Question 1

A thin plate moves between two parallel, horizontal, stationary flat surfaces at a constant velocity of 5 m/s as shown in the figure. The two stationary surfaces are spaced 4 cm apart, and the medium between them is filled with oil whose viscosity is 0.9 $N \cdot s/m^2$. The part of the plate immersed in oil at any given time is 2-m long and 0.5-m wide. If the plate moves through the mid-plane between the surfaces, determine the force required to maintain this motion. What would your response be if the plate was 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1)?

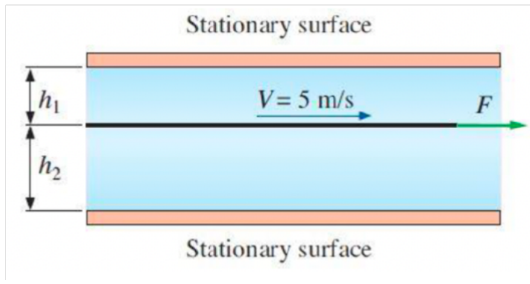

Figure 1: Plate moving between two parallel surfaces.

(a)

The plate moves through the mid-plane between the surfaces.

Assumptions:

- Oil is a Newtonian fluid
- The velocity profile is linear

Since the velocity profile is linear, the shear stress can be calculated using the following equation:

$$\tau = \mu \frac{du}{dy} \tag{1}$$

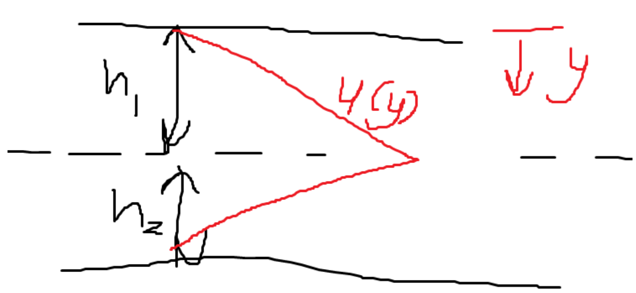

Figure 2: Velocity profile for Question 1.

The velocity as a function of y is:

$$u(y) = \frac{u_{max}}{h_1} y \tag{2}$$

$$\frac{du}{dy} = \frac{u_{max}}{h_1} \tag{3}$$

Substituting Equation (3) into Equation (1) yields:

$$\begin{aligned}\tau_{top} &= \mu \frac{u_{max}}{h_1} \\ &= 0.9 \frac{5}{0.02} \\ &= 225\,\mathrm{N/m^2}\end{aligned}$$

Similarly, $\tau_{bottom} = 225\,\mathrm{N/m^2}$.

The force required to maintain the motion is:

$$\begin{aligned}F &= \tau_{top} A \\ &= 225 \cdot 2 \cdot 0.5 \\ &= 225\mathrm{N}\end{aligned}$$

By symmetry, the force required to maintain the motion is the same for the bottom surface. Therefore,

$$\boxed{F = 450\,\mathrm{N}}$$

(b)

The plate is 1 cm from the bottom surface (h_2) and 3 cm from the top surface (h_1).

By similar methods,

$$\begin{aligned}F = F_{top} + F_{bottom} &= \tau_{top} A + \tau_{bottom} A \\ &= \mu A \left(\frac{u_{max}}{h_1} + \frac{u_{max}}{h_2} \right) \\ &= 0.9 \cdot 2 \cdot 0.5 \left(\frac{5}{0.03} + \frac{5}{0.01} \right) \\ &= 0.9(166.67 + 500) \\ &= \boxed{600\,\mathrm{N}}\end{aligned}$$

Question 2

A gas is contained in a vertical, frictionless piston-cylinder device. The piston has a mass of 5 kg and a cross-sectional area of 35 cm^2. A compressed spring above the piston exerts a force of 75 N on the piston. If the atmospheric pressure is 95 kPa, determine the pressure inside the cylinder.

