

A Distribution For Magnetic Fields

Diego Ortiz, UNIR, Wladimir Banda, Hamburg University,
Federico Zertuche, CENBIO, UTE.

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A Distribution For Magnetic Fields

The team:

Diego Ortiz, UNIR (Math).

Wladimir Banda, Hamburg University (Physics).

Me, UTE (Applied Math).

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What we study:

Role of galactic winds and outflows in galaxy evolution.

Remove gas and metals from the disk and nuclear regions of star-forming galaxies and deposit them in the circumgalactic medium.

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What we want to understand:

The presence of cold gas (clouds) in such outflows.

The Wind/Shock - Cloud simulations:

Transport via momentum transfer from hot gas?

- In purely hydrodynamic regimes: Too many instabilities, cloud gets destroyed rapidly.
- Recent simulations show that magnetic stresses can aid cloud acceleration and survival.

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In this talk:

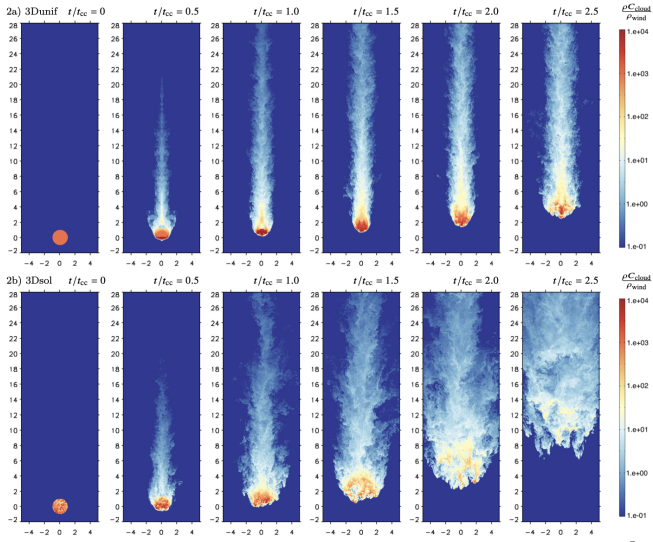
- Tools for a systematic statistical study of the effect of magnetic fields.

Need: Probaility distribution over magnetic fileds.

(Herr W.: For magnetic fields with compact support, sometimes symmetric and div-free! And, depending on the day, turbelent too
- Kazantsev if dinamos.)

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Need: Probability distribution over magnetic fields.



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Need: Probability distribution over $f : \mathbf{R}^m \rightarrow \mathbf{R}^n$.

Gaussian Process: A proba. distribution over a function space.

$$f(x) \sim \mathbf{GP}(0, k(x, x'))$$

For any $\mathbf{x} := [x_1, \dots, x_n]^T$,

$$\mathbf{f}(\mathbf{x}) \sim \mathbf{N} \left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} k(x_1, x_1) & \cdots & k(x_1, x_n) \\ & \ddots & \\ k(x_n, x_1) & \cdots & k(x_n, x_n) \end{bmatrix} \right)$$

where $\mathbf{f}(\mathbf{x}) := [f(x_1), \dots, f(x_n)]^T$.

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What does that mean? To simulate:

1. Choose a kernel k .
2. Fix a set of input points $\mathbf{x} := [x_1, \dots, x_n]^T$.
3. To simulate $\mathbf{f}(\mathbf{x})$, build the covariance matrix $\mathbf{K}(\mathbf{x}, \mathbf{x})$, draw from $\mathbf{N}(\mathbf{0}, \mathbf{K}(\mathbf{x}, \mathbf{x}))$.

where $\mathbf{K}(\mathbf{x}, \mathbf{x})[i, j] = k(x_i, x_j)$.

