# A Distribution For Magnetic Fields

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# A Distribution For Magnetic Fields

The team:

- Diego Ortiz, UNIR (Math).
- Wladimir Banda, Hamburg University (Physics).
- Me, UTE (Applied Math).

# A Distribution For Magnetic Fields

**What we study:**

Role of galactic winds and outflows in galaxy evolution.

Remove gas and metals from the disk and nuclear regions of star-forming galaxies and deposit them in the circumgalactic medium.

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**What we want to understand:**

The role of magnetic fields in the dense phase.

The presence of cold gas (clouds) in such outflows.

**The Wind/Shock - Cloud simulations:**

Transport via momentum transfer from hot gas?

- In purely hydrodynamic regimes: Too many instabilities, cloud gets destroyed rapidly.
- Recent simulations show that magnetic stresses can aid cloud acceleration and survival.

# A Distribution For Magnetic Fields

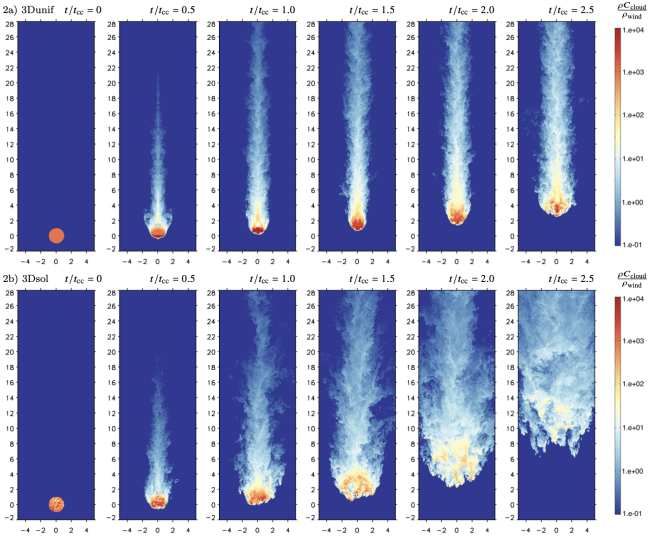
**In this talk:**

- Tools for a systematic statistical study of the effect of magnetic fields.

**Need: Probaility distribution over magnetic fileds.**
(Herr W.: For magnetic fields with compact support, sometimes symmetric and div-free! And, depending on the day, turbelent too - Kazantsev if dinamos.)

# A Distribution For Magnetic Fields

**Need: Probaility distribution over magnetic fileds.**



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**Need: Probaility distribution over $f : \mathbf{R}^m \rightarrow \mathbf{R}^n$.**
**Gaussian Process:** A proba. distribution over a function space.

$$f(x) \sim \mathbf{GP}\left(0,\ k(x, x')\right)$$

For any $\mathbf{x} := [x_1, ..., x_n]^T$,

$$\mathbf{f(x)} \sim \mathbf{N}\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix},\ \begin{bmatrix} k(x_1, x_1) \cdots k(x_1, x_n) \\ \ddots \\ k(x_n, x_1) \cdots k(x_n, x_n) \end{bmatrix}\right)$$

where $\mathbf{f(x)} := [f(x_1), ..., f(x_n)]^T$.

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**What does that mean?** To simulate:

1. Choose a kernel $k$.

2. Fix a set of input points $\mathbf{x} := [x_1, ..., x_n]^T$.

3. To simulate $\mathbf{f}(\mathbf{x})$, build the covariance matrix $\mathbf{K}(\mathbf{x}, \mathbf{x})$, draw from $\mathbf{N}\left(\mathbf{0},\ \mathbf{K}(\mathbf{x}, \mathbf{x})\right)$.

   where $\mathbf{K}(\mathbf{x}, \mathbf{x})[i, j] = k(x_i, x_j)$.

