MEC E 301
Lab 3: Displacement Transducers

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Section: D21

Question 1

(a)

The plot of displacement over voltage can be found in Fig. 1. The slope, intercept, and R^2 value of the linear regression were determined using `=LINEST()` in Excel. The results can be found in Table 1.

Table 1: Linear regression of displacement over voltage

Slope (mm/V)	Intercept (mm)	R^2
12.67	-1.50	1.00

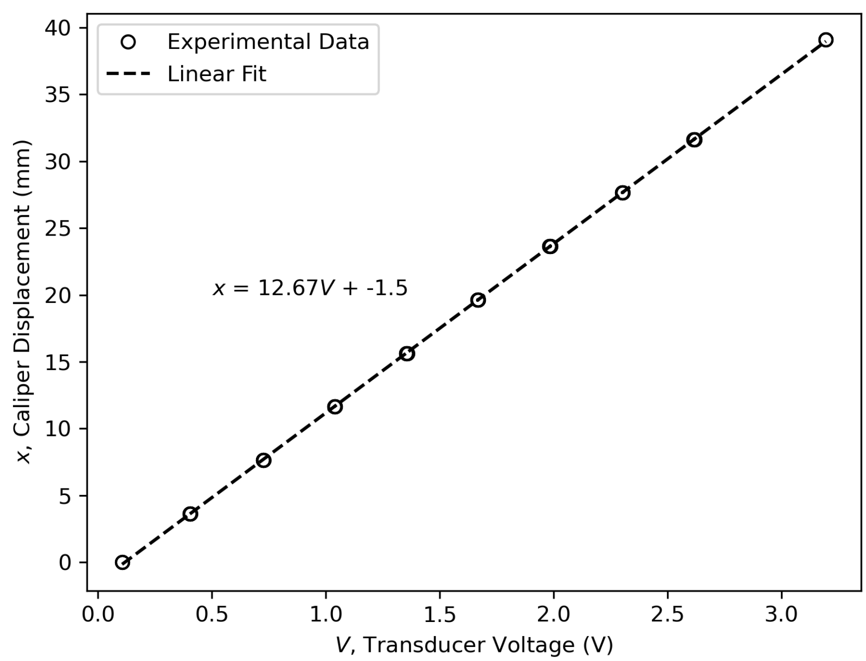

Figure 1: Displacement over voltage for the potentiometer

Note that 2 decimal places were chosen as linear regression involved sum of squares, and the limiting decimal place was 2, which comes from the caliper measurements.

The function is:

$$\boxed{x = 12.67V - 1.50 \ \ [\mathrm{mm}]} \tag{1}$$

The sensitivity is the inverse of the slope,

$$\boxed{\text{Sensitivity} = \frac{1}{12.67} = 0.07891\,\mathrm{mm\,V^{-1}}}$$

The span is the difference between the maximum and minimum displacement. The span is therefore,

$$\boxed{\text{Span} = 39.10\,\mathrm{mm}}$$

(b)

First, all the voltages were converted to displacements using the linear regression function, Eq. (1) discussed in the previous section. The displacement table can be found in Appendix A in Table A.11.

The deviation table can be found in Table 2. The deviation curve can be found in Fig. 2.

Table 2: Deviation table of potentiometer after conversion to displacement.

Caliper Reading (mm)	Up 1 (mm)	Down 1 (mm)	Up 2 (mm)	Down 2 (mm)	Up 3 (mm)	Down 3 (mm)
0.00		-0.16		-0.16		-0.16
3.64	-0.03	-0.03	-0.03	0.01	-0.03	0.01
7.64	0.05	0.05	0.05	0.05	0.05	0.09
11.64	0.05	0.05	0.05	0.02	0.02	0.02
15.64	0.06	0.02	0.02	0.02	0.02	0.06
19.64	0.01	0.01	-0.03	-0.03	0.01	-0.03
23.64	0.02	-0.02	0.02	-0.02	0.02	-0.02
27.64	0.06	0.02	0.02	0.02	0.06	0.02
31.64	0.03	-0.01	0.03	-0.01	0.03	0.03
39.10	-0.12		-0.12		-0.12	

The linear fit for the deviation curve was found by using `=LINEST()` in Excel. The results can be found in Table 3.

