MEC E 331 Formula Sheet

1. Pre-midterm Stuff

1.1 Variable Definitions and Terms

- Fluid: A fluid is a substance that deforms continuously under the application of shear (tangential) stress
- Fluid Mechanics: deals with fluids at rest (fluid statics) and fluids in motion (fluid dynamics)
- Internal flow: fluid is bounded by solid surface i.e. flow in a pipe or duct
- External flow: unbounded flow or a fluid over a surface i.e. airplane wing air
- Laminar flow: smooth orderly layered flow - low velocities, high viscosity fluids
- Turbulent flow: highly disordered flow patterns - low viscosity and/or high speed
- Specific Gravity: the ratio of the density of a substance to the density of some standard substance at a specified temp
- Specific weight: the weight of a unit volume of a substance. Is equal to density times gravity
- Viscosity: A property that represents the internal resistance of a fluid to motion or the "fluidity". Viscosity relates the local stress in a moving fluid to the strain rate of the fluid element
- Newtonian fluids: When shear stress is proportional to strain rate
- Pseudoplastic: the more the fluid is sheared the less viscous it becomes
- Dilatant: The more the fluid is sheared the more viscous it becomes
- Eulerian: concerned with the fluid properties at a specific space-time point
- Lagrangian: concerned with a particular particle of fluid as it moves through space at time
- Streamline: A streamline is a curve that is everywhere tangent to the instantaneous local velocity vector
- Pathlines: A pathline is the actual path traveled by an individual fluid particle
- Streaklines: A streakline is the locus of fluid particles that have passed sequentially through a prescribed point in the flow
- Timeline: A set of adjacent fluid particles marked as the same (earlier) instant in time
- Streamtube: consists a bundle of streamlines
- Control system: consists of a fixed amount of mass, and no mass can cross the boundary
- Control volume: a volume in space
- Reynolds Transport Theorem: the relationship between time rates of change of extensive property for a system and for a control volume
- 1st Law of Thermodynamics: Energy cannot be created or destroyed
- 2nd Law of Thermodynamics: For a spontaneous process, the entropy of the universe increases

1.2 Formulas

1.2.1 Steady Flow

Steady laminar flow means linear velocity profile, Newtonian fluid stuff:

$$\tau = \mu \frac{du}{dy}$$

$$u = \frac{V-0}{h-0} y$$

For force,

$$F = \tau A = \mu \frac{du}{dy} A$$

1.2.2 Manometer

Manometer stuff:

i) Start at one end of known pressure. If open to atmosphere then use P_{atm}.

ii) If going down, add $\rho g h$ to the pressure. If going up, subtract $\rho g h$ from the pressure.

$$P_1 + \sum \rho_i g h_i = P_2$$

1.2.3 Vector Fields

Vector field stuff: Stagnation point when $\vec{V} = 0$. Acceleration field:

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Streamline: For 2D, you can check exactness by

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$

you can solve for streamline with

$$udx + vdy = 0$$

by solving for y in terms of x.

1.2.4 Reynolds Transport Theorem

The statement of the Reynolds Transport Theorem is

$$\frac{dN_{\text{sys}}}{dt} = \frac{d}{dt}\int_{\text{CV}} \eta\rho dV + \int_{\text{CS}} \eta\rho(\vec{V}\cdot\hat{n})dA$$

which says time rate of change of property N = time rate of change of property N in CV + net rate of flux of N out of CV.

5. & 8. Bernoulli-Energy Methods

5.1 General Procedure

1. There are 2 equations that are generally useful for these types of problems:
   i) Bernoulli's equation. Valid in regions of steady, incompressible flow where net frictional forces are negligible.
   ii) Mass flow rate: $\dot{m} = \rho A\dot{x} = \rho V$
2. Identify the assumptions so the appropriate equations can be used.
3. Try and cancel out as many terms as possible from the Bernoulli equation. Use mass flow rate to determine $\dot{x}$.
4. Use energy methods to determine the pressure head if necessary.

