MEC E 403
Lab 1: Centrifugal Pumps

by: Alex Diep

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Instructor	Lisa Kinsale
TAs	Enrique Millones
	Simin Shabani
Group Members	Ahmad, Safiya
	Allegretto, Luca
	Colabella, James
	Dadhania, Karan
	Sammam, S M Faiaz
CCID	abdiep
Student ID	1664334
Section	H41
Group	13

Abstract

Centrifugal pumps are used in many applications, including water supply, irrigation, and sewage systems. The general purpose of a pump is to increase the pressure of a fluid. Due to their wide application, the likelyhood of encountering a centrifugal pump in engineering design is significant. Understanding pump performance, limitations, and key parameters is important for the design and operation of these systems.

This lab experiment was conducted to investigate the performance of a centrifugal pump, comparing them to the manufacturer's specifications. The pump was tested in single, parallel, and series configurations at speeds of 1800, 2700, and 3600 RPM. The tests were performed by setting the pump in the desired configuration (eg. single, parallel, or series), and then measuring the time it took to fill 200 lb of water. The weight was measured with a scale, pressure measured with a transducer, torque measured with a dyno mass, and pump speed with a strobotach.

Generally, the manufacturer's performance was higher than the experimental performance. This is illustrated in multiple plots, where the manufacturer specifications had higher head coefficient for a given flow coefficient, at most with an error of 39.7% with respect to the manufacturer at the fully open 3600 RPM configuration. In addition, in the efficiency plot, the manufacturer's efficiency was higher than the experimental efficiency for all pump speeds.

Parallel and series pump performance had poor agreement between the theoretical and experimental data. The theoretical head and flow had an error of 30.9% and 33.7% respectively for the parallel pump, and 23.1% and 13.0% respectively for the series pump. This suggests that the theoretical model does not accurately predict the parallel pump performance, and somewhat accurately predicts the series pump performance.

An analysis of the specification sheet, through plotting head coefficient - flow coefficient curves, suggested that the pump impeller width and height were not geometrically similar to the impeller diameter. For example, this suggests that the specification for a 96mm impeller diameter had the same width and height as a 102mm impeller diameter.

The investigation found that the performance of the pump was lower than the manufacturer's specifications. The history of the experimental pump was not known, and it is possible that the pump was not operating at its peak performance. It is critical to test the pump before designing a pump system, as the manufacturer's specifications may not be accurate. Further work in improving flow rate measurements through utilizing a calibrated digital flow meter should be considered to reduce the uncertainty in the results.

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1 Nomenclature

Symbol	Description	Units
b	Blade height at the exit	m
β_2	Blade angle	rad
D_1	Pump suction line diameter	m
D_2	Pump discharge line diameter	m
g	Gravitational acceleration	m/s^2
H	Stagnation head	m
H'_{ideal}	Ideal shutoff head	m
H_t	Transducer head	m
H'_{thumb}	Rule of thumb shutoff head	m
$\dot{m}$	Mass flow rate of water	kg/s
N	Number of pump impeller blades	-
ΔP	Pressure change	Pa
Q	Volumetric flow rate	m^3/s
r_2	Impeller radius	m
t	Time	s
T	Torque exerted by the pump impeller	Nm
U	Impeller tip speed	m/s
v_{2r}	Radial component of the exit velocity	m/s
$v_{2\theta}$	Tangential component of the exit velocity	m/s
w	Blade width	m
η	Pump efficiency	%

ρ	Density of water	kg/m^3
Ω	Pump impeller angular speed	rad/s
Φ	Flow coefficient	-
Ψ	Head coefficient	-

2 Introduction

The focus of this lab was to analyze the performance of centrifugal pumps at different operating speeds, valve orientations, along with both parallel and series configurations. The data obtained from the lab testing is used to determine head, flow, and efficiency. The lab results are compared with the manufacturer provided data sheets and theoretical calculations to evaluate their validity. The parameters measured during each test are the moment arm, time to collect water (200lbs), and the transducer reading. The goal of determining the performance of different pumping configurations is useful for understanding how to achieve optimal efficiency in a piping system, and the effect certain factors (i.e., pump speed) have [1].

