MEC E 301
Lab 4: Displacement Transducers

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Question 1

(a) Quarter Bridge

Table 1: Quarter bridge results for various mass loads

Mass (kg)	E_{nw} (V)	E_w (V)	ΔE_w (V)	ΔE_0 (V)	ϵ
0.050	0.870	0.886	0.016	5.33E-06	3.09E-06
0.100	0.864	0.912	0.048	1.60E-05	9.28E-06
0.200	0.870	0.941	0.071	2.37E-05	1.37E-05
0.500	0.870	1.073	0.203	6.77E-05	3.92E-05
1.000	0.851	1.270	0.419	1.40E-04	8.10E-05
1.000	0.844	1.257	0.413	1.38E-04	7.98E-05
1.000	0.864	1.276	0.412	1.37E-04	7.96E-05
1.000	0.773	1.192	0.419	1.40E-04	8.10E-05
1.000	0.773	1.196	0.423	1.41E-04	8.18E-05
1.000	0.777	1.202	0.425	1.42E-04	8.22E-05

Sample calculation for E_0 of 0.05 kg load:

$$\begin{aligned} E_0 &= \frac{E_\text{w} - E_\text{nw}}{G} \\ &= \frac{0.886 - 0.870}{3000} \\ &= \boxed{5.33 \times 10^{-6}\,\text{V}} \end{aligned}$$

Sample calculation for ϵ of 0.05 kg load:

$$E_0 = \frac{1}{4} E_\text{in} F_\text{g} (\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4) \tag{1}$$

From Fig. 1, we can see that $\epsilon_2 = \epsilon_3 = \epsilon_4 = 0$ for the quarter bridge configuration. Therefore, the

Table 2: Half bridge results for various mass loads

Mass (kg)	E_{nw} (V)	E_w (V)	ΔE_w (V)	ΔE_0 (V)	ϵ
0.050	0.400	0.442	0.042	1.40E-05	4.06E-06
0.100	0.396	0.483	0.087	2.90E-05	8.41E-06
0.200	0.403	0.567	0.164	5.47E-05	1.59E-05
0.500	0.400	0.819	0.419	1.40E-04	4.05E-05
1.000	0.396	1.234	0.838	2.79E-04	8.10E-05
1.000	0.393	1.234	0.841	2.80E-04	8.13E-05
1.000	0.396	1.231	0.835	2.78E-04	8.07E-05
1.000	0.390	1.237	0.847	2.82E-04	8.19E-05
1.000	0.390	1.234	0.844	2.81E-04	8.16E-05
1.000	0.396	1.237	0.841	2.80E-04	8.13E-05

equation simplifies to:

$$
\begin{aligned}
E_0 &= \frac{1}{4} E_{\text{in}} F_{\text{g}}(\epsilon_1) \\
\implies \epsilon &= \frac{4E_0}{E_{\text{in}} F_{\text{g}}} \\
\epsilon &= \frac{4 \times 5.33E-06}{3.30 \times 2.09} \\
&= \boxed{qty3.09E-06}
\end{aligned}
$$

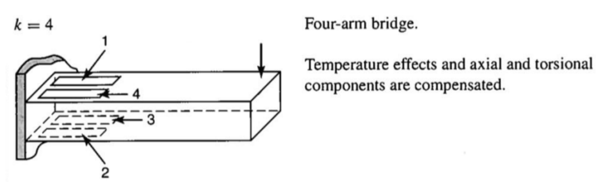

Figure 1: Strain gage configuration

(b) Half Bridge

Sample calculations for E_0 are identical to those for the quarter bridge configuration.

Sample calculation for ϵ of 0.05 load is similar to that of the quarter bridge configuration. From

Eq. (1) and Fig. 1, we can see that $\epsilon_3 = \epsilon_4 = 0$ for the half bridge configuration. Also, $\epsilon_1 - \epsilon_2 = 2\epsilon$. Therefore, the equation simplifies to:

$$\begin{aligned} E_0 &= \frac{1}{2} E_{\text{in}} F_{\text{g}}(\epsilon) \\ \implies \epsilon &= \frac{2E_0}{E_{\text{in}} F_{\text{g}}} \\ \epsilon &= \frac{2 \times 1.40E-05}{3.30 \times 2.09} \\ &= \boxed{qty4.06E-06} \end{aligned}$$

