

8. Internal Forced Convection

8.1. General Procedure

- Find fluid properties from Appendix 1 at bulk mean temperature $T_b = (T_i + T_e)/2$
 - $\rho, \mu, k, c_p, Pr, \nu$
- Determine mean velocity V_{avg}
- Determine the type of flow (laminar or turbulent)
 - Laminar: $Re < 2300$
 - Turbulent: $Re > 4000$
- Determine the Nusselt number, Nu , using the appropriate correlation
 - Check if $l_{h,laminar}$ and $l_{t,laminar}$ is less than L . If so, use Table 1
 - Else, use empirical correlations
- Determine the heat transfer coefficient h using Nu , k , and A_s

8.2. Variable Definitions

- Nu : Nusselt number
- Re : Reynolds number
- Pr : Prandtl number
- μ : Dynamic viscosity
- ν : Kinematic viscosity
- k : Thermal conductivity
- h : Convection heat transfer coefficient
- D_h : Hydraulic diameter
- A_s : Surface area
- A_c : Cross-sectional area
- V_{avg} : Average velocity
- T_b : Bulk mean temperature
- T_i : Inlet temperature
- T_e : Exit temperature
- \dot{m} : Mass flow rate
- \dot{q} : Heat flux
- ΔT_{lm} : Log mean temperature difference

8.3. Formulas

8.3.1. General Formulas

$$\begin{aligned}\dot{m} &= \rho V_{avg} A_c \\ Re &= \frac{\rho V_{avg} D_h}{\mu} = \frac{V_{avg} D_h}{\nu} \\ D_h &= \frac{4A_c}{\text{Perimeter}} = D|_{\text{circular}} = a|_{\text{square}} \\ &= \frac{2ab}{a+b}|_{\text{rectangular}} = \frac{4ab}{a+b}|_{\text{channel}} \\ Nu &= \frac{hD_h}{k} \\ A_s &= \pi DL|_{\text{circular}} = 4ab|_{\text{rectangular}} \\ A_c &= \pi \frac{D^2}{4}|_{\text{circular}} = ab|_{\text{rectangular}} \\ l_{h,laminar} &= 0.05ReD_h \\ l_{t,laminar} &= 0.05RePrD_h = Prl_{h,laminar} \\ l_{h,turbulent} &\approx l_{t,turbulent} = 10D_h\end{aligned}$$

8.3.2. Constant \dot{q}

$$\begin{aligned}T_e &= T_i + \frac{\dot{q}}{\dot{m}c_p} \\ \dot{q} &= h(T_s - T_b)\end{aligned}$$

8.3.3. Constant T_s

$$\begin{aligned}T_e &= T_s - (T_s - T_i) \exp\left(-\frac{hA_s}{\dot{m}c_p}\right) \\ T_s &= \frac{T_e - T_i \exp\left(-\frac{\dot{m}c_p}{hA_s}\right)}{1 - \exp\left(-\frac{\dot{m}c_p}{hA_s}\right)} \\ \dot{Q} &= hA_s \Delta T_{lm} \\ T_{lm} &= \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]}\end{aligned}$$

8.3.4. Correlations for Nu

For fully developed laminar flow, use Table 1.

For entry region in a circular tube where $T_s = \text{constant}$, use:

$$(\text{Edwards et al., 1979}) \quad Nu = 3.66 + \frac{0.0658(D/L)RePr}{1 + 0.04[(D/L)RePr]^2/3}$$

For entry region in a circular tube where the difference between T_s and T_b is large, use:

$$(\text{Sieder and Tate, 1936}) \quad Nu = 1.86 \left(\frac{RePrD}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_s} \right)^{0.14}$$

$$0.6 < Pr < 5, \quad 0.0044 < \frac{\mu_b}{\mu_s} < 9.75$$

All properties for Sieder and Tate should be evaluated at T_b except μ_s which should be evaluated at T_s .