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**Different covariance functions, different function spaces.**

**Which function space?** You get choose by chossing/building the covariance function $k$.

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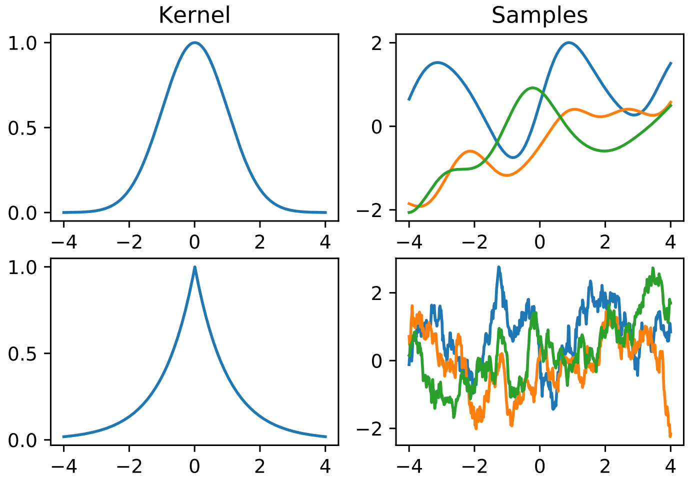
**Which function space?** Regularity depends on $k$.



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**Algebra of covariance functions** Can combine covariance functions to encode other characteristics:

- $\alpha k$ is a covariance function.
- $k(\phi(x),\ \phi(x'))$ is a covariance function.
- $k_1 \times k_2$ is a covariance function - like an AND covariance function, high vals if both are.
- $k_1 + k_2$ is a covariance function - like an OR covariance function, high vals if one is.

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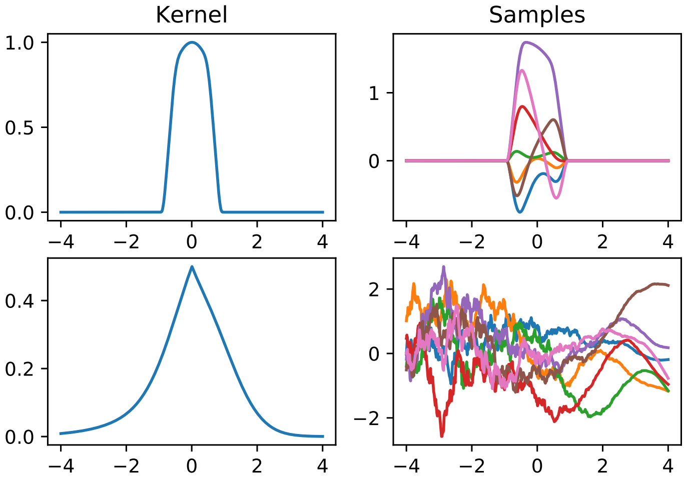
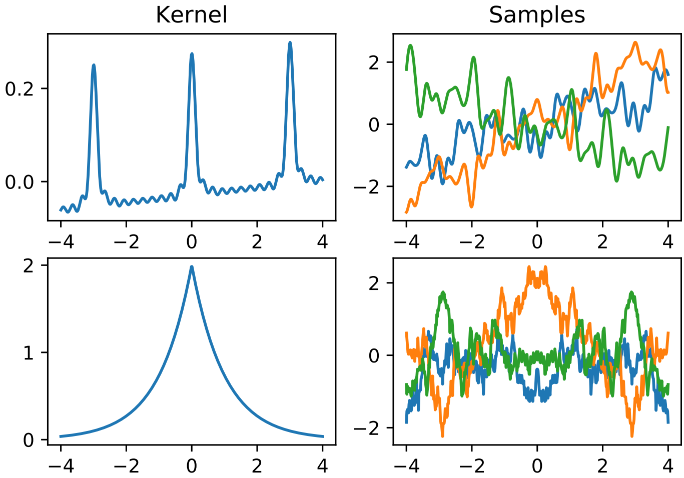
**Algebra of covariance functions** Can combine covariance functions to encode other characteristics: $k_1 \times k_2$



