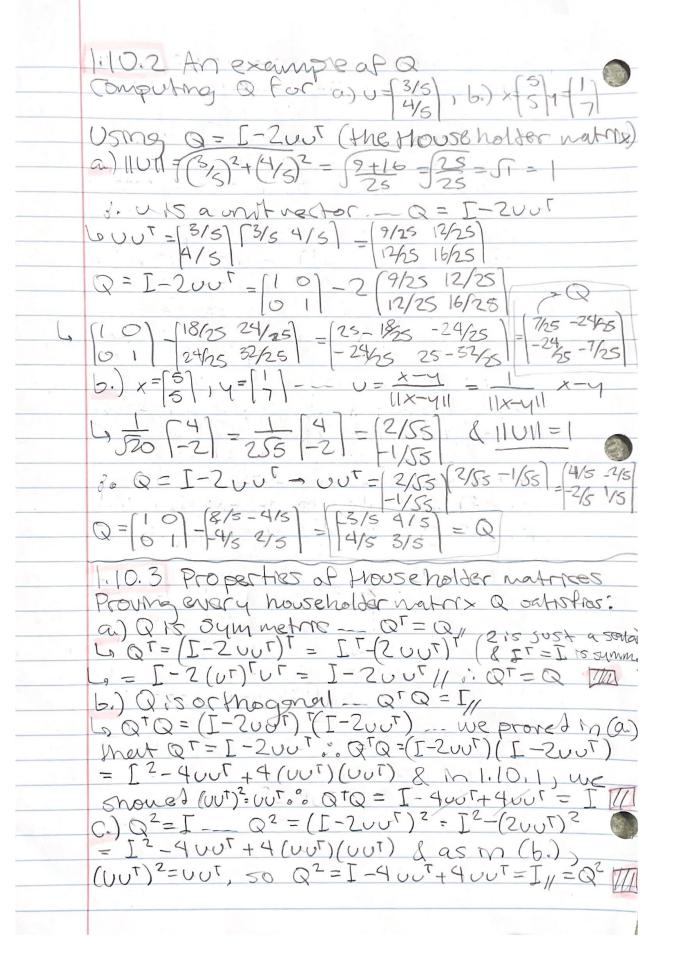
The 2x2 marrix a Showing that the standard matrix Q of the reflection of utils a = [1-22] -2212] = I-2001 direction of utils a = [1-22] -2212] = I-2001 . The reflection of x m the the perpendice to x-y: U= x=1 unerou; a aunit rector

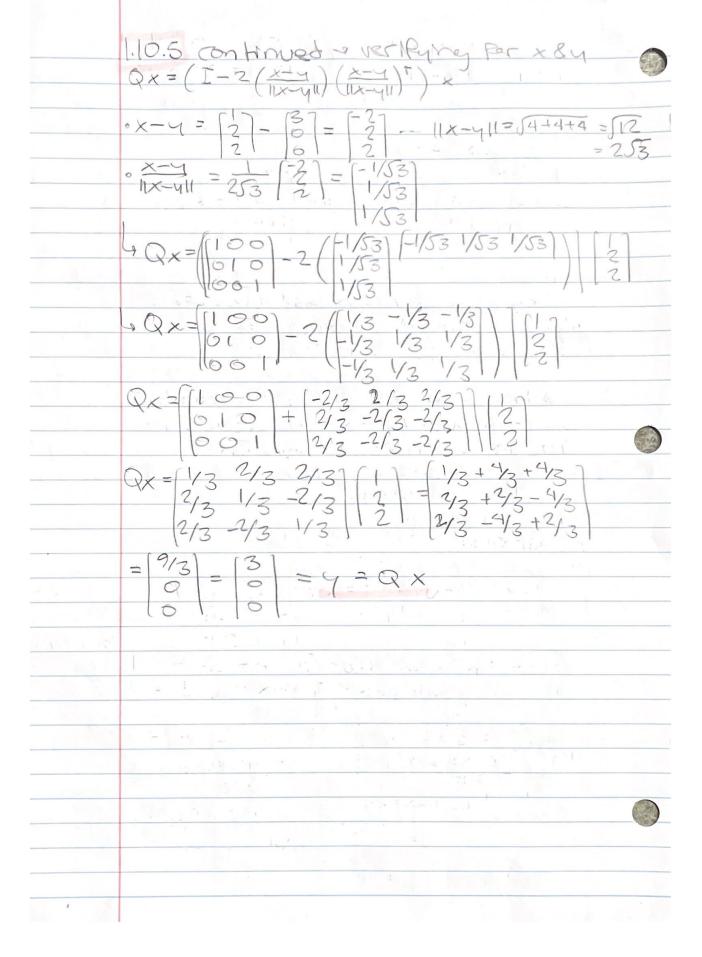
In the troothy -yil of x-y.

