MEC E 301
Lab 7: Flow

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Question 1

Table 1 shows the ambient pressure and temperature used for air density calculation.

Table 1: Ambient pressure and temperature

Barometer (mmHg)	Temperature (°C)	Correction for Temperature (mmHg)	Correction for Latitude (mmHg)	Corrected Pressure (mmHg)
711.0	22.0	2.55	0.4805	708.9

Given that the gas constant for air is $R = 0.287\,\mathrm{kJ\,kg^{-1}\,K^{-1}}$, the density of air is

$$
\begin{aligned}
PV &= mRT \\
\implies \rho &= \frac{P}{RT} \\
&= \frac{708.9/7.501}{0.287 \times (22 + 273.15)} \\
&= \boxed{1.12\,\mathrm{kg\,m^{-3}}}
\end{aligned}
$$

Question 2

The maximum velocity is given by

$$
v = \sqrt{\frac{2P_v}{\rho}} \tag{1}
$$

where P_v is the dynamic pressure and ρ is the density of air. The maximum pressure from Table 2 is $P_v = 20.5\,\mathrm{mmH_2O}$. From hydrostatic pressure,

$$
P = h\rho_w g = 1\,\mathrm{mm} \times 997\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}} = 9.78\,\mathrm{Pa} = 1\,\mathrm{mmH_2O}
$$

where ρ_w is the density of water [1]. Thus in (1),

$$
v = \sqrt{\frac{2 \times 20.5 \times 9.78}{1.12}} = \boxed{19.0\,\mathrm{m\,s^{-1}}}
$$

Question 3

Similarly, the velocity at a yaw angle of $-45°$ is given by (1) with $P_v = 14.5\,\mathrm{mmH_2O}$. The velocity is

$$v = \sqrt{\frac{2 \times 14.5 \times 9.78}{1.12}} = \boxed{13.6\,\mathrm{m\,s^{-1}}}$$

The error is

$$err = 19.0 - 13.6 = \boxed{5.4\,\mathrm{m\,s^{-1}}}$$
$$\%err = \frac{19.0 - 13.6}{19.0} \times 100\% = \boxed{28.4\%}$$

Question 4

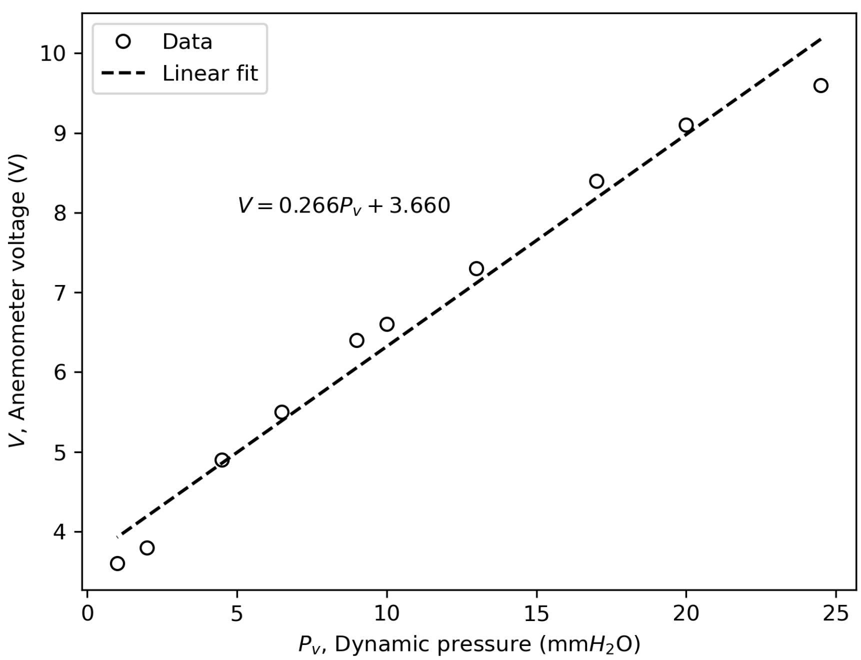

Figure 1: Anemometer voltage response to dynamic pressure

From the linear regression $y = ax + b$ in Figure 1, the sensitivity is

$$sensitivity = a = \boxed{0.27\,\mathrm{V\,m^{-1}\,s^{-1}}}$$

Question 5

The pressure of the orifice is given in the units of inches of alcohol. Converting using $\rho_{alcohol} = 790\,\mathrm{kg\,m^{-3}}$ [2].

