Question 1

The distribution of stress in an aluminum machine component is given by:

$$\sigma = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} = \begin{bmatrix} y+2z^2 & 3z^2 & 2y^2 \\ 3z^2 & x+z & x^2 \\ 2y^2 & x^2 & 3x+y \end{bmatrix} \text{MPa}$$

Calculate the state of strain of a point positioned at $(1, 2, 4)$. Use $E = 70\,\text{GPa}$ and $\nu = 0.3$.

Solution

From the generalized Hooke's law, the stress-strain relation is given by:

$$\begin{aligned} \epsilon_x &= \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) \\ \epsilon_y &= \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) \\ \epsilon_z &= \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) \\ \gamma_{xy} &= \frac{1}{2G}\tau_{xy} \\ \gamma_{yz} &= \frac{1}{2G}\tau_{yz} \\ \gamma_{xz} &= \frac{1}{2G}\tau_{xz} \end{aligned}$$

Where

$$G = \frac{E}{2(1+\nu)} = \frac{70}{2(1+0.3)} = 26.923\,\text{GPa}$$

Evaluate the stress state at $(1, 2, 4)$:

$$\sigma = \begin{bmatrix} 2+2(4)^2 & 3(4)^2 & 2(2)^2 \\ 3(4)^2 & 1+4 & 1^2 \\ 2(2)^2 & 1^2 & 3(1)+2 \end{bmatrix} = \begin{bmatrix} 34 & 48 & 8 \\ 48 & 5 & 1 \\ 8 & 1 & 5 \end{bmatrix} \text{MPa}$$

Using the stress-strain relations, the strain state is given by:

$$\begin{aligned}
\epsilon &= \begin{bmatrix} \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) & \frac{1}{2G}\tau_{xy} & \frac{1}{2G}\tau_{xz} \\ \frac{1}{2G}\tau_{yx} & \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) & \frac{1}{2G}\tau_{yz} \\ \frac{1}{2G}\tau_{zx} & \frac{1}{2G}\tau_{zy} & \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{70\times 10^3}(34 - 0.3(5+5)) & \frac{1}{2(26.923)\times 10^3}48 & \frac{1}{2(26.923)\times 10^3}8 \\ \frac{1}{2(26.923)\times 10^3}48 & \frac{1}{70\times 10^3}(5 - 0.3(34+5)) & \frac{1}{2(26.923)\times 10^3}1 \\ \frac{1}{2(26.923)\times 10^3}8 & \frac{1}{2(26.923)\times 10^3}1 & \frac{1}{70\times 10^3}(5 - 0.3(34+5)) \end{bmatrix} \\
&= \begin{bmatrix} 4.43\times 10^{-4} & 8.91\times 10^{-4} & 1.49\times 10^{-4} \\ 8.91\times 10^{-4} & -9.57\times 10^{-5} & 1.86\times 10^{-5} \\ 1.49\times 10^{-4} & 1.86\times 10^{-5} & -9.57\times 10^{-5} \end{bmatrix}
\end{aligned}$$

Question 2

The aluminum rectangular parallelepiped ($E = 70$ GPa and $\nu = \frac{1}{3}$) shown in Fig. 1 has dimensions $a = 150\,\text{mm}$, $b = 100\,\text{mm}$, and $c = 75\,\text{mm}$. It is subjected to tri-axial stresses $\sigma_x = 70\,\text{MPa}$, $\sigma_y = -30\,\text{MPa}$, and $\sigma_z = -15\,\text{MPa}$ acting on the x, y, and z faces, respectively. Determine,

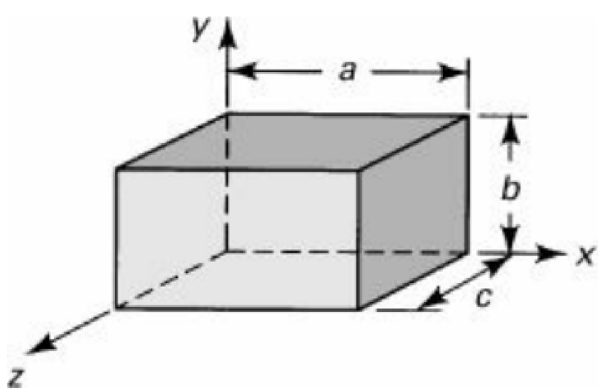

Figure 1: Rectangular parallelepiped subjected to tri-axial stresses.