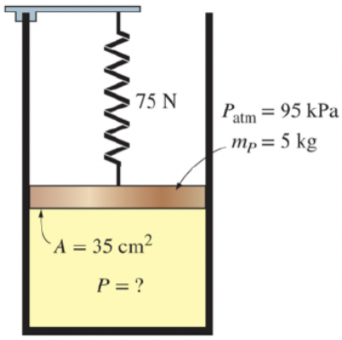

Figure 3: Piston-cylinder device

The freebody diagram is:

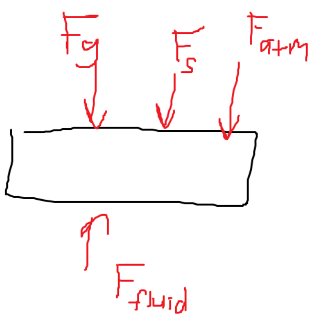

Figure 4: Freebody diagram of the piston

The force balance is:

$$F_{fluid} = F_s + F_{atm} + F_g \tag{4}$$
$$P_{fluid}A_p = F_s + P_{atm}A_p + mg \tag{5}$$
$$P_{fluid} = \frac{F_s}{A_p} + P_{atm} + \frac{mg}{A_p} \tag{6}$$
$$\tag{7}$$

Substituting values:

$$P_{fluid} = \frac{75}{0.0035} + 95000 + \frac{5 \times 9.81}{0.0035} \tag{8}$$
$$= 130\,442.857\,143\,\text{Pa} \tag{9}$$
$$= \boxed{130.4\,\text{kPa}} \tag{10}$$

Question 3

The system shown in the figure is used to accurately measure changes when the pressure is increased by ΔP in the water pipe. When $\Delta h = 70$ mm, what is the change in the pipe pressure?

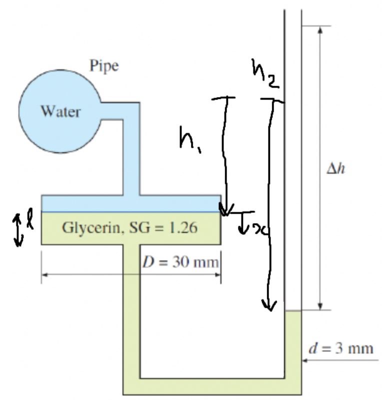

Figure 5: Convoluted manometer diagram

Assumptions:

- x is less than l
- Density is constant
- The manometer is open to the atmosphere

By conservation of mass, the volume displaced on the right side of the manometer is equal to the volume displaced on the left side of the manometer.

$$\Delta V_{left} = \Delta V_{right}$$
$$\Delta h \left(\frac{\pi}{4} D_{right}^2\right) = x \left(\frac{\pi}{4} D_{left}^2\right)$$
$$\implies x = \Delta h \left(\frac{D_{right}}{D_{left}}\right)^2$$
$$= 70 \times \left(\frac{3}{30}\right)^2$$
$$= 0.7\,\text{mm}$$

In the initial state, the pressure balance going from the water to the air is (look at where the fluid starts and ends; downwards displacement is positive, upwards displacement is negative):

$$P_{w,1} + \rho_w g h_1 + \rho_g g(h_2 - h_1) = P_{atm} \tag{11}$$

In the final state, the pressure balance is:

$$P_{w,2} + \rho_w g(h_1 + x) + \rho_g g(h_2 - \Delta h - x - h_1) = P_{atm} \tag{12}$$

Subtracting (11) from (12) yields:

$$P_{w,2} - P_{w,1} = -\rho_w g x + \rho_g g(\Delta h + x) \tag{13}$$

To find the density of glycerin given SG = 1.26, we can use the following equation:

$$\rho_g = \rho_{H_2O} \times SG = 1000 \times 1.26 = 1260\,\mathrm{kg\,m^{-3}}$$

Substituting this into (13) yields:

$$\Delta P = -1000 \times 9.81 \times \frac{0.7}{1000} + 1260 \times 9.81 \times \frac{70 + 0.7}{1000} = \boxed{867.0\,\mathrm{Pa}}$$

Question 4

The gauge pressure of the air in the tank shown in the figure below is measured to be 50 kPa. Determine the differential height h of the mercury column.