3 Procedure

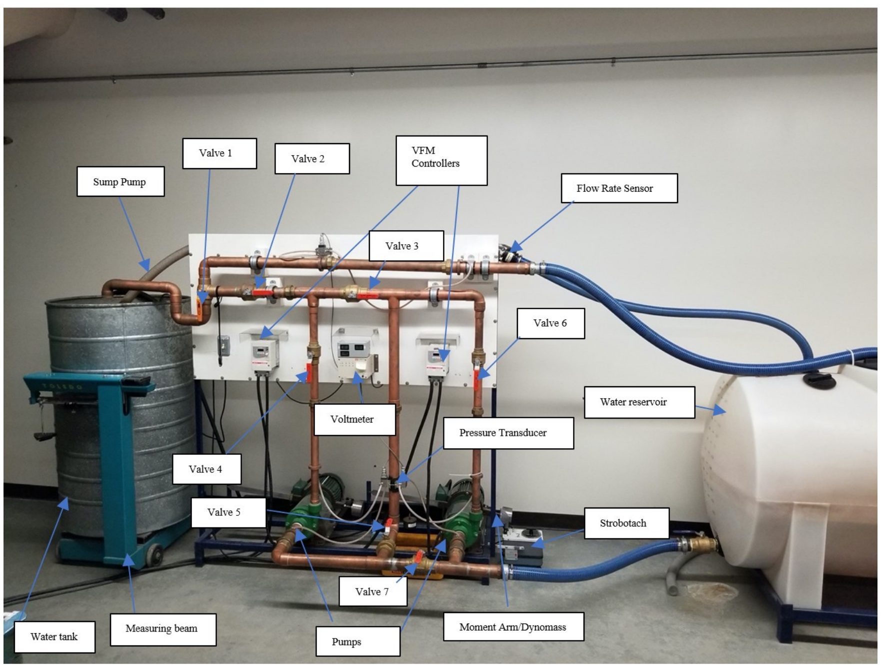

Figure 1: Experimental setup for the pump system

3.1 Equipment

- Pump system that can be configured as a single pump or as two pumps in series or parallel operation. This is the system being studied.
- Stopwatch to measure time to fill a container
- Strobotach to verify pump rotational speed
- Mass scale to measure 200 lbs of water

- 1.401 kg mass to measure moment arm (torque) of motor
- Pressure transducers to measure pump pressure differential

3.2 Procedure

1. The pump system was tested in single pump configuration at 1800, 2700, and 3600 RPM. The speed of the pump was verified using the strobotach, once, after changing the pump speed.

2. At each pump speed (1800, 2700, and 3600 RPM), the following measurements at four different pressures were recorded (closed, full open, and two other equally spaced intermediate settings):

   i. Time to fill a container to 200 lb of water was recorded three times per given speed and pressure setting. After opening valve 1 (Fig.3), the ”200 lb” weight was placed on the measuring beam. As the mass of the container increased, the beam was raised. The time was recorded when the beam was level. Then, the water container was cleared with the sump pump.

      A total of 36 time measurements were recorded for the single pump configuration, 12 measurements for the parallel, and 12 measurements for the series configurations.

   ii. Pressure transducers voltage output was recorded. For the single pump configuration, only the output of the transducer for the active pump was recorded.

      The transducers were measured once per given speed and pressure setting. A total of 12 pressure transducer measurements were recorded for the single pump configuration, 8 measurements for the parallel, and 8 measurements for the series configurations.

   iii. The moment arm were recorded to determine the torque produced by the motor. The dynomass was balanced along the beam attached to the pump until level. The distance of the dyno mass was recorded.

      The moment arm was recorded once per given speed and pressure setting. The dyno mass was recorded once per given speed. A total of 12 moment arm and dyno mass measurements were recorded. The moment arm was neglected for the parallel and series configurations.

3. Item (2) was repeated for parallel and series system configurations with both pumps set at 2700 RPM (neglect the 1800 and 3600 RPM configurations for parallel and series systems). The moment arm was neglected and the output of both pressure transducers were recorded.

4 Theory

4.1 Euler's Turbomachinery Equations

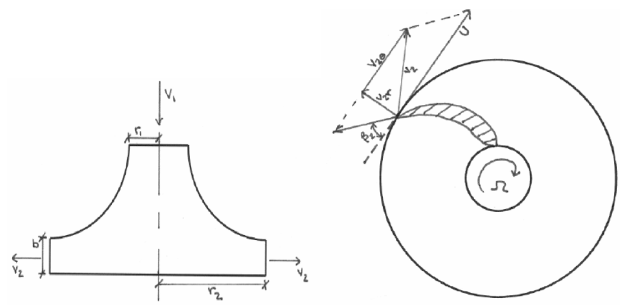

Figure 2: Impeller force and acceleration diagrams.

