

Question 1

A thin plate moves between two parallel, horizontal, stationary flat surfaces at a constant velocity of 5 m/s as shown in the figure. The two stationary surfaces are spaced 4 cm apart, and the medium between them is filled with oil whose viscosity is $0.9 \text{ N} \cdot \text{s}/\text{m}^2$. The part of the plate immersed in oil at any given time is 2-m long and 0.5-m wide. If the plate moves through the mid-plane between the surfaces, determine the force required to maintain this motion. What would your response be if the plate was 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1)?

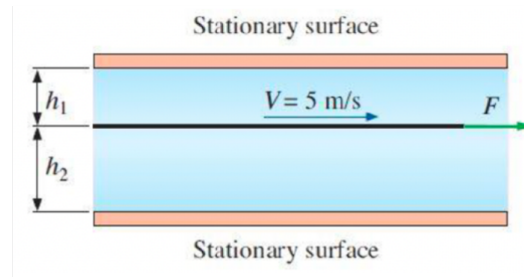


Figure 1: Plate moving between two parallel surfaces.

(a)

The plate moves through the mid-plane between the surfaces.

Assumptions:

- Oil is a Newtonian fluid
- The velocity profile is linear

Since the velocity profile is linear, the shear stress can be calculated using the following equation:

$$\tau = \mu \frac{du}{dy} \quad (1)$$

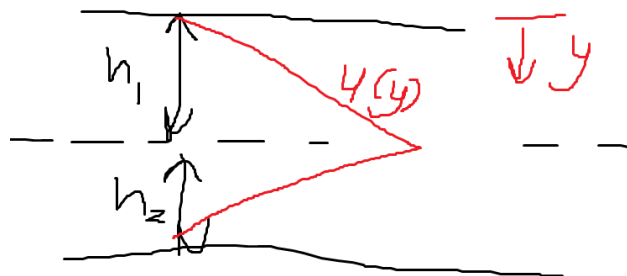


Figure 2: Velocity profile for Question 1.

The velocity as a function of y is:

$$u(y) = \frac{u_{max}}{h_1} y \quad (2)$$

$$\frac{du}{dy} = \frac{u_{max}}{h_1} \quad (3)$$

Substituting Equation (3) into Equation (1) yields:

$$\begin{aligned} \tau_{top} &= \mu \frac{u_{max}}{h_1} \\ &= 0.9 \frac{5}{0.02} \\ &= 225 \text{ N/m}^2 \end{aligned}$$

Similarly, $\tau_{bottom} = 225 \text{ N/m}^2$.

The force required to maintain the motion is:

$$\begin{aligned} F &= \tau_{top} A \\ &= 225 \cdot 2 \cdot 0.5 \\ &= 225 \text{ N} \end{aligned}$$

By symmetry, the force required to maintain the motion is the same for the bottom surface. Therefore,

$$\boxed{F = 450 \text{ N}}$$

(b)

The plate is 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1).

By similar methods,

$$\begin{aligned} F &= F_{top} + F_{bottom} = \tau_{top} A + \tau_{bottom} A \\ &= \mu A \left(\frac{u_{max}}{h_1} + \frac{u_{max}}{h_2} \right) \\ &= 0.9 \cdot 2 \cdot 0.5 \left(\frac{5}{0.03} + \frac{5}{0.01} \right) \\ &= 0.9(166.67 + 500) \\ &= \boxed{600 \text{ N}} \end{aligned}$$

Question 2

A gas is contained in a vertical, frictionless piston-cylinder device. The piston has a mass of 5 kg and a cross-sectional area of 35 cm². A compressed spring above the piston exerts a force of 75 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.

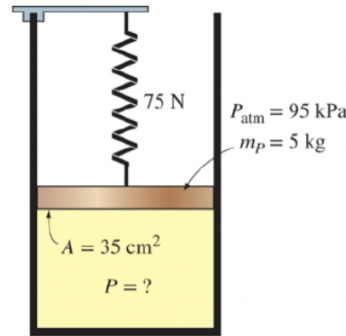


Figure 3: Piston-cylinder device

The freebody diagram is:

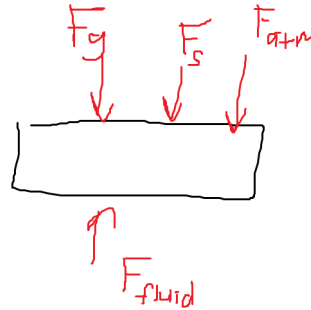


Figure 4: Freebody diagram of the piston

The force balance is:

$$F_{fluid} = F_s + F_{atm} + F_g \quad (4)$$

$$P_{fluid}A_p = F_s + P_{atm}A_p + mg \quad (5)$$

$$P_{fluid} = \frac{F_s}{A_p} + P_{atm} + \frac{mg}{A_p} \quad (6)$$

$$(7)$$

Substituting values:

$$P_{fluid} = \frac{75}{0.0035} + 95000 + \frac{5 \times 9.81}{0.0035} \quad (8)$$

$$= 130\,442.857\,143 \text{ Pa} \quad (9)$$

$$= \boxed{130.4 \text{ kPa}} \quad (10)$$

Question 3

The system shown in the figure is used to accurately measure changes when the pressure is increased by ΔP in the water pipe. When $\Delta h = 70 \text{ mm}$, what is the change in the pipe pressure?

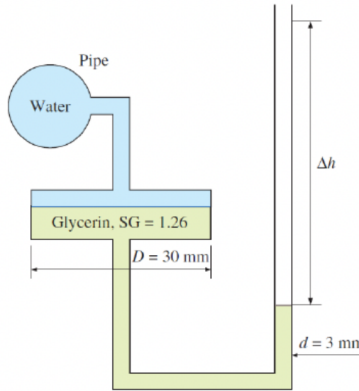


Figure 5: Convoluted manometer diagram

For some pressure of water P_w and a datum to measure the height of the glycerin h_2 , the pressure balance is given by:

$$P_{w,1} - P_{atm} = \rho_g g h_2 \quad (11)$$

$$P_{w,2} - P_{atm} = \rho_g g (h_2 + \Delta h) \quad (12)$$

Observe that the width of the manometer does **not** contribute to the pressure balance.

Subtracting (11) from (12) yields:

$$\Delta P = \rho_g g \Delta h \quad (13)$$

To find the density of glycerin given $SG = 1.26$, we can use the following equation:

$$\rho_g = \rho_{H_2O} \times SG = 1000 \times 1.26 = 1260 \text{ kg m}^{-3} \quad (14)$$

Substituting this into (13) yields:

$$\Delta P = 1260 \times 9.81 \times 0.07 = \boxed{865.2 \text{ Pa}}$$