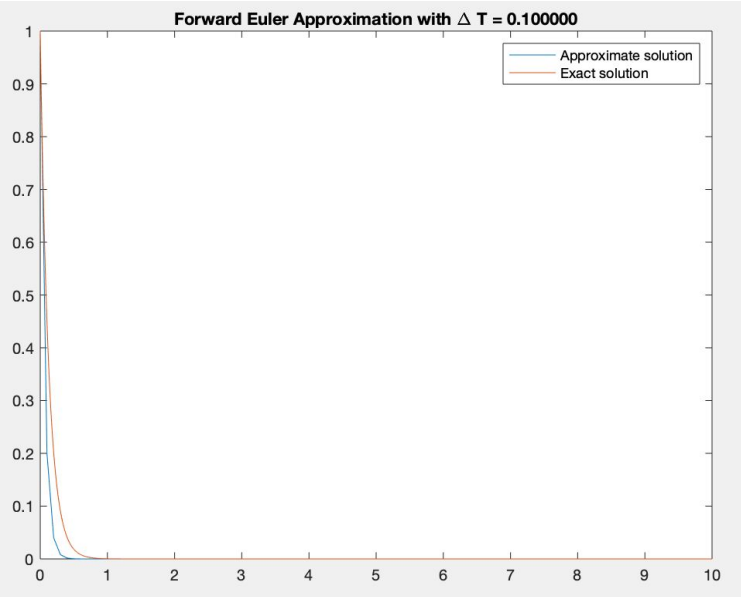
Abelardo Riojas
ISC 4232C Lab 2
Prof. Quaife Fall 2020

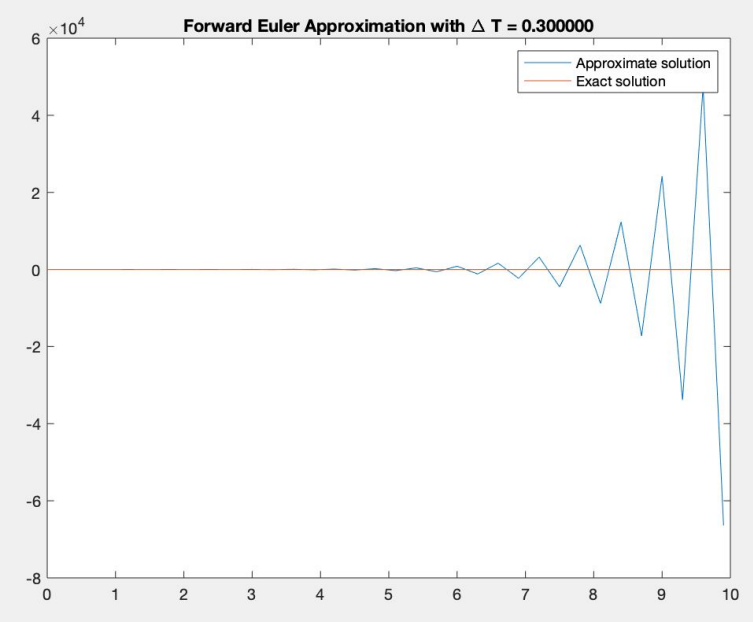
Lab 2

1. Figures

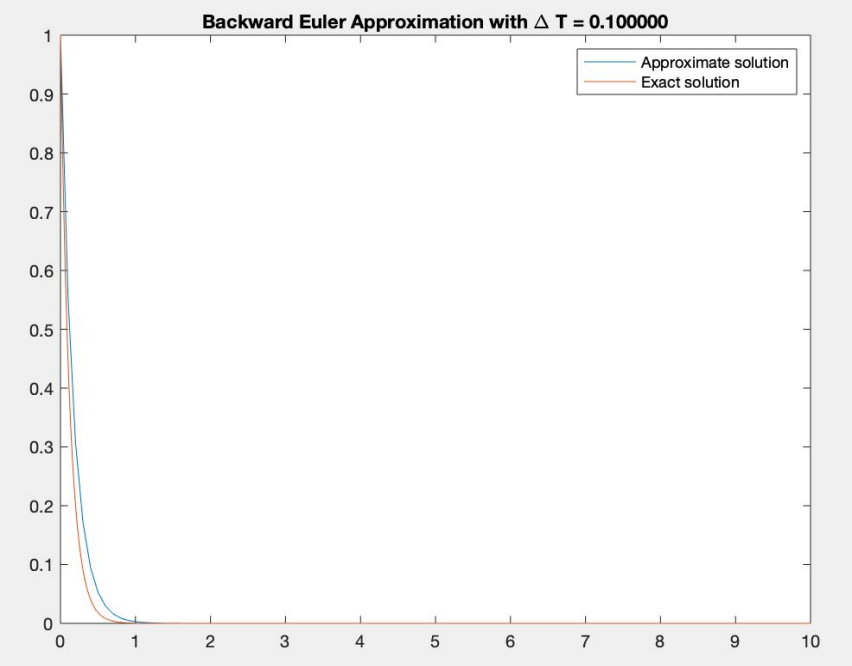
a.