The fit function for deviation is therefore,

$$\Delta x = 5.29 \times 10^{-5} x - 1.49 \times 10^{-2} \quad [\mathrm{mm}]$$

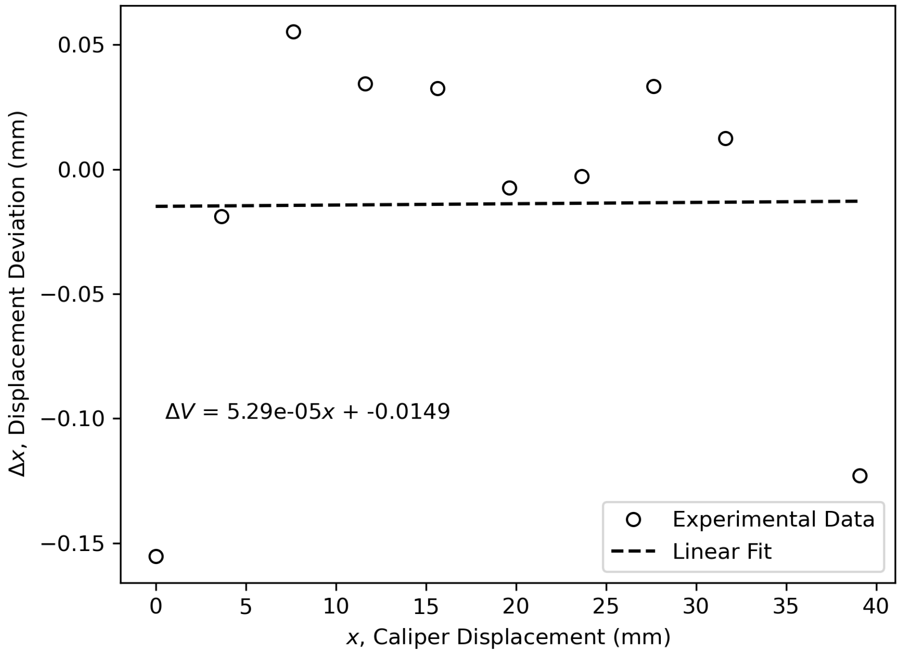

Figure 2: Displacement deviation curve of potentiometer with respect to displacement (independent linearity).

Table 3: Linear fit of deviation curve of potentiometer with respect to displacement (independent linearity).

Slope (V/mm)	Intercept (V)	R^2
5.29E-05	-1.49E-02	9.11E-05

Evaluating the fit function at all displacements results in difference deviation Table 4.

A hysteresis table can be generated by subtracting the up and down readings. The hysteresis table can be found in Table 5.

Table 4: Difference table of nominal deviation and linear fit of deviation.

Caliper Reading (mm)	Nominal Deviation (mm)	Independent Fit (mm)	Difference (mm)
0.00	-0.16	-0.015	0.14
3.64	-0.03	-0.015	0.00
7.64	0.05	-0.014	-0.07
11.64	0.05	-0.014	-0.05
15.64	0.06	-0.014	-0.05
19.64	0.01	-0.014	-0.01
23.64	0.02	-0.014	-0.01
27.64	0.06	-0.013	-0.05
31.64	0.03	-0.013	-0.03
39.10	-0.12	-0.013	0.11

Table 5: Hysteresis table of potentiometer.

