MEC E 301
Lab 2: Dimensional Measurement Techniques

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Contents

1 Introduction

The primary objective of this lab is to investigate and familiarize elements of digital measurements such as word length (number of bits), quantization, resolution, conversion rates, sampling, accuracy, and signal conditioning through the use of an Arduino Uno.

2 Procedure

A schematic of the experimental setup is shown in Figure [insert figure] The precis provided by the MEC E 301 course was followed [?]. All measurements were taken from the serial monitor of the Arduino IDE. During calibration, different circuit components and reference voltages were used to measure the voltage output of the PCB.

3 Results and Discussion

3.1 Calibration of the Arduino Uno Results

The results can be found in A in Table 1.

3.2 Calibration of the Arduino Uno Discussion

3.2.1 Trade-off Between Resolution and Range

The trade-off between resolution and range is that as the range increases, the resolution decreases. This is because the number of bits available to represent the range is fixed. From Appendix C, if the range is multiplied by a factor of k, then the resolution is also multiplied by a factor of k, increasing resolution error.

3.2.2 Repeatability Compared to Accuracy

Repeatability is the maximum deviation between two measurements of the same reference quantity. Accuracy is the maximum deviation between the measured value and the true value. Repeatability is a measure of precision while accuracy is a measure of correctness.

3.2.3 Accuracy of Arduino Uno with 3.3V Reference Voltage

The accuracy for the 5V reference voltage is 54.00mV while the accuracy for the 3.3V reference voltage is 17.00mV. The accuracy of the Arduino Uno is improved using the 3.3V reference voltage. Many computers do not supply exactly 5V over the USB cable and the Arduino's A/D converter assumes the voltage is 5V. Supplying a known voltage to the AREF pin from the MEC E 301 Shield allows for more accurate measurements.

3.2.4 Accuracy of Voltage Sources

From Appendix D, if we use a conservative estimate for the accuracy of the voltage source, which occurs at 2.500V, then the accuracy is 1.250mV. ANSI/ISA 51.1 states that the accuracy of the standard can be ignored should the accuracy be one tenth of the instrument tested.

Comparing this to the accuracy of the Arduino Uno, in Table 1, the accuracy of the voltage source is sufficient for all reference values except the setup with the 3.3V reference voltage and 10x amplifier, which has an experimental accuracy of 0.000mV.

3.2.5 Accuracy of Arduino Uno Compared to Manufacturer's Accuracy

Referring to Appendix B, the experimental accuracy was less than the manufacturer's accuracy for all configurations except the 3.3V reference voltage with a 10x amplifier and the 3.3V reference voltage with a [-10, 10]V range.

The 5V reference voltage has an accuracy of 54.00mV while the manufacturer's accuracy is 9.766mV. This may be attributed to the variable reference voltage supplied by the computer

3.2.6 Additional Error from Voltage Divider, Voltage Scaler, and Amplifier

The calculation is shown in Appendix E. The error at 0.102V is 1.020mV and the error at 2.500V is 25.00mV. The error at 0.102 is not significant while the error at 2.500V is significant by ANSI/ISA 51.1.

3.3 Time Varying Voltage Discussion

A Appendix: Arduino Uno Calibration Results

Table 1: Range, Resolution, Repeatability, Accuracy, and Manufacturer's Accuracy for Various Ranges of the Arduino Uno

Arduino Config.	Range (V)	Resolution (mV/LSB)	Repeatability (mV)	Acc. (mV)	Manuf. Acc. (mV)
5V Ref.	0.000 - 5.000	4.883	44.00	54.00	9.766
3.3V Ref.	0.000 - 3.300	3.223	4.000	17.00	6.445
3.3V Ref., 10x VDiv	0.00 - 33.00	32.23	32.00	83.00	64.45
3.3V Ref., [-10, 10]V	-10.00 - 10.00	19.53	0.000	24.00	39.06
3.3V Ref., 10x Amp.	0.000 - 0.330	0.3223	0.000	0.000	0.6445

B Appendix: Arduino Uno Accuracy

Table 2 summarizes the range, resolution, repeatability, accuracy, and manufacturer's accuracy for various ranges of the Arduino Uno. Sample calculations for the 5V reference voltage are shown below. Note, the manufacturer's accuracy is ± 2 LSBs.

