MEC E 301
Lab 3: Displacement Transducers

by: Alex Diep

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CCID: abdiep
Student ID: 1664334
Section: D21

Question 1

(a)

The plot of displacement over voltage can be found in Fig. 1. The slope, intercept, and R^2 value of the linear regression were determined using `=LINEST()` in Excel. The results can be found in Table 1.

Table 1: Linear regression of displacement over voltage

Slope (mm/V)	Intercept (mm)	R^2
12.67	-1.50	1.00

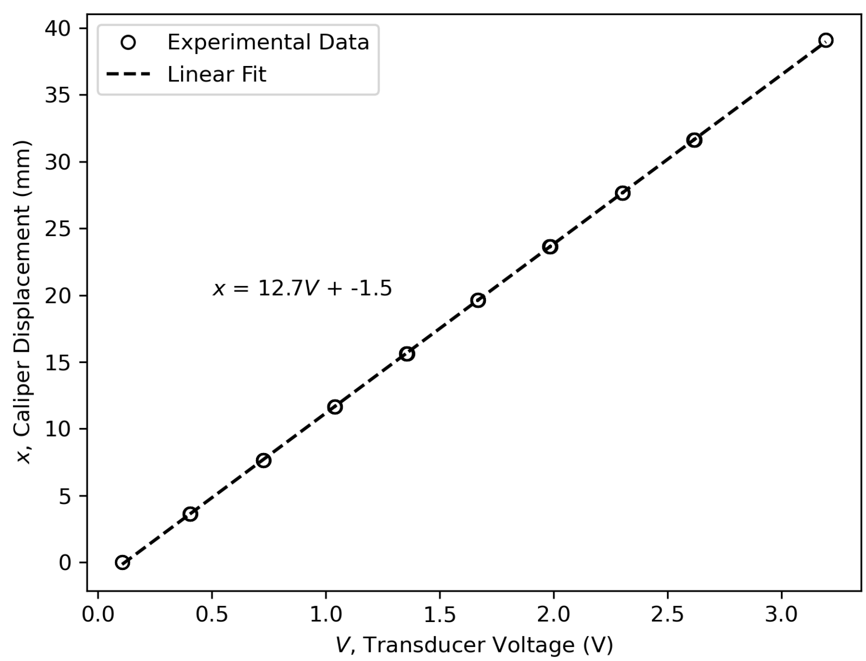

Figure 1: Displacement over voltage for the potentiometer

(b)

First, all the voltages were converted to displacements using the linear regression equation found in the previous section. The displacement table can be found in Appendix B in Table B.3.

The deviation table can be found in Table 2. The deviation curve can be found in Fig. 2.

Table 2: Deviation table of potentiometer

Caliper Reading (mm)	Up 1 (mm)	Down 1 (mm)	Up 2 (mm)	Down 2 (mm)	Up 3 (mm)
0.00		-0.155		-0.155	
3.64	3.608	3.608	3.608	3.646	3.608
7.64	7.689	7.689	7.689	7.689	7.727
11.64	11.693	11.693	11.693	11.655	11.655
15.64	15.698	15.660	15.660	15.660	15.660
19.64	19.652	19.652	19.614	19.614	19.652
23.64	23.656	23.618	23.656	23.618	23.656
27.64	27.699	27.661	27.661	27.661	27.699
31.64	31.665	31.627	31.665	31.627	31.665
39.10	38.977		38.977		38.977