Caliper Reading (mm)	Up 1 - Down 1 (mm)	Up 2 - Down 2 (mm)	Up 3 - Down 3 (mm)
3.64	0.00	-0.04	-0.04
7.64	0.00	0.00	-0.04
11.64	0.00	0.04	0.00
15.64	0.04	0.00	-0.04
19.64	0.00	0.00	0.04
23.64	0.04	0.04	0.04
27.64	0.04	0.00	0.04
31.64	0.04	0.04	0.00

(c)

A summary table is provided below:

Table 6: Summary of results for potentiometer.

	Nominal (mm)	Relative (%)
Accuracy	$\pm$ 0.16	$\pm$ 0.40
Repeatability	$\pm$ 0.04	$\pm$ 0.10
Linearity	0.14	0.36
Resolution	0.038	0.097
Hysteresis	0.04	0.10

Accuracy can be found by taking the max value absolute value of the deviation Table 2. The accuracy is therefore,

$$\boxed{\text{Accuracy} = \max(\text{Table}) = \pm 0.16\,\text{mm}}$$

Relative accuracy is

$$\boxed{\text{Relative Accuracy} = \frac{\text{Accuracy}}{\text{Span}} = \frac{0.16}{39.10} = \pm 0.40\%}$$

The repeatability is simply the maximum deviation between repeated measurements of the same caliper reading approached from the same direction.

$$\boxed{\begin{aligned}\text{Repeatability} &= \max(\text{Table}) \\ &= \max\left(\begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.04 \\ 0.04 & 0.04 \\ 0.04 & 0.04 \\ 0.04 & 0.04 \\ 0.00 & 0.04 \\ 0.04 & 0.00 \\ 0.00 & 0.00 \\ 0.00 & 0.04 \end{bmatrix}\right) \\ &= \pm 0.04\,\text{mm}\end{aligned}}$$

The relative repeatability is

$$\boxed{\text{Relative Repeatability} = \frac{\text{Repeatability}}{\text{Span}} = \frac{0.04}{39.10} = \pm 0.1\%}$$

The linearity is the maximum difference between the nominal deviation and the linear fit of the deviation. The linearity is therefore,

$$\boxed{\begin{aligned}\text{Linearity} &= \max(\text{Table})\\ &= \max\left(\begin{bmatrix}0.14\\0.00\\-0.07\\-0.05\\-0.05\\-0.01\\-0.01\\-0.05\\-0.03\\0.11\end{bmatrix}\right)\\ &= 0.14\,\text{mm}\end{aligned}}$$

The relative linearity is

$$\boxed{\text{Relative Linearity} = \frac{\text{Linearity}}{\text{Span}} = \frac{0.14}{39.10} = 0.36\%}$$

A resistor is a transducer that converts electrical energy to heat energy. The resolution of a transducer is

$$\text{Resolution} = \frac{V_{\text{Span}}}{2^n G\eta}$$

where G is the gain, and η is the sensitivity. The resolution of this system is therefore,

$$\boxed{\text{Resolution} = \frac{3.3}{2^{10} \times 1 \times 0.07891} = 0.038\,\text{V}}$$

The relative resolution is

$$\boxed{\text{Relative Resolution} = \frac{\text{Resolution}}{\text{Span}} = \frac{0.038}{39.10} = 0.097\%}$$

The hysteresis is the maximum difference between the up and down readings. The hysteresis can be found by applying `=MAX(ABS(Table))` in Excel on Table 5. The hysteresis is therefore,

$$\boxed{\text{Hysteresis} = 0.04\,\text{mm}}$$

The relative hysteresis is

$$\boxed{\text{Relative Hysteresis} = \frac{\text{Hysteresis}}{\text{Span}} = \frac{0.04}{39.10} = 0.1\%}$$

(d)

The plot of displacement over time and velocity over time can be found in Fig. 3 and Fig. 4 respectively.