$$\begin{aligned}
\text{Resolution} &= \frac{V_{\text{ru}} - V_{\text{rl}}}{2^n} \\
&= \frac{5.000 - 0.000}{2^{10}} \\
&= \boxed{4.883\,\text{mV/LSB}} \\
\text{Repeatability} &= \max(\text{Max Deviation}) \\
&= \max(\langle 5.00, 0.00, 9.00, 44.00 \rangle) \\
&= \boxed{44.00\,\text{mV}} \\
\text{Accuracy} &= \max(\text{Deviation}) \\
&= \max\left(\begin{bmatrix} -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.004 & -0.009 \\ 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 \\ 0.016 & 0.021 & 0.012 & 0.016 & 0.012 & 0.012 & 0.016 & 0.016 & 0.012 & 0.016 \\ 0.02 & 0.024 & 0.02 & 0.024 & 0.01 & 0.02 & \mathbf{0.054} & 0.024 & 0.034 & 0.02 \end{bmatrix}\right) \\
&= \boxed{54.00\,\text{mV}} \\
\text{Manuf. Acc.} &= 2\,\text{LSB} \times \text{Resolution} \\
&= \boxed{9.766\,\text{mV}}
\end{aligned}$$

For significant figures, since the range is given to 3 decimal places, the resolution is given to 3 decimal places, more often than not, the number of significant figures is 4. This is because addition and subtraction do not take into account the number of significant figures but rather the number of decimal places.

Table 2: Range, Resolution, Repeatability, Accuracy, and Manufacturer's Accuracy for Various Ranges of the Arduino Uno

Arduino Config.	Range (V)	Resolution (mV/LSB)	Repeatability (mV)	Acc. (mV)	Manuf. Acc. (mV)
5V Ref.	0.000 - 5.000	4.883	44.00	54.00	9.766
3.3V Ref.	0.000 - 3.300	3.223	4.000	17.00	6.445
3.3V Ref., 10x VDiv	0.00 - 33.00	32.23	32.00	83.00	64.45
3.3V Ref., [-10, 10]V	-10.00 - 10.00	19.53	0.000	24.00	39.06
3.3V Ref., 10x Amp.	0.000 - 0.330	0.3223	0.000	0.000	0.6445

C Appendix: Derivation of Resolution

The trade-off between resolution and range is that as the range increases, the resolution decreases. This is because the number of bits available to represent the range is fixed. For example, the Arduino Uno has 10 bits to represent the range of voltages. This means that the Arduino Uno can represent $2^{10} = 1024$ different voltages. If the range is 0 to 5V, then the resolution is 4.883mV/LSB. If the range is 0 to 10V, then the resolution is 9.766mV/LSB. This can be shown mathematically. Then the resolution is given by:

$$\text{Resolution} = \frac{V_{\text{ru}} - V_{\text{rl}}}{2^n}$$

Let us assume the upper range V_{ru} and lower range V_{rl} are both multiplied by a factor of k.

$$\begin{aligned}\text{Resolution'} &= \frac{kV_{\text{ru}} - kV_{\text{rl}}}{2^n} \\ &= \frac{k(V_{\text{ru}} - V_{\text{rl}})}{2^n} \\ &= k\text{Resolution}\end{aligned}$$

D Appendix: Voltage Source Accuracy

The accuracy of the voltage source is 0.05%. A table deviation from the true value is shown below.

Table 3: Deviation from True Value for Voltage Source with 0.05% Accuracy

Voltage (V)	Deviation from True Value (mV)
0.102	0.051
1.024	0.512
1.800	0.900
2.500	**1.250**

As one can see, the voltage source deviation is 10x smaller than the accuracy of the Arduino Uno. This means that the voltage source is a suitable reference voltage for the Arduino Uno.

Table 4: Accuracy of Voltage Source Compared to Arduino Uno

Configuration	Accuracy (mV)
Voltage Source	1.250
5V Ref.	54.00
3.3V Ref.	17.00
3.3V Ref., 10x VD	83.00
3.3V Ref., [-10, 10]V	24.00
3.3V Ref., 10x Amp.	0.000

E Appendix: Circuit Component Accuracy

Below is a table of the additional error from the voltage divider, voltage scaler, and amplifier. The error is calculated by multiplying the voltage by 1%.

Table 5: Additional Error from Voltage Divider, Voltage Scaler, and Amplifier

Voltage (V)	Error (mV)
0.102	1.020
2.500	25.00

F Time-Varying Voltage Measurements

F.1 Time-Varying Voltage Period Measurements

Below is a table of the time-varying period measurements for the 10-bit and 5-bit ADCs.

Table 6: Time-Varying Period Measurements for 10-bit and 5-bit ADCs

	Period (μs)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit	21000	21132	21132	21124	21400
5-bit	20912	21104	21136	21096	21304

Obtaining the nominal value (mean), standard deviation, and T-distribution inverse was done through Excel. The mean was calculated using the `=AVERAGE()` function across 5 cycles, which was approximately 60 data points. The standard deviation was calculated using the `=STDEV.S()` function. The T-distribution inverse was calculated using the `=T.INV()` function, where $\alpha = 0.05$ and $n = 5$. The results are shown in Table 7.

