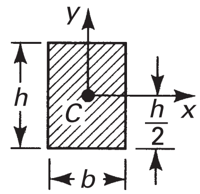
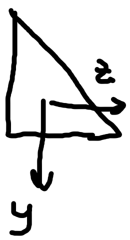
TABLE C.1. *Properties of Some Plane Areas*

1. Rectangle

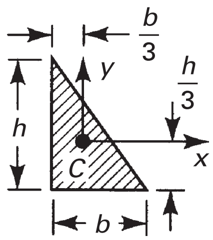


$$A = bh$$
$$I_x = \frac{bh^3}{12}$$
$$J_c = \frac{bh(b^2 + h^2)}{12}$$

$I_{yz} = +\frac{b^2h^2}{72}$

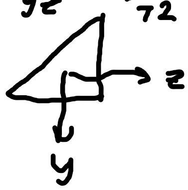


2. Right triangle

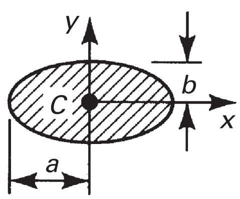


$$A = \frac{bh}{2}$$
$$I_x = \frac{bh^3}{36} \quad I_{xy} = -\frac{b^2h^2}{72}$$
$$J_c = \frac{bh(b^2 + h^2)}{36}$$

$I_{yz} = -\frac{b^2h^2}{72}$

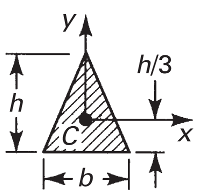


3. Ellipse



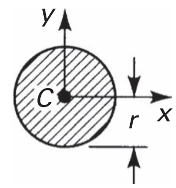
$$A = \pi ab$$
$$I_x = \frac{\pi ab^3}{4}$$
$$J_c = \frac{\pi ab(a^2 + h^2)}{4}$$

4. Isosceles triangle



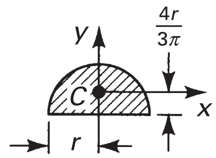
$$A = \frac{bh}{2}$$
$$I_x = \frac{bh^3}{36} \quad I_y = \frac{hb^3}{48}$$
$$J_c = \frac{bh}{144}(4h^2 + 3b^2)$$

5. Circle



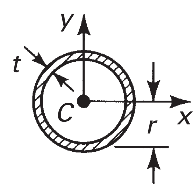
$$A = \pi r^2$$
$$I_x = \frac{\pi r^4}{4}$$
$$J_c = \frac{\pi r^4}{2}$$

6. Semicircle



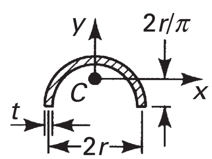
$$A = \frac{\pi r^2}{2}$$
$$I_x = 0.110r^4$$
$$I_y = \frac{\pi r^4}{8}$$

7. Thin tube



$$A = 2\pi rt$$
$$I_x = \pi r^3 t$$
$$J_c = 2\pi r^3 t$$

8. Half of thin tube



$$A = \pi rt$$
$$I_x \approx 0.095\pi r^3 t$$
$$I_y = 0.5\pi r^3 t$$

FIGURE C.1. *Plane area A with centroid C.*

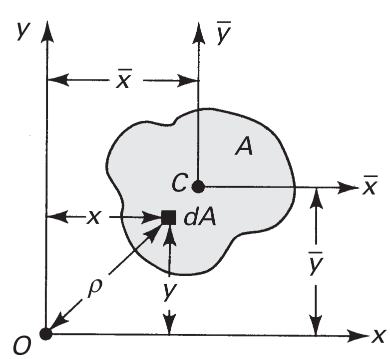
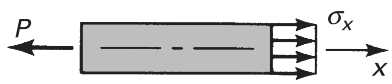
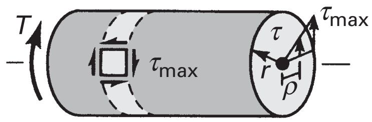


Table 1.1. *Commonly Used Elementary Formulas for Stress*[a]

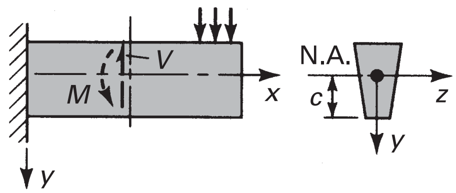
1. *Prismatic Bars of Linearly Elastic Material*



Axial loading: $\sigma_x = \dfrac{P}{A}$ (a)



