

Project Report

Mechanics of Material

Project:

Basic Pressure Hull Design & Analysis

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Problem Statement

A submarine has to be designed to perform tests at Mariana Trench seabed. Design a thick pressure vessel that can carry an average living being to the depth. During your design incorporate the hydrodynamic effects to settle the base at a maximum possible speed and come back to the top with the same.

Submarines' Main Parts

Hull

A submarine hull has two major components, the *light hull* and the *pressure hull*. The light hull of a submarine is the outer non-watertight hull which provides a hydrodynamically efficient shape. The pressure hull is the inner hull of a submarine that maintains structural integrity with the difference between outside and inside pressure at depth.

We are choosing a double hull submarine out of the three possible types. These hulls bear high amounts of pressures during the life time of the submarine and are most prone to failure. During this project we will only be considering these parts and running our analysis on them.

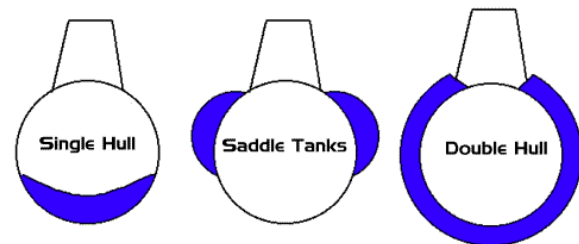


Figure 1 - Types of submarines

Ballast Tanks

Ballast tanks control the buoyancy of the submarine which in turn controls the diving down and the rising up. Ballast tanks can be flooded with water which can later be pumped out.

Working

The working aspects under consideration for this analysis are only diving into the depth of water and rising up to the surface.

Diving

While diving, the ballast tanks are flooded with water using flood ports. The water adds to the weight of the submarine to the point where the buoyant force is not enough to keep the submarine afloat and the submarine submerges further into the depth of the ocean/sea.

$$F_{\text{buoyant}} < W_{\text{submarine}}$$

The depth of the submarine from the surface of the ocean/sea is controlled by the amount of water being flooded into the ballast tanks. The diving speed is controlled by the rate at which the water floods the ballast tanks which is further controlled by the opening and closing of flood ports.

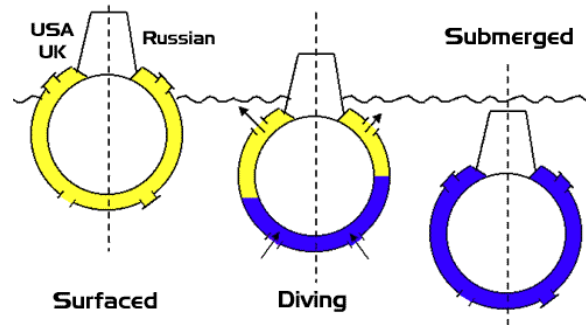


Figure 2 - Submarine diving

Rising

Rising up to the surface is similar to diving in a sense that it is also controlled by the amount of water in the ballast tanks. While rising, the water previously added to the tanks is now pumped out by immensely powerful pumps. These pumps pump the water out of the submarine and back into the ocean/sea and replace it with air. This decreases the weight of the submarine and the buoyant force pushes the submarine in upward direction.

$$F_{\text{buoyant}} > W_{\text{submarine}}$$

The submarine rises up to the level where the buoyant force is balanced by the weight of the submarine. The speed at which the submarine rises is controlled by the rate at which the water is being pumped out.

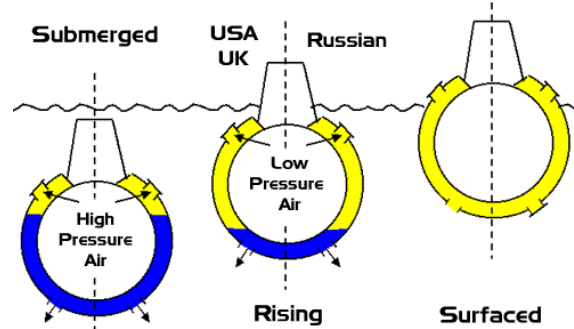


Figure 3 - Submarine surfacing

Assumptions

Water is perfectly still

The water was considered to be still and all surface and underwater waves were ignored. All underwater wave variables were ignored during the calculation of the stresses and pressures including:

1. Turbulence caused by the waves
2. Acceleration/retardation caused by waves in same/opposite direction

Ignored all seismic variables

All underwater disturbances generated by seismic activity within the earth were ignored including all underwater volcanic activity.

Water's density remains constant

As we go deep underwater the density of salt water is subject to change with the depth. It was assumed that water has constant density throughout the depth of ocean.

No variation in gravity occurs

Variation was gravity, however minute, was ignored as we go down in water.

No material imperfections

Material imperfections (if any) for our vessel were ignored and it was assumed that the material used for our vehicle is absolutely perfect.

Constant pressure at sea bed

It was assumed that at sea bed, pressures experienced from all sides of the vehicle are constant as they vary minutely due to the height.

Stagnation pressure was ignored

Stagnation pressure was calculated and was neglected because its value was pretty nominal.

Perfect manufacturing

It is assumed that vehicle is manufactured ideally and no leakage is present.

Spherical Thick-Walled Vessel Stress Analysis

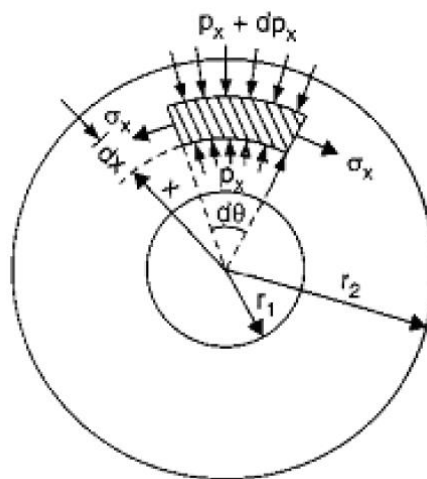


Figure 4 - Thick-walled vessel cross-section

Equating Strains

e_y = circumferential strain

e_x = radial strain

$$e_y = \frac{\text{Final Circumference} - \text{Original Circumference}}{\text{Original Circumference}}$$

$$e_y = \frac{2\pi(x + u) - 2\pi x}{2\pi x} = \frac{u}{x}$$

$$e_x = \frac{(dx + du) - dx}{dx} = \frac{du}{dx}$$

$$u = xe_y$$

Force Resolution

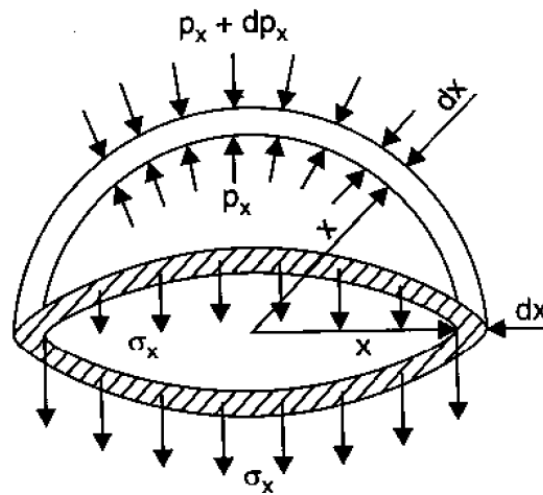


Figure 5 - Stresses in thick-walled vessel

Equating the vertical forces we get:

P = pressure

$$\pi x^2 p = \pi (x + dx)^2 (p + dp) + 2\pi x dx \sigma_x$$

Simplifying and neglecting higher powers of derivatives:

$$\sigma_x = -p - \frac{x}{2} \frac{dp}{dx}$$

Differentiating w.r.t x:

$$\frac{d\sigma_x}{dx} = -\frac{dp}{dx} - \frac{1}{2} \left(x \frac{d^2 p}{dx^2} + \frac{dp}{dx} \right)$$

Now we have 3 principal stresses:

1. Radial Stress p
2. Hoop Stress

3. Same Hoop Stress in a perpendicular in plane direction

Using Hooks Law we can find another equation for the strains:

$$e_x = \frac{1}{E}(p + 2\nu\sigma_x)$$

$$e_y = \frac{1}{E}(\sigma_x - \nu\sigma_x + \nu p)$$

By substitution:

$$e_y = \frac{u}{x}$$

$$e_x = \frac{du}{dx}$$

$$\frac{u}{x} = \frac{1}{E} \left[\sigma_x \left[\frac{m-1}{m} \right] + \frac{p}{m} \right]$$

$$\frac{du}{dx} = - \left[\frac{p}{E} + \frac{2\sigma_x}{mE} \right]$$

$$\frac{du}{dx} = -p - \frac{x}{2} \frac{dp}{dx}$$

$$\frac{du}{dx} = \frac{1}{E} \left[\sigma_x \left[\frac{m-1}{m} \right] + \frac{p}{m} \right] + \frac{1}{E} x \left[\left[\frac{m-1}{m} \right] \frac{d\sigma_x}{dx} + \frac{1}{m} \frac{dp}{dx} \right] = \frac{-1}{E} \left[p + \frac{2\sigma_x}{m} \right]$$

$$-p - \frac{2\sigma_x}{m} = \left[\frac{m-1}{m} \right] \sigma_x + \frac{p}{m} + \left[\frac{m-1}{m} \right] x \frac{d\sigma_x}{dx} + \frac{x}{m} \frac{dp}{dx}$$

$$-p - \frac{2}{m} \left[p + \frac{x}{2} Z \right] = \left[\frac{m-1}{m} \right] \left[-p - \frac{x}{2} Z \right] + \frac{p}{m} + \left[\frac{m-1}{m} \right] x \left[-Z - \frac{1}{2} \left[x \frac{dZ}{dx} + Z \right] \right] + \frac{x}{m} Z$$

$$-p + \frac{2p}{m} + \frac{x}{m} Z = \left[\frac{1-m}{m} \right] \left[p + \frac{x}{2} Z \right] + \frac{p}{m} + \frac{x}{m} Z + \left[\frac{m-1}{m} \right] \left[-xZ - \frac{x^2}{2} \frac{dZ}{dx} - \frac{x}{2} Z \right]$$

$$-p + \frac{2p}{m} + \frac{x}{m} Z = \left[\frac{m-1}{m} \right] \left[-xZ - \frac{x^2}{2} \frac{dZ}{dx} - \frac{x}{2} Z - p - \frac{x}{2} Z \right] + \frac{p}{m} + \frac{x}{m} Z$$

$$-pm + 2p + xZ = [m-1] \left[-2xZ - \frac{x^2}{2} \frac{dZ}{dx} - p \right] + p + xZ$$

$$-pm + 2p + xZ = -2mxZ - \frac{x^2 m}{2} \frac{dZ}{dx} - pm + 2xZ + \frac{x^2}{2} \frac{dZ}{dx} + p + p + xZ$$

$$0 = -2mxZ - \frac{x^2 m}{2} \frac{dZ}{dx} + 2xZ + \frac{x^2}{2} \frac{dZ}{dx}$$

$$0 = -2mxZdx - \frac{x^2 m}{2} dZ + 2xZdx + \frac{x^2}{2} dZ$$

$$0 = -2xZdx[m-1] + \frac{x^2}{2} dZ[m-1]$$

$$2xZdx[m-1] = \frac{x^2}{2} dZ[m-1]$$

$$2xZdx = \frac{x^2}{2} dZ$$

$$4Zdx = x dZ$$

$$4 \frac{dx}{x} = \frac{dZ}{Z}$$

Integrating the above:

$$\ln(Z) = 4\ln x + A$$

$$\ln(Z) = \ln \frac{B}{x^4}$$

Where A is a constant and $Z = dp/dx$

We can simplify to:

$$\frac{dp}{dx} = \frac{B}{x^4}$$

Integrating we get:

$$p = -\frac{B}{3x^3} + C$$

Substitution the value of p we can get the value of the hoop stress as:

$$\sigma_x = -\left(-\frac{B}{3x^3} + C\right) - \frac{x}{2} \frac{dp}{dx}$$

Simplifying this further we get:

$$\sigma_x = -\frac{B}{6x^3} - C$$

If we substitute $B = -6b$ and $C = -a$ we can simplify both equations to:

$$\sigma_x = \frac{b}{x^3} + a$$

$$p = \frac{2b}{x^3} - a$$

The constants are to be determined by the boundary conditions.

Material Selection

Hull and Ballistic Tanks

While designing a hull even if the shape is the same, the forces that work on the object are very different. Pressure from the outside compresses the material while pressure from inside

of the object stretches the material. This difference is very important, because materials have different compressive and tensile strengths. The strength of a submarine hull is not merely that of absolute strength, but rather yields strength. We are considering four materials for the main hull. These four materials are HY-steel 100, HY-Steel 80, aluminum and titanium.

Steel

Since the industrial revolution steel has become one of main construction materials. Steel has both high compressive and tensile strength and can be produced as strips, plates, wires, profiles, beams and columns in various shapes. These properties make it an ideal construction material for a lot of applications. The "HY" steels are designed to possess high yield strength (strength in resisting permanent plastic deformation). Modern steel manufacturing methods that can precisely control time/temperature during processing of HY steels has made the cost to manufacture more economical.

HY-80

HY-80 is high tensile, high yield strength, low alloy steel. It was developed for use in naval applications and is still currently used in many naval applications. It is valued for its strength to weight ratio. Weldments of HY-80 are noted for good ductility, notch toughness and strength. HY-80 is considered to have good corrosion resistance and has good formability to supplement being weldable.

HY-100

HY 100 is a high yield steel with minimum yield strength of 100 ksi. While many steels offer high yield strength, HY 100 also offers exceptionally high impact strength as well. This combination makes HY 100 unique among steels. This material is used extensively in marine defense and petrochemical applications. High Strength HY100 steel has a well-documented history in pressure vessel applications particularly in submarine hull construction. Under arduous working conditions the steel has to have excellent low cycle fatigue properties and in conjunction with a high resistance to crack propagation.

Aluminum

Aluminum has a very high strength to weight ratio. It happens to be more expensive than steel. Al is difficult to weld; you need to be very careful not to lose your shielding gas. Al loves to react with the air and water when it is being welded. It is best if it is done indoors. It makes it very expensive to make a submarine/ship indoor. Aluminum is very expensive. Also, aluminum loses the strength near the weld.

Titanium

Titanium is recognized for its high strength-to-weight ratio. It is a strong metal with low density that is quite ductile. It also has a very high ultimate tensile strength. The corrosion rates are very low for titanium. However, titanium is extremely expensive. While we can ensure a better design by using less amount of titanium, the price would still be extremely high.

Properties Comparison

Material	Poisson Ratio	E [GPa]	K [GPa]	G [GPa]	Yield Strength [MPa]	Density [kg/m ³]	Corrosion in sea water	Price
HY-Steel 80	.30	207	172	79	550	7746	Low	Acceptable
HY-Steel 100	.30	207	172	79	690	7748	Low	Acceptable
Aluminum	.33	69	68-70	26	276	2700	Low	High
Titanium	.342	113.8	110	44	880	4506	Very Low	Very High

Conclusion

Based on the above-mentioned accounts we can tell that HY-100 steel and titanium are the possible choice for the construction of ballistic missile and the pressure hull. Titanium is however more expensive whereas the HY-100 steel can provide desirable strength with suitable design specifications. Hence, the material that we have chosen for Pressure Hull and Ballistic Cylinders is steel HY-100.

See Through Pane

We planned to include a see through pane in the submarine that would help the living creature see the marine life with naked eye. For that we used *poly-methyl-methacrylate* (PMMA), also known as acrylic or acrylic glass. It is a transparent thermoplastic often used in sheet form as a lightweight or shatter-resistant alternative to glass. It was preferred because of its moderate properties, easy handling and processing, and low cost. PMMA is a strong, tough, and lightweight material. However, in order to increase its impact strength, monomers such as butyl acrylate can be added.

Given below are the various properties of PMMA:

Properties	Values [MPa]
Tensile Strength	39.3
Flexure Strength	58.6
Flexure Modulus	1770

Shape

Hull

We have observed that a sphere is the optimal form to withstand outside or inside pressure, it uses the least material with a minimum surface area to encompass a certain volume. Most of the time however, another shape is chosen over a sphere for various reasons. Pressure vessels use the spherical shape to withstand large pressure loads coming from inside the tank, instead of pressure from the outside. Steel can also be produced as a sphere, but this is very

difficult and not commonly done, which makes it expensive. Pressure vessels are often cylindrical because of the high expenses involved with the construction of a sphere. These cylinders are capped off with a dish or dome to form a pill or can shape. A lot of studies have gone into optimizing these shapes with the least thickness and best cap shape. The wall tension in a cylindrical pressure vessel is twice as big as in a spherical vessel, therefore the larger the pressure vessel, or the higher the pressure gets, it becomes more and more beneficial to invest in the more expensive dome shape. Another way of reducing the costs of the dome shape is to divide it into segments. But this is only profitable if the sphere is large enough, because more segments means the dome takes a longer time to construct. Steel can be welded, bolted or riveted. Welding creates a continuous connection between two steel components. This gives steel great flexibility when it comes to combining different elements to create a certain shape. Welding Bolts are often used to connect steel to other materials, such as wood, but can also be a sustainable choice to allow the structure to be disassembled.

