8. Internal Forced Convection

8.1. General Procedure

1. Find fluid properties from Appendix 1 at bulk mean temperature $T_b = (T_i + T_e)/2$
   - ρ, μ, k, c_p, Pr, ν
2. Determine mean velocity V_{avg}
3. Determine the type of flow (laminar or turbulent)
   - Laminar: $\text{Re} < 2300$
   - Turbulent: $\text{Re} > 4000$
4. Determine the Nusselt number, Nu, using the appropriate correlation
   - Check if $l_{h,\text{laminar}}$ and $l_{t,\text{laminar}}$ is less than L. If so, use Table 1
   - Else, use empirical correlations
5. Determine the heat transfer coefficient h using Nu, k, and A_s

8.2. Variable Definitions

- Nu: Nusselt number
- Re: Reynolds number
- Pr: Prandtl number
- μ: Dynamic viscosity
- ν: Kinematic viscosity
- k: Thermal conductivity
- h: Convection heat transfer coefficient
- D_h: Hydraulic diameter
- A_s: Surface area
- A_c: Cross-sectional area
- V_{avg}: Average velocity
- T_b: Bulk mean temperature
- T_i: Inlet temperature
- T_e: Exit temperature
- $\dot{m}$: Mass flow rate
- $\dot{q}$: Heat flux
- ΔT_{lm}: Log mean temperature difference

8.3. Formulas

8.3.1. General Formulas

$$\begin{aligned}
\dot{m} &= \rho V_{\text{avg}} A_c \\
\text{Re} &= \frac{\rho V_{\text{avg}} D_h}{\mu} = \frac{V_{\text{avg}} D_h}{\nu} \\
D_h &= \frac{4A_c}{\text{Perimeter}} = D|_{\text{circular}} = a|_{\text{square}} \\
&= \left.\frac{2ab}{a+b}\right|_{\text{rectangular}} = \left.\frac{4ab}{a+b}\right|_{\text{channel}} \\
\text{Nu} &= \frac{hD_h}{k} \\
A_s &= \pi DL|_{\text{circular}} = 4ab|_{\text{rectangular}} \\
A_c &= \pi\frac{D^2}{4}|_{\text{circular}} = ab|_{\text{rectangular}} \\
l_{h,\text{laminar}} &= 0.05\text{Re}D_h \\
l_{t,\text{laminar}} &= 0.05\text{RePr}D_h = \text{Pr}l_{h,\text{laminar}} \\
l_{h,\text{turbulent}} &\approx l_{t,\text{turbulent}} = 10D_h
\end{aligned}$$

8.3.2. Constant $\dot{q}$

$$\begin{aligned}
T_e &= T_i + \frac{\dot{q}}{\dot{m}c_p} \\
\dot{q} &= h(T_s - T_b)
\end{aligned}$$

8.3.3. Constant T_s

$$\begin{aligned}
T_e &= T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \\
T_s &= \frac{T_e - T_i \exp\left(-\frac{\dot{m}C_p}{hA_s}\right)}{1 - \exp\left(-\frac{\dot{m}C_p}{hA_s}\right)} \\
\dot{Q} &= hA_s\Delta T_{\text{lm}} \\
T_{\text{lm}} &= \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]}
\end{aligned}$$

8.3.4. Correlations for Nu

For fully developed laminar flow, use Table 1.

For entry region in a circular tube where T_s = constant, use:

$$\text{(Edwards et al., 1979) Nu} = 3.66 + \frac{0.0658(D/L)\text{RePr}}{1 + 0.04[(D/L)\text{RePr}]^{2/3}}$$

For entry region in a circular tube where the difference between T_s and T_b is large, use:

$$\text{(Sieder and Tate, 1936) Nu} = 1.86\left(\frac{\text{RePr}D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

$$0.6 < \text{Pr} < 5, \quad 0.0044 < \frac{\mu_b}{\mu_s} < 9.75$$

All properties for Sieder and Tate should be evaluated at T_b except μ_s which should be evaluated at T_s.

