

# MEC E 301

## Lab 8: Rotation Formulas

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## Question 1

Table 1 shows the ambient pressure and temperature used for air density calculation.

Table 1: Ambient pressure and temperature

Barometer	Temperature	Correction for Tem- perature	Correction for Lati- tude	Corrected Pressure
(mmHg)	(°C)	(mmHg)	(mmHg)	(mmHg)
711.0	22.0	2.55	0.4805	708.9

Given that the gas constant for air is  $R = 0.287 \text{ kJ kg}^{-1} \text{ K}^{-1}$ , the density of air is

$$\begin{aligned}
 PV &= mRT \\
 \Rightarrow \rho &= \frac{P}{RT} \\
 &= \frac{708.9/7.501}{0.287 \times (22 + 273.15)} \\
 &= \boxed{1.12 \text{ kg m}^{-3}}
 \end{aligned}$$

## Question 2

The maximum velocity is given by

$$v = \sqrt{\frac{2P_v}{\rho}} \quad (1)$$

where  $P_v$  is the dynamic pressure and  $\rho$  is the density of air. The maximum pressure from Table 2 is  $P_v = 20.5 \text{ mmH}_2\text{O}$ . From hydrostatic pressure,

$$P = h\rho_w g = 1 \text{ mm} \times 997 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} = 9.78 \text{ Pa} = 1 \text{ mmH}_2\text{O}$$

where  $\rho_w$  is the density of water [1]. Thus in (1),

$$v = \sqrt{\frac{2 \times 20.5 \times 9.78}{1.12}} = \boxed{19.0 \text{ m s}^{-1}}$$

### Question 3

Similarly, the velocity at a yaw angle of  $-45^\circ$  is given by (1) with  $P_v = 14.5 \text{ mmH}_2\text{O}$ . The velocity is

$$v = \sqrt{\frac{2 \times 14.5 \times 9.78}{1.12}} = \boxed{13.6 \text{ m s}^{-1}}$$

The error is

$$\begin{aligned} err &= 19.0 - 13.6 = \boxed{5.4 \text{ m s}^{-1}} \\ \%err &= \frac{19.0 - 13.6}{19.0} \times 100\% = \boxed{28.4\%} \end{aligned}$$

### Question 4

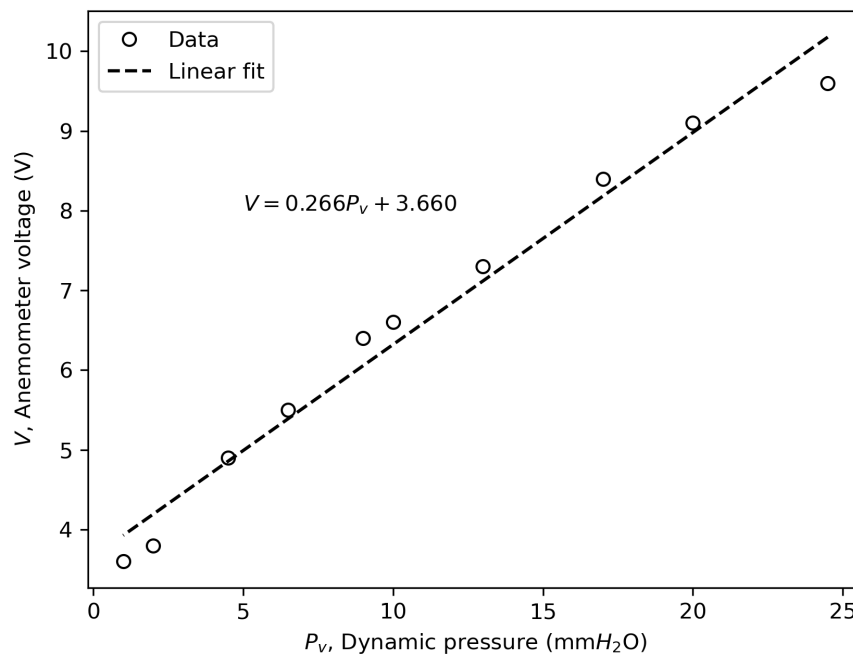


Figure 1: Anemometer voltage response to dynamic pressure

From the linear regression  $y = ax + b$  in Figure 1, the sensitivity is

$$sensitivity = a = \boxed{0.27 \text{ V m}^{-1} \text{ s}^{-1}}$$

## Question 5

The pressure of the orifice is given in the units of inches of alcohol. Converting using  $\rho_{alcohol} = 790 \text{ kg m}^{-3}$  [2].

