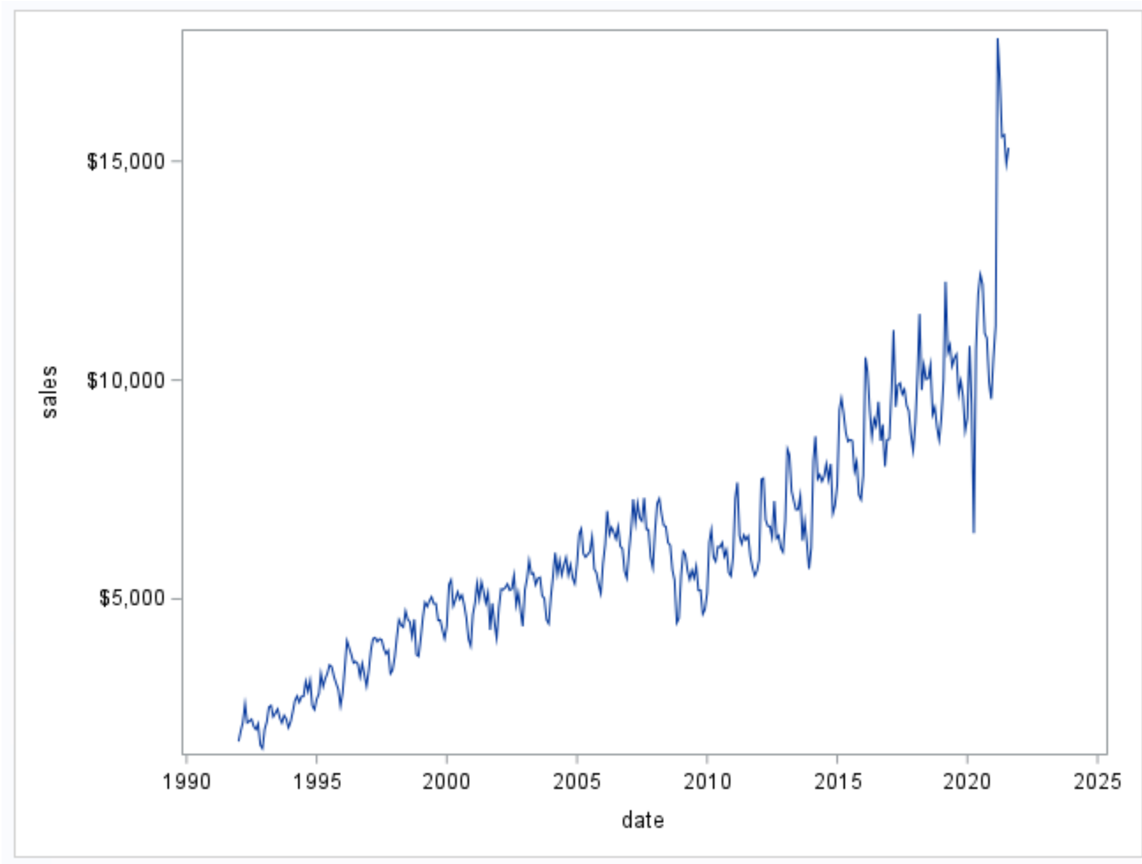


Time Series Analysis

USED CAR SALES

David Arthur
Tim Clanton
John Williams

ST 534 | November 2021

1. Introduction

Our goal with this report is to demonstrate the use of time series analysis for fitting a model to real-world data and using that model to predict future events.

Time series analysis is the use of statistical reasoning to identify behaviors in data which includes time as an explanatory variable. The inclusion of time as a measurement of the data creates a natural ordering for our data and permits the assumption that earlier measurements of our response variable may have some effect on the measurement of that variable in the future. Importantly, this means that our measurements are inherently not independent, a common assumption in other instances of statistical analysis.

2. The Data Set

We will be analyzing retail sales numbers (in millions of U.S. Dollars) of used car dealers in the United States, using data from the Research division of the Federal Reserve Bank of St. Louis, Federal Reserve Economic Data (https://fred.stlouisfed.org). The sales figures are reported as monthly data, and we will be examining the period from January 1992 through August 2021, which is the most recent data available as of the time of this report.

More information on the data set can be found at https://fred.stlouisfed.org/series/MRTSSM44112USN.

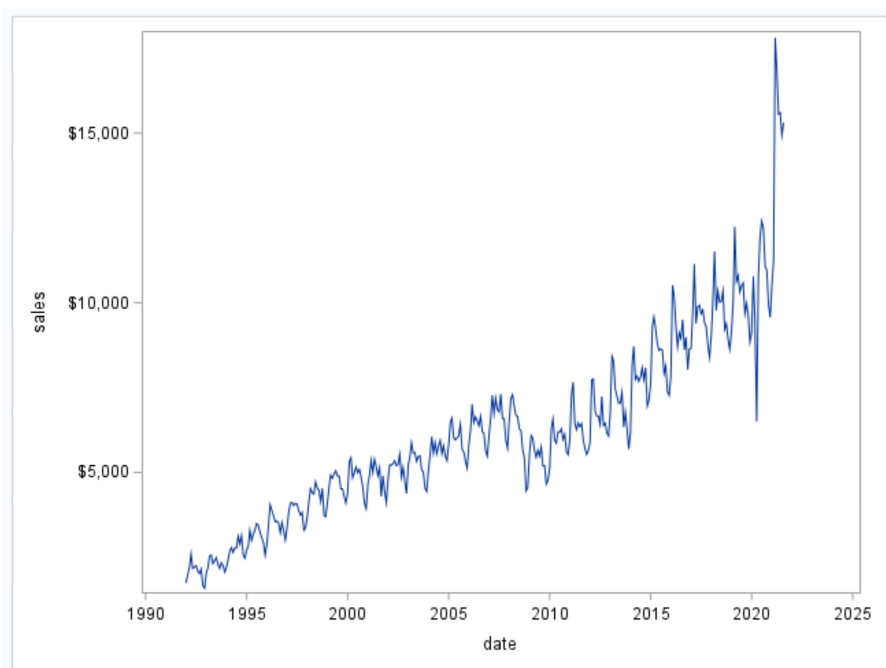


Figure 1: Used Car Sales (January 1992 - August 2021)

A visual examination of these data in Figure 1 reveals several important factors we need to consider. First, we can see that the fluctuations gradually increase in magnitude as we proceed forward through time with a marked jump in variance beginning in 2009-2010 and another beginning in 2020. Before we can begin to work with these data, we will first need to ensure that the variance remains relatively constant throughout. To address this, we will use Box-Cox analysis, which we discuss in the next section.

The dramatic increase in fluctuation beginning in 2020, likely due to economic shifts caused by the COVID-19 pandemic, suggests that an intervening shift may have altered Used Car sales in a manner beyond what would be expected from random real-world noise. Therefore, we will also conduct an intervention analysis and examine whether that produces a better model for these data.

We also notice a general upward trend through the data, suggesting that the mean of Used Car sales has not remained constant over time. If that is the case, we will need to address this piece of our model for the data before we can use software to estimate the remaining parameters. We accomplish this using the Dickey-Fuller analysis, which will be discussed in further detail in Section 3.2.

3. Exploring an ARIMA Model

3.1. Box Cox Analysis

To stabilize the variance across all our data, we will simply transform the data to even out the fluctuations. While such a transformation will change the value of the sales data, and therefore, also alter the mean and variance of that data, it will not meaningfully change the relationships between the sales in different months.

We will use Box-Cox analysis to determine the stabilizing factor we will use to conduct this transformation. The estimated factor provided by our Box-Cox analysis below is $\lambda = 0.25$, indicating that we should take a fourth root of our sales data to stabilize the variance across our dataset. We note that the 95% confidence interval for this estimate is very narrow suggesting we can be reasonably confident that this transformation will provide the necessary stabilization.

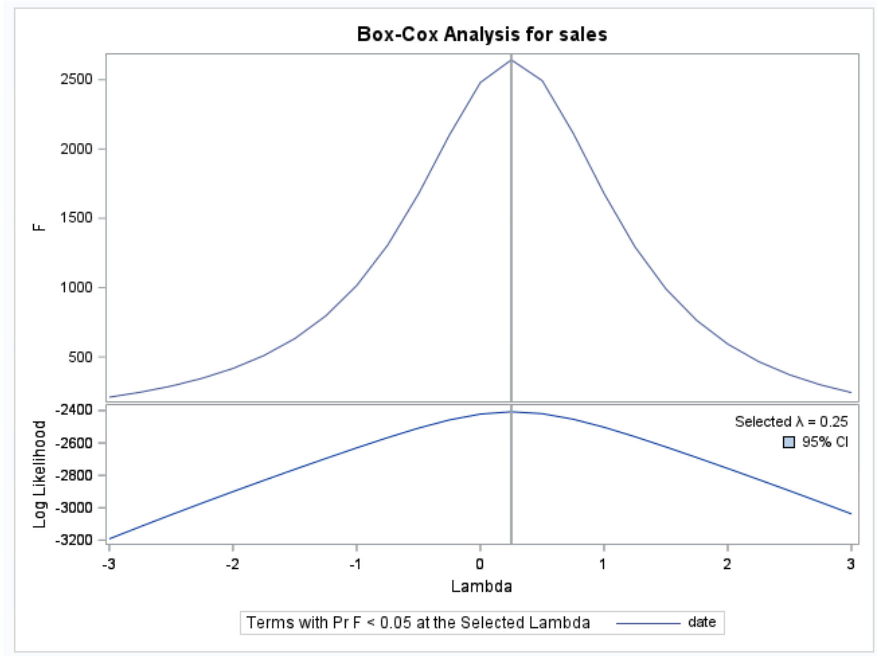


Figure 2

After transforming the data, we obtain a plot of the new data which is included in Figure 3. The transformation appears to have been successful; the variance in our new sales data remains reasonably consistent throughout. There remains a significant increase in fluctuation in the most recent data which is not unexpected. However, for the purposes of this initial analysis, we will assume these data points will not dramatically affect our analysis.