(c) Full Bridge

Table 3: Full bridge results for various mass loads

Mass (kg)	E_{nw} (V)	E_w (V)	ΔE_w (V)	ΔE_0 (V)	ϵ
0.050	0.274	0.358	0.084	2.80E-05	4.06E-06
0.100	0.271	0.435	0.164	5.47E-05	7.93E-06
0.200	0.271	0.609	0.338	1.13E-04	1.63E-05
0.500	0.271	1.115	0.844	2.81E-04	4.08E-05
1.000	0.271	1.969	1.698	5.66E-04	8.21E-05
1.000	0.274	1.972	1.698	5.66E-04	8.21E-05
1.000	0.274	1.966	1.692	5.64E-04	8.18E-05
1.000	0.271	1.966	1.695	5.65E-04	8.19E-05
1.000	0.274	1.969	1.695	5.65E-04	8.19E-05
1.000	0.277	1.969	1.692	5.64E-04	8.18E-05

Sample calculations for E_0 are identical to those for the quarter bridge configuration.

Sample calculation for ϵ of 0.05 load is similar to that of the quarter bridge configuration. From

Eq. (1) and Fig. 1, we can see that $\epsilon_1 - \epsilon_2 - \epsilon_3 - \epsilon_4 = 4\epsilon$. Therefore, the equation simplifies to:

$$\begin{aligned} E_0 &= E_{\text{in}} F_{\text{g}}(\epsilon) \\ \implies \epsilon &= \frac{E_0}{E_{\text{in}} F_{\text{g}}} \\ \epsilon &= \frac{1.40E-05}{3.30 \times 2.09} \\ &= \boxed{4.06 \times 10^{-6}} \end{aligned}$$

Question 2

(a) Theoretical Strain

The theoretical strains at various loads are given in Table 4.

Table 4: Theoretical strain value at a given loading

Mass (kg)	M (Nm)	σ (Pa)	ϵ (m/m)
0.050	0.099	2.9E+05	4.2E-06
0.100	0.199	5.75E+05	8.35E-06
0.200	0.398	1.15E+06	1.67E-05
0.500	0.994	2.88E+06	4.17E-05
1.000	1.99	5.75E+06	8.35E-05

(b) Sample Calculation

The beam measurements are given in Table 5.

The moment of inertia of the beam is calculated as follows:

$$\begin{aligned} I &= \frac{bh^3}{12} \\ &= \frac{(12.8116 \times 10^{-3})(12.7256 \times 10^{-3})^3}{12} \\ &= 2.20017 \times 10^{-9}\ \text{m}^4 \end{aligned}$$

Table 5: Beam dimension measurements

Dimension	Measurement Number 1 (mm)	2 (mm)	3 (mm)	4 (mm)	5 (mm)	Nominal Mean (mm)
L	203.5	202.5	202.5	202.5	202.5	202.7
b	12.810	12.813	12.811	12.812	12.812	12.812
h	12.726	12.729	12.724	12.724	12.725	12.726

For the 1 kg load, the theoretical strain is calculated as follows:

$$
\begin{aligned}
M &= mgl \\
&= (1)(9.81)(0.2027) \\
&= 1.99\,\mathrm{Nm} \\
\sigma &= \frac{Mh}{2I} \\
&= \frac{(1.99)(12.726 \times 10^{-3})}{2(2.20017 \times 10^{-9})} \\
&= 5.75\,\mathrm{MPa} \\
\epsilon &= \frac{\sigma}{E} \\
&= \frac{5.75 \times 10^{6}}{68.9 \times 10^{9}} \\
&= \boxed{8.35 \times 10^{-5}\,\mathrm{m/m}}
\end{aligned}
$$

Question 3

(a) Plot of Bridge Output Voltage vs Theoretical Strain

The plot of bridge output voltage vs theoretical strain is shown in Figure 2.

(b) Sensitivity of Each Bridge

The sensitivity of each bridge is given in Table 6.

