MEC E 301
Lab 2: Dimensional Measurement Techniques

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1 Introduction

The primary objective of this lab is to investigate and familiarize elements of digital measurements such as word length (number of bits), quantization, resolution, conversion rates, sampling, accuracy, and signal conditioning. The Arduino Uno will be used to measure a voltage source which will undergo signal conditioning and be converted to a digital signal. Understanding precise digital measurements is important for the design of control systems, data acquisition, and signal processing.

2 Procedure

A schematic of the experimental setup is shown in Figure [insert figure] The precis provided by the MEC E 301 course was followed [**?**]. All measurements were taken from the serial monitor of the Arduino IDE. During calibration, different circuit components and reference voltages were used to measure the voltage output of the PCB.

3 Results and Discussion

3.1 Calibration of the Arduino Uno Results

Table 1: Range, Resolution, Repeatability, Accuracy, and Manufacturer's Accuracy for Various Ranges of the Arduino Uno

Arduino Config.	Range (V)	Resolution (mV/LSB)	Repeatability (mV)	Acc. (mV)	Manuf. Acc. (mV)
5V Ref.	0.000 - 5.000	4.883	44.00	54.00	9.766
3.3V Ref.	0.000 - 3.300	3.223	4.000	17.00	6.445
3.3V Ref., 10x VDiv	0.00 - 33.00	32.23	32.00	83.00	64.45
3.3V Ref., [-10, 10]V	-10.00 - 10.00	19.53	0.000	24.00	39.06
3.3V Ref., 10x Amp.	0.000 - 0.330	0.3223	0.000	0.000	0.6445

3.2 Calibration of the Arduino Uno Discussion

3.2.1 Trade-off Between Resolution and Range

The trade-off between resolution and range is that as the range increases, the resolution decreases. This is because the number of bits available to represent the range is fixed. For example, the Arduino Uno has 10 bits to represent the range of voltages. This means that the Arduino Uno can represent $2^{10} = 1024$ different voltages.

From Appendix B, if the range is multiplied by a factor of k, then the resolution is also multiplied by a factor of k, increasing resolution error.

3.2.2 Repeatability Compared to Accuracy

Repeatability is the maximum deviation between two measurements of the same reference quantity. Accuracy is the maximum deviation between the measured value and the true value. Repeatability is a measure of precision while accuracy is a measure of correctness. Both measure different aspects of the measurement system.

3.2.3 Accuracy of Arduino Uno with 3.3V Reference Voltage

The accuracy for the 5V reference voltage is 54.00mV while the accuracy for the 3.3V reference voltage is 17.00mV. The accuracy of the Arduino Uno is improved using the 3.3V reference voltage. Many computers do not supply exactly 5V over the USB cable and the Arduino's A/D converter assumes the voltage is 5V. Supplying a known voltage to the AREF pin from the MEC E 301 Shield allows for more accurate measurements.

3.2.4 Accuracy of Voltage Sources

From Appendix C, if we use a conservative estimate for the accuracy of the voltage source, which occurs at 2.500V, then the accuracy is 1.250mV. ANSI/ISA 51.1 states that the accuracy of the standard can be ignored should the accuracy be one tenth of the instrument tested.

Comparing this to the accuracy of the Arduino Uno, in Table 1, the accuracy of the voltage source is sufficient for all reference values except the setup with the 3.3V reference voltage and 10x amplifier, which has an experimental accuracy of 0.000mV.

3.2.5 Accuracy of Arduino Uno Compared to Manufacturer's Accuracy

Referring to Appendix A, the experimental accuracy was less than the manufacturer's accuracy for all configurations except the 3.3V reference voltage with a 10x amplifier and the 3.3V reference voltage with a [-10, 10]V range.

The 5V reference voltage has an accuracy of 54.00mV while the manufacturer's accuracy is 9.766mV. This may be attributed to the variable reference voltage supplied by the computer

3.2.6 Additional Error from Voltage Divider, Voltage Scaler, and Amplifier

The calculation is shown in Appendix D. The error at 0.102V is 1.020mV and the error at 2.500V is 25.00mV. The error at 0.102 is not significant while the error at 2.500V is significant by ANSI/ISA 51.1.