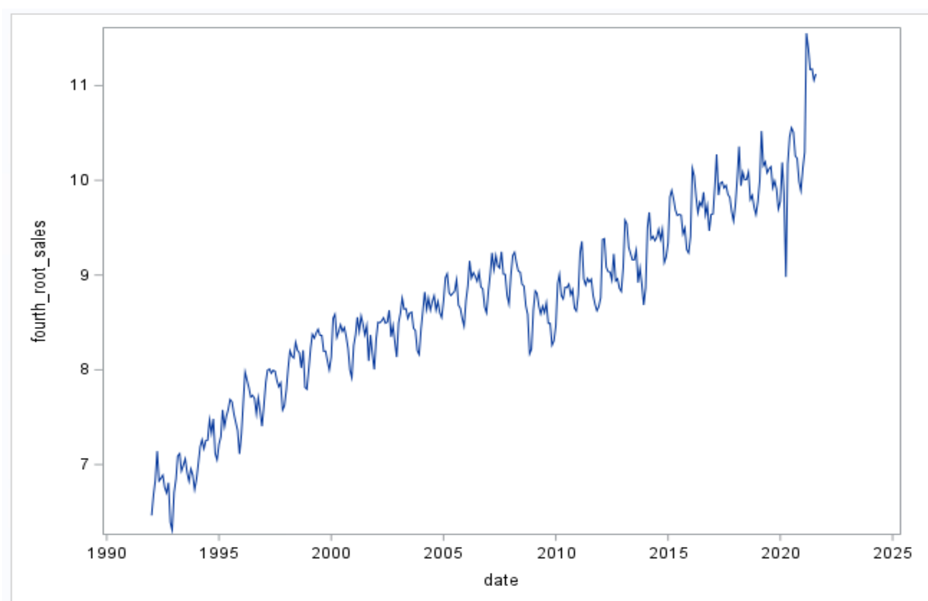


Figure 3: Data after using fourth root transformation

3.2. DICKEY-FULLER TESTS

Our next step is to determine if we need to be concerned with any unit roots in the polynomial representation of the process which describes our data. Such factors would indicate a one-to-one relationship between each data point and those adjacent to it. Unfortunately, the software struggles to identify and estimate such factors and therefore, we must remove them prior to estimating the other autoregressive (AR) and moving average (MA) factors. We resolve this complication be simply taking the difference of every pair of adjacent points and performing the next phase of analysis on those differences instead.

We use the Dickey-Fuller Unit Root Test to determine if this "differencing" is necessary. Notably, the null hypothesis of the Dickey-Fuller Unit Root Test is that differencing is required.

Based on the graph of the transformed data above, we can assume the mean follows a consistent linear trend. This makes sense given that this is economic data and will therefore be affected by inflation and general economic growth, even if there is no specific growth within the used car dealer industry. Looking in the "Trend" section of the table below, we note that the probability of obtaining a value for Tau less than or equal to that which we found is nearly zero. Therefore, we reject the hypothesis that differencing is needed.

Augmented Dickey-Fuller Unit Root Tests							
Type	**Lags**	**Rho**	**Pr < Rho**	**Tau**	**Pr < Tau**	**F**	**Pr > F**
Zero Mean	0	0.4305	0.7888	0.94	0.9081		
	1	0.4162	0.7851	0.93	0.9060		
	2	0.4347	0.7899	1.13	0.9337		
Single Mean	0	-8.1776	0.2062	-1.98	0.2971	2.64	0.3955
	1	-7.6169	0.2358	-1.83	0.3636	2.33	0.4748
	2	-5.2177	0.4129	-1.44	0.5631	1.88	0.5900
Trend	0	-71.7816	0.0007	-6.23	<.0001	19.44	0.0010
	1	-85.5565	0.0007	-6.38	<.0001	20.37	0.0010
	2	-75.7645	0.0007	-5.62	<.0001	15.82	0.0010

Figure 4

3.3. TREND AND CORRELATION ANALYSIS

Having decided against differencing this data, we obtain the Trend and Correlation Analysis presented in Figure 5. This analysis includes a plot of our transformed data as well as plot of the Autocorrelation Function (ACF), the Partial Autocorrelation Function (PACF), and the Inverse Autocorrelation Function (IACF), which all provide information regarding the relationship between our sales data.

Looking at these plots, the first thing we notice is the gradual decline in the ACF, which appears to plateau every 12 lags before resuming its decline. We also notice spikes in the PACF every 12 lags. These two features together suggest that we should conduct yearly seasonal differencing: we will take the difference of each data point with that twelve months prior. Such differencing will eliminate any one-to-one relationship between sales numbers which are 12 months apart.

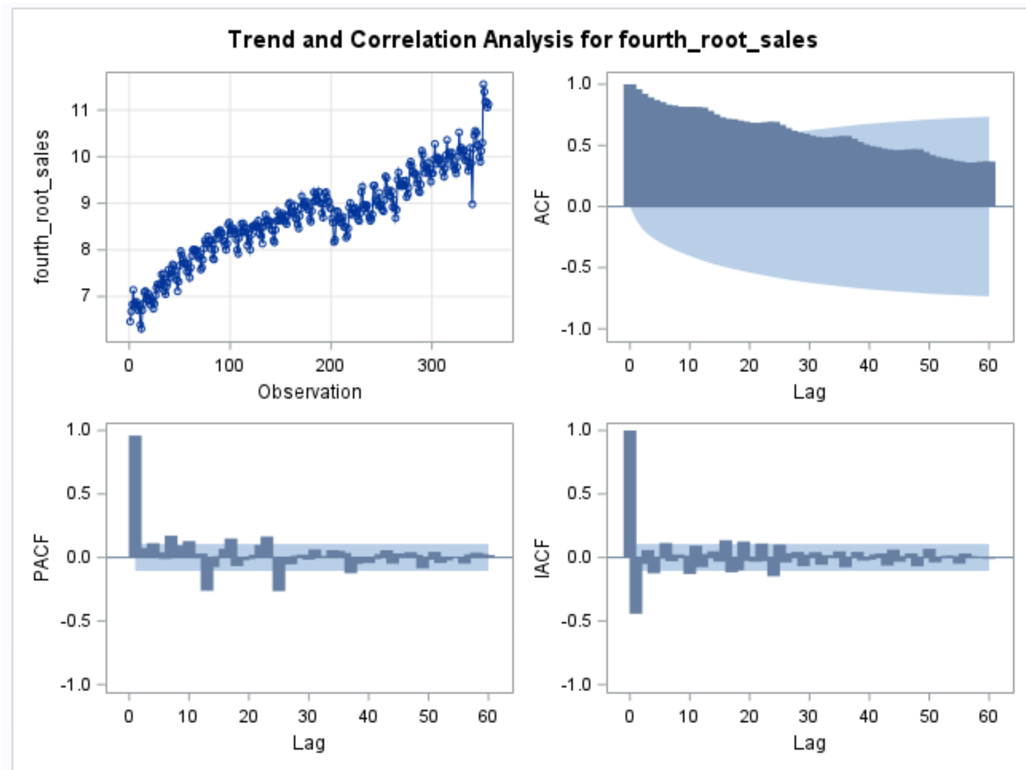


Figure 5

3.4. After Seasonal Differencing

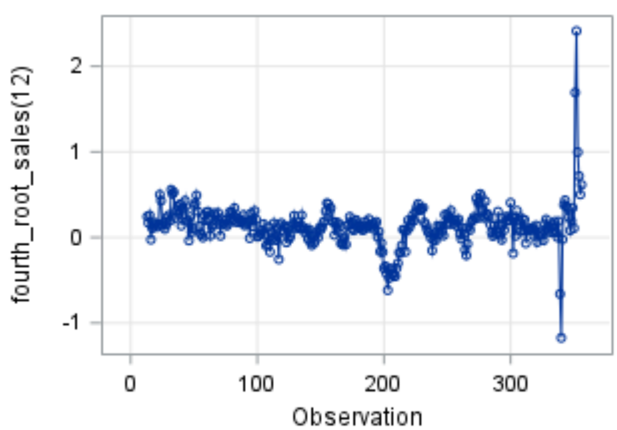


Figure 6: Data after yearly seasonal differencing

3.4.1. Dickey-Fuller Test

After removing the 12-month seasonal unit root, we want to ensure that the seasonality was not masking the need for regular differencing. Therefore, we once again run a Dickey-Fuller Unit Root Test. This time, we look to the "Zero Mean" portion of the table produced by the software, as the seasonal differencing will have eliminated any constant terms within the process that would have moved the mean away from zero.

Once again, the test rejects the null hypothesis that differencing is necessary, indicating that we can proceed with the next step of our analysis.

Augmented Dickey-Fuller Unit Root Tests							
Type	Lags	Rho	Pr < Rho	Tau	Pr < Tau	F	Pr > F
Zero Mean	0	-77.5051	<.0001	-6.51	<.0001		
	1	-77.2152	<.0001	-6.08	<.0001		
	2	-48.9585	<.0001	-4.60	<.0001		
Single Mean	0	-101.841	0.0001	-7.63	<.0001	29.14	0.0010
	1	-110.373	0.0001	-7.30	<.0001	26.65	0.0010
	2	-75.0383	0.0016	-5.64	<.0001	15.93	0.0010
Trend	0	-101.814	0.0001	-7.62	<.0001	29.08	0.0010
	1	-110.314	0.0001	-7.29	<.0001	26.58	0.0010
	2	-74.8944	0.0007	-5.63	<.0001	15.93	0.0010

3.4.2. Trend and Correlation Analysis

Now that we have finished transforming our data, we can begin attempting to fit models to the transformed data. These models are produced by using software to find estimates of the coefficients in each model framework. Therefore, we need to examine the Trend and Correlation Analysis of our fully transformed data, presented as Figure 7 below, and determine what models should be considered.

Beginning with the ACF plot, we notice a gradual decline beginning at lag 1, with a significant spike at lag 12 and nothing significant following that point. Furthermore, we notice a spike at lag 1 in the PACF plot and declining spikes at every 12 lags. These two facts together suggest that a good model may contain one AR factor and one seasonal MA factor at lag 12. This would be the most parsimonious model we could find, so it is where we will begin.

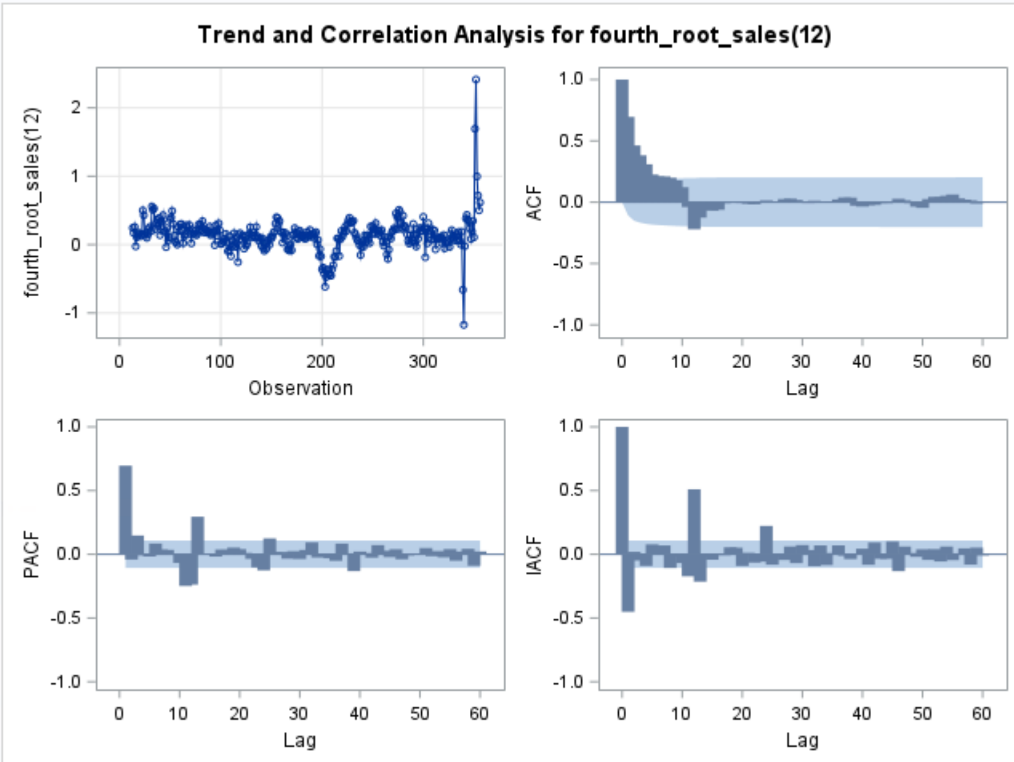


Figure 7

3.5. Model Fitting

Model 1: ARIMA(1,0,0)×(0,1,1)$_{12}$

For this simple model, we notice that all the estimates are found to be significant. Next, we note that none of the estimates are highly correlated with any other. Therefore, there is no reason to believe that interplay between them is interfering with our analysis.

Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	0.12525	0.01608	7.79	<.0001	0
MA1,1	0.69841	0.04393	15.90	<.0001	12
AR1,1	0.84255	0.03104	27.14	<.0001	1

Correlations of Parameter Estimates			
Parameter	MU	MA1,1	AR1,1
MU	1.000	-0.029	0.015
MA1,1	-0.029	1.000	0.256
AR1,1	0.015	0.256	1.000

Unfortunately, the Chi-Square test of the residuals for white noise rejects the null hypothesis that there is no correlation among the residual values.

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	13.76	4	0.0081	-0.099	-0.089	0.060	0.015	-0.045	0.125
12	38.51	10	<.0001	-0.008	0.086	0.104	0.195	-0.021	-0.112
18	48.43	16	<.0001	-0.038	0.138	0.007	-0.029	0.054	0.055
24	56.05	22	<.0001	-0.013	0.064	0.020	-0.030	0.034	0.117
30	58.85	28	0.0006	-0.044	0.037	0.032	0.001	0.055	0.008
36	62.19	34	0.0022	-0.020	0.077	-0.014	0.006	0.005	0.046
42	66.34	40	0.0055	0.019	0.085	-0.045	-0.021	0.025	0.004
48	71.55	46	0.0093	0.056	-0.001	-0.036	0.089	-0.008	0.026
54	79.92	52	0.0077	0.017	-0.108	0.088	0.021	-0.012	0.020
60	83.96	58	0.0145	0.070	-0.038	0.024	0.020	-0.045	-0.019

This is further supported by the Residual Correlation analysis, which shows significant correlations in the ACF, PACF, and IACF, and shows a low probability of white noise.

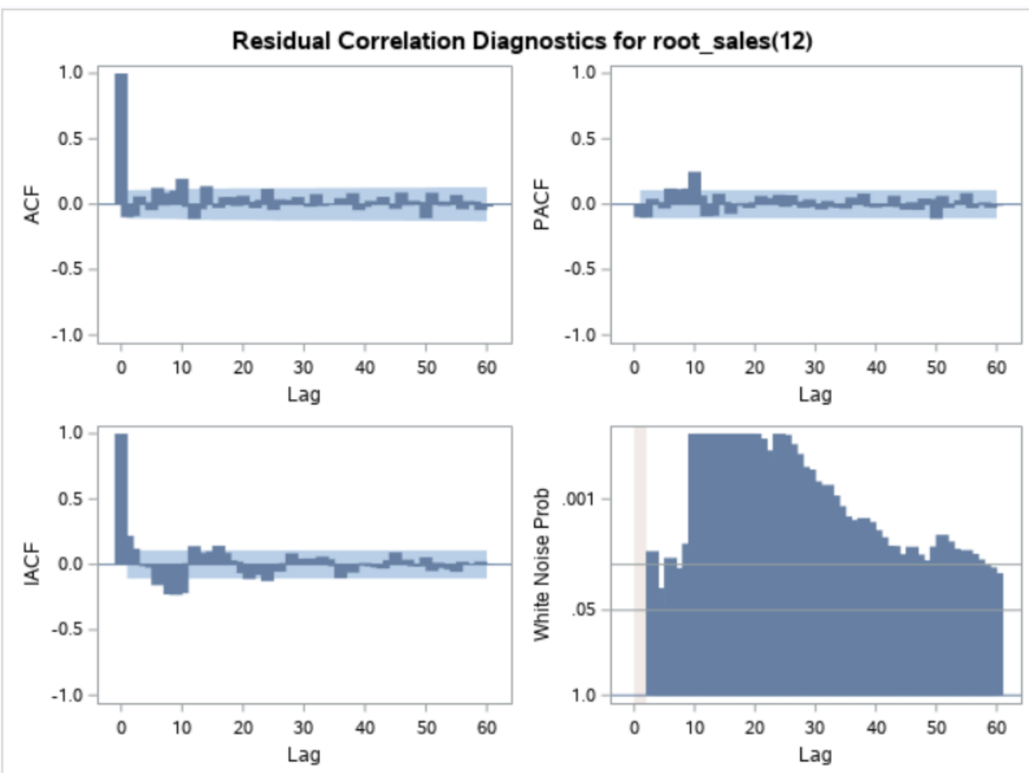


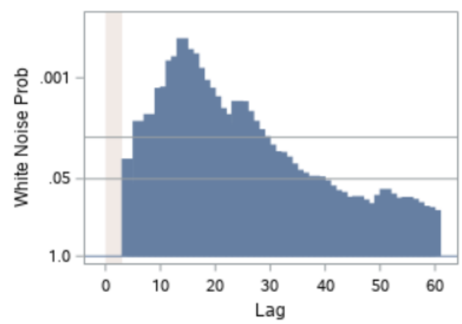
Figure 8

Therefore, this model is not adequate for describing the relationship of monthly used car sales to that of prior months.   Looking at the ACF, PACF, and IACF plots in Figure 8, we can see small spikes at lags 3, 6, 9, and 11. Therefore, we will attempt to fit several models with each of these factors in search of a model that adequately represents these data. As before, we will work with the lags that appear to have the strongest correlative effects first and progress onto more complex models only as necessary.

There are many variations that can be attempted: different combinations of factors at the aforementioned lags and different combinations of MA and/or AR factors where appropriate. Not every model is included in this report; in particular, many of those with factors which were found to be not significant have been omitted.

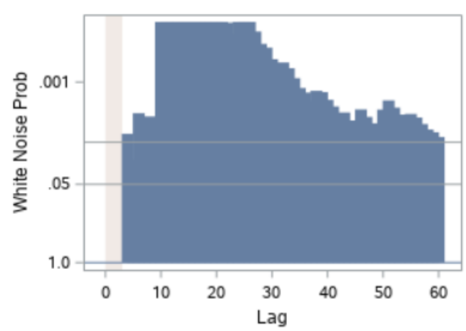
MODEL 2: AR(1,9)MA(12)[1]

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	12.69	3	0.0053	-0.063	-0.013	0.093	0.045	-0.035	0.142
12	29.59	9	0.0005	0.014	0.112	-0.001	0.131	-0.073	-0.110
18	37.46	15	0.0011	-0.078	0.099	-0.041	-0.058	0.012	0.025
24	43.84	21	0.0025	-0.042	0.036	-0.008	-0.043	-0.011	0.110
30	45.82	27	0.0133	-0.061	0.016	0.005	-0.021	0.028	0.004
36	48.59	33	0.0393	-0.024	0.055	-0.036	-0.013	-0.017	0.043
42	52.16	39	0.0773	-0.005	0.067	-0.053	-0.034	0.012	-0.021
48	55.87	45	0.1285	0.043	-0.011	-0.048	0.061	-0.029	0.020
54	63.26	51	0.1165	0.013	-0.103	0.080	-0.007	-0.013	0.029
60	67.16	57	0.1680	0.071	-0.039	0.022	0.013	-0.043	-0.020



MODEL 3: AR(1,9)MA(9,12)

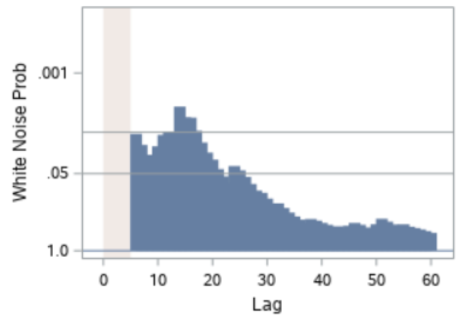
Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	13.71	3	0.0033	-0.102	-0.068	0.065	0.038	-0.042	0.129
12	38.00	9	<.0001	0.005	0.102	0.041	0.210	-0.013	-0.109
18	48.52	15	<.0001	-0.037	0.147	-0.019	-0.025	0.052	0.049
24	55.90	21	<.0001	-0.010	0.072	-0.014	-0.018	0.025	0.117
30	58.33	27	0.0004	-0.043	0.041	0.017	0.001	0.051	0.002
36	62.20	33	0.0016	-0.012	0.079	-0.042	0.010	0.003	0.043
42	66.84	39	0.0036	0.017	0.085	-0.055	-0.022	0.027	-0.008
48	73.09	45	0.0051	0.064	0.000	-0.056	0.089	-0.012	0.021
54	81.07	51	0.0047	0.025	-0.108	0.083	0.013	-0.014	0.016
60	85.53	57	0.0086	0.078	-0.039	0.011	0.023	-0.042	-0.027



[1] All ARIMA models in Section 3.5 include a seasonal difference at lag 12.

MODEL 4: AR(1,6,9,12)MA(12)

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	6.53	1	0.0106	-0.053	-0.010	0.093	0.069	0.044	0.022
12	18.57	7	0.0096	-0.070	0.058	-0.026	0.107	-0.089	-0.075
18	26.48	13	0.0147	-0.016	0.126	-0.036	-0.066	-0.005	0.007
24	31.32	19	0.0372	-0.019	0.033	-0.002	-0.025	0.016	0.104
30	32.96	25	0.1320	-0.041	0.023	0.007	-0.006	0.044	-0.010
36	34.59	31	0.3003	-0.013	0.056	-0.016	0.003	-0.015	0.022
42	38.33	37	0.4088	-0.004	0.076	-0.039	-0.031	0.032	-0.015
48	44.50	43	0.4084	0.060	-0.023	-0.073	0.071	-0.032	0.002
54	51.03	49	0.3938	0.013	-0.090	0.085	-0.008	-0.007	0.022
60	54.18	55	0.5061	0.068	-0.011	0.033	0.031	-0.024	-0.016



MODEL 5: AR(1,2,6,9,12)MA(11,12)

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	.	0	.	-0.007	0.042	0.004	-0.008	-0.029	0.016
12	9.29	5	0.0982	-0.073	0.088	-0.046	0.063	-0.027	-0.057
18	13.96	11	0.2352	-0.044	0.086	-0.023	-0.052	-0.020	0.001
24	19.02	17	0.3276	-0.020	0.024	-0.008	-0.035	0.060	0.088
30	19.87	23	0.6497	-0.021	0.026	-0.009	-0.018	0.027	-0.003
36	22.16	29	0.8134	-0.002	0.058	-0.040	-0.004	0.020	0.024
42	25.85	35	0.8696	-0.001	0.075	-0.052	-0.021	0.016	-0.020
48	32.21	41	0.8353	0.065	-0.015	-0.081	0.068	-0.017	-0.010
54	37.42	47	0.8400	0.018	-0.084	0.065	-0.011	0.003	0.032
60	40.52	53	0.8955	0.076	-0.018	0.026	0.018	-0.017	-0.015

At this point, our chi-square test for white noise fails to reject white noise at the 5% alpha level for all lags up to 60 so this model may be adequate. However, our plot of white noise probability (see Figure 9) still shows a low probability at lower lags, indicating that this model could be better.