5.2 Variable Definitions and Terms

- P: Pressure
- $\dot{x}$: Velocity
- z: Elevation
- V: Volume
- C_d: Discharge coefficient
- β: Ratio of throat diameter to pipe diameter d/D
- Bernoulli's eqn = Provides a good approximation and is valid in regions of steady, incompressible flow where net frictional forces are negligible
- Static pressure = the actual thermodynamic pressure
- Dynamic pressure = represents the pressure rise when fluid motion is brought to a stop isentropically
- Hydrostatic pressure = value depends on the reference level selected where z=0. A 'potential' pressure
- Total pressure = = static pressure + dynamic pressure hydrostatic pressure
- Pressure head = height of fluid needed to produce a static pressure P
- Velocity head = height of fluid needed to produce velocity V during a free, frictionless vertical fall
- Elevation head = height relative to some reference plate z=0
- Hydraulic grade line (HGL) = the line that represents pressure and elevation heads
- Energy grade line (EGL) = the line that represents total pressure head

5.3 Formulas

Classic Bernoulli Equations:

Bernoulli's Equation: $\frac{P_1}{\rho} + \frac{\dot{x}_1^2}{2} + gz_1 = \frac{P_2}{\rho} + \frac{\dot{x}_2^2}{2} + gz_2$

Mass Conservation: $\Delta m_{\text{CV}} = \dot{m}_{\text{in}} - \dot{m}_{\text{out}}$

Mass Flow Rate: $\dot{m} = \rho A\dot{x} = \rho V$

Obstruction flowmeter:

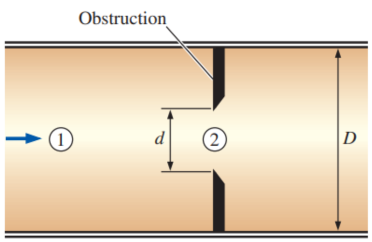

Figure 1: Obstruction flowmeter

Obstruction flowmeter: $\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$

Mass Balance : $\implies \dot{x}_1 = (d/D)^2 \dot{x}_2$

Head Loss: $h_L = \frac{P_1}{\rho g} + \frac{\dot{x}_1^2}{2g} + z_1 - \frac{P_2}{\rho g} - \frac{\dot{x}_2^2}{2g} - z_2$

6. Momentum Analysis of Flow Systems

6.1 General Procedure

1. Utilize the Bernoulli equation to obtain the $P_{1,\text{gage}}$
2. $\sum \vec{F}$ represents external forces acting on the system. Some examples are:
   i) Pressure: $P_{1,\text{gage}} A_1$
   ii) Reaction force: F_R
3. Use momentum equation to obtain forces. For uniform flow, $\beta = 1$. If not given, it is expected to assume uniform flow.

6.2 Variable Definitions

- β: Momentum-flux correction factor. It's a correction factor for the surface integral.

[1] $(\vec{V} \cdot \nabla)$ is the convective derivative operator, not $\text{div}(\vec{V})$

6.3 Formulas

Momentum Equation: $\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$

Momentum Correction Factor: $\beta = \begin{cases} 4/3 & \text{laminar} \\ 1 & \text{turbulent} \end{cases}$

8. Internal Flow: Pipes and Ducts

8.2 Variable Definitions

- Pressure drop: A pressure drop due to viscous effects represents an irreversible pressure loss
- Head loss: Additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe
- Major loss: The viscous (frictional) losses for fully-developed flow through straight pipe section
- Minor loss: Additional losses at the pipe.
- K_L = Loss coefficient
- α = Kinetic energy correction factor

8.3 Formulas

$$D_h = \frac{4A_c}{\text{wetted perimeter}}$$

$$\text{Re} = \frac{\rho V D_h}{\mu}$$

$$L_{h,\text{laminar}} = 0.05\text{Re}D$$

$$L_{h,\text{turbulent}} = 10D$$

Pressure stuff

$$\Delta P = \frac{32\mu L \dot{x}_{\text{avg}}}{D^2}, \text{ Laminar}$$

$$\Delta P_L = f\frac{L}{D}\frac{\rho \dot{x}_{\text{avg}}^2}{2}, \text{ Either}$$

$$h_L = f\frac{L}{D}\frac{\dot{x}_{\text{avg}}^2}{2g}, \text{ Either}$$

$$f = \frac{8\tau_w}{\rho \dot{x}_{\text{avg}}^2} \overset{\text{laminar}}{=} \frac{64}{\text{Re}}$$

Horizontal pipe, laminar (Re< 2300):

$$\dot{x}_{\text{avg}} = \frac{\Delta P D^2}{32\mu L}$$

$$\dot{V}_{\text{avg}} = \frac{\Delta P \pi D^4}{128\mu L}$$

Turbulent flow through pipe:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\epsilon}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Energy equation (**$\alpha = 2$ for laminar, $\alpha = 1.05$ for turbulent**)

$$\frac{P_1}{\rho g} + \alpha_1\frac{\dot{x}_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2\frac{\dot{x}_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