Using the following assumptions:

1. Viscous effects are negligible
2. Velocity profile is uniform at the exit
3. All work done by the pump is transferred to the fluid

Then by conservation of angular momentum,

$$T = \dot{m} r_2 v_{2\theta} \tag{1}$$

where T is the torque, $\dot{m}$ is the mass flow rate, r_2 is the radius of the impeller, and $v_{2\theta}$ is the tangential velocity at the exit. From Figure 2, we can see that

$$v_{2\theta} = U - v_{2r} \cot \beta_2 \tag{2}$$

where U is the tip speed, v_{2r} is the radial velocity at the exit, and β_2 is the blade angle. Combining (1) and (2), we get

$$T = \dot{m} r_2 (U - v_{2r} \cot \beta_2) \tag{3}$$

By Assumption 3, we can say that

$$T\Omega = \dot{m} g H \tag{4}$$

where Ω is the impeller angular velocity, g is the acceleration due to gravity, and H is the total head rise across the pump. By combining (3), (4), and the kinematic relationship $U = r_2\Omega$, we get

$$\frac{Hg}{U^2} = 1 - \frac{v_{2r}}{U}\cot\beta_2 \tag{5}$$

Defining the head coefficient Ψ and the flow coefficient Φ as

$$\Psi = \frac{Hg}{U^2}, \quad \Phi = \frac{v_{2r}}{U} \tag{6}$$

then (5) becomes

$$\Psi = 1 - \Phi\cot\beta_2 \tag{7}$$

also, v_{2r} can be expressed by the continuity equation as

$$v_{2r} = \frac{Q}{A_2} = \frac{Q}{b(2\pi r_2 - Nw)} \tag{8}$$

where Q is the flow rate, A_2 is the area of the exit, b is the blade height at the exit, N is the number of blades, and w is the width of the blade.

4.2 Shutoff Head

The ideal shutoff head can be obtained as

$$H'_{\text{ideal}} = \frac{U^2}{g} \tag{9}$$

by setting $\Phi = 0$ in (7). If it is assumed that all kinetic energy is lost due to friction, then

$$\frac{KE}{\text{unit weight}} = \frac{U^2}{2g}$$

where KE is the kinetic energy. Therefore, the rule of thumb for the shutoff head is

$$\begin{aligned} H'_{\text{thumb}} &= \frac{U^2}{g} - \frac{U^2}{2g} \\ H'_{\text{thumb}} &= \frac{U^2}{2g} = \frac{1}{2}H'_{\text{ideal}} \end{aligned} \tag{10}$$

4.3 Affinity Laws

For large Reynolds numbers, the flow is dynamically similar in geometrically similar machines when the flow and head coefficients are the same. For geometrically similar machines operating at different conditions (i) and (ii) such that the head and flow coefficients are the same. First, equating the head coefficients, we get

$$\Psi_i = \Psi_{ii}$$

so expanding, we get

$$\begin{aligned} \frac{H_i}{U_i^2} &= \frac{H_{ii}}{U_{ii}^2} \\ \frac{H_i}{H_{ii}} &= \left(\frac{U_i}{U_{ii}}\right)^2 \approx \left(\frac{D_i \Omega_i}{D_{ii}\Omega_{ii}}\right)^2 \end{aligned} \tag{11}$$

where D is the diameter of the impeller. Next, equating the flow coefficients, we get

$$\Phi_i = \Phi_{ii}$$

$$\frac{v_{2ri}}{U_i} = \frac{v_{2rii}}{U_{ii}} \tag{12}$$

$$\frac{b_i}{D_i} = \frac{b_{ii}}{D_{ii}} \tag{13}$$

It can be shown that

$$\frac{Q_i}{Q_{ii}} = \frac{\Omega_i}{\Omega_{ii}} \left(\frac{D_i}{D_{ii}}\right)^3 \tag{14}$$

For geometrically similar machines, the impelle width, w, is also scaled by impeller diameter, so

$$\frac{w_i}{D_i} = \frac{w_{ii}}{D_{ii}} \tag{15}$$

4.4 Transducer Head Adjustment

In this experiment, since the inlet and outlet pipe diameters are different, the transducer head, H_t, must be corrected for flow kinetic energy to give the pump stagnation head, H, as

$$
\begin{aligned}
H &= H_t + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \\
H &= H_t + \frac{v_2^2}{2g}\left(1 - \left(\frac{v_1}{v_2}\right)^2\right) \qquad (16)
\end{aligned}
$$

where H_t is the transducer head, v_1 is the velocity at the inlet, and v_2 is the velocity at the outlet. The volume flow rate through the system can be written as

$$Q = \frac{v_2 \pi D_2^2}{4} \qquad (17)$$

and with the pipe diameters D_1 at the inlet and D_2 at the outlet, mass conservation requires that

$$v_2 \pi D_2^2 = v_1 \pi D_1^2 \qquad (18)$$

With (17) and (18), (16) becomes

$$H = H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) \qquad (19)$$

4.5 Pumps in Series and Parallel

When pumps are connected in series, the total head is the sum of the individual heads, and the flow rate is the same as the individual flow rates.

