Section 1.3

First, we prove that similarity is transitive, i.e. if A is similar to B and B is similar to C, then A is similar to C.

Suppose A is similar to B and B is similar to C, then there exists invertible matrices P and Q such that

$$B = P^{-1}AP \quad , \quad C = Q^{-1}BQ$$

Then, we can write

$$C = Q^{-1}(P^{-1}AP)Q$$

$$C = (PQ)^{-1}A(PQ)$$

since P & Q are invertible, PQ is also invertible

Since PQ is an invertible matrix, this shows that A is similar to C.

To prove that A_k is similar to A for all $k \geq 1$, we will use mathematical induction and transitivity of similar matrices.

Base case : $k = 1$

We have proved in section 1.1 that A is similar to A_1. So the base case has been verified.

Inductive step

We need to prove that $A_{k+1} = R_k Q_k$ is similar to A assuming that A_k is similar to A.

We have the QR factorization of A_k as follows:

$$A_K = Q_K R_K$$

We know that

$$A_{K+1} = R_K Q_K$$

$$A_{K+1} = R_K Q_K = Q_K^T A_K Q_K$$ since Q_K is orthogonal

By the inductive hypothes, A_K is similar to A, so we get:

$$A_{K+1} = Q_K^T (P_K^{-1} A P_K) Q_K$$

let $P_{K+1} = P_K Q_K$, then

$$A_{K+1} = P_{K+1}^{-1} A P_{K+1}$$

Hence, A_{K+1} is similar to A, and by the principal of mathematical induction, A_K is similar to A for all $K \geq 1$.