Table 7: Time-Varying Period Uncertainty Measurements for 10-bit and 5-bit ADCs

	Nominal Value (μs)	STDEV (μs)	T-Inv	P_x (μs)
10-bit	21158	146.66	2.7764	182.10
5-bit	21110	139.42	2.7764	173.11

Sample calculations for the **10-bit** ADC are shown. To calculate the random uncertainty, the following equation was used:

$$\begin{aligned} P_x &= t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} \\ &= 2.7764\frac{146.66\,\mu\text{s}}{\sqrt{5}} \\ &= \boxed{182.10\,\mu\text{s}} \end{aligned}$$

Table 8: Time-Varying Frequency Results for 10-bit and 5-bit ADCs

	Frequency (Hz)	
	Nominal Value	Uncertainty
10-bit	47.264	0.40680
5-bit	47.370	0.38844

F.2 Time-Varying Voltage Frequency Calculations

The equation relating frequency and period is shown below. Sample calculation for the 10-bit ADC is shown below.

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{21158\,\mu\text{s}} \\ &= \boxed{47.264\,\text{Hz}} \end{aligned}$$

Utilizing error propagation, the random uncertainty in frequency is:

$$\begin{aligned} P_{x'} &= \sqrt{\left(\frac{\partial f}{\partial T}P_x\right)^2} \\ &= \left|\frac{\partial f}{\partial T}P_x\right| \\ &= \left|-\frac{1}{T^2}P_x\right| \\ &= \frac{P_x}{T^2} \\ &= \frac{182.10\,\mu\text{s}}{21158\,\mu\text{s}^2} \\ &= \boxed{0.40680\,\text{Hz}} \end{aligned}$$

F.3 Time-Varying Voltage Peak to Peak Voltage Calculations

Below are the resulting peaks and troughs for the 10-bit and 5-bit ADCs for 5 cycles.
The nominal values for troughs and peaks were calculated using the `=AVERAGE()` function in Excel, the standard deviation was calculated using the `=STDEV.S()` function, and the T-distribution inverse was calculated using the `=T.INV()` function, where $\alpha = 0.05$ and $n = 5$. The systematic uncertainty was taken to be the accuracy of the 3.3V reference voltage for 10-bit ADC. and half the resolution for 5-bit ADC.

Table 9: Time-Varying Peak and Trough Voltage Measurements for 10-bit and 5-bit ADCs

	Peak and Trough Voltage (V)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit Trough	1.212	1.234	1.234	1.206	1.199
5-bit Trough	1.225	1.237	1.228	1.192	1.215
10-bit Peak	1.763	1.772	1.763	1.743	1.766
5-bit Peak	1.76	1.772	1.766	1.747	1.76

Table 10: Time-Varying Peak and Trough Voltage Uncertainty Measurements for 10-bit and 5-bit ADCs

	Nominal Value (V)	STDEV (V)	T-Inv	P_x (V)	B_x (V)	U_x (V)
10-bit Trough	0.544	0.01111	2.7764	0.01380	0.01700	0.0219
5-bit Trough	0.542	0.008307	2.7764	0.01031	0.05156	0.0526
10-bit Peak	1.761	0.009274	2.7764	0.01151	0.01700	0.0205
5-bit Peak	1.761	0.01092	2.7764	0.01356	0.05156	0.0533

The total uncertainty, U_x, was calculated using the root sum square (RSS) method. An example calculation for the 10-bit trough is shown below:

$$\begin{aligned} U_x &= \sqrt{P_x^2 + B_x^2} \\ &= \sqrt{0.01380^2 + 0.01700^2} \\ &= \boxed{0.0219\,\mathrm{V}} \end{aligned}$$

Obtaining peak to peak voltage was done by subtracting the trough from the peak. The results are shown in Table 11. A sample calculation for 10-bit ADC and equation for peak to peak voltage is shown below:

$$\begin{aligned} V_{\text{p-p}} &= V_{\text{ru}} - V_{\text{rl}} \\ &= 1.761\,\mathrm{V} - 0.544\,\mathrm{V} \\ &= \boxed{1.217\,\mathrm{V}} \end{aligned}$$

Table 11: Time-Varying Peak to Peak Voltage Results for 10-bit and 5-bit ADCs

	Nominal Value (V)	U_x (V)
10-bit	1.217	0.0381
5-bit	1.219	0.0981

Error propagation was used to calculate the uncertainty in peak to peak voltage. The equation for peak to peak voltage is shown below with sample calculations for the 10-bit ADC.

$$
\begin{aligned}
U_{x'} &= \sqrt{\left(\frac{\partial V_{\text{p-p}}}{\partial V_{\text{ru}}} U_{x,ru}\right)^2 + \left(\frac{\partial V_{\text{p-p}}}{\partial V_{\text{rl}}} U_{x,rl}\right)^2} \\
&= \sqrt{1^2 0.0219^2 + (-1)^2 0.0205^2} \\
&= \boxed{0.0381\,\text{V}}
\end{aligned}
$$