$$H_{\text{series},t} = H_{1,t} + H_{2,t} \qquad (20)$$

$$Q_{\text{series}} = Q_1 = Q_2 \qquad (21)$$

When pumps are connected in parallel, the total head is the average of the individual heads, and the total flow rate is the sum of the individual flow rates.

$$H_{\text{parallel},t} = \frac{H_{1,t} + H_{2,t}}{2} \qquad (22)$$

$$Q_{\text{parallel}} = Q_1 + Q_2 \qquad (23)$$

4.6 Pump Efficiency

The pump efficiency is defined as

$$\begin{aligned}\eta &= \frac{\text{useful work done}}{\text{total work done}} \\ &= \frac{\rho g Q H}{T\Omega}\end{aligned} \tag{24}$$

5 Results and Discussion

5.1 Main Results

Table 2 shows the experimental and manufacturer pump performance summary. The pump configuration, valve configuration, pump speed, volumetric flow rate, corrected head, head coefficient, and flow coefficient are shown. Sample calculations for the single pump in Appendix A. Parallel and series pump performance analysis is shown in Appendix B.

Table 2: Experimental and Manufacturer Pump Performance Summary

Pump Config.	Valve Config.	Pump Speed, Ω (RPM)	Volumetric Flow Rate, Q ($\mathrm{m^3\,s^{-1}}$)	Corrected Head, H (m)	Head Coefficient, Ψ	Flow Coefficient, Φ
Single	Fully open	1800	0.003159	2.91	0.275	0.12
Single	Partial 1	1800	0.003003	3.16	0.300	0.11
Single	Partial 2	1800	0.001492	5.09	0.482	0.05
Single	Closed	1800	-	5.49	0.520	0.00
Single	Fully open	2700	0.004858	5.91	0.249	0.12
Single	Partial 1	2700	0.004749	6.31	0.265	0.12
Single	Partial 2	2700	0.002911	10.6	0.445	0.07
Single	Closed	2700	-	12.4	0.522	0.00
Single	Fully open	3600	0.006474	9.72	0.230	0.12

Single	Partial 1	3600	0.006240	11.2	0.266	0.11
Single	Partial 2	3600	0.002454	20.8	0.491	0.05
Single	Closed	3600	-	21.8	0.515	0.00
Single	Manufacturer	1800	0.0040	2.58	0.244	0.15
Single	Manufacturer	1800	0.0033	4.14	0.392	0.12
Single	Manufacturer	1800	0.0026	4.96	0.470	0.10
Single	Manufacturer	1800	0.0020	5.40	0.511	0.07
Single	Manufacturer	2700	0.0060	5.77	0.243	0.15
Single	Manufacturer	2700	0.0050	9.12	0.384	0.12
Single	Manufacturer	2700	0.0040	11.0	0.463	0.10
Single	Manufacturer	2700	0.0030	12.1	0.509	0.07
Single	Manufacturer	2700	0.0020	12.8	0.539	0.05
Single	Manufacturer	3600	0.0080	10.3	0.244	0.15
Single	Manufacturer	3600	0.0067	16.1	0.381	0.12
Single	Manufacturer	3600	0.0054	19.2	0.454	0.10
Single	Manufacturer	3600	0.0040	21.6	0.511	0.07
Single	Manufacturer	3600	0.0027	22.8	0.540	0.05
Parallel	Fully open	2700	0.007266	11.3	0.474	0.18
Parallel	Partial 1	2700	0.006283	11.7	0.492	0.15
Parallel	Partial 2	2700	0.002217	12.5	0.528	0.05
Parallel	Closed	2700	-	12.5	0.526	0.00
Series	Fully open	2700	0.005584	9.15	0.385	0.14
Series	Partial 1	2700	0.005449	10.4	0.439	0.13
Series	Partial 2	2700	0.002983	21.9	0.921	0.07

Series	Closed	2700	-	24.9	1.048	0.00

5.2 Single Pump Performance

5.2.1 Head vs. Flow Rate

Using the data from Table 2, the head vs. flow rate for the experimental single pump data was plotted in Figure 3. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix A. Plot was made using Matplotlib in Python [2].

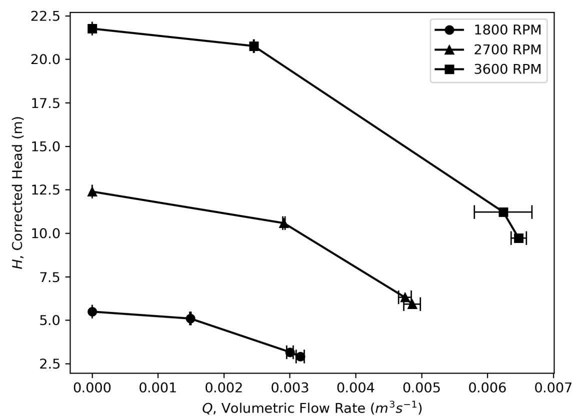

Figure 3: Single pump experimental head vs. flow rate plot.