# A Distribution For Magnetic Fields

**Algebra of covariance functions** Can combine covariance functions to encode other characteristics: $k_1 + k_2$



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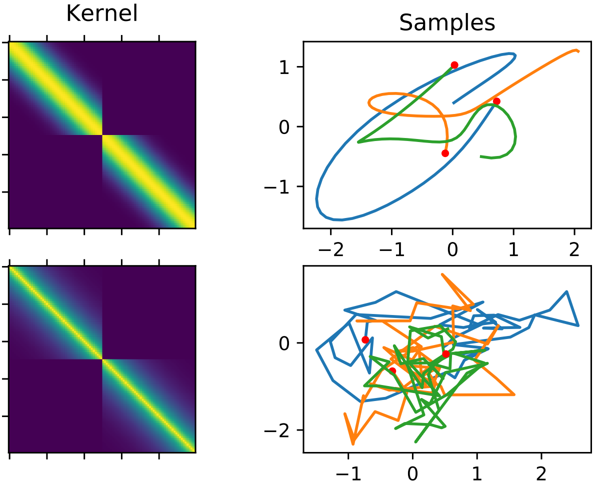
**A process** $R \rightarrow R^2$ Is a distribution over doodles.



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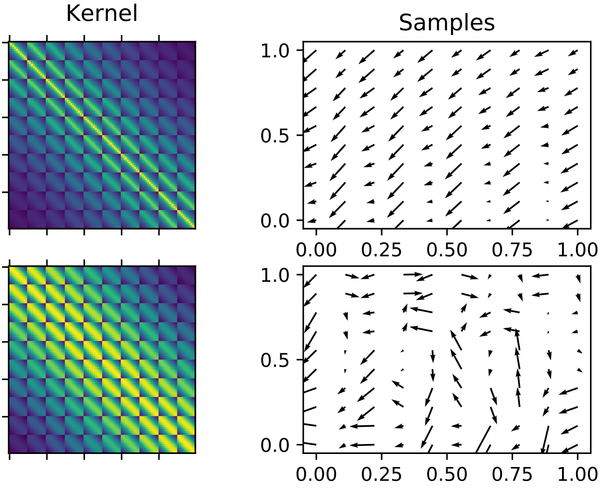
**A process** $R^2 \to R^2$ Is a distribution over vector fields.



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In fact you can encode more:

**Div-Free**:

$$\nabla f = 0$$

i.e. Processes that satisfy a linear contraint.

$$\mathcal{L} f = 0 \tag{1}$$

(Idea: Find an operator $\mathcal{G}$ such that $\mathcal{L}\mathcal{G} = 0$. Then, $\mathcal{G}f$ satisfies (1).)

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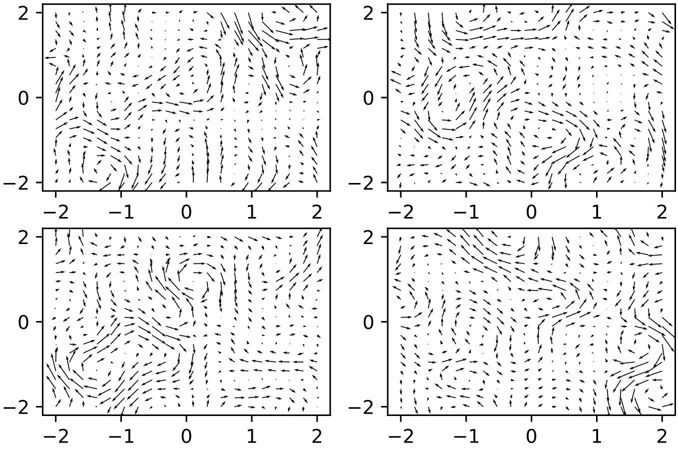
**Div-Free**: Samples from a div-free **GP**.