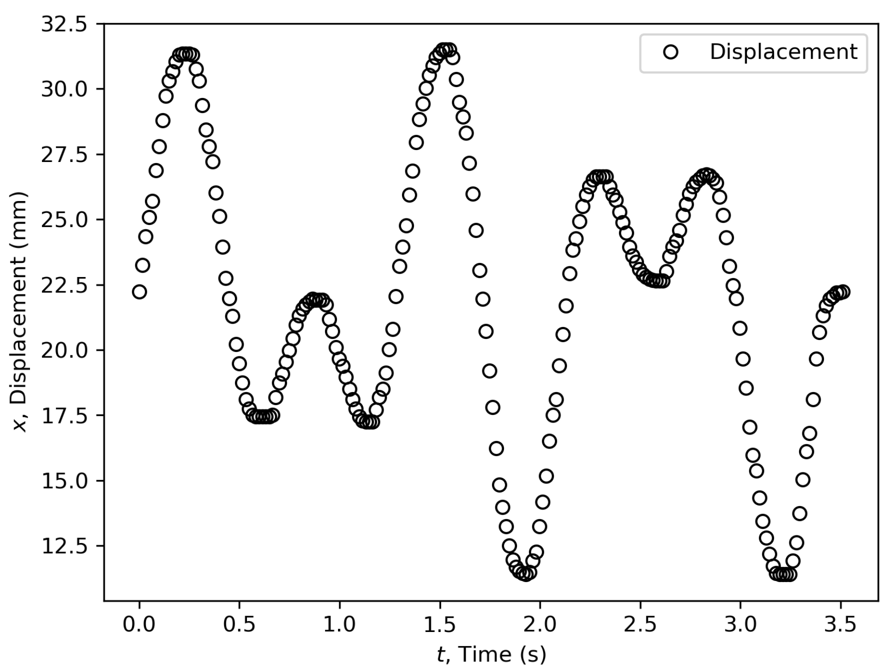

Figure 3: Displacement over time of potentiometer.

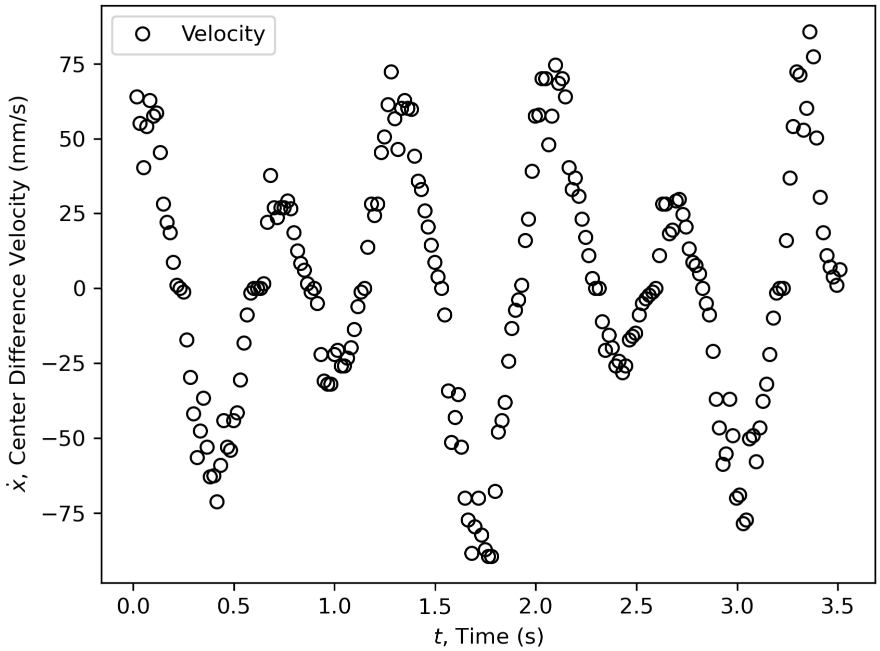

Figure 4: Velocity over time of potentiometer.

