IRT, MC data analysis, Astrophysical Plasma

October 10, 2024, Meudon

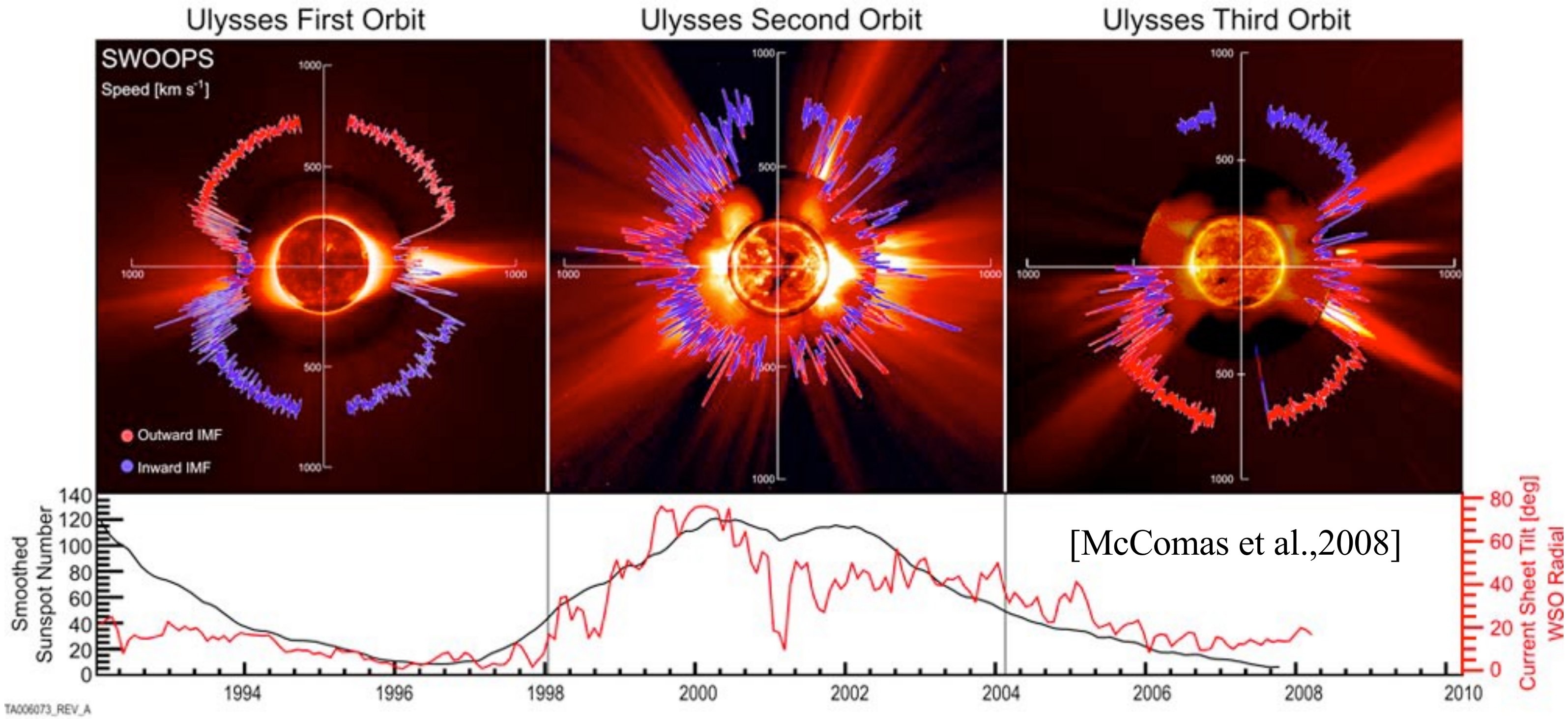
Spectral analysis of in-situ time series measurements

Olga Alexandrova

Astronomer @ Observatoire de Paris/LESIA

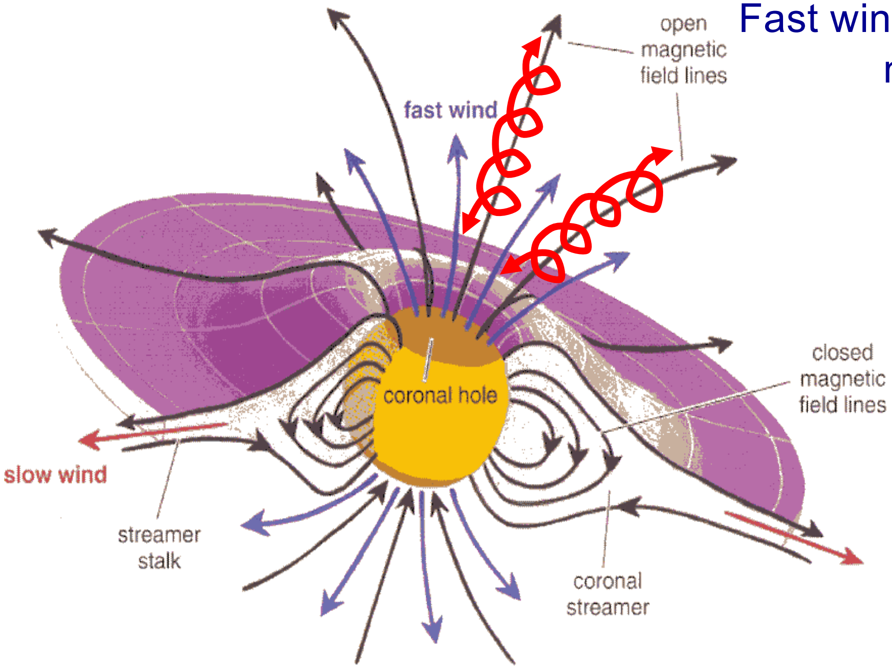
Measurements in astrophysics

- Remote sensing (images with telescopes or spacecraft, spectro-polarimetry …)
- In-situ measurements with space missions

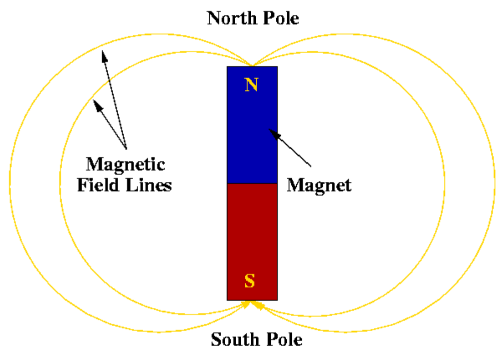


Example of remote + in-situ (SOHO + Ulysses). The joint ESA-NASA Ulysses deep-space mission was designed to study the heliosphere - the region of space influenced by the Sun and its magnetic field.

Magnetic field of the Sun: slow and fast streams and 11 years cycle



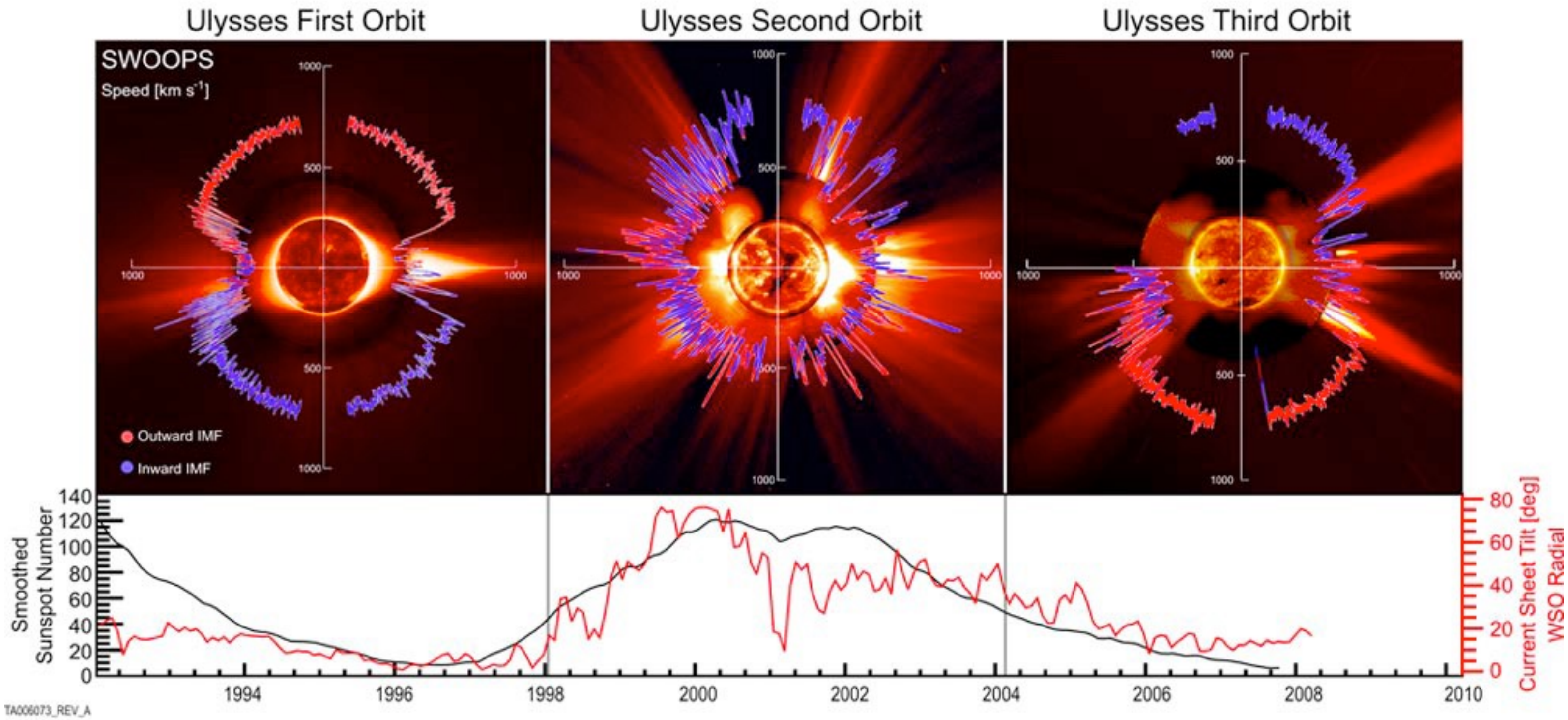
Fast wind: V = 600-800 km/s,
n=3 cm-3, Tp=5.10^{5}K



Slow wind: V = 300-400 km/s,
n=7 cm-3, Tp=2.10^{5}K

- Fast wind blows out from the coronal holes (open field lines). Slow wind – from the coronal streamers, above the closed field lines and along the heliospherical current sheet (c.f. purple zone).
- Every 11 years magnetic dipole of the Sun reverses.

Slow and fast winds and the 11 years solar cycle

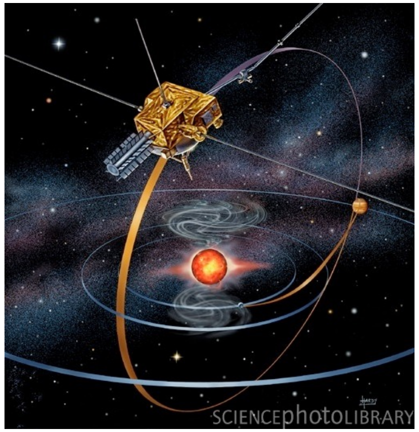


Slow wind: V = 300-400 km/s, n=7 cm^{-3}, T=$2x10^5$K
Fast wind: V = 600-800 km/s, n=3 cm^{-3}, T=$3x10^5$K
Mean free path ~ 1 AU (Sun-Earth distance)