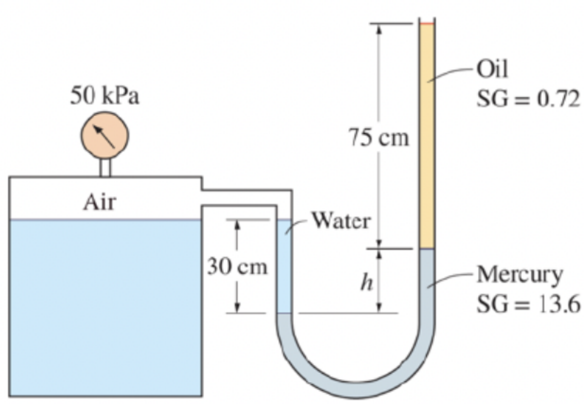

Figure 6: Monometer with multiple fluids

The pressure balance for the system is given by:

$$P_{\text{air}} + P_{\text{w}} - P_{\text{Hg}} - P_{\text{oil}} = P_{\text{atm}}$$

Isolating for P_{gauge}:

$$
\begin{aligned}
P_{\text{gauge}} &= P_{\text{air}} - P_{\text{atm}} \\
&= P_{\text{Hg}} + P_{\text{oil}} - P_{\text{w}} \\
&= \rho_{\text{Hg}} g h_1 + \rho_{\text{oil}} g h_2 - \rho_{\text{w}} g h_3 \\
\implies h_1 &= \frac{P_{\text{gauge}} + \rho_{\text{w}} g h_3 - \rho_{\text{oil}} g h_2}{\rho_{\text{Hg}} g} \qquad (14)
\end{aligned}
$$

Finding densities

$$
\begin{aligned}
\rho_w &= 1000\,\mathrm{kg\,m^{-3}} \\
\rho_{\text{Hg}} &= 13\,600\,\mathrm{kg\,m^{-3}} \\
\rho_{\text{oil}} &= 720\,\mathrm{kg\,m^{-3}}
\end{aligned}
$$

Substituting into (14)

$$
\begin{aligned}
h_1 &= \frac{50000 + 1000 \times 9.81 \times 0.3 - 720 \times 9.81 \times 0.75}{13600 \times 9.81} \\
&= \boxed{0.357\,\mathrm{m}}
\end{aligned}
$$

Question 5

A steady, incompressible, two-dimensional velocity field is given by $\vec{V} = (0.523 - 1.88x + 3.94y)\hat{i} + (-2.44 + 1.26x + 1.88y)\hat{j}$, calculate the acceleration at the point $(x, y) = (-1.55, 2.07)$.

Since $\vec{V} = \langle u, v \rangle$ is a function of (x, y, t), the acceleration is given by the multivariable chain rule:

$$
\begin{aligned}
\vec{a} &= \frac{\partial \vec{V}}{\partial t}\cancel{\frac{dt}{dt}} + \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} \\
&= \cancel{\frac{\partial \vec{V}}{\partial t}} + \frac{\partial \vec{V}}{\partial x}u + \frac{\partial \vec{V}}{\partial y}v \\
&= \begin{bmatrix} -1.88 \\ 1.26 \end{bmatrix} u + \begin{bmatrix} 3.94 \\ 1.88 \end{bmatrix} v
\end{aligned} \qquad (15)
$$

Plugging in the given values to evaluate for u and v at the point $(x, y) = (-1.55, 2.07)$:

$$
\begin{aligned}
u &= 0.523 - 1.88(-1.55) + 3.94(2.07) \\
&= 11.5928 \\
v &= -2.44 + 1.26(-1.55) + 1.88(2.07) \\
&= -0.5014
\end{aligned}
$$

Evaluating for acceleration:

$$\boxed{\vec{a} = \begin{bmatrix} -23.77 \\ 13.66 \end{bmatrix} [\hat{i} + \hat{j}]}$$