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***Abstract*—This paper presents significant extensions to the distributed agreement problem in stochastic communication networks, building upon the foundational work of coordination through stochastic channels.**

***Index Terms*—Distributed agreement, stochastic networks, approximate consensus, multidimensional algorithms, conditional analysis, asymmetric channels**

I. Introduction

The work made by Fraigniaud, Patt-Shamir, and Rajsbaum [1] in distributed alogrithms established optimal stochastic algorithms, introducing the Agreed Meeting Point (AMP) and Flip Value (FV) algorithms with proven optimality bounds.

Based on this foundation, this paper presents extensions that address several limitations and open questions from the original work. Specifically, we contribute:

1) **New Range of values and options**: We expanded the accepted values for the process and the number of porcesses that the algorithms can take.
2) **Multidimensional Agreement Algorithms**: Extension to arbitrary dimension.
3) **Conditional Analysis Framework**: Theoretical and experimental analysis of agreement protocols conditioned on the event that at least one message is delivered per round.
4) **New Algorithm Variants**: Introduction of MIN, New AMP, and RECURSIVE AMP algorithms with theoretical analysis and experimental validation.

The motivation for these extensions stems from practical distributed systems where processes often operate on multidimensional data, communication channels exhibit asymmetric properties, and conditional guarantees significantly impact system design decisions.

II. Background and Related Work

A. Original work

The foundational model considers n synchronous processes communicating over a complete graph where each message is delivered with probability $p \in [0, 1]$. Processes start with binary input values and must decide values in $[0, 1]$ minimizing the expected discrepancy.

The original work established two optimal algorithms:

- **AMP (Agreed Meeting Point)**: Optimal for $p > 0.5$ with expected discrepancy $E[D] = 1 - p$ for two processes in one round.
- **FV (Flip Value)**: Optimal for $p \leq 0.5$ with expected discrepancy $E[D] = p^2 + (1-p)^2$ for two processes in one round.

B. Theoretical Foundations

The *Integrality Lemma* proved that for two processes, optimal algorithms need only consider binary outputs $\{0, 1\}$. However, this property does not extend to $n > 2$ processes, necessitating more sophisticated approaches for larger networks.

The expected discrepancy for k rounds follows:

$$E[D_k^{AMP}] \leq (1-p)^k \tag{1}$$

$$E[D_k^{FV}] \leq (p^2 + (1-p)^2)^k \tag{2}$$

III. Extended Value Domains and Process Scalability

A. Beyond Binary Values: Continuous Domain Extension

The original work focused primarily on binary input values $\{0, 1\}$ with discrepancy values constrained to the unit interval $[0, 1]$. Our first extension generalizes this to arbitrary continuous domains.

1) Continuous Value Domains: We extend the problem formulation to allow:

- **Input values**: $x_i \in [a, b] \subset \mathbb{R}$
- **Meeting point**: m as a configurable parameter rather than a fixed value

Algorithm Adaptation:

For the AMP algorithm with an arbitrarily scheduled meeting point $m \in [a, b]$ the algorithm remained the same as the original, but new values were introduced for the meeting point.

For the FV algorithm, the algorithm stayed the same.

2) Theoretical Implications: The fundamental results extend naturally to continuous domains. For processes with initial values $x_A, x_B \in [a, b]$, the expected discrepancy formulas become:

[ECUACIÓN PENDIENTE: Fórmula de discrepancia AMP para dominios continuos - debería ser proporcional a —xA - xB—] [ECUACIÓN PENDIENTE: Fórmula de discrepancia FV para dominios continuos - similar escalamiento]

B. Scalable Process Count: For more than 2 processes

1) Multi-Process Generalization: The original analysis focused primarily on two-process scenarios. We implement and analyze the algorithms for arbitrary process counts $n \geq 2$.

Communication Model: Each process i sends its current value to all other processes. Each message is delivered independently with probability p.

Algorithm Execution: In each round:

1) Process i receives a subset R_i of messages from other processes
2) If R_i contains any value different from x_i, the algorithm rule applies
3) Otherwise, process i maintains its current value

2) Expected Discrepancy for n Processes: Following the approach of the original paper for n processes, we implement the formulas using the functions $A(n, m)$ and $B(n, m)$ as defined in the original work.