Looking at the estimates themselves, we note that all our factors are found to be significant, and none is highly correlated with any other.

Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	0.12862	0.02486	5.17	<.0001	0
MA1,1	0.12024	0.04611	2.61	0.0095	11
MA1,2	0.66212	0.05074	13.05	<.0001	12
AR1,1	0.64888	0.04293	15.12	<.0001	1
AR1,2	0.15859	0.04746	3.34	0.0009	3
AR1,3	0.17818	0.05060	3.52	0.0005	6
AR1,4	0.21432	0.05563	3.85	0.0001	9
AR1,5	-0.26825	0.05532	-4.85	<.0001	12

Correlations of Parameter Estimates								
Parameter	MU	MA1,1	MA1,2	AR1,1	AR1,2	AR1,3	AR1,4	AR1,5
MU	1.000	-0.046	-0.004	-0.025	-0.020	0.030	0.056	0.030
MA1,1	-0.046	1.000	-0.077	0.075	0.078	-0.098	0.243	-0.152
MA1,2	-0.004	-0.077	1.000	0.106	-0.075	-0.134	-0.125	0.467
AR1,1	-0.025	0.075	0.106	1.000	-0.502	-0.106	-0.246	0.116
AR1,2	-0.020	0.078	-0.075	-0.502	1.000	-0.239	0.025	-0.260
AR1,3	0.030	-0.098	-0.134	-0.106	-0.239	1.000	-0.350	-0.237
AR1,4	0.056	0.243	-0.125	-0.246	0.025	-0.350	1.000	-0.455
AR1,5	0.030	-0.152	0.467	0.116	-0.260	-0.237	-0.455	1.000

Examining the updated ACF and PACF in Figure 9, it is difficult to say what factor may be missing from this model. However, the IACF suggests that a factor may be missing at lag 8, but it is unclear whether it should be an AR factor or an MA factor.

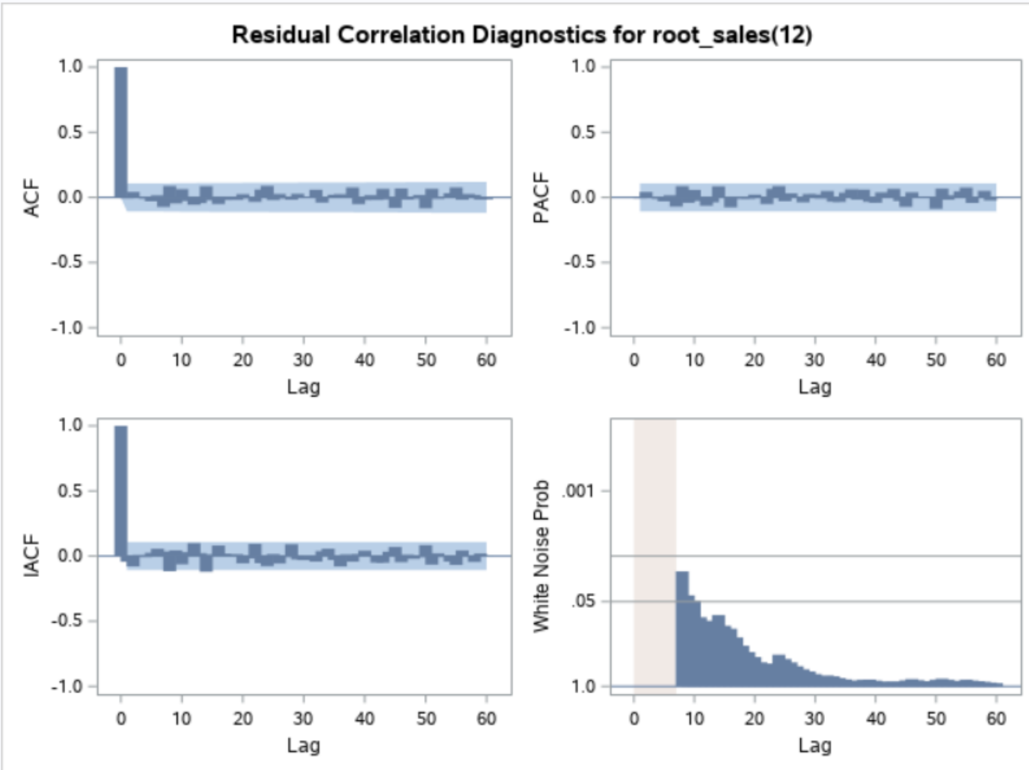


Figure 9

MODEL 6: AR(1,3,6,9,12)MA(8,11,12)

Taking the results of Model 5 into consideration, we next attempt to include a MA factor at lag 8. This factor is found to be significant and does not remove the significance of any of our prior factors.

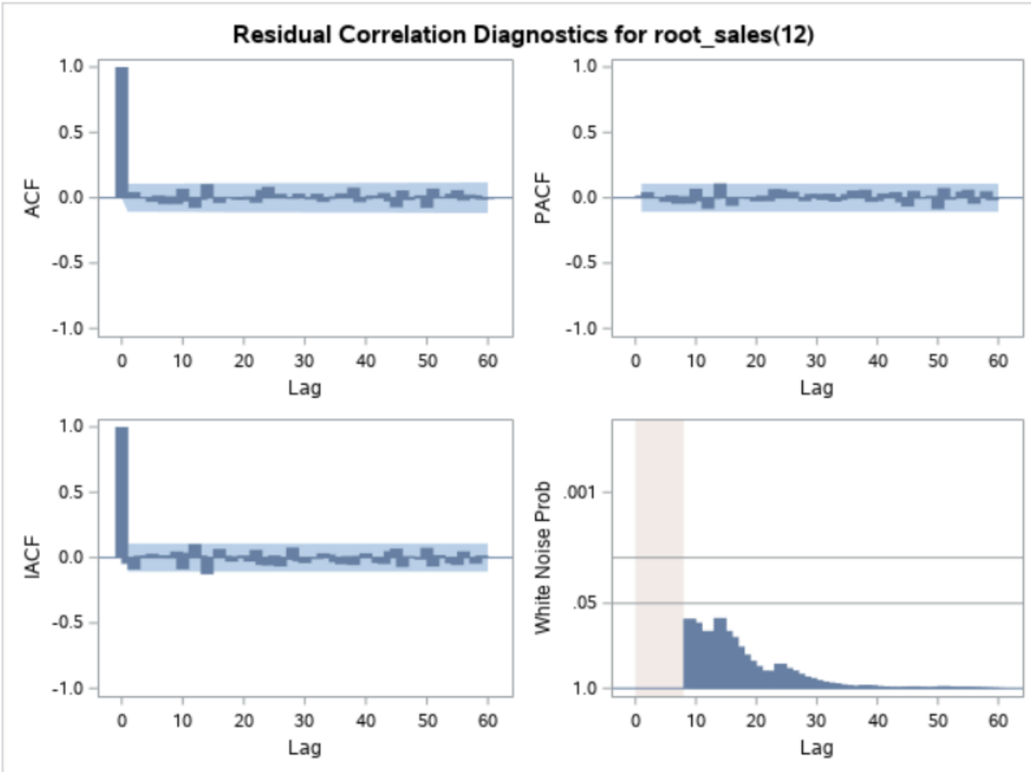
Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	0.12845	0.02538	5.06	<.0001	0
MA1,1	-0.11666	0.04601	-2.54	0.0117	8
MA1,2	0.10362	0.04685	2.21	0.0277	11
MA1,3	0.62720	0.05143	12.19	<.0001	12
AR1,1	0.63861	0.04275	14.94	<.0001	1
AR1,2	0.16576	0.04687	3.54	0.0005	3
AR1,3	0.13158	0.05298	2.48	0.0135	6
AR1,4	0.23276	0.05602	4.15	<.0001	9
AR1,5	-0.28066	0.05539	-5.07	<.0001	12

Furthermore, none of the factors are highly correlated with each other.

Correlations of Parameter Estimates									
Parameter	MU	MA1,1	MA1,2	MA1,3	AR1,1	AR1,2	AR1,3	AR1,4	AR1,5
MU	1.000	0.003	-0.053	0.003	-0.029	-0.022	0.019	0.049	0.036
MA1,1	0.003	1.000	0.024	0.150	0.080	-0.030	0.278	-0.115	0.004
MA1,2	-0.053	0.024	1.000	-0.187	0.097	-0.003	-0.045	0.286	-0.219
MA1,3	0.003	0.150	-0.187	1.000	0.124	-0.090	-0.073	-0.120	0.466
AR1,1	-0.029	0.080	0.097	0.124	1.000	-0.495	-0.091	-0.172	0.104
AR1,2	-0.022	-0.030	-0.003	-0.090	-0.495	1.000	-0.237	-0.011	-0.221
AR1,3	0.019	0.278	-0.045	-0.073	-0.091	-0.237	1.000	-0.362	-0.182
AR1,4	0.049	-0.115	0.286	-0.120	-0.172	-0.011	-0.362	1.000	-0.441
AR1,5	0.036	0.004	-0.219	0.466	0.104	-0.221	-0.182	-0.441	1.000

Our chi-square test of the residuals fails to reject white noise up to lag 60 and the plot of white noise probability appears to show that the probability is sufficiently high at all lags as well. This suggests that this model is adequate for describing our data.

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	.	0	.	0.017	0.044	0.006	-0.001	-0.031	0.017
12	7.10	4	0.1307	-0.049	0.012	-0.049	0.069	-0.030	-0.077
18	11.75	10	0.3023	-0.016	0.105	-0.001	-0.041	-0.001	0.002
24	16.47	16	0.4204	-0.015	-0.015	0.010	-0.039	0.061	0.084
30	17.25	22	0.7493	0.005	0.030	0.006	-0.008	0.032	0.007
36	18.60	28	0.9100	-0.013	0.031	-0.033	-0.010	0.011	0.034
42	21.60	34	0.9510	0.014	0.077	-0.035	-0.011	0.017	-0.005
48	26.03	40	0.9569	0.037	-0.029	-0.074	0.056	-0.019	0.001
54	31.02	46	0.9556	0.018	-0.078	0.070	-0.003	0.009	0.031
60	33.08	52	0.9811	0.058	-0.021	0.024	0.018	-0.014	-0.010



MODEL 7: AR(1,3,6,8,9,12)MA(11,12)

Here in Model 7, we will switch the MA factor at lag 8 to an AR factor at lag 8. As with Model 6, the parameter at lag 8 is significant along with all others.