For entry region between two isothermal parallel plates, use:

$$\text{(Edwards et al., 1979) } \mathrm{Nu} = 7.54 + \frac{0.03(D_h/L)\mathrm{RePr}}{1 + 0.016[(D_h/L)\mathrm{RePr}]^{2/3}}$$

$$\mathrm{Re} \leq 2800$$

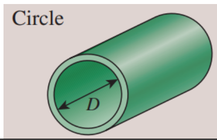
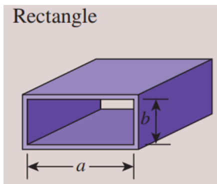
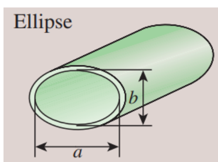
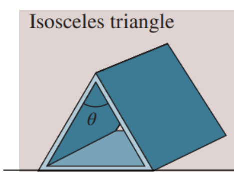
For turbulent flow in a circular tube, use:

$$\text{(Dittus-Boelter, 1930) } \mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{n}$$

$$n = 0.4 \text{ (Heating)}, \quad n = 0.3 \text{ (Cooling)}$$

Tables

Table 1: Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/P$, $Re = V_{\text{avg}} D_h/\nu$, and $\text{Nu} = hD_h/k$) (**Table 8-1 in textbook**)

		Nu		
Tube Geometry	a/b or $\theta°$	T_s = constant	$\dot{q}_s$ = constant	f
	—	4.36	3.66	64/Re
	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
	$\theta°$			
	10	1.61	2.45	50.80/Re
	30	2.26	2.91	52.28/Re
	60	2.47	3.11	53.32/Re
	90	2.34	2.98	52.60/Re
	120	2.00	2.68	50.96/Re

9. Natural Convection

9.1. General Procedure

9.1.1. Over Surfaces

1. Find Rayleigh number, Ra_L, using fluid properties at film temperature $T_f = (T_s + T_\infty)/2$
2. Use the appropriate correlation in Table 2 to find the Nusselt number, Nu
3. Determine the heat transfer coefficient h using Nu, k, and L_c

9.1.2. In Enclosures

1. Find Rayleigh number, Ra_L, using fluid properties at average temperature $T_{\text{avg}} = (T_1 + T_2)/2$ where T_1 and T_2 are the temperatures of the hot and cold surfaces respectively.
2. Use the appropriate correlation to find the Nusselt number, Nu
3. Determine the heat transfer coefficient h using Nu, k, and L_c

9.2. Variable Definitions

- Ra_L: Rayleigh number
- Gr_L: Grashof number
- T_s: Surface temperature
- T_∞: Ambient temperature
- T_f: Film temperature
- L_c: Characteristic length
- β: Coefficient of volume expansion
- ν: Kinematic viscosity
- α: Thermal diffusivity
- g: Gravitational acceleration

9.3. Formulas

$$\mathrm{Ra}_L = \mathrm{Gr}_L \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr}$$

$$\beta = \frac{1}{T}, \quad \text{for ideal gases}$$

$$h = \frac{k\mathrm{Nu}}{L_c}$$

9.3.1. Over Surfaces

For convection

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Use Table 2 to find Nu.

9.3.2. In Rectangular Enclosures

For convection in rectilinear enclosures,

$$\dot{Q} = hA_s(T_1 - T_2)$$

where T_1 and T_2 are the temperatures of the hot and cold surfaces respectively.