5.2.2 Head Coefficient vs. Flow Coefficient

The head coefficient and flow coefficient for the experimental, manufacturer, and ideal pump data are shown in Figure 4. The ideal pump data was calculated from ideal pump equation (Eq. 7) using the impeller angle. Impeller angle was determined visually, shown in Appendix C. Sample calculations for the experimental and manufacturer head and flow coefficients are shown in Appendix A.

For the single experimental pump, the head and flow coefficients appear to fall onto the same curve. This suggests that the non-dimensionalization of the head and flow coefficients is appropriate. A non-linear relationship is observed, where the head coefficient decreases as the flow coefficient increases. Most points of different speeds are within error bars of each other. The largest source of error was the precision error caused by the reaction time of the stopwatch operator.

The manufacturer head and flow coefficients also appear to fall onto the same, but different than the experimental, curve. At 3600 RPM, $Phi = 0.12$, the error between the experimental and manufacturer was 39.7%. In general manufacturer head coefficient is higher than the experimental head coefficient for the same flow coefficient. This suggests that the manufacturer has over rated their pump.

The ideal head and flow coefficients are shown as a straight line. The ideal head coefficient is much higher than the experimental and manufacturer head coefficient for the same flow coefficient. The ideal pump neglects all losses and is overly optimistic. The linear trend does not follow the experimental and manufacturer data.

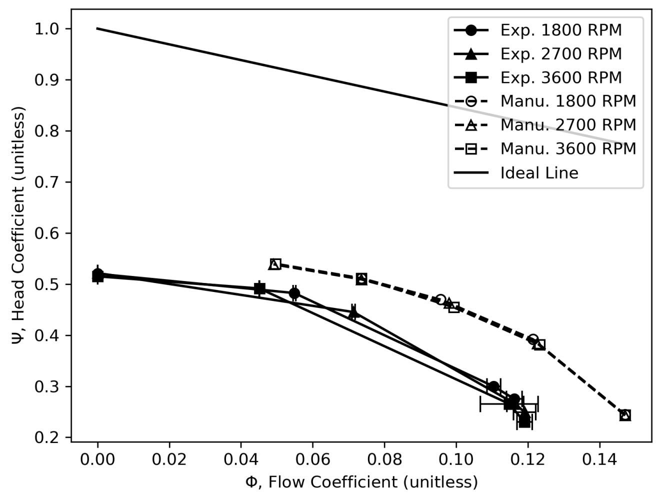

Figure 4: Single pump experimental, manufacturer, and ideal head coefficient vs. flow coefficient plot.

5.2.3 Ideal and Rule of Thumb Shutoff Head

Table 3: Ideal, rule of thumb, and experimental shutoff head for the single pump at 3600 RPM.

Ideal Shutoff Head, H'_{ideal} (m)	Rule of Thumb Shutoff Head, H'_{thumb} (m)	Experimental Shutoff Head, H'_{exp} (m)

42.2	21.1	21.8

The ideal and rule of thumb shutoff head for the single pump at 3600 RPM was calculated and compared to the experimental shutoff head (single closed valve configuration). The ideal and rule of thumb shutoff head was calculated using Eq. 9 and 10. The experimental shutoff head was taken from Table 2. Sample calculations for the ideal and rule of thumb shutoff head are shown in Appendix D and the experimental shutoff head in Appendix A.

The ideal shutoff head assumes all kinetic energy is lost too friction and is overly optimistic. The error was calculated to be 48.3%. This suggests poor agreement between the experimental and ideal shutoff head.

The rule of thumb shutoff head is a more realistic estimate and is within 3.3% of the experimental shutoff head. This suggests good agreement between the experimental and rule of thumb shutoff head.

5.3 Parallel and Series Pump Performance

The head vs. flow rate for the experimental parallel and series pump data was plotted in Figure 5 and 6. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Sample calculations are shown in Appendix B.

The parallel curves have poor agreement. The theoretical head was higher than the experimental head for the same flow. For example, in the fully open configuration, the flow error was 33.7% and the head error was 30.9%. This suggests the theoretical model does not accurately predict the parallel pump performance.