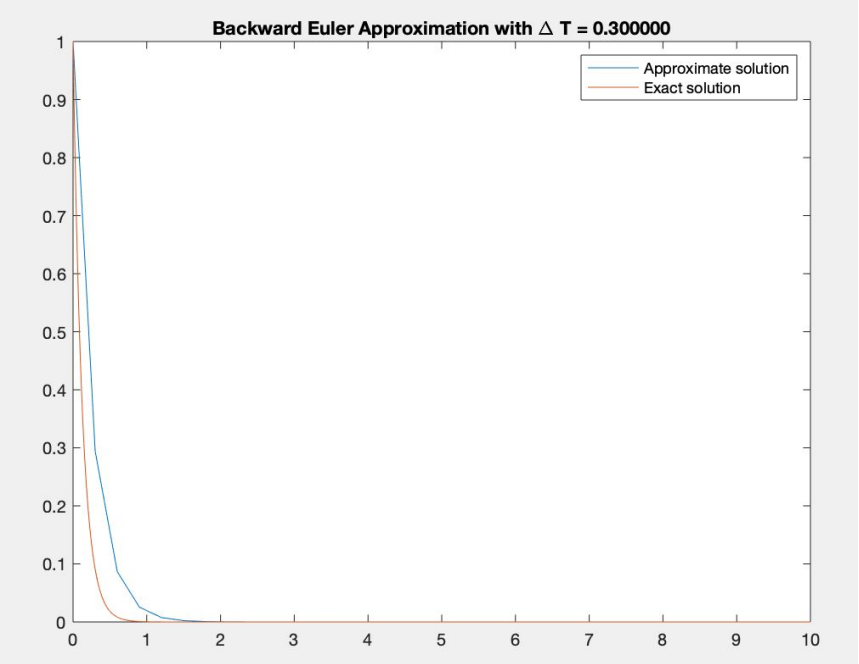
b.



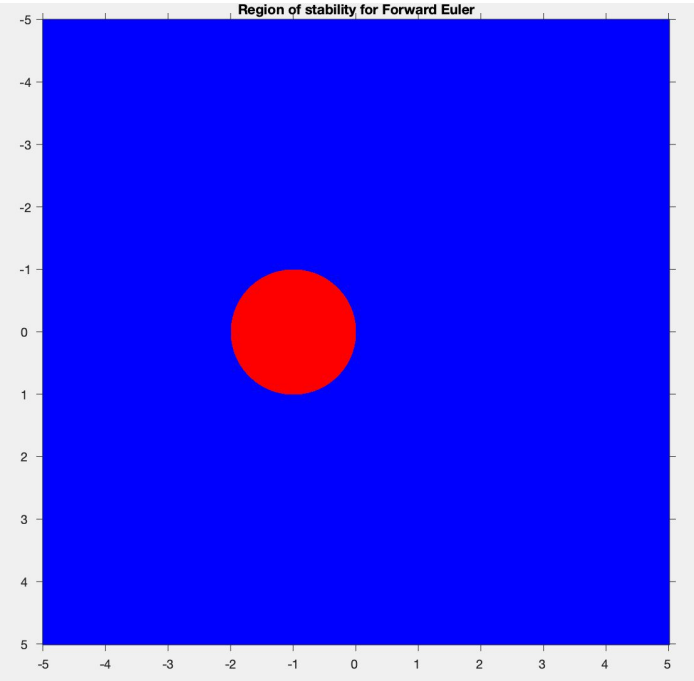
c.


Backward Euler Approximation with △ T = 0.100000
Approximate solution
Exact solution
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

