A Distribution For Magnetic Fields

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A Distribution For Magnetic Fields

The team:

- Diego Ortiz, UNIR (Math).
- Me, UTE (Applied Math).
- Wladimir Banda, Hamburg University (Physics).

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What we study:

Role of galactic winds and outflows in galaxy evolution.

Remove gas and metals from the disk and nuclear regions of star-forming galaxies and deposit them in the circumgalactic medium.

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What we want to understand:

The presence of cold gas (clouds) in such outflows.

The Wind/Shock - Cloud simulations:

Transport via momentum transfer from hot gas?

- In purely hydrodynamic regimes: Too many instabilities, cloud gets destroyed rapidly.
- Recent simulations show that magnetic stresses can aid cloud acceleration and survival.

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In this talk:

- Tools for a systematic statistical study of the effect of magnetic fields.

Need: Probaility distribution over magnetic fileds.
(Herr W.: For magnetic fields with compact support, sometimes symmetric and div-free! And, depending on the day, turbelent too.)

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Need: Probaility distribution over $f : \mathbf{R}^m \to \mathbf{R}^n$.
Gaussian Process: A proba. distribution over a function space.

$$f(x) \sim \mathbf{GP}\left(0,\ k(x, x')\right)$$

For any $\mathbf{x} := [x_1, ..., x_n]^T$,

$$\mathbf{f}(\mathbf{x}) \sim \mathbf{N}\left(\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix},\ \begin{bmatrix} k(x_1, x_1) \cdots k(x_1, x_n) \\ \ddots \\ k(x_n, x_1) \cdots k(x_n, x_n) \end{bmatrix}\right)$$

where $\mathbf{f}(\mathbf{x}) := [f(x_1), ..., f(x_n)]^T$.

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What does that mean? To simulate:

1. Choose a vector space - a covariance function k.
2. Fix a set of input points $\mathbf{x} := [x_1, ..., x_n]^T$.
3. To simulate $\mathbf{f}(\mathbf{x})$, build the covariance matrix $\mathbf{K}(\mathbf{x}, \mathbf{x})$, draw from $\mathbf{N}\left(\mathbf{0},\ \mathbf{K}(\mathbf{x}, \mathbf{x})\right)$.

   where $\mathbf{K}(\mathbf{x}, \mathbf{x})[i, j] = k(x_i, x_j)$.

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Different covariance functions, different function spaces.

Which function space? You get choose by chossing/building the covariance function k.

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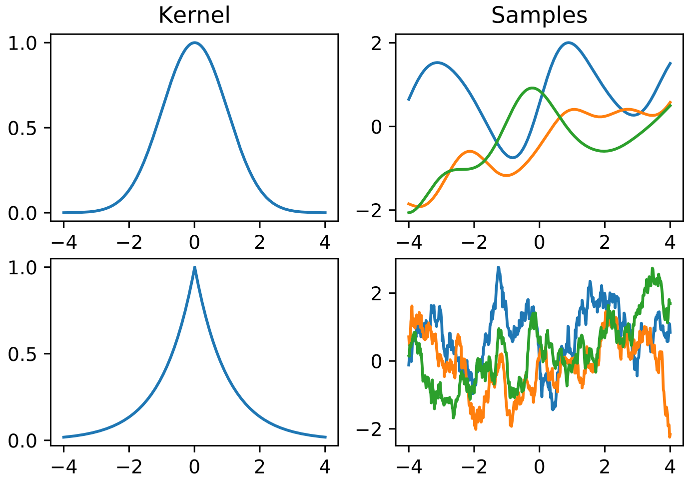
Which function space? Regularity depends on k.



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Algebra of covariance functions Can combine covariance functions to encode other characteristics:

- αk is a covariance function.
- $k(\phi(x),\ \phi(x'))$ is a covariance function.
- $k_1 \times k_2$ is a covariance function - like an AND covariance function, high vals if both are.
- $k_1 + k_2$ is a covariance function - like an OR covariance function, high vals if one is.

