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ISC 4323 Lab 3 Report
Prof. Quaife Fall 2020

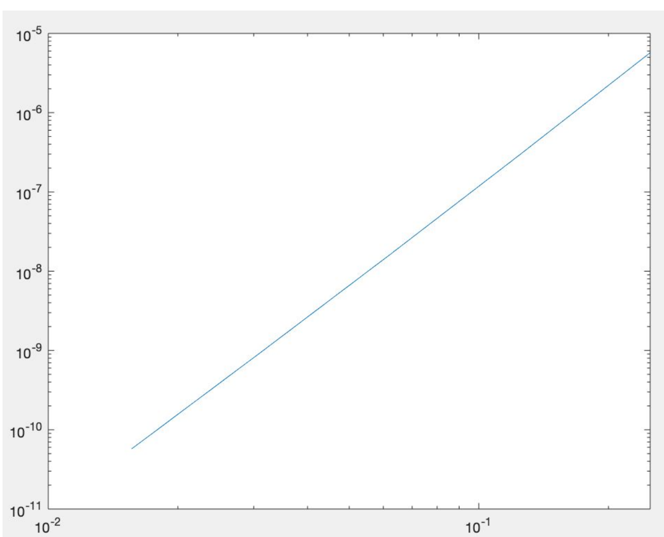
Lab 3 Report

Figures

1.2.1 Tabulation of Relative Errors for RK4 Method

Δt	*Relative Error*
1/4	$3.6 * 10^{-6}$
1/8	$1.8 * 10^{-7}$
1/16	$1.0 * 10^{-8}$
1/32	$6.0 * 10^{-10}$
1/64	$3.6 * 10^{-11}$

1.2.2 loglog plot of Δt vs global error



1.3.2 Derivation of Amplification Factor for RK4 Method

$$k_1 = \Delta t \lambda y = zy$$

$$k_2 = \Delta t(\lambda(y + a_{21} zy)) = zy + a_{21} z^2 y$$

$$k_3 = z(y + a_{31} zy + a_{32} zy + a_{32} a_{21} z^2 y)$$

$$= zy + a_{31} z^2 y + a_{32} z^2 y + a_{32} a_{21} z^3 y$$

$$k_4 = z(y + a_{41} zy + a_{42} zy + a_{42} a_{21} z^2 y + a_{43} zy + a_{43} a_{31} z^2 y + a_{43} a_{32} z^2 y + a_{43} a_{32} a_{21} z^3 y)$$

$$k_4 = zy + a_{41} z^2 y + a_{42} z^2 y + a_{42} a_{21} z^3 y + a_{43} z^2 y + a_{43} a_{31} z^3 y + a_{43} a_{32} z^3 y + a_{43} a_{32} a_{21} z^4 y$$

$$y_{n+1} = y_n + b_1 z y_n + b_2 z y_n + b_2 a_{21} z^2 y_n + b_3 z y_n + b_3 a_{31} z^2 y_n + b_3 a_{32} z^2 y_n + b_3 a_{32} a_{21} z^3 y_n + b_4 z y_n + b_4 a_{41} z^2 y_n + b_4 a_{42} z^2 y_n + b_4 a_{42} a_{21} z^3 y_n + b_4 a_{43} z^2 y_n + b_4 a_{43} a_{31} z^3 y_n + b_4 a_{43} a_{32} z^3 y_n + b_4 a_{43} a_{32} a_{21}$$

$$y_{n+1} = y_n(1 + b_1 z + b_2 z + b_2 a_{21} z^2 + b_3 z + b_3 a_{31} z^2 \; z^4 y_n) + b_3 a_{32} z^2 + b_3 a_{32} a_{21} z^3 + b_4 z + b_4 a_{41} z^2 + b_4 a_{42} z^2 + b_4 a_{42} a_{21} z^3 + b_4 a_{43} z^2 y_n + b_4 a_{43} a_{31} z^3 y_n + b_4 a_{43} a_{32} z^3 + b_4 a_{43} a_{32} a_{21} z^4)$$

$$\Lambda(z) = (1 + \tfrac{1}{6} z + \tfrac{1}{3} z + \tfrac{1}{6} z^2 + \tfrac{1}{3} z + \tfrac{1}{6} z^2 + \tfrac{1}{12} z^3 + \tfrac{1}{6} z + \tfrac{1}{6} z^2 + \tfrac{1}{12} z^3 + \tfrac{1}{24} z^4)$$