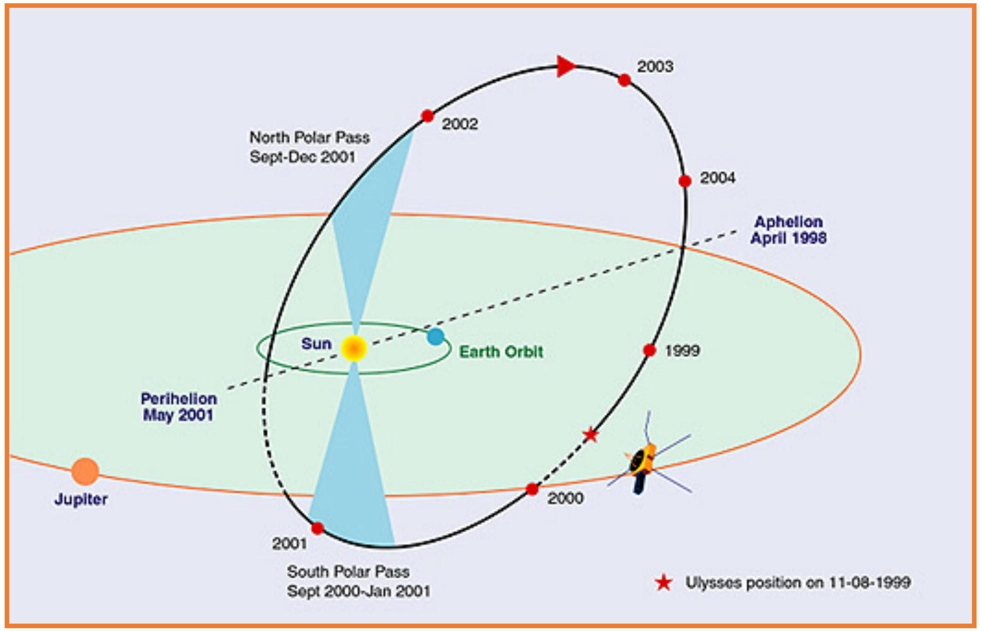
Ulysses

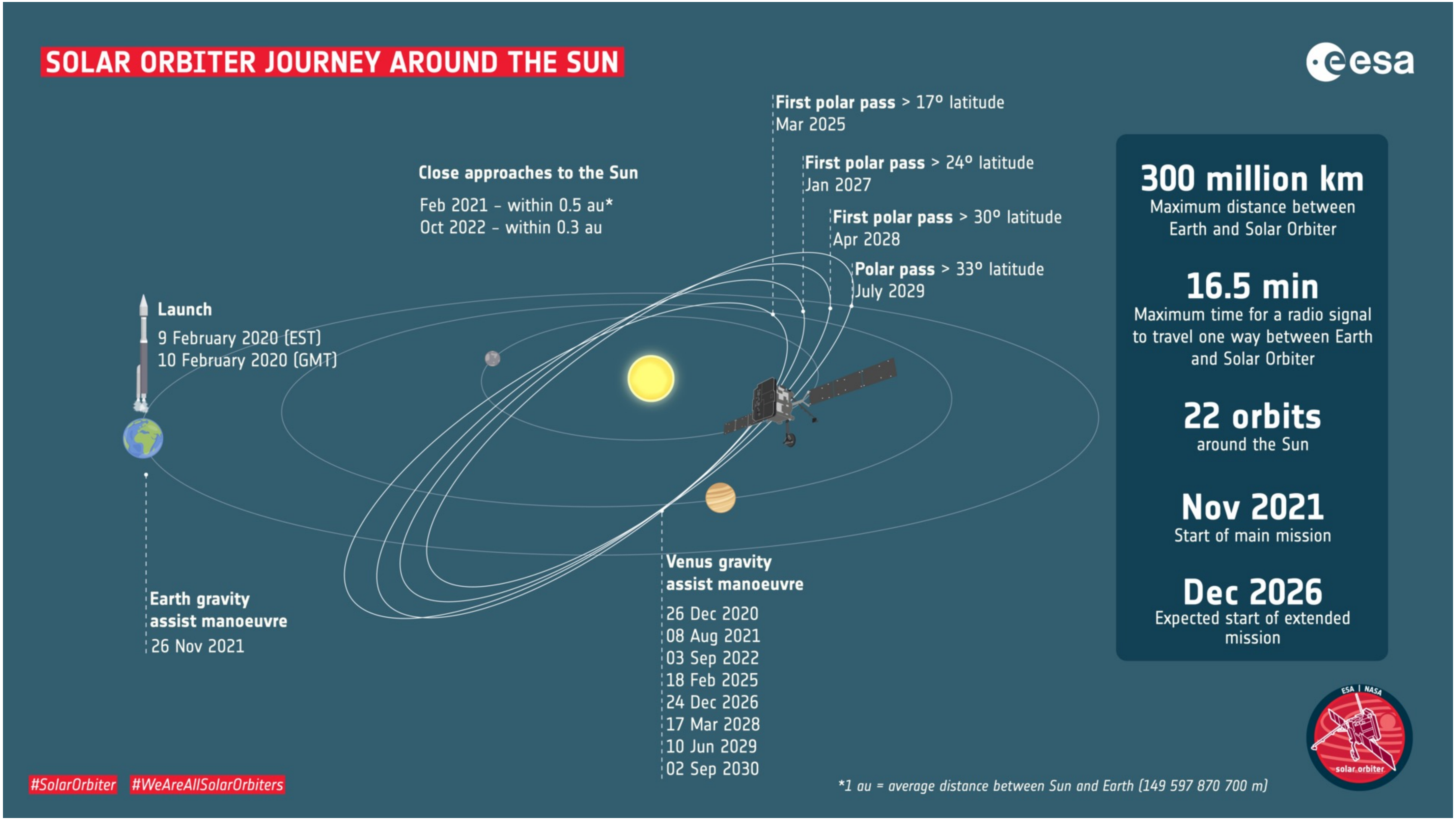
http://sci.esa.int/ulysses/

http://smsc.cnes.fr/ULYSSES/Fr/



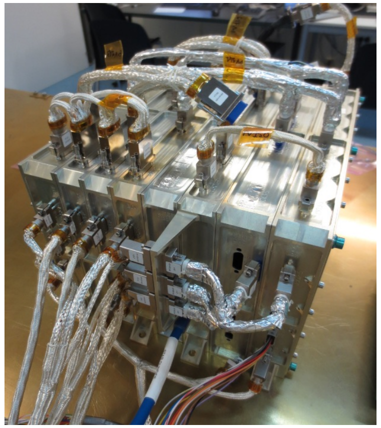
- Launched in 1990, initially planed for 5 years, extended once by 5 years and once by 9 years
- First to explore high latitudes: up to 80º (Jupiter: gravity assistance in 1992 to get it out of the ecliptic)
- Eccentric orbit: 1.3 - 5.4 AU



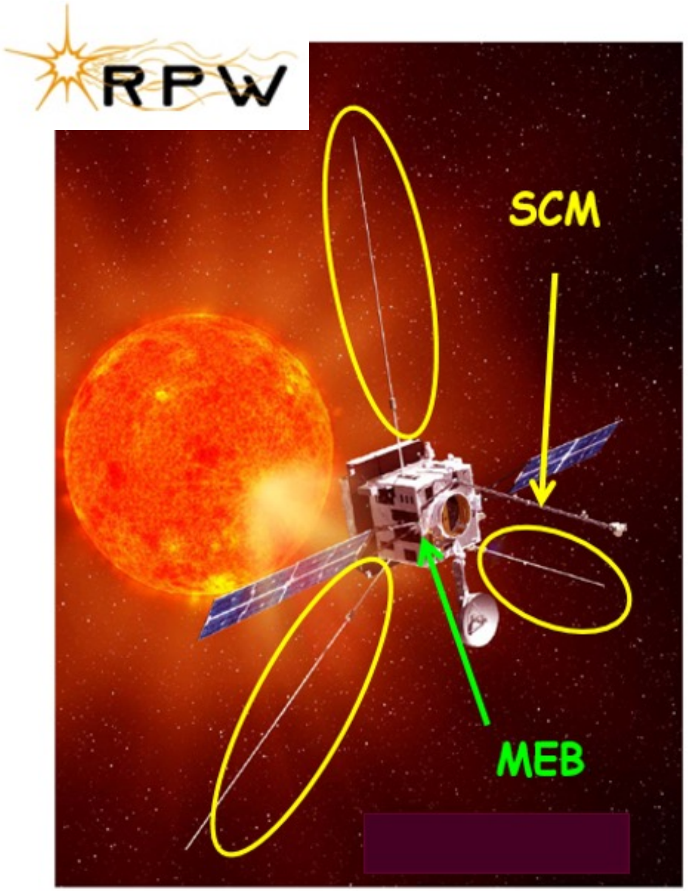

SOLAR ORBITER JOURNEY AROUND THE SUN
esa
First polar pass > 17° latitude
Mar 2025
First polar pass > 24° latitude
Jan 2027
First polar pass > 30° latitude
Apr 2028
Polar pass > 33° latitude
July 2029
Close approaches to the Sun
Feb 2021 – within 0.5 au*
Oct 2022 – within 0.3 au
Launch
9 February 2020 (EST)
10 February 2020 (GMT)
Earth gravity
assist manoeuvre
26 Nov 2021
Venus gravity
assist manoeuvre
26 Dec 2020
08 Aug 2021
03 Sep 2022
18 Feb 2025
24 Dec 2026
17 Mar 2028
10 Jun 2029
02 Sep 2030
300 million km
Maximum distance between Earth and Solar Orbiter
16.5 min
Maximum time for a radio signal to travel one way between Earth and Solar Orbiter
22 orbits
around the Sun
Nov 2021
Start of main mission
Dec 2026
Expected start of extended mission
ESA | NASA
solar orbiter
#SolarOrbiter #WeAreAllSolarOrbiters
*1 au = average distance between Sun and Earth (149 597 870 700 m)

ESA/Solar Orbiter/RPW (Radio & Plasma Waves)
PIship LESIA
Mesures in-situ => 0.3 UA

SCM (Search Coil Magnetometer), LPC2E



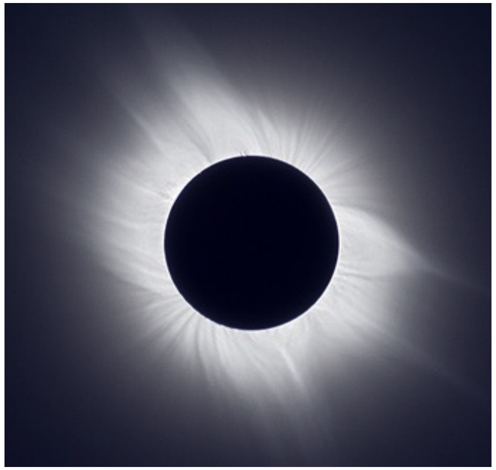
MEB (Main Electronic Box)



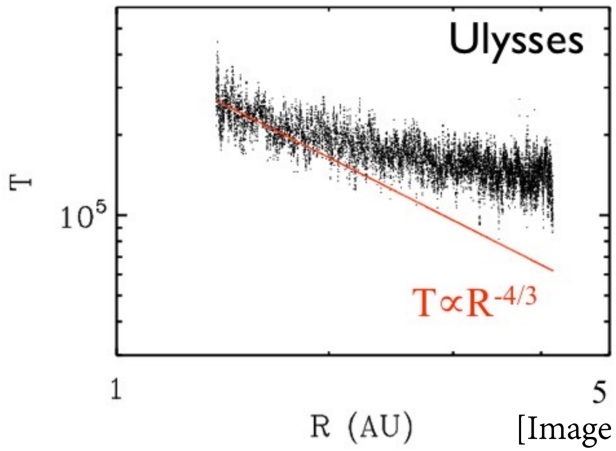
Electric antennas

The solar wind

- Expansion of the solar corona in interplanetary space
- Best natural laboratory of astrophysical plasmas, which can be explored with space missions



Wind temperature (T_{ions}) decays less than adiabatic ($\sim R^{-4/3}$) => heating !



[Image credit: Lorenzo Matteini]

- Essentially electrons and protons (~5% of heavier ions)
- Supersonic and superalfvénic in interplanetary space (mean speed ~500 km/s)
- Mean temperature (e^-,p^+) at 1 AU ~20 eV
- Mean density at 1 AU ~5 cm^{-3}
- **Few collisions (1 collision/1 AU)** ⇒ viscosity ~ 0 ⇒ magnetic field is frozen in plasma

Focus on in-situ measurements in Heliosphere

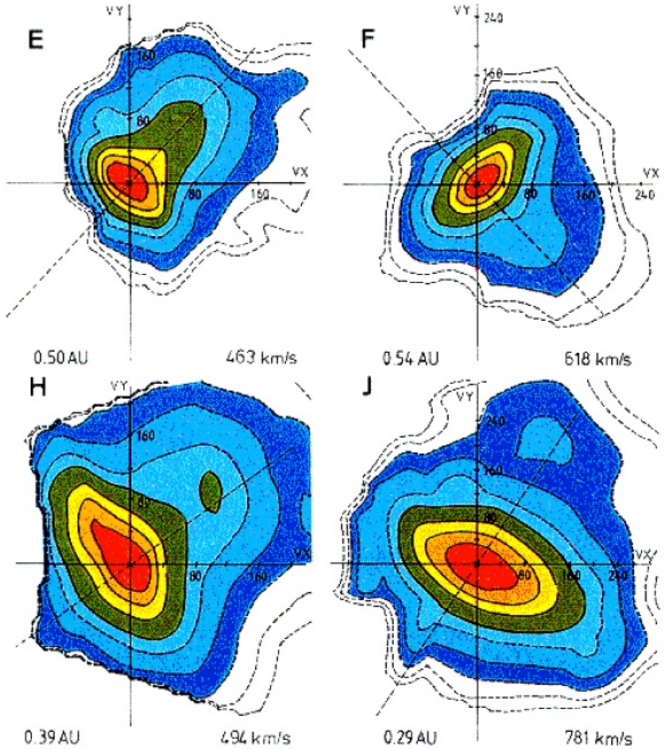
- Electromagnetic field and its fluctuations as functions of time $\vec{B}, \delta\vec{B}, \vec{E}, \delta\vec{E}$
- Particles distribution functions and their moments $f_i(\vec{v},\vec{r},t), \langle v^n . f_i(\vec{v},\vec{r},t)\rangle$

Examples of the space missions to study solar wind and Heliospheric plasmas

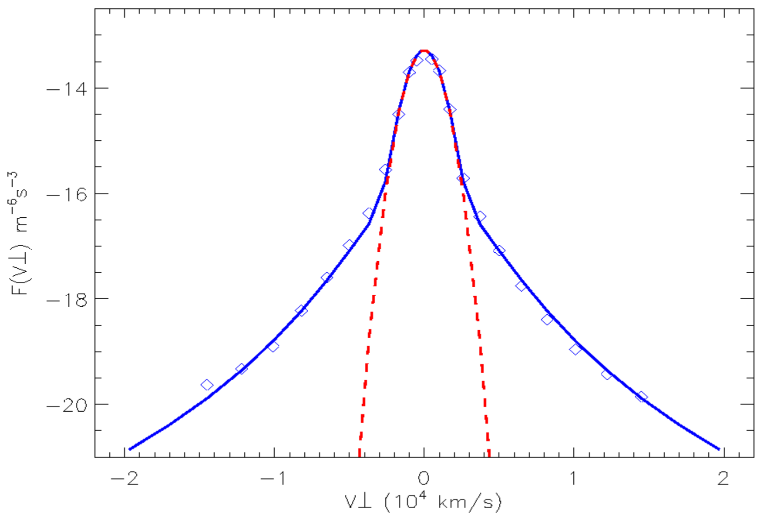
- …
- Helios (1976)
- Voyager (1977)
- …
- Ulysses (1990)
- Wind (1994)
- Cassini (1997)
- Cluster (2000)
- …
- Parker Solar Probe (2018)
- Solar Orbiter (2020)

Proton and electron distribution functions in the solar wind

Helios mission (~1972).
Protons : bi-Maxwellian + beam

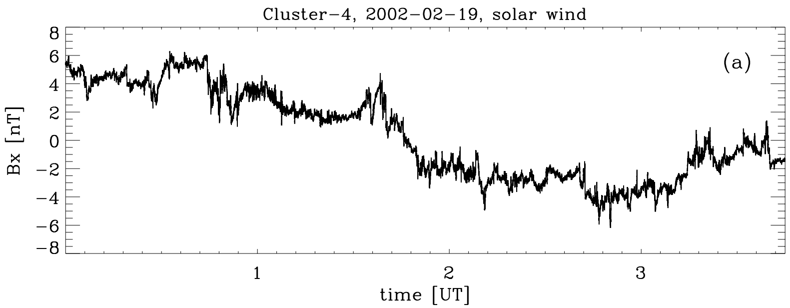


Ulysse mission. Electrons:
Kappa distributions + beam

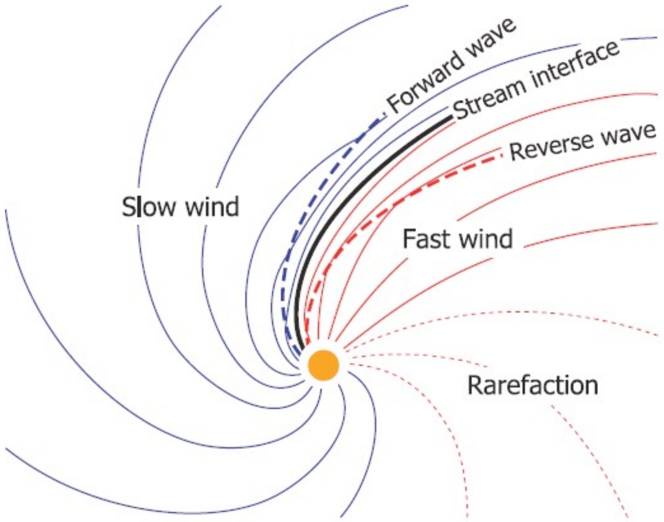


- far to be Maxwellian...