The velocity was calculated by taking the central difference of the displacement. This was chosen because the central difference method has an error of $O(h^2)$, whereas the forwards and backwards difference methods have an error of $O(h)$. A downside is that the first and last points cannot be calculated. However, due to the large number of points, this was negligible.

Question 2

(a)

The coefficients of the polynomial of the form $y = ax^3 + bx^2 + cx + d$ were taken using the `=LINEST(x, y^{1, 2, 3}, True, True)` function in Excel. The coefficients are shown in Table 7.

Table 7: Coefficients of third degree fitting polynomial for Hall-effect sensor measurements

Coefficient	a	b	c	d
Value	-0.34	2.97	-9.48	11.06

The equation is therefore,

$$\boxed{x = -0.34V^3 + 2.97V^2 - 9.48V + 11.06 \quad [\mathrm{mm}]} \tag{2}$$

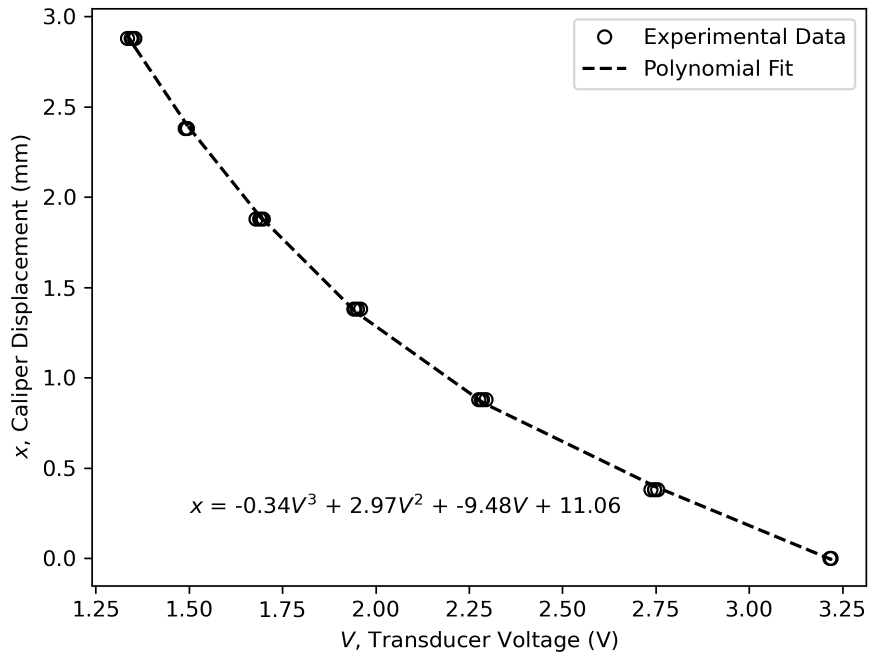

Figure 5: Hall-effect sensor data

(b)

Using Eq. (2), the Hall-effect sensor displacement was calculated. The table of Hall-effect sensor displacement is shown in Appendix B in Table B.12. The plot of these points is shown in Fig. 6.

A deviation of the Hall-effect sensor displacement from the caliper displacement was calculated. The deviation is shown in Table 8.

The accuracy of the Hall-effect sensor was found by taking the maximum of the absolute value of the deviations.