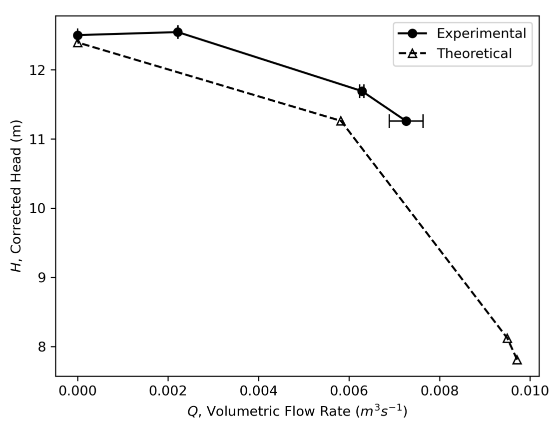

Figure 5: Parallel pump experimental and theoretical head vs. flow rate plot.

The series curves have little agreement. The theoretical head was lower than the experimental head for the same flow. For example, in the fully open configuration, the flow error was 13.0% and the head error was 23.1%. These deviations were smaller than the parallel pump. This suggests the theoretical model does somewhat accurately predicts the series pump performance.

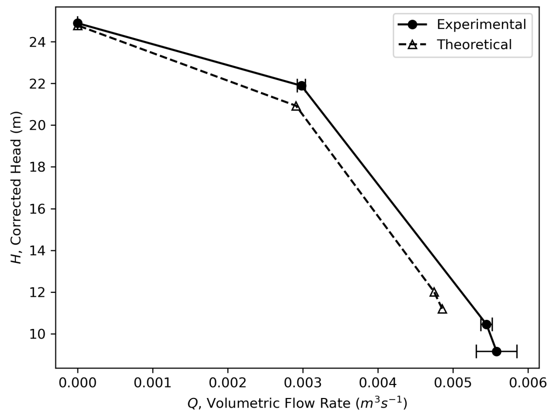

Figure 6: Series pump experimental and theoretical head vs. flow rate plot.

5.4 Geometric Similarity in Manufacturer's Specifications

The manufacturer's specifications are for geometrically dissimilar pumps. To investigate the effects of assuming similar geometry, two plots were produced. Sample calculations for the head and flow coefficients are shown in Appendix E.

The first plot, Figure 7, shows the head coefficient vs. flow coefficient for geometrically similar pumps where impeller blade height, b, and impeller width, w, were scaled by the impeller diameter, D. An observation is that the head coefficient decreases for a given flow coefficient as the impeller diameter decreases.

The second plot, Figure 8, shows the head coefficient vs. flow coefficient for geometrically dissimilar pumps where impeller blade height, b, and impeller width, w, were not scaled by the impeller diameter, D. An observation is that the head coefficient decreases for a given flow coefficient as the impeller diameter decreases.

The head and flow coefficients for the geometrically similar pumps fall only somewhat collapse onto the same curve. At low flow coefficients, the head coefficient decreases as the impeller diameter decreases. For higher flow coefficients, the head coefficient increases as the impeller diameter decreases. This is not expected as the curves should collapse onto the same curve. The head and flow coefficients for the geometrically dissimilar pumps fall more so onto the same curve. The same trend where at low flow coefficients, the head coefficient decreases for a given flow coefficient as the impeller diameter decreases and for higher flow coefficients, the head coefficient increases for a given flow coefficient as the impeller diameter decreases. This trend is much less pronounced. This suggests that the pumps are geometrically dissimilar, which is consistent with the manufacturer's specifications.

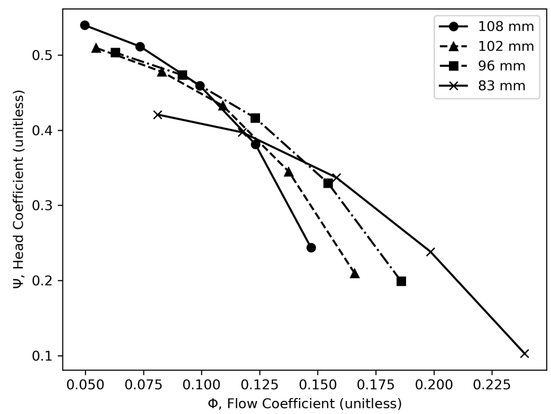

Figure 7: Head coefficient vs. flow coefficient for geometrically similar pumps.

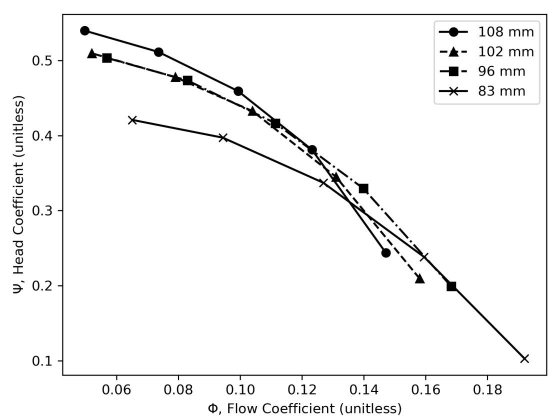

Figure 8: Head coefficient vs. flow coefficient for geometrically dissimilar pumps.