- reflection of x across theme: O(x)=x-2proj(x) - (Pro/U(X)=(UTX)U) - Q(X)= x-2(UTX)U (Since U 18 a unit vector ||Ui||= | Peralli= |-n Q(x)=x-2(utx)u must be true for all x 000 = I - 200 Tunero I 15 2×2 160= (95) -> 001= (95) (9195) - (915) 9195 (0) [22/1² 20/10²] = (1-0/1² -20/10²] // Q 18 Plects x across the line perpendicular X-y : Q is orthogonal Q = (I-2001) T (I-2001) = I -2(UT) TUT (I-2005) $= T - 200^{\dagger} (I - 200^{\dagger}) = I - 400^{\dagger} + 4(00^{\dagger})(00^{\dagger})$ -. A unit vector multiplies by itself too not change the product: $(uu^{\tau})^2 = uu^{\tau} & uu^{\tau} \wedge s$ oyumetric $(uu^{\tau} = L)$, thus Q is arthogonal L, (x = x - 2) ($u^{\tau} \times u^{\tau}$) unith reflects x÷. [1-2]? -2didz] = [-UUT = Q -2didz 1-2d2]

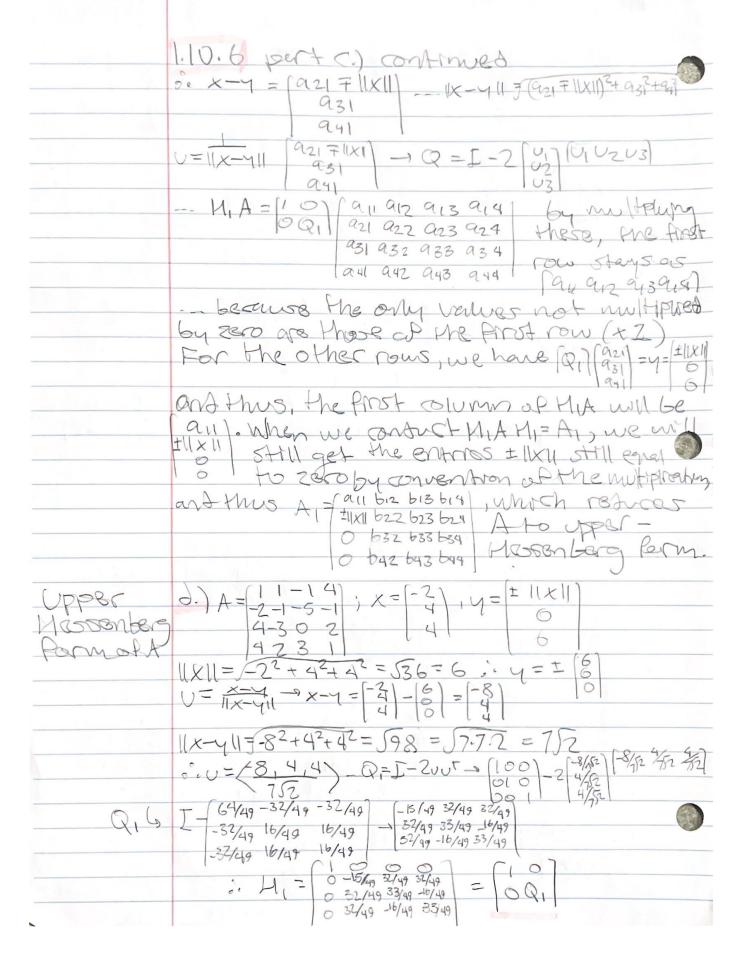


.10.4 comporting QV for some vectors y Proving if a is a Householder matrix corresponding to the unit vector u, then Qu-(-V if vis in spangu).