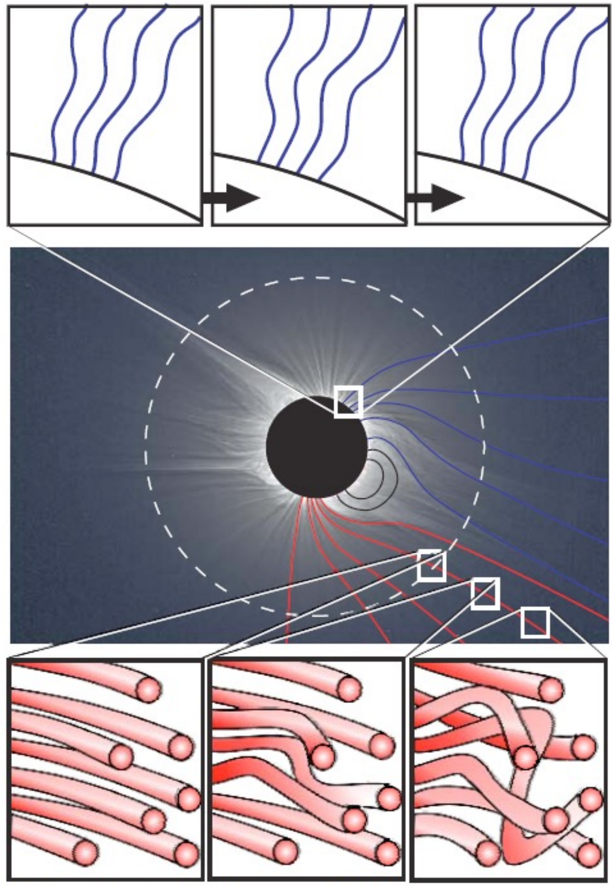
Magnetic field measurements



$$\boxed{\mathbf{B}(t) = \mathbf{B}_0(t) + \delta\mathbf{B}(t)}$$



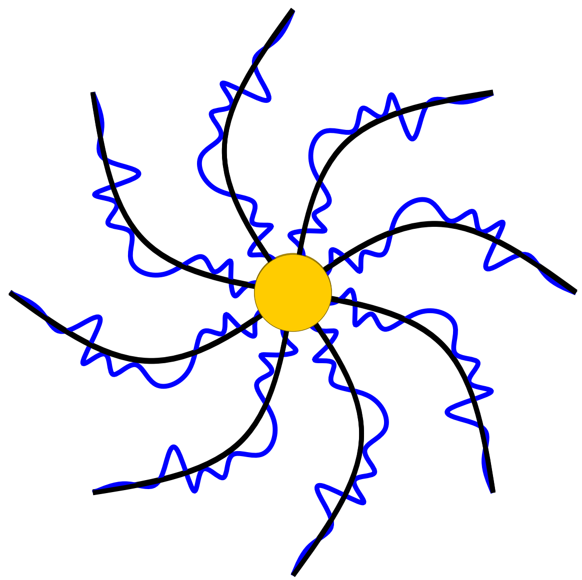
Taylor hypothèses : $\boxed{x = t \cdot V_{sw}}$



Heliospheric field is tangled by waves and/ or turbulence generated in transit

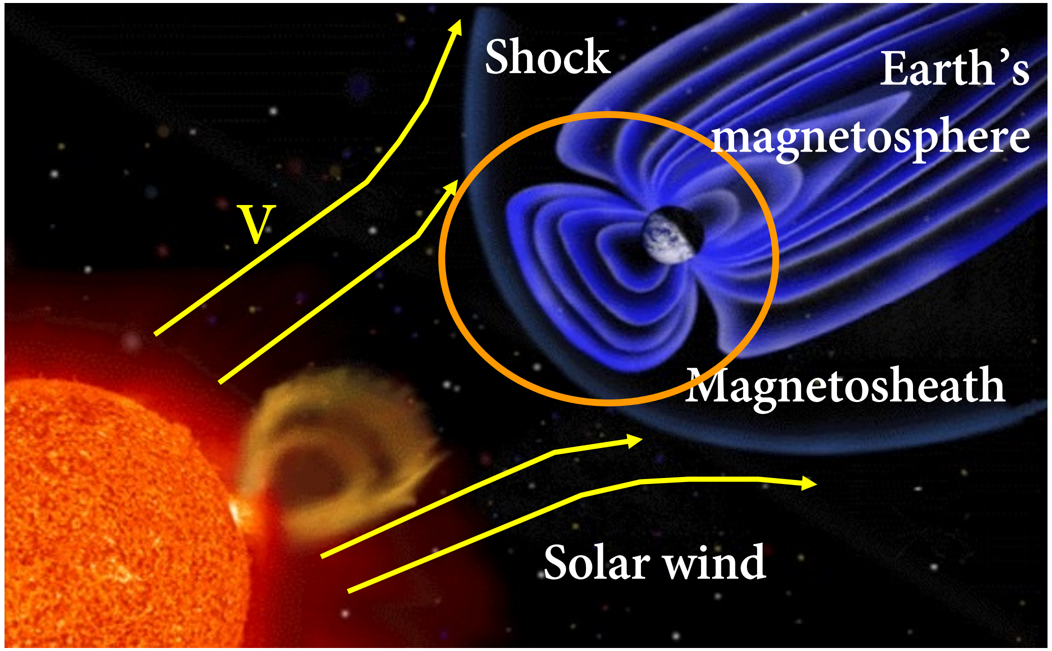
[Owens & Forsyth, 2013]

Parker Spiral



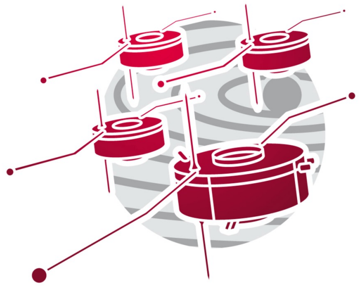
Magnetic field is turbulent

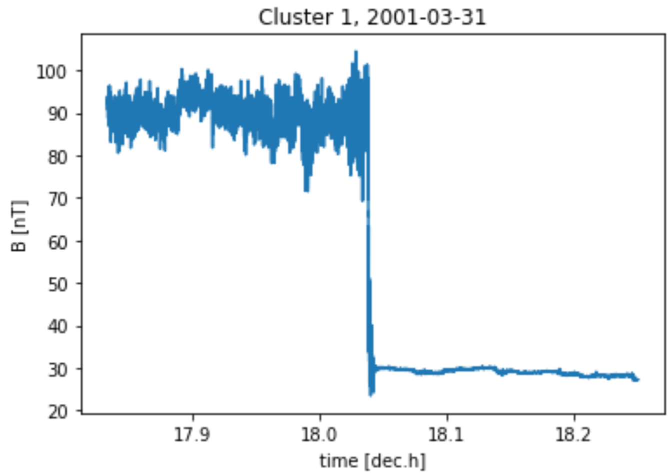
Solar wind and the Earth's magnetosphere



In-situ data: time series of plasma parameters.

Ex: Bow-shock crossing by Cluster-1 satellite





V = 300-800 km/s, $V_{mag.sonor}$ ~ 100 km/s

$V > V_{mag.sonor}$:super-sonic flow

Interaction of the supersonic solar wind with the Earth's magnetic field =>

- Formation of the magnetosphere
- Generation of the shock wave in front of the magnetosphere

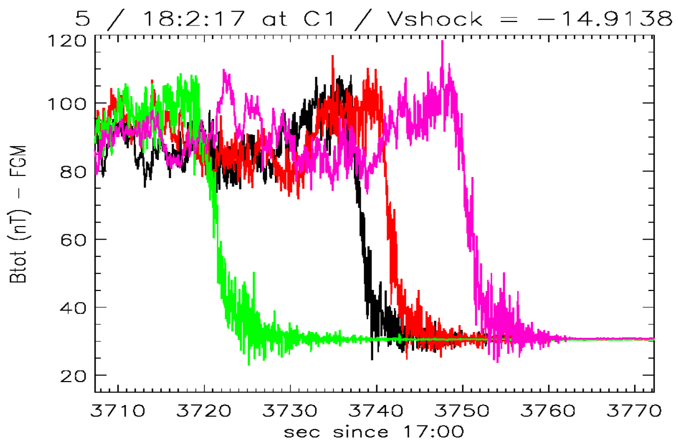
Why spectral analysis of in-situ satellite measurements ?

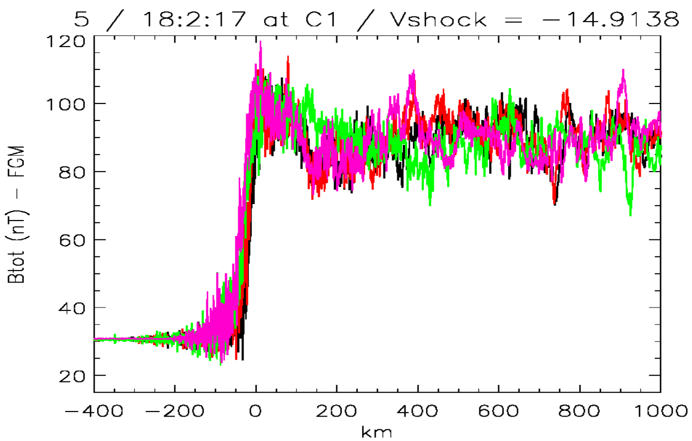
Multi-scale phenomena in space plasma
=> decomposition in frequencies/scales:

$$Y(t) \Rightarrow y(f)$$
$$Y(t) \Rightarrow y(t,f)$$

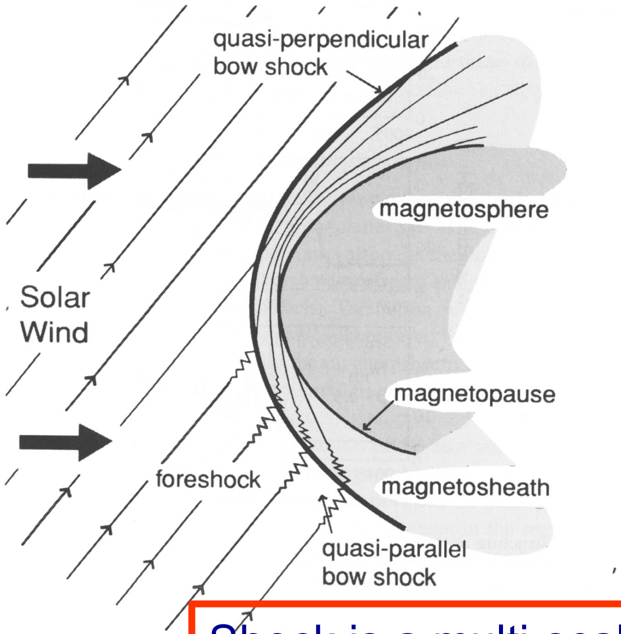
Earth's bow-shock: fast magnetosonic discontinuity

- Magnetospheric boundary at which V_{sw} becomes subsonic, kinetic energy decreases, all other types of energy (magnetic, thermal, turbulent) increase…
- Thickness << mean free path, of the order of the electron Larmor radius ~ 5 km





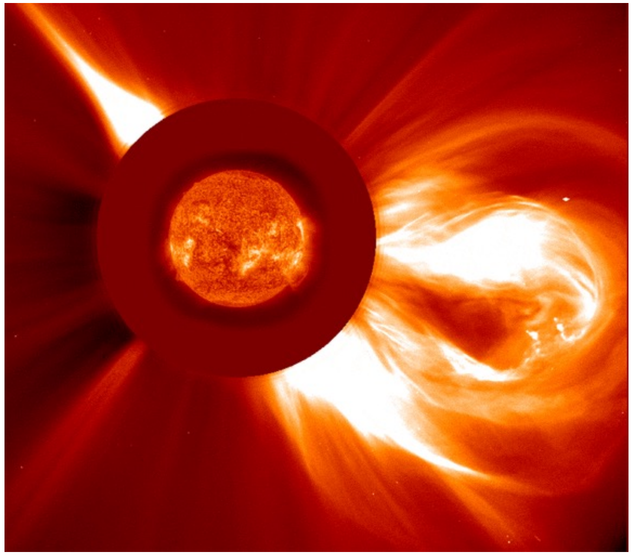
Shock coves scales from the electron Larmor radius and up to the size of the magnetosphere => multi-scale phenomenon…

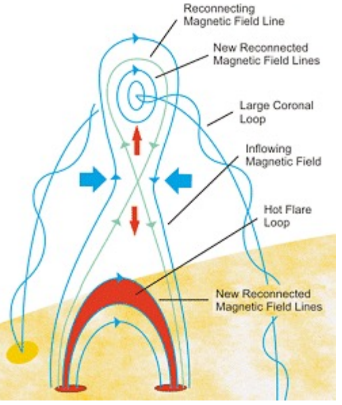


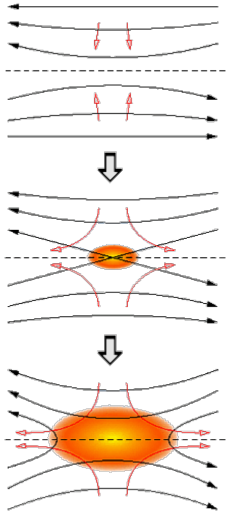
Shock is a multi-scale phenomenon.

Magnetic reconnection and Coronal Mass Ejections (CME)

Dynamics of magnetic loops linked to solar spots (reconnection of field lines) is at the origin of coronal mass ejections (CME)

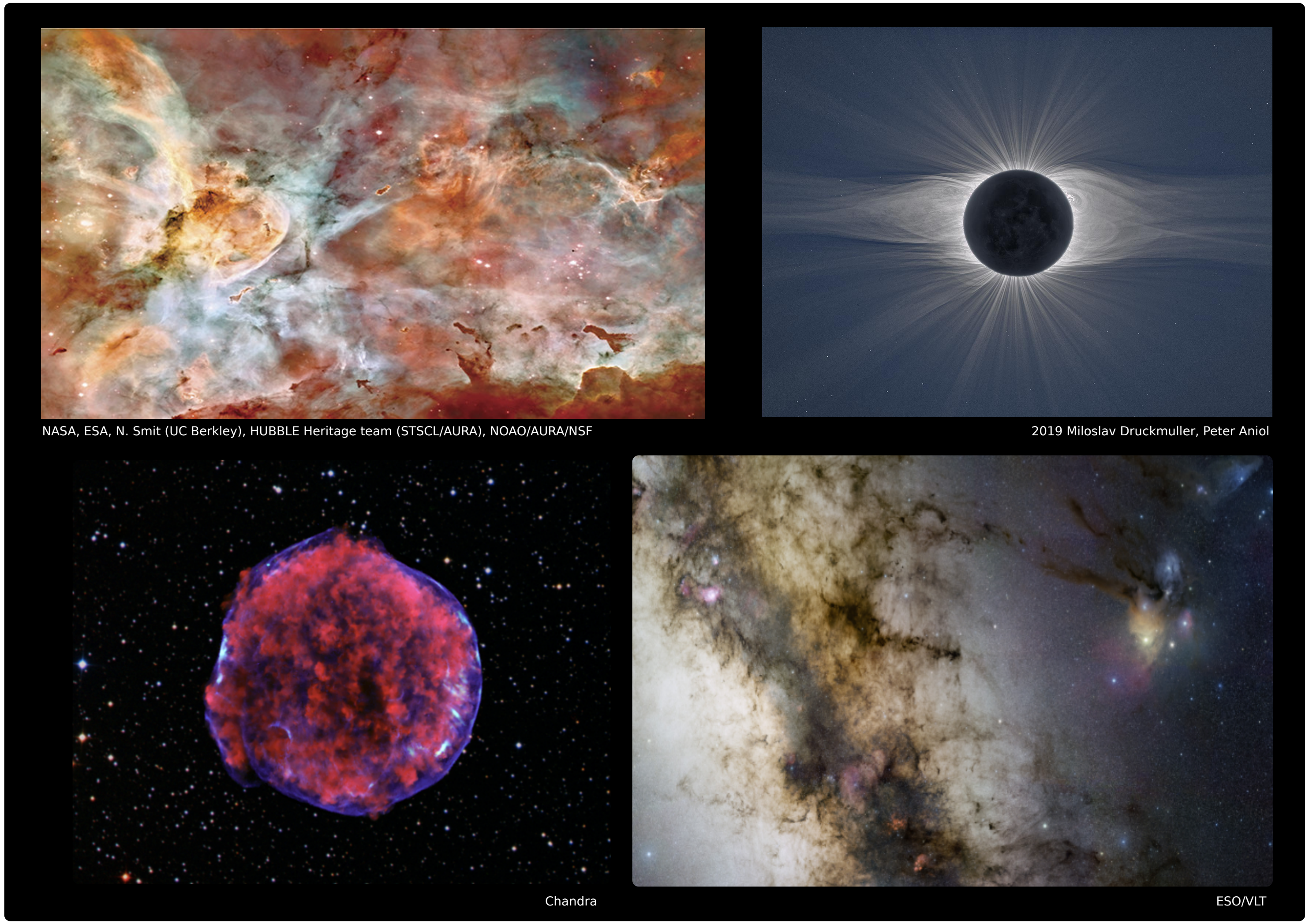






Magnetic reconnection is a multi-scale phenomenon.

Atrophysical plasmas are generally turbulent



Turbulence develops in any flow where energy is injected at scales L_0 >> the scale of dissipation L_d ($L_0/L_d = R_e^{3/4}$)

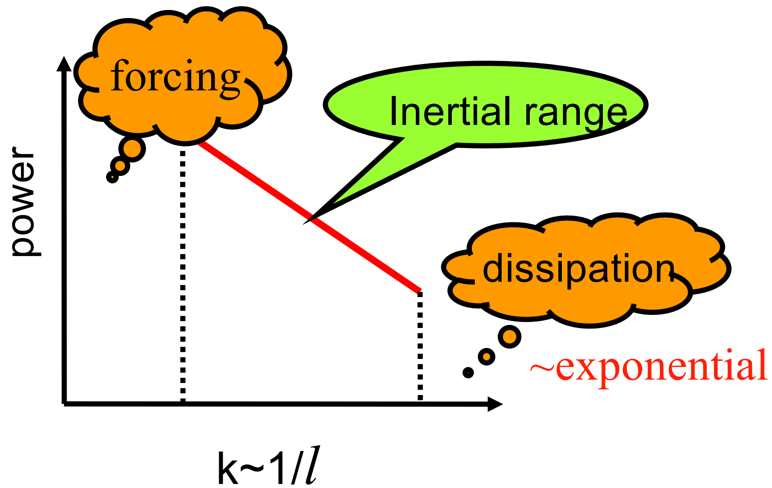
What kind of order emerges from this apparent chaos ?