See-Through Pane

See-through Panes on submarines are generally made of acrylic plastic. In the case of deep diving submarines, these panes can be several inches thick. The edge of the acrylic is usually conically tapered such that the external pressure forces the acrylic window against the seat. Usually such windows are flat rather than spherically dished. This decreases the area that can be viewed, but eliminates distortion associated with curved glass.

Pressure at Sea Bed

The depth of the Mariana Trench is 10,994 meters. The density of salt water is 1029 kg/m^3 .

Since:

$$P = \rho gh = 1029 \times 9.81 \times 10994 = 111 \text{ MPa}$$

Or 1110 Bars. We will assume that the pressure experienced from all sides of the sea bed is constant since the height varies only minutely as compared to the depth of the Mariana trench.

Hoop and Radial Pressure

Ballast Sphere

Since water was filled at roughly the sea level.

We have our equations as:

$$\sigma_x = \frac{b}{x^3} + a$$

$$p = \frac{2b}{x^3} - a$$

We get two simultaneous equations:

$$111\text{MPa} = \frac{2b}{x_{\text{external}}^3} - a$$

$$0\text{Pa} = \frac{2b}{x_{\text{internal}}^3} - a$$

Which means we have to solve the matrix:

$$\begin{bmatrix} \frac{2}{x^3} & -1 \\ \frac{2}{x^3} & -1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 111\text{MPa} \\ 0 \end{bmatrix}$$

For different values of x and solving in excel we get:

$P_{\text{external}} = 111\text{MPa}$

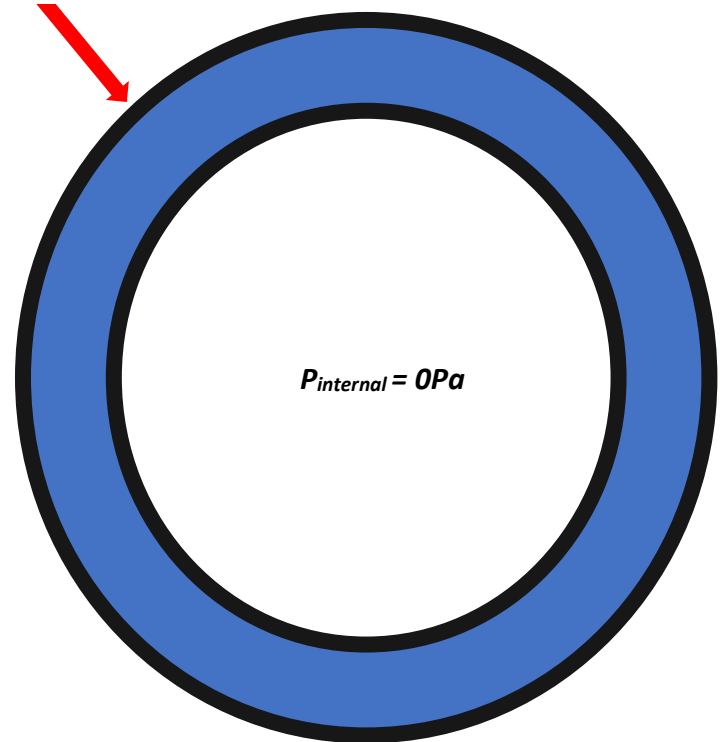


Figure 6 - Gauge pressures inside and outside the submarine

Ballast Sphere							
External Radius	1.2	$2/x^3$	1.157407407	-1 External Pressure	111	b	-131.7362637
Internal Radius	1	$2/x^3$	2	-1 Internal Pressure	0	a	-263.4725275
Ballast Sphere							
External Radius	2.2	Matrix	0.1878287	-1 External Pressure	111	b	-1785.389728
Internal Radius	2		0.25	-1 Internal Pressure	0	a	-446.347432
Ballast Sphere							
External Radius	3.2	Matrix	0.061035156	-1 External Pressure	111	b	-8512.976422
Internal Radius	3		0.074074074	-1 Internal Pressure	0	a	-630.590846
Ballast Sphere							
External Radius	4.2	Matrix	0.026994925	-1 External Pressure	111	b	-26086.49643
Internal Radius	4		0.03125	-1 Internal Pressure	0	a	-815.2030135
Ballast Sphere							
External Radius	5.2	Matrix	0.014223942	-1 External Pressure	111	b	-62497.94977
Internal Radius	5		0.016	-1 Internal Pressure	0	a	-999.9671963
Ballast Sphere							
External Radius	6.2	Matrix	0.008391796	-1 External Pressure	111	b	-127959.3364
Internal Radius	6		0.009259259	-1 Internal Pressure	0	a	-1184.808671
Ballast Sphere							
External Radius	0.915618816	Matrix	2.605468751	-1 External Pressure	111	b	-85.33333341
Internal Radius	0.8		3.90625	-1 Internal Pressure	0	a	-333.3333336

This tells us that increasing the radii and decreasing the thickness increases the Hoop Stress.

Now we have a few variables that we need to play with:

- Optimal Hoop Stress
- Optimal Thickness for Buoyancy
- Cost of material

We know that yielding will occur if our tensile stress goes over 700MPa.

Design Considerations and Dimensions

The outer shape of the submarine is to be a perfect sphere. The following reasons support this decision:

1. Sphere is the shape with the minimum surface-area-to-volume ratio
2. For the same pressure conditions, the sphere has lesser stresses than a cylinder of corresponding dimensions
3. In case of the sphere the longitudinal stresses are equal to the hoop stress which is less than longitudinal stresses in cylinders
4. Spheres have uniform pressure distributions as compared to cylinders
5. No stress concentrations on edges as spheres don't have any as opposed to cylinders

The outer sphere or the outer hull is to be unpressurized while the inner hull is to be kept pressurized.

The dimensions are as follows:

The inner radius of the pressure hull = 0.8m

This gives a sphere of internal diameter 1.6m which is more than enough to house the required equipment, the air cylinders and a human being in sitting position.

The thickness of the outer light hull was kept at minimal because it is not pressurized under any conditions. Thus, we chose an arbitrary value for the thickness. The outer light hull has a thickness of half an inch.

The thickness of the outer hull = 0.0127m

These parameters were fixed and the excel solver was used to find the values of other variables considering the following parameters:

1. When the ballast tank of the submarine is completely filled, it should go down at a velocity which is the maximum possible velocity under the given conditions.
2. When the ballast tanks are evacuated of water and filled with compressed air only, the submarine should ascend with the same constant velocity.

The volume of the ballast tank was calculated by considering the above-mentioned constraints.

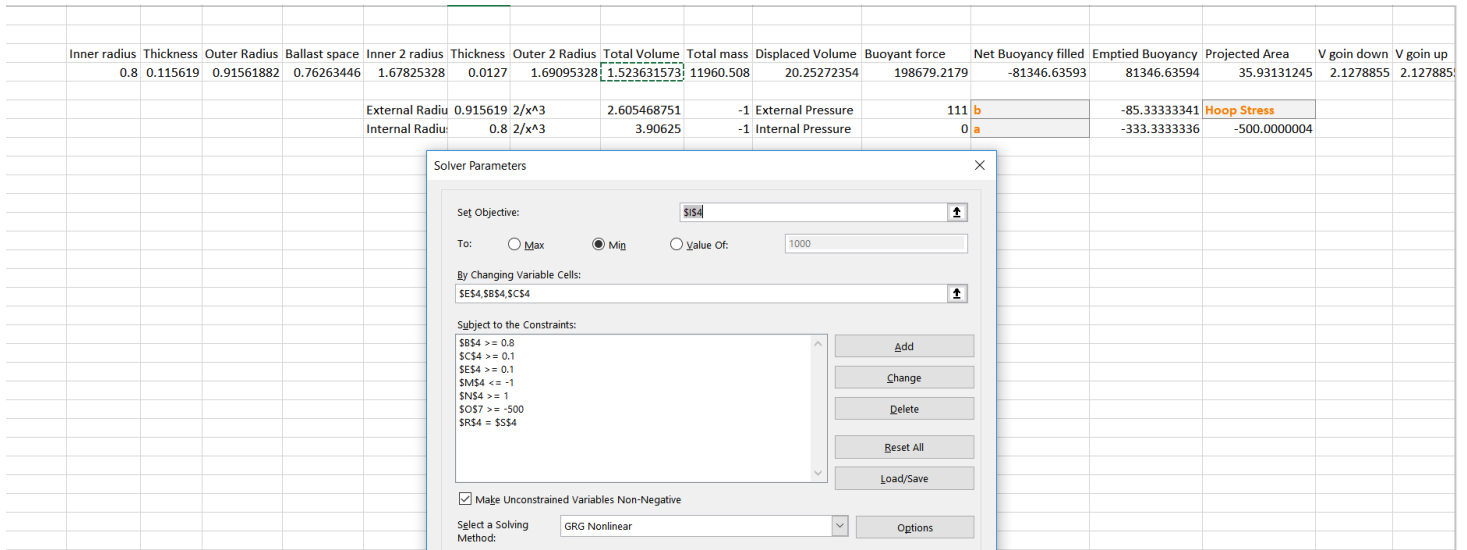


Figure 7 - Determining dimensions for the submarine

Through these calculations, we found out the following optimal values for the dimensions of the submarine.