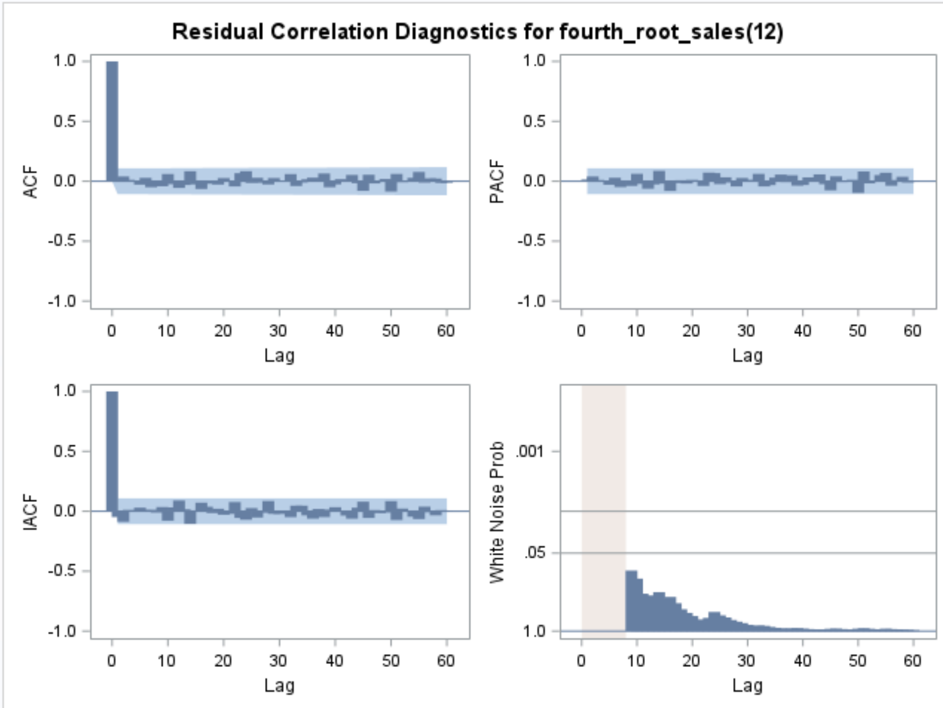
Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	0.12884	0.02608	4.94	<.0001	0
MA1,1	0.13030	0.04654	2.80	0.0054	11
MA1,2	0.65448	0.05145	12.72	<.0001	12
AR1,1	0.63733	0.04296	14.83	<.0001	1
AR1,2	0.16599	0.04727	3.51	0.0005	3
AR1,3	0.13799	0.05340	2.58	0.0102	6
AR1,4	0.13720	0.06366	2.16	0.0318	8
AR1,5	0.14282	0.06456	2.21	0.0276	9
AR1,6	-0.28479	0.05596	-5.09	<.0001	12

Also, none of the parameters considered are highly correlated.

Correlations of Parameter Estimates									
Parameter	MU	MA1,1	MA1,2	AR1,1	AR1,2	AR1,3	AR1,4	AR1,5	AR1,6
MU	1.000	-0.051	-0.004	-0.025	-0.020	0.024	0.006	0.046	0.030
MA1,1	-0.051	1.000	-0.126	0.049	0.095	-0.119	0.095	0.163	-0.184
MA1,2	-0.004	-0.126	1.000	0.129	-0.087	-0.085	-0.141	-0.045	0.484
AR1,1	-0.025	0.049	0.129	1.000	-0.505	-0.056	-0.127	-0.148	0.137
AR1,2	-0.020	0.095	-0.087	-0.505	1.000	-0.248	0.067	-0.009	-0.269
AR1,3	0.024	-0.119	-0.085	-0.056	-0.248	1.000	-0.333	-0.112	-0.166
AR1,4	0.006	0.095	-0.141	-0.127	0.067	-0.333	1.000	-0.512	-0.168
AR1,5	0.046	0.163	-0.045	-0.148	-0.009	-0.112	-0.512	1.000	-0.304
AR1,6	0.030	-0.184	0.484	0.137	-0.269	-0.166	-0.168	-0.304	1.000

As with Models 5 and 6, we can accept the hypothesis that the residuals are white noise; thus, we can conclude that Model 7 is adequate.

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	.	0	.	0.015	0.040	0.012	0.000	-0.028	0.028
12	5.37	4	0.2516	-0.050	0.004	-0.042	0.060	-0.024	-0.056
18	10.18	10	0.4250	-0.031	0.083	-0.027	-0.065	-0.022	0.000
24	15.64	16	0.4782	-0.028	0.026	-0.004	-0.044	0.067	0.083
30	16.62	22	0.7840	-0.015	0.028	-0.013	-0.027	0.026	-0.001
36	19.04	28	0.8969	0.003	0.059	-0.040	-0.011	0.016	0.029
42	22.04	34	0.9433	0.007	0.064	-0.050	-0.026	0.017	-0.004
48	27.08	40	0.9409	0.052	-0.017	-0.080	0.053	-0.019	-0.003
54	32.07	46	0.9407	0.018	-0.087	0.060	-0.008	0.003	0.028
60	35.13	52	0.9648	0.077	-0.011	0.024	0.022	-0.015	-0.013



Note, if we were to consider adding both the AR and MA parameter at lag 8 to Model 5, neither parameter is significant:

Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	0.12854	0.02620	4.91	<.0001	0
MA1,1	-0.06903	0.06202	-1.11	0.2665	8
MA1,2	0.12193	0.04712	2.59	0.0101	11
MA1,3	0.64080	0.05117	12.52	<.0001	12
AR1,1	0.63672	0.04294	14.83	<.0001	1
AR1,2	0.16721	0.04710	3.55	0.0004	3
AR1,3	0.12829	0.05358	2.39	0.0172	6
AR1,4	0.08150	0.08686	0.94	0.3488	8
AR1,5	0.18426	0.07586	2.43	0.0157	9
AR1,6	-0.28487	0.05628	-5.06	<.0001	12

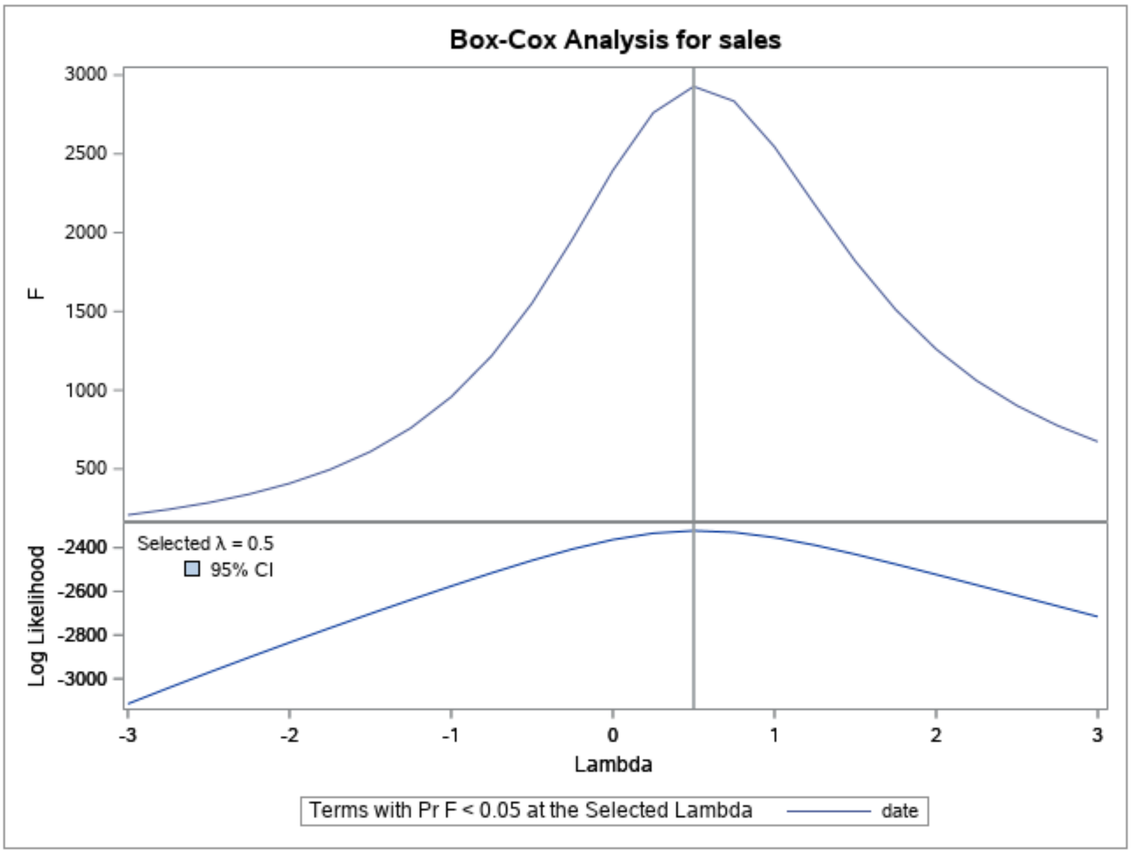
Thus, we will not consider this possible model further; only one parameter at lag 8 (either AR or MA) creates an adequate model, not both together.

