Question 3

$x \in [a, b]$

Number of Intervals : n

Interval Size : $\frac{b-a}{n}$

$I = \int_a^b \sin x dx$

Exact solution of the above integral is 2

In [1]:
```
import numpy as np



def WriteDataToFile(list,myfile):
    myfile.write(', '.join(str(item) for item in list)+'\n')



F = lambda x: np.sin(x)

I_exact = 2.
xb = np.pi
xa = 0.

def TrapezoidalRuleIntegration(n):
    x = np.linspace(xa,xb,n+1) ##(n + 1)Plus one because linspace creates n point w
    s=0
    [s:=s+2*F(i) for i in x]
    return (s-F(xb)-F(xa))*0.5*(xb-xa)/n

def SimpsonRuleIntegration(n):
    x = np.linspace(xa,xb,n+1) ##(n + 1)Plus one because linspace creates n point w
    s=F(xa)+F(xb)
    [s:=s+4*F(x[i]) for i in range(1,len(x)-1,2)]
    [s:=s+2*F(x[i]) for i in range(2,len(x)-1,2)]
    return s*(xb-xa)/n/3




myfile = open("output_data\\trap.csv", 'a')
myfile.seek(0)
myfile.truncate()
WriteDataToFile(["n","h","EstimateIntegral","eh","e2h/eh"],myfile)

firstiter = TrapezoidalRuleIntegration(1)
WriteDataToFile([1,(xb-xa),firstiter,-firstiter+I_exact,"-"],myfile)

n = np.array([2**i for i in range(0,8)])
error_vec = np.zeros(len(n))


for i in n[1:]:
    T_h = TrapezoidalRuleIntegration(i)
    eh = abs(T_h - I_exact)
    np.append(error_vec,eh)
```

```
    eh2 = abs(firstiter - I_exact)
    firstiter = T_h
    WriteDataToFile([i,(xb-xa)/i,T_h,eh,eh2/eh],myfile)


myfile.close()


myfile = open("output_data\\SimpingForNami.csv", 'a')
myfile.seek(0)
myfile.truncate()
WriteDataToFile(["n","h","EstimateIntegral","eh","e2h/eh"],myfile)

firstiter = SimpsonRuleIntegration(1)
WriteDataToFile([1,(xb-xa),firstiter,-firstiter+I_exact,"-"],myfile)

n = np.array([2**i for i in range(0,8)])
error_vec = np.zeros(len(n))


for i in n[1:]:
    T_h = SimpsonRuleIntegration(i)
    eh = abs(T_h - I_exact)
    np.append(error_vec,eh)
    eh2 = abs(firstiter - I_exact)
    firstiter = T_h
    WriteDataToFile([i,(xb-xa)/i,T_h,eh,eh2/eh],myfile)


myfile.close()
```

Results

Part(a)

n	h	EstimateIntegral	eh	e2h/eh
1	3.141592653589793	1.9236706937217898e-16	1.9999999999999998	-
2	1.5707963267948966	1.570796326794897	0.429203673205103	4.659792366325491
4	0.7853981633974483	1.8961188979370398	0.1038811020629602	4.131681939078501
8	0.39269908169872414	1.9742316019455508	0.025768398054449193	4.031337215586981
16	0.19634954084936207	1.9935703437723395	0.006429656227660452	4.007741182739018
32	0.09817477042468103	1.998393360970144	0.0016066390298559163	4.001929561139234
64	0.04908738521234052	1.9995983886400375	0.00040161135996252817	4.000482033192044
128	0.02454369260617026	1.9998996001842038	0.00010039981579623714	4.000120486053522

As seen our value of $\frac{e_{2h}}{e_h}$ approaches to **4**. That actually makes sence because order of convergance for Trapozoidal method of integration is 2 .

$$Global\ Error = \frac{(a-b)h^2 f"(\xi)}{12}$$

Beside that Degree of Precesion for Trapozoidal rule is 1

Part (b)

n	h	EstimateIntegral	eh	e2h/eh
1	3.141592653589793	1.2824471291478598e-16	1.9999999999999998	-
2	1.5707963267948966	2.0943951023931953	0.09439510239319526	21.18753991779319
4	0.7853981633974483	2.0045597549844207	0.0045597549844207386	20.701792687482968
8	0.39269908169872414	2.000269169948388	0.0002691699483881038	16.940059660175127
16	0.19634954084936207	2.0000165910479355	1.6591047935499148e-05	16.223806322213832
32	0.09817477042468103	2.0000010333694127	1.0333694127062643e-06	16.05529226189237
64	0.04908738521234052	2.0000000645300013	6.453000134243325e-08	16.01378260047776
128	0.02454369260617026	2.0000000040322576	4.032257638897363e-09	16.00344202214203

As seen our value of $\frac{e_{2h}}{e_h}$ approaches to **16**. That actually makes sence because order of convergance for Simpson method of integration is 4 .

$Global\ Error = \frac{(a-b)h^4 f""(\xi)}{2880}$

Beside that Degree of Precesion for Simpson rule is 3

Part (c)

From above to analysis it is very evident that convergance condition can be checked by ratio $\frac{e_{2h}}{e_h}$, In case we are not aware of the exact definate integral we can just put stopping condition on that ration

Stopping condition

$\frac{e_{2h}}{e_h} \rightarrow 2^{order}$

X	Trapezoidal Rule	SipmpsonRule
Order	2	4