Space for the ballast tank along the radial direction = 0.76m

Inner Radius of the ballast tank = 1.67m

Thickness of the pressure hull = 0.11m

The isometric cross-sectional view of submarine looks like¹:

¹ The ballast sphere is attached to the inner pressure hull via ribs and webs but these have been excluded from the picture for clarity.

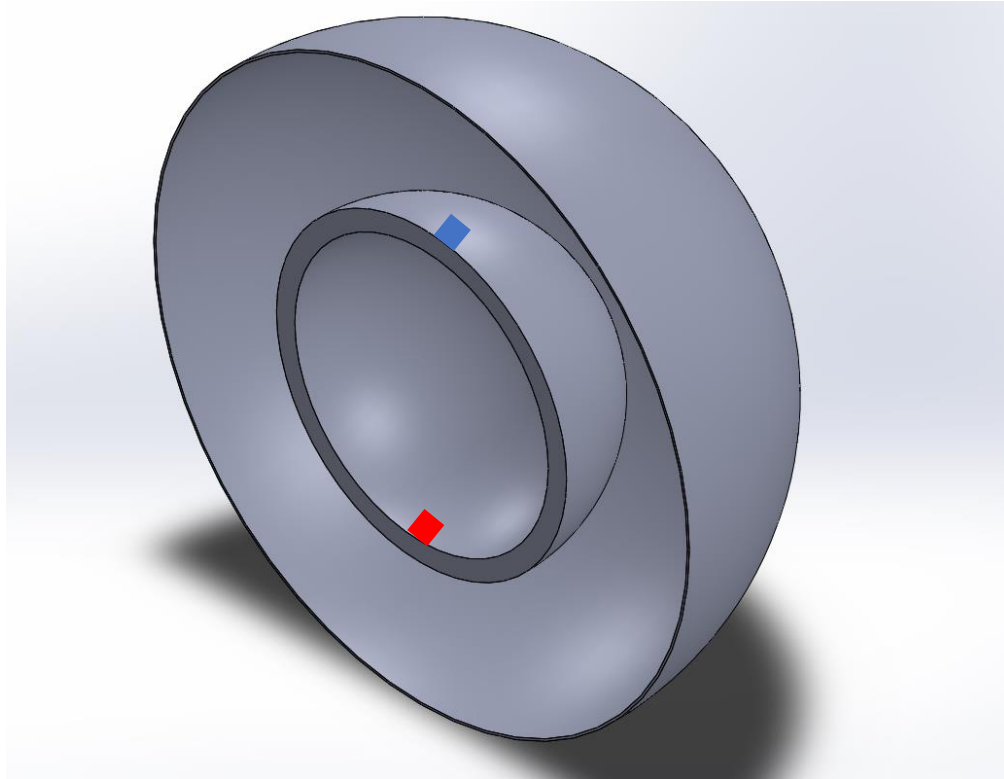


Figure 8 - Isometric view of the submarine parts under consideration

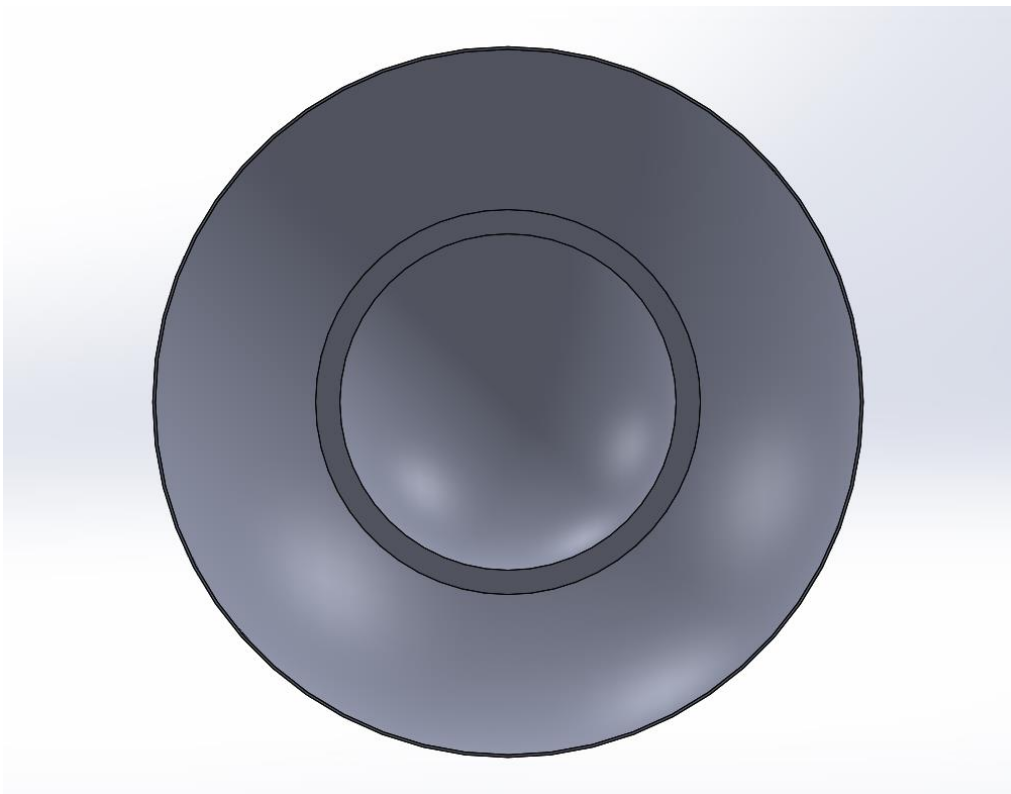


Figure 9 - Front view of the submarine parts under consideration

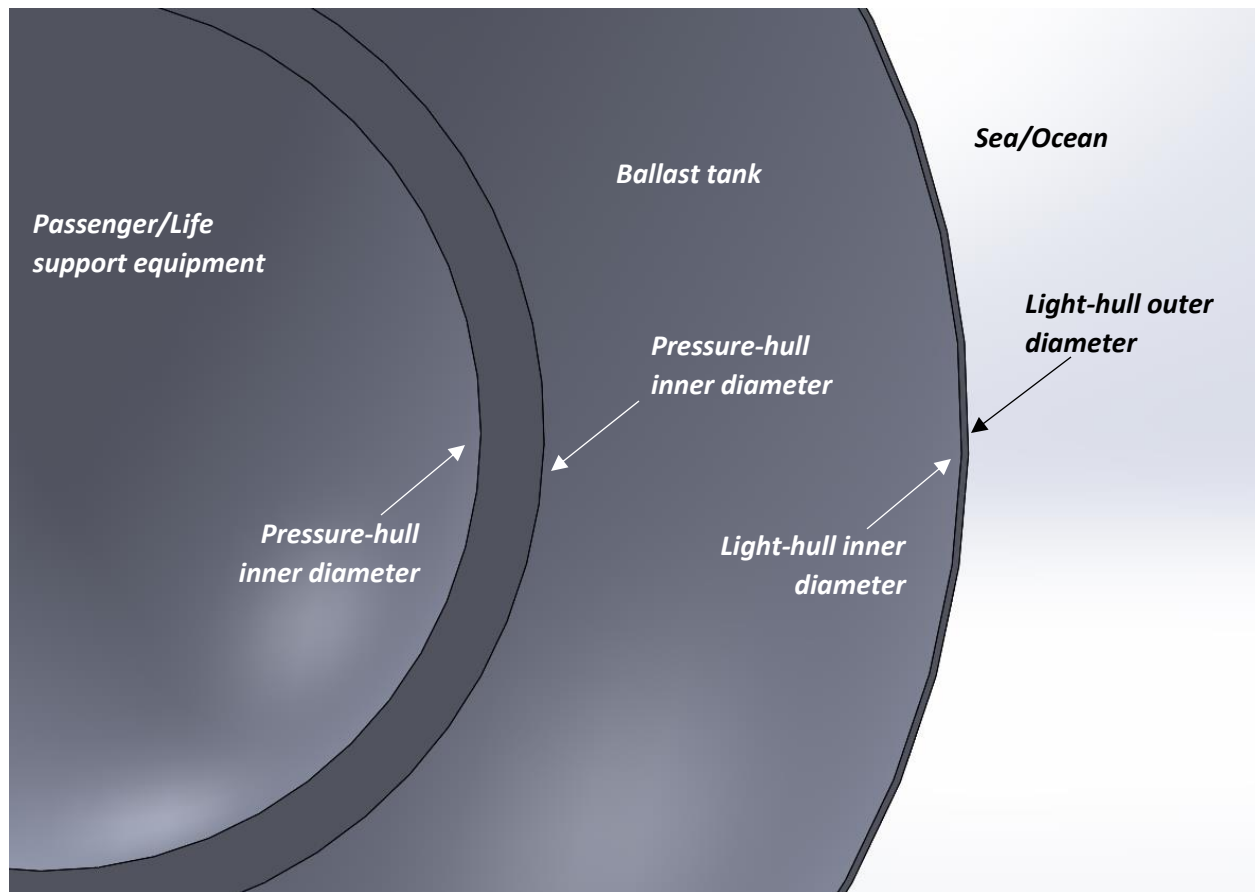


Figure 10 - Submarine front-view close-up

The final calculations are tabulated below (all units in SI):