4. Exploring an Intervention Model

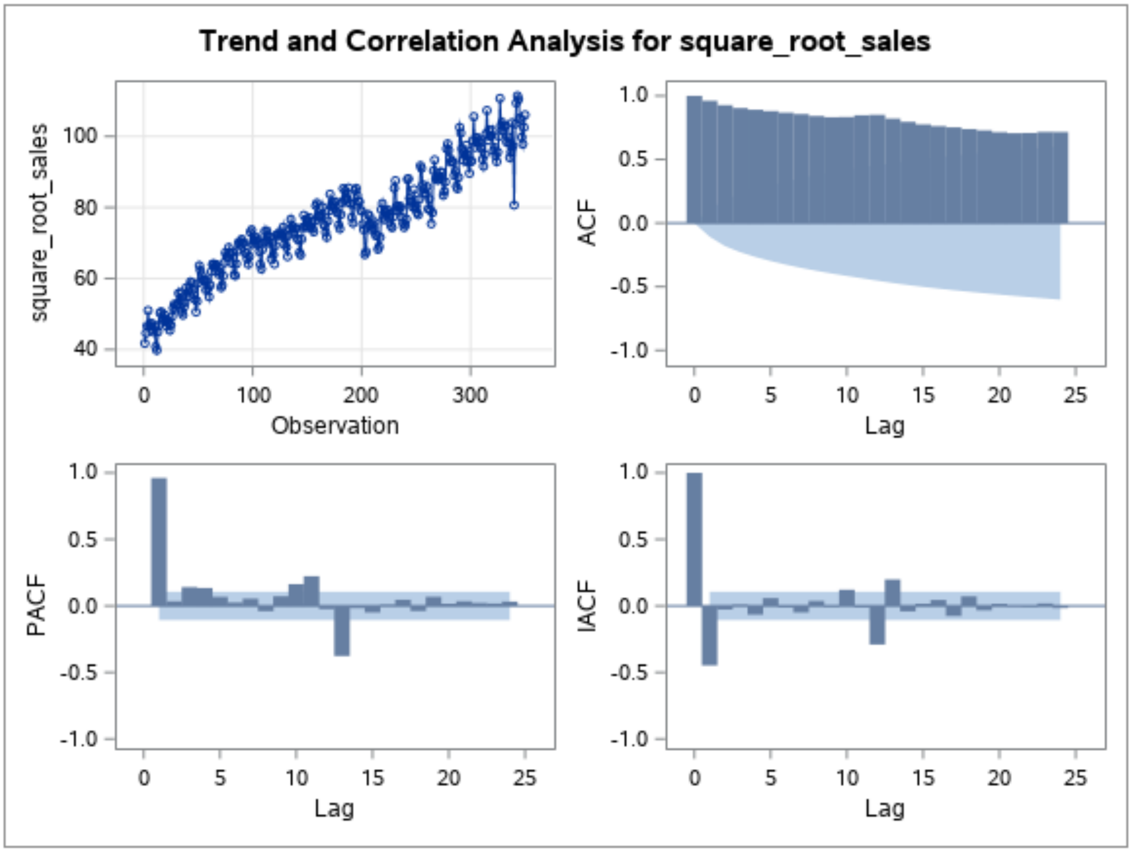
Reducing the residuals to white noise in the previous section resulted in an undesirably complex model. Noticing Figure 1 displays a drastic step up in sales beginning March 2021, we now examine whether a model incorporating an intervention effect will fit the data well with greater parsimony. First, we fit a model to the data from before the jump, then apply that model plus an intervention term to the full data set.

4.1. Model for data before jump

We start with a Box-Cox analysis of whether a variance-stabilizing transformation is needed:



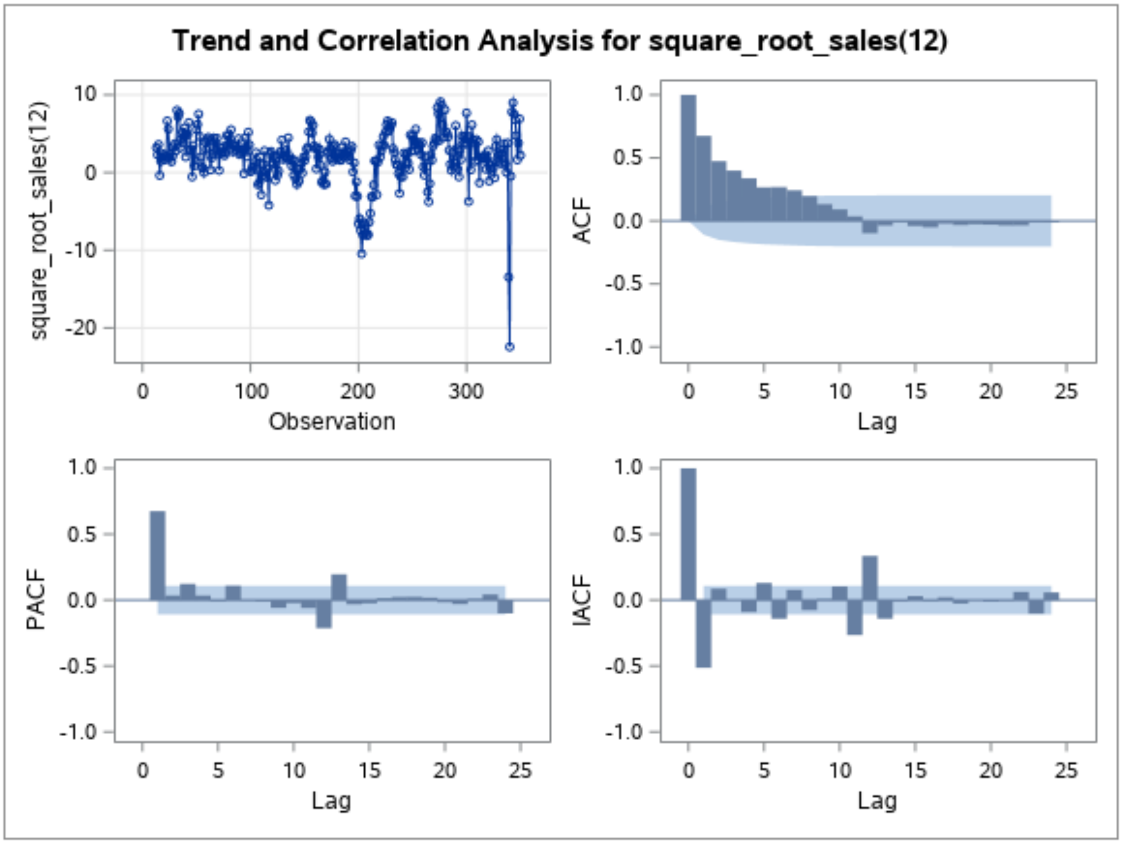
The resulting estimate of $\lambda = 0.5$ indicates the need for a square root transformation of the data. We apply that transformation, examine the Trend and Correlation Analysis, and conduct a Dickey-Fuller Test of the need for differencing:



Augmented Dickey-Fuller Unit Root Tests							
Type	**Lags**	**Rho**	**Pr < Rho**	**Tau**	**Pr < Tau**	**F**	**Pr > F**
Zero Mean	0	0.4081	0.7830	0.46	0.8146		
	1	0.4116	0.7839	0.48	0.8192		
	2	0.4903	0.8041	0.69	0.8647		
Single Mean	0	-9.9232	0.1351	-2.30	0.1725	3.10	0.2773
	1	-9.2500	0.1591	-2.17	0.2187	2.79	0.3566
	2	-6.5869	0.3011	-1.82	0.3679	2.25	0.4955
Trend	0	-88.7400	0.0007	-7.15	<.0001	25.53	0.0010
	1	-108.286	0.0001	-7.34	<.0001	26.95	0.0010
	2	-94.9827	0.0007	-6.44	<.0001	20.74	0.0010

Figure 10

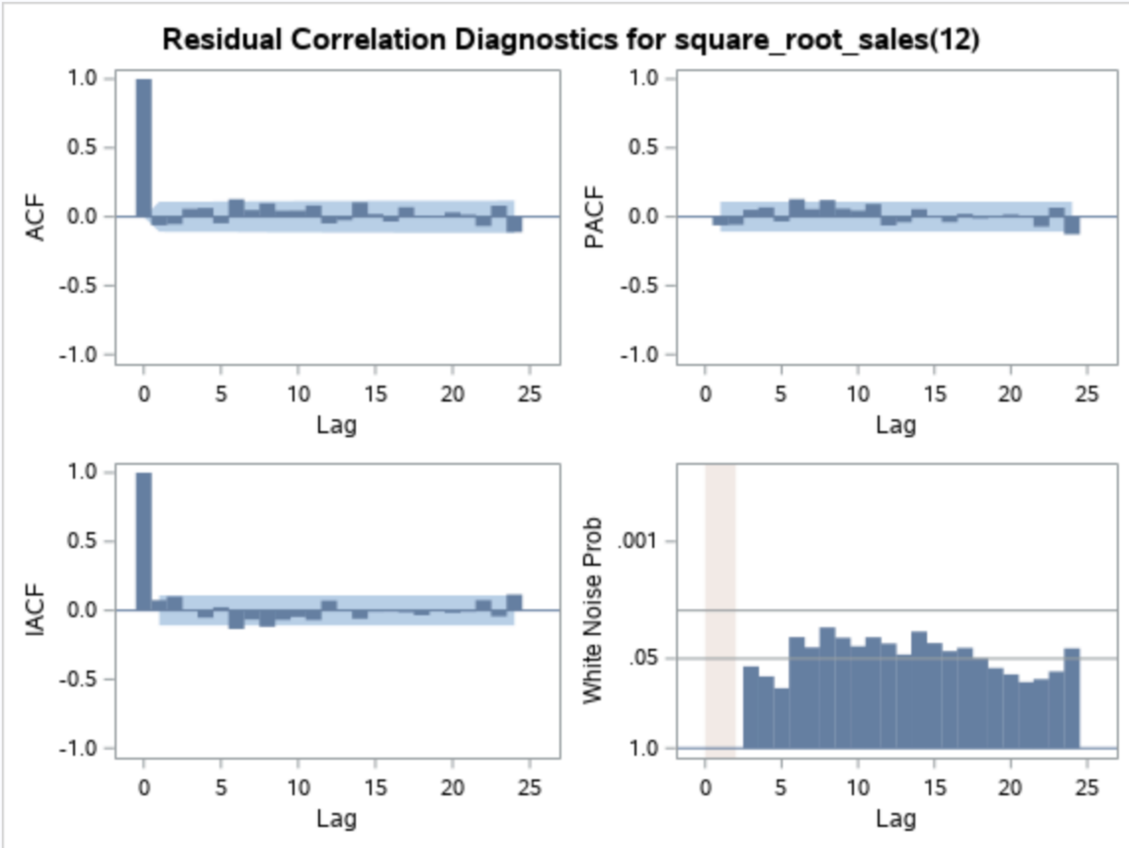
Considering the clear trend in the data, we look at the "Trend" section of the Dickey-Fuller Test results and conclude that we can reject the need for regular differencing. However, the ACF plot suggests we need to take a seasonal difference with a period of 12. We do that and inspect the updated Trend and Correlation Analysis:



The seasonally differenced data now appears stationary. The ACF and PACF plots look like an AR(1)(12), so we try estimating that model:

MODEL 8: ARIMA(1,0,0)×(1,1,0)$_{12}$

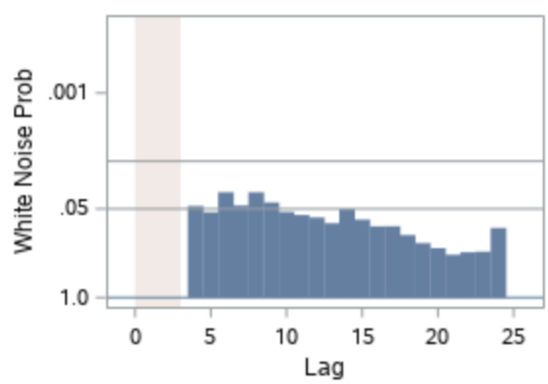
Conditional Least Squares Estimation					
Parameter	**Estimate**	**Standard Error**	**t Value**	**Approx Pr > |t|**	**Lag**
MU	2.03580	0.33480	6.08	<.0001	0
AR1,1	0.73870	0.03744	19.73	<.0001	1
AR2,1	-0.44669	0.06614	-6.75	<.0001	12