A Figures

(a) Schematic of the system

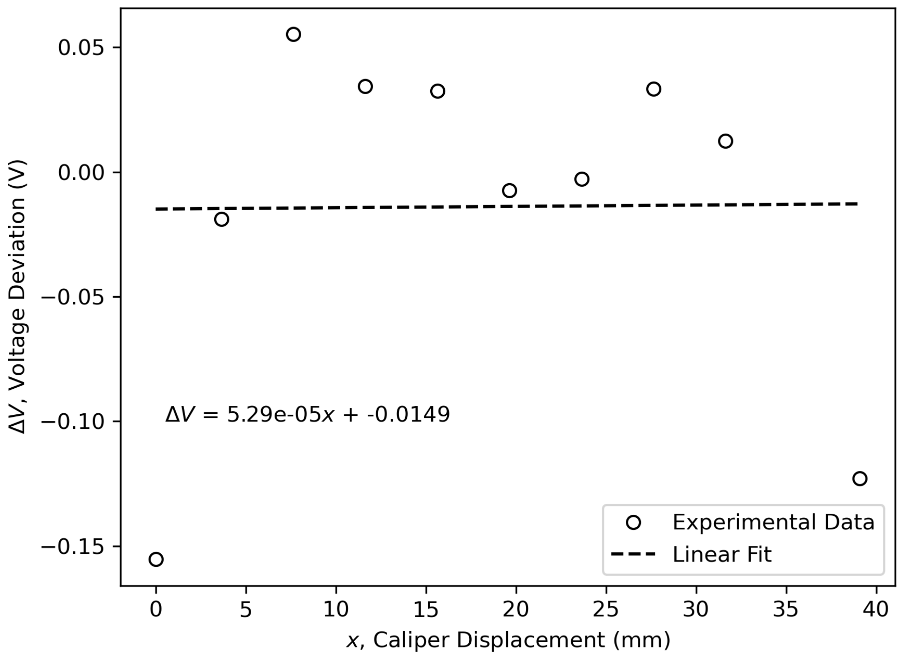

Figure 2: Deviation curve of potentiometer with respect to displacement (independent linearity)

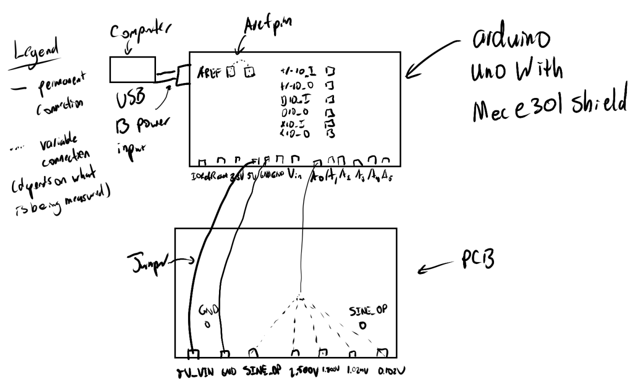

Figure A.3: Schematic of the system without a reference voltage used to measure the voltage across the PCB

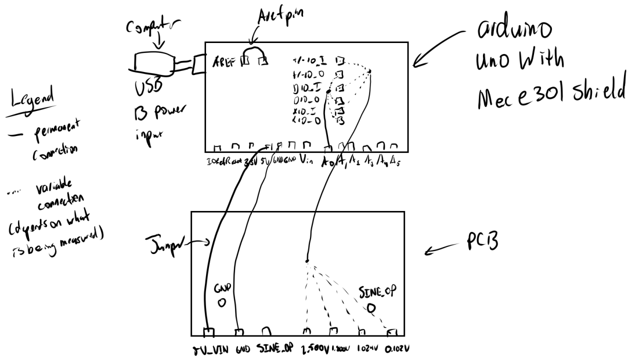

Figure A.4: Schematic of the system with a 3.3V reference voltage used to measure the voltage across the PCB

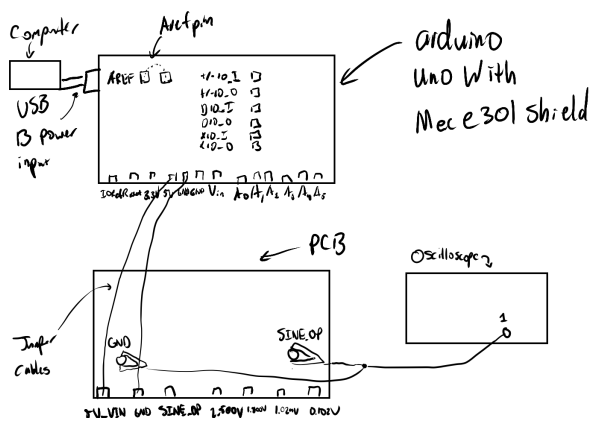

Figure A.5: Schematic of the system used to measure the voltage across the PCB with an oscilloscope

