Alireza Hosseini 810100125 AI-HW2

Bayes Nets

سوال اول :

الف)

join O $\longrightarrow$

O	W	P(W,O)
+	+	0.45
-	+	0.1
+	-	0.05
-	-	0.4

Eliminate O $\longrightarrow$

W	P(W)
+	0.55
-	0.45

ب)

$P(+o, -w, +f, -r, +a) =$

$P(+O)\, P(-r)\, P(+a|-r,+f)\, P(+f|-w,+o)\, P(-w|+o)$

$= 0.5 \times 0.8 \times 0.7 \times 0.6 \times 0.1 = \underline{0.0168}$

ج)

General rule: Each node is conditionally independent of its non-descendents given its parents

1. صحیح $P(A|F,O) = P(A|F)$
2. غلط $P(F|A) \;!= P(F|A,R)$
3. غلط $P(F|O) \;!= P(F|O,A)$
4. صحیح $P(R|F) = P(R)$

د)

$P(O|-a)$ $\boxed{P(O)\ P(W|O)\ P(F|O,W)\ P(-a|F)}$

$P(O)$, $P(W|O)$ $\xrightarrow{\times}$ $P(W,O)$

$P(F|O,W)$, $P(-a|F)$ $\xrightarrow{\times}$ $P(-a,F|O,W)$ $\xrightarrow{\Sigma}$ $P(-a|O,W)$

$P(-a|O,W)$, $P(O,W)$ $\xrightarrow{\times}$ $P(O,W,-a)$ $\xrightarrow{\Sigma}$ $P(O,-a)$ $\xrightarrow{\text{nomalize}}$ $P(O|-a)$

HMM

سوال اول :

الف)

$$A = \begin{array}{cccc|c} s & a & h & r & \\ 0.4 & 0.1 & 0 & 0.5 & s \\ 0.4 & 0.4 & 0.2 & 0 & a \\ 0 & 0.1 & 0.5 & 0.4 & h \\ 0.2 & 0 & 0.2 & 0.6 & r \end{array} \qquad B = \begin{array}{cccc|c} B & H & P & L & \\ 0.8 & 0 & 0 & 0.2 & s \\ 0 & 1 & 0 & 0 & a \\ 0 & 0.1 & 0.9 & 0 & h \\ 0.2 & 0 & 0.1 & 0.7 & r \end{array}$$

$$\pi = \begin{array}{cccc} s & a & h & r \\ 0.25 & 0.25 & 0.25 & 0.25 \end{array}$$ ⟵ محاسبه شده از روی A و احساب توزیع احتمالی

ب)

$O = \{B, B, L, H\}$

$$P(O) = \sum_{i=1}^{4} \alpha_4(i) \quad , \quad \alpha_t(j) = \sum_{i=1}^{4} \alpha_{t-1}(i)\, a_{ij}\, b_j(O_t)$$

$\alpha_1(s) = \pi_s\, b_s(B) = 0.25 \times 0.8 = 0.2$

$\alpha_1(a) = \pi_a\, b_a(B) = 0.25 \times 0 = 0$

$\alpha_1(h) = \pi_h\, b_h(B) = 0.25 \times 0 = 0$

$\alpha_1(r) = \pi_r\, b_r(B) = 0.25 \times 0.2 = 0.05$

$$\alpha_2(s) = \overset{0.2}{\alpha_1(s)}\, \underset{0.4}{a_{ss}}\, \underset{0.8}{b_s(B)} + \overset{0.05}{\alpha_1(r)}\, \underset{0.2}{a_{rs}}\, \overset{0.8}{b_s(B)} = 0.064 + 0.008 = 0.072$$

$\alpha_2(a) = 0$

$\alpha_2(h) = 0$

$$\alpha_2(r) = \overset{0.2}{\alpha_1(s)}\, \underset{0.5}{a_{sr}}\, \overset{0.2}{b_r(B)} + \overset{0.05}{\alpha_1(r)}\, \underset{0.6}{a_{rr}}\, \overset{0.2}{b_r(B)} = 0.002 + 0.006 = 0.008$$