Turbulence ?

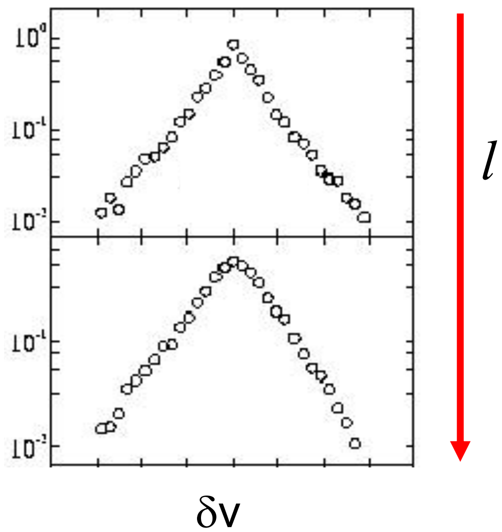
Leonardo da Vinci,
water studies (1510-1512)

Locally unpredictable, but **statistical properties are predictable and universal**

1) Velocity field energy~$k^{-5/3}$ (scale invariance, same physics at all scales l)



2) Intermittency : deviation from the Gaussianity at small l



Why does turbulence develop ?

"Navier-Stokes equation probably contains all of turbulence"
(Uriel Frisch, 1995)

ν → kinematic viscosity

$$\frac{\partial u}{\partial t} + (u.\nabla)u = -\nabla p + \nu \Delta u$$

non-linear term dissipation

Non-linear term >> term of dissipation ⇒ Turbulence

Why does turbulence develop ?

"Navier-Stokes equation probably contains all of turbulence"
(Uriel Frisch, 1995)

ν → kinematic viscosity

$$\frac{\partial u}{\partial t} + (u.\nabla)u = -\nabla p + \nu \Delta u$$

non-linear term dissipation

Non-linear term >> term of dissipation ⇒ Turbulence

If the energy injection scale L > L_d - scale of dissipation (~m.f.p.), turbulence develops !

Reynolds number
[1883]

$$R = \left.\frac{(u.\nabla)u}{\nu \Delta u}\right|_{\text{At largest scale L}}$$

$$\boxed{R = \frac{UL}{\upsilon}}$$

Reynolds number :

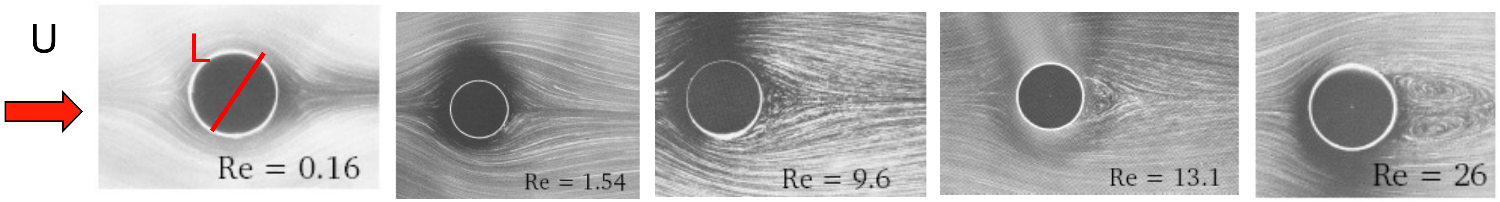
a measure of the strength of the turbulence

$$R = \frac{(u.\nabla)u}{\nu \Delta u}$$

At largest scale L

$$\boxed{R = \frac{UL}{\upsilon}}$$

Reynolds number is the ratio of NL-term over dissipation one (at the largest scale)

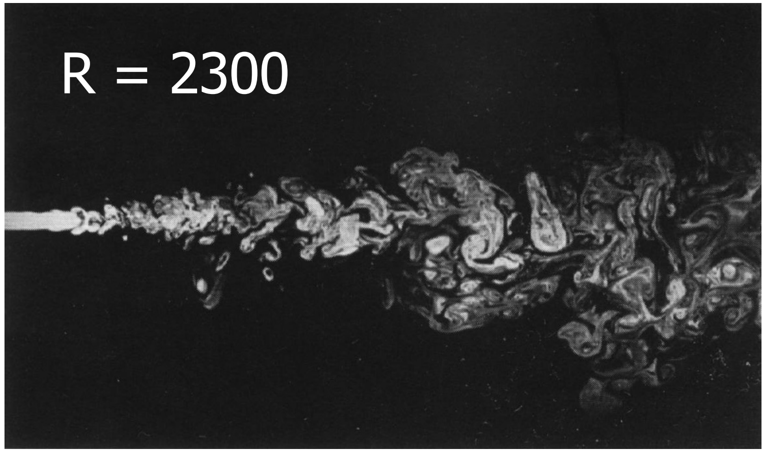


A fluid flow becomes “turbulent” as the Reynolds number increases

The higher R, the stronger the turbulence !

Example of fully developed turbulence in a jet

Here R reaches its critical value: there is no laminar flow anymore



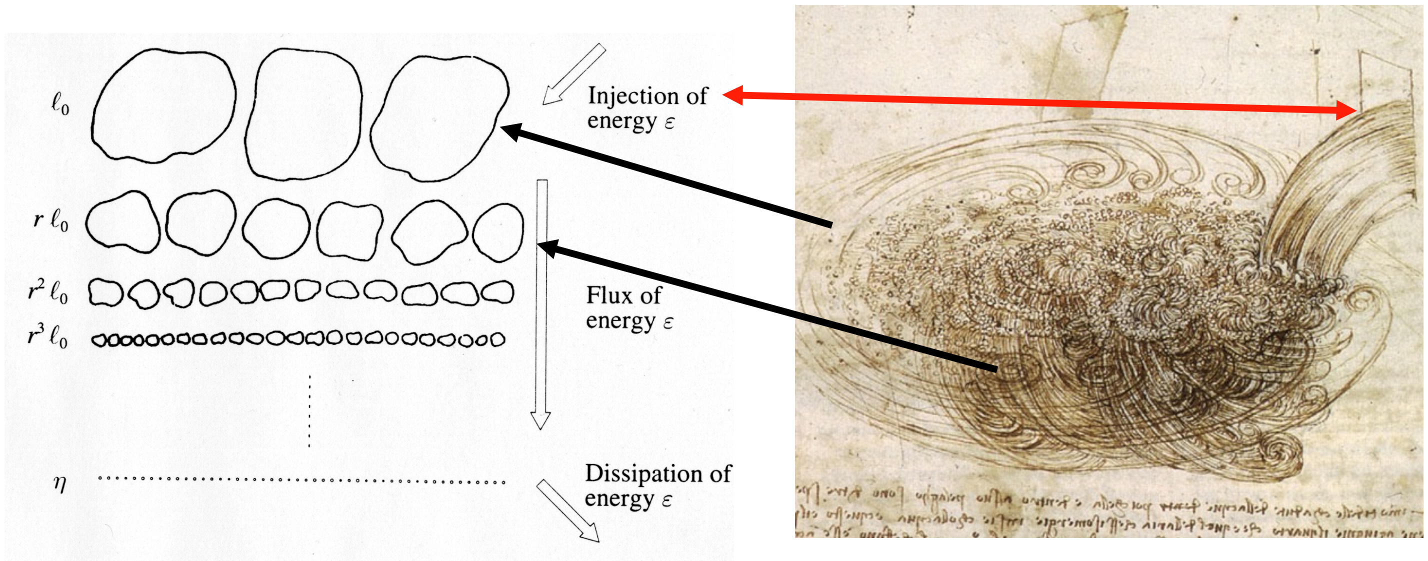
Typical Reynold numbers:

$R \approx 10^3$	$R \approx 10^4$	$R \approx 10^6$	$R \approx 10^{10}$
Blood flow in aorta	Laboratory	Atmosphere	Astrophysical fluid flows

How does this happen?

Since dissipation is efficient only at very small scales, there is an energy transfer to small scales:

nonlinear energy cascade

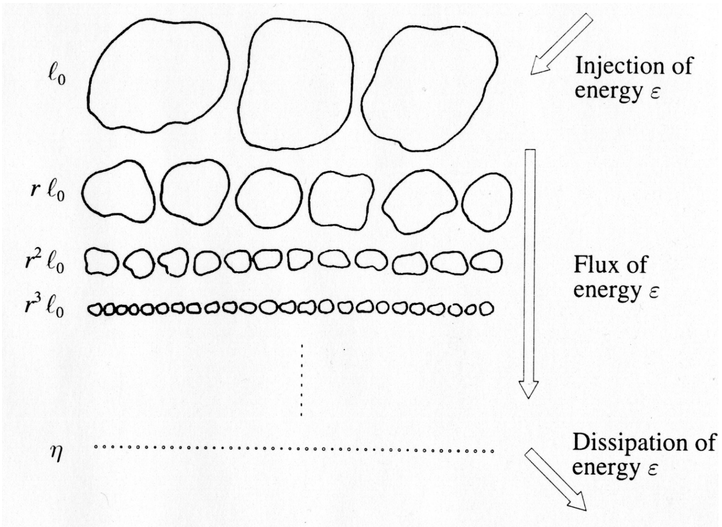


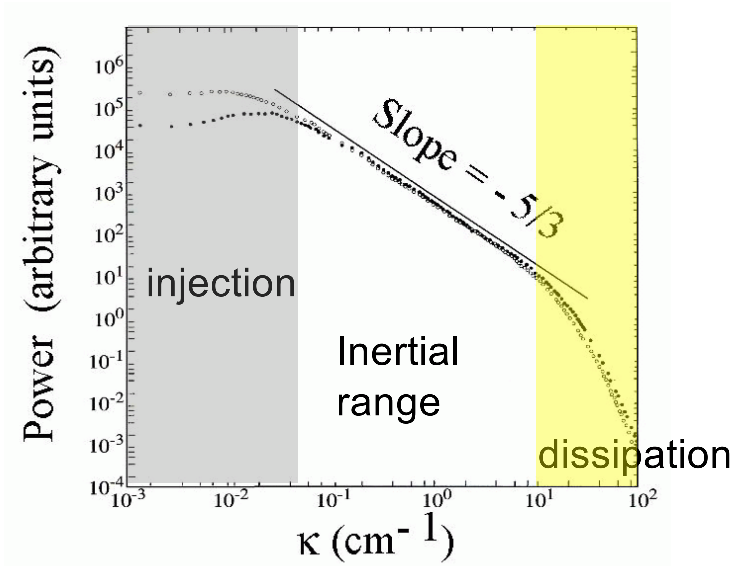
The Richardson's phenomenology: breaks down (distortion) of eddies at large scales and transfer of energy to small scales.

Energy cascade in the Fourier space

Dynamical properties of turbulence are random, but statistical properties are predictable and universal !

Universal turbulent spectrum : $\boxed{E \sim k^{-5/3}}$





Dissipation range: $\boxed{E(k) = Ak^{-\alpha}\exp(-k/k_d)}$

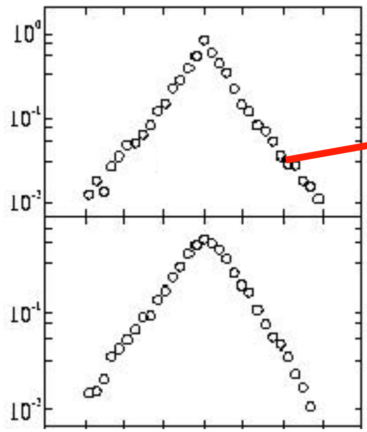
Intermittency (HD) : non-homogeneity of turbulence

- non-Gaussianity of turbulent fluctuations
- dependence of the non-Gaussianity on scale
- appearance of coherent structures

PDF(δv)

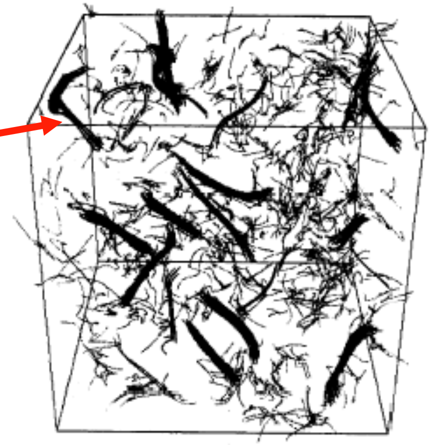
Small scale

Large scale



$\delta v_l = v(r+l) - v(r)$

[She et al., 1991]



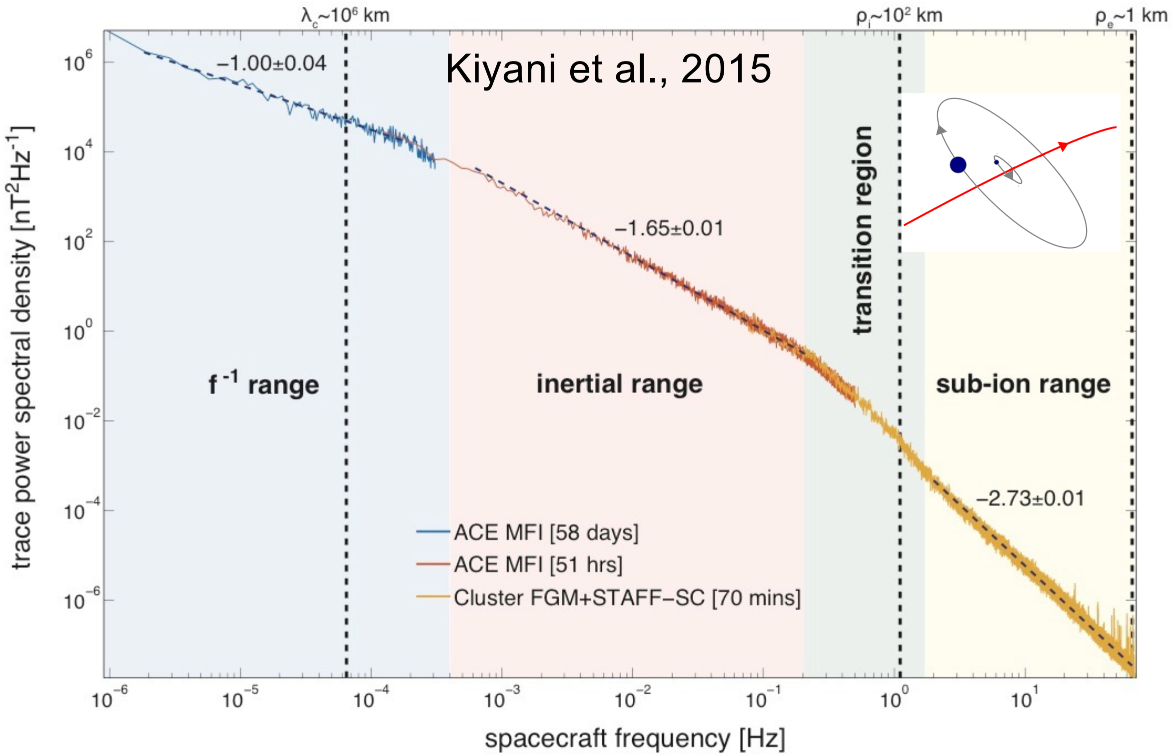
3D Simulations HD : filaments of vorticity with cross-section ~ $L_{dissipation}$, and length ~ $L_{injection}$

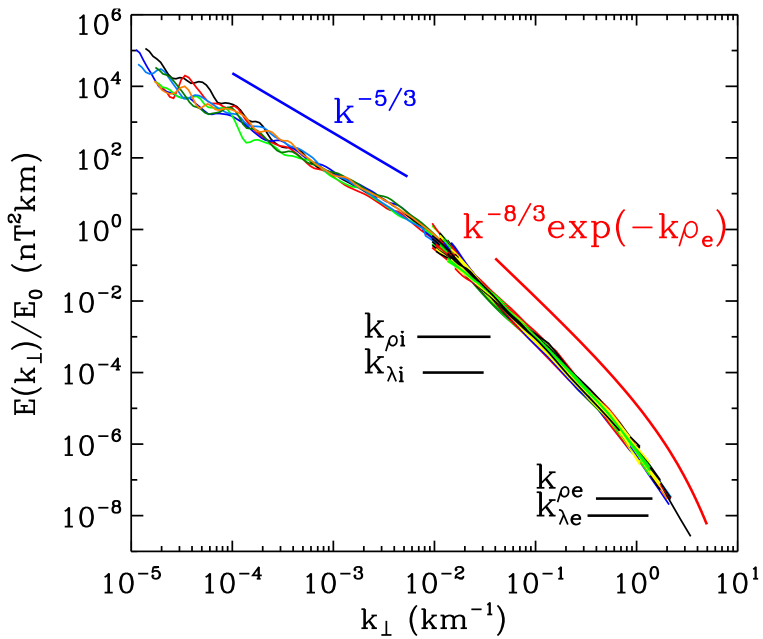
And what about space plasma turbulence?