Loss coefficient

$$h_{L,\text{minor}} = K_L\frac{\dot{x}^2}{2g}$$

for sudden expansion,

$$K_L = \alpha\left(1 - \frac{d_{\text{small}}}{d_{\text{large}}}\right)^2$$

pump work,

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P$$

9. Differential Analysis of Fluid Flow

9.1 General Procedure

1. Most problems will be simplifiable to 2D or 1D because full form Navier-Stokes equations are too difficult to solve. The art of these problems is to simplify the equations to a form that can be solved.
2. Common assumptions are: steady, laminar, incompressible, constant viscosity, constant pressure, constant temperature, and parallel flow (velocity only in one direction). Gravity typically acts in the negative z-direction (unless it's like an inclined plane where you'd set your coordinates to be tangential and normal to the plane).
3. Check the problem statement for these key words:
   i) Steady: All $\frac{\partial}{\partial t} = 0$
   ii) Laminar: Generally implies parallel flow, flow in one direction only.
   iii) Incompressible: $\text{div}(\vec{V}) = \nabla \cdot \vec{V} = 0$, $\frac{\partial \rho}{\partial t} = 0$
   iv) Pressure acts in only one-direction: $\frac{\partial P}{\partial x} = 0$, $\frac{\partial P}{\partial y} = 0$, or $\frac{\partial P}{\partial z} = 0$
   v) Parallel flow: Velocity in the direction of motion is non-zero, velocity in the other directions is zero.
   vi) Gravity only in z-direction: $\vec{g} = -g\hat{k}$
4. Boundary conditions:
   i) No-slip: $\vec{V}_{\text{fluid}} = \vec{V}_{\text{boundary}}$ at an interface boundary.
   ii) No-shear at a : $\tau_{\text{fluid}} = \tau_{\text{boundary}} \approx 0$ at a free surface boundary with small surface tension like air.
5. Try to simplify the continuity equation first. Use the results in simplifying the Navier-Stokes equation.
6. Solve for whatever is asked for in the problem statement.

9.2. Operator Definitions

- ∇: The gradient operator, $\nabla = \frac{\partial}{\partial x}\hat{\imath} + \frac{\partial}{\partial y}\hat{\jmath} + \frac{\partial}{\partial z}\hat{k}$

- $\frac{\partial \vec{V}}{\partial x}$: The vector partial derivative, $\frac{\partial \vec{V}}{\partial x} = \frac{\partial u}{\partial x}\hat{i} + \frac{\partial v}{\partial x}\hat{j} + \frac{\partial w}{\partial x}\hat{k}$
- $\frac{D}{Dt}$: The material derivative, $\frac{D\vec{T}}{Dt} = \frac{\partial \vec{T}}{\partial t} + (\vec{V}\cdot\nabla)\vec{T}$ [1]
- $(\vec{V}\cdot\nabla)$: The convective derivative, $(\vec{V}\cdot\nabla)\vec{T} = u\frac{\partial \vec{T}}{\partial x} + v\frac{\partial \vec{T}}{\partial y} + w\frac{\partial \vec{T}}{\partial z}$
- ∇^2: The Laplacian operator, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

9.3 Variable Definitions

- $\vec{V}$: Velocity vector, $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$
- ρ: Density
- μ: Viscosity
- P: Pressure

9.4 Formulas

Continuity Equation: $\dfrac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{V}) = 0$

Special Case 1: Steady Compressible Flow: $\nabla\cdot(\rho\vec{V}) = 0$

Special Case 2: Incompressible Flow: $\nabla\cdot\vec{V} = 0$

Incompressible flow, Newtonian, Navier-Stokes Equation:

$$\rho\frac{D\vec{V}}{Dt} = -\nabla P + \rho\vec{g} + \mu\nabla^2\vec{V}$$

For example in x-direction:

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

9.5 General Terms

- Control volume analysis: very useful tool to engineering for flow analysis. Gives 'engineering analysis' answer, sometimes crude approximation, but always useful. A method of analysis in which a volume in space is selected and the conservation of mass, momentum, and energy are applied to the volume
- Differential analysis: in principle can be used for any problems. In practice, limited cases where exact analytical solutions exist. Nowadays, CFD (computational fluid dynamics) simulations are widely performed based on differential analysis. Involves application of differential equations of fluid motion to any and every point in the flow field over a region called the flow domain. Experimental (dimensional) analysis: based on the results of experiments. Technique to derive the most use out of the fewest number of experiments (which cost time and money)

10. Boundary Layer Approximation

10.1 General Procedure

1. Identify the type of flow using the Reynolds number. If the Re $> 5\times10^5$, the flow is turbulent. If the Re $< 5\times10^5$, the flow is laminar.
2. Use Table 1 to determine whatever you need.