$$P_v = 1\,\mathrm{inalcohol} \times \frac{1\,\mathrm{m}}{39.37\,\mathrm{in}} \times 790\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}} = 196.8\,\mathrm{Pa}$$

Table 2: Orifice calibration

K	A_c ($\mathrm{m^2}$)	ΔP (Pa)	Q ($\mathrm{m^3 s^{-1}}$)
0.60116	5.15×10^{-4}	1.66×10^{3}	0.0169
0.60144	5.15×10^{-4}	1.31×10^{3}	0.0150
0.60162	5.15×10^{-4}	1.08×10^{3}	0.0136
0.60186	5.15×10^{-4}	822	0.0119
0.60234	5.15×10^{-4}	616	0.0103
0.60290	5.15×10^{-4}	421	0.00852
0.60348	5.15×10^{-4}	313	0.00736
0.60458	5.15×10^{-4}	201	0.00590
0.60647	5.15×10^{-4}	108	0.00434
0.60950	5.15×10^{-4}	48.9	0.00294

Sample calculations for the first row of Table 2 are shown below.

$$P_v = 33.5 \times 196.8 = 6594.48\,\mathrm{Pa}$$

$$A_c = \frac{\pi}{4} \times (27.3/1000)^2 = 5.15 \times 10^{-4}\,\mathrm{m^2}$$

$$K = \frac{0.6011 - 0.6013}{35 - 30}(33.5 - 30) + 0.6013 = 0.60116$$

$$Q = KA_c\sqrt{\frac{2P_v}{\rho}} = 0.60116 \times 5.15 \times 10^{-4} \times \sqrt{\frac{2 \times 6594.48}{1.12}} = \boxed{0.0169\,\mathrm{m^3\,s^{-1}}}$$

Table 3: Venturi and nozzle discharge coefficients

Q ($\mathrm{m^3s^{-1}}$)	P_v (Pa)	β	A_c ($\mathrm{m^2}$)	C_d
0.0383	1662.7	0.4062	0.000721	0.960
0.0383	3208.6	0.3432	0.000515	0.974

Question 6

For the venturi meter,

$$\begin{aligned}
P_v &= 170 \times 9.78 = 1662.7\,\mathrm{Pa} \\
\beta &= \frac{d}{D} = \frac{30.3}{74.6} = 0.4062 \\
A_c &= \frac{\pi}{4} \times (30.3/1000)^2 = 0.000721\,\mathrm{m}^2 \\
C_d &= \frac{Q_{\text{actual}}\sqrt{1-\beta^4}}{A_c}\sqrt{\frac{\rho}{2P_v}} \\
&= \frac{0.0383 \times \sqrt{1-0.4062^4}}{0.000721}\sqrt{\frac{1.12}{2 \times 1662.7}} = \boxed{0.960}
\end{aligned}$$

For the venturi meter theoretical discharge coefficient, $\boxed{C_{d,\text{theory}} = 0.9965}$.

