

Question 1

A tube acts as a water siphon. Determine the speed of jet and the minimum pressure of water in the bend (at the point A).

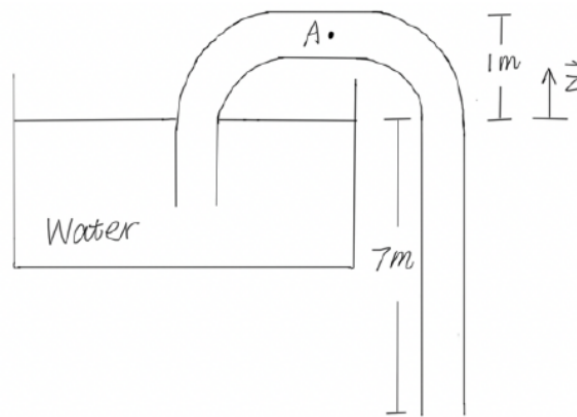


Figure 1: Tube with water siphon

Solution

Assumptions:

- Steady flow
- Incompressible flow
- Negligible viscous effects
- Negligible diameter change
- Reservoir is large enough to be considered infinite

By the Bernoulli equation, the speed of jet is given by:

$$\underbrace{\frac{P_1}{\rho} - \frac{P_2}{\rho}}_{\text{Both exposed to atm}} + \frac{v_2^2}{2} - \underbrace{\frac{v_1^2}{2}}_{\text{large reservoir}} + g(z_2 - z_1) = 0$$

$$\frac{v_2^2}{2} + g(z_2 - z_1) = 0$$

Which results in

$$\begin{aligned} v_2 &= \sqrt{2g(z_1 - z_2)} \\ &= \sqrt{2(9.81)(7)} \\ &= \boxed{11.7 \text{ m s}^{-1}} \end{aligned}$$

In the pipe, by the assumptions, the velocity is constant. Therefore taking the Bernoulli equation between the pipe exit and the bend,

$$\frac{P_A}{\rho} - \frac{P_2}{\rho} + \frac{v_A^2}{2} - \frac{v_2^2}{2} + g(z_A - z_2) = 0$$

$$\frac{P_A}{\rho} - \frac{P_2}{\rho} + g(z_A - z_2) = 0$$

Rearranging for P_A ,

$$\begin{aligned} P_A &= \rho g(z_2 - z_A) + P_2 \\ &= (1000)(9.81)(-7 - 1) + 101325 \\ &= \boxed{22.84 \text{ kPa}} \end{aligned}$$

Question 2

A pitot-static probe is used to measure the speed of an aircraft flying at 3000 m. If the differential pressure reading is 3 kPa, determine the speed of the aircraft. (The density of the atmosphere at an elevation of 3000 m is $\rho = 0.909 \text{ kg m}^{-3}$!)

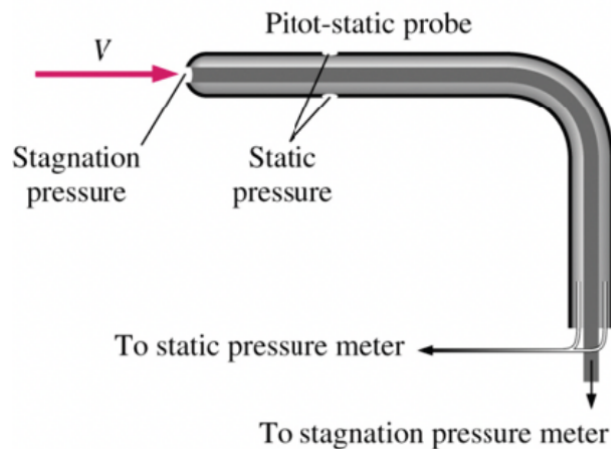


Figure 2: Pitot-static probe

Solution

Assumptions:

- Steady flow
- Incompressible flow
- Negligible viscous effects

At the stagnation point, the velocity is zero. By the Bernoulli equation,

$$\frac{P_1}{\rho} + \frac{P_2}{\rho} + \frac{v_1^2}{2} - \frac{v_2^2}{2} + \cancel{g(z_1 - z_2)} = 0$$

$$\frac{\Delta P}{\rho} + \frac{v_1^2}{2} = 0$$

Rearranging for v_1 ,

$$v_1 = \sqrt{2 \frac{\Delta P}{\rho}}$$

$$= \sqrt{2 \frac{3000}{0.909}}$$

$$= \boxed{81.2 \text{ m s}^{-1}}$$

Question 3

Water enters a tank of diameter 1 m steadily at a mass flow rate of \dot{m} . An orifice at the bottom with diameter D_o allows water to escape. The orifice has a rounded entrance, so the frictional losses are negligible. If the tank is initially empty, (a) determine the maximum height that the water will reach in the tank and (b) obtain a relation for water height z

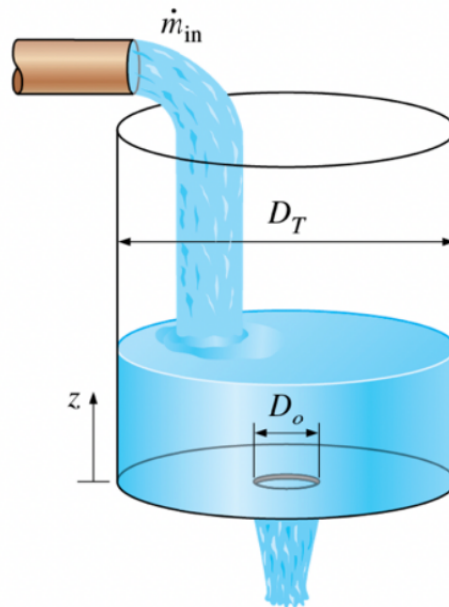


Figure 3: Water tank

Solution

(a)

Let the height of the water at the maximum height be h . At the maximum height, equilibrium is reached, so the mass flow rate in is equal to the mass flow rate out.

By the Bernoulli equation,

$$\underbrace{\frac{P_1}{\rho} - \frac{P_2}{\rho}}_{\text{Both at atm}} + \underbrace{\frac{v_1^2}{2}}_{\text{Sufficiently large}} - \frac{v_2^2}{2} + g(z_1 - z_2) = 0$$

$$h = \frac{v_2^2}{2g}$$

By the continuity equation,

$$\begin{aligned} \dot{m} &= \rho A_0 v_2 \\ \implies v_2 &= \frac{\dot{m}}{\rho A_0} \\ &= \frac{\dot{m}}{\rho \pi D_0^2 / 4} \end{aligned}$$

Substituting into the Bernoulli equation,

$$\begin{aligned} h &= \frac{v_2^2}{2g} \\ &= \frac{\left(\frac{\dot{m}}{\rho \pi D_0^2 / 4} \right)^2}{2g} \\ &= \boxed{\frac{8\dot{m}^2}{\rho^2 \pi^2 g D_0^4}} \end{aligned}$$

(b)

Let the height of the water at time t be $z(t)$. At a given time, the exit velocity is given by the Bernoulli equation,

$$\underbrace{\frac{P_1}{\rho} - \frac{P_2}{\rho}}_{\text{Both at atm}} + \underbrace{\frac{v_1^2}{2}}_{\text{Sufficiently large}} - \frac{v_2^2}{2} + g(z_1 - z_2) = 0$$

$$\implies v_2 = \sqrt{2gz(t)}$$

Accumulation of mass in the tank is given by the continuity equation,

$$\dot{V} = \dot{m} - A_0 v_2$$

Substituting expressions for V , A_0 , and v_2 ,

$$\begin{aligned}\frac{d}{dt} \left(\frac{\pi D_T^2}{4} z(t) \right) &= \dot{m} - \frac{\pi D_0^2}{4} \sqrt{2gz(t)} \\ \frac{\pi D_T^2}{4} \dot{z}(t) &= \dot{m} - \frac{\pi D_0^2}{4} \sqrt{2gz(t)}\end{aligned}$$

Observe this is a constant coefficient, first order, non-homogeneous, non-linear differential equation. I do not know how to solve this, nor does it seem particularly useful to solve this with tools such as ODE45. As such, I will just box the implicit relationship.

$$\boxed{\frac{\pi D_T^2}{4} \dot{z}(t) = \dot{m} - \frac{\pi D_0^2}{4} \sqrt{2gz(t)}}$$