Both AR terms are highly significant. We test several variations on this model to see if we can increase the probability of white noise for the residuals, and reduce AIC:

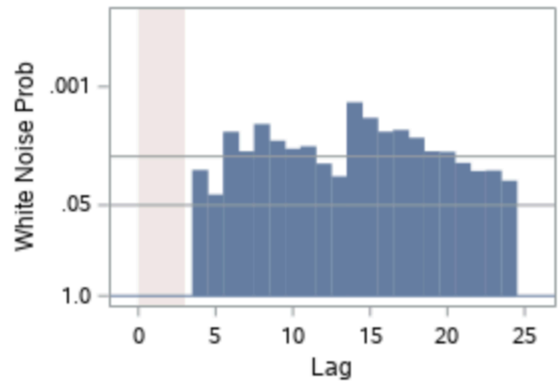
MODEL 9: ARIMA(1,0,1)×(1,1,0)$_{12}$

Constant Estimate	0.531431
Variance Estimate	5.412246
Std Error Estimate	2.326423
AIC	1533.947
SBC	1549.239
Number of Residuals	338



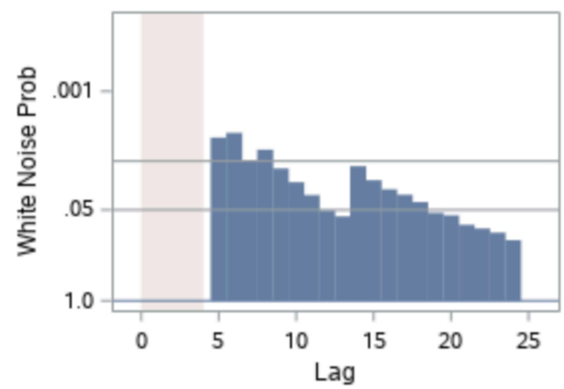
MODEL 10: ARIMA(1,0,0)×(1,1,1)$_{12}$

Constant Estimate	0.482796
Variance Estimate	5.320168
Std Error Estimate	2.306549
AIC	1528.147
SBC	1543.439
Number of Residuals	338



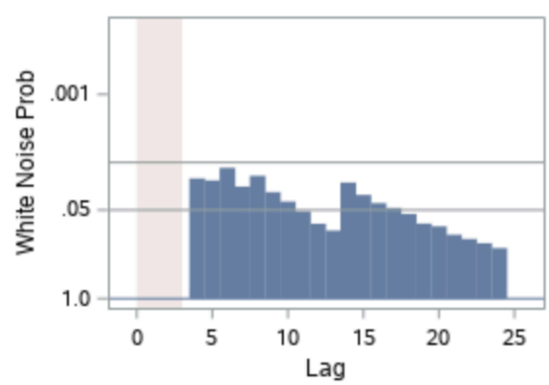
MODEL 11: ARIMA(1,0,1)×(1,1,1)$_{12}$

Constant Estimate	0.28001
Variance Estimate	5.231813
Std Error Estimate	2.287316
AIC	1523.473
SBC	1542.588
Number of Residuals	338



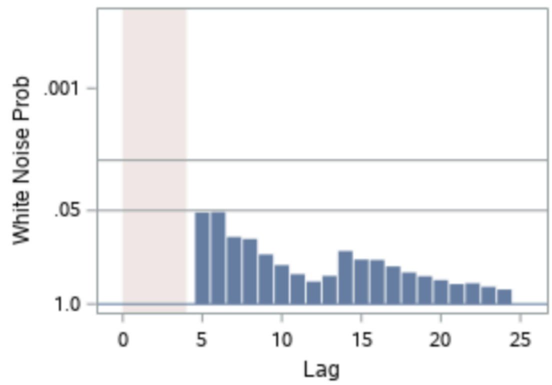
MODEL 12: ARIMA(1,0,1)×(0,1,1)$_{12}$

Constant Estimate	0.261076
Variance Estimate	5.223076
Std Error Estimate	2.285405
AIC	1521.922
SBC	1537.214
Number of Residuals	338



MODEL 13: ARIMA(2,0,1)×(0,1,1)$_{12}$

Constant Estimate	0.036919
Variance Estimate	5.057942
Std Error Estimate	2.248987
AIC	1512.049
SBC	1531.165
Number of Residuals	338

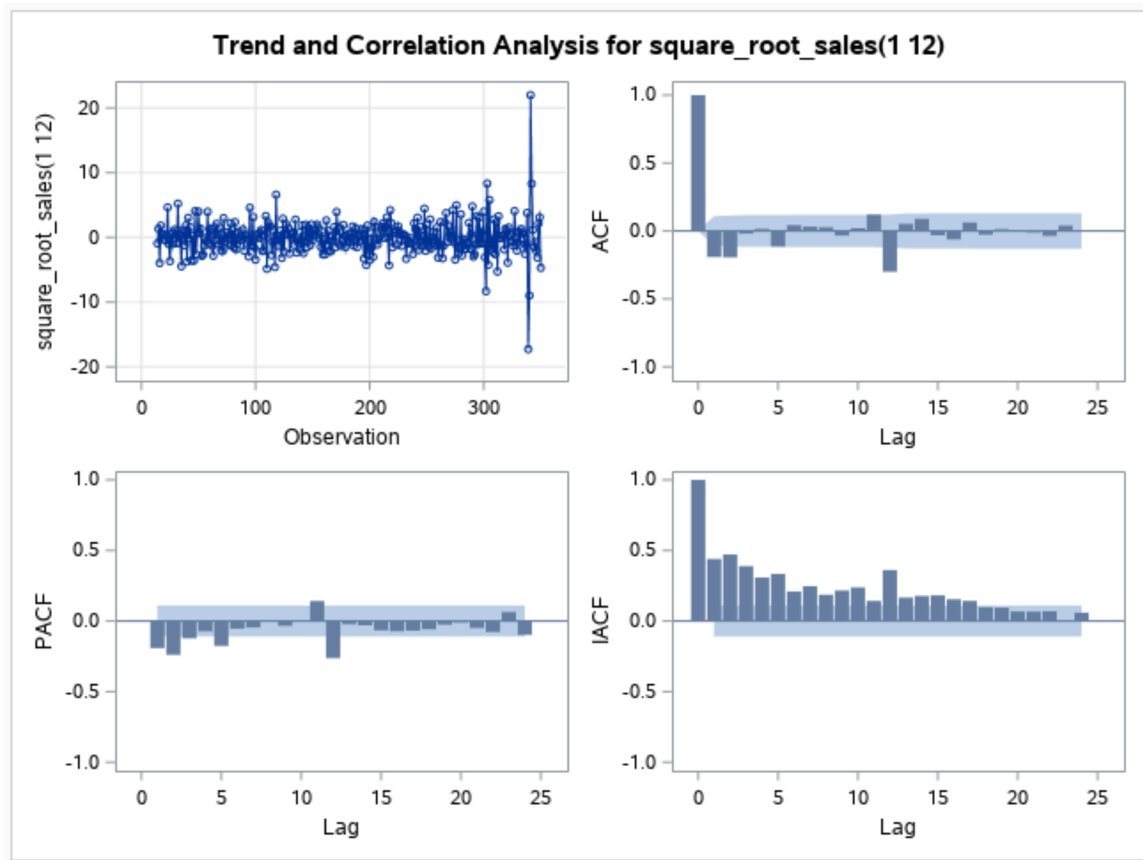


Model 13 has the greatest probability of white noise residuals, and the smallest AIC. We inspect the model's coefficients:

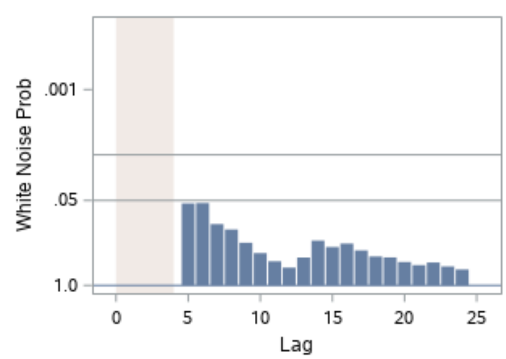
Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	2.01564	0.45577	4.42	<.0001	0
MA1,1	0.80938	0.08378	9.66	<.0001	1
MA2,1	0.62893	0.06021	10.45	<.0001	12
AR1,1	1.43546	0.10993	13.06	<.0001	1
AR1,2	-0.45378	0.09972	-4.55	<.0001	2

Looking at the AR terms, we see that $(1 - 1.435B + 0.454B^2)$ has a root equal to 1.036, which is very close to 1, suggesting that the model may need a regular difference in addition to the seasonal difference, despite the results of the Dickey-Fuller Test from Figure 10. We try a variation of Model 13 including both a regular and seasonal difference:

Model 14: ARIMA(2,1,1)×(0,1,1)$_{12}$



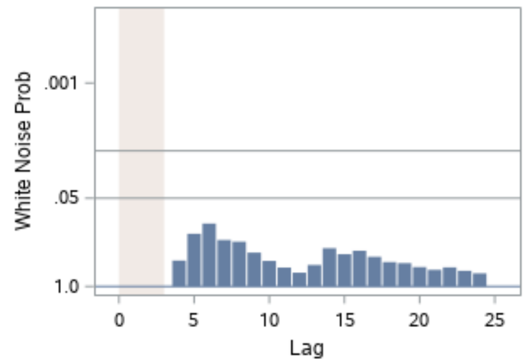
Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	-0.0027551	0.01370	-0.20	0.8408	0
MA1,1	0.84571	0.05515	15.33	<.0001	1
MA2,1	0.64153	0.05585	11.49	<.0001	12
AR1,1	0.50134	0.07750	6.47	<.0001	1
AR1,2	-0.03294	0.06445	-0.51	0.6096	2



This model results in white noise residuals, but the AR(2) term is no longer significant, so we fit a model without that term:

MODEL 15: ARIMA(1,1,1)×(0,1,1)$_{12}$

Conditional Least Squares Estimation					
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag
MU	-0.0028789	0.01305	-0.22	0.8255	0
MA1,1	0.86278	0.04435	19.45	<.0001	1
MA2,1	0.64341	0.05557	11.58	<.0001	12
AR1,1	0.50700	0.07572	6.70	<.0001	1



Constant Estimate	-0.00142
Variance Estimate	5.136612
Std Error Estimate	2.26641
AIC	1511.805
SBC	1527.086
Number of Residuals	337

This model has residuals with the highest probability of white noise, and lowest AIC, so we choose it as our model for the data prior to March 2021. We will apply it to the full data set with the addition of an intervention effect, as described in the next subsection.

