An Investigation of Simple Harmonic Oscillations of a Damped System

by: Alex Diep

Date: March 20, 2023

Instructor	Lisa Kinsale
TAs	Ahmed Elsherbiny
	Marjan Darabi
Group Members	Ahmad, Safiya
	Allegretto, Luca
	Colabella, James
	Dadhania, Karan
	Sammam, S M Faiaz
CCID	abdiep
Student ID	1664334
Section	H41
Group	13

Abstract

This will be a summary of the lab. It will include the purpose of the lab, the methods used, and the results obtained. This will be completed after the lab is finished.

Contents

List of Figures

List of Tables

1 Nomenclature

will do this after the lab is done because I don't know what all the variables used will be yet.

asd

2 Introduction

2.1 Background

TO DO: Reword and find sources for information.

3 Procedure

4 Theory

4.1 Free Vibrations

The experimental setup for free vibrations is modelled in Figure 1. The system consists of a mass m_e attached to a spring with stiffness k_e.

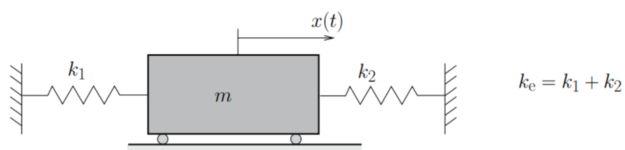

Figure 1: Spring-Mass System

If x is the displacement of the mass from its equilibrium position, the equation of motion is given by

$$m_e\ddot{x} + k_e x = 0 \tag{1}$$

The solution to Equation 1 is given by

$$x(t) = \frac{v_0}{p}\sin(pt) + x_0\cos(pt) \tag{2}$$

where v_0 is the initial velocity, x_0 is the initial displacement, and $p = \sqrt{\frac{k_e}{m_e}}$ is the natural frequency of the system. The natural frequency is the frequency at which the system will oscillate if it is displaced and released. The period of the system is given by

$$\tau = \frac{2\pi}{p} \tag{3}$$

4.2 Forced Vibrations

The experimental setup for forced vibrations is modelled in Figure 2. The system consists of a mass m_e attached to a spring with stiffness k_e. The force is $F(t) = kY_0\sin(\omega t)$.

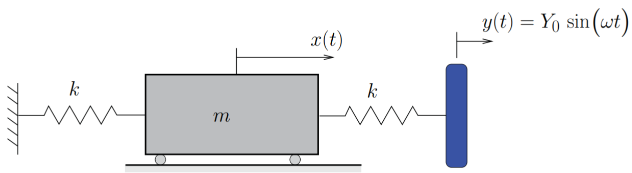

Figure 2: Forced Damped Vibrations System

The equation of motion for the system is given by

$$m_e\ddot{x} + k_e x = F_0 \cos(\omega t) \tag{4}$$

where $F_0 = kY_0$. The time-dependent solution to Equation 4 is

$$x(t) = \frac{Y_0}{2}\left[\frac{1}{1-\left(\frac{\omega}{p}\right)^2}\right]\sin(\omega t) \tag{5}$$

$$\tag{6}$$

where $p = \sqrt{\frac{k_e}{m_e}}$ is the natural frequency of the system. The DMF is given by

$$\text{DMF} = \frac{1}{\left|1-\left(\frac{\omega}{p}\right)^2\right|} \tag{7}$$

Plotting the DMF against ω/p will give the frequency response of the system, as shown in Figure 3. At $\omega/p < \sqrt{2}$, the DMF > 1, which means the system amplifies the input force. At $\omega/p > \sqrt{2}$, the DMF < 1, which means the system attenuates the input force. The system is in resonance at $\omega/p = 1$. Defining static deflection as

$$\mathbb{X}_0 = \frac{F_0}{k_e} \tag{8}$$

we can see that the $Y_0/2$ term in Eq. 5 is static deflection.