$$\boxed{\text{Accuracy} = \max(|\text{Deviations}|) = 0.05 \quad [\text{mm}]}$$

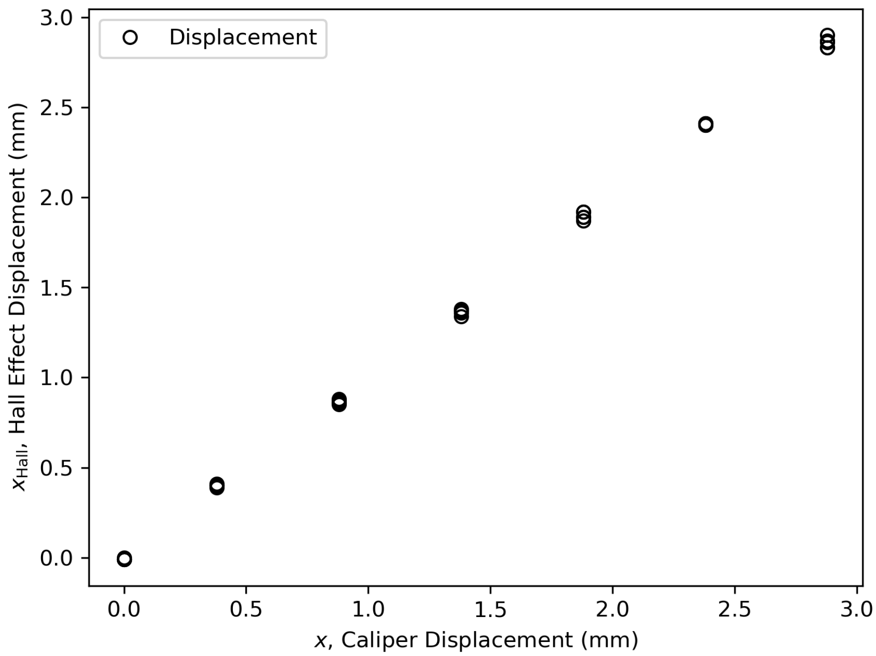

Figure 6: Hall-effect sensor displacement vs. caliper displacement

Table 8: Deviation of Hall-effect sensor displacement from caliper displacement

	Reading				
Displacement	1	2	3	4	5
(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
0.00	-0.01	-0.01	0.00	-0.01	-0.01
0.38	0.02	0.03	0.02	0.02	0.01
0.88	-0.02	0.00	-0.01	-0.02	-0.03
1.38	-0.02	-0.04	0.00	-0.01	-0.01
1.88	0.04	0.04	0.01	0.01	-0.01
2.38	0.02	0.03	0.03	0.03	0.02
2.88	-0.05	0.02	-0.01	-0.05	-0.02

(c)

Repeating section a), with the Hall-effect voltage as the dependent variable and the caliper reading as the independent variable, the coefficients of the polynomial of the form $y = ax^3 + bx^2 + cx + d$ were taken using the `=LINEST(x, y^{1, 2, 3}, True, True)`. The plot of the Hall-effect voltage vs. caliper reading is shown in Fig. 7.