PRESSURE HULL		
Inner Radius	Thickness	Outer Radius
0.800	0.116	0.916
Volume	Mass	
1.071	8,295.974	

BALLAST SPHERE	
Ballast Space	Ballast Volume
0.727	15.363
Mass when filled with water	Column1
15,808.932	

LIGHT HULL		
Inner Radius	Thickness	Outer Radius
1.643	0.013	1.656
Volume	Mass	
0.434	3,363.888	

SUBMARINE PROPERTIES	
Total Mass of the metal	11,659.862
Total Weight of the metal	114,383.243
Total Volume displaced	19.013
Weight of the filled ballast tank	155,085.622
Total Weight with filled ballast tank	269,468.865
Total Weight with emptied ballast tank	114,383.243
Buoyant Force	191925.9398

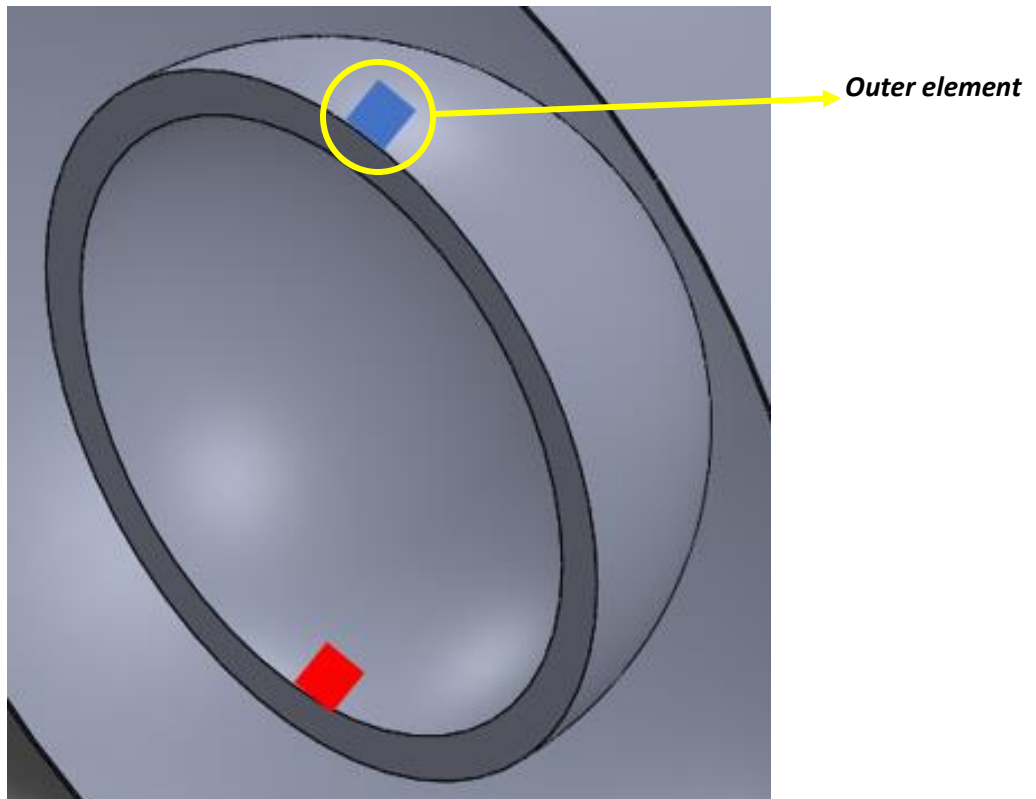
Failure Criterion:

While taking the element on the outside surface of the pressure hull, we have calculated the hoop and radial stresses for the pressure hull.

$$\text{Hoop Stress} = -444.5 \text{ MPa}$$

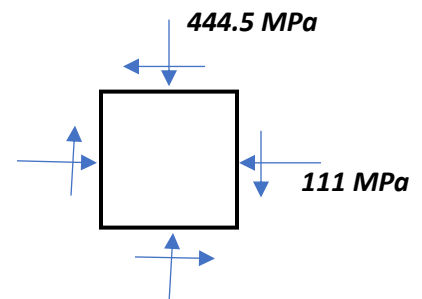
$$\text{Radial Stress} = -111 \text{ MPa}$$

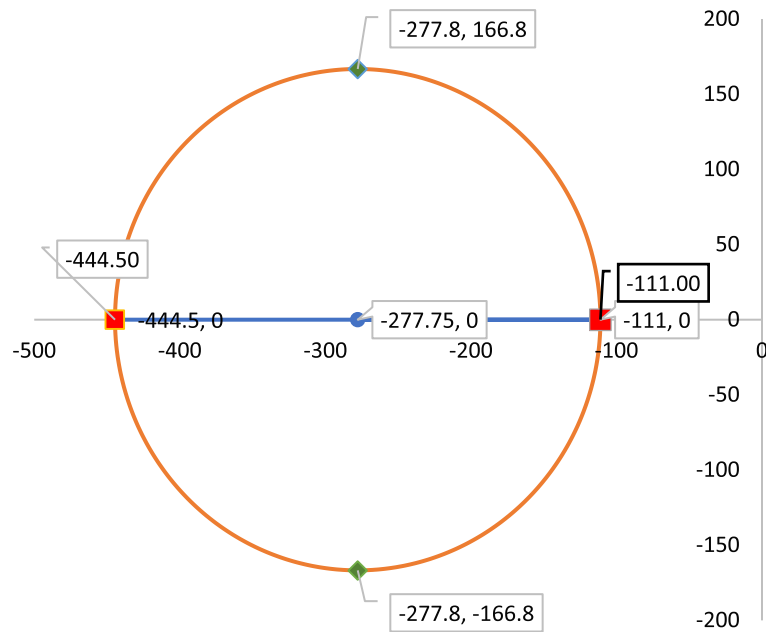
Element under Consideration



Mohr Circle

We construct a Mohr's Circle for the above-mentioned stresses





Principal stresses $\sigma_2 = -444.5 \text{ MPa}$ $\sigma_1 = -111 \text{ MPa}$ $\tau_{max} = -277.8 \text{ MPa}$

Tresca Failure Theory:

From the Mohr's circle, we get $\tau_{max} = 277.8 \text{ MPa}$. According to the Tresca criterion, the factor of safety is given as:

$$n = \frac{Sy/2}{\tau_{max}} = \frac{689/2}{277.8} = 1.24$$

Von Mises Theory:

Calculating the Von Mises Stress for the pressure hull,

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

$$\sigma' = 400.7 \text{ MPa}$$

We compare this Von Mises stress to the yield strength of the material and hence the factor of safety is