4.3 Damped Spring Mass System

An energy dissipation method is added to the system to model the energy loss in the system. The most common approach is to add viscous damping, which is proportional to the velocity of the mass. The equation of motion from Eq. 1 is modified to include damping as

$$m_e\ddot{x} + c_e\dot{x} + k_e x = 0 \tag{9}$$

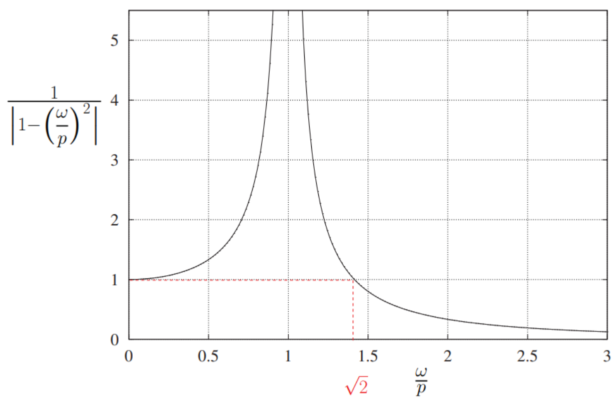

Figure 3: DMF vs. ω/p

where c_e is the damping coefficient. Assuming the mass is given an initial displacement and zero initial velocity, the solution to Eq. 9 is given by

$$x(t) = Ae^{\zeta t} \cos\left(\sqrt{1-\zeta^2}t\right) \tag{10}$$

where,

$$\zeta = \frac{c_e}{2m_e p} = \frac{c_e}{2\sqrt{k_e m_e}} \tag{11}$$

The solution to Eq. 9 is a decaying sinusoidal function, plotted in Figure 4. It can be shown that the peaks can be related by

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = \frac{2\pi}{\sqrt{1-\zeta^2}} \tag{12}$$