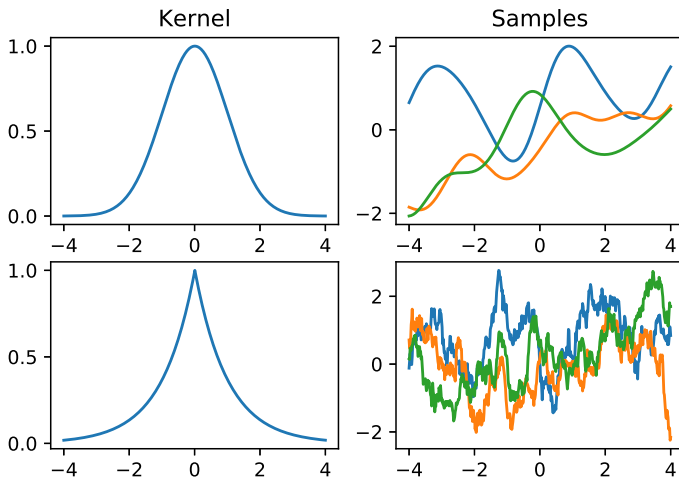
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Different covariance functions, different function spaces.

Which function space? You get choose by chossing/building the covariance function k .

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Which function space? Regularity depends on k .



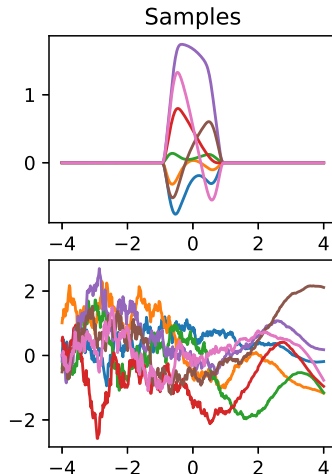
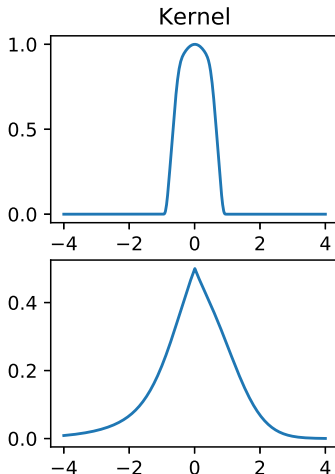
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Algebra of covariance functions Can combine covariance functions to encode other characteristics:

- αk is a covariance function.
- $k(\phi(x), \phi(x'))$ is a covariance function.
- $k_1 \times k_2$ is a covariance function - like an AND covariance function, high vals if both are.
- $k_1 + k_2$ is a covariance function - like an OR covariance function, high vals if one is.

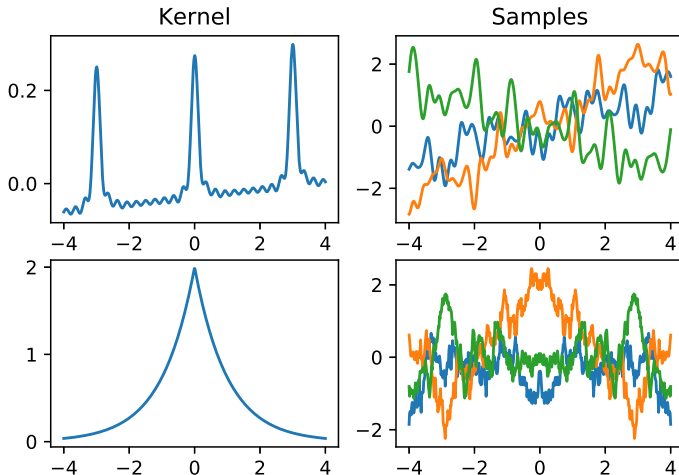
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Algebra of covariance functions Can combine covariance functions to encode other characteristics: $k_1 \times k_2$