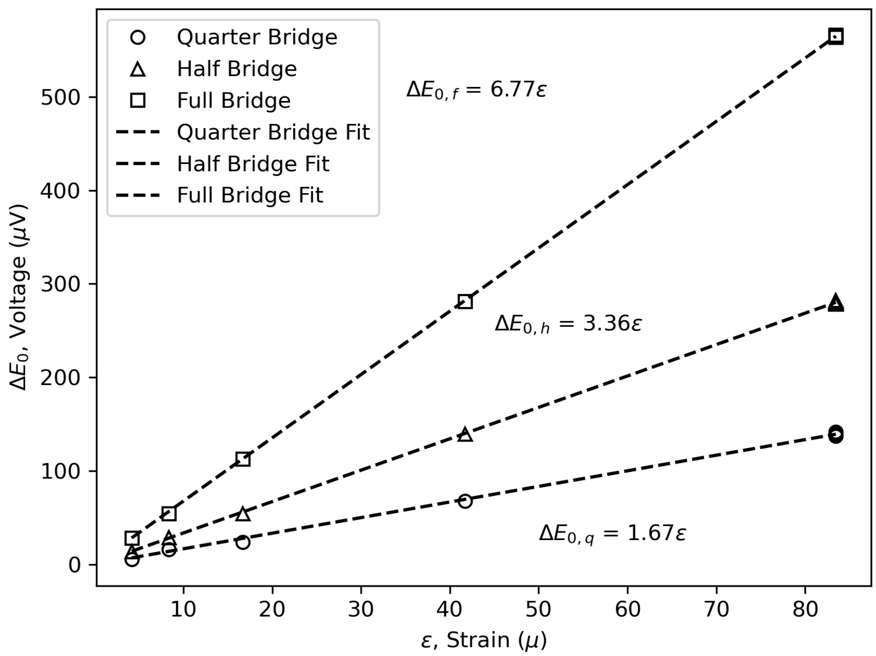

Figure 2: Plot of bridge output voltage vs theoretical strain

Table 6: Sensitivity of each bridge

Bridge	Sensitivity (V)
Quarter Bridge	1.67
Half Bridge	3.36
Full Bridge	6.77

(c) Discussion

The full bridge has the highest sensitivity because it has the most strain gauges. For example, a displacement on the top will increase the measurement of the strain gauges ϵ_1 and ϵ_4. A displacement on the bottom will increase the measurement of the strain gauges ϵ_2 and ϵ_3. This results in $\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4 = 4\epsilon$.

The least sensitive bridge is the quarter bridge because it only has one strain gauge. A displacement on the top will increase the measurement of the strain gauge ϵ_1. None of the other strain gauges will be affected. This results in $\epsilon_1 = \epsilon$.z

Question 4

Table 7: Uncertainty in measurements of b, h, and L

Dimension	STDEV (mm)	T_{INV}	P_x (mm)	B_x (mm)	U_x (mm)
L	0.4	2.7764	0.5	1	1
b	0.001	2.7764	0.001	0.003	0.003
h	0.002	2.7764	0.002	0.003	0.004

A sample calculation of the uncertainty in L is shown below. The uncertainty in b and h are calculated in a similar manner.

Using Excel's `STDEV.S` function applied to Table 5, `T.INV` function with $\alpha = 0.05$ and $n = 5$, the precision uncertainty P_x is calculated as:

$$\begin{aligned} P_x &= \frac{S_x}{\sqrt{n}} \times t_{\alpha/2,n-1} \\ &= \frac{0.4}{\sqrt{5}} \times 2.7764 \\ &= 0.5\,\text{mm} \end{aligned}$$

The bias uncertainty B_x was given to be 1 mm. The total uncertainty U_x is calculated as:

$$\begin{aligned} U_x &= \sqrt{P_x^2 + B_x^2} \\ &= \sqrt{(0.5)^2 + (1)^2} \\ &= \boxed{1\,\text{mm}} \end{aligned}$$

Question 5

The relevant equations are:

$$M = mgL$$
$$\sigma = \frac{Mh}{2I}$$
$$I = \frac{bh^3}{12}$$
$$\epsilon = \frac{\sigma}{E}$$

Combining these equations, we get:

$$\epsilon = \frac{6mgL}{bh^2E}$$

Computing the partials,

$$\frac{\partial \epsilon}{\partial m} = \frac{6gL}{bh^2E}$$
$$\frac{\partial \epsilon}{\partial L} = \frac{6mg}{bh^2E}$$
$$\frac{\partial \epsilon}{\partial b} = -\frac{6mgL}{b^2h^2E}$$
$$\frac{\partial \epsilon}{\partial h} = -\frac{12mgL}{bh^3E}$$
$$\frac{\partial \epsilon}{\partial E} = -\frac{6mgL}{bh^2E^2}$$