In **horizontal rectangular enclosures** ($L_c = L$, where L is the gap between plates),

$$\mathrm{Nu} = 1 + 1.44\left[1 - \left(\frac{1708}{\mathrm{Ra}_L}\right)\right]^+ + \left[\frac{\mathrm{Ra}_L}{18} - 1\right]^+$$

$$\mathrm{Ra}_L < 10^8 \text{ (gases)}, \quad \mathrm{Ra}_L < 10^5 \text{ (liquids)}$$

For large aspect ratios ($H/L \geq 12$), this equation (Hollands et al., 1976) correlates experimental data extremely well for tilt angles up to 70°. $[\,]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero.

$$\mathrm{Nu}_L = 1 + 1.44\left[1 - \left(\frac{1708}{\mathrm{Ra}_L \cos\theta}\right)\right]^+ + \left[\frac{\mathrm{Ra}_L \cos\theta}{18} - 1\right]^+$$

$$\mathrm{Ra}_L < 10^8 \text{ (gases)}, \quad \mathrm{Ra}_L < 10^5 \text{ (liquids)},$$

$$0 < \theta < 70°, \quad \frac{H}{L} \geq 12$$

In **vertical rectangular enclosures** ($L_c = L$, where L is the gap between plates),

$$\mathrm{Nu}_L = 0.18\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29}$$

$$\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}} > 10^3, \; 1 < \frac{H}{L} < 2$$

or

$$\mathrm{Nu}_L = 0.22\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-0.25}$$

$$\mathrm{Ra}_L < 10^{10}, \; 2 < \frac{H}{L} < 10$$

or

$$\mathrm{Nu}_L = 0.42\left(\frac{\mathrm{Pr}}{0.2 + \mathrm{Pr}}\mathrm{Ra}_L\right)^{0.25}\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$$1 < \mathrm{Pr} < 2 \times 10^4, \; 10^4 < \mathrm{Ra}_L < 10^7, \; 10 < \frac{H}{L} < 40$$

9.3.3. In Concentric Horizontal Cylinders

In **concentric horizontal cylinders** ($L_c = (D_o - D_i)/2$, where D_o and D_i are the outer and inner diameters respectively),

$$\dot{Q}_{\text{cylinder}} = \frac{2\pi k\mathrm{Nu}}{\ln(D_o/D_i)}(T_i - T_o)$$

where T_i and T_o are the temperatures of the inner and outer surfaces respectively.

$$\mathrm{Nu} = \max\left\{1, \; 0.386\left(\frac{\mathrm{Pr}}{0.861 + \mathrm{Pr}}\right)^{0.25}(F_{\text{cyl}}\mathrm{Ra}_L)^{0.25}\right\}$$

9.3.4. Combined Natural and Forced Convection

For combined natural and forced convection,

$$\mathrm{Nu}_{\mathrm{Overall}} = \left(\mathrm{Nu}^n_{\mathrm{Forced}} \pm \mathrm{Nu}^n_{\mathrm{Natural}}\right)^{1/n}$$

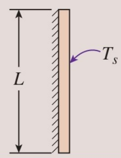
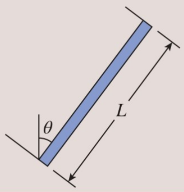
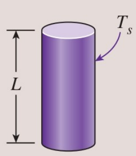
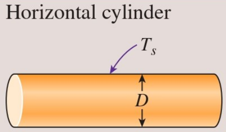
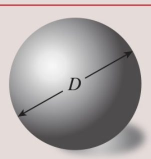
where the plus sign is for assisting flows and the minus sign is for opposing flows. $n = 3.5$ for horizontal plates and $n = 4$ for cylinders and spheres. Else use $n = 3$.

Tables

Table 2: Empirical correlations for the average Nusselt number for natural convection over surfaces (**Table 9-1 in textbook**)