5.5 Pump Efficiency

The pump efficiencies for the experimental data was calculated in Appendix F. The pump effiecncies for the manufacturer data was given in the manufacturer's specifications. The plot of the experimental and manufacturer pump efficiencies is shown in Figure 9.

The pump had the highest efficiency of 49.9% when operating at 2700 RPM in a partially closed valve configuration. The pump had the lowest efficiency of 37.0% when operating at 1800 RPM in a partially closed valve configuration.

The actual pump efficiency was lower than the manufacturer pump efficiency for all pump speeds. It was difficult to compare directly since the flow coefficients of the experimenal and manufacturer seldom matched. The largest deviation was approximately 29% at 1800 RPM and a flow coefficient of 0.12. Again, this suggests the manufacturer has over rated their pump.

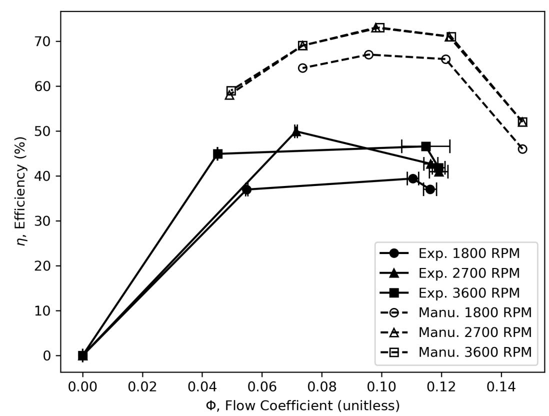

Figure 9: Single pump experimental and manufacturer flow coefficient vs. efficiency plot.

5.6 Elevation Effects

The vertical elevation of the pump lines have an effect on the head of the pump. This variation was not accounted for in the modelling. The transducers did not measure pressure immediately at the outlet due to the flow needing to develop fully. Some distance from the inlet was required due to the pressure gradient in the pump.

The error caused by the vertical displacements between the inlet and outlet was less than 0.2m (see Fig.3). This would have added a small error of 0.2m to the head which is 2.6% of the average head measured in the single pump configuration, 9.25m. This is a small error and was neglected.

6 Conclusion

The focus of this lab was to investigate and understand the performance of centrifugal pumps. The pump performance was studied in single, parallel, and series configurations. The pump efficiency was also calculated.

The head and flow coefficients were plotted for the experimental data. The head and flow coefficients for the single pump fell onto the same curve. The manufacturer's head and flow coefficients also fell onto a single, but different, curve. The manufacturer head coefficient was higher than the experimental head coefficient for the same flow coefficient. The ideal head and flow coefficients were linear and did not agree with the experimental and manufacturer data.

The rule of thumb shutoff head was within 3.3% of the experimental shutoff head. This suggests good agreement between the experimental and rule of thumb shutoff head. The ideal shutoff head was 48.3% higher than the experimental shutoff head. This suggests poor agreement between the experimental and ideal shutoff head.

The parallel pump performance had poor agreement between the theoretical and experimental data. The theoretical head was higher than the experimental head for the same flow. The series pump performance had little agreement between the theoretical and experimental data. The theoretical head was lower than the experimental head for the same flow.

The head and flow coefficients for the geometrically similar pumps fell only somewhat collapse onto the same curve. The head and flow coefficients for the geometrically dissimilar pumps fell more so onto the same curve. This suggests that the pumps are geometrically dissimilar, which is consistent with the manufacturer's specifications.

The pump efficiencies were calculated for the experimental and manufacturer data. The pump was most efficient when operating at 2700 RPM in a partially closed valve configuration. The pump was least efficient when operating at 1800 RPM in a partially closed valve configuration. The actual pump efficiency was lower than the manufacturer pump efficiency for all pump speeds.

The vertical elevation of the pump lines have an effect on the head of the pump. This variation was not accounted for in the modelling. It was assumed that this variation was negligible.

6.1 Technical Recommendations

This investigation revealed how pump performance can vary significantly from the manufacturer's specifications. It is critical to test the pump before designing a pump system, as the manufacturer's specifications may not be accurate. Due to performance issues and unknown history, replacing or repairing the pump is recommended.

Further work in improving flow rate measurements through utilizing a calibrated digital flow meter should be considered to reduce the uncertainty in the results. The largest source of error for all calculated parameters was the precision error from time. This was due to the reaction time of the stopwatch operator. To reduce this error, a digital flow meter should be used to measure the flow rate. This would reduce the error in the flow rate and the head.

The pressure transducer was assumed to have no precision error since calibration was not performed and only one measurement was taken. To account for this error, the pressure transducer should be calibrated and multiple measurements should be taken.