A Appendix: Arduino Uno Accuracy

Table 2 summarizes the range, resolution, repeatability, accuracy, and manufacturer's accuracy for various ranges of the Arduino Uno. Sample calculations for the 5V reference voltage are shown below. Note, the manufacturer's accuracy is ± 2 LSBs.

$$\begin{aligned}
\text{Resolution} &= \frac{V_{\text{ru}} - V_{\text{rl}}}{2^n} \\
&= \frac{5.000 - 0.000}{2^{10}} \\
&= \boxed{4.883\,\text{mV/LSB}} \\
\text{Repeatability} &= \max(\text{Max Deviation}) \\
&= \max(\langle 5.00, 0.00, 9.00, 44.00 \rangle) \\
&= \boxed{44.00\,\text{mV}} \\
\text{Accuracy} &= \max(\text{Deviation}) \\
&= \max\left(\begin{bmatrix} -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.009 & -0.004 & -0.009 \\ 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 \\ 0.016 & 0.021 & 0.012 & 0.016 & 0.012 & 0.012 & 0.016 & 0.016 & 0.012 & 0.016 \\ 0.02 & 0.024 & 0.02 & 0.024 & 0.01 & 0.02 & \mathbf{0.054} & 0.024 & 0.034 & 0.02 \end{bmatrix}\right) \\
&= \boxed{54.00\,\text{mV}} \\
\text{Manuf. Acc.} &= 2\,\text{LSB} \times \text{Resolution} \\
&= \boxed{9.766\,\text{mV}}
\end{aligned}$$

For significant figures, since the range is given to 3 decimal places, the resolution is given to 3 decimal places, more often than not, the number of significant figures is 4. This is because addition and subtraction do not take into account the number of significant figures but rather the number of decimal places.

Table 2: Range, Resolution, Repeatability, Accuracy, and Manufacturer's Accuracy for Various Ranges of the Arduino Uno

Arduino Config.	Range (V)	Resolution (mV/LSB)	Repeatability (mV)	Acc. (mV)	Manuf. Acc. (mV)
5V Ref.	0.000 - 5.000	4.883	44.00	54.00	9.766
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3.3V Ref., [-10, 10]V	-10.00 - 10.00	19.53	0.000	24.00	39.06
3.3V Ref., 10x Amp.	0.000 - 0.330	0.3223	0.000	0.000	0.6445

B Appendix: Derivation of Resolution

The trade-off between resolution and range is that as the range increases, the resolution decreases. This is because the number of bits available to represent the range is fixed. For example, the Arduino Uno has 10 bits to represent the range of voltages. This means that the Arduino Uno can represent $2^{10} = 1024$ different voltages. If the range is 0 to 5V, then the resolution is 4.883mV/LSB. If the range is 0 to 10V, then the resolution is 9.766mV/LSB. This can be shown mathematically. Then the resolution is given by:

$$\text{Resolution} = \frac{V_{\text{ru}} - V_{\text{rl}}}{2^n}$$

Let us assume the upper range V_{ru} and lower range V_{rl} are both multiplied by a factor of k.

$$\begin{aligned}\text{Resolution'} &= \frac{kV_{\text{ru}} - kV_{\text{rl}}}{2^n} \\ &= \frac{k(V_{\text{ru}} - V_{\text{rl}})}{2^n} \\ &= k\text{Resolution}\end{aligned}$$

C Appendix: Voltage Source Accuracy

The accuracy of the voltage source is 0.05%. A table deviation from the true value is shown below.

Table 3: Deviation from True Value for Voltage Source with 0.05% Accuracy

Voltage (V)	Deviation from True Value (mV)
0.102	0.051
1.024	0.512
1.800	0.900
2.500	**1.250**

As one can see, the voltage source deviation is 10x smaller than the accuracy of the Arduino Uno. This means that the voltage source is a suitable reference voltage for the Arduino Uno.

Table 4: Accuracy of Voltage Source Compared to Arduino Uno

Configuration	Accuracy (mV)
Voltage Source	1.250
5V Ref.	54.00
3.3V Ref.	17.00
3.3V Ref., 10x VD	83.00
3.3V Ref., [-10, 10]V	24.00
3.3V Ref., 10x Amp.	0.000

D Appendix: Circuit Component Accuracy

Below is a table of the additional error from the voltage divider, voltage scaler, and amplifier. The error is calculated by multiplying the voltage by 1%.