$$P_v = 1 \text{ in alcohol} \times \frac{1 \text{ m}}{39.37 \text{ in}} \times 790 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} = 196.8 \text{ Pa}$$

Table 2: Orifice calibration

$K$	$A_c$ (m <sup>2</sup> )	$\Delta P$ (Pa)	$Q$ (m <sup>3</sup> s <sup>-1</sup> )
0.60116	$5.15 \times 10^{-4}$	$1.66 \times 10^3$	0.0169
0.60144	$5.15 \times 10^{-4}$	$1.31 \times 10^3$	0.0150
0.60162	$5.15 \times 10^{-4}$	$1.08 \times 10^3$	0.0136
0.60186	$5.15 \times 10^{-4}$	822	0.0119
0.60234	$5.15 \times 10^{-4}$	616	0.0103
0.60290	$5.15 \times 10^{-4}$	421	0.00852
0.60348	$5.15 \times 10^{-4}$	313	0.00736
0.60458	$5.15 \times 10^{-4}$	201	0.00590
0.60647	$5.15 \times 10^{-4}$	108	0.00434
0.60950	$5.15 \times 10^{-4}$	48.9	0.00294

Sample calculations for the first row of Table 2 are shown below.

$$P_v = 33.5 \times 196.8 = 6594.48 \text{ Pa}$$

$$A_c = \frac{\pi}{4} \times (27.3/1000)^2 = 5.15 \times 10^{-4} \text{ m}^2$$

$$K = \frac{0.6011 - 0.6013}{35 - 30} (33.5 - 30) + 0.6013 = 0.60116$$

$$Q = K A_c \sqrt{\frac{2P_v}{\rho}} = 0.60116 \times 5.15 \times 10^{-4} \times \sqrt{\frac{2 \times 6594.48}{1.12}} = \boxed{0.0169 \text{ m}^3 \text{ s}^{-1}}$$

Table 3: Venturi and nozzle discharge coefficients

$Q$ (m <sup>3</sup> s <sup>-1</sup> )	$P_v$ (Pa)	$\beta$	$A_c$ (m <sup>2</sup> )	$C_d$
0.0383	1662.7	0.4062	0.000721	0.960
0.0383	3208.6	0.3432	0.000515	0.974

## Question 6

For the venturi meter,

$$P_v = 170 \times 9.78 = 1662.7 \text{ Pa}$$

$$\beta = \frac{d}{D} = \frac{30.3}{74.6} = 0.4062$$

$$A_c = \frac{\pi}{4} \times (30.3/1000)^2 = 0.000721 \text{ m}^2$$

$$C_d = \frac{Q_{\text{actual}} \sqrt{1 - \beta^4}}{A_c} \sqrt{\frac{\rho}{2P_v}}$$

$$= \frac{0.0383 \times \sqrt{1 - 0.4062^4}}{0.000721} \sqrt{\frac{1.12}{2 \times 1662.7}} = \boxed{0.960}$$

For the venturi meter theoretical discharge coefficient,  $C_{d,\text{theory}} = 0.9965$ .