Evaluating the partials times the uncertainty in each variable,

$$\frac{\partial \epsilon}{\partial m}\delta m = 4.17 \times 10^{-7}$$
$$\frac{\partial \epsilon}{\partial L}\delta L = 4.60 \times 10^{-7}$$
$$\frac{\partial \epsilon}{\partial b}\delta b = 2.12 \times 10^{-8}$$
$$\frac{\partial \epsilon}{\partial h}\delta h = 4.96 \times 10^{-8}$$
$$\frac{\partial \epsilon}{\partial E}\delta E = 1.21 \times 10^{-6}$$

The total uncertainty is the RSS of the partials times the uncertainty in each variable:

$$\begin{aligned}\delta\epsilon &= \sqrt{(4.17 \times 10^{-7})^2 + (4.60 \times 10^{-7})^2 + (2.12 \times 10^{-8})^2 + (4.96 \times 10^{-8})^2 + (1.21 \times 10^{-6})^2} \\ &= \boxed{\pm 1.36 \times 10^{-6}}\end{aligned}$$

The calculations were handled by Matlab, and the code is shown below:

```
clc; clear; close all;
syms m g L b h E

epsilon = 6*m*g*L/(b*h^2*E)

delEdelm = diff(epsilon, m)
delEdelL = diff(epsilon, L)
delEdelb = diff(epsilon, b)
delEdelh = diff(epsilon, h)
delEdelE = diff(epsilon, E)

delEdelm_delta_m = double(subs(delEdelm, [m g L b h E], [1 9.81 0.2027
   0.012811 0.0127256 68.9*10^9]) * 0.005)
delEdelL_delta_L = double(subs(delEdelL, [m g L b h E], [1 9.81 0.2027
   0.012811 0.0127256 68.9*10^9]) * 1.11654678249935/1000)
delEdelb_delta_b = double(subs(delEdelb, [m g L b h E], [1 9.81 0.2027
   0.012811 0.0127256 68.9*10^9]) * 0.00325628602303491/1000)
delEdelh_delta_h = double(subs(delEdelh, [m g L b h E], [1 9.81 0.2027
```

```
    0.012811 0.0127256 68.9*10^9]) * 0.00378200336150775/1000)
delEdelE_delta_E = double(subs(delEdelE, [m g L b h E], [1 9.81 0.2027
    0.012811 0.0127256 68.9*10^9]) * 1*10^9)

delE_delta_E = sqrt(delEdelm_delta_m^2 + delEdelL_delta_L^2 +
    delEdelb_delta_b^2 + delEdelh_delta_h^2 + delEdelE_delta_E^2)
```

Question 6

(a) Uncertainty in ΔE_w

Table 8: Uncertainty in ΔE_w

Dimension	STDEV (V)	T-INV	P_x (V)	B_x (V)	U_x (V)
ΔE_w	2.683E-03	2.5706	2.8E-03	3.223E-03	4.3E-03

Again, standard deviation is calculated using Excel's `STDEV.S` function. T-INV is calculated using Excel's `T.INV` function with $\alpha = 0.05$ and $n = 6$. The precision uncertainty P_x is calculated as:

$$\begin{aligned} P_x &= \frac{S_x}{\sqrt{n}} \times t_{\alpha/2,n-1} \\ &= \frac{2.683E-03}{\sqrt{6}} \times 2.5706 \\ &= 2.8 \times 10^{-3}\,\mathrm{V} \end{aligned}$$

The bias uncertainty B_x is the resolution,

$$\begin{aligned} B_x &= \frac{3.3}{2^{10}} \\ &= 3.223 \times 10^{-3}\,\mathrm{V} \end{aligned}$$

The total uncertainty U_x is calculated as:

$$\begin{aligned} U_x &= \sqrt{P_x^2 + B_x^2} \\ &= \sqrt{(2.8 \times 10^{-3})^2 + (3.223 \times 10^{-3})^2} \\ &= \boxed{4.3 \times 10^{-3}\,\mathrm{V}} \end{aligned}$$

(b) Uncertainty in ΔE_0

The equation for ΔE_0 is given by:

$$\begin{aligned}\Delta E_0 &= \frac{\Delta E_w}{G} \\ &= \frac{4.3 \times 10^{-3}}{2.09} \\ &= \boxed{2.06 \times 10^{-3}\,\mathrm{V}}\end{aligned}$$

Calculating the partials,

$$\begin{aligned}\frac{\partial \Delta E_0}{\partial \Delta E_w} &= \frac{1}{G} \\ \frac{\partial \Delta E_0}{\partial G} &= -\frac{\Delta E_w}{G^2}\end{aligned}$$