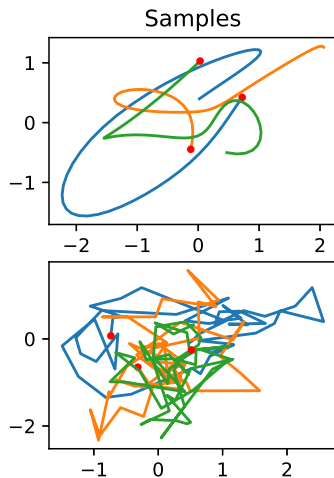
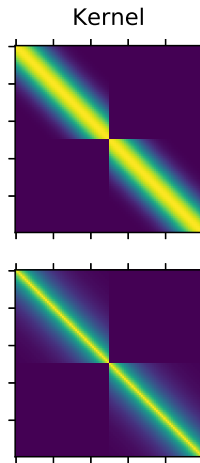
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Algebra of covariance functions Can combine covariance functions to encode other characteristics: $k_1 + k_2$



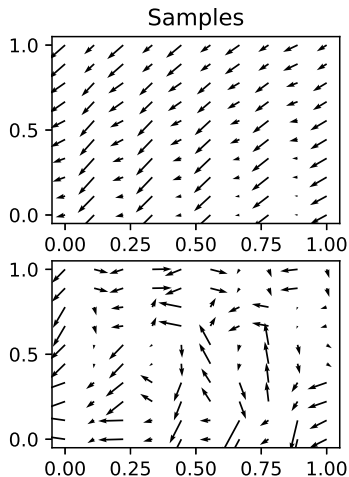
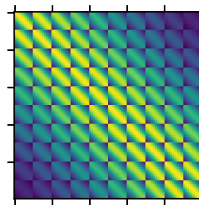
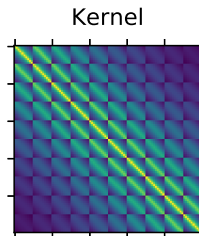
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A process $R \rightarrow R^2$ is a distribution over doodles.



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A process $R^2 \rightarrow R^2$ Is a distribution over vector fields.



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In fact you can encode more:

Div-Free:

$$\nabla f = 0$$

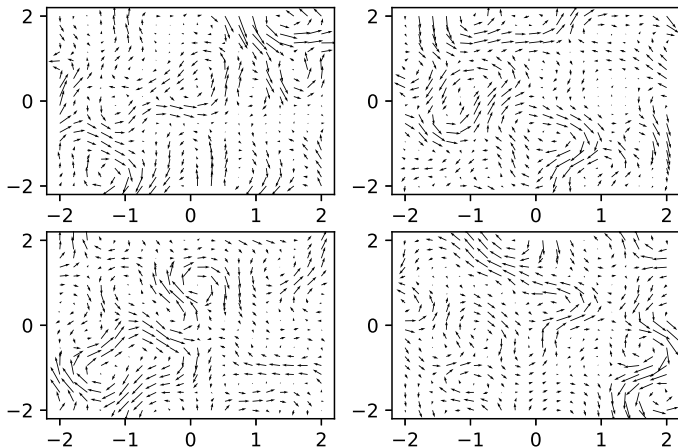
i.e. Processes that satisfy a linear constraint.

$$\mathcal{L}f = 0 \tag{1}$$

(Idea: Find an operator \mathcal{G} such that $\mathcal{L}\mathcal{G} = 0$. Then, $\mathcal{G}f$ satisfies (1).)

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Div-Free: Samples from a div-free **GP**.