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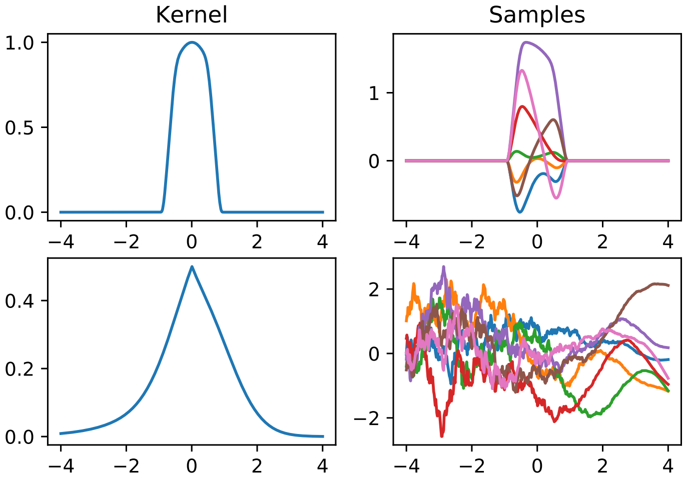
Algebra of covariance functions Can combine covariance functions to encode other characteristics: $k_1 \times k_2$



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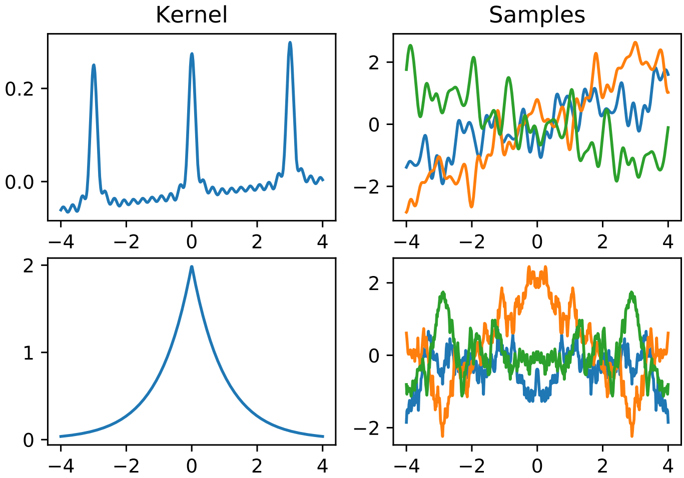
Algebra of covariance functions Can combine covariance functions to encode other characteristics: $k_1 + k_2$



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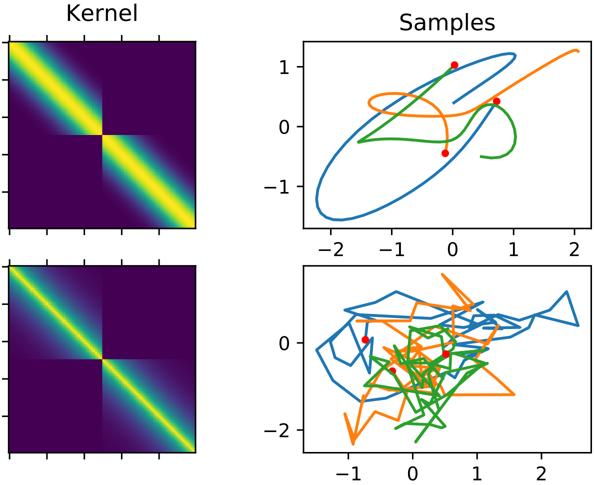
A process $R \rightarrow R^2$ Is a distribution over doodles.



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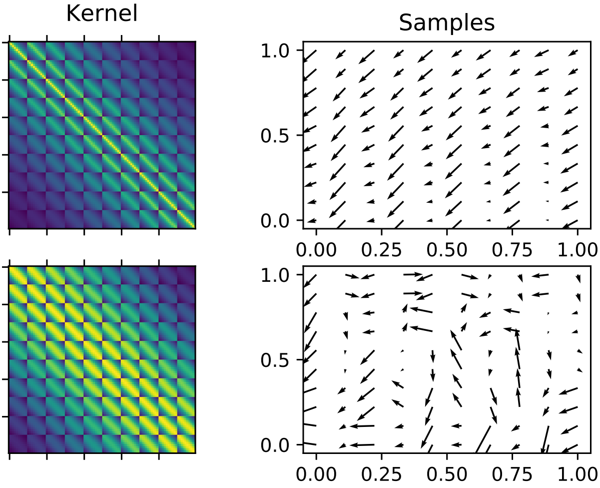
A process $R^2 \to R^2$ Is a distribution over vector fields.



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In fact you can encode more:

Div-Free:

$$\nabla f = 0$$

i.e. Processes that satisfy a linear contraint.

$$\mathcal{L} f = 0 \tag{1}$$

(Idea: Find an operator $\mathcal{G}$ such that $\mathcal{L}\mathcal{G} = 0$. Then, $\mathcal{G} f$ satisfies (1).)