- How is it different from HD turbulence?
- Does it share the above universal characteristics, as power-law spectra and intermittency ?
- What about dissipation ?

Solar wind turbulent spectrum (at 1 AU)

Magnetic fluctuations **cover 8 decades in frequencies** (or in k). Characteristic plasma scales (Larmor radius of charged particles) separate turbulent cascade in different regimes:





Alexandrova et al., 2012, APJ

Taylor hypothesis: $\boxed{\omega_{obs} = kV \rightarrow k = 2\pi f/V}$

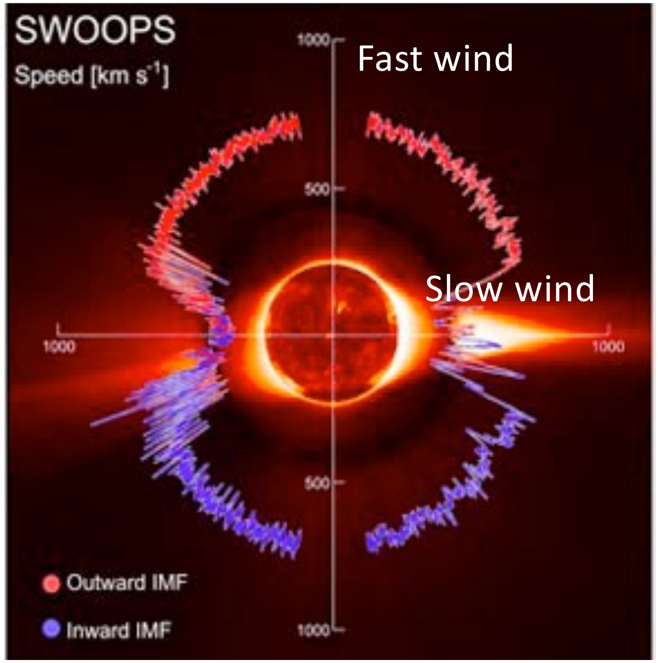
Turbulence is a multi-scale phenomenon.

Intermittency in space plasma turbulence

$$\delta b = B_r(t+\tau) - B_r(t), \text{ with } B_r \text{ radial component of B-field}$$

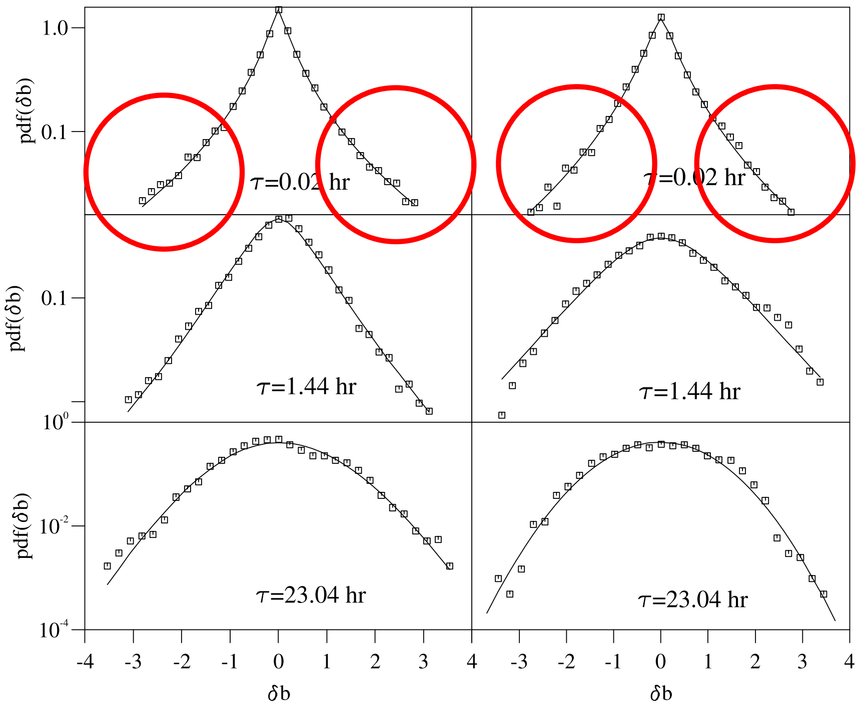
Turbulent fluctuations = Increments of magnetic field

[Sorriso-Valvo et al. 1999]



Fast wind

Slow wind



τ = 72 sec

τ = 26 min

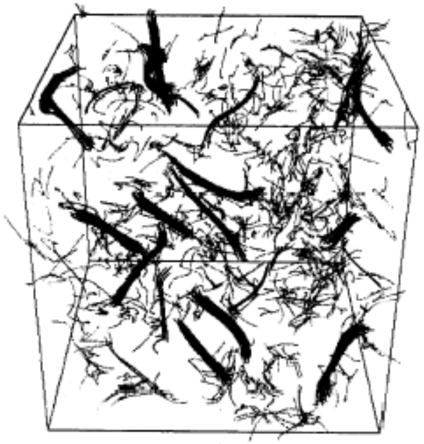
τ = 23 h

Non Gaussian pdf's : turbulence is not a noise !

High tails : coherent structures

Definition of coherent structure (intermittent events)

[She et al., 1991]



1. Farge & Schneider (2015): 'Everything that is not noise'

2. Inspired by Fiedler (1988) & Lesieur (1993)

- **High amplitude event localized in space** =>
- Delocalisation in Fourier space
- **Particular topology** =>
- Phase coupling over a large range of scales
- **Life time** ≫ life time of random fluctuations

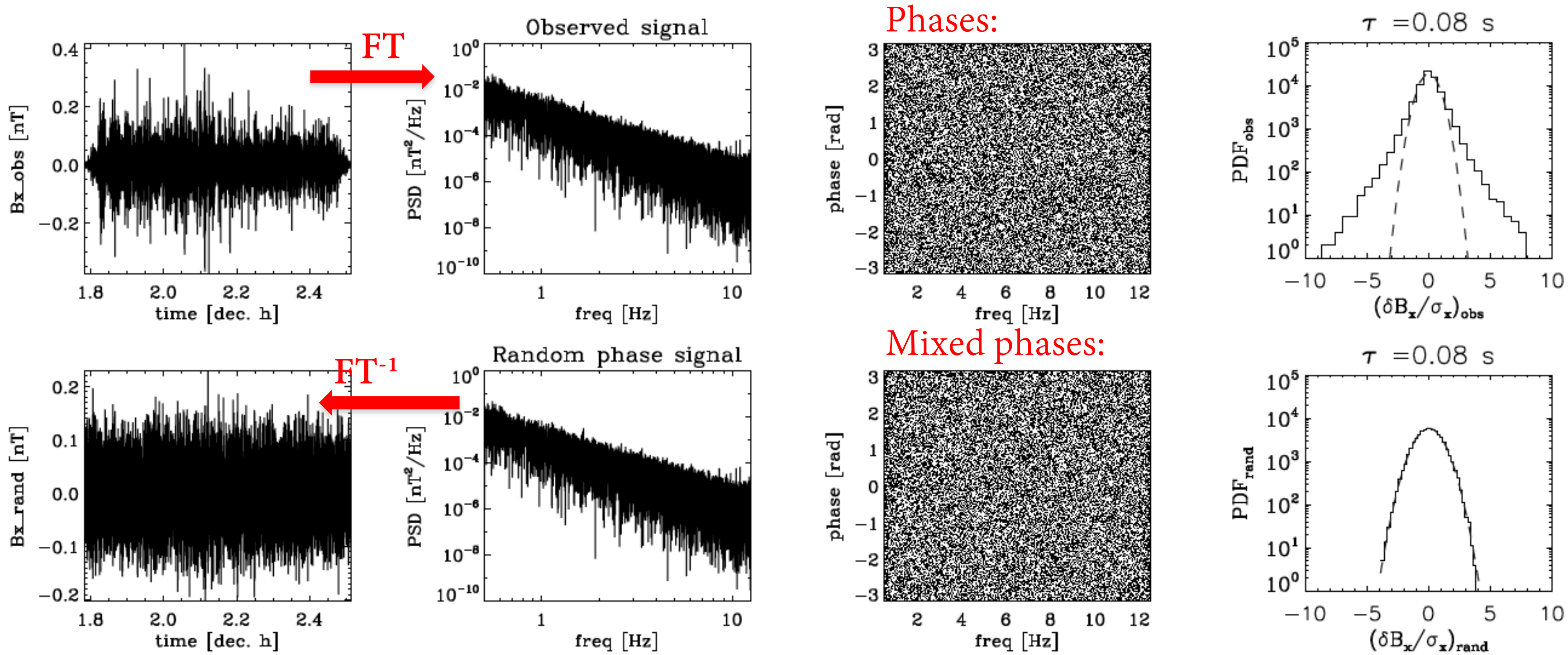
Phase coupling and non-Gaussianity

From the observed signal we construct a signal with random phases but with the same spectrum.

[Einaudi & Velli 1999, Hada et al. 2003; Koga & Hada, 2003; Sahraoui, 2008]

Observed signal

Random phase signal

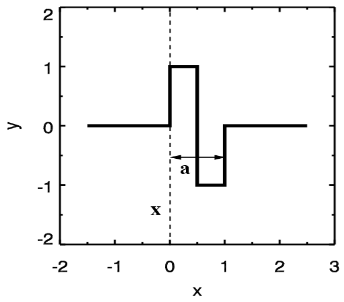


[Claudia Rossi, Tesi di Laurea, 2011]

Non-Gaussian tails ⇔ coupled phases !

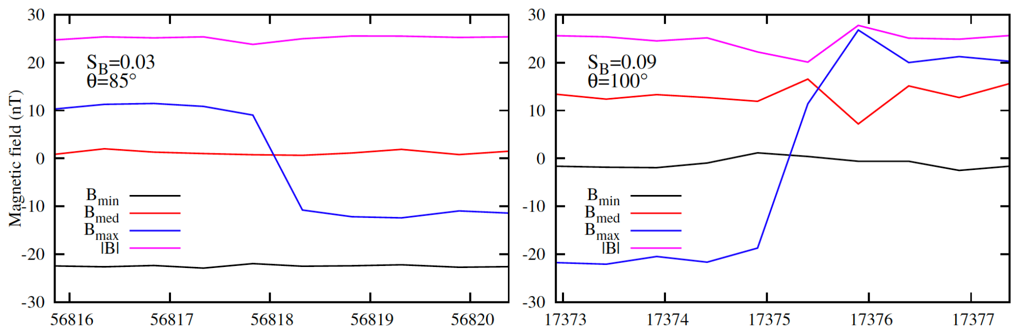
Intermittent structures in solar wind turbulence

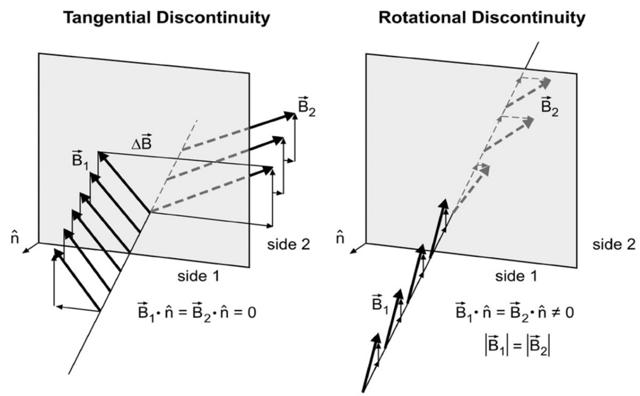
Turbulent fluctuations = Increments of magnetic field => Partial Variance of Increments (PVI) Method = Haar Wavelets (step function)



$$\mathrm{PVI}(t,\tau) = \frac{|\Delta \boldsymbol{B}(t,\tau)|}{\sqrt{\langle|\Delta \boldsymbol{B}(t,\tau)|^2\rangle}}$$

- High B-increments => planar discontinuities (current sheets and magnetosonic shocks)



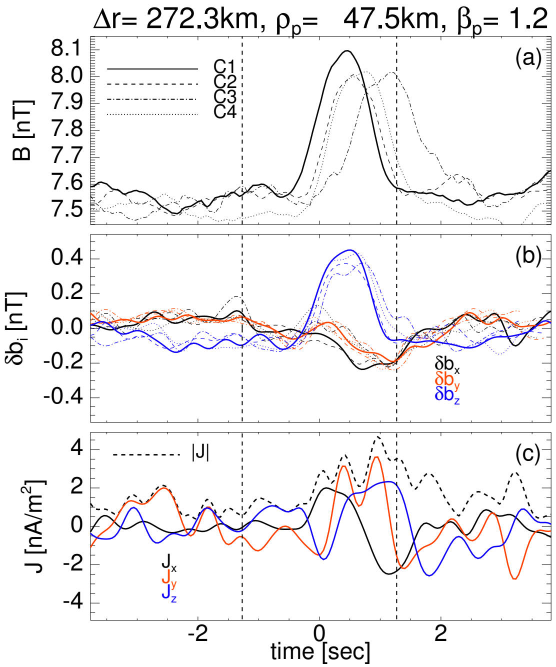


[e.g., Veltri & Mangeney 1999, Servidio, et al. 2008, Greco, et al. 2009, 2012, 2014, Perri et al. 2012]

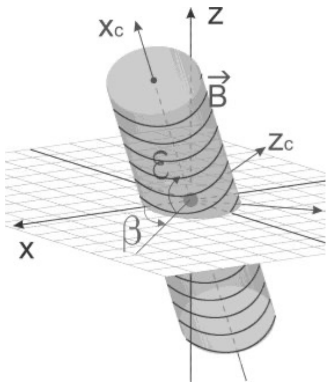
Intermittent structures in solar wind turbulence

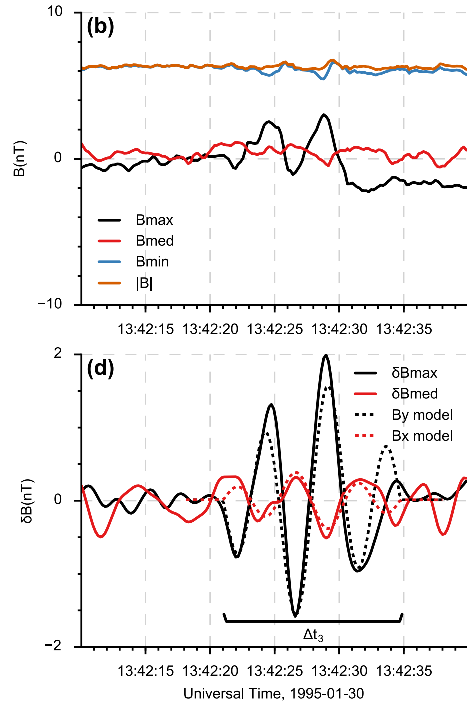
- Turbulent fluctuations = Morlet wavelet coefficients
- **High Morlet wavelet coefficients => more complex topologies, such as Alfven vortices, magneric holes & magnetosonic solitons** [e.g., Lion et al. 2016, Perrone 2016, 2017, 2021, Roberst 2016].

