Section 1.1

$A = QR$
$A_1 = RQ$

Q is an orthogonal matrix since A is a square matrix

We can write

$$Q^T A = R$$

since Q is orthogonal and hence invertible

Then, we get

$$A_1 = (Q^T A) Q$$

Let $P = Q$, then

$$A_1 = P^T A P$$
$$= P^{-1} A P$$

since Q is an orthonogonal matrix as A is square

Hence, A_1 is similar to A.

Let λ be an eigenvalue of A with corresponding eigenvector v. we have

$$A_1 = P^T A P$$

$$A_1 - \lambda I = P^T A P - \lambda I$$

$$A_1 - \lambda I = P^{-1} A P - \lambda P^{-1} P$$
$$= P^{-1}(AP - \lambda P)$$
$$= P^{-1}(A - \lambda)P$$

$$\det(A_1 - \lambda I) = \det[P^{-1}(A - \lambda I)P]$$
$$= \det(P^{-1}) \det(A - \lambda I) \det(P)$$

using properties of determinants.

Since $\det(P^{-1})\det(P) = \det(P^{-1}P) = \det(I) = 1$, we have
$$\det(A_1 - \lambda I) = \det(A - \lambda I)$$

Which proves that they have the same eigenvalues.