Question 1

A thin plate moves between two parallel, horizontal, stationary flat surfaces at a constant velocity of 5 m/s as shown in the figure. The two stationary surfaces are spaced 4 cm apart, and the medium between them is filled with oil whose viscosity is 0.9 N $\cdot$ s/m^2. The part of the plate immersed in oil at any given time is 2-m long and 0.5-m wide. If the plate moves through the mid-plane between the surfaces, determine the force required to maintain this motion. What would your response be if the plate was 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1)?

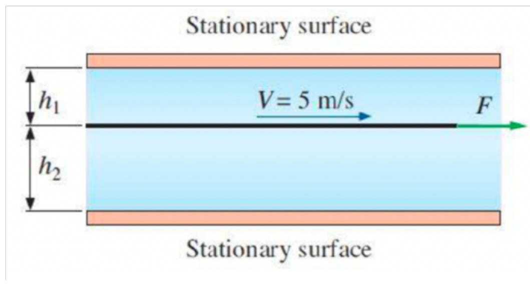

Figure 1: Problem diagram for Question 1.

(a)

The plate moves through the mid-plane between the surfaces.

Assumptions:

- Oil is a Newtonian fluid
- The velocity profile is linear

Since the velocity profile is linear, the shear stress can be calculated using the following equation:

$$\tau = \mu \frac{du}{dy} \tag{1}$$

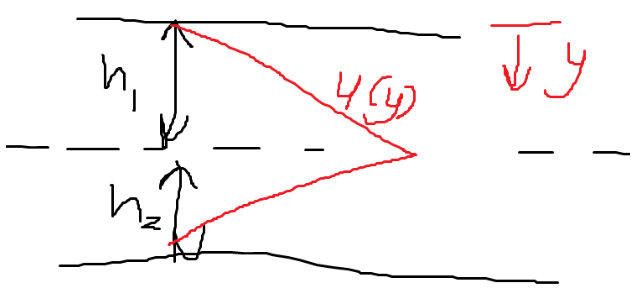

Figure 2: Velocity profile for Question 1.

The velocity as a function of y is:

$$u(y) = \frac{u_{max}}{h_1} y \tag{2}$$

$$\frac{du}{dy} = \frac{u_{max}}{h_1} \tag{3}$$

Substituting Equation (3) into Equation (1) yields:

$$\begin{aligned} \tau_{top} &= \mu \frac{u_{max}}{h_1} \\ &= 0.9 \frac{5}{0.02} \\ &= 225\,\mathrm{N/m^2} \end{aligned}$$

Similarly, $\tau_{bottom} = 225\,\mathrm{N/m^2}$.

The force required to maintain the motion is:

$$\begin{aligned} F &= \tau_{top} A \\ &= 225 \cdot 2 \cdot 0.5 \\ &= 225\mathrm{N} \end{aligned}$$

By symmetry, the force required to maintain the motion is the same for the bottom surface. Therefore,

$$\boxed{F = 450\,\mathrm{N}}$$

(b)

The plate is 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1).

By similar methods,

$$\begin{aligned} F = F_{top} + F_{bottom} &= \tau_{top} A + \tau_{bottom} A \\ &= \mu A \left(\frac{u_{max}}{h_1} + \frac{u_{max}}{h_2} \right) \\ &= 0.9 \cdot 2 \cdot 0.5 \left(\frac{5}{0.03} + \frac{5}{0.01} \right) \\ &= 0.9(166.67 + 500) \\ &= \boxed{600\,\mathrm{N}} \end{aligned}$$