MEC E 301
Lab 7: Flow

by: Alex Diep

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CCID: abdiep
Student ID: 1664334
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Question 1

Question 2

For frequency

$$\omega = \frac{2\pi n}{Nt_{\text{gate}}}$$

$$\delta\omega = \sqrt{\left(\frac{\partial \omega}{\partial n}\delta n\right)^2}$$

$$\delta\omega = \left|\frac{2\pi}{Nt_{\text{gate}}}\right|\delta n$$

For period

$$\omega = \frac{2\pi}{NT}$$

$$\delta\omega = \sqrt{\left(\frac{\partial \omega}{\partial T}\delta T\right)^2}$$

$$\delta\omega = \left|\frac{2\pi}{NT^2}\right|\delta T$$

Also for frequency

$$\omega = \frac{2\pi f}{N}$$

$$\delta\omega = \sqrt{\left(\frac{\partial \omega}{\partial f}\delta f\right)^2}$$

$$\delta\omega = \left|\frac{2\pi}{N}\right|\delta f$$

For single measurement uncertainty for the Sensor 1, first row of data,

$$\delta T \approx 2\sigma_e = 2 \times 8.69 = 17.4\,\mu\text{s}$$

$$\delta\omega = \left|\frac{2\pi}{NT^2}\right|\delta T = \frac{2\pi}{12 \times 63356} \times 17.4 \times 10^6 = 0.002267\,\text{rad}\,\text{s}^{-1}$$

Angular acceleration, for the forward difference method,

$$\alpha_i = \frac{\omega_i - \omega_{i-1}}{T_i}$$
$$\delta\alpha_i = \sqrt{\left(\frac{\partial \alpha_i}{\partial \omega_i}\delta\omega_i\right)^2 + \left(\frac{\partial \alpha_i}{\partial \omega_{i-1}}\delta\omega_{i-1}\right)^2 + \left(\frac{\partial \alpha_i}{\partial T_i}\delta T_i\right)^2}$$
$$\delta\alpha_i = \sqrt{\left(\frac{1}{T_i}\delta\omega_i\right)^2 + \left(\frac{-1}{T_i}\delta\omega_{i-1}\right)^2 + \left(\frac{\omega_i - \omega_{i-1}}{T_i^2}\delta T_i\right)^2}$$

For period bias for Sensor 1,

$$\omega = \frac{2\pi}{NT}$$
$$B_{x,\omega} = \sqrt{\left(\frac{\partial \omega}{\partial T}\delta T\right)^2}$$
$$B_{x,\omega} = \left|\frac{2\pi}{NT^2}\right|\delta T = \frac{2\pi}{12 \times 4250^2} \times 4 \times 10^6 = 0.116\,\mathrm{rad\,s^{-1}}$$

For frequency bias for Sensor 1,

$$\omega = \frac{2\pi n}{N t_{\text{gate}}}$$
$$B_{x,\omega} = \sqrt{\left(\frac{\partial \omega}{\partial n}\delta n\right)^2}$$
$$B_{x,\omega} = \left|\frac{2\pi}{N t_{\text{gate}}}\right|\delta n = \frac{2\pi}{12 \times 1000} \times 1 \times 10^3 = 0.524\,\mathrm{rad\,s^{-1}}$$

For period precision for Sensor 1,

$$P_{x,T} = t_{\alpha/2,\nu}\frac{S_{x,T}}{\sqrt{N}} = 2.262 \times \frac{8.69}{\sqrt{10}} = 6.22\,\mu\mathrm{s}$$
$$P_{x,\omega} = \left|\frac{2\pi}{NT^2}\right|P_{x,T} = \frac{2\pi}{12 \times 4250^2} \times 6.22 \times 10^6 = 0.180\,\mathrm{rad\,s^{-1}}$$

For frequency precision for Sensor 1,

$$P_{x,f} = t_{\alpha/2,\nu}\frac{S_{x,f}}{\sqrt{N}} = 2.262 \times \frac{0.316}{\sqrt{10}} = 0.226\,\mathrm{Hz}$$
$$P_{x,\omega} = \left|\frac{2\pi}{N}\right|P_{x,f} = \frac{2\pi}{12} \times 0.226 = 0.118\,\mathrm{rad\,s^{-1}}$$

For period total uncertainty for Sensor 1,

$$U_{x,\omega} = \sqrt{P_{x,\omega}^2 + B_{x,\omega}^2} = \sqrt{0.180^2 + 0.116^2} = 0.214\,\mathrm{rad\,s^{-1}}$$

For frequency total uncertainty for Sensor 1,

$$U_{x,\omega} = \sqrt{P_{x,\omega}^2 + B_{x,\omega}^2} = \sqrt{0.118^2 + 0.524^2} = 0.537\,\mathrm{rad\,s^{-1}}$$