Torsion: $\tau = \dfrac{T\rho}{J}, \qquad \tau_{\max} = \dfrac{Tr}{J}$ (b)



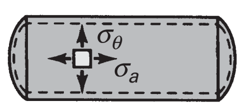
Bending: $\sigma_x = -\dfrac{My}{I}, \qquad \sigma_{\max} = \dfrac{Mc}{I}$ (c)

Shear: $\tau_{xy} = \dfrac{VQ}{Ib}$ (d)

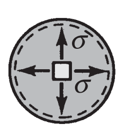
where

σ_x = normal axial stress
τ = shearing stress due to torque
τ_{xy} = shearing stress due to vertical shear force
P = axial force
T = torque
V = vertical shear force
M = bending moment about z axis
A = cross-sectional area
y, z = centroidal principal axes of the area
I = moment of inertia about neutral axis (N.A.)
J = polar moment of inertia of circular cross section
b = width of bar at which τ_{xy} is calculated
r = radius
Q = first moment about N.A. of the area beyond the point at which τ_{xy} is calculated

2. *Thin-Walled Pressure Vessels*



Cylinder: $\sigma_\theta = \dfrac{pr}{t}, \qquad \sigma_a = \dfrac{pr}{2t}$ (e)



Sphere: $\sigma = \dfrac{pr}{2t}$ (f)

where

σ_θ = tangential stress in cylinder wall
σ_a = axial stress in cylinder wall
σ = membrane stress in sphere wall
P = internal pressure
t = wall thickness
r = mean radius

[a]Detailed derivations and limitations of the use of these formulas are discussed in Sections 1.6, 5.7, 6.2, and 13.14.

5. Bending of Beams

5.1. General Procedure

General procedure of asymmetric bending problems

1. Identify the location of the centroid of the cross-section, and define it as the origin of the (y, z) coordinate system. If the centroid is unknown, set an arbitrary origin and use parallel axis theorem to find the centroid.

2. Define the orientation of (y, z) axes of the cross-section wisely so that all required moments of inertia I_y, I_z, and I_{yz} can be obtained (from Table) or calculated easily.

3. Determine bending moments M_z and M_y at your cross-section. Use elementary beam theory to find the bending moments if given a load.

4. Use the relations to find the stress σ_x and the neutral axis.

5.2. Formulas

Centroid equations:

$$\bar{x} = \frac{\sum \bar{x}_i A_i}{\sum A_i}$$

where $\bar{x}_i$ is the x-coordinate of the centroid of the i-th area, and A_i is the area of the i-th area.

Moment equations:

$$M_y = P_z L$$
$$M_z = P_y L$$

where P_z and P_y are positive in the positive z and y directions, respectively. Parallel axis theorem:

$$\bar{z} = \frac{\sum \bar{z}_i A_i}{\sum A_i}$$
$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$
$$I_z = \sum (I_{\bar{z},i} + A_i d_{y,i}^2)$$
$$I_y = \sum (I_{\bar{y},i} + A_i d_{z,i}^2)$$
$$I_{yz} = \sum (I_{\bar{y}\bar{z},i} + A_i d_{y,i} d_{z,i})$$

where $I_{\bar{z},i}$, $I_{\bar{y},i}$, and $I_{\bar{y}\bar{z},i}$ are the moments of inertia about the centroidal axes, and $d_{y,i}$ and $d_{z,i}$ are the distances from the centroidal axes to the parallel axes. Note: $I_{yz} = 0$ if there is symmetry about **either** the y or z direction.

Moment to stress:

$$\tau = \frac{VQ}{Ib} \overset{\text{rect}}{=} \frac{3V}{2A_c}$$
$$\sigma_x = \frac{(M_y I_z + M_z I_{yz}) d_z - (M_y I_{yz} + M_z I_y) d_y}{I_y I_z - I_{yz}^2}$$
$$\tan\phi = \frac{M_y I_z + M_z I_{yz}}{M_z I_y + M_y I_{yz}}$$

stress is maximum at the furthest point from the neutral axis on the cross-section. For σ_x, d_y and d_z are the signed displacements ($\pm$) from the centroid to the point of interest in the y and z directions.

Method of integration

$$EI\frac{d^4 v}{dx^4} = p$$
$$EI\frac{d^3 v}{dx^3} = -V$$
$$EI\frac{d^2 v}{dx^2} = M$$
$$EI\frac{dv}{dx} = \int M$$

also slope $\theta = dv/dx$ and deflection is v.

Singularity functions