Question 2

Multidimensions Newton raphson

Given System

$x_1^3 - 2x_2 - 2 = 0$

$x_1^3 - 5x_3^2 + 7 = 0$

$x_2 x_3^2 - 1 = 0$

Initial guess : $\mathbf{x} = [1, 1, 1]$

$\epsilon = 5 \times 10^{-12}$

Exact solution : $\mathbf{x} = [\sqrt[3]{3}, 0.5, \sqrt{2}]$

In [1]:
```
import numpy as np
from matplotlib import pyplot as plt



def WriteDataToFile(list,myfile):
    myfile.write(', '.join(str(item) for item in list)+'\n')



def F(x):
    return np.array([x[0]**3 - 2*x[1] - 2,x[0]**3 - 5*x[2]**2 + 7, x[1]*x[2]**2 - 1

def J(x):
    return np.matrix([[3*x[0]**2, -2, 0],
                      [3*x[0]**2,  0, -10*x[2]],
                      [0, x[2]**2 , 2*x[1]*x[2]]
                     ])

max_iter= 10
tol = 5e-12
x0 = np.array([1,1,1])

myfile = open("output_data\\mnr.csv", 'a')
myfile.seek(0)
myfile.truncate()
WriteDataToFile(["ilteration","x(1)","x(2)","x(3)","Infinity Norm F(x)"],myfile)
WriteDataToFile([0,x0[0],x0[1],x0[2],np.linalg.norm(F(x0),np.inf)],myfile)


for i in range(0,max_iter):
    v = -np.linalg.solve(J(x0), F(x0))
    x0 = x0 + v
    WriteDataToFile([i,x0[0],x0[1],x0[2],np.linalg.norm(F(x0),np.inf)],myfile)
    if(np.linalg.norm(v,np.inf) < 1e-12):
        break

myfile.close()
```

Result

ilteration	x(1)	x(2)	x(3)	Infinity Norm F(x)
0	1	1	1	3.0
0	1.4285714285714286	0.1428571428571428	1.4285714285714286	0.708454810495627
1	1.4401111728738187	0.4930516953863257	1.4133129516398	0.015152165728195222
2	1.4422553387582202	0.5000080621820467	1.4142149902141499	1.987236353251376e-05
3	1.4422495703352232	0.5000000000147997	1.414213562375909	1.439719454765509e-10
4	1.4422495703074083	0.49999999999999994	1.414213562373095	1.7763568394002505e-15

Solution :
$[1.4422495703074083, 0.49999999999999994, 1.414213562373095] \approx [\sqrt[3]{3}, 0.5, \sqrt{2}]$