For the nozzle meter,

$$P_v = 16.3 \times 196.8 = 3208.6 \text{ Pa}$$

$$\beta = \frac{d}{D} = \frac{25.63}{74.6} = 0.3432$$

$$A_c = \frac{\pi}{4} \times (25.63/1000)^2 = 0.000515 \text{ m}^2$$

$$C_d = \frac{Q_{\text{actual}} \sqrt{1 - \beta^4}}{A_c} \sqrt{\frac{\rho}{2P_v}}$$

$$= \frac{0.0383 \times \sqrt{1 - 0.3432^4}}{0.000515} \sqrt{\frac{1.12}{2 \times 3208.6}} = \boxed{0.974}$$

For the nozzle meter theoretical discharge coefficient,

$$V = \frac{Q}{A_c} = \frac{0.0383}{0.000515} = 8.76 \text{ m s}^{-1}$$

$$\text{Re}_D = \frac{\rho V D}{\mu} = \frac{1.12 \times 8.76 \times 74.6/1000}{1.8347 \times 10^{-5}} = 3.97 \times 10^5$$

$$C_{d,\text{theory}} = 0.9965 - 6.53 \sqrt{\frac{0.3432}{3.97 \times 10^5}} = \boxed{0.977}$$

where  $\mu$  is the dynamic viscosity of air at 23°C [3].

**The nozzle is closer to the theory than the venturi meter.**

## Question 7

Error prop for air density,  $\rho$ ,

$$\rho = P^1 R^{-1} T^{-1}$$

$$\frac{\delta \rho}{|\rho|} = \sqrt{\left( (1) \frac{\delta P}{|P|} \right)^2 + \left( (-1) \frac{\delta T}{|T|} \right)^2}$$

$$= \sqrt{(1 \times 0.01)^2 + ((-1) \times 0.01)^2}$$

$$= \pm 0.0141$$

Error prop for  $Q$ ,

$$Q = C_d^1 A_c^1 (1 - \beta^4)^{-1} \sqrt{2 P_v^{\frac{1}{2}} \rho^{-\frac{1}{2}}}$$

$$\frac{\delta Q}{|Q|} = \sqrt{\left( (1) \frac{\delta C_d}{|C_d|} \right)^2 + \left( \left( \frac{1}{2} \right) \frac{\delta P_v}{|P_v|} \right)^2 + \left( \left( -\frac{1}{2} \right) \frac{\delta \rho}{|\rho|} \right)^2}$$

For the orifice meter,  $\delta C_d/|C_d| = 0.005$ ,  $\delta P_v/|P_v| = 0.01$ , and  $\delta \rho/|\rho| = 0.0141$ . Thus,

$$\frac{\delta Q}{|Q|} = \sqrt{(0.005)^2 + \left( \frac{1}{2} \times 0.01 \right)^2 + \left( -\frac{1}{2} \times 0.0141 \right)^2}$$

$$= \pm 0.01$$

For the venturi meter,  $\delta C_d/|C_d| = 0.02$ ,  $\delta P_v/|P_v| = 0.01$ , and  $\delta \rho/|\rho| = 0.0141$ . Thus,

$$\frac{\delta Q}{|Q|} = \sqrt{(0.02)^2 + \left( \frac{1}{2} \times 0.01 \right)^2 + \left( -\frac{1}{2} \times 0.0141 \right)^2}$$

$$= \pm 0.02$$

For the nozzle meter,  $\delta C_d/|C_d| = 0.01$ ,  $\delta P_v/|P_v| = 0.01$ , and  $\delta \rho/|\rho| = 0.0141$ . Thus,

$$\begin{aligned}\frac{\delta Q}{|Q|} &= \sqrt{(0.01)^2 + \left(\frac{1}{2} \times 0.01\right)^2 + \left(-\frac{1}{2} \times 0.0141\right)^2} \\ &= \pm 0.02\end{aligned}$$

For the orifice meter, the  $\rho$  term is the dominant term as

$$\begin{aligned}\left(\left(-\frac{1}{2}\right) \frac{\delta \rho}{|\rho|}\right)^2 &> \left(\left(\frac{1}{2}\right) \frac{\delta P_v}{|P_v|}\right)^2 = \left(\frac{\delta C_d}{|C_d|}\right)^2 \\ \iff \left(\frac{1}{2} \times 0.0141\right)^2 &> \left(\frac{1}{2} \times 0.01\right)^2 = (0.005)^2\end{aligned}$$

