Question 1

A thin plate moves between two parallel, horizontal, stationary flat surfaces at a constant velocity of 5 m/s as shown in the figure. The two stationary surfaces are spaced 4 cm apart, and the medium between them is filled with oil whose viscosity is 0.9 $\mathrm{N \cdot s/m^2}$. The part of the plate immersed in oil at any given time is 2-m long and 0.5-m wide. If the plate moves through the mid-plane between the surfaces, determine the force required to maintain this motion. What would your response be if the plate was 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1)?

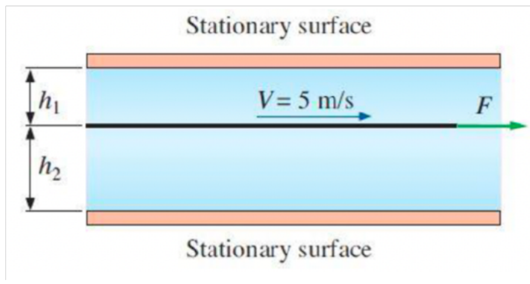

Figure 1: Plate moving between two parallel surfaces.

(a)

The plate moves through the mid-plane between the surfaces.

Assumptions:

- Oil is a Newtonian fluid
- The velocity profile is linear

Since the velocity profile is linear, the shear stress can be calculated using the following equation:

$$\tau = \mu \frac{du}{dy} \tag{1}$$

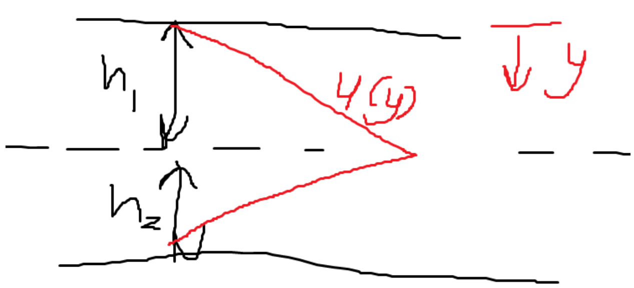

Figure 2: Velocity profile for Question 1.

The velocity as a function of y is:

$$u(y) = \frac{u_{max}}{h_1} y \tag{2}$$

$$\frac{du}{dy} = \frac{u_{max}}{h_1} \tag{3}$$

Substituting Equation (3) into Equation (1) yields:

$$\begin{aligned} \tau_{top} &= \mu \frac{u_{max}}{h_1} \\ &= 0.9 \frac{5}{0.02} \\ &= 225\,\mathrm{N/m^2} \end{aligned}$$

Similarly, $\tau_{bottom} = 225\,\mathrm{N/m^2}$.

The force required to maintain the motion is:

$$\begin{aligned} F &= \tau_{top} A \\ &= 225 \cdot 2 \cdot 0.5 \\ &= 225\mathrm{N} \end{aligned}$$

By symmetry, the force required to maintain the motion is the same for the bottom surface. Therefore,

$$\boxed{F = 450\,\mathrm{N}}$$

(b)

The plate is 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1).

By similar methods,

$$\begin{aligned} F = F_{top} + F_{bottom} &= \tau_{top} A + \tau_{bottom} A \\ &= \mu A \left(\frac{u_{max}}{h_1} + \frac{u_{max}}{h_2} \right) \\ &= 0.9 \cdot 2 \cdot 0.5 \left(\frac{5}{0.03} + \frac{5}{0.01} \right) \\ &= 0.9(166.67 + 500) \\ &= \boxed{600\,\mathrm{N}} \end{aligned}$$

Question 2

A gas is contained in a vertical, frictionless piston-cylinder device. The piston has a mass of 5 kg and a cross-sectional area of 35 cm^2. A compressed spring above the piston exerts a force of 75 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.