10.2 Variable Definitions

- Re_x = Reynolds number, the ratio of inertial forces to viscous forces, at x
- δ = Boundary layer thickness is the distance from the wall to the point where the velocity is 99% of the free stream velocity.
- $\delta*$ = Displacement thickness is the distance that a streamline just outside of the boundary layer is deflected away from the wall due to the effect of the boundary layer.
- θ = Momentum thickness, defined as the loss of momentum flux per unit width decided by ρU^2 due to the presence of the growing boundary layer.
- τ_w = Wall shear stress, the force per unit area exerted by the fluid on the wall.
- C_f = Local friction coefficient, the ratio of the wall shear stress to the dynamic pressure.
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10.3 Formulas

$\mathrm{Re}_x = \dfrac{\rho V x}{\mu} = \dfrac{Vx}{\nu}$

Boundary Layer Thickness: $\dfrac{\delta}{x} = 4.91\sqrt{\mathrm{Re}_x}$

Wall Shear Stress: $\tau_w = \dfrac{0.332\rho U^2}{\sqrt{\mathrm{Re}_x}}$

Local Friction Coefficient: $C_f = \dfrac{\tau_w}{\frac{1}{2}\rho U^2} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$

Displacement Thickness: $\delta^* = \displaystyle\int_0^\infty \left(1 - \frac{u}{U}\right)dy = \frac{1.72x}{\sqrt{\mathrm{Re}_x}}$

Momentum Thickness: $\theta = \displaystyle\int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right)dy = \frac{0.664x}{\sqrt{\mathrm{Re}_x}}$

Drag Force: $F_D = \displaystyle\int_A \tau_w dA = \int_0^L \tau_w w dx$

Continuity Equation: $\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

Momentum Equation: $u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = U\dfrac{dU}{dx} + \nu\dfrac{\partial^2 u}{\partial y^2}$

Table 1: Boundary Layer Approximation for flat plate.

Laminar	Turbulent
$\frac{\delta}{x} = 4.91\sqrt{\mathrm{Re}_x}$	$\frac{\delta}{x} = \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$
$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} = \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$
$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} = \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$
$C_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_f = \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$
$\tau_w = \frac{0.332\rho U^2}{\sqrt{\mathrm{Re}_x}}$	$\tau_w = \frac{0.013\rho U^2}{(\mathrm{Re}_x)^{1/7}}$

11. External Flow: Drag and Lift

11.1 General Procedure

1. Determine whether to consider drag and/or lift.
2. Find the Reynolds number, Re_x to determine whether the flow is laminar or turbulent.
3. Determine the drag coefficient, C_D using a table.
4. Make sure to use the frontal area for drag and planform area for lift.
5. If a composite body is given, use superposition, i.e. $C_D = \sum C_{D_i}$.

11.2 Variable Definitions and Terms

- F_D = Drag force, the force component in the direction of the flow velocity.
- F_L = Lift force, the force component normal to the flow velocity.
- Frontal Area = The area projected onto a plane normal to the flow direction.
- Planform Area = The area seen by a person looking down on the object.
- Pressure drag = The difference between the high pressure in the front stagnation region and the low pressure in the shear separated region causes a large drag contribution
- Skin friction drag = Drag induced by τ_w.
- Flow separation = At sufficiently high velocities, the fluid stream detaches itself from the surface of the body.
- Seperated region = The low pressure region behind the body where recirculating and backflows occur
- Wake = The region of flow trailing the body where the effects of the body on velocity are felt
- The Magnus effect = The phenomenon of producing lift by the rotation of a solid body

11.3 Formulas

Drag, F_D:

$$C_{D,\text{friction}} = \frac{2F_{D,\text{friction}}}{\rho\dot{x}^2 A}$$

$$C_{D,\text{pressure}} = \frac{2F_{D,\text{pressure}}}{\rho\dot{x}^2 A}$$

$$C_D = C_{D,\text{friction}} + C_{D,\text{pressure}}$$

$$F_D = F_{D,\text{friction}} + F_{D,\text{pressure}}$$

$$W_D = F_D\dot{x}$$

Lift, F_L:

$$F_L = \frac{1}{2}\rho\dot{x}^2 A C_L$$