O = I-2 unit where u is {V if V · V · U = 0} · Q = I-2001 WASCE () 15 a unit rector (IIVII = · NEIKU - showed that NIO we the span Enj Lo Let V= CU For some scalar C; V= CU M QV L. QV = Q(CU) = CQ(V) (By ASSOCIATIVE Law) -- Q(U) = (I - 2005) U = U-2005U (UTU=||U||^2=1) Lo Q(U)= U-ZU=-U __ QV=(-U) =-50 1P V Espen (U), Q1=-1777 · vou=0. Vou=0 means vou are attaggrand to each other. so it must be true that ut v = 0 the of thegonal naturx Q is equal to QV = ([=200] V = V-200[V _ (UTV=0 : 00[V = 0) Lo QV= V-2(0) = V .: if V. U=0, then QV=V [1] .10.5 From that Qx=y x 7 y W 11 x 11 = 11 y 11 & U= 1x y 11 . Householder notrix Q satisfies Qx=4 Parx 2 84=13 $L_{y} Qx = (I - 2uv^{T})x = \left[I - 2\left(\frac{x-y}{\|x-y\|}\right)\left(\frac{x-y}{\|x-y\|}\right)^{T}\right]x$ Lo Qx = I-2 v (xT-yT) (1x-y11 is just a scalar; transpose (Qx = x - 20 x x - y x = x - 20 x · x - y · x (y x is a scalar) $\log x = x - \sqrt{2x^2 - 2y \cdot x} = (4x + ||x||^2 + ||x||^2 - 2y \cdot x)$ $= \frac{||x - y||}{||x - y||} = \frac{||x - y||}{||x - y||} = \frac{||x - y||}{||x - y||}$ (11x11=11y11) -9 Qx=x-U 11x112+114112-24x-> (11x-4112) $Qx = x - 0 \frac{\|x - y\|^2}{\|x - y\|} \rightarrow Qx = x - 0 \frac{\|x - y\|}{\|x - y\|}$ 0x=x-U 11x-411 -90x=x-x-411 11x-411 4Qx=x-(x-4) → Ox=x-x+4 → Qx=4



1.10.6 Reduction to Upper Messenberg Form an air air ara ara upper Mossentery motors 931 932 938 934 941 942 943 949 a) why Mis on orthogonal, symmetric notif · 1 = 10 , where Q= qu q12 913 091192913 0 921922 923 931 932933 0 931 932 933 Also, Q = 5-2001, where u; sa ont vector (Q) is orthogonal (QTP)=I, as proved in MIM = [10 0] = [10] = I also symmetric in 1.10.3, such that Q1=Q1, Thus Hi must also be symmetic-Thus M, is orthogonal & symmetric. exgenialis, if It, is orthogonal as provedin part a), then the engenielles of A & A, most be the same -- MIAMI = AI => MIAHI = A are shuller metrices (as proved in [.]) -- 50 the eigenvalues of A, are equal to mass of A. C.) Shooting A, = M, AHI is a matrix of the form A1 = = | 612 b13 b14 1 A1 = = | 1 | 1 | b22 b23 b24 0 632 683 634 0921922923 0 642 643 644 0 931 932 933 1 11X11 = 921 +9512 +941 Q = 1-2m, mers 941



.10.6 Case of Symmetrie Matrices; tridiagran native 2.) Why a Symmetric mentrix A has an upper Mensonberg form that is tritraggray! For the 5x5 · All symmetre notrees satisfy A = A, neering every entry aij = aji (which is unround arous the tragonal of that neverte) · An upper the soon berg form neutrix has all zeros below the subdiagonal entry for each column (that is, for every 1>j+1, aij=0 o Given Mis, we can infer that a symmetime matrix A will have an Upper Mosson berg Form that rople cts the symmetry in it's without form, across the draggner of the Upper Messenberg form Lo specifically the entries on the nown tradgment (every aii) will remain nonzero, the entires on the first upper taggered will be nonzero (a;;+1,i=1,2-n-1), & the entros on the frost Subdiagonal will be nonzero (ai, i-1), & due to the symmetry, ai, i-1= ai-1,i Ly Thus we can expect for entires above & Gelow each first upper a subtragonal will all be ZBTO; Par Sx5 symmetry natros, this will result in the form of A as seen above verall, the only nonzero entires for a symmetric natrix's upper Messenterg form Lie on, through alove, and directly below the diagonal entires, and by convention, his rectos a tribiagonal warra. So Any Upper Versectory form of a symmetric