(b) Plots of Time-Varying Signals

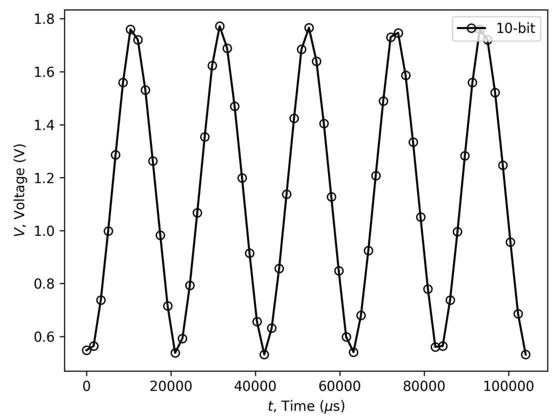

Figure A.6: Time varying signal of PCB 18 Measured with 10-bit ADC of the Arduino Uno

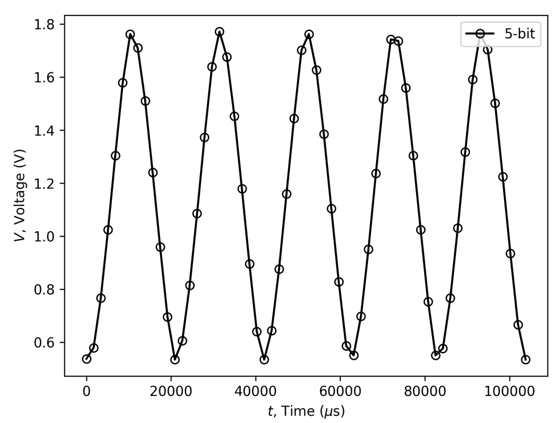

Figure A.7: Time varying signal of PCB 18 Measured with 5-bit ADC of the Arduino Uno

B Appendix: Displacement Table of Potentiometer

Table B.3: Displacement table of potentiometer

Caliper Reading (mm)	Up 1 (mm)	Down 1 (mm)	Up 2 (mm)	Down 2 (mm)	Up 3 (mm)
0.00		-0.155		-0.155	
3.64	3.608	3.608	3.608	3.646	3.608
7.64	7.689	7.689	7.689	7.689	7.727
11.64	11.693	11.693	11.693	11.655	11.655
15.64	15.698	15.660	15.660	15.660	15.660
19.64	19.652	19.652	19.614	19.614	19.652
23.64	23.656	23.618	23.656	23.618	23.656
27.64	27.699	27.661	27.661	27.661	27.699
31.64	31.665	31.627	31.665	31.627	31.665
39.10	38.977		38.977		38.977

C Appendix: Derivation of Resolution

The trade-off between resolution and range is that as the range increases, the resolution decreases. This is because the number of bits available to represent the range is fixed. For example, the Arduino Uno has 10 bits to represent the range of voltages. This means that the Arduino Uno can represent $2^{10} = 1024$ different voltages. If the range is 0 to 5V, then the resolution is 4.883mV/LSB. If the range is 0 to 10V, then the resolution is 9.766mV/LSB.

This can be shown mathematically. Then the resolution is given by:

$$\text{Resolution} = \frac{V_{\text{ru}} - V_{\text{rl}}}{2^n}$$

Let us assume the upper range V_{ru} and lower range V_{rl} are both multiplied by a factor of k.

$$\begin{aligned}\text{Resolution'} &= \frac{kV_{\text{ru}} - kV_{\text{rl}}}{2^n} \\ &= \frac{k(V_{\text{ru}} - V_{\text{rl}})}{2^n} \\ &= k\text{Resolution}\end{aligned}$$

D Appendix: Voltage Source Accuracy

The accuracy of the voltage source is 0.05%. A table deviation from the true value is shown below.