$\alpha_3(s) = \alpha_2(s)\, a_{ss}\, b_s(L) + \alpha_2(r)\, a_{rs}\, b_s(L) = 0.00576 + 0.00032 = 0.00608$

$\alpha_3(r) = \alpha_2(s)\, a_{sr}\, b_r(L) + \alpha_2(r)\, a_{rr}\, b_r(L) = 0.0252 + 0.00336 = 0.02856$

$\alpha_4(a) = \alpha_3(s)\, a_{sa}\, b_a(H) + \alpha_3(r)\, a_{ra}\, b_a(H) = 0.000608 + 0 = 0.000608$

$\alpha_4(h) = \alpha_3(s)\, a_{sh}\, b_h(H) + \alpha_3(r)\, a_{rh}\, b_h(H) = 0 + 0.0005712$

$P(O) = 0.000608 + 0.0005712 = 0.0011792$ ✓

ج) در مراجعه دوم (t = 2) کاربر در حالت روحی غمگین (s)

$$\text{smoothing}: \quad P(\overset{X_2}{S} \mid O) = \alpha \underbrace{P(\overset{X_2}{s} \mid B,B)}_{\text{forward}} \underbrace{P(L,H \mid \overset{X_2}{s})}_{\text{backward}}$$

$$P(X_2 = s \mid B,B) = \sum_{x} P(x,B)\, P(s \mid x)\, P(B \mid s)$$

$$= P(B \mid s)\left(\alpha_1(s)P(s \mid s) + \alpha_1(a)P(s \mid a) + \alpha_1(h)P(s \mid h) + \alpha_1(r)P(s \mid r)\right)$$

$$= 0.8 \times \left(0.2 \times 0.4 + 0 + 0 + 0.05 \times 0.2\right) = \underline{0.072}$$

$$P(L,H \mid s) = \beta_2(s) = a_{as}\, b_s(L)\, \beta_3(a) + a_{rs}\, b_s(L)\, \cancelto{0}{\beta_3(r)} = 0.2 \times 0.4 \times 0.6 = 0.048$$

$$\beta_3(s) = \sum_j a_{js}\, b_s(H)\, \cancelto{1}{\beta_4(j)} = a_{as}\, \cancelto{0}{b_s(H)} = 0$$

$$\beta_3(a) = \sum_j a_{ja}\, b_a(H)\, \beta_4(j) = 0.6$$

$$\beta_3(r) = \sum_j a_{jr}\, b_r(H)\, \cancelto{1}{\beta_4(j)} = a_{hr}\, b_r(H) = 0$$

$$\alpha = 1 \implies \boxed{P(s \mid O) = 0.072 \times 0.048 = 0.003456} \checkmark$$

د) رشته روحی که بالاترین احتمال را دارد.

$\boxed{s, s, s, a \quad \text{:محتمل ترین}}$

$V_1(s) = \pi_s b_s(B) = 0.2$ $\qquad V_1(a) = \pi_a b_a(B) = 0$

$V_1(h) = \pi_h b_h(B) = 0$ $\qquad V_1(r) = \pi_r b_r(B) = 0.05$

$V_2(s) = V_1(s)\, a_{ss}\, b_s(B) = 0.2 \times 0.4 \times 0.8 = 0.064$ $\qquad V_2(a) = V_2(h) = 0$

$V_2(r) = V_1(s)\, a_{sr}\, b_r(B) = 0.2 \times 0.5 \times 0.2 = 0.02$

$V_3(s) = V_2(s)\, a_{ss}\, b_s(L) = 0.064 \times 0.4 \times 0.2 = 0.00512$

$V_3(r) = V_2(s)\, a_{sr}\, b_r(L) = 0.0224$

$\boxed{V_4(a) = V_3(s)\, a_{sa}\, \cancelto{1}{b_a(H)} = 0.000512 \longrightarrow \text{max probability}}$

$V_4(h) = V_3(r)\, \underset{0.1}{a_{\cancel{rh}}}\, \cancel{b_h(H)} = 0.0224 \times 0.2 \times 0.1 = 0.000448$