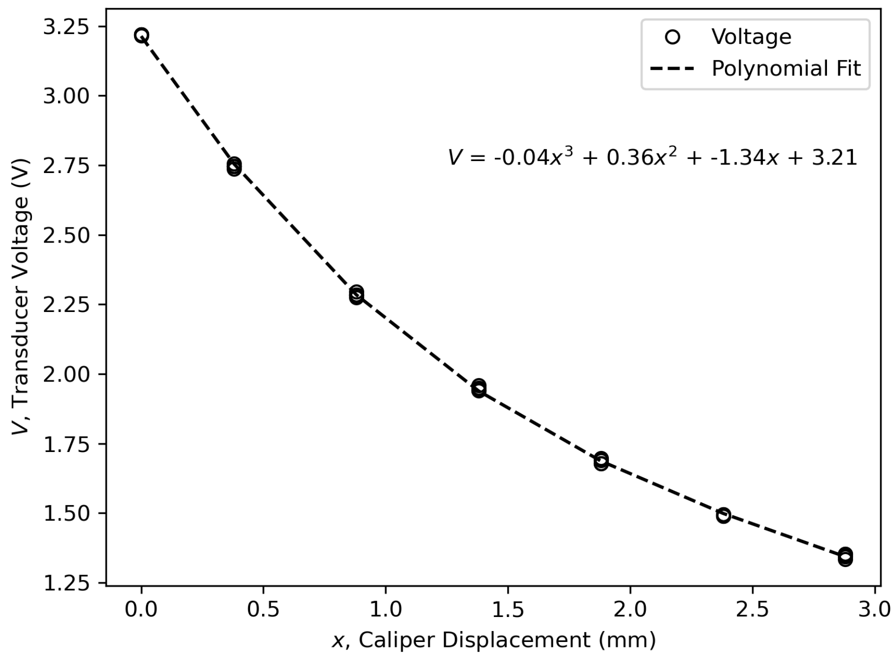

Figure 7: Hall-effect voltage vs. caliper reading

The fit function is

$$V = -0.04x^3 + 0.36x^2 - 1.34x + 3.21 \ \ [\mathrm{V}] \tag{3}$$

The derivative of Eq. (3) is

$$\frac{dV}{dx} = -0.12x^2 + 0.72x - 1.34 \ \ [\mathrm{V/mm}] \tag{4}$$

The sensitivity when the displacement is zero is

$$\boxed{\text{Sensitivity at } x = 0\,\mathrm{mm} = -1.34\,\mathrm{V/mm}}$$

The sensitivity when the displacement is 3mm is

$$\boxed{\text{Sensitivity at } x = 3\,\mathrm{mm} = -0.30\,\mathrm{V/mm}}$$

(d)

The Hall-effect sensor has the highest sensitivity. The sensitivity of the Hall-effect sensor at 0 mm and 3 mm is -1.34 V/mm and -0.30 V/mm respectively, both of which are higher than the sensitivity of the potentiometer at 0.07891 V/mm for all measurements in its span.

The Hall-effect sensor would be better at measuring small displacements because it has a higher sensitivity. A small change in displacement would result in a larger change in voltage, which would be easier to detect.

Question 3

(a)

The accuracy and repeatability of the sensor was calculated for each displacement of the large and small target. The results are shown in Table 9.

Table 9: Accuracy and repeatability at each displacement of the large and small target

	Large Target		Small Target	
Displacement (mm)	Accuracy (± mm)	Repeatability (± mm)	Accuracy (± mm)	Repeatability (± mm)
100	21	2	6	3
200	24	2	9	2
300	23	3	7	8
400	24	3	35	15
500	28	5	125	116
600	33	11	267	307
700	36	14	285	358
800	41	27	255	392
900	69	73	258	470
1000	70	135	168	253

A sample calculation for the accuracy and repeatability of the large and small targets at 100mm is shown below. First a deviation table was created. The deviation table is shown in Table 10.

Table 10: Deviation table for large and small targets at 100mm

	Deviation Reading	
Displacement (mm)	Large (mm)	Small (mm)
100	20	-4
100	20	-3
100	20	-4
100	20	-3
100	20	-4
100	20	-4
100	20	-4
100	21	-6
100	20	-3
100	19	-3

Accuracy is the maximum deviation, while repeatability is the difference between the maximum and minimum deviation.

$$\boxed{\text{Accuracy}_{\text{large}} = \max(\text{Deviation}_{\text{large}}) = \pm 21\text{ mm}}$$

$$\boxed{\text{Repeatability}_{\text{large}} = \max(\text{Deviation}_{\text{large}}) - \min(\text{Deviation}_{\text{large}}) = 21 - 19 = \pm 2\text{ mm}}$$

The accuracy and repeatability of the sensor was plotted as a function of target displacement for each target size. The results are shown in Fig. 8 and Fig. 9 respectively.