Table D.4: Deviation from True Value for Voltage Source with 0.05% Accuracy

Voltage (V)	Deviation from True Value (mV)
0.102	0.051
1.024	0.512
1.800	0.900
2.500	**1.250**

As one can see, the voltage source deviation is 10x smaller than the accuracy of the Arduino Uno. This means that the voltage source is a suitable reference voltage for the Arduino Uno.

Table D.5: Accuracy of Voltage Source Compared to Arduino Uno

Configuration	Accuracy (mV)
Voltage Source	1.250
5V Ref.	54.00
3.3V Ref.	17.00
3.3V Ref., 10x VD	83.00
3.3V Ref., [-10, 10]V	24.00
3.3V Ref., 10x Amp.	0.000

E Appendix: Circuit Component Accuracy

Below is a table of the additional error from the voltage divider, voltage scaler, and amplifier. The error is calculated by multiplying the voltage by 1%.

Table E.6: Additional Error from Voltage Divider, Voltage Scaler, and Amplifier

Voltage (V)	Error ($\pm$mV)
0.102	1.020
2.500	25.00

F Time-Varying Voltage Measurements

Below in Table F.7 are the nominal measurements for the time-varying voltage for the 10-bit and 5-bit ADCs.

Table F.7: Nominal Measurements of Time-Varying Voltage

	Mean Measurements of Time-Varying Voltage		
	Frequency (Hz)	Peak to Peak (V)	Mean Voltage (V)
10-bit	47.266	1.217	1.135
5-bit	47.372	1.2194	1.144
Oscilloscope	48.54	1.28	1.17

(a) Mean Sample Calculation

The mean voltage for the 10-bit and 5-bit ADCs was calculated using Excel using the `=AVERAGE()` function across 5 cycles, which was approximately 60 data points.

(b) Time-Varying Voltage Period Measurements

Below is a table of the time-varying period measurements for the 10-bit and 5-bit ADCs.

Table F.8: Time-Varying Period Measurements for 10-bit and 5-bit ADCs

	Period (μs)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit	21000	21132	21132	21124	21400
5-bit	20912	21104	21136	21096	21304

Obtaining the nominal value (mean), standard deviation, and T-distribution inverse was done through Excel. The mean was calculated using the `=AVERAGE()` function across 5 cycles, which was approximately 60 data points. The standard deviation was calculated using the `=STDEV.S()` function. The T-distribution inverse was calculated using the `=T.INV()` function, where $\alpha = 0.05$ and $n = 5$. The results are shown in Table F.9.

Sample calculations for the **10-bit** ADC are shown. To calculate the random uncertainty, the

Table F.9: Time-Varying Period Uncertainty Measurements for 10-bit and 5-bit ADCs

	Nominal Value (μs)	STDEV (μs)	T-Inv	P_x ($\pm\mu$s)
10-bit	21158	146.66	2.7764	182.10
5-bit	21110	139.42	2.7764	173.11

following equation was used:

$$\begin{aligned} P_x &= t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} \\ &= 2.7764\frac{146.66\,\mu\text{s}}{\sqrt{5}} \\ &= \boxed{182.10\,\mu\text{s}} \end{aligned}$$

(c) Time-Varying Voltage Frequency Calculations

Table F.10: Time-Varying Frequency Results for 10-bit and 5-bit ADCs

	Frequency (Hz) Nominal Value	Frequency ($\pm$Hz) Uncertainty
10-bit	47.264	0.40680
5-bit	47.370	0.38844

The equation relating frequency and period is shown below. Sample calculation for the 10-bit ADC is shown below.