For entry region between two isothermal parallel plates,

use:

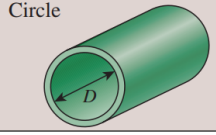
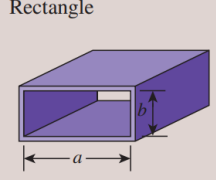
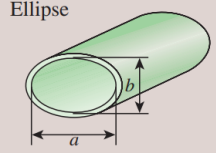
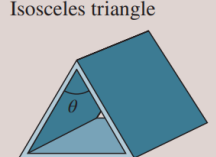
$$\text{(Edwards et al., 1979) } \text{Nu} = 7.54 + \frac{0.03(D_h/L)\text{RePr}}{1 + 0.016[(D_h/L)\text{RePr}]^{2/3}}$$
$$\text{Re} \leq 2800$$

For turbulent flow in a circular tube, use:

$$\text{(Dittus-Boelter, 1930) } \text{Nu} = 0.023\text{Re}^{0.8}\text{Pr}^n$$
$$n = 0.4 \text{ (Heating), } \quad n = 0.3 \text{ (Cooling)}$$

Tables

Table 1: Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/P$, $Re = V_{\text{avg}}D_h/\nu$, and $Nu = hD_h/k$) (**Table 8-1 in textbook**)

Tube Geometry	a/b or θ°	Nu		f 64/Re
		$T_s = \text{constant}$	$\dot{q}_s = \text{constant}$	
Circle 	—	4.36	3.66	64/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles triangle 	θ°			
	10	1.61	2.45	50.80/Re
	30	2.26	2.91	52.28/Re
	60	2.47	3.11	53.32/Re
	90	2.34	2.98	52.60/Re
	120	2.00	2.68	50.96/Re

9. Natural Convection

9.1. General Procedure

9.1.1. Over Surfaces

1. Find Rayleigh number, Ra_L , using fluid properties at film temperature $T_f = (T_s + T_\infty)/2$
2. Use the appropriate correlation in Table 3 to find the Nusselt number, Nu
3. Determine the heat transfer coefficient h using Nu , k , and L_c

9.1.2. In Enclosures

1. Find Rayleigh number, Ra_L , using fluid properties at average temperature $T_{avg} = (T_1 + T_2)/2$ where T_1 and T_2 are the temperatures of the hot and cold surfaces respectively.
2. Use the appropriate correlation to find the Nusselt number, Nu
3. Determine the heat transfer coefficient h using Nu , k , and L_c

9.2. Variable Definitions

- Ra_L : Rayleigh number
- Gr_L : Grashof number
- T_s : Surface temperature
- T_∞ : Ambient temperature
- T_f : Film temperature
- L_c : Characteristic length
- β : Coefficient of volume expansion
- ν : Kinematic viscosity
- α : Thermal diffusivity
- g : Gravitational acceleration

9.3. Formulas

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} Pr$$

$$\beta = \frac{1}{T}, \quad \text{for ideal gases}$$

$$h = \frac{kNu}{L_c}$$

9.3.1. Over Surfaces

For convection

$$\dot{Q} = hA_s(T_s - T_\infty)$$

Use Table 3 to find Nu .

9.3.2. In Rectangular Enclosures

For convection in rectilinear enclosures,

$$\dot{Q} = hA_s(T_1 - T_2)$$

where T_1 and T_2 are the temperatures of the hot and cold surfaces respectively.

In **horizontal rectangular enclosures** ($L_c = L$, where L is the gap between plates),

$$Nu = 1 + 1.44 \left[1 - \left(\frac{1708}{Ra_L} \right) \right]^+ + \left[\frac{Ra_L}{18} - 1 \right]^+$$

$$Ra_L < 10^8 \text{ (gases)}, \quad Ra_L < 10^5 \text{ (liquids)}$$

For large aspect ratios ($H/L \geq 12$), this equation (Hollands et al., 1976) correlates experimental data extremely well for tilt angles up to 70° . $[\]^+$ indicates that if the quantity in the bracket is negative, it should be set equal to zero.

$$Nu_L = 1 + 1.44 \left[1 - \left(\frac{1708}{Ra_L \cos \theta} \right) \right]^+ + \left[\frac{Ra_L \cos \theta}{18} - 1 \right]^+$$

$$Ra_L < 10^8 \text{ (gases)}, \quad Ra_L < 10^5 \text{ (liquids)},$$

$$0 < \theta < 70^\circ, \quad \frac{H}{L} \geq 12$$

In **vertical rectangular enclosures** ($L_c = L$, where L is the gap between plates),

$$Nu_L = 0.18 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.29}$$

$$\frac{Pr}{0.2 + Pr} > 10^3, \quad 1 < \frac{H}{L} < 2$$

or

$$Nu_L = 0.22 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.28} \left(\frac{H}{L} \right)^{-0.25}$$

$$Ra_L < 10^{10}, \quad 2 < \frac{H}{L} < 10$$

or

$$Nu_L = 0.42 \left(\frac{Pr}{0.2 + Pr} Ra_L \right)^{0.25} Pr^{0.012} \left(\frac{H}{L} \right)^{-0.3}$$

$$1 < Pr < 2 \times 10^4, \quad 10^4 < Ra_L < 10^7, \quad 10 < \frac{H}{L} < 40$$