From which, the damping ratio can be determined as

$$\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}} \tag{13}$$

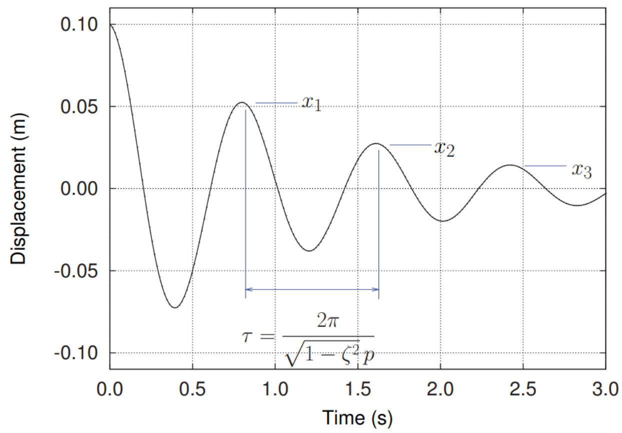

Figure 4: Damped Response of a Spring-Mass System

5 Results and Discussion

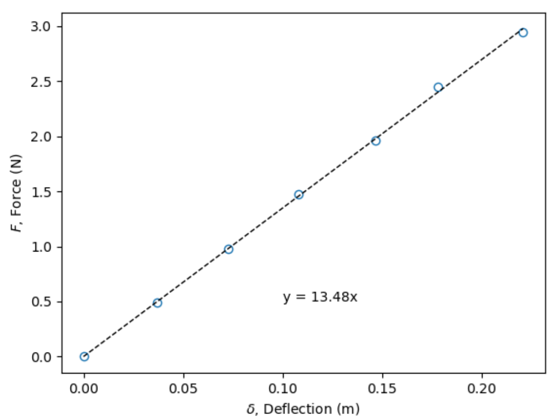

Figure 5: Force vs. Deflection

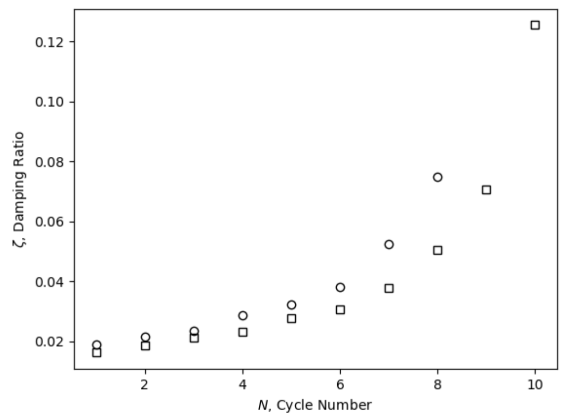

Figure 6: Big Cart Damping Ratio

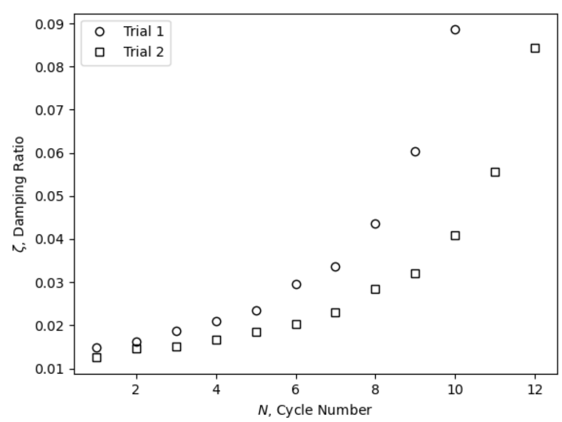

Figure 7: Small Cart Damping Ratio

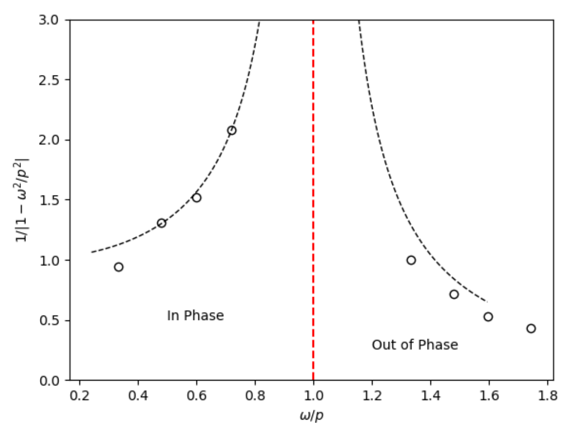

Figure 8: Big Cart DMF vs. ω/p

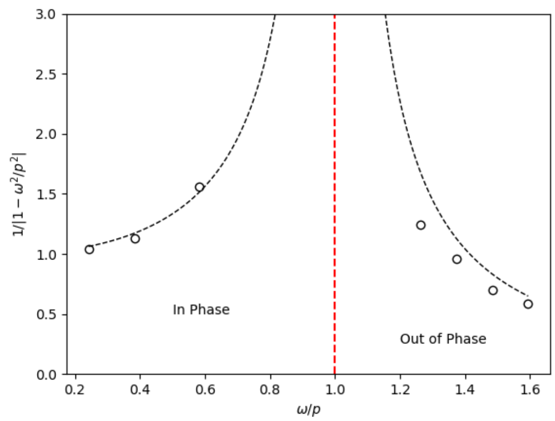

Figure 9: Small Cart DMF vs. ω/p

6 Conclusion

- Summarize the results of the lab
- Discuss the significance of the results
- Discuss the sources of error
- Discuss the limitations of the experiment
- Discuss the implications of the results
- Discuss the future work

6.1 Technical Recommendations

7 References

The references section will be auto populated with Bibtex.