(a) the changes Δa, Δb, and Δc in the dimensions of the block.

(b) the change ΔV in the volume.

Solution

(a)

From the generalized Hooke's law, the stress-strain relation is given by:

$$\begin{aligned}\epsilon_x &= \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) \\ \epsilon_y &= \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) \\ \epsilon_z &= \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y))\end{aligned}$$

From the definition of strain,

$$\begin{aligned}\Delta x &= \epsilon_x a \\ \Delta y &= \epsilon_y b \\ \Delta z &= \epsilon_z c\end{aligned}$$

By direct substitution,

$$\begin{aligned}\Delta x &= \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z))a \\ &= \frac{1}{70 \times 10^3}(70 - \frac{1}{3}(-30 - 15))150 \\ &= \boxed{0.182\,\text{mm}} \\ \Delta b &= \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z))b \\ &= \frac{1}{70 \times 10^3}(-30 - \frac{1}{3}(70 - 15))100 \\ &= \boxed{-0.0690\,\text{mm}} \\ \Delta c &= \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y))c \\ &= \frac{1}{70 \times 10^3}(-15 - \frac{1}{3}(70 - 30))75 \\ &= \boxed{-0.0304\,\text{mm}}\end{aligned}$$

(b)

The change in volume is given by:

$$\Delta V = eV_0$$

Where dilation e is given by:

$$\begin{aligned}e = \epsilon_x + \epsilon_y + \epsilon_z = &= \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) \\ &= \frac{1 - \frac{2}{3}}{70 \times 10^3}(70 - 30 - 15) \\ &= 1.1905 \times 10^{-4}\end{aligned}$$

Therefore,

$$\begin{aligned}\Delta V &= eV_0 \\ &= (1.1905 \times 10^{-4})(150)(100)(75) \\ &= \boxed{134\,\mathrm{mm}^3}\end{aligned}$$

Question 3

A rectangular plate is subjected to uniform tensile stress σ along its upper and lower edges as shown in Fig. 2. Determine the displacements u and v in terms of x, y, and material properties (E, ν) using Eqns. (1) and (2) and the appropriate conditions at the origin.

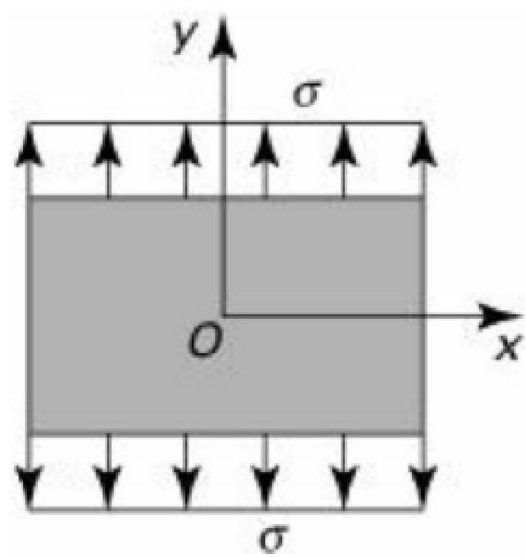

Figure 2: Rectangular plate subjected to uniform tensile stress.

$$\epsilon_x = \frac{\nabla u}{\nabla x}, \quad \epsilon_y = \frac{\nabla v}{\nabla y} \tag{1}$$

$$\gamma_{xy} = \alpha_x - \alpha_y = \frac{\nabla u}{\nabla y} + \frac{\nabla v}{\nabla x} \tag{2}$$