For the nozzle meter,

$$\begin{aligned}
P_v &= 16.3 \times 196.8 = 3208.6\,\mathrm{Pa} \\
\beta &= \frac{d}{D} = \frac{25.63}{74.6} = 0.3432 \\
A_c &= \frac{\pi}{4} \times (25.63/1000)^2 = 0.000515\,\mathrm{m}^2 \\
C_d &= \frac{Q_{\text{actual}}\sqrt{1-\beta^4}}{A_c}\sqrt{\frac{\rho}{2P_v}} \\
&= \frac{0.0383 \times \sqrt{1-0.3432^4}}{0.000515}\sqrt{\frac{1.12}{2 \times 3208.6}} = \boxed{0.974}
\end{aligned}$$

For the nozzle meter theoretical discharge coefficient,

$$V = \frac{Q}{A_c} = \frac{0.0383}{0.000515} = 8.76\,\mathrm{m\,s^{-1}}$$

$$\mathrm{Re}_D = \frac{\rho V D}{\mu} = \frac{1.12 \times 8.76 \times 74.6/1000}{1.8347 \times 10^{-5}} = 3.97 \times 10^5$$

$$C_{d,\text{theory}} = 0.9965 - 6.53\sqrt{\frac{0.3432}{3.97 \times 10^5}} = \boxed{0.977}$$

where μ is the dynamic viscosity of air at 23°C [3].

The nozzle is closer to the theory than the venturi meter.

Question 7

Error prop for air density, ρ,

$$\rho = P^1 R^{-1} T^{-1}$$

$$\begin{aligned}\frac{\delta\rho}{|\rho|} &= \sqrt{\left((1)\frac{\delta P}{|P|}\right)^2 + \left((-1)\frac{\delta T}{|T|}\right)^2} \\ &= \sqrt{(1 \times 0.01)^2 + ((-1) \times 0.01)^2} \\ &= \pm 0.0141\end{aligned}$$

Error prop for Q,

$$Q = C_d^1 A_c^1 (1-\beta^4)^{-1} \sqrt{2} P_v^{\frac{1}{2}} \rho^{-\frac{1}{2}}$$

$$\frac{\delta Q}{|Q|} = \sqrt{\left((1)\frac{\delta C_d}{|C_d|}\right)^2 + \left(\left(\frac{1}{2}\right)\frac{\delta P_v}{|P_v|}\right)^2 + \left(\left(-\frac{1}{2}\right)\frac{\delta\rho}{|\rho|}\right)^2}$$

For the orifice meter, $\delta C_d/|C_d| = 0.005$, $\delta P_v/|P_v| = 0.01$, and $\delta\rho/|\rho| = 0.0141$. Thus,

$$\begin{aligned}\frac{\delta Q}{|Q|} &= \sqrt{(0.005)^2 + \left(\frac{1}{2} \times 0.01\right)^2 + \left(-\frac{1}{2} \times 0.0141\right)^2} \\ &= \pm 0.01\end{aligned}$$

For the venturi meter, $\delta C_d/|C_d| = 0.02$, $\delta P_v/|P_v| = 0.01$, and $\delta\rho/|\rho| = 0.0141$. Thus,

$$\begin{aligned}\frac{\delta Q}{|Q|} &= \sqrt{(0.02)^2 + \left(\frac{1}{2} \times 0.01\right)^2 + \left(-\frac{1}{2} \times 0.0141\right)^2} \\ &= \pm 0.02\end{aligned}$$

For the nozzle meter, $\delta C_d/|C_d| = 0.01$, $\delta P_v/|P_v| = 0.01$, and $\delta\rho/|\rho| = 0.0141$. Thus,

$$\begin{aligned}\frac{\delta Q}{|Q|} &= \sqrt{(0.01)^2 + \left(\frac{1}{2}\times 0.01\right)^2 + \left(-\frac{1}{2}\times 0.0141\right)^2} \\ &= \pm 0.02\end{aligned}$$

For the orifice meter, the ρ term is the dominant term as

$$\begin{aligned} &\left(\left(-\frac{1}{2}\right)\frac{\delta\rho}{|\rho|}\right)^2 > \left(\left(\frac{1}{2}\right)\frac{\delta P_v}{|P_v|}\right)^2 = \left(\frac{\delta C_d}{|C_d|}\right)^2 \\ \iff &\left(\frac{1}{2}\times 0.0141\right)^2 > \left(\frac{1}{2}\times 0.01\right)^2 = (0.005)^2\end{aligned}$$