$$FOS = \frac{689}{400.7}$$

$$FOS = 1.7$$

We have used the following equation for the DE theory

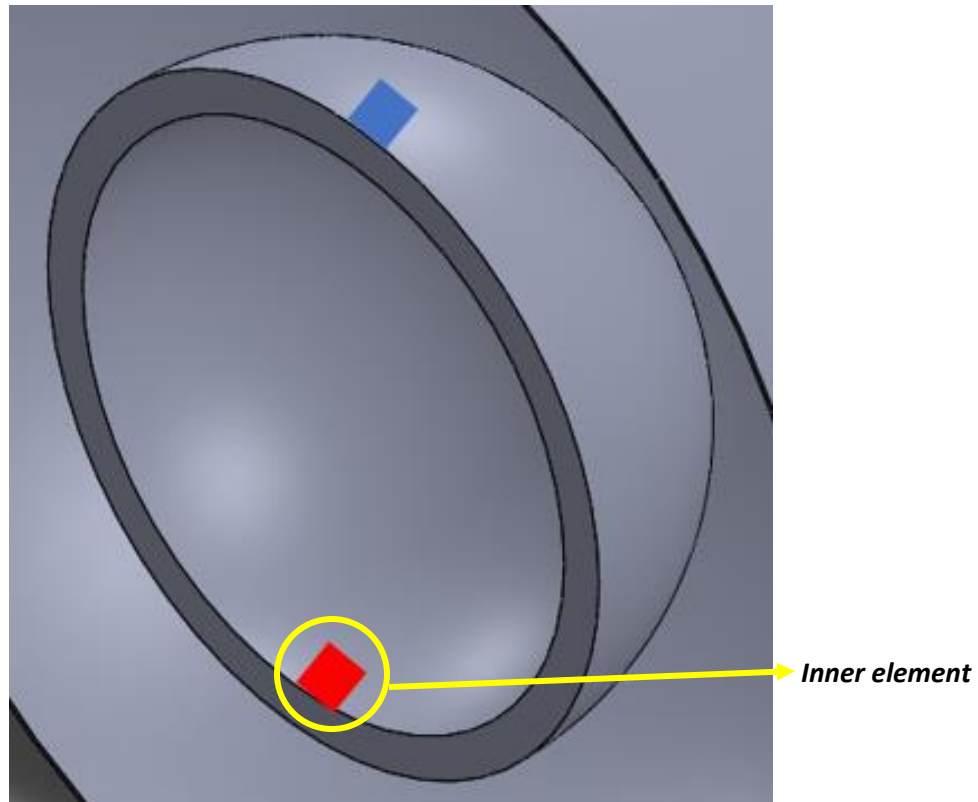
$$689^2 = x^2 - xy + y^2$$

The safe regions according to Von Mises Theory, Tresca and Rankine have been plotted. Our most critical element lies within the safe region according to all three theories of failure.

While taking the element on the inside surface of the pressure hull, we have calculated the hoop and radial stresses for the pressure hull. There is no radial stress acting on the inside of the sphere.

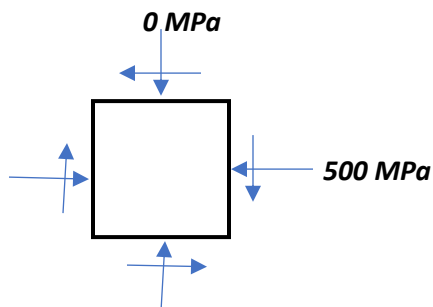
$$\text{Hoop Stress} = -500\text{MPa}$$

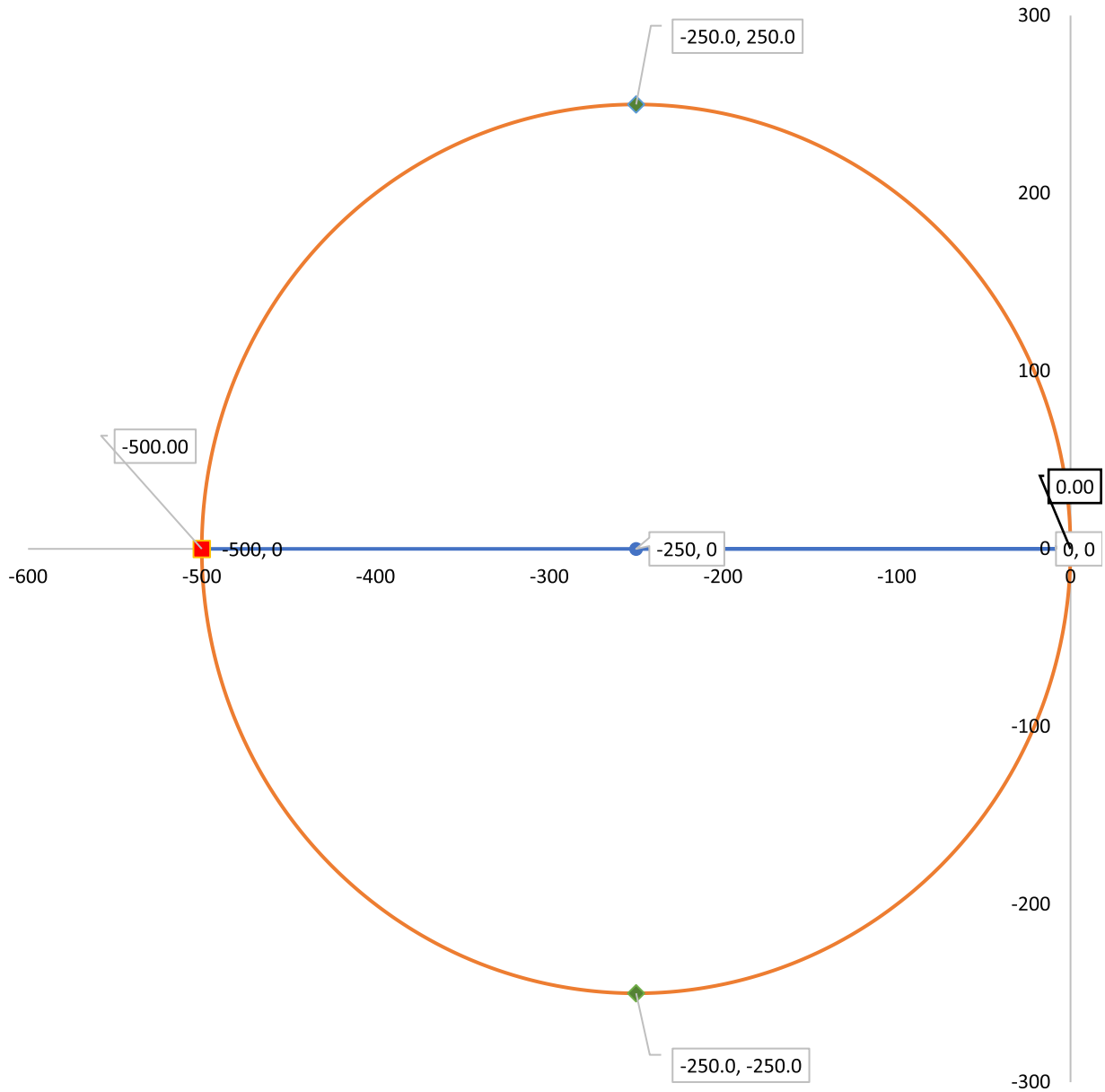
Element under Consideration



Mohr Circle:

We construct a Mohr's Circle for the above-mentioned stresses





Principal stresses $\sigma_1 = -500 \text{ MPa}$ $\sigma_2 = 0 \text{ MPa}$ $\tau_{max} = -250 \text{ MPa}$

Tresca Failure Theory:

From the Mohr's circle, we get $\tau_{max} = 250 \text{ MPa}$. According to the Tresca criterion, the factor of safety is given as:

$$n = \frac{S_y/2}{\tau_{max}} = \frac{689/2}{250} = 1.378$$

Von Mises Theory:

Calculating the Von Mises Stress for the pressure hull,

$$\sigma' = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

$$\sigma' = 500 \text{ MPa}$$

We compare this Von Mises stress to the yield strength of the material and hence the factor of safety is

