2} = \frac{G_F^2}{2} \sum_{\text{spins}} \left|\frac{-4i}{\sqrt{2}}\right|^2 \bar{u}(p_4)(\gamma_\nu P_L) u(p_2) \cdot \bar{u}(p_3)(\gamma^\nu P_L) u(p_1) \cdot \bar{u}(p_1)(\gamma^\mu P_L) u(p_3)$$

$$\times\, \bar{u}(p_2)(\gamma_\mu P_L) u(p_4)$$

$$= 4G_F^2 \sum_{\text{spins}} \bar{u}(p_4)_a (\gamma_\nu P_L)_{ab} u(p_2)_b \cdot \bar{u}(p_3)_c (\gamma^\nu P_L)_{cd} u(p_1)_d \cdot \bar{u}(p_1)_i (\gamma^\mu P_L)_{ij} u(p_3)_j$$

$$\times\, \bar{u}(p_2)_m (\gamma_\mu P_L)_{mn} u(p_4)_n$$

$$= 4G_F^2 \, tr[\underbrace{u_n(p_4)\bar{u}_a(p_4)}_{=(\not{p}_4)_{na}}(\gamma_\nu P_L)_{ab}\underbrace{u_b(p_2)\bar{u}_m(p_2)}_{=(\not{p}_2+m_e)_{bm}}(\gamma_\mu P_L)_{mn}] \cdot tr[\cdots]$$

$$= 4G_F^2 \, tr[\not{p}_4(\gamma_\nu \underbrace{P_L}_{\frac{1}{2}(1-\gamma_5)})(\not{p}_2+m_e)(\gamma_\mu P_L)] \, tr[(\not{p}_3+m_\mu)(\gamma^\nu \underbrace{P_L}_{\frac{1}{2}(1-\gamma_5)})\not{p}_1(\gamma^\mu P_L)]$$

e) $$= \frac{G_F^2}{4} tr[\not{p}_4\gamma_\nu(1-\gamma_5)(\not{p}_2+m_e)\gamma_\mu(1-\gamma_5)] \, tr[(\not{p}_3+m_\mu)\gamma^\nu(1-\gamma_5)\not{p}_1\gamma^\mu(1-\gamma_5)]$$

ignor m_e, m_μ

$P_L^2 = P_L$

$$= \frac{G_F^2}{4} tr[\underbrace{\not{p}_4\gamma_\nu(1-\gamma_5)\not{p}_2\gamma_\mu}_{A_{\nu\mu}}(1-\gamma_5)] \, tr[\not{p}_3\gamma^\nu(1-\gamma_5)\not{p}_1\gamma^\mu(1-\gamma_5)]$$

$= 2(1-\gamma_5)$

$$A_{\nu\mu} = tr[\not{p}_4\gamma_\nu(1-\gamma^5)(\not{p}_2+m_e)\gamma_\mu]$$

$$= tr[\not{p}_4\gamma_\nu\not{p}_2\gamma_\mu + \underbrace{\not{p}_4\gamma_\nu\gamma_\mu m_e}_{=0} - \not{p}_4\gamma_\nu\gamma^5\not{p}_2\gamma_\mu - \underbrace{m_e\not{p}_4\gamma_\nu\gamma^5\gamma_\mu}_{=0}]$$

$= \vdots$

$$= \frac{G_F^2}{4}\left[tr(\not{p}_4\gamma_\nu\not{p}_2\gamma_\mu) - tr[\not{p}_4\gamma_\nu\not{p}_2\gamma_\mu\gamma_5] - tr[\not{p}_4\gamma_\nu\gamma_5\not{p}_2\gamma_\mu] + tr[\not{p}_4\gamma_\nu\gamma_5\not{p}_2\gamma_\mu\gamma_5]\right.$$
$$\left.\times[(4,2)\leftrightarrow(3,1)]\right]$$

$$= G_F^2[\,tr(\not{p}_4\gamma_\nu\not{p}_2\gamma_\mu) - tr(\not{p}_4\gamma_\nu\not{p}_2\gamma_\mu\gamma_5)] \times tr[(4,2)\leftrightarrow(3,1)]$$

$$= 4p_4^\alpha p_2^\beta(g_{\alpha\nu}g_{\beta\mu} - g_{\alpha\beta}g_{\nu\mu} + g_{\alpha\mu}g_{\nu\beta})$$

$$= 4(p_{4\nu}\cdot p_{2\mu} - p_4\cdot p_2\cdot g_{\nu\mu} + p_{4\mu}p_{2\nu})$$

$$= -4i\,\epsilon_{\alpha\nu\beta\mu}p_4^\alpha p_2^\beta$$

product $= 0$

symm in $\mu\nu$ anti-symm in $\mu\nu$

$$= 4G_F^2\left[\{(p_4)_\nu\cdot(p_2)_\mu - p_4\cdot p_2\, g_{\mu\nu} + (p_4)_\mu(p_2)_\nu\} + i\epsilon_{\alpha\nu\beta\mu}p_4^\alpha p_2^\beta\right]$$

$$\times\left[\{(p_3)^\nu(p_1)^\mu - p_3\cdot p_1\cdot g^{\mu\nu} + (p_3)^\mu(p_1)^\nu\} + i\epsilon^{\lambda\nu\sigma\mu}(p_3)_\lambda(p_1)_\sigma\right]$$