TABLE 9–1

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic Length L_c	Range of Ra	Nu	
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$\text{Nu} = 0.59\,\text{Ra}_L^{1/4}$ $\text{Nu} = 0.1\,\text{Ra}_L^{1/3}$ $\text{Nu} = \left\{0.825 + \dfrac{0.387\,\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)	**(9–19)** **(9–20)** **(9–21)**
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g with $g\cos\theta$ for $0 < \theta < 60°$	
Horizontal plate (surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	(a) $\text{Nu} = 0.54\,\text{Ra}_L^{1/4}$ (a) $\text{Nu} = 0.15\,\text{Ra}_L^{1/3}$ (b) $\text{Nu} = 0.27\,\text{Ra}_L^{1/4}$	**(9–22)** **(9–23)** **(9–24)**
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{\text{Gr}_L^{1/4}}$	
Horizontal cylinder 	D	$\text{Ra}_D \leq 10^{12}$	$\text{Nu} = \left\{0.6 + \dfrac{0.387\,\text{Ra}_D^{1/6}}{[1 + (0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2$	**(9–25)**
Sphere 	D	$\text{Ra}_D \leq 10^{11}$ $(\text{Pr} \geq 0.7)$	$\text{Nu} = 2 + \dfrac{0.589\,\text{Ra}_D^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$	**(9–26)**

12. Thermal Radiation Fundamentals

12.1. General Procedure

Most questions will ask for irradiation given some geometry and temperatures.

1. Determine absorptivity α, reflectivity ρ, and transmissivity τ using the geometry and material properties. Note that if two of these are known, the third can be found using $\alpha + \rho + \tau = 1$.
2. Use solid angle ω, irradiation G, and intensity I to find whatever is asked for

12.2. Variable Definitions

- ω: Solid angle
- G: Irradiation
- I: Intensity
- J: Radiosity
- $\dot{Q}$: Heat transfer rate
- α: Absorptivity
- ρ: Reflectivity
- τ: Transmissivity
- ϵ: Emissivity
- σ: Stefan-Boltzmann constant

12.3. Formulas

For $A \ll r^2$ (small area, far away from surface),

$$\begin{aligned}
\omega_{2-1} &:= \frac{A_2 \cos\theta_2}{r^2} \\
J_1 &:= \pi I_{1,e+r} = E_b + G_{\text{ref}} = \epsilon\sigma T_1^4 + \rho G_1 \\
\dot{Q}_{1-2} &:= I_1(A_1 \cos\theta_1)\omega_{2-1} \\
G_2 &:= \frac{\dot{Q}_{1-2}}{A_2}
\end{aligned}$$

Combining the above equations into G_2,

$$G_2 = \frac{(\epsilon\sigma T_1^4 + \rho G_1)A_1 \cos\theta_1 \cos\theta_2}{\pi r^2}$$

For $\theta_1 = \theta_2 = \rho = 0$ and $\epsilon = 1$ for a blackbody,

$$\begin{aligned}
G_2 &= \frac{\sigma T_1^4 A_1}{\pi r^2} \\
\implies T_1 &= \left(\frac{G_2 \pi r^2}{\sigma A_1}\right)^{1/4}
\end{aligned}$$

13. Thermal Radiation Heat Transfer

13.1. General Procedure

Most questions will ask for irradiation given some geometry and temperatures.

1. Determine view factors F_{ij} from inspection using the geometry. Check for special cases such as:
   - $F_{ii} = 0$ when the surface i is planar or convex
   - $F_{ij} = 1$ when the surface i is a concave and fully encloses surface j
2. Check Figures section for view factors that cannot be obtained from inspection
3. Use the reciprocity relation and summation rule to find the remaining view factors

Solvability condition: Given N surfaces, there are N^2 view factors. $N(N-1)/2$ view factors must be given by inspection, problem statement, or tables. The rest can be determined using reciprocity and summation.

13.2. Variable Definitions

- F_{ij}: View factor, the fraction of radiation leaving surface i that strikes surface j
- ω: Solid angle
- G: Irradiation, the rate of radiant energy incident on a surface per unit area
- I: Intensity
- J: Radiosity, the total rate of radiant energy leaving a surface per unit area
- E_b: Black body emissive power
- $\dot{Q}$: Heat transfer rate
- α: Absorptivity
- ρ: Reflectivity
- τ: Transmissivity
- ϵ: Emissivity
- σ: Stefan-Boltzmann constant

13.3. Formulas

Reciprocity relation:

$$A_{1,s}F_{12} = A_{2,s}F_{21}$$

Summation rule:

$$\sum_{j=1}^{n} F_{ij} = 1$$

Crossed-strings method:

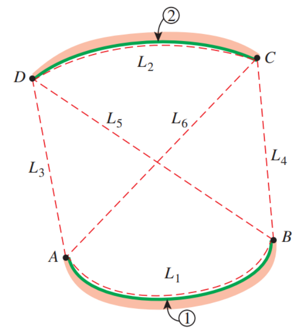


Figure 1: Determination of the view factor F_{12} by the application of the crossed-strings method.