1.3.2 Stability plot of RK4 Method

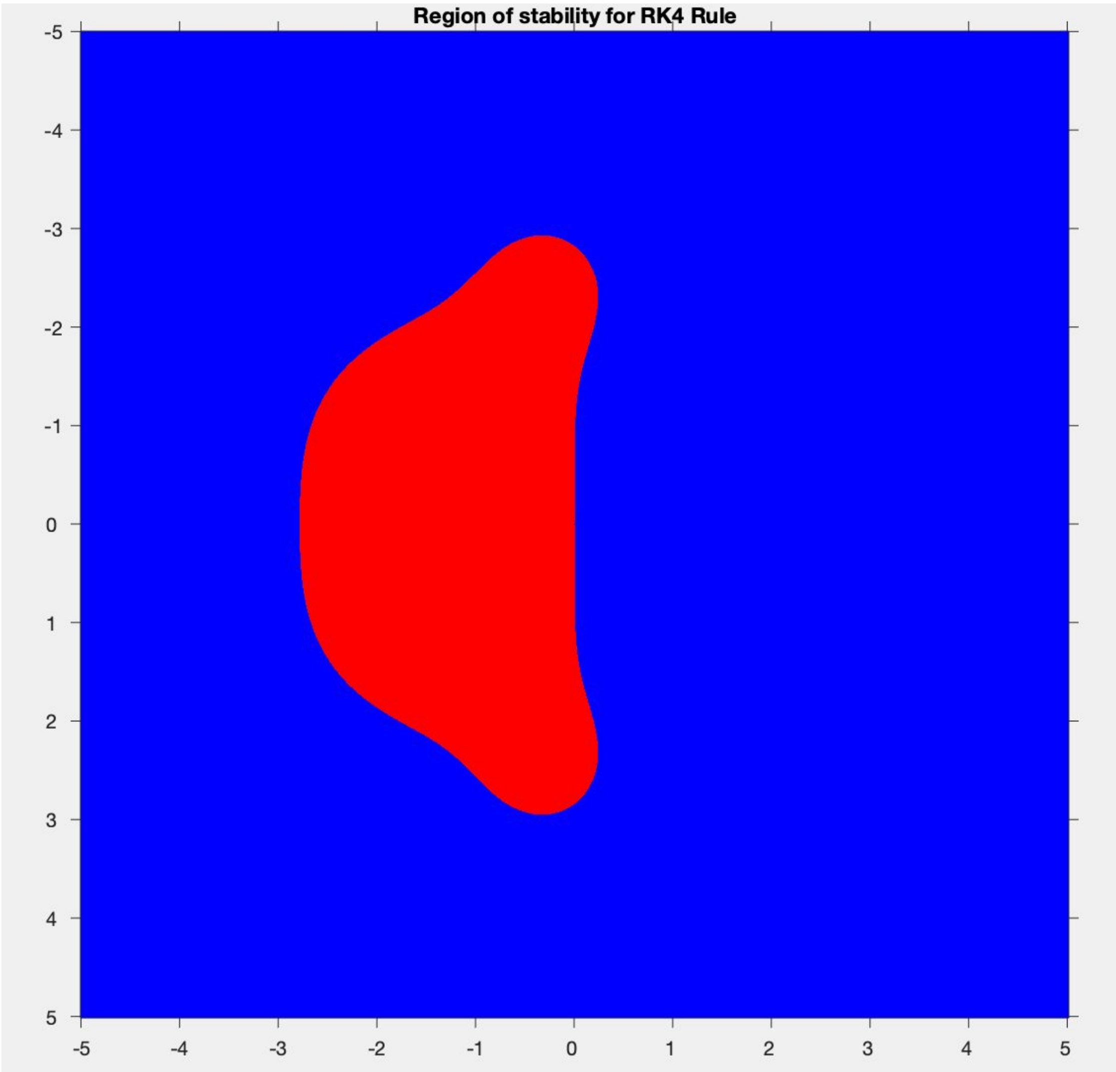
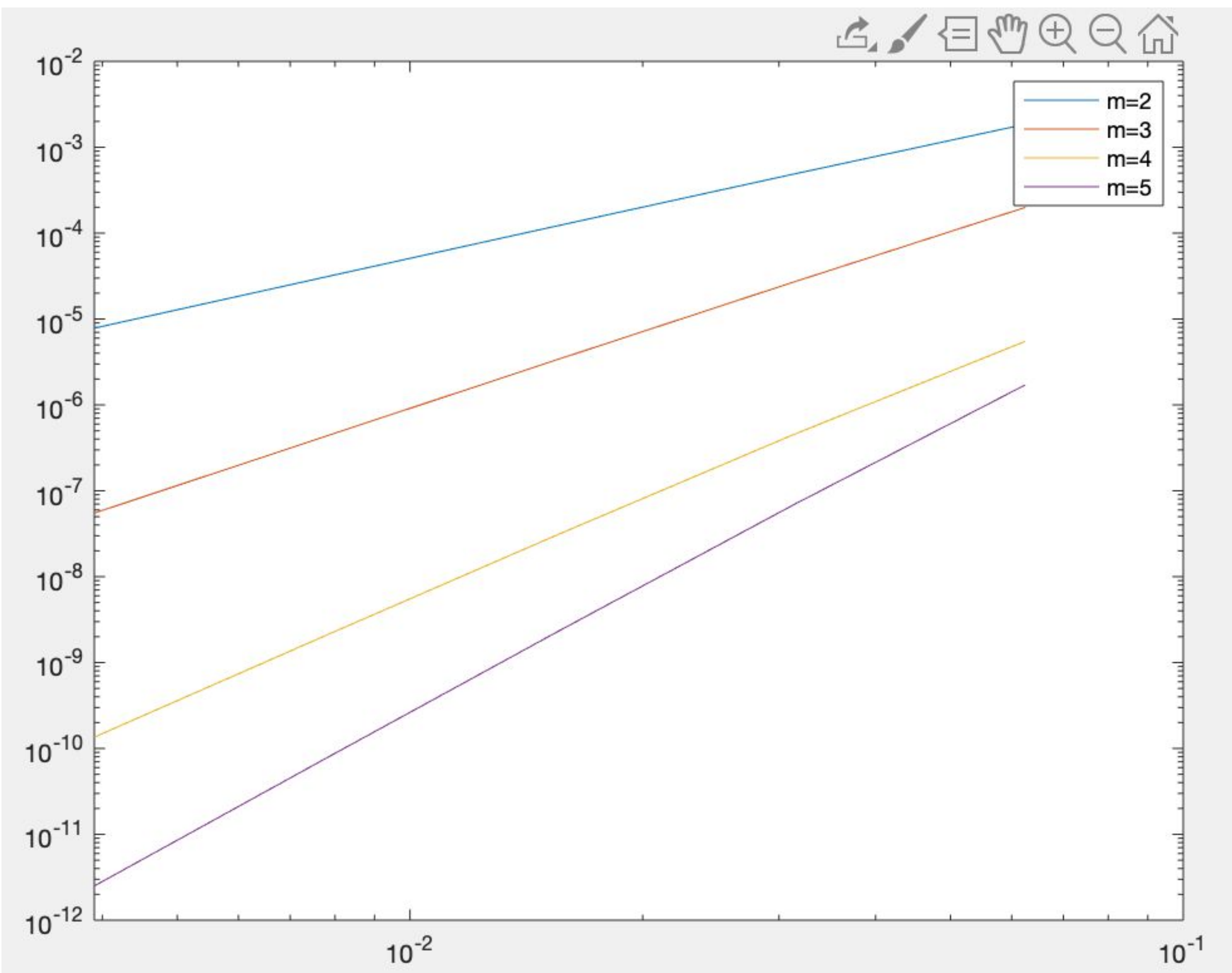