Compressible soliton in slow wind [Perrone et al. 2016]

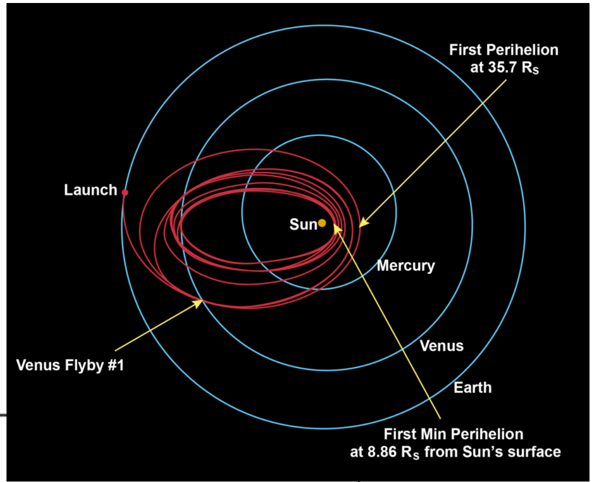


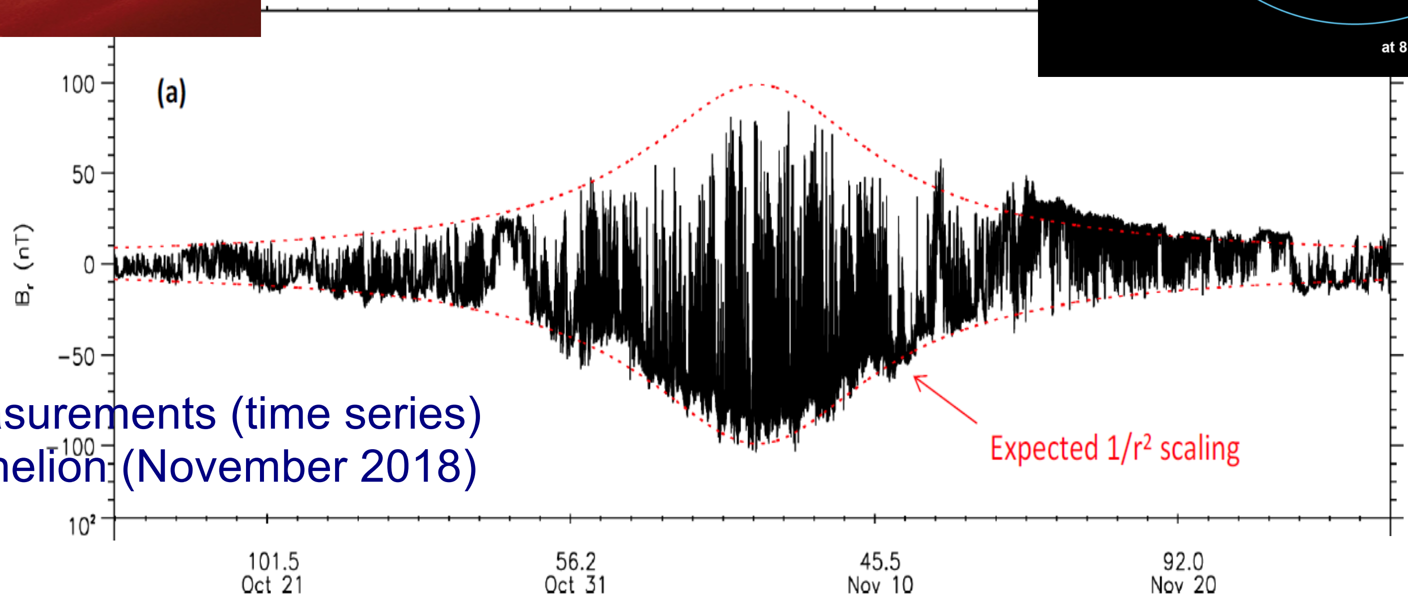
Alfven vortex in fast wind [Lion et al. 2016]





Parker Solar Probe : first in-situ measurements at 10 R_{Sun}

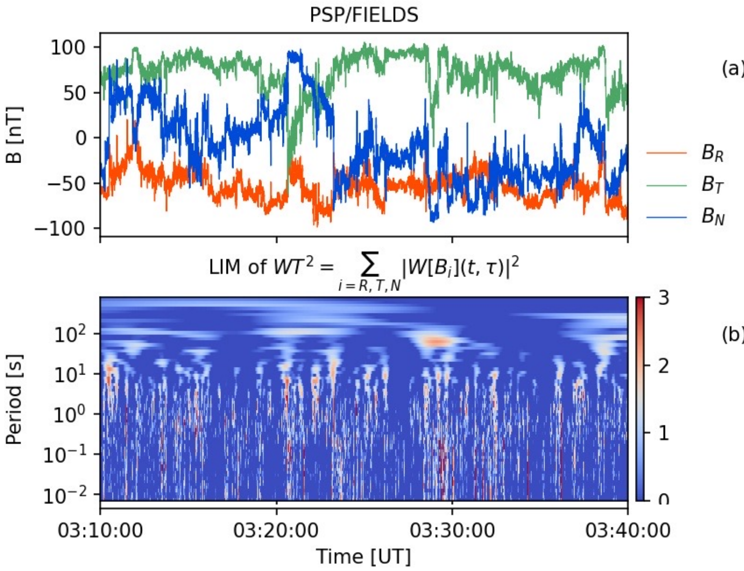


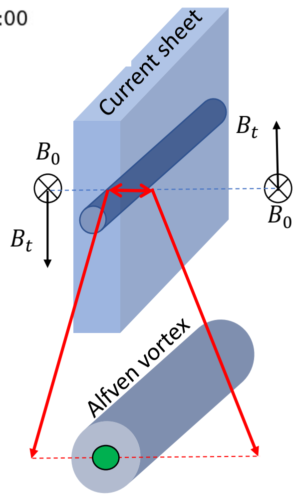


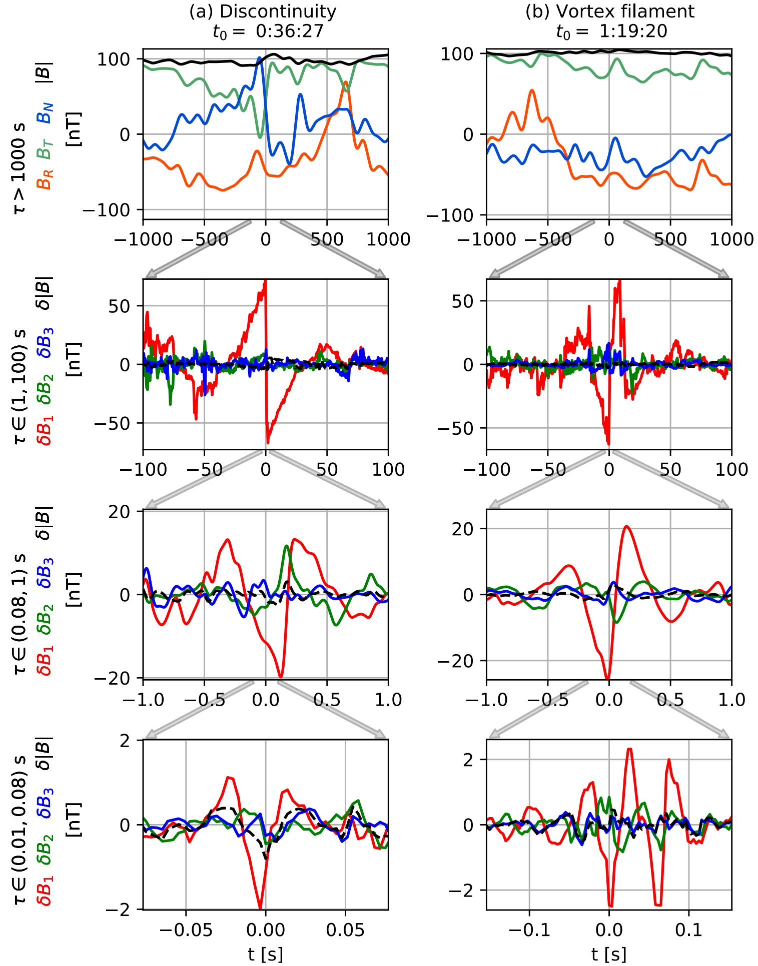
Magnetic field measurements (time series) during the 1st perihelion (November 2018)

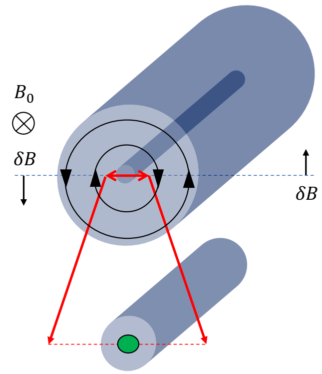
Bale et al., Nature 2019

Embedded intermittent events at 0.17 au from the Sun









[Vinogradov et al., 2024]

- As we can see, physical processes in astro plasmas, such as shocks, magnetic reconnection and turbulence are **multi-scale** non-linear phenomena.

- What are methods to treat in-situ measurements to get maximum information possible from the signal?

- Fourier vs time-frequency analysis.

Fourier Transform (FT)

Real data are discrete time series $u[j]=u(t_j)=u(t_0+j*dt)$ of N measurements during time T with time resolution $dt = T/N$, and $t_j=j*dt$, $j=0,1,2,\ldots,N-1$.

FT of u[j] can be defined as: $$\hat{u}[n] = \frac{1}{N}\sum_{j=0}^{N-1} u[j]e^{-2\pi i\frac{nj}{N}}$$

where n is an index of frequencies and j is an index of points measured in time.

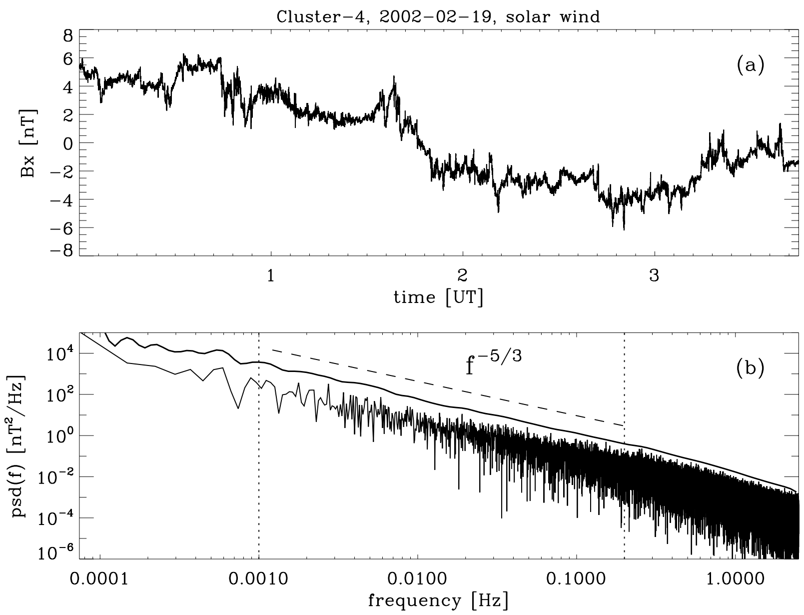
- If u(t) is real function, then $$\hat{u}[N-n] = (\hat{u}[n])^*$$

- FT preserves energy (Parseval's theorem): $$\frac{1}{N}\sum_{j=0}^{N-1}|u[j]|^2 = \sum_{n=0}^{N-1}|\hat{u}[n]|^2$$

Energy distribution in the signal as a function of frequency is given by **power spectral density** (PSD), defined as:

$$S[n] = 2T|\hat{u}[n]|^2, \quad n = 0, 1, 2, \ldots, N/2$$
$$f_n = n/T, \quad n = 0, 1, 2, \ldots, N/2$$

Magnetic fluctuations and turbulent spectrum



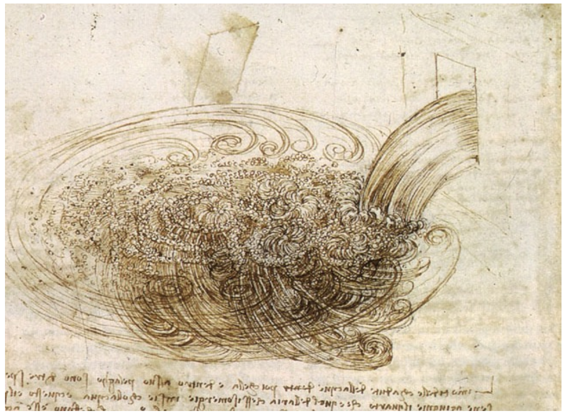
Fourier transform of $B_x(t)$:

$$\hat{B}_x(\omega) = \int_{-\infty}^{\infty} B_x(t) \exp(-i\omega t) dt$$

Power Spectral Density (PSD):

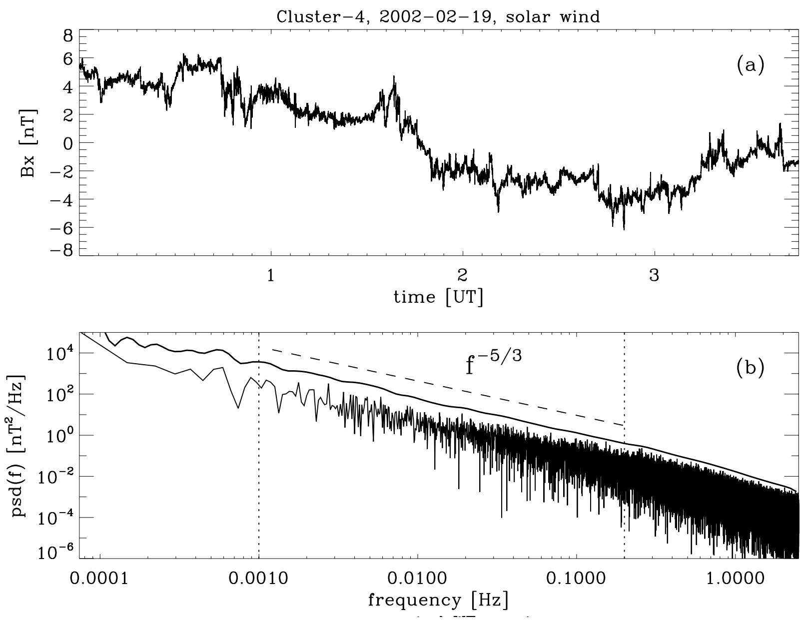
$$S(f) = 2T|\hat{B}_x(f)|^2, \; \omega = 2\pi f$$

=> power-law spectrum



- In-situ measurements are time series
- In Fourier space we get frequency, f, spectra.
- Taylor hypothesis: $r=tV_{sw}$ and $k = 2\pi f/V_{sw}$.
- Kolomogorov-like turbulence with PSD ~ $k^{-5/3}$

Magnetic fluctuations and turbulent spectrum



Fourier transform of Bx(t):

$$\boxed{\hat{B}_x(f) = \frac{1}{N} \sum_{j=0}^{N-1} B_x(t_j) e^{-2\pi i f t_j}}$$

Here, we compare our signal with sine waves of const amplitudes. Is it relevant for the real signal ?