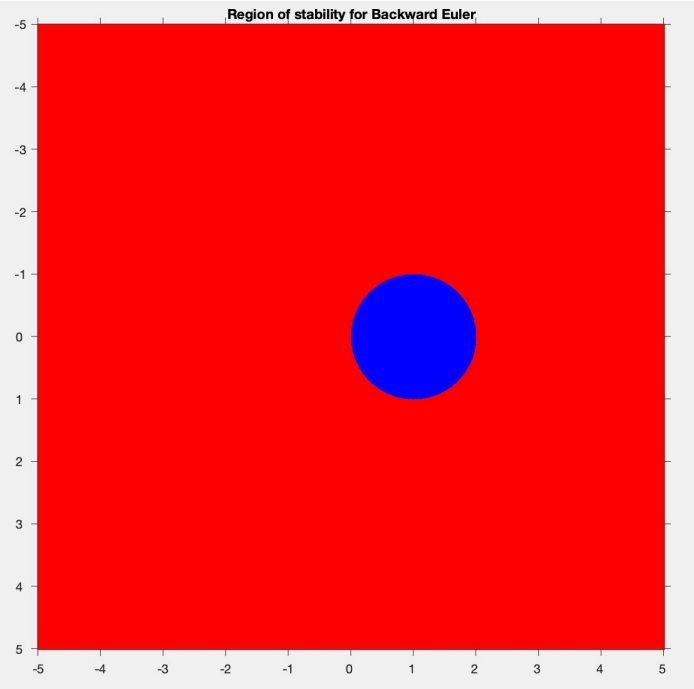
d.

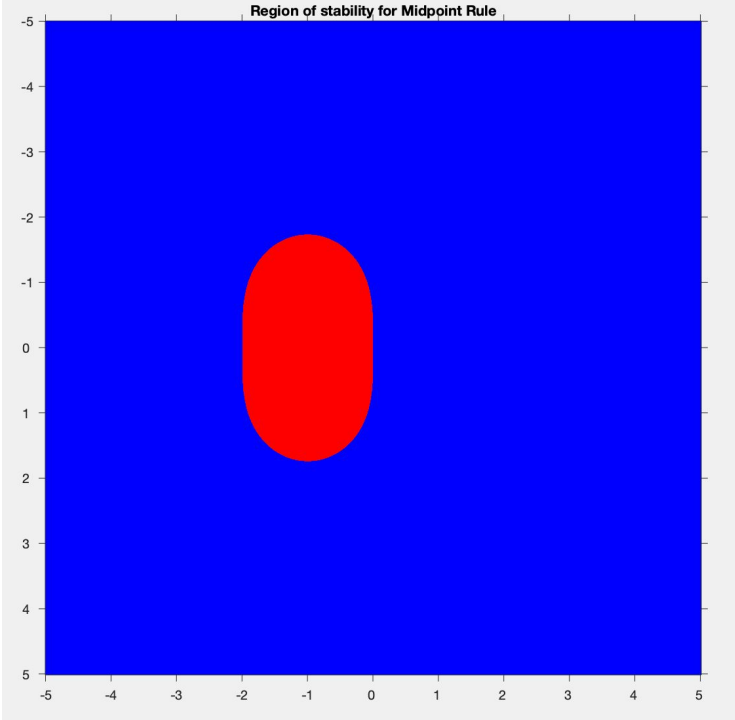

Backward Euler Approximation with △ T = 0.300000
Approximate solution
Exact solution
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10

e.


Region of stability for Forward Euler
-5
-4
-3
-2
-1
0
1
2
3
4
5
-5
-4
-3
-2
-1
0
1
2
3
4
5

f.


Region of stability for Backward Euler
-5
-4
-3
-2
-1
0
1
2
3
4
5
-5
-4
-3
-2
-1
0
1
2
3
4
5

g. 

Derivation of Midpoint Rule amplification factor on next page.

$$y_{n+1} = y_n + \Delta t\, F\left(y_n + \frac{\Delta t}{2} F(y_n, t_n),\ t_n + \frac{\Delta t}{2}\right)$$

$$F(y,t) = \lambda t$$

$$y_0 = y_0$$

$$y_1 = y_0 + \Delta t \lambda \left(y_0 + \frac{\Delta t}{2} \lambda y_0\right)$$

$$y_1 = y_0 \left(1 + \lambda \Delta t + \left(1 + \frac{\Delta t}{2} \lambda\right)\right)$$

$$y_1 = y_0 \left(1 + \lambda \Delta t + \frac{\lambda^2 \Delta t^2}{2}\right)$$

$$y_1 = y_0 \left(1 + z + \frac{z^2}{2}\right) \qquad z = \lambda \Delta t$$

~~$y_2 = y_1 + \Delta t \lambda \left(y_1 + \frac{\Delta t}{2} \lambda y_1\right)$~~

~~$y_2 = y_0 \left(1 + z + \frac{z^2}{2}\right) + \lambda \Delta t \left(y_0 \left(1 + z + \frac{z^2}{2}\right) + \frac{z}{2} y_1\right)$~~

~~$\equiv y_1 + z\left(y_1 + \frac{z}{2} y_1\right)$~~

$$y_{n+1} = y_{n-0} \left(1 + z + \frac{z^2}{2}\right)$$

Amplification FACTOR!