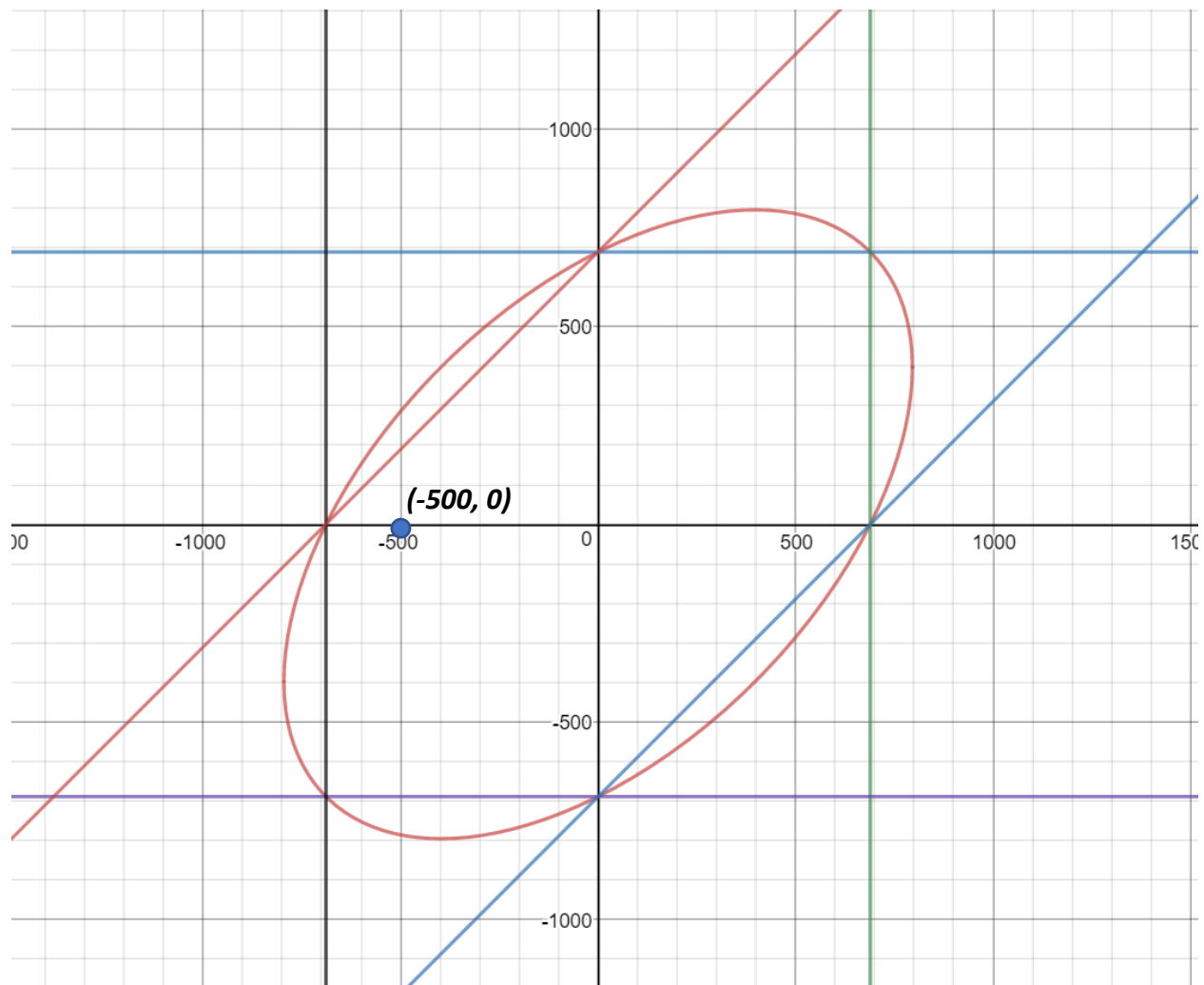
$$FOS = \frac{689}{500}$$

$$FOS = 1.378$$

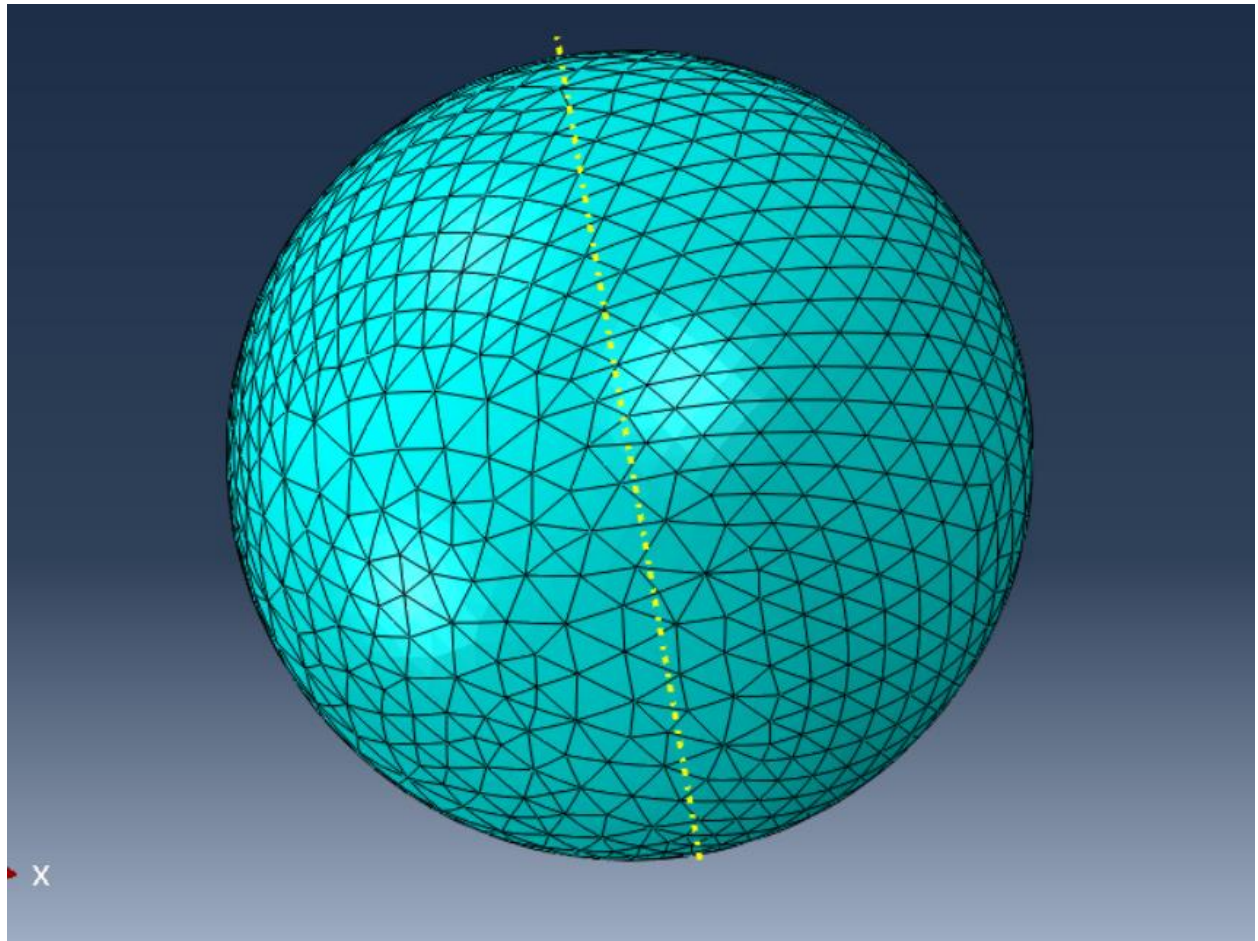
We have used the following equation for the DE theory

