9. Differential Analysis of Fluid Flow

8.1. General Procedure

1. a

8.2. Variable Definitions

- ∇: The gradient operator, $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
- $\frac{D}{Dt}$: The material derivative, $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla$
- $\vec{V}$: Velocity vector, $\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$
- ∇^2: The Laplacian operator
- P: Pressure

8.3. Formulas

Continuity Equation: $\dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$

Cartesian: $\dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho u) + \dfrac{\partial}{\partial y}(\rho v) + \dfrac{\partial}{\partial z}(\rho w) = 0$

Cylindrical: $\dfrac{\partial \rho}{\partial t} + \dfrac{1}{r}\dfrac{\partial (r\rho u)}{\partial r} + \dfrac{\partial (\rho v)}{\partial \theta} + \dfrac{\partial (\rho w)}{\partial z} = 0$

Special Case 1: Steady Compressible Flow: $\nabla \cdot (\rho \vec{V}) = 0$

Special Case 2: Incompressible Flow: $\nabla \cdot \vec{V} = 0$

Incompressible flow, constant μ, Navier-Stokes Equation:

$$\rho \frac{D\vec{V}}{Dt} = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{V}$$

Constant $\dot{q}$:

$$T_e = T_i + \frac{\dot{q}}{\dot{m} c_p}$$

$$\dot{q} = h(T_s - T_b)$$

Constant T_s:

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{\dot{m} C_p}{h A_s}\right)$$

$$T_s = \frac{T_e - T_i \exp\left(-\frac{\dot{m} C_p}{h A_s}\right)}{1 - \exp\left(-\frac{\dot{m} C_p}{h A_s}\right)}$$

$$\dot{Q} = h A_s \Delta T_{\mathrm{lm}}$$

$$T_{\mathrm{lm}} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]}$$

For fully developed laminar flow, use Table **??**.

For entry region in a circular tube where $T_s =$ constant, use:

(Edwards et al., 1979) $\mathrm{Nu} = 3.66 + \dfrac{0.0658(D/L)\mathrm{RePr}}{1 + 0.04[(D/L)\mathrm{RePr}]^{2/3}}$

For entry region in a circular tube where the difference between T_s and T_b is large, use:

(Sieder and Tate, 1936) $\mathrm{Nu} = 1.86 \left(\dfrac{\mathrm{RePr}D}{L}\right)^{1/3} \left(\dfrac{\mu_b}{\mu_s}\right)^{0.14}$

$$0.6 < \mathrm{Pr} < 5, \quad 0.0044 < \frac{\mu_b}{\mu_s} < 9.75$$

All properties for Sieder and Tate should be evaluated at T_b except μ_s which should be evaluated at T_s.

For entry region between two isothermal parallel plates, use:

(Edwards et al., 1979) $\mathrm{Nu} = 7.54 + \dfrac{0.03(D_h/L)\mathrm{RePr}}{1 + 0.016[(D_h/L)\mathrm{RePr}]^{2/3}}$

$$\mathrm{Re} \leq 2800$$

For turbulent flow in a circular tube, use:

$$\text{(Dittus-Boelter, 1930) } \mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^n$$

$$n = 0.4 \text{ (Heating)}, \quad n = 0.3 \text{ (Cooling)}$$

8.4 General Terms

- Control volume analysis: A method of analysis in which a volume in space is selected and the conservation of mass, momentum, and energy are applied to the volume
- Differential analysis: involves application of differential equations of fluid motion to any and every point in the flow field over a region called the flow domain.