

## Lab 7 Report

We can use this to construct a linear system by assigning ranking points  $r_i$  to team  $T_i$  via:

$$r_1 - r_2 = 4$$

$$r_3 - r_1 = 9$$

$$r_1 - r_4 = 6$$

$$r_3 - r_4 = 3$$

$$r_2 - r_4 = 7$$

This is an overdetermined system and does not even have a unique least squares solution because we could always add a constant  $c$  to any solution  $[r_1 + c, r_2 + c, r_3 + c, r_4 + c]^T$  and still satisfy all the equations equally well. This can be fixed by adding another equation like,

$$r_1 + r_2 + r_3 + r_4 = 20.$$

1. Given these equations, use LLS to rank the teams

```
A = [  
    1 -1 0 0;  
    -1 0 1 0;  
    1 0 0 -1;  
    0 0 1 -1;  
    0 1 0 -1;  
    1 1 1 1;  
    ];  
  
f = [4 9 6 3 7 20]';  
  
ahat = (A'*A)\(A'*f)  
ahat =  
    5.2500  
    4.6250  
    9.1250  
    1.0000
```

Rankings (Best to worst): Team 3 (9.125 points), Team 1 (5.25 points), Team 2 (4.62 points), Team 4 (1 point).

Vary the step-size  $h$  between  $10^{-15}$  to  $10^{-1}$  (consider using the `logspace` command), and construct a log-log plot of absolute error and  $h$ . Note that we can compute the absolute error, since we know that the true value of  $f'(\pi/3) = 0.5$ .

Using the graphs above, find the (approximately) optimum value of  $h$  for the two difference formula, and compare the absolute errors of the two numerical formulae.

2.

```
h = 10.^(-15:-1)';
trueval = .5;

errorCDF = abs(trueval - centerdf(pi/3,h));
errorFDF = abs(trueval - forwarddf(pi/3,h));

loglog(h,errorCDF)
xlabel('h');
ylabel('Absolute Error Center Difference Formula');

figure
loglog(h,errorFDF)
xlabel('h');
ylabel('Absolute Error Forward Difference Formula');

function val = centerdf(x,h)
val = (sin(x+h) - sin(x-h))./(2.*h);
end

function val = forwarddf(x,h)
val = (sin(x+h) - sin(x))./h;
end
```

Optimal  $h$  for center distance formula:  $10^{-5}$

Optimal  $h$  for forward distance formula:  $10^{-8}$

Error at optimal  $h$  CDF:  $7.827 \times 10^{-12}$

Error at optimal  $h$  FDF:  $3.039 \times 10^{-9}$ .

Center distance formula has less error at optimal  $h$  compared to forward distance formula.

Plots below.