Calculating the partials times the uncertainty (the nominal value for ΔE_w is 1.160 V),

$$\begin{aligned}\frac{\partial \Delta E_0}{\partial \Delta E_w}\delta \Delta E_w &= \frac{1}{3000} \times 4.3 \times 10^{-3} \\ &= 1.4 \times 10^{-6} \\ \frac{\partial \Delta E_0}{\partial G}\delta G &= -\frac{1.160}{3000^2} \times 1 \times 3000 \\ &= -3.9 \times 10^{-6}\end{aligned}$$

The total uncertainty is the RSS of the partials times the uncertainty in each variable:

$$\begin{aligned}\delta \Delta E_0 &= \sqrt{(1.4 \times 10^{-6})^2 + (-3.9 \times 10^{-6})^2} \\ &= \boxed{3.6 \times 10^{-5}\,\mathrm{V}}\end{aligned}$$

Question 7

The equation for strain for the full bridge is:

$$\epsilon = \frac{\Delta E_0}{F_g V_{\text{in}}}$$

The partials are:

$$\frac{\partial \epsilon}{\partial \Delta E_0} = \frac{1}{F_g V_{\text{in}}}$$
$$\frac{\partial \epsilon}{\partial F_g} = -\frac{\Delta E_0}{F_g^2 V_{\text{in}}}$$
$$\frac{\partial \epsilon}{\partial V_{\text{in}}} = -\frac{\Delta E_0}{F_g V_{\text{in}}^2}$$

Evaluating the partials times the uncertainty of each variable is shown in Table 9. For example,

$$\begin{aligned}\frac{\partial \epsilon}{\partial \Delta E_0}\delta \Delta E_0 &= \frac{1}{2.09 \times 3.3} \times 5.65 \times 10^{-4} \\ &= 5.21 \times 10^{-6}\end{aligned}$$

Table 9: Partials times uncertainty for variables in ϵ

Variable	Nominal Value	Uncertainty	$\frac{\partial \epsilon}{\partial x_i}\delta x_i$
ΔE_0	5.65E-04	3.59E-05	5.21E-06
F_g	2.09	0.02	-7.83E-07
V_{in}	3.3	0.017	-1.35E-06

The total uncertainty in ϵ is the RSS of the partials times the uncertainty in each variable:

$$\begin{aligned}U_x &= \sqrt{\left(\frac{\partial \epsilon}{\partial \Delta E_0}\delta \Delta E_0\right)^2 + \left(\frac{\partial \epsilon}{\partial F_g}\delta F_g\right)^2 + \left(\frac{\partial \epsilon}{\partial V_{\text{in}}}\delta V_{\text{in}}\right)^2} \\ &= \sqrt{(5.21 \times 10^{-6})^2 + (-7.83 \times 10^{-7})^2 + (-1.35 \times 10^{-6})^2} \\ &= \boxed{5.4 \times 10^{-6}}\end{aligned}$$

A Appendix: Displacement Table of Potentiometer

Table A.10: Displacement table of potentiometer

Caliper Reading (mm)	Up 1 (mm)	Down 1 (mm)	Up 2 (mm)	Down 2 (mm)	Up 3 (mm)
0.00		-0.16		-0.16	
3.64	3.61	3.61	3.61	3.65	3.61
7.64	7.69	7.69	7.69	7.69	7.73
11.64	11.69	11.69	11.69	11.66	11.66
15.64	15.70	15.66	15.66	15.66	15.70
19.64	19.65	19.65	19.61	19.61	19.65
23.64	23.66	23.62	23.66	23.62	23.66
27.64	27.70	27.66	27.66	27.66	27.70
31.64	31.67	31.63	31.67	31.63	31.67
39.10	38.98		38.98		38.98

B Appendix: Hall Effect Sensor Displacement Table

Table B.11: Hall effect sensor displacement table

	Sensor Reading Number				
Caliper Reading (mm)	1 (mm)	2 (mm)	3 (mm)	4 (mm)	5 (mm)
0.00	-0.01	-0.01	0.00	-0.01	-0.01
0.38	0.40	0.41	0.40	0.40	0.39
0.88	0.86	0.88	0.87	0.86	0.85
1.38	1.36	1.34	1.38	1.37	1.37
1.88	1.92	1.92	1.89	1.89	1.87
2.38	2.40	2.41	2.41	2.41	2.40
2.88	2.83	2.90	2.87	2.83	2.86