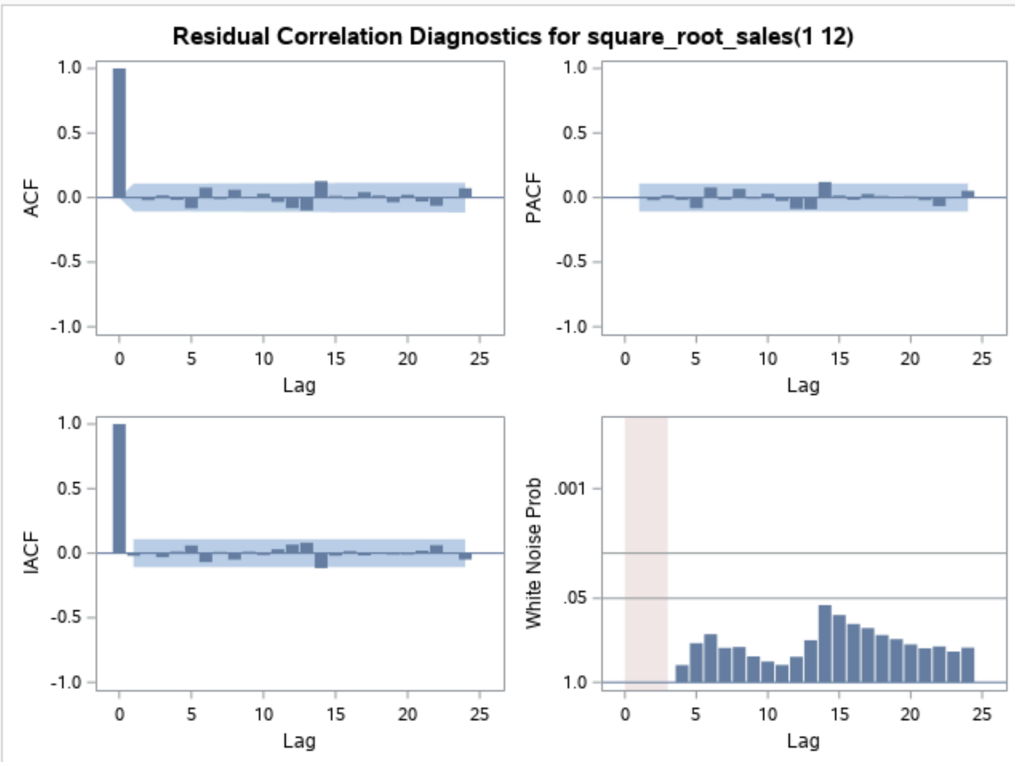
4.2. INTERVENTION MODEL

The intervention effect will account for the jump in total sales that occurs in March of 2021 and lasts through August 2021, the end of our data set. The interaction effect is modeled with a deterministic input variable represented by a step function, $I_t^{(T)} = 0$ if $t < T$, and $I_t^{(T)} = 1$ if $t \geq T$, where T is March 2021.

MODEL 16: ARIMA(1,1,1)×(0,1,1)$_{12}$ + I_1

Maximum Likelihood Estimation							
Parameter	Estimate	Standard Error	t Value	Approx Pr > |t|	Lag	Variable	Shift
MA1,1	0.84743	0.04780	17.73	<.0001	1	square_root_sales	0
MA2,1	0.79965	0.03756	21.29	<.0001	12	square_root_sales	0
AR1,1	0.48361	0.07763	6.23	<.0001	1	square_root_sales	0
NUM1	21.69224	1.95273	11.11	<.0001	0	supply_chain	0

Autocorrelation Check of Residuals									
To Lag	Chi-Square	DF	Pr > ChiSq	Autocorrelations					
6	4.89	3	0.1800	0.008	-0.020	0.017	-0.018	-0.083	0.078
12	9.36	9	0.4044	-0.011	0.061	-0.000	0.030	-0.034	-0.082
18	19.68	15	0.1844	-0.100	0.128	0.014	-0.007	0.043	0.016
24	24.05	21	0.2906	-0.035	0.022	-0.030	-0.062	0.003	0.073
30	26.06	27	0.5153	-0.038	0.005	0.019	-0.045	0.038	-0.010
36	29.23	33	0.6555	-0.033	0.061	-0.037	-0.039	-0.000	0.022
42	33.98	39	0.6977	0.036	0.059	-0.045	-0.064	0.023	-0.027
48	37.12	45	0.7919	0.045	-0.008	-0.064	0.038	-0.004	-0.017



This model fits well, with all terms highly significant, and the residuals appearing reasonably to be white noise. Under the square root transformation of the data, the model can be represented as:

$$(1-0.484B)Z_t = 21.692I_t(t) + \frac{(1-0.847B)(1-0.800B^{12})}{(1-B)(1-B^{12})}a_t$$

5. Choosing the "Best" Model

One criterion for comparing models is Akaike's Information Criteria (AIC). AIC measures the quality of a model's fit by balancing how closely the model fits the data with the number of parameters in the model. This approach avoids overfitting, with the goal of fitting the signal, not the noise. Schwartz's Bayesian Criterion (SBC) is similar to AIC but imposes a steeper penalty on increasing the number of parameters. For K = # of parameters, n = # of observations, and L is the likelihood of the model:

$$AIC(K) = -2log(L) + 2K$$
$$SBC(K) = -2log(L) + Klog(n)$$

Smaller values are better for both AIC and BIC.

One limitation of AIC and SBC is that they cannot be used to compare models that use different transformations. Therefore, we cannot compare the AIC of our non-intervention models (fourth-root transformation) with our intervention model (square root transformation).

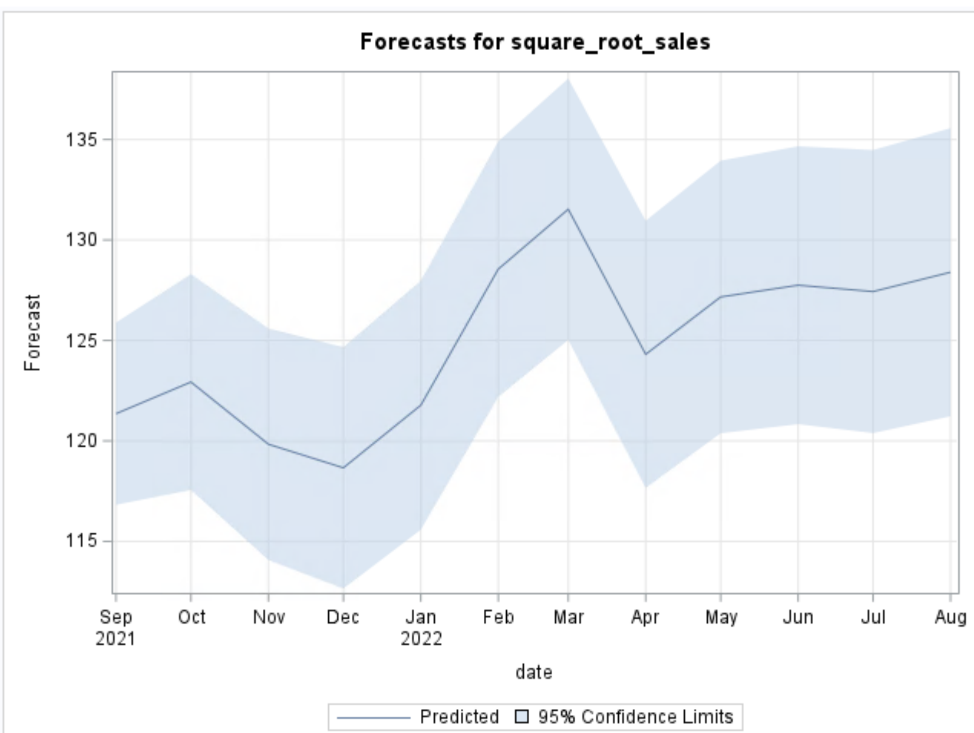
The table below compares the AIC and SBC of the non-intervention models that adequately produced residuals as white noise. Our non-intervention model with the lowest AIC was Model 6. Model 6 and Model 16 (the intervention model) were similarly effective in producing residuals that resemble white noise, but Model 16 is more parsimonious, so we choose Model 16 as our "best" model.

Adequate Non-Intervention Models	AIC	SBC
Model 5	-383.8	-353.6
Model 6	-386.8	-352.3
Model 7	-386.6	-352.0

6. Forecast – 1 Year Out

Using our "best" model (Model 16), we can forecast used car sales for the next 12 months. The output provides 95% confidence intervals for each month's forecasted sales. Note that these forecasts and confidence limits are measured in square-root dollars. We must square the forecasts and limits to get the estimates in dollars.

Forecasts for variable square_root_sales				
Obs	Forecast	Std Error	95% Confidence Limits	
357	121.3551	2.3123	116.8229	125.8872
358	122.9313	2.7406	117.5598	128.3028
359	119.8351	2.9400	114.0729	125.5974
360	118.6608	3.0653	112.6529	124.6687
361	121.7784	3.1611	115.5828	127.9740
362	128.5492	3.2434	122.1922	134.9063
363	131.5286	3.3190	125.0235	138.0336
364	124.3089	3.3906	117.6634	130.9544
365	127.1634	3.4597	120.3825	133.9443
366	127.7484	3.5270	120.8356	134.6612
367	127.4329	3.5928	120.3912	134.4746
368	128.3946	3.6572	121.2265	135.5627



Since the Federal Reserve Economic Data has been updated after our original analysis of pre-September 2021 data, we have access to actual sales for September 2021: $15,536. Let's see if our "best" model accurately forecasted used car sales for September 2021. Model 16 estimates used car sales of $(\$121.36)^2 = \$14{,}727$, with a 95% confidence interval of $13,648 to $15,848. Yes, the actual sales fall within that 95% confidence interval.

The table below summarizes how well a sample of the models considered predict September 2021 used car sales. The intervention model (Model 16) has the closest forecast to actual; it also includes actual in its 95% confidence interval. The only ARIMA model to capture actual sales in its 95% confidence interval was Model 5 (but just barely).

SEP 2021 Actual: $15,536	**SEP 2021 Forecast**	**Did 95% CI contain actual?**
Model 5	$14,168	Yes
Model 6	$13,963	No
Model 7	$14,009	No
Model 16	$14,727	Yes

7. Conclusion

Given the results of our forecasting analysis, it appears that our original choice of best model has been confirmed. Model 16 produces the greatest likelihood of white noise residual values (at worst, on par with Model 6) among our better fitting models. And of the best models, it is the only one to successfully capture the actual September value in its 95% confidence forecast interval.

From this, we can say with a measure of confidence that used car sales have been impacted by an intervening cause in 2020. Furthermore, we can immediately see the impact of such a cause on traditional time series analysis. Our original attempts to model the data resulted in overfitting, as the models tried to compensate for the wild swings post-2020. With the intervention model, we were able to obtain a relatively simple seasonal model that adequately models the data. This parsimonious model avoids the problem of overfitting while still allowing us to successfully predict future measurements.