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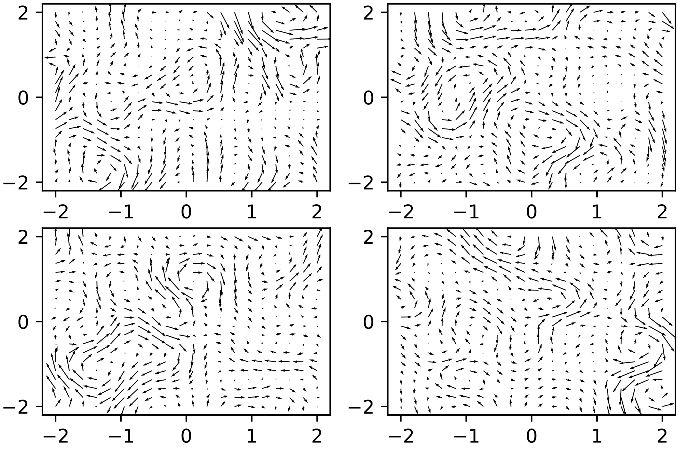
Div-Free: Samples from a div-free **GP**.



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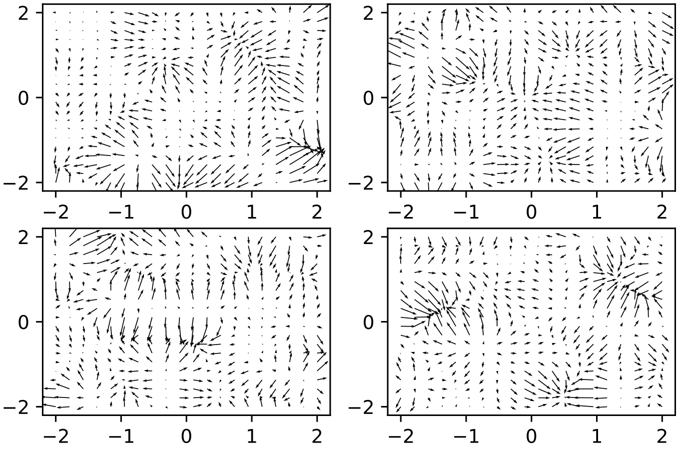
Curl-Free: Samples from a curl-free **GP**.



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So far:

- Distributions over function spaces.
- Mathematically Expressive: Can encode regularity, symmetries, changes, support.

In progress:

- Computationally NOT Expressive: Autodiff code in progress.
- Turbulence NOT clear: Studying the relationship between kernels and energy spectral decay.

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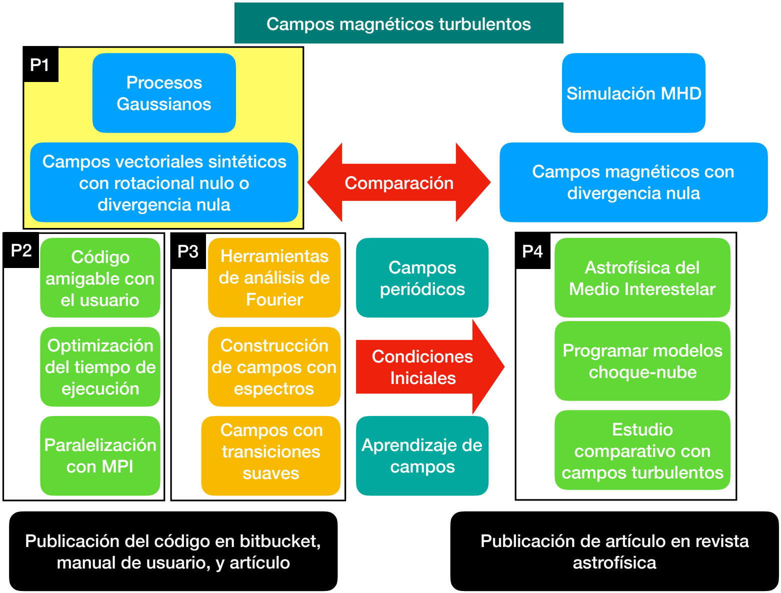
A bigger picture:



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A bigger picture: (In alphabetical order)

- Computer Science, Math, Physics.
- Australia, Ecuador, Germany.