9.3.3. In Concentric Horizontal Cylinders

In **concentric horizontal cylinders** ($L_c = (D_o - D_i)/2$, where D_o and D_i are the outer and inner diameters respectively),

$$\dot{Q}_{cylinder} = \frac{2\pi kNu}{\ln(D_o/D_i)} (T_i - T_o)$$

where T_i and T_o are the temperatures of the inner and outer surfaces respectively.

$$Nu = \max \left\{ 1, 0.386 \left(\frac{Pr}{0.861 + Pr} \right)^{0.25} (F_{cyl} Ra_L)^{0.25} \right\}$$

9.3.4. Combined Natural and Forced Convection

For combined natural and forced convection,

$$\text{Nu}_{\text{Overall}} = (\text{Nu}_{\text{Forced}}^n \pm \text{Nu}_{\text{Natural}}^n)^{1/n}$$

where the plus sign is for assisting flows and the minus sign is for opposing flows. $n = 3.5$ for horizontal plates and $n = 4$ for cylinders and spheres. Else use $n = 3$.

12. Thermal Radiation Fundamentals

9.1. General Procedure

Most questions will ask for irradiation given some geometry and temperatures.

1. Determine absorptivity α , reflectivity ρ , and transmissivity τ using the geometry and material properties. Note that if two of these are known, the third can be found using $\alpha + \rho + \tau = 1$.
2. Use solid angle ω , irradiation G , and intensity I to find whatever is asked for

9.2. Variable Definitions

- ω : Solid angle
- G : Irradiation
- I : Intensity
- J : Radiosity
- \dot{Q} : Heat transfer rate
- α : Absorptivity
- ρ : Reflectivity
- τ : Transmissivity
- ϵ : Emissivity
- σ : Stefan-Boltzmann constant

9.3. Formulas

For $A \ll r^2$ (small area, far away from surface),

$$\omega_{2-1} := \frac{A_2 \cos \theta_2}{r^2}$$

$$J_1 := \pi I_{1,e+r} = \pi(E_b + G_{\text{ref}}) = \pi(\epsilon \sigma T_1^4 + \rho G_1)$$

$$\dot{Q}_{1-2} := I_1 (A_1 \cos \theta_1) \omega_{2-1}$$

$$G_2 := \frac{\dot{Q}_{1-2}}{A_2}$$

Combining the above equations into G_2 ,

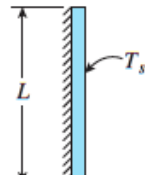
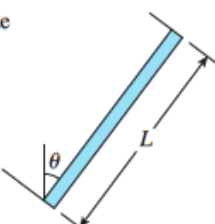

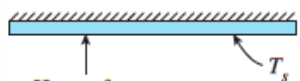
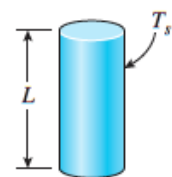
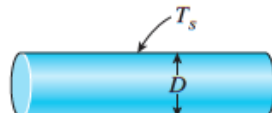
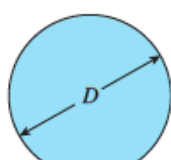
$$G_2 = \frac{(\epsilon \sigma T_1^4 + \rho G_1) A_1 \cos \theta_1 \cos \theta_2}{\pi r^2}$$

Since $\theta_1 = \theta_2 = \rho = 0$ and $\epsilon = 1$ for a blackbody,

$$\begin{aligned} G_2 &= \frac{\sigma T_1^4 A_1}{\pi r^2} \\ \implies T_1 &= \left(\frac{G_2 \pi r^2}{\sigma A_1} \right)^{1/4} \end{aligned}$$

Table 3: Empirical correlations for the average Nusselt number for natural convection over surfaces (**Table 9-1 in textbook**)

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4-10^9 $10^{10}-10^{13}$ Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{ 0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g \cos \theta$ for $0 < \theta < 60^\circ$
Horizontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4-10^7 10^7-10^{11} 10^5-10^{11}	$Nu = 0.59Ra_L^{1/4}$ (9-22) $Nu = 0.1Ra_L^{1/3}$ (9-23) $Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{ 0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)