$$= 4 G_F^2 \{ (p_3\cdot p_4)(p_1\cdot p_2) - (p_1\cdot p_3)(p_2\cdot p_4) + (p_2\cdot p_3)(p_1\cdot p_4) + i\epsilon^{\lambda\nu\sigma\mu}(p_4)_\nu (p_2)_\mu (p_3)_\lambda (p_1)_\sigma$$

$$-(p_2\cdot p_4)(p_1\cdot p_3) + 4(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_3)\cancel{(p_4\cdot p_2)}$$

$$+(p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)\cancel{(p_2\cdot p_4)} + (p_1\cdot p_2)(p_3\cdot p_4) + i\epsilon^{\lambda\nu\sigma\mu}(p_4)_\mu (p_2)_\nu (p_3)_\lambda (p_1)_\sigma$$

$$+ i\epsilon_{\alpha\nu\beta\mu} p_4^\alpha p_2^\beta (p_3)^\nu (p_1)^\mu + i\epsilon_{\alpha\nu\beta\mu} p_4^\alpha p_2^\beta p_3^\mu p_1^\nu$$

$$- \epsilon_{\alpha\nu\beta\mu}\epsilon^{\lambda\nu\sigma\mu}\, p_{1\sigma} p_2^\beta p_{3\lambda} p_4^\alpha \}$$

$$= \epsilon_{\mu\nu\beta\alpha}\epsilon^{\mu\nu\sigma\lambda} = 2!\,\delta^{\sigma\lambda}_{\beta\alpha} = 2!(\delta^\sigma_\beta\delta^\lambda_\alpha - \delta^\sigma_\alpha\delta^\lambda_\beta)$$

Terms with (one) ϵ:

$$\sim \epsilon^{\lambda\nu\sigma\mu}(p_1)_\sigma (p_2)_\mu (p_3)_\lambda (p_4)_\nu + \epsilon^{\lambda\nu\sigma\mu}(p_1)_\sigma (p_2)_\nu (p_3)_\lambda (p_4)_\mu$$

$$+ \epsilon_{\alpha\nu\beta\mu}(p_1)^\mu p_2^\beta (p_3)^\nu (p_4)^\alpha + \epsilon_{\alpha\nu\beta\mu} p_1^\nu p_2^\beta p_3^\mu p_4^\alpha$$

$$= 0$$

$$= 4G_F^2 [2(p_1\cdot p_2)\cancel{(p_3\cdot p_4)} - 2(p_1\cdot p_3)(p_2\cdot p_4) + 2(p_1\cdot p_4)(p_2\cdot p_3)$$

$$- 2\cancel{(p_1\cdot p_2)}(p_3\cdot p_4) + 2(p_1\cdot p_4)(p_2\cdot p_3)]$$

$$= 16 G_F^2 [(p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)(p_2\cdot p_4)]$$

USUALLY, one should have sth. similar in weak sector if $m_i \equiv 0$

$$64 G_F^2 (p_1\cdot p_2)(p_3\cdot p_4)$$

f) $p_1 = (E_1, 0, 0, p)$ $\quad p_2 = (E_2, 0, 0, -p)$

$p_3 = (E_3, 0, p\sin\theta, p\cos\theta)$ $\quad p_4 = (E_4, 0, -p\sin\theta, -p\cos\theta)$

$$E_1 + E_2 = E_3 + E_4 = \sqrt{s}$$

$$\rightarrow (p_1\cdot p_4) = E_1 E_4 - p^2\cos\theta$$

$$(p_1\cdot p_3) = E_1 E_3 + p^2\cos\theta$$

$$\rightarrow (p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_3)(p_2\cdot p_4)$$

$$= (E_1 E_4 - p^2\cos\theta)^2 - (E_1 E_3 + p^2\cos\theta)^2$$

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}\,\frac{1}{2}\sum|M|^2 \overset{!}{=} \frac{G_F^2}{\pi}$$

$$\rightarrow \frac{1}{2}\sum|M|^2 = 16\cancel{\pi} s^2 \frac{G_F^2}{\cancel{\pi}}$$

$$\rightarrow \overline{|M|^2} = 16 G_F^2 s^2 \quad \rightarrow \frac{d\sigma}{dt} = \frac{\overline{|M|^2}}{16\pi^2 s} \quad \Rightarrow \sigma = \frac{G_F^2 s}{\pi}$$

kinda expected from non-rel. theory

(g) $G_F = \frac{\sqrt{2}}{8} \frac{g^2}{m_W^2}$,

$$\rightarrow \sigma(s) = \frac{1}{32\pi} \frac{g^4}{m_W^4} \frac{s}{\frac{s}{m_W^2}+1}$$

$$= \frac{1}{32\pi} \left(\frac{8G_F}{\sqrt{2}}\right)^2 \frac{s}{\frac{s}{m_W^2}+1}$$

$s \to \infty$ $\longrightarrow \frac{1}{32\pi} \left(\frac{8G_F}{\sqrt{2}}\right)^2$

YES!.

So if $s/m_W^2 << 1$, then $\rightarrow$ (2)

$\Leftrightarrow s << m_W^2$