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{21158\,\mu\text{s}} \\ &= \boxed{47.264\,\text{Hz}} \end{aligned}$$

Table F.11: Time-Varying Peak and Trough Voltage Measurements for 10-bit and 5-bit ADCs

	Peak and Trough Voltage (V)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit Trough	1.212	1.234	1.234	1.206	1.199
5-bit Trough	1.225	1.237	1.228	1.192	1.215
10-bit Peak	1.763	1.772	1.763	1.743	1.766
5-bit Peak	1.76	1.772	1.766	1.747	1.76

Utilizing error propagation, the random uncertainty in frequency is:

$$\begin{aligned}
P_{x'} &= \sqrt{\left(\frac{\partial f}{\partial T}P_x\right)^2} \\
&= \left|\frac{\partial f}{\partial T}P_x\right| \\
&= \left|-\frac{1}{T^2}P_x\right| \\
&= \frac{P_x}{T^2} \\
&= \frac{182.10\,\mu\text{s}}{21158\,\mu\text{s}^2} \\
&= \boxed{0.40680\,\text{Hz}}
\end{aligned}$$

(d) Time-Varying Voltage Peak to Peak Voltage Calculations

Below are the resulting peaks and troughs for the 10-bit and 5-bit ADCs for 5 cycles.

The nominal values for troughs and peaks were calculated using the `=AVERAGE()` function in Excel, the standard deviation was calculated using the `=STDEV.S()` function, and the T-distribution inverse was calculated using the `=T.INV()` function, where $\alpha = 0.05$ and $n = 5$. The systematic uncertainty was taken to be the accuracy of the 3.3V reference voltage for 10-bit ADC. and half the resolution for 5-bit ADC.

Table F.12: Time-Varying Peak and Trough Voltage Uncertainty Measurements for 10-bit and 5-bit ADCs

	Nominal Value (V)	STDEV (V)	T-Inv	P_x ($\pm$V)	B_x ($\pm$V)	U_x ($\pm$V)
10-bit Trough	0.544	0.01111	2.7764	0.01380	0.01700	0.0219
5-bit Trough	0.542	0.008307	2.7764	0.01031	0.05156	0.0526
10-bit Peak	1.761	0.009274	2.7764	0.01151	0.01700	0.0205
5-bit Peak	1.761	0.01092	2.7764	0.01356	0.05156	0.0533

The total uncertainty, U_x, was calculated using the root sum square (RSS) method. An example calculation for the 10-bit trough is shown below:

$$\begin{aligned} U_x &= \sqrt{P_x^2 + B_x^2} \\ &= \sqrt{0.01380^2 + 0.01700^2} \\ &= \boxed{0.0219\,\text{V}} \end{aligned}$$

Obtaining peak to peak voltage was done by subtracting the trough from the peak. The results are shown in Table F.13. A sample calculation for 10-bit ADC and equation for peak to peak voltage is shown below:

$$\begin{aligned} V_{\text{p-p}} &= V_{\text{ru}} - V_{\text{rl}} \\ &= 1.761\,\text{V} - 0.544\,\text{V} \\ &= \boxed{1.217\,\text{V}} \end{aligned}$$

Table F.13: Time-Varying Peak to Peak Voltage Results for 10-bit and 5-bit ADCs

	Nominal Value (V)	U_x ($\pm$V)
10-bit	1.217	0.0381
5-bit	1.219	0.0981

Error propagation was used to calculate the uncertainty in peak to peak voltage. The equation for peak to peak voltage is shown below with sample calculations for the 10-bit ADC.

$$\begin{aligned} U_{x'} &= \sqrt{\left(\frac{\partial V_{\text{p-p}}}{\partial V_{\text{ru}}} U_{x,ru}\right)^2 + \left(\frac{\partial V_{\text{p-p}}}{\partial V_{\text{rl}}} U_{x,rl}\right)^2} \\ &= \sqrt{1^2 0.0219^2 + (-1)^2 0.0205^2} \\ &= \boxed{0.0381\,\text{V}} \end{aligned}$$