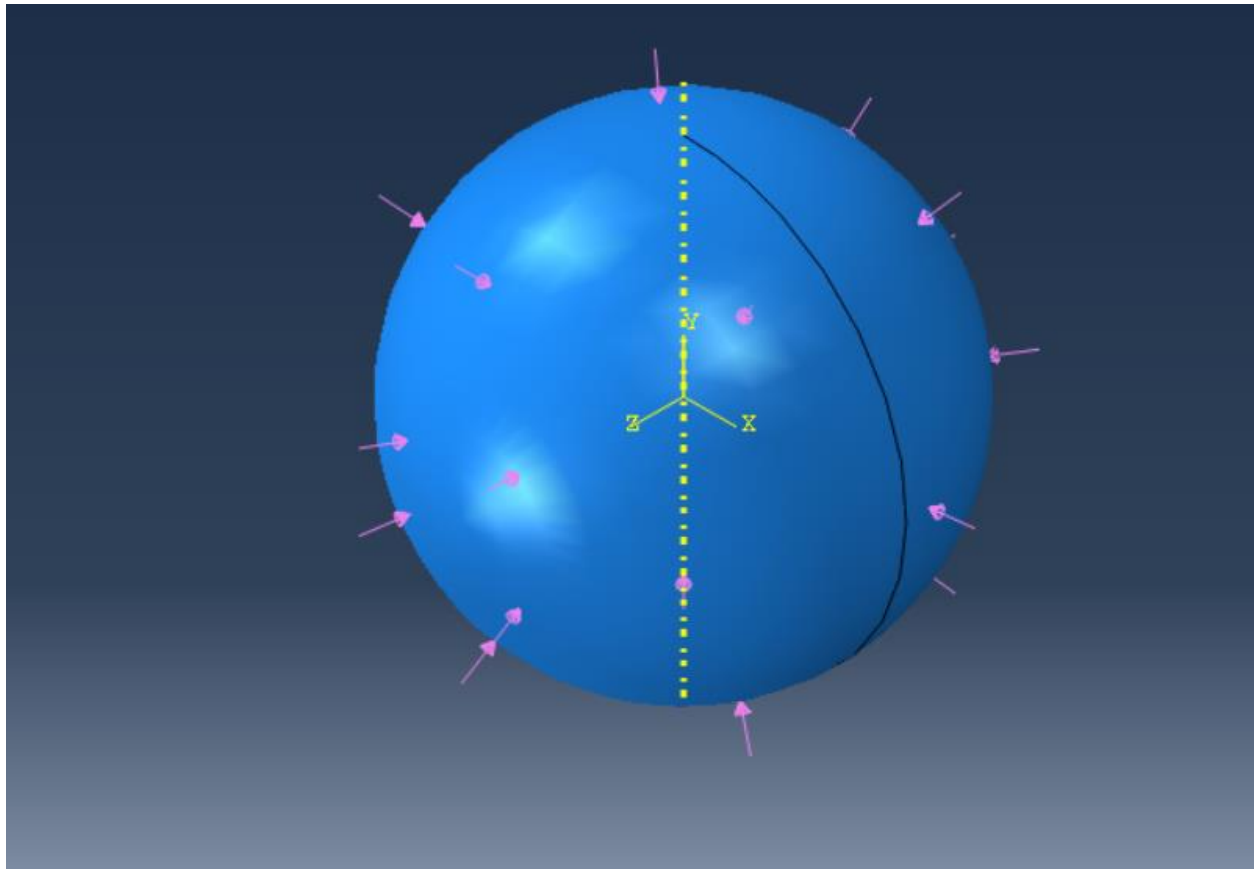
$$689^2 = x^2 - xy + y^2$$

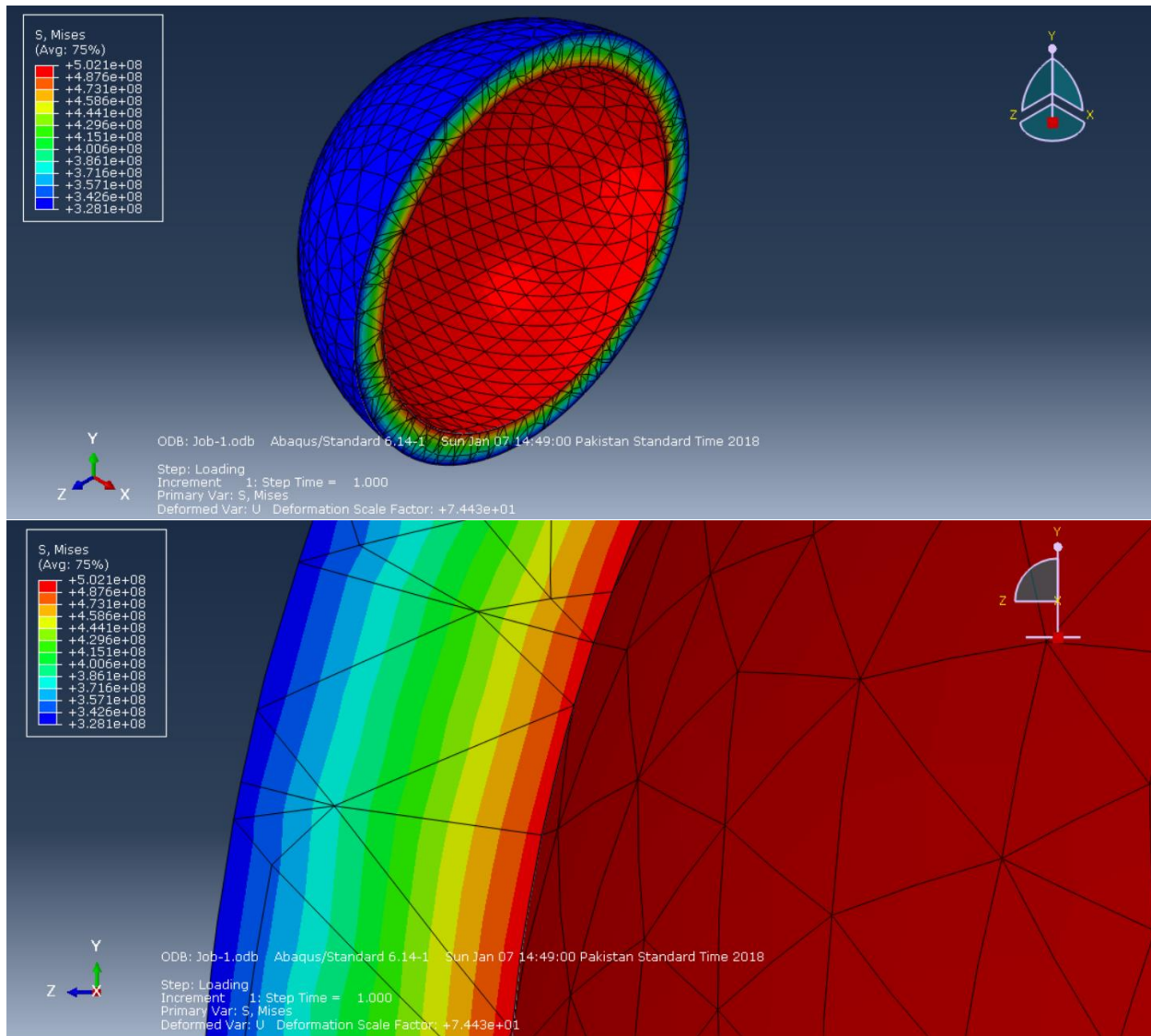
The safe regions according to Von Mises Theory, Tresca and Rankine have been plotted. Our most critical element lies within the safe region according to all three theories of failure.



Finite Element Analysis with ABAQUS







This shows that the principal element which lies at the inner radius of our pressurized hull is equal to 502MPa.

This is accurate within:

$$Error = \frac{502 - 500}{500} * 100 = 0.4\%$$

And a factor of safety of:

$$FOS = \frac{689}{502} = 1.4$$

Diving to the Sea Bed:

When the ballast tank of the submarine is initially filled with water, it starts sinking. The following calculations correspond to this state.

$$\textbf{\textit{Total weight of the submarine}} = \textbf{\textit{Weight of the sub}} + \textbf{\textit{Weight of the water in tank}}$$

$$\textbf{\textit{Total Weight}} = 269468N$$

$$\textbf{\textit{Buoyant Force}} = 191925N$$

$$\textbf{\textit{Net force in the downward direction}} = W - F_B$$

$$\textbf{\textit{Net force in the downward direction}} = 77543N$$

Due to this difference of forces, the submarine keeps accelerating until it is slowed by the drag to the point when the net force acting on it becomes zero and now it moves with a constant terminal velocity.

$$\textbf{\textit{Drag on the submarine}} = D = C_d \frac{\rho}{2} V^2 A$$

$$C_d = 1.02$$

Where

$$Re = \frac{\rho v D}{\mu}$$

Under balanced conditions:

$$W = D + F_B$$

$$\textbf{\textit{Velocity of the submarine while descending}} = \sqrt{\frac{4g}{3C_d} \left(\frac{\rho_s - \rho}{\rho} \right)}$$

$$\textbf{\textit{Velocity of the submarine while descending}} = \frac{2.12 \text{ m}}{s} = \frac{7.63 \text{ Km}}{h}$$

The summarized values are given below

DURING DESCEND	
Equilibrium Equation	$W=F+D$
Weight acting downwards	269,468.865
Buoyancy acting upwards	191,925.940
Drag acting upwards	77,542.925
Terminal Velocity	2.120

Stationary at the Sea Bed:

To keep the submarine stationary at the sea bed or at any level, the level of water in ballast tanks is maintained in such a way that the buoyance force equals the weight and net force acting on it becomes zero. Under these conditions, the submarine floats at a constant depth. The key is to control the ratio of water and compressed air in the tank. This is done with the help of high power pumps and vents installed in the ballast tanks.

To keep the submarine stationary at a level:

$$Velocity = 0 \frac{m}{s}$$

$$Weight\ of\ the\ submarine = Buoyant\ force\ acting\ on\ the\ submarine$$

$$W - F_B = 0$$

$$Net\ force\ acting\ on\ the\ submarine = 0\ N$$

Rising Up to the Surface

When the submarine is at the sea bed and it has to rise up, the ballast tanks are emptied. Compressed air pumps out the water and it causes the buoyance force to overcome the weight of the submarine. Under this condition, the submarine ascends. The following calculations correspond to this state.

$$Weight\ of\ the\ submarine = 114383N$$

$$Buoyant\ Force = 191925N$$

$$Net\ force\ in\ the\ Upward\ direction = 77542N$$

Due to this difference of forces, the submarine keeps accelerating until it is slowed by the drag to the point when the net force acting on it becomes zero and now it moves with a constant terminal velocity.

$$\text{Drag on the submarine} = D = C_d \frac{\rho}{2} V^2 A$$

$$C_d = 1.02$$

Where

$$Re = \frac{\rho v D}{\mu}$$

$$\text{Velocity of the submarine while ascending} = \sqrt{\frac{4g}{3C_d} \left(\frac{\rho_s - \rho}{\rho} \right)}$$

$$\text{Velocity of the submarine while ascending} = \frac{2.12m}{s} = \frac{7.63Km}{h}$$

The summarized values are tabulated below

DURING ASCEND	
Equilibrium Equation	F=W+D
Weight acting downwards	114,383.243
Buoyancy acting upwards	191,925.940
Drag acting upwards	77,542.697
Terminal Velocity	2.120

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