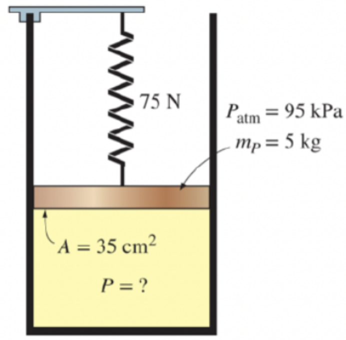

Figure 3: Piston-cylinder device

The freebody diagram is:

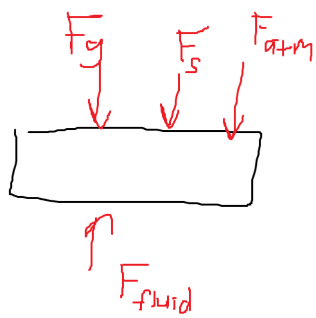

Figure 4: Freebody diagram of the piston

The force balance is:

$$F_{fluid} = F_s + F_{atm} + F_g \tag{4}$$
$$P_{fluid}A_p = F_s + P_{atm}A_p + mg \tag{5}$$
$$P_{fluid} = \frac{F_s}{A_p} + P_{atm} + \frac{mg}{A_p} \tag{6}$$
$$\tag{7}$$

Substituting values:

$$
\begin{aligned}
P_{fluid} &= \frac{75}{0.0035} + 95000 + \frac{5 \times 9.81}{0.0035} && (8)\\
&= 130\,442.857\,143\,\mathrm{Pa} && (9)\\
&= \boxed{130.4\,\mathrm{kPa}} && (10)
\end{aligned}
$$

Question 3

The system shown in the figure is used to accurately measure changes when the pressure is increased by ΔP in the water pipe. When $\Delta h = 70$ mm, what is the change in the pipe pressure?

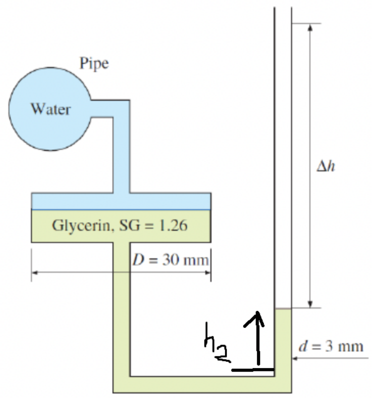

Figure 5: Convoluted manometer diagram

For some pressure of water P_w and a datum to measure the height of the glycerin h_2, the pressure balance is given by:

$$
\begin{aligned}
P_{w,1} - P_{atm} &= \rho_g g h_2 && (11)\\
P_{w,2} - P_{atm} &= \rho_g g (h_2 + \Delta h) && (12)
\end{aligned}
$$

Observe that the width of the manometer does **not** contribute to the pressure balance.

Subtracting (11) from (12) yields:

$$\Delta P = \rho_g g \Delta h \quad (13)$$

To find the density of glycerin given SG = 1.26, we can use the following equation:

$$\rho_g = \rho_{H_2O} \times SG = 1000 \times 1.26 = 1260\,\mathrm{kg\,m^{-3}} \quad (14)$$

Substituting this into (13) yields:

$$\Delta P = 1260 \times 9.81 \times 0.07 = \boxed{865.2\,\mathrm{Pa}}$$

Question 4

The gage pressure of the air in the tank shown in the figure below is measured to be 50 kPa. Determine the differential height h of the mercury column.

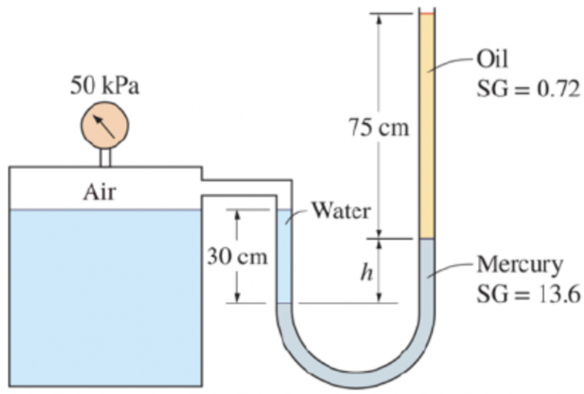

Figure 6: Cylinder rolling on a board.