$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_4)}{2L_1}$$

On a black body, net radiation heat transfer is

$$E_{bi} = \sigma T_i^4$$
$$\dot{Q}_{12} = A_{1,s}F_{12}\sigma(T_1^4 - T_2^4) = A_{2,s}F_{21}\sigma(T_1^4 - T_2^4)$$

On a grey body ($\epsilon_i = \alpha_i$, $\tau = 0$, $\alpha_i + \rho_i = 1$), net radiation heat transfer is

$$J_i = \pi I_i = \epsilon_i \sigma T_i^4 + \rho_i G_i$$
$$\dot{Q}_{12} = \frac{A_{i,s}\epsilon_i}{1-\epsilon_i}(E_{bi} - J_i)$$

For radiation in two-surface enclosures, net radiation heat transfer is

$$\dot{Q}_{12} = \dot{Q}_1 = -\dot{Q}_2$$
$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_{1,s}} + \frac{1}{A_{2,s}F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_{2,s}}} = \frac{E_{b1} - E_{b2}}{R_1 + R_{12} + R_2}$$

For some familiar geometries, the above equation simplifies, which is given in Table 3.

Figures

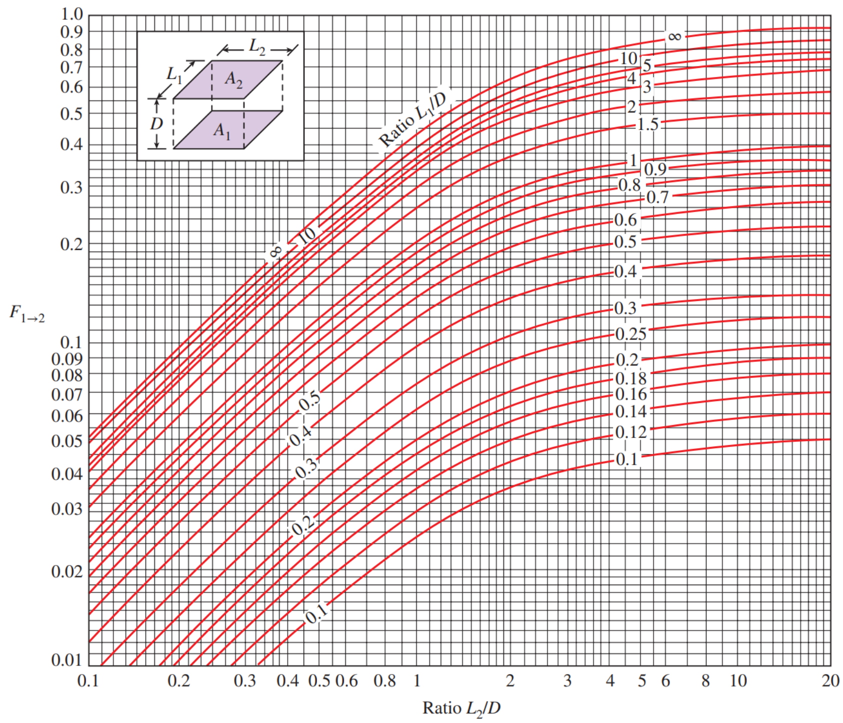


Figure 2: View factor between two aligned parallel rectangles of equal size

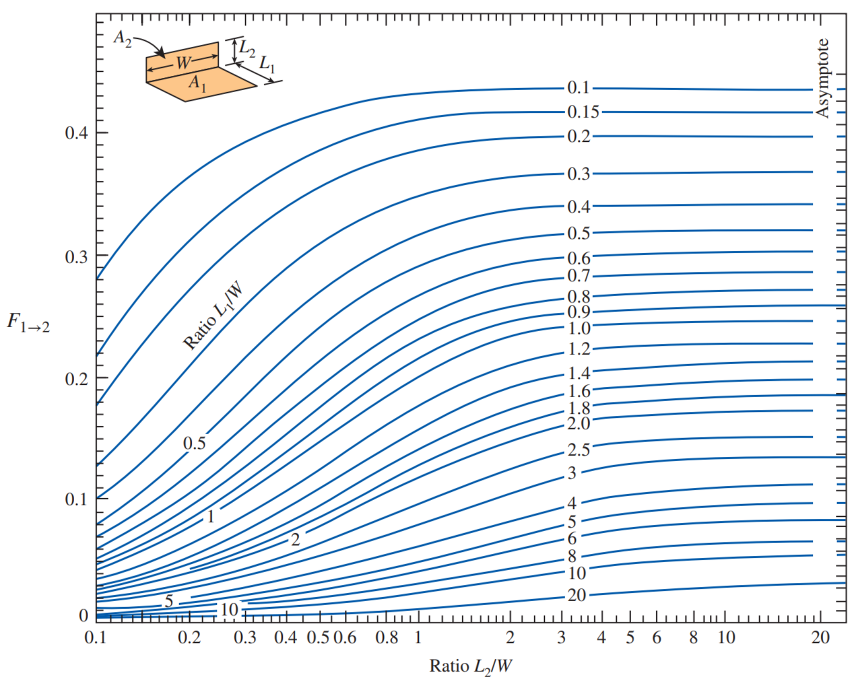


Figure 3: View factor between two perpendicular rectangles with a common edge

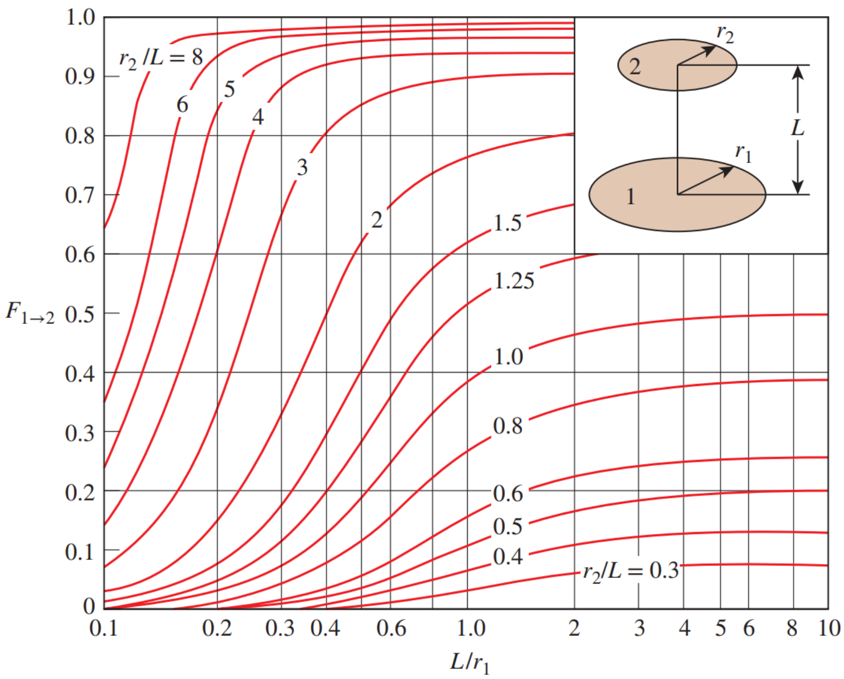


Figure 4: View factor between two coaxial parallel disks

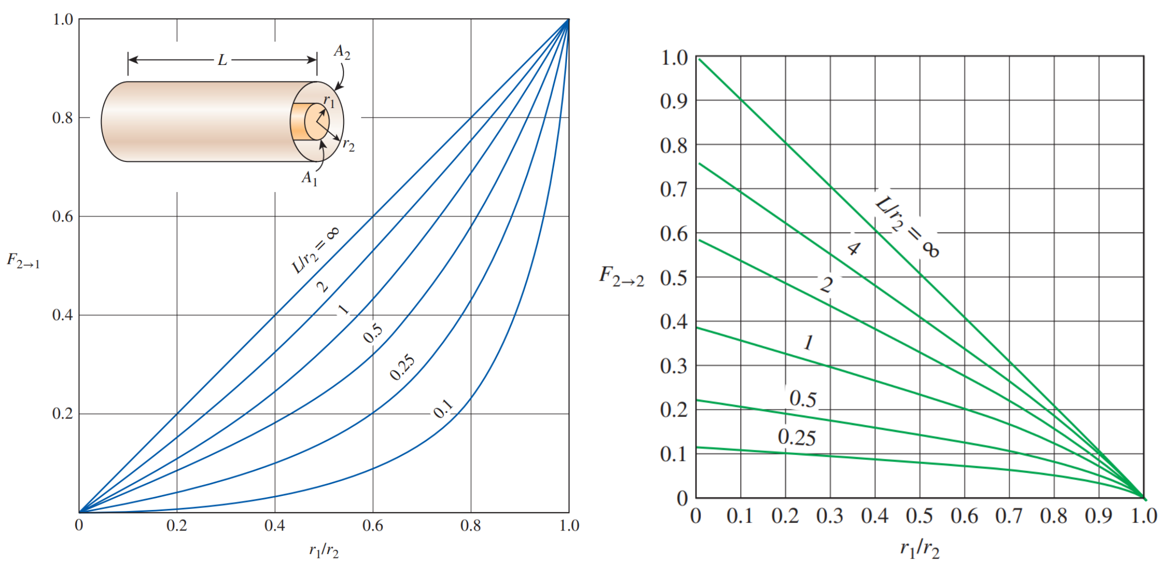


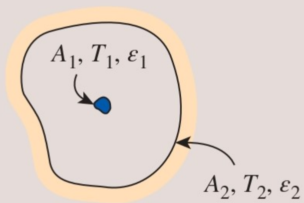
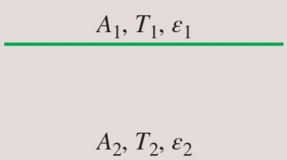
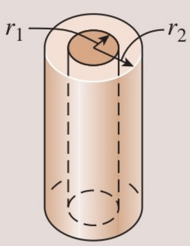
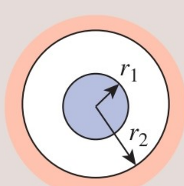
Figure 5: View factors for two concentric cylinders of finite length: (a) outer cylinder to inner cylinder; (b) outer cylinder to itself.

Tables

Table 3: Radiation heat transfer relations for some familiar two-surface arrangements

TABLE 13–3

Radiation heat transfer relations for some familiar two-surface arrangements

Small object in a large cavity		$\frac{A_1}{A_2} \approx 0$ $F_{12} = 1$	$\dot{Q}_{12} = A_1 \sigma \varepsilon_1 (T_1^4 - T_2^4)$	**(13–37)**
Infinitely large parallel plates		$A_1 = A_2 = A$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$	**(13–38)**
Infinitely long concentric cylinders		$\frac{A_1}{A_2} = \frac{r_1}{r_2}$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)}$	**(13–39)**
Concentric spheres		$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$ $F_{12} = 1$	$\dot{Q}_{12} = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1 - \varepsilon_2}{\varepsilon_2}\left(\dfrac{r_1}{r_2}\right)^2}$	**(13–40)**