Etude 11 - 1-D Solitaire

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1 Introduction

One-dimensional solitaire is played on an infinite row of holes, initially containing some pegs. A move involves hopping a peg over an adjacent peg into an empty hole, removing the hopped-over peg. The goal is to win by reducing the board to a single peg. This report explores the strategy and findings while attempting to generate distinct winnable starting positions with 7 pegs.

2 Thought Process

Let ● = peg and ○ = a hole

Firstly, we began this project with the concept of working in reverse. We would start at the final winning board ● and then work our way back to starting position via unhops.

If we simulate the unhop of, ● , we get ○●● or ●●○. Since left or right peg can hop over the other peg to leave a board state of ●○○ or ○○●, which is a winning position.

$●○○ \xrightarrow{unhop} ○●●$

$○○● \xrightarrow{unhop} ●●○$

Furthermore, the state of ●●○ and ○●● can also be unhopped again. Both pegs in the middle had to have hopped over another peg to reach the centre.
Ergo, if we unhop the left centre ● we get ●●○●○, likewise, we get ○●○●● if we unhop the right ●. We can also unhop both at the same time(from ○●●○) to form ●●○○●●.

$○●● \xrightarrow{unhop\,left} ●●○●○$ since unhop(○○●)● = ●●○●○

$●●○ \xrightarrow{unhop\,right} ○●○●●$ since ●unhop(●○○) = ○●○●●

$○●●○ \xrightarrow{unhop\,both} ●●○○●●$ since unhop(○○●)unhop(●○○) = ●●○○●●

This also extends to holes that are in the centre of the board. If we go back to the example ●●○○●●, we can unhop left at index 0 to form, ●●○●○○●● and also unhop at index 3 to form ●●○○●●●●. Interestingly, unhopping from the left shifts the ○○ to the left.

From this we find that every time you get a scenario in where you have ○○●, we can replace this with ●●○ with a left unhop and can turn ●○○ into ○●● with a right unhop. But also only one ○○ is present in the board as we can only create it by doing a unhop both using the extra space from not having one if you try to do it again you won't have enough holes/space.

3 Magical Number 7

Using what we have now figured out, we know that if n <4, then there is only 1 distinct solvable state for each.

If n = 1 ●○
If n = 2 ●●○
If n = 3 ●●○●

To work out by hand we simply unhopped every situation we could find from n=3 using:

$\bullet\circ\circ \xrightarrow{unhop} \circ\bullet\bullet$

$\circ\circ\bullet \xrightarrow{unhop} \bullet\bullet\circ$

The following are winnable board states with 7 pegs:

●●○●○●○●○●○●

●●○●○●●●○●

●○●●●●●●

●●○●○●○●○○●●

●●○●●●○○●●

●●○●○●○○●○●●

●●○●○○●●●●

●●●●○●●●

4 Tid Bits

4.1 Patterns

After coding up a solution to the problem we found that interesting pattern emerged. As seen in the table below, the difference between the distinct states at n and n-1 (d) go up by a constant (1) as n increases. Ergo, when you plot the n vs d this results in a linear relationships, as seen in the plot below.

Number of Pegs (n)	Distinct States (S)	S(n) - S(n-1)
4	2	
5	3	1
6	5	2
7	8	3
8	12	4
9	17	5
10	23	6
11	30	7
12	38	8
13	47	9
14	57	10
15	68	11
16	80	12
17	93	13
18	107	14
19	122	15
20	138	16