For the venturi, the dominant term is  $C_d$  as

$$\begin{aligned}\left(\frac{\delta C_d}{|C_d|}\right)^2 &> \left(\frac{1}{2} \times \frac{\delta \rho}{|\rho|}\right)^2 > \left(\frac{1}{2} \times \frac{\delta P_v}{|P_v|}\right)^2 \\ \iff (0.02)^2 &> \left(\frac{1}{2} \times 0.0141\right)^2 > \left(\frac{1}{2} \times 0.01\right)^2\end{aligned}$$

Lastly, for the nozzle, the dominant term is  $C_d$  as

$$\begin{aligned}\left(\frac{\delta C_d}{|C_d|}\right)^2 &> \left(\frac{1}{2} \times \frac{\delta \rho}{|\rho|}\right)^2 > \left(\frac{1}{2} \times \frac{\delta P_v}{|P_v|}\right)^2 \\ \iff (0.01)^2 &> \left(\frac{1}{2} \times 0.0141\right)^2 > \left(\frac{1}{2} \times 0.01\right)^2\end{aligned}$$

## Question 8

From the linear regression  $y = ax + b$  in Figure 2, the sensitivity is

$$\boxed{\text{sensitivity} = 1.7 \times 10^4 \text{ Hz s m}^{-3}}$$

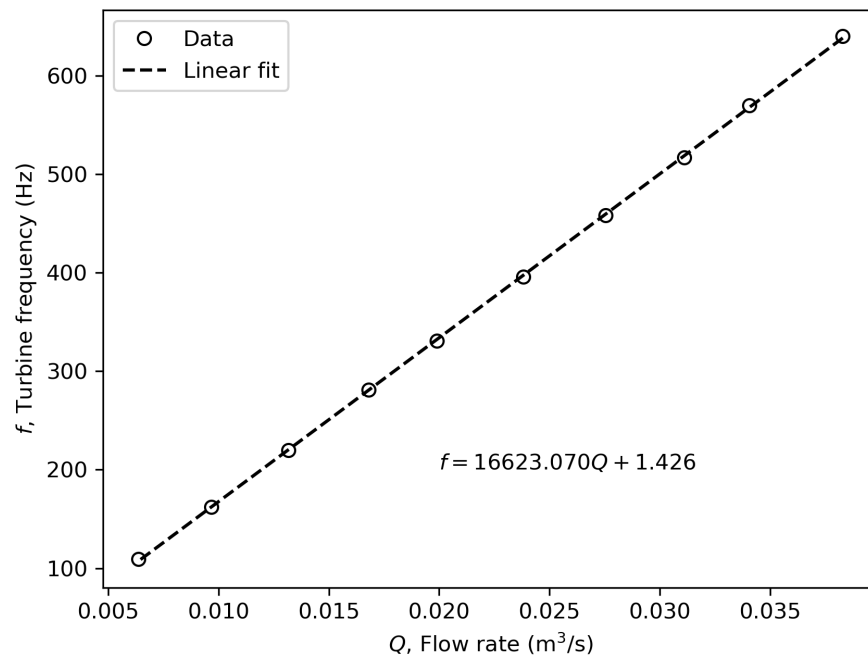


Figure 2: Turbine meter frequency response to orifice meter flow rate



## References

- [1] P. H. Bigg, “Density of water in SI units over the range 0-40°C,” *British Journal of Applied Physics*, vol. 18, no. 4, p. 521, Apr. 1967. [Online]. Available: <https://dx.doi.org/10.1088/0508-3443/18/4/315>
- [2] R. E. Ferner and J. Chambers, “Alcohol intake: measure for measure,” *BMJ : British Medical Journal*, vol. 323, no. 7327, pp. 1439–1440, Dec. 2001. [Online]. Available: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1121897/>
- [3] W. N. Bond, “The viscosity of air,” *Proceedings of the Physical Society*, vol. 49, no. 3, p. 205, May 1937. [Online]. Available: <https://dx.doi.org/10.1088/0959-5309/49/3/301>