All the information on time is lost (different signals may have the same spectra)…

- Fourier spectrum is a mean distribution of energy among scales. Homogeneity of turbulence ? Time dependence of the spectrum ?

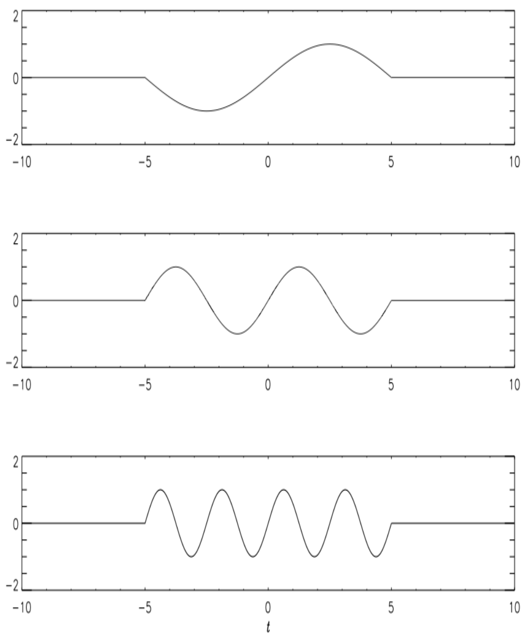
- => time-frequency analysis $\boxed{B_j(t) \rightarrow \hat{B}_j(f,t)}$

Time-frequency analysis
1. Windowed Fourier Transform

$$Gf(\omega, t) = \int f(u) g(u-t) \exp(-i\omega u) du$$

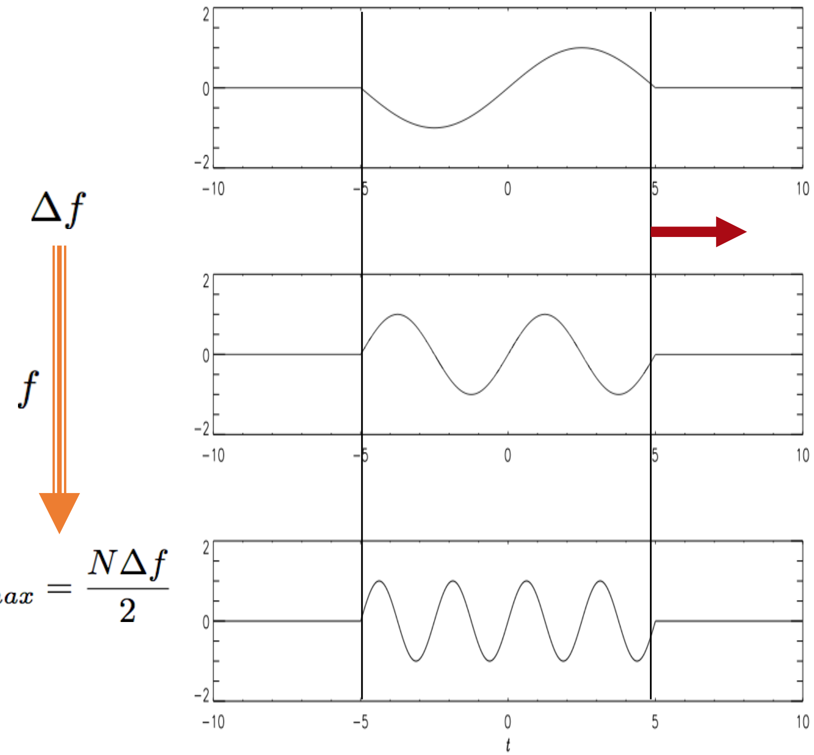
With g(t) being the window function (real, even, positiv).
Usually g(t) is a Gaussian. Example with g(t) = rectangular window:

Problem: time resolution decreases with increasing frequency…



Windowed FT vs wavelets

Windowed Fourier Transform

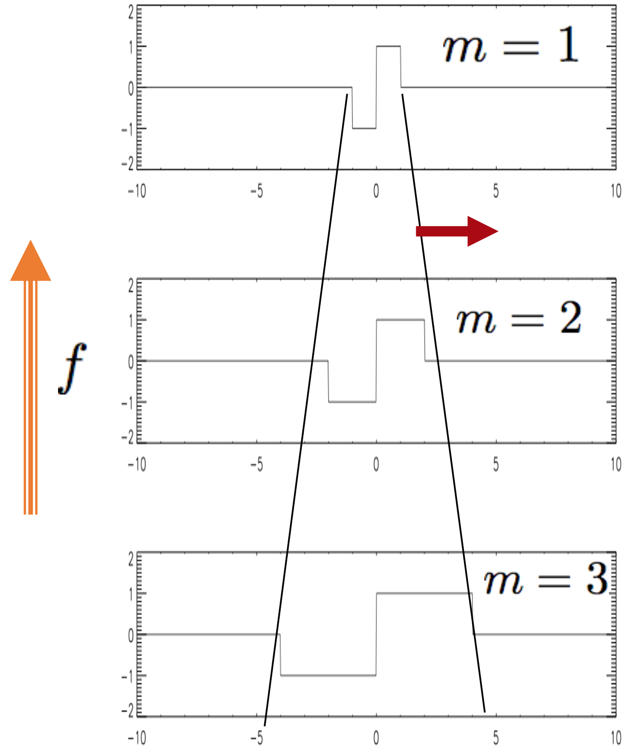


$-T/2$ $\quad$ $T/2$

$$N = \frac{T}{dt} \qquad \Delta t = T \qquad \Delta f = 1/T$$

$$f_{max} = \frac{N\Delta f}{2}$$

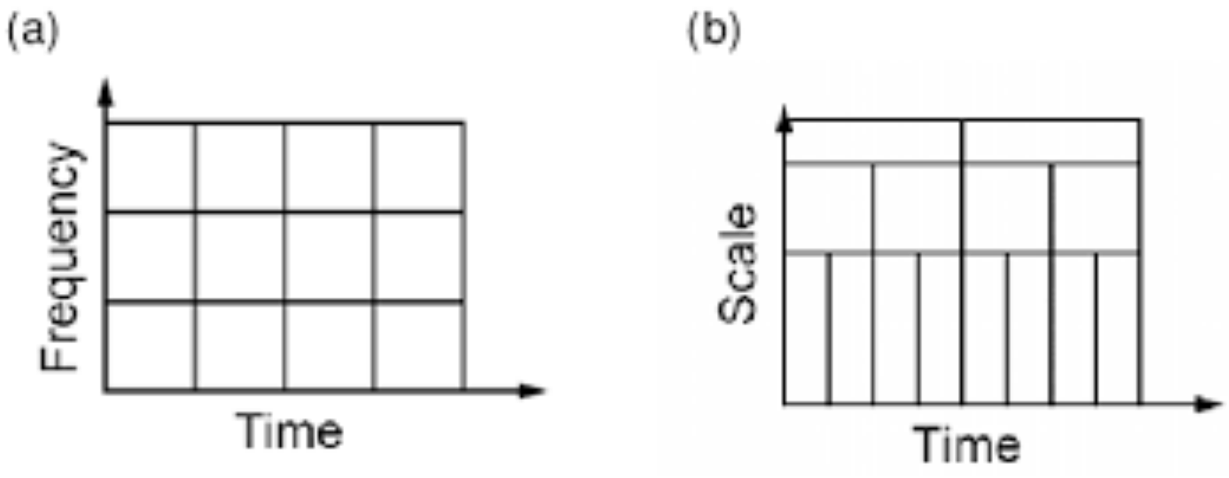
Haar wavelet transform



$$\Delta t_m = 2^m dt$$

$$\Delta f_m \propto f_m = 1/\Delta t_m$$

Windowed Fourier Transform vs Wavelet Transform



It is clear that the frequency resolution Δf (or time scale $\Delta\tau = 1/\Delta f$) and the temporal resolution Δt can not be small simultaneously but are linked by the uncertainty principle :

$$\Delta f \Delta T \sim const. \tag{1}$$

In wavelet transforms the analysis windows are specially adapted to the scales. In the Fourier transform with sliding window the temporal resolution decreases as the frequency increases

Time-frequency analysis: 2. Wavelet Transform

$$\boxed{W(a,b) = \frac{1}{\sqrt{a}} \int f(t) \Psi^*(\frac{t-b}{a}) dt}$$

- **Ψ(a,b)** are called wavelets.
- **a** is a scale parameter, ~ inverse frequency **a=1/f**
- **b** is a location parameter on the time axis

The function *Ψ(t)* can be used as a wavelet (or mother wavelet), if it satisfies following conditions:

1. **Zero mean** (admissibility condition). It ensures that *Ψ(t)* is a wave-like.
Ψ(t) is a wavelet of M-order if for any m ≤ M, m moments are zero:

$$\int t^m \Psi(t) dt = 0$$

2. It has a **compact support** (i.e., sufficiently fast decay to obtain localization in time)
3. **Self-similarity**: all the functions are obtained by rescaling (a) & translation (b) of the Mother function.

$$\Psi_{a,b}(t) = \frac{1}{\sqrt{a}} \Psi(\frac{t-b}{a})$$

The normalizing const is chosen so that : $\int |\Psi_{a,b}(t)|^2 dt = \int |\Psi(t)|^2 dt$

Low f has large analysing box, high f has small analysing box.

Parseval theorem and Scalogram

The WT is an energy preserving transformation [Kumar, 1994]:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{C_\Psi} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{|W(a,b)|^2}{a^2} da db$$

Where $C_\Psi = \frac{1}{2\pi} \int_0^\infty \frac{|\hat{\Psi}(\omega)|^2}{|\omega|} d\omega < \infty$ and Ψ^ = FT(Ψ)

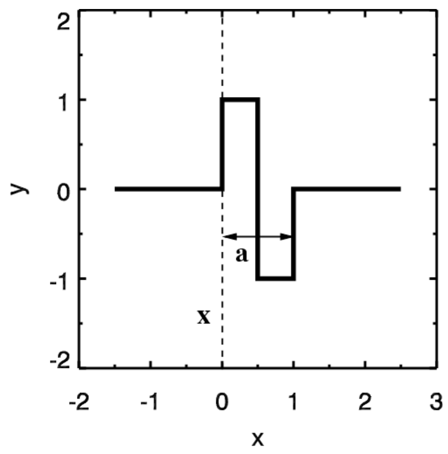
$\frac{1}{C_\Psi} \frac{|W(a,b)|^2}{a^2}$ - can be considered as an energy density function on

the (a,b) plane, i.e. it gives the energy on the scale interval Δa and time interval Δb centred around scale a and time b.

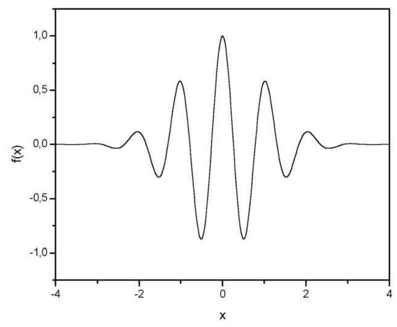
We call $|W(a,b)|^2$ a scalogram.

Examples of Mother Wavelets

Haar Wavelet Transform



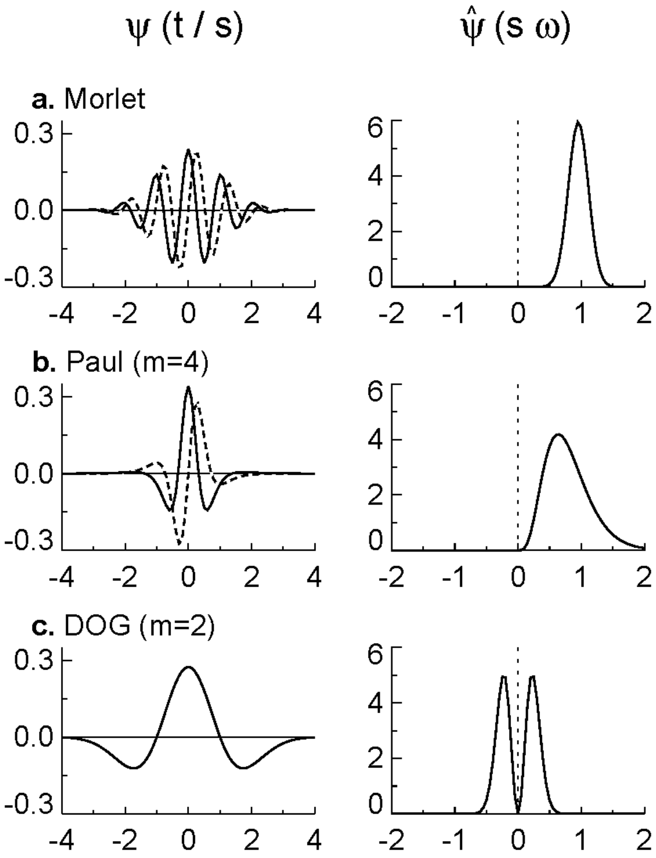
Morlet Wavelet Transform

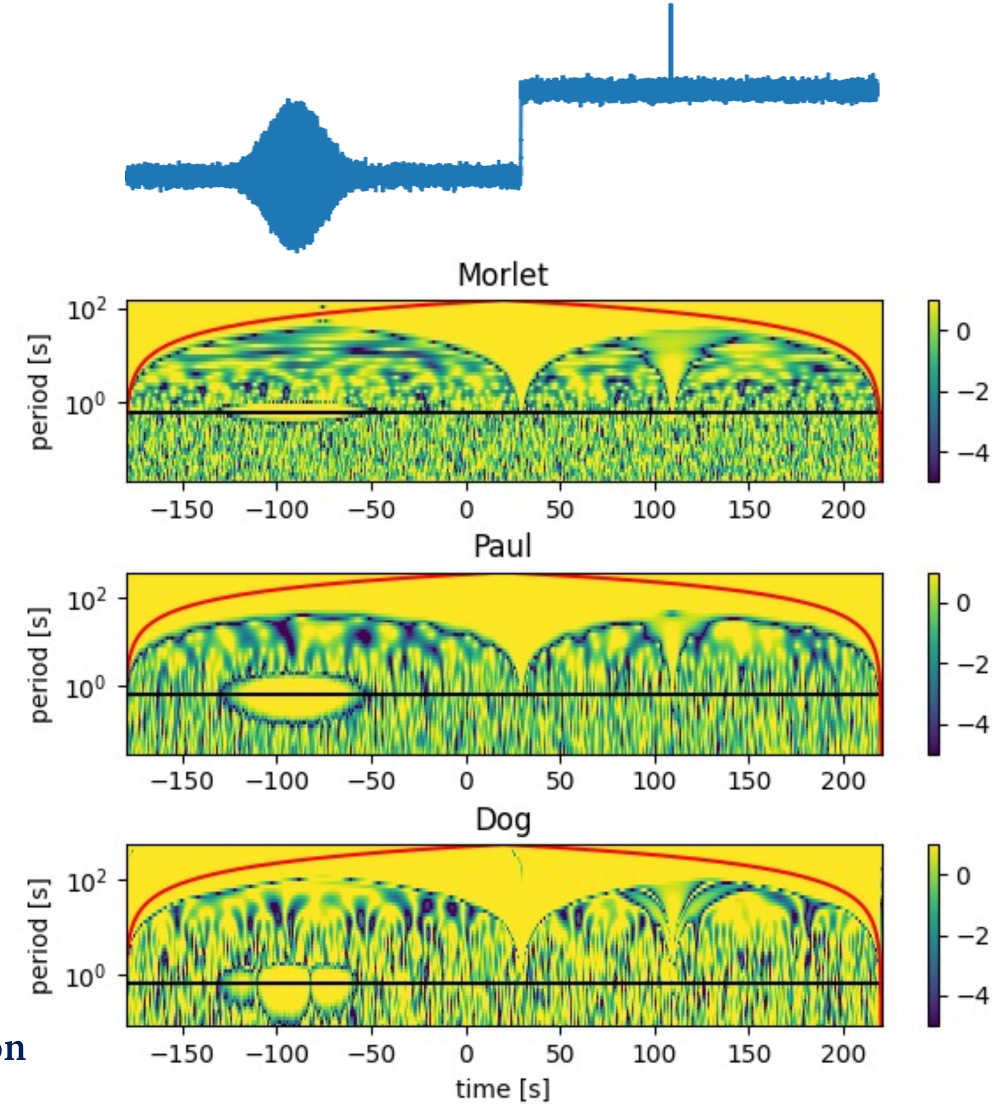


$$\psi_0(t) = \pi^{-1/4} e^{-i\omega_0 t} e^{-t^2/2}$$

- Haar WT: has the best time resolution, worse frequency-resolution
- Morlet WT: complex WT; with $\omega_0 \geq 5$, zero mean. Good compromise in t-f localization.
- Morlet WT is similar to Gabor function in Windowed Fourier Transform, but, in WT we fix number of oscillations, e.g. $\omega_0=6$, and it remains constant for any scale a.

