P2.43.

The distribution of stress in an aluminum machine component is given by Eqn. (1).

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} = \begin{bmatrix} y + 2z^2 & 3z^2 & 2y^2 \\ 3z^2 & x + z & x^2 \\ 2y^2 & x^2 & 3x + y \end{bmatrix} \text{MPa} \tag{1}$$

Calculate the state of strain of a point positioned at $(1, 2, 4)$. Use $E = 70\,\text{GPa}$ and $\nu = 0.3$.

P2.47.

The aluminum rectangular parallelepiped ($E = 70\,\text{GPa}$ and $\nu = \frac{1}{3}$) shown in Fig. 1 has dimensions $a = 150\,\text{mm}$, $b = 100\,\text{mm}$, and $c = 75\,\text{mm}$. It is subjected to tri-axial stresses $\sigma_x = 70\,\text{MPa}$, $\sigma_y = -30\,\text{MPa}$, and $\sigma_z = -15\,\text{MPa}$ acting on the x, y, and z faces, respectively. Determine,

(a) the changes Δa, Δb, and Δc in the dimensions of the block.

(b) the change ΔV in the volume.

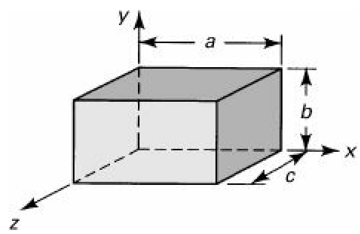

Figure 1: A aluminum rectangular parallelepiped

P2.49.

A rectangular plate is subjected to uniform tensile stress σ along its upper and lower edges as shown in Fig. 2. Determine the displacements u and v in terms of x, y, and material properties (E, ν) using Eqn. (2) and the appropriate conditions at the origin.

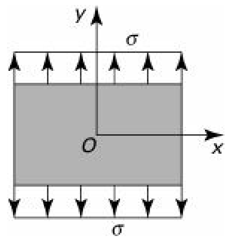

Figure 2: A rectangular plate

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y} \tag{2a}$$

$$\gamma_{xy} = \alpha_x - \alpha_y = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{2b}$$

P2.50.

The stress field in an elastic body is given by

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} = \begin{bmatrix} cy^2 & 0 \\ 0 & -cx^2 \end{bmatrix}$$

where c is a constant. Derive expressions for the displacement components $u\left(x, y\right)$ and $v\left(x, y\right)$ in the body.

P2.53.

A uniform bar of rectangular cross section $2h \times b$ and specific weight γ hangs in the vertical plane as shown in Fig. 3. Its weight results in displacements shown in Eqn. (3). Demonstrate whether this solution satisfies the 15 equations of elasticity and the boundary conditions.

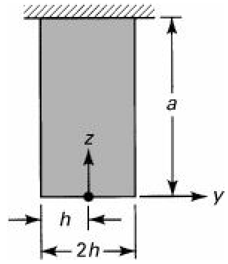

Figure 3: A uniform rectangular bar

$$u = -\frac{\nu\gamma}{E}xz \tag{3a}$$

$$v = -\frac{\nu\gamma}{E}yz \tag{3b}$$

$$w = \frac{\gamma}{2E}\left[\left(z^2 - a^2\right) + \nu\left(x^2 + y^2\right)\right] \tag{3c}$$

P2.58.

A round bar is composed of three segments of the same material as shown in Fig. 4. The diameter is d for the lengths $\overline{BC}$ and $\overline{DE}$ and $n \times d$ for length $\overline{CD}$, where n is the ratio of the two diameters. Neglecting the stress concentrations, verify that the strain energy of the bar when subjected to axial load P is

$$U = \frac{1 + 3n^2}{4n^2} \frac{P^2 L}{2AE}$$

where $A = \frac{\pi d^2}{4}$. Compare the results for $n = 1$ with those for $n = \frac{1}{2}$ and $n = 2$.

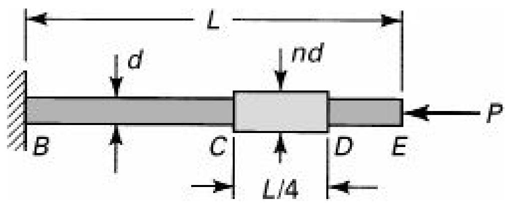

Figure 4: A round bar