For the venturi, the dominant term is C_d as

$$\begin{aligned} &\left(\frac{\delta C_d}{|C_d|}\right)^2 > \left(\frac{1}{2}\times\frac{\delta\rho}{|\rho|}\right)^2 > \left(\frac{1}{2}\times\frac{\delta P_v}{|P_v|}\right)^2 \\ \iff &(0.02)^2 > \left(\frac{1}{2}\times 0.0141\right)^2 > \left(\frac{1}{2}\times 0.01\right)^2\end{aligned}$$

Lastly, for the nozzle, the dominant term is C_d as

$$\begin{aligned} &\left(\frac{\delta C_d}{|C_d|}\right)^2 > \left(\frac{1}{2}\times\frac{\delta\rho}{|\rho|}\right)^2 > \left(\frac{1}{2}\times\frac{\delta P_v}{|P_v|}\right)^2 \\ \iff &(0.01)^2 > \left(\frac{1}{2}\times 0.0141\right)^2 > \left(\frac{1}{2}\times 0.01\right)^2\end{aligned}$$

Question 8

From the linear regression $y = ax + b$ in Figure 2, the sensitivity is

$$\boxed{\mathit{sensitivity} = 1.7\times 10^4\,\mathrm{Hz\,s\,m^{-3}}}$$

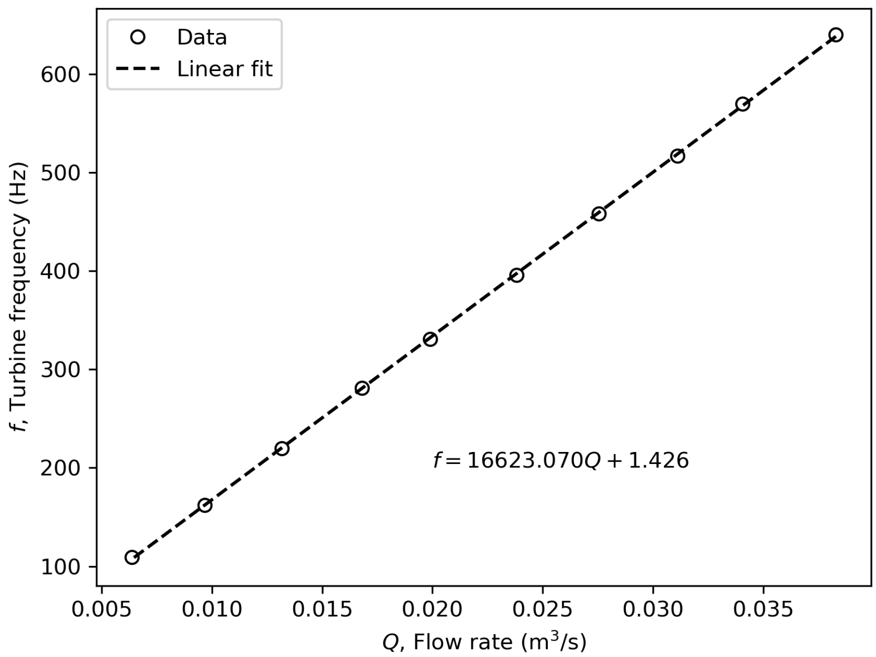

Figure 2: Turbine meter frequency response to orifice meter flow rate

References

[1] P. H. Bigg, "Density of water in SI units over the range 0-40°C," *British Journal of Applied Physics*, vol. 18, no. 4, p. 521, Apr. 1967. [Online]. Available: https://dx.doi.org/10.1088/0508-3443/18/4/315

[2] R. E. Ferner and J. Chambers, "Alcohol intake: measure for measure," *BMJ : British Medical Journal*, vol. 323, no. 7327, pp. 1439–1440, Dec. 2001. [Online]. Available: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1121897/

[3] W. N. Bond, "The viscosity of air," *Proceedings of the Physical Society*, vol. 49, no. 3, p. 205, May 1937. [Online]. Available: https://dx.doi.org/10.1088/0959-5309/49/3/301