For n processes in one round:

[ECUACIÓN PENDIENTE: Fórmulas completas usando A(n,m) y B(n,m) del paper original] [NOTA: Revisar páginas 14-15 del paper para las fórmulas exactas]

C. Multi-Round Analysis

1) Convergence Over Multiple Rounds: For r rounds with n processes, the expected discrepancy follows:

$$E[D_r^{AMP}] \leq \left(E[D_1^{AMP}]\right)^r \tag{3}$$

$$E[D_r^{FV}] \leq \left(E[D_1^{FV}]\right)^r \tag{4}$$

This exponential convergence property extends from the two-process case, though the base factors now depend on the specific n-process formulas.

D. Implementation Validation

1) Continuous Domain Experiments: We validate the continuous domain extension through systematic experiments:

[TABLA PENDIENTE: Resultados experimentales para diferentes rangos de valores] [NOTA: Incluir datos del simulador con rangos [0,1], [0,10], [-5,5], etc.]

2) Multi-Process Scaling: Performance scaling with process count demonstrates expected theoretical behavior:

[TABLA PENDIENTE: Resultados de escalamiento con 2,3,4,6,8 procesos] [NOTA: Mostrar como mejora la convergencia con más procesos]

IV. New Algorithm Variants

Building upon the foundational AMP and FV algorithms, we introduce new algorithmic variants that extend the capabilities of distributed agreement protocols. These new algorithms address specific limitations and provide enhanced functionality for practical applications.

A. MIN Algorithm: Distributed Minimum Selection

The MIN algorithm implements a distributed minimum-finding protocol that differs fundamentally from the original approaches.

1) Basic Operation: Unlike AMP and FV which make immediate decisions upon receiving messages, the MIN algorithm:

- **Accumulates information** across multiple rounds
- **Maintains a knowledge set** of all observed values
- **Delays final decision** until termination
- **Selects the minimum** from all known values

Algorithm Description:

1) Initialize: Each process maintains its initial value and a set of known values
2) During rounds: Collect received values but maintain current value
3) At termination: Select the minimum value from the accumulated knowledge set
4) Tie-breaking: Use coordinate sum minimization with lexicographic ordering

2) Convergence Properties: The MIN algorithm guarantees convergence to the global minimum value under connectivity assumptions:

Key advantage: Provides deterministic convergence to a specific target (global minimum) rather than approximate agreement.

B. RECURSIVE AMP: Enhanced Meeting Point Protocol

The RECURSIVE AMP extends the original AMP concept by applying the meeting point rule iteratively to multiple received values.

1) Enhanced Meeting Point Calculation: Rather than using a fixed meeting point, RECURSIVE AMP:

- **Computes dynamic range** from all received values
- **Applies meeting point rule** within the observed value range
- **Scales with dimensionality** for multidimensional spaces
- **Maintains AMP properties** while being more adaptive

Range-Based Meeting Point:

2) Relationship to Original AMP: RECURSIVE AMP generalizes the original AMP by: - Maintaining the meeting point principle - Adapting the meeting point to observed data - Extending naturally to multidimensional cases - Preserving convergence properties

C. Algorithm Comparison Overview

1) Performance Characteristics: Each algorithm exhibits distinct performance profiles:

- **AMP**: Fast convergence, configurable meeting point, optimal for $p > 0.5$
- **FV**: Simple implementation, optimal for $p \leq 0.5$, direct value adoption
- **MIN**: Deterministic target, accumulative learning, global minimum convergence
- **RECURSIVE AMP**: Adaptive meeting points, range-based decisions, multidimensional support

2) Use Case Guidelines: **Algorithm Selection Criteria:** - Use **AMP/FV** for classic approximate agreement with proven optimality - Use **MIN** when convergence to a specific extremal value is desired - Use **RECURSIVE AMP** for adaptive scenarios with unknown value distributions

3) Computational Complexity: This algorithmic diversity provides practitioners with a toolkit of agreement protocols suitable for different distributed computing scenarios, from traditional consensus to specialized optimization problems.

REFERENCES

[1] P. Fraigniaud, B. Patt-Shamir, and S. Rajsbaum, "Coordination through stochastic channels," in *Proceedings of the 37th International Symposium on Distributed Computing*, 2023.