A wave, a discontinuity and a Dirac function with different wavelets

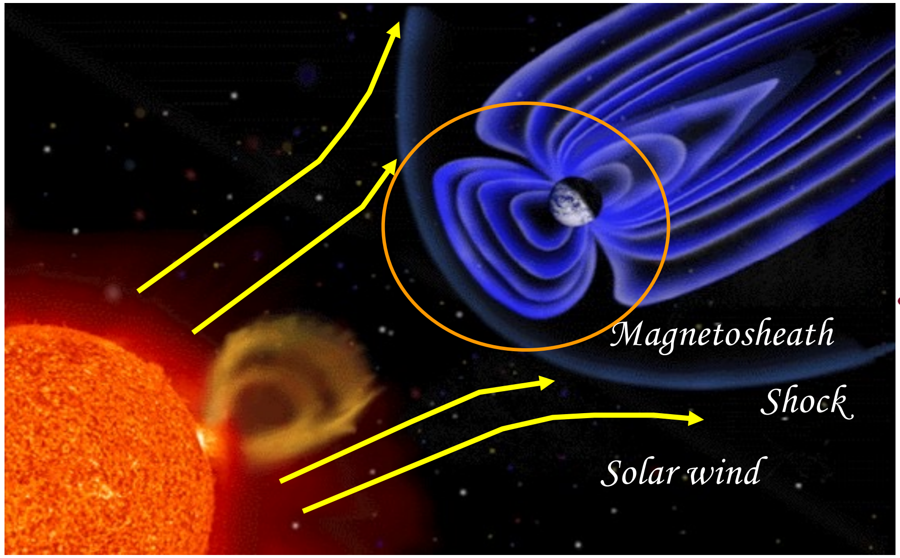


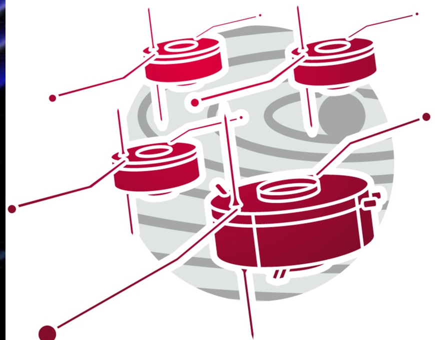


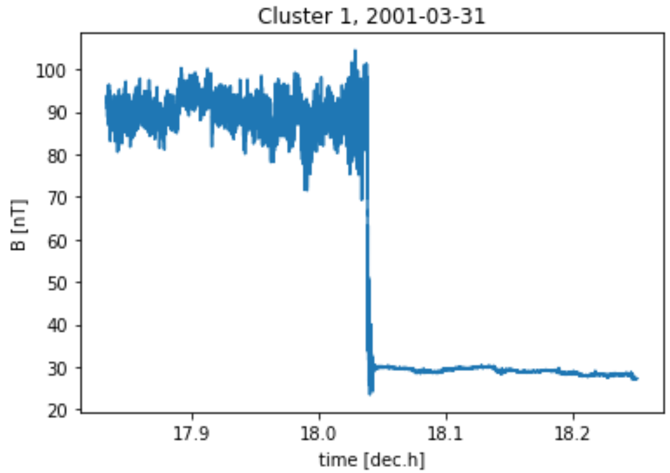
Morlet wavelet is a good compromise for time-frequency resolution

Cluster mission ESA/NASA, 4 s/c, since 2000

- Multi-satellite mission to study magnetosphere/solar wind connection





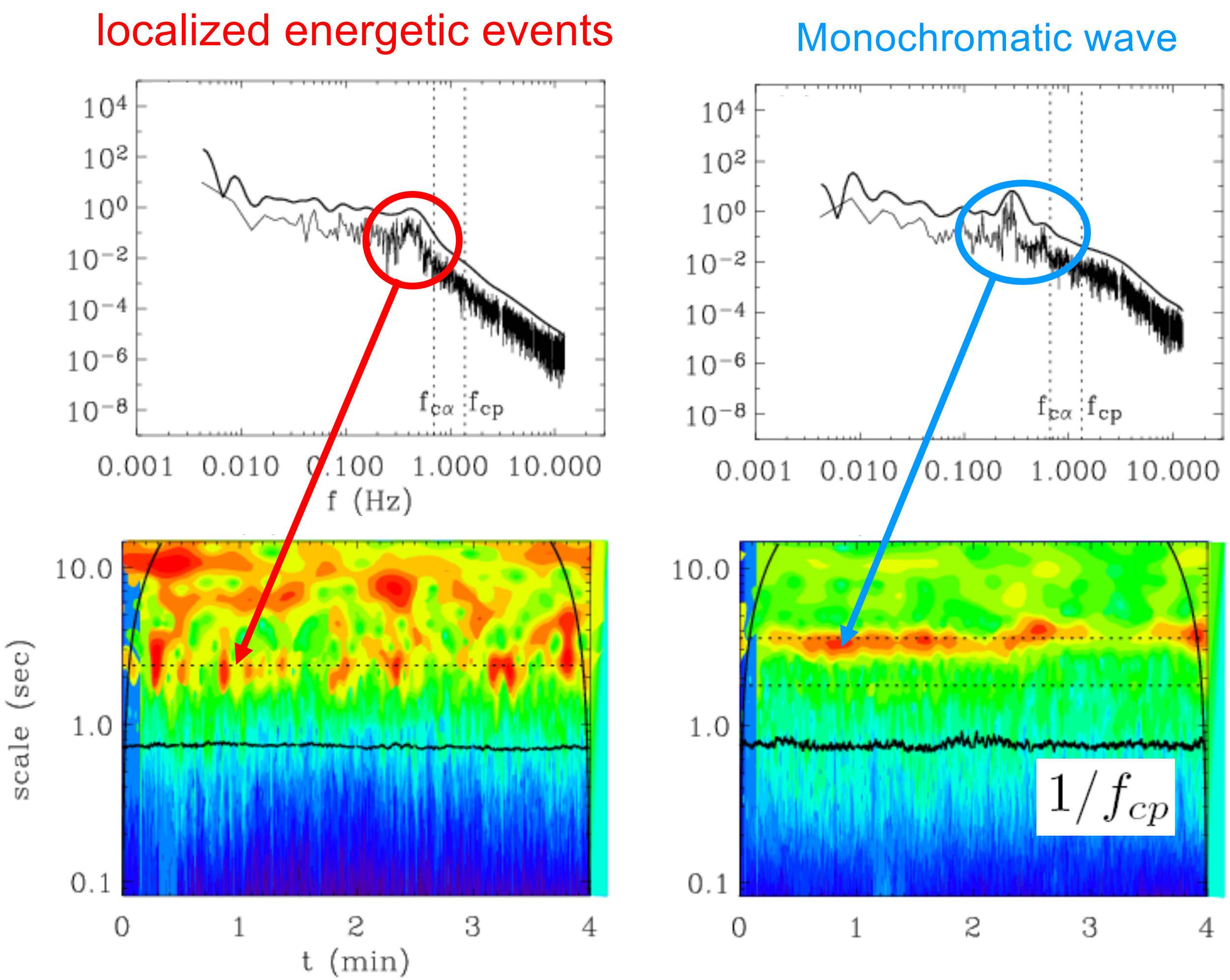


Example of shock crossing : magnetosheath – solar wind transition (studied during practical work). More details can be found in [Alexandrova et al. 2004, JGR]

Fourier PSD and scalogramms in the magnetosheath (downstream of the shock):

Fourier and wavelet spectra:

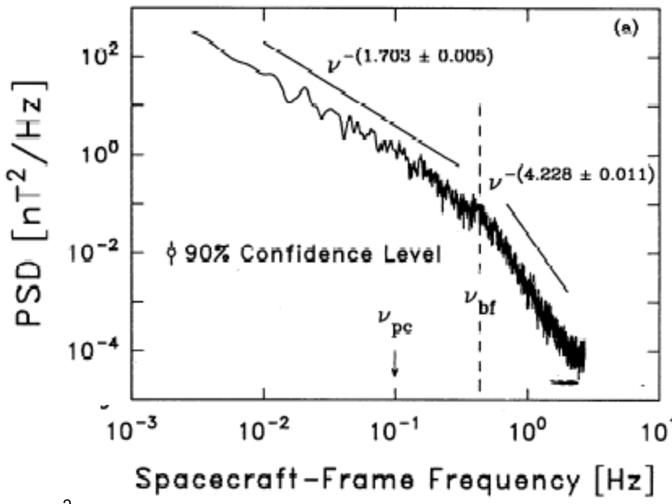
Wavelet scalograms (energy in time and scales):

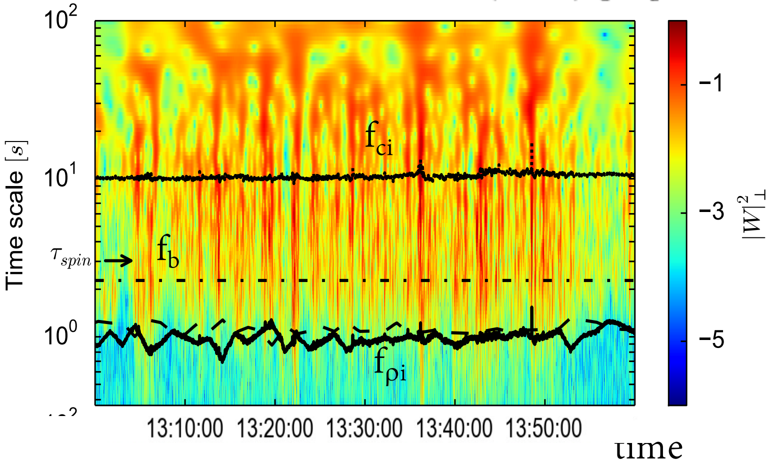


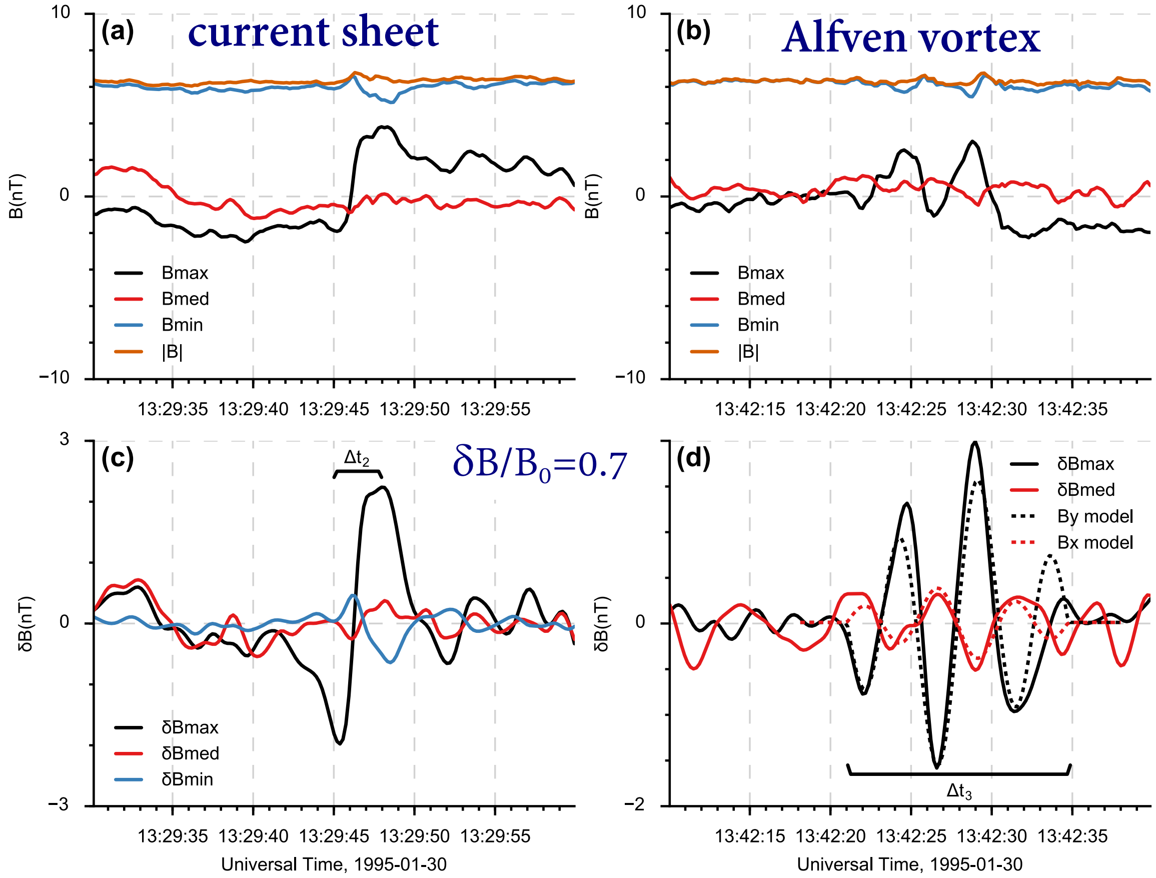
[Alexandrova et al. 2004, JGR]

Non-homogeneity of turbulence and detection of localised events with wavelets

[Lion, Alexandrova & Zaslavskiy, 2016, APJ]







At the times of localised events in scalogram in physical space we see current sheets and Alfven vortices

Conclusion - discussion

- Space and Astro plasmas is turbulent, events like shocks and magnetic reconnection are frequent => multi-scale approach is needed.

- Fourier Transform & PSD: information of repartition of energy of the signal among scales (for a stationary, homogeneous, ~‘infinite’/periodic signal).

- Time-frequency analysis: gives information on time dependence of the energy repartition among scales

- Wavelets has better time-frequency resolution than Fourier Window Transform. **Wavelets are applicable for non-stationary and non-homogeneous signals** =>
  - detection of energetic events
  - wavelet spectrum is well suited for turbulent signals.

Appendix: Taylor hypothesis

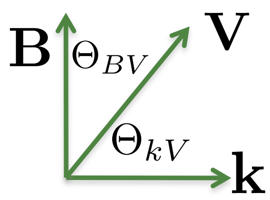
$$\omega_{obs} = \omega_0 + \mathbf{k} \cdot \mathbf{V}$$

Supposing that ω_0 << k.V, ($V\varphi$ << V) :

$$\omega_{obs} = \mathbf{k} \cdot \mathbf{V} = kV \cos(\Theta_{kV})$$

We don't know the angle between **k** and **V** => assumption of **k** || **V**:

$$\boxed{\omega_{obs} = kV \rightarrow k = 2\pi f / V}$$



For 2D turbulence: $\mathbf{k} \perp \mathbf{B} \rightarrow \cos \Theta_{kV} = \sin \Theta_{BV}$

$$\boxed{\omega_{obs} = \mathbf{k}_\perp \cdot \mathbf{V} = k_\perp V \cos(\Theta_{kV}) = k_\perp V \sin(\Theta_{VB})}$$

[Leamon+2000, Mangeney+2006, Bourouain+2012]