Question 1

Water at 10°C ($\rho = 999.7$ kg/m^3 and $\mu = 1.307 \times 10^{-3}$ N·s/m^2) is flowing steadily in a 0.12-cm-diameter, 15-m-long pipe at an average velocity of 0.9 m/s. Determine (a) the pressure drop, (b) the head loss, and (c) the pumping power requirement to overcome this pressure drop.

(a)

First, find the Reynolds number

$$\begin{aligned}\mathrm{Re} &= \frac{\rho v D}{\mu} \\ &= \frac{999.7 \times 0.9 \times 0.0012}{1.307 \times 10^{-3}} \\ &= 826.07\end{aligned}$$

Since this is less than the critical Reynolds number of 2300, the flow is laminar.

For laminar flow in a circular pipe, the friction factor is given by

$$\begin{aligned}f &= \frac{64}{\mathrm{Re}} \\ &= \frac{64}{826.07} \\ &= 0.077475\end{aligned}$$

Then, the pressure drop is given by

$$\begin{aligned}\Delta P &= f \frac{L}{D} \frac{\rho v^2}{2} \\ &= 0.077475 \times \frac{15}{0.0012} \times \frac{999.7 \times 0.9^2}{2} \\ &= 392099.522344 \\ &= \boxed{392 \text{ kPa}}\end{aligned}$$

(b)

Head loss is given by

$$\begin{aligned}h_L &= \frac{\Delta P}{\rho g} \\ &= \frac{392099.522344}{999.7 \times 9.81} \\ &= \boxed{39.98 \text{ m}}\end{aligned}$$

(c)

Pumping power is given by

$$\begin{aligned}\dot{W}_{\text{pump},L} = \dot{V}\Delta P &= \frac{\pi D^2}{4} v \Delta P \\ &= \frac{\pi(0.0012)^2}{4} \times 0.9 \times 392099.522344 \\ &= \boxed{0.3991 \text{ W}}\end{aligned}$$

Question 2

Oil with a density of 850 kg/m^3 and kinematic viscosity of 0.00062 m^2/s is being discharged by an 8-mm-diameter, 40-m-long horizontal pipe from a storage tank open to the atmosphere. The height of the liquid level above the center of the pipe is 4 m. Disregarding the minor losses, determine the flow rate of oil through the pipe.

Assume the flow is laminar. We will verify this assumption later.

For flow in a laminar horizontal pipe,

$$\dot{V} = \frac{\Delta P \pi D^4}{128 \mu L}$$

Since $\mu = \rho\nu$ and the pressure drop for an open tank is $\Delta P = \rho g h$, we have

$$\begin{aligned}\dot{V} &= \frac{\rho g h \pi D^4}{128 \rho \nu L} \\ &= \frac{g h \pi D^4}{128 \nu L} \\ &= \frac{9.81 \times 4 \times \pi \times (0.008)^4}{128 \times 0.00062 \times 40} \\ &= \boxed{1.591 \times 10^{-7} \text{ m}^3/\text{s}}\end{aligned}$$

This corresponds to a velocity of $v = \frac{\dot{V}}{\pi D^2/4} = 0.00316$ m/s. The Reynolds number is

$$\begin{aligned}\text{Re} &= \frac{vD}{\nu} \\ &= \frac{0.00316 \times 0.008}{0.00062} \\ &= \boxed{0.04077}\end{aligned}$$

Since this is less than the critical Reynolds number of 2300, the flow is laminar, and our assumption is valid.

Question 3

A horizontal pipe has an abrupt expansion from $D_1 = 5$ cm to $D_2 = 10$ cm. The water velocity in the smaller section is 8 m/s and the flow is turbulent. The pressure in the smaller section is $P_1 = 410$ kPa. Taking the kinetic energy correction factor to be 1.06 at both the inlet and the outlet, determine the downstream pressure P_2, and estimate the error that would have occurred if Bernoulli's equation had been used.

The energy equation is

$$\frac{P_1}{\rho g} + \alpha_1 \frac{v_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{v_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

Since $\alpha_1 = \alpha_2 = 1.06$, $z_1 = z_2$, $h_{\text{pump},u} = h_{\text{turbine},e} = 0$,

$$\begin{aligned} \frac{P_1}{\rho g} + \alpha \frac{v_1^2}{2g} &= \frac{P_2}{\rho g} + \alpha \frac{v_2^2}{2g} + h_L \\ \implies P_2 &= P_1 + \rho \left(\alpha \frac{v_1^2 - v_2^2}{2} - g h_L \right) \end{aligned}$$

The loss coefficient for an abrupt expansion is given by

$$\begin{aligned} K_L &= \alpha \left(1 - \left(\frac{D_1}{D_2} \right)^2 \right)^2 \\ &= 1.06 \left(1 - \left(\frac{0.05}{0.1} \right)^2 \right)^2 \\ &= 0.59625 \end{aligned}$$

Then, the head loss is given by

$$\begin{aligned} h_L &= K_L \frac{v_1^2}{2g} \\ &= 0.59625 \times \frac{8^2}{2 \times 9.81} \\ &= 1.9450 \text{ m} \end{aligned}$$

v_2 is given by the continuity equation

$$\begin{aligned} v_2 &= \frac{A_1}{A_2} v_1 \\ &= \frac{D_1^2}{D_2^2} v_1 \\ &= \frac{0.05^2}{0.1^2} \times 8 \\ &= 2 \text{ m/s} \end{aligned}$$

Finally, the downstream pressure P_2 is given by

$$\begin{aligned}
P_2 &= P_1 + \rho\left(\alpha\frac{v_1^2 - v_2^2}{2} - gh_L\right)\\
&= 410\times10^3 + 1000\times\left(1.06\times\frac{8^2-2^2}{2} - 9.81\times1.9450\right)\\
&= \boxed{422.7\text{ kPa}}
\end{aligned}$$

The Bernoulli equation can be found by modifying the energy equation to let $\alpha = 1$, $h_{\text{pump},u} = h_{\text{turbine},e} = 0$, and $h_L = 0$. Arising,

$$\begin{aligned}
P_2 &= P_1 + \rho\left(\frac{v_1^2 - v_2^2}{2}\right)\\
&= 410\times10^3 + 1000\times\left(\frac{8^2-2^2}{2}\right)\\
&= \boxed{440\text{ kPa}}
\end{aligned}$$

The error is

$$\begin{aligned}
\text{err} &= 440 - 422.7 = \boxed{17.3\text{ kPa}}\\
\%\text{err} &= \frac{440-422.7}{422.7}\times100\% = \boxed{4.09\%}
\end{aligned}$$