Figure 2.2.1 Tabulation of Relative Error for Adams Bashforth Method with m = 2 to 5

m parameter	*2*	*3*	*4*	*5*
$\Delta t = 1/16$	$1.1 * 10^{-3}$	$1.2 * 10^{-4}$	$3.5 * 10^{-6}$	$1.0 * 10^{-6}$
$\Delta t = 1/32$	$3.0 * 10^{-4}$	$1.7 * 10^{-5}$	$2.8 * 10^{-7}$	$4.3 * 10^{-8}$
$\Delta t = 1/64$	$7.8 * 10^{-5}$	$2.2 * 10^{-6}$	$2.0 * 10^{-8}$	$1.5 * 10^{-9}$
$\Delta t = 1/128$	$2.0 * 10^{-5}$	$2.8 * 10^{-7}$	$1.3 * 10^{-9}$	$4.9 * 10^{-11}$
$\Delta t = 1/256$	$4.9 * 10^{-6}$	$3.5 * 10^{-8}$	$8.5 * 10^{-11}$	$1.5 * 10^{-12}$

Figure 2.2.2 loglog plot of Δt vs global error for each m (2,3,4,5)



Discussions

The numerical rate of convergence for the RK4 method is O(dt^4). We can see this in figure 1.2.2. For every decrease of the timestep by a factor of 2, the global truncation error is decreased by a factor of 16 (more or less). This implies that the slope of the loglog graph is 4, which it is.

The numerical rate of convergence for the Adams Bashforth method depends on which m parameter you use.

For m = 2, the rate of convergence is O(dt^2). For every decrease of the timestep by a factor of 2, the global truncation error decreases by a factor of 4.

For m = 3, the rate of convergence is O(dt^3). For every decrease of the timestep by a factor of 2, the global truncation error decreases by a factor of 8.

For m = 4, the rate of convergence is O(dt^4). For every decrease of the timestep by a factor of 2, the global truncation error decreases by a factor of 16.

For m = 5, the rate of convergence is O(dt^5). For every decrease of the timestep by a factor of 2, the global truncation error decreases by a factor of 32.