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Lab 8 Report

1. Consider the integral

$$I = \int_0^1 \sqrt{x}\log(x)dx$$

The exact value is $I = -4/9$. Use trapezoidal rule with $n = 1, 2, 4,$ and 8 equal intervals to numerically evaluate the integral. Report the relative and absolute error.

1.

```
for i = [1 2 4 8]
    a = trap(@f,0,1,i)
    absE = abs(-4/9 - a)
    relE = absE/(4/9)
end

function val = f(x)
val = sqrt(x)*log(x);
end

function val = trap(func,a,b,n)
h = (b-a)/n;
sum = 0;
for i = 1:n-1
    sum = sum + func(a+i*h);
end

val = (func(a) + func(b) + 2*sum) * ((b-a)/(2*n));
end
```

```
a =
     0
absE =
    0.4444
relE =
     1
a =
   -0.2451
absE =
    0.1994
relE =
    0.4488
```

```
a =
   -0.3581
absE =
    0.0863
relE =
    0.1943
a =
   -0.4081
absE =
    0.0364
relE =
    0.0818
```

2. Evaluate the integral

$$I = \int_0^2 \exp(-x^2)dx,$$

which is related to the "error function" (but differs by $2/\sqrt{\pi}$), defined intrinsically in Matlab as erf.

(a) Use $n = 4$ equispaced intervals, and apply Simpson's 1/3 rule to evaluate the integral.

(b) Using Gauss quadrature with 4 nodes, and recompute the integral above.

In both cases report the absolute error.

2.

```
valSimp = simp(@g,0,2,4)
absEsimp = erf(2) - (2/sqrt(pi))*simp(@g,0,2,4)

valQuad = quadr(@g,0,2)
absEquad = erf(2) - (2/sqrt(pi))*quadr(@g,0,2)

function val = g(x)
val = exp(-(x^2));
end
function val = simp(func,a,b,n)
h = (b-a)/n;
sum1 = 0;
sum2 = 0;
for j = 1:(n/2)-1
    sum1 = sum1 + func(a+(2*j)*h);
end
for j = 1:(n/2)
    sum2 = sum2 + func(a+(2*j)*h);
end
val = (func(a) + func(b) + 2*sum1 + 4*sum2)*(h/3);
```

```
end

function I = quadr(func,a,b)
quadWi = [(18-sqrt(30))/36; (18+sqrt(30))/36; (18+sqrt(30))/36;
(18-sqrt(30))/36];
quadXi = [ 0.8611363115940525; 0.3399810435848563; -0.3399810435848563;
-0.8611363115940525];

wi = (b-a)/2 .* quadWi;
xi = ((b-a).*quadXi + (b+a))/2;
I = 0;
for i = 1:4
    I = I + wi(i)*func(xi(i));
end

valSimp =

    0.5498


absEsimp =

    0.3749


valQuad =

    0.8822


absEquad =

  -1.6667e-04
```

3.

(i) Use 4-point Gauss quadrature formula to compute the integrals:

$$\text{(a) } I_a = \int_{-1}^{1} \frac{1}{e^x + e^{-x}} dx, \qquad \text{(b) } I_b = \int_0^2 \frac{1}{e^x + e^{-x}} dx$$

i.

```
Ia = quadr(@h,-1,1)
Ib = quadr(@h,0,2)

function val = h(x)
val = 1/(exp(x) + exp(-x));
end
```

```
Ia =

    0.8657


Ib =

    0.6509
```

(ii) Suppose we wish to evaluate the integral:

$$I = \int_1^\infty \frac{\sin x}{x^2} dx,$$

using one of the methods that we studied in class. How would you deal with the "infinity" in the upper limit of integration? Be as specific as possible.

ii.

Solution:

Integrate by parts:

I = fg - integral(f*g')

f = sin(x)

g'(x) = 1/x^2

I = sin(x)/x - integral(-cos(x)/x)

integral(-cos(x)/x) = -Ci(x) where Ci(x) = 0.57721 + ln(x) + integral from 0 to x(cos(t-1)/t)

I = Ci(x) - sin(x)/x

= (Ci(inf) - sin(inf)/inf) - (Ci(1) - sin(1)/1)

= sin(1) - Ci(1)

= sin(1) - (.57721 + ln(1) + $\int_0^1 cos(t - 1)/t \, dt$

The problem has been transformed into one without a singularity.

(iii) Consider two integrals

$$
\begin{aligned}
I_1 &= \int_0^1 \int_0^1 e^{-x_1^3 + x_2^3}\, dx_1 dx_2 \\
I_2 &= \iint_D e^{-x_1^3 + x_2^3}\, dx_1 dx_2
\end{aligned}
$$

where D is the region described by $x_1^2 + 2x_2^2 \leq 1$. Would you recommend using Monte Carlo to evaluate both integrals? Explain your choice in detail.

iii.

Both integrals above describe a 2D integrand which offers no benefits for convergence over a rule like Simpson's ⅓ rule.

For a MC integral, the error is equal to 1/sqrt(n) while for Simpson's 1/3rd, we get error = 1/n^(4/d) where d = the dimension of the problem.

Hence, 1/n^2 << 1/n^.5 and thus MC would give more error and slow convergence.