A Appendix: Table Dump

Will format later, formatting tables is aids

```
Mass Force Intitial Position, $a$ Final Position, $b$ Deflection
(g) (N) (cm) (cm) (m)
0 0 91.1 91.1 0
50 0.4905 91.1 87.4 0.037
100 0.981 91.1 83.8 0.073
150 1.4715 91.1 80.3 0.108
200 1.962 91.1 76.4 0.147
250 2.4525 91.1 73.3 0.178
300 2.943 91.1 69 0.221
```

```
Small mass (kg) 0.2648 Big mass (kg) 0.4465
Trial w/o measuring system w/ measuring system w/o measuring system w/ measuring syste
1 8.81 11.13 11.49 13.7
2 8.71 10.98 11.67 13.14
3 8.85 11.01 11.69 13.31
4 8.88 10.9 11.84 13.19
5 8.93 10.95 11.61 13.21
avg, 10 tau 8.836 10.994 11.66 13.31
tau 0.8836 1.0994 1.166 1.331
natural freq, p, derived from period (rad/s) 7.110893286 5.715103972 5.388666644
4.720650118
natural freq, p, derived from period (Hz) 1.131733816 0.909587047 0.857632933
0.751314801
natural freq, p, derived from using k_e (rad/s) 7.134339805 5.494169447
natural freq, p, derived from using k_e (Hz) 1.135465446 0.874424226
```

```
Pulley Diamater (mm) Mass Groove Depth (mm)
Big 195.98 259.9 1.5
Small 52.61 13.6 1.5
```

```
dataset peak number amplitude delta \zeta
big 1 1 8.2775 0.119385043 0.018997291
big 1 2 7.346 0.134348497 0.021377341
```

```
big 1 3 6.4225 0.146730503 0.02334652
big 1 4 5.546 0.179764425 0.028598694
big 1 5 4.6335 0.202002469 0.032133089
big 1 6 3.786 0.240058119 0.038178581
big 1 7 2.978 0.328811927 0.052260531
big 1 8 2.1435 0.471264045 0.074793917
big 1 9 1.338
big 2 1 9.4465 0.101586118 0.01616582
big 2 2 8.534 0.116963123 0.018612035
big 2 3 7.592 0.132252668 0.021044005
big 2 4 6.6515 0.145034028 0.023076735
big 2 5 5.7535 0.17320361 0.027555743
big 2 6 4.8385 0.192437655 0.030613049
big 2 7 3.9915 0.23835321 0.037907825
big 2 8 3.145 0.317698849 0.05049883
big 2 9 2.289 0.444895542 0.070630487
big 2 10 1.467 0.79495422 0.125520247
big 2 11 0.6625
small 1 1 9.473 0.093317023 0.014850228
small 1 2 8.629 0.102608739 0.016328511
small 1 3 7.7875 0.118031863 0.018782041
small 1 4 6.9205 0.131541363 0.020930872
small 1 5 6.0675 0.147293322 0.023436022
small 1 6 5.2365 0.185362555 0.029488537
small 1 7 4.3505 0.211971847 0.033717185
small 1 8 3.5195 0.27362217 0.043507086
small 1 9 2.677 0.379561461 0.060299159
small 1 10 1.8315 0.55872893 0.088574955
small 1 11 1.0475
small 2 1 23.182 0.078630971 0.012513528
small 2 2 21.429 0.091411866 0.014547111
small 2 3 19.557 0.095540303 0.015203954
small 2 4 17.775 0.10451699 0.016632095
small 2 5 16.011 0.1158877 0.018440964
small 2 6 14.259 0.126950746 0.020200716
small 2 7 12.559 0.144982969 0.023068616
```

```
small 2 8 10.864 0.178279663 0.028362675
small 2 9 9.09 0.201245362 0.032012778
small 2 10 7.433 0.257773714 0.040991478
small 2 11 5.744 0.350674281 0.055724823
small 2 12 4.045 0.532063149 0.084378491
small 2 13 2.376
```

```
DMF stuff peak number amplitude doing analysis in pandas cause this is aids
```

```
dataset frequency dmf freq/p
small 0.22hz 0.22 1.039647887 0.241868
small 0.35hz 0.35 1.128692153 0.38479
small 0.53hz 0.53 1.559769526 0.582682
small 1.15hz 1.15 1.242669933 1.26431
small 1.25hz 1.25 0.958841941 1.37425
small 1.35hz 1.35 0.701959584 1.48419
small 1.45hz 1.45 0.586846295 1.59413
big 0.25hz 0.25 0.94371831 0.33275
big 0.36hz 0.36 1.308779343 0.47916
big 0.45hz 0.45 1.517589984 0.59895
big 0.54hz 0.54 2.080947503 0.71874
big 1.00hz 1 1.002852113 1.331
big 1.11hz 1.11 0.712910798 1.47741
big 1.20hz 1.2 0.532661231 1.5972
big 1.31hz 1.31 0.42843729 1.74361
```