Question 1

Consider the following steady, incompressible, two-dimensional velocity field:

$$\vec{V} = (u, v) = (4.35 + 0.656x)\hat{(i)} + (-1.22 - 0.656y)\hat{\jmath}$$

Generate an analytical expression for the flow streamlines.

(a)

Since we're in two dimensions, the differential that governs the streamline is:

$$\frac{dx}{u} = \frac{dy}{v}$$
$$\frac{dx}{4.35 + 0.656x} = \frac{dy}{-1.22 - 0.656y}$$
$$(1.22 + 0.656y)dx + (4.35 + 0.656x)dy = 0$$

The existence of a function $f(x, y)$ such that $df = 0$ must be verified. By exactness,

$$\frac{\partial}{\partial y}(1.22 + 0.656y) = \frac{\partial}{\partial x}(4.35 + 0.656x)$$
$$0.656 = 0.656$$

Therefore, $f(x, y)$ exists. Integrating the dx term:

$$\int (1.22 + 0.656y)dx = 1.22x + 0.656xy + g(y)$$

Differentiating with respect to y:

$$\frac{\partial}{\partial y}(1.22x + 0.656xy + g(y)) = (4.35 + 0.656x)$$
$$0.656x + g'(y) = 4.35 + 0.656x$$
$$g'(y) = 4.35$$
$$g(y) = 4.35y + C$$

Therefore, the streamlines are given by:

$$\boxed{f(x, y) = 1.22x + 0.656xy + 4.35y = C}$$

Question 2

A system is shown in the figure. Air ($\rho = 0.97\,\mathrm{kg/m^3}$) flows into surface 1 and flows out of surface 2. If $v_1 = 5.6\,\mathrm{m/s}$, determine:

(a) the mass flow rate of air, and

(b) v_2.

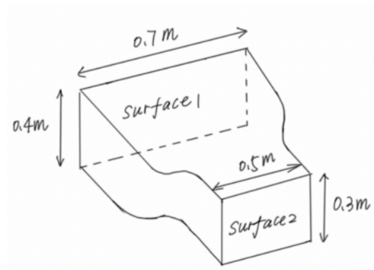

Figure 1: Piston-cylinder device

(a)

Assumptions

1. Steady flow
2. Incompressible flow
3. Constant density

The mass flow rate of air is given by:

$$\begin{aligned}\dot{m} &= \rho A_1 v_1 \\ &= (0.97)(0.4 \times 0.7)(5.6) \\ &= \boxed{1.521\,\mathrm{kg\,s^{-1}}}\end{aligned}$$

(b)

By conservation of mass, the mass flow rate at surface 2 is equal to the mass flow rate at surface 1, assuming steady flow.

$$\begin{aligned}v_2 = \frac{\dot{m}}{\rho A} &= \frac{1.521}{(0.97)(0.5 \times 0.3)} \\ &= \boxed{10.454\,\mathrm{m\,s^{-1}}}\end{aligned}$$

Question 3

Consider the flow of an incompressible Newtonian fluid between two parallel plates that are 4 mm apart. If the upper plate moves to the right with $u_1 = 5\,\mathrm{m/s}$ while the bottom one moves to the left with $u_2 = 1.5\,\mathrm{m/s}$, what would be the net flow rate at a cross-section between the two plates? Take the plate width to be 5 cm.

(a)

Assumptions

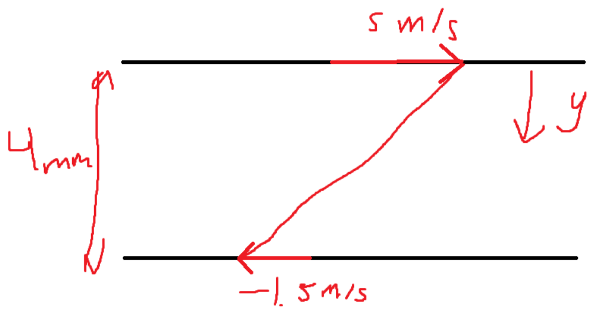

Figure 2: Flow between two parallel plates.

1. Steady flow
2. Incompressible flow
3. No slip at the boundaries

First, denote y as the direction normal to the top plate to the bottom plate. Given the boundary conditions, the velocity profile is linear, and is given by:

$$\begin{aligned}
u(y) &= \frac{u2 - u1}{h}(y) + u_1 \\
&= \frac{-1.5 - 5}{0.004}y + 5 \\
&= -1625y + 5 \; [\mathrm{m\,s^{-1}}]
\end{aligned}$$

The net flow rate of a cross-section between the two plates is given by the integral of the velocity profile times width at the cross-section:

$$\begin{aligned}
Q &= \int_0^{0.04} u(y)w\,dy \\
&= \int_0^{0.04} (-1625y + 5)(0.05)\,dy \\
&= 0.00035 \\
&= \boxed{3.50 \times 10^{-4}\,\mathrm{m^3\,s^{-1}}}
\end{aligned}$$

Question 4

Air enters a nozzle steadily at $\rho = 2.21\,\mathrm{kg\,m^{-3}}$ and $u = 20\,\mathrm{m\,s^{-1}}$ and leaves at $\rho = 0.762\,\mathrm{kg\,m^{-3}}$ and $u = 150\,\mathrm{m\,s^{-1}}$. If the inlet area of the nozzle is $A_1 = 60\,\mathrm{cm^2}$, determine

(a) the mass flow rate through the nozzle, and

(b) the exit area of the nozzle.

(a)

First determine the mass flow rate through the nozzle. The mass flow rate is given by:

$$\begin{aligned}\dot{m} &= \rho_1 A_1 u_1 \\ &= 2.21 \times 60(0.01)^2 \times 20 \\ &= \boxed{0.265\,\mathrm{kg\,s^{-1}}}\end{aligned}$$

(b)

Next, determine the exit area of the nozzle. Since mass flow rate is constant throughout the nozzle, the mass flow rate at the exit is the same as the mass flow rate at the inlet.

$$\begin{aligned}A_2 &= \frac{\dot{m}}{\rho_2 u_2} \\ &= \frac{0.265}{0.762 \times 0.01^2 \times 150} \\ &= \boxed{23.2\,\mathrm{cm}^2}\end{aligned}$$