Table 5: Additional Error from Voltage Divider, Voltage Scaler, and Amplifier

Voltage (V)	Error (mV)
0.102	1.020
2.500	25.00

E Time-Varying Voltage Measurements

E.1 Time-Varying Voltage Results

Table 6: Time-Varying Period Measurements for 10-bit and 5-bit ADCs

	Period (μs)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit	21000	21132	21132	21124	21400
5-bit	20912	21104	21136	21096	21304

Table 7: Time-Varying Frequency Measurements for 10-bit and 5-bit ADCs

	Frequency (Hz)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit	47.619	47.295	47.331	47.357	46.729
5-bit	47.819	47.384	47.313	47.402	46.940

Table 8: Time-Varying Peak to Peak Voltage Measurements for 10-bit and 5-bit ADCs

	Peak to Peak Voltage (V)				
	Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5
10-bit	1.212	1.234	1.234	1.206	1.199
5-bit	1.225	1.237	1.228	1.192	1.215

E.2 Mean Sample Calculation

The mean for the 10-bit and 5-bit ADCs was calculated using Excel using the `=AVERAGE()` function across 5 cycles, which was approximately 60 data points.

	Mean Measurements of Time-Varying Voltage		
	Frequency (Hz)	Peak to Peak (V)	Voltage (V)
10-bit	47.266	1.217	1.135
5-bit	47.372	1.2194	1.144
Oscilloscope	48.54	1.28	1.17

E.3 Uncertainty Analysis

E.3.1 Frequency Uncertainty

Sample calculations for the 10-bit ADC are shown below. Note that the 5-bit ADC calculations are similar.

For frequency, standard deviation, σ, was calculated from Excel using `=STDEV.S()`. For the T-distribution inverse, $t_{\alpha/2,n-1}$, the `=T.INV()` function was used, where $\alpha = 0.05$ and $n = 5$.
Using $\sigma = 0.32582\,\text{Hz}$ and $t_{\alpha/2,n-1} = 2.7764$, the uncertainty in frequency is:

$$\begin{aligned} u(f) &= t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} \\ &= 2.7764\frac{0.32582\,\text{Hz}}{\sqrt{5}} \\ &= 0.40456\,\text{Hz} \end{aligned}$$

E.3.2 Peak to Peak Voltage Uncertainty

For peak to peak voltage, $\sigma = 0.0162$ and $t_{\alpha/2,n-1} = 2.7764$, so the uncertainty in peak to peak voltage is:

$$\begin{aligned} P_x &= t_{\alpha/2,n-1}\frac{\sigma}{\sqrt{n}} \\ &= 2.7764\frac{0.0162\,\text{V}}{\sqrt{5}} \\ &= 0.0201\,\text{V} \end{aligned}$$

For B_x, error propagation was used. For a single measurement, the systematic uncertainty was $B_x = 0.01700\,\text{V}$, which was taken from the calibration of the 3.3V reference section. Since the governing equation is:

$$V_{\text{p-p}} = V_{\text{ru}} - V_{\text{rl}}$$

The propagated systematic uncertainty is:

$$
\begin{aligned}
B_{x'} &= \sqrt{\frac{\partial V_{\text{p-p}}}{\partial V_{\text{ru}}}^2 B_{\text{ru}}^2 + \frac{\partial V_{\text{p-p}}}{\partial V_{\text{rl}}}^2 B_{\text{rl}}^2} \\
&= \sqrt{1^2 0.017002 + (-1)^2 0.017002} \\
&= \sqrt{2}(0.01700) \\
&= 0.02404\,\text{V}
\end{aligned}
$$

For total uncertainty of peak to peak voltage, RSS was used:

$$
\begin{aligned}
U_x &= \sqrt{P_x^2 + B_x^2} \\
&= \sqrt{0.0201\,\text{V}^2 + 0.02404\,\text{V}^2} \\
&= 0.0313\,\text{V}
\end{aligned}
$$

Note: For the 5-bit ADC, the accuracy is half the resolution, $B_x = 0.05156\,\text{V}$.

Table 9: Uncertainty Results for Time-Varying Voltage Measurements

	Frequency (Hz)	Peak to Peak Voltage (V)		
	P_x	P_x	B_x	U_x
10-bit	0.4046	0.0201	0.02404	0.0313
5-bit	0.3886	0.0214	0.07292	0.0760