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ISC 4220C Spring 2020
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Lab 2 Report

1. Consider the functions f(x) = ln(x) and g(x) = $25x^3 - 6x^2 + 7x - 88$. Use zero through third-order Taylor series expansions around the point x = 1 for both these functions. Use these to evaluate f(3) and g(2.5). Compute the true relative error for each approximation.

   Taylor expansion formula: $\sum_{k=0}^{order} \frac{1}{k!} f^{(k)}(c)(x - c)^k$

   f(x) = ln(x)
   0th Order: f(x) = 1/0! * ln(1) * $(x - 1)^0$
   1st Order: f(x) = 1/0! * ln(1) * $(x - 1)^0$ + 1/1! * 1/(1) * $(x - 1)^1$
   2nd Order: f(x) = 1/0! * ln(1) * $(x - 1)^0$ + 1/1! * 1/(1) * $(x - 1)^1$ + 1/2! * -1/$(1)^2$ * $(x - 1)^2$
   3rd Order: f(x) = 1/0! * ln(1) * $(x - 1)^0$ + 1/1! * 1/(1) * $(x - 1)^1$ + 1/2! * -1/$(1)^2$ * $(x - 1)^2$ + 1/3! * 2/$(1)^3$ * $(x - 1)^3$

   f(3) =
   0th Order = 0
   1st Order = 2
   2nd Order = 0
   3rd Order = 2.6666….

   g(x) = $25x^3 - 6x^2 + 7x - 88$
   0th Order: g(x) = 1/0! * (5$(1)^3$ - 6$(1)^2$ + 7(1) - 88) * $(x - 1)^0$
   1st Order: g(x) = 1/0! * (5$(1)^3$ - 6$(1)^2$ + 7(1) - 88) * $(x - 1)^0$ + 1/1! * (75$(1)^2$ - 12(1) + 7) * $(x - 1)^1$
   2nd Order: g(x) = 1/0! * (5$(1)^3$ - 6$(1)^2$ + 7(1) - 88) * $(x - 1)^0$ + 1/1! * (75$(1)^2$ - 12(1) + 7) * $(x - 1)^1$ + 1/2! * (150(1) - 12) * $(x - 1)^2$
   3rd Order: g(x) = 1/0! * (5$(1)^3$ - 6$(1)^2$ + 7(1) - 88) * $(x - 1)^0$ + 1/1! * (75$(1)^2$ - 12(1) + 7) * $(x - 1)^1$ + 1/2! * (150(1) - 12) * $(x - 1)^2$ + 1/3! * (150) * $(x - 1)^3$

   g(2.5)
   0th Order = -82
   1st Order = 23
   2nd Order = 175.25
   3rd Order = 262.625

2. Consider the function f(x) = 1 + sin(x). Can the bisection method be used to find it roots? Why or why not? Can Newton's method be used? What order of convergence do you expect, and why?

   Bisection cannot be used because there is no interval where the function is negative to give a boundary such that bisection would find a root. You need an interval where on one side it's negative and on the other it's positive.

   Newton's method can be used so long as you don't pick an interval where the derivative is zero. You would expect the order of convergence to be quadratic since the error is squared.

$$E_{t,i+1} = -\frac{f''(\xi)}{2f'(x_i)}E_{t,i}^2$$

3. Explain whether the following claim is true or false: If x^* is the solution to f(x) = 0, then $x^* + \alpha$ is a solution of $f(x) - \alpha = 0$.

   All the alpha is doing is shifting each point of the graph down by alpha. The roots are the 'same' but moved down, so x* + alpha would give you the solution.

   Computer Lab

```
[value2, iter] = newton(2,1,2)
[value5, iter] = newton(5,1,5)

function valx = f(x,a,n)
    valx = x^n - a;
end

function valx = f_prime(x,n)
    valx = n*x^(n-1);
end

function [x, iter] = newton(a,xguess,n)
    iter = 0;
    x = xguess - (f(xguess,a,n)/f_prime(xguess,n));
    while abs(x - xguess) >= .0001
        x = xguess;
        xguess = xguess - (f(xguess,a,n)/f_prime(xguess,n));
        iter = iter + 1;
    end
end
```

RESULTS: `value2 = 1.414215686274510 value5 = 1.379731505359937`