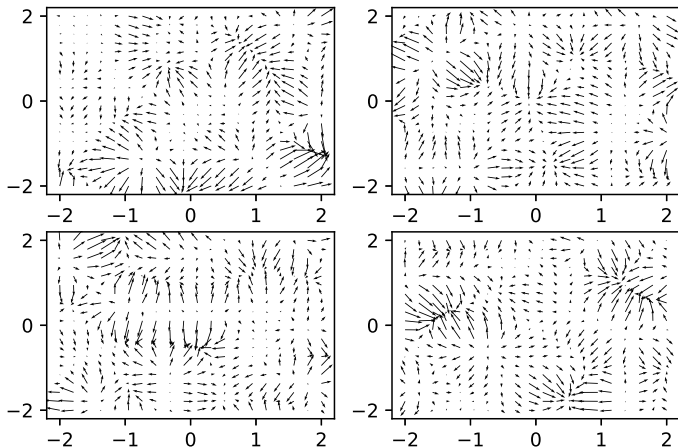


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MHD simulation:

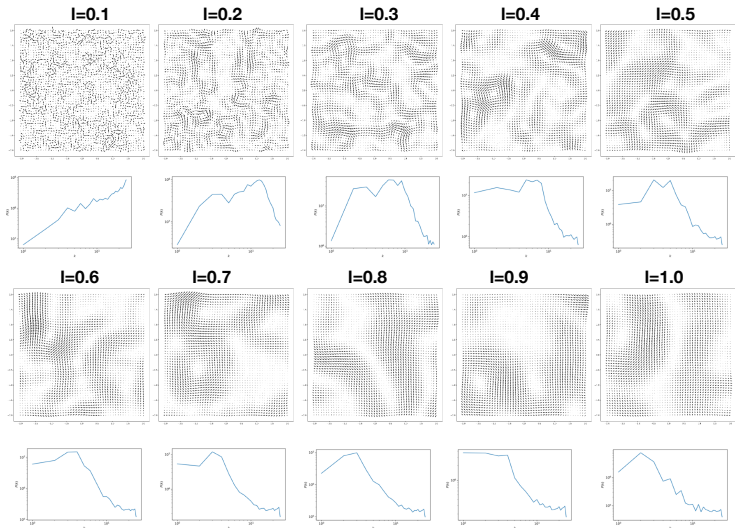
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Curl-Free: Samples from a curl-free **GP**.



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Turbulence and Kernels:



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So far:

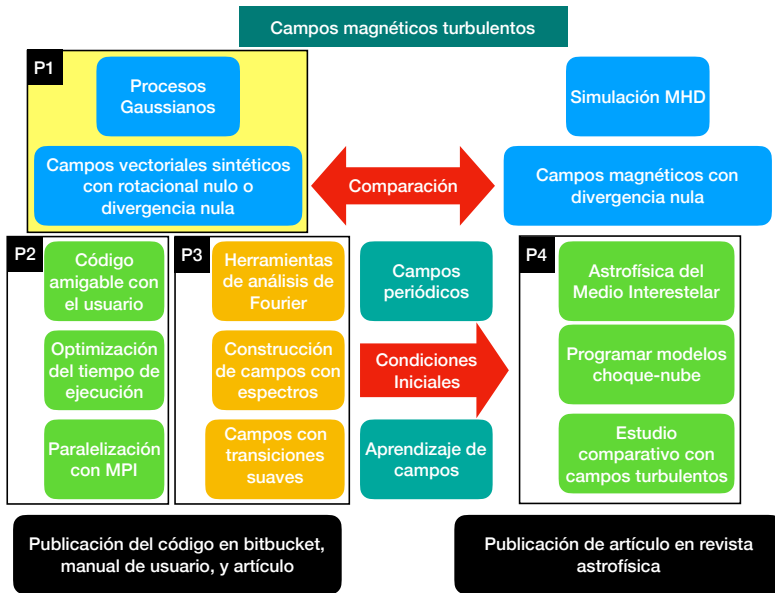
- Distributions over function spaces.
- Mathematically Expressive: Can encode regularity, symmetries, changes, support.

In progress:

- Computationally NOT Expressive: Autodiff code in progress.
- Turbulence NOT clear: Studying the relationship between kernels and energy spectral decay.

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A bigger picture:



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A bigger picture: (In alphabetical order)

- Computer Science, Math, Physics.
- Australia, Ecuador, Germany.

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Some references:

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