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**MHD simulation**:

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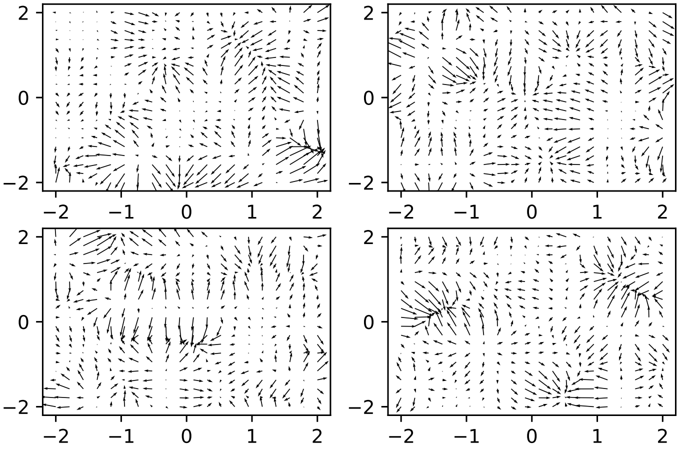
**Curl-Free**: Samples from a curl-free **GP**.



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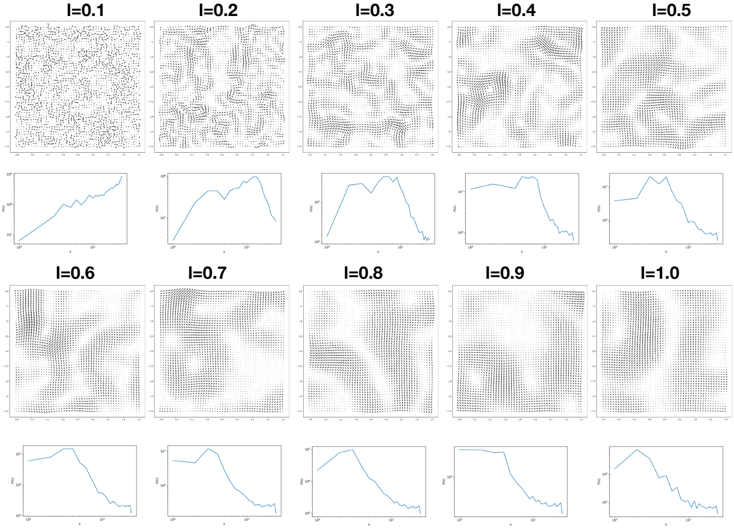
## Trubulence and Kernels:



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**So far:**

- Distributions over function spaces.
- Mathematically Expressive: Can encode regularity, symmetries, changes, support.

**In progress:**

- Computationally NOT Expressive: Autodiff code in progress.
- Turbulence NOT clear: Studying the relationship between kernels and energy spectral decay.

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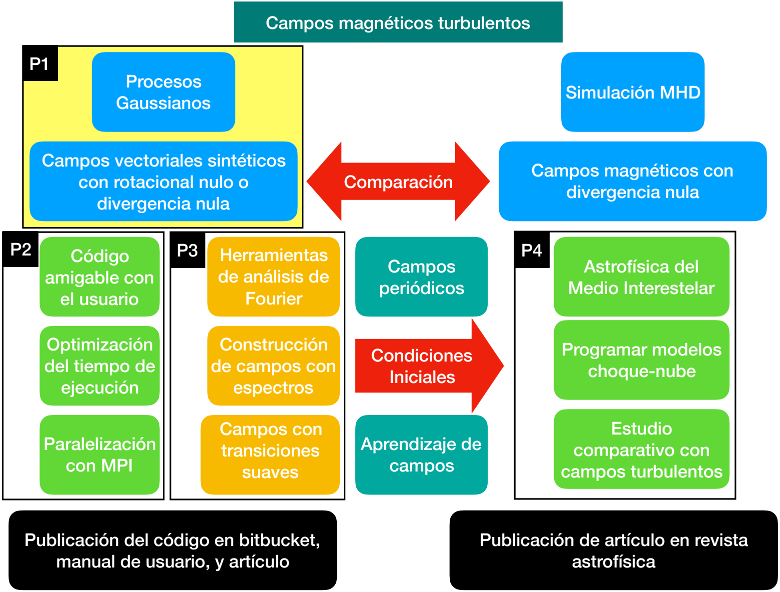
## A bigger picture:



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**A bigger picture:** (In alphabetical order)

- Computer Science, Math, Physics.
- Australia, Ecuador, Germany.

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**Some references:**


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