Handout-2022-23-Sem-II-CSF402-ComputationalGeometry

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1 Introduction

The Part-I of the handout in the Bulletin 2021[1] has included several topics, as shown below.

CS F402 Computational Geometry **3 0 3**

Introduction to Computational Geometry, degeneracies and robustness, convex hull in 2D, line-segment intersection, doubly-connected edge list, computing the overlay of two subdivisions, art gallery theorem, guarding and triangulation, monotone polygons, partitioning arbitrary polygon into monotone polygons, triangulating a monotone polygon, range search problem, Kd-trees, range trees, fractional cascading, point location problem, trapezoidal maps, randomized incremental algorithm to compute trapezoidal map, post-office problem, Voronoi diagram and its properties, Algorithm to compute Voronoi diagram, Delaunay triangulation and relation with Voronoi diagram, Computing Delaunay triangulation, line and point duality, arrangement of lines, application of computational geometry.

Popular perception[2] is that the main impetus for the development of computational geometry as a discipline was progress in computer graphics and CAD/CAM. Other popular applications of computational geometry include motion planning and visibility problems in robotics, geometrical location and search, route planning using GIS, integrated circuit geometry (layout) design and verification, mesh generation in computer-aided engineering (CAE) and computational fluid dynamics (CFD), 3D reconstruction for computer vision, etc. The part-I of the handout that is outlined above reflects this perception and emphasis.

However, there are distinct but closely linked approaches to computational geometry that retain the core of basic problems and techniques listed above, but organize them around integrated conceptual frameworks rather than seemingly diverse applications.

These approaches are, mainly,

1. Algorithmic Geometry: Also called combinatorial computational geometry, deals with geometric objects as discrete entities. Objects of interest are convex hulls and convex hull algorithms, low-dimensional randomized linear programming, point set triangulation for two- and three-dimensional data, arrangements of hyperplanes, of line segments, and of triangles, Voronoi diagrams, and Delaunay triangulations. The algorithms in this list can be called (perhaps augmented with more) together "basic geometric algorithms".

[1] https://www.bits-pilani.ac.in/Uploads/Goa_Upload/Bulletin/Bulletin_2020-21.pdf
[2] https://en.wikipedia.org/wiki/Computational_geometry

2. Numerical Computational Geometry: Also called machine geometry, computer-aided geometric design (CAGD), or geometric modeling, deals with CAD/CAM systems. The applications, techniques and algorithms listed in the Part I of the Handout in the Bulletin are precisely this.

Apart from these two main branches, there is a third approach that can be common to both. This approach looks at geometric algorithms that seek to solve combinatorial problems by linear programming and related techniques with efficiency sans guarantees. The ellipsoid method and the classical *geometry of numbers* of Minkowski.[Grötschel et al., 1993] A combination of these two methods yields a theoretical framework in which the polynomial time solvability of a large number of combinatorial optimization problems can be shown quite easily. It establishes the algorithmic equivalence of problems which are "dual" in various senses. The number theoretic applications of these lead to *Discrepancy Theory*: the theory of *irregularities of distribution*. Typical questions are finding the "most uniform" way of distributing n points in the unit square; measuring the "irregularity" necessarily present in any such distribution, etc. Applications go beyond number theory (Ramsey Theory, hypergraphs etc.) to numerical integration and similar tasks for financial calculations, computer graphics, computational physics, etc.

1.1 The Approach to be Taken in This Semester

Mainly combinatorial geometry, or the *Discrete Geometry* of [Matoušek, 2002]: Questions typically involve finite sets of points, lines, circles, planes, or other simple geometric objects. e.g., what is the largest number of regions into which n lines can partition the plane, or what is the minimum possible number of distinct distances occurring among n points in the plane? And more complicated objects such as convex polytopes or finite families of convex sets. The emphasis is on "combinatorial" properties: Which of the given objects intersect, or how many points are needed to intersect all of them, etc.

2 Books and Readings

TextBook T1 Of course [Matoušek, 2002]. Its chapters 1,3,4 are the main content. Chapters 13,14 are my soft targets, but not with full rigour.

Reference Books **[R1]** [Grötschel et al., 1993],
[R2] [Mulmuley, 1994],
[R3] Any one of the classics [de Berg et al., 2008], [Boissonnat et al., 1998],
[R4] Visibility Algorithms in the Plane - Subir Kumar Ghosh (Cambridge University Press, 2007)[Ghosh, 2007].

Others I will share from time to time.

3 Lecture Plan

#	Module	Topics	Lecture Hours	Readings
1	Convexiy	Linear and Affine Subspaces, General Position; Convex Sets, Convex Combinations, Separation; Radon's Lemma and Helly's Theorem; Center-point and Ham Sandwich	1-15	T1 Ch.1
2	Independent Sets	The Erdos-Szekeres Theorem; Horton Sets	16-21	T1 Ch.3
3	Incidence Problems	Lower Bounds: Incidences and Unit Distances; Point-Line Incidences via Crossing Numbers; Distinct Distances via Crossing Numbers; Point-Line Incidences via Cuttings; A Weaker Cutting Lemma; The Cutting Lemma: A Tight Bound	22-40	T1 Ch.4
4	Applications	Similarity and difference between linear and integer programming will be the focus. Connections between greedy methods (and problems in $\mathbb{P}$ solvable by greedy algorithms) and geometry will be demonstrated.	Throughout, interspersed	TBA

4 Evaluation Scheme

1. Midsem and Compre: 35% and 40%. It will be few long-time solvable analytical questions, not essaying but problem solving and proofs.

2. Take home assignments 25%, based on programs and other materials given for practical applications and demonstrations. Largely experiential credit, not any test as such. Doing sincere and sufficient effort to go through the materials will fetch credit. Simple quiz – mostly viva-voce – will be conducted just to verify the effort.

References

[Boissonnat et al., 1998] Boissonnat, J.-D., Yvinec, M., and Brönnimann, T. H. (1998). *Algorithmic Geometry*. Cambridge University Press.

[de Berg et al., 2008] de Berg, M., Cheong, O., van Kreveld, M., and Overmars, M. (2008). *Computational Geometry: Algorithms and Applications*. Springer, 3 edition.

[Ghosh, 2007] Ghosh, S. K. (2007). *Visibility Algorithms in the Plane*. Cambridge University Press.

[Grötschel et al., 1993] Grötschel, M., Lovász, L., and Schrijver, A. (1993). *Geometric Algorithms and Combinatorial Optimization*. Springer-Verlag, second corrected edition.

[Matoušek, 2002] Matoušek, J. (2002). *Lectures on Discrete Geometry*. Springer.

[Mulmuley, 1994] Mulmuley, K. (1994). *Computational Geometry: An Introduction Through Randomized Algorithms*. Prentice Hall.