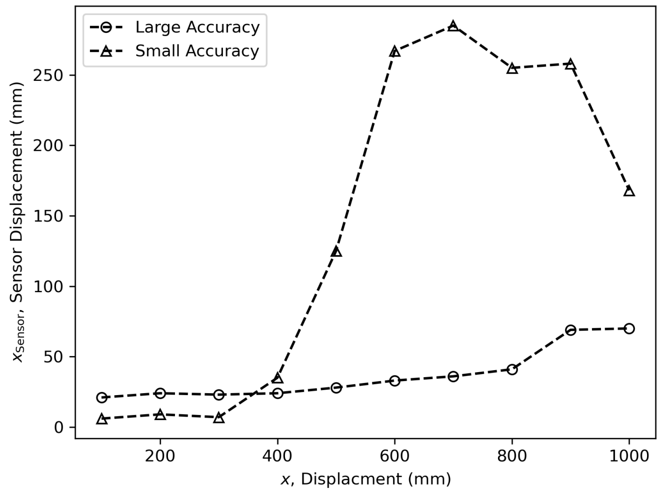

Figure 8: Accuracy of the sensor as a function of target displacement for each target size

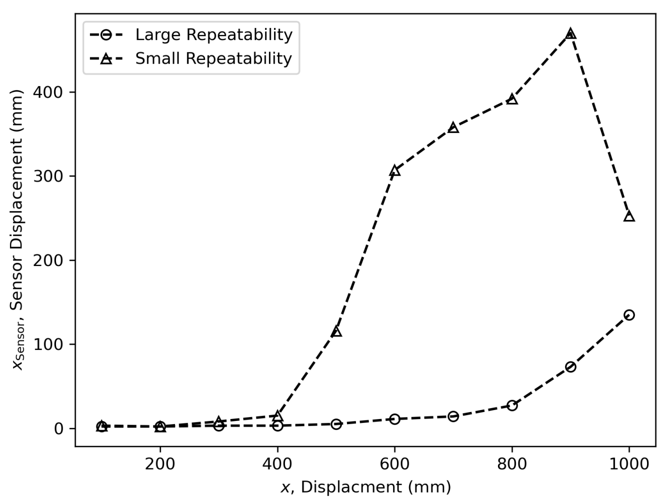

Figure 9: Repeatability of the sensor as a function of target displacement for each target size

(b)

The accuracy of the sensor increases with target size and decreases with target displacement. This is because the larger target has a bigger surface area, so more light is reflected back to the sensor.

Observing the target signal strengths, the large target had a higher signal strength than the small target for a given displacement. Status 2 was reached at a lower displacement for the smaller target than the larger target, meaning the sensor had less confidence in the reading provided.

In Fig. 8, the accuracy of the smaller target was slightly higher than the larger target at 100mm, 200mm, and 300mm. This diverges quickly as the displacement increases.

A Appendix: Displacement Table of Potentiometer

Table A.11: Displacement table of potentiometer

Caliper Reading (mm)	Up 1 (mm)	Down 1 (mm)	Up 2 (mm)	Down 2 (mm)	Up 3 (mm)
0.00		-0.16		-0.16	
3.64	3.61	3.61	3.61	3.65	3.61
7.64	7.69	7.69	7.69	7.69	7.73
11.64	11.69	11.69	11.69	11.66	11.66
15.64	15.70	15.66	15.66	15.66	15.70
19.64	19.65	19.65	19.61	19.61	19.65
23.64	23.66	23.62	23.66	23.62	23.66
27.64	27.70	27.66	27.66	27.66	27.70
31.64	31.67	31.63	31.67	31.63	31.67
39.10	38.98		38.98		38.98

B Appendix: Hall Effect Sensor Displacement Table

Table B.12: Hall effect sensor displacement table

	Sensor Reading Number				
Caliper Reading (mm)	1 (mm)	2 (mm)	3 (mm)	4 (mm)	5 (mm)
0.00	-0.01	-0.01	0.00	-0.01	-0.01
0.38	0.40	0.41	0.40	0.40	0.39
0.88	0.86	0.88	0.87	0.86	0.85
1.38	1.36	1.34	1.38	1.37	1.37
1.88	1.92	1.92	1.89	1.89	1.87
2.38	2.40	2.41	2.41	2.41	2.40
2.88	2.83	2.90	2.87	2.83	2.86