7 References

[1] V. S. Lobanoff and R. R. Ross, *Centrifugal Pumps: Design and Application*, 2nd ed. Houston, TX: Gulf Publishing, May 1992.

[2] J. D. Hunter, "Matplotlib: A 2d graphics environment," *Computing in Science & Engineering*, vol. 9, no. 3, pp. 90–95, 2007.

A Appendix: Single Pump Analysis

This section will discuss the single pump analysis. Experimental discharge, head, head coefficient, and flow coefficient will be calculated and compared to the manufacturer's specifications. The uncertainty in the discharge and head will also be calculated.

A.1 Single Pump Discharge and Heads

Table A.4: Single pump experimental transducer output and time to collect water for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Transducer Output, V_t (V)	Time to collect water (s) Trial 1	Trial 2	Trial 3	Nominal time (s)
Fully open	1800	0.75	28.55	28.99	28.79	28.78
Partial 1	1800	0.83	30.14	30.51	30.16	30.27
Partial 2	1800	1.43	61.12	60.80	60.86	60.93
Closed	1800	1.56	-	-	-	-
Fully open	2700	1.50	18.66	18.93	18.55	18.71
Partial 1	2700	1.62	19.23	19.23	18.96	19.14
Partial 2	2700	2.94	31.15	31.30	31.22	31.22
Closed	2700	3.52	-	-	-	-
Fully open	3600	2.44	14.13	13.93	14.06	14.04
Partial 1	3600	2.89	15.04	14.35	14.31	14.57
Partial 2	3600	5.85	37.01	37.04	37.07	37.04
Closed	3600	6.18	-	-	-	-

Table A.5: Single pump experimental discharge and heads for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Volume Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Transducer Head, H_t (m)	Corrected Head, H (m)
Fully open	1800	0.003159	2.64	2.91
Partial 1	1800	0.003003	2.92	3.16
Partial 2	1800	0.001492	5.04	5.09
Closed	1800	-	5.49	5.49
Fully open	2700	0.004858	5.28	5.91
Partial 1	2700	0.004749	5.70	6.31
Partial 2	2700	0.002911	10.4	10.6
Closed	2700	-	12.4	12.4
Fully open	3600	0.006474	8.59	9.72
Partial 1	3600	0.006240	10.2	11.2
Partial 2	3600	0.002454	20.6	20.8
Closed	3600	-	21.8	21.8

Sample calculations are evaluated for the fully open configuration at 1800 RPM. Starting with time,

$$t = \frac{\sum t_i}{n} = \frac{28.55\,\mathrm{s} + 28.99\,\mathrm{s} + 28.79\,\mathrm{s}}{3} = 28.78\,\mathrm{s}$$

The volumetric flow rate was calculated using water which had a mass of $m = 200\,\mathrm{lb} = 90.7\,\mathrm{kg}$ and a density of $\rho = 998\,\mathrm{kg\,m^{-3}}$. Then,

$$Q = \frac{m}{\rho t} = \frac{90.7\,\mathrm{kg}}{998\,\mathrm{kg\,m^{-3}} \times 28.78\,\mathrm{s}} = 0.003159\,\mathrm{m^3\,s^{-1}}$$

The transducer head was found by

$$H_t = \frac{\Delta P}{\rho g} = \frac{0.75\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} = 2.64\,\mathrm{m}$$

The corrected head was found using inlet diameter, $D_1 = 0.0508\,\mathrm{m}$, and outlet diameter, $D_2 = 0.0381\,\mathrm{m}$, by

$$\begin{aligned} H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 2.64\,\mathrm{m} + \frac{8 \times (0.003159\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.0381\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.0381\,\mathrm{m}}{0.0508\,\mathrm{m}}\right)^4\right] \\ &= 2.91\,\mathrm{m} \end{aligned}$$

A.1.1 Single Pump Discharge and Head Uncertainty

Table A.6: Single pump experimental time uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed (RPM)	Time STDEV, S_t (s)	Time Precision, P_t (±s)	Time Bias, B_t (±s)	Time Uncertainty, δ_t (±s)
Fully open	1800	0.2203	0.5473	0.01	0.55
Partial 1	1800	0.2081	0.5169	0.01	0.52
Partial 2	1800	0.1701	0.4225	0.01	0.42
Closed	1800	-	-	-	-
Fully open	2700	0.1955	0.4857	0.01	0.49
Partial 1	2700	0.1559	0.3872	0.01	0.39
Partial 2	2700	0.0751	0.1864	0.01	0.19
Closed	2700	-	-	-	-
Fully open	3600	0.1015	0.2521	0.01	0.25
Partial 1	3600	0.4104	1.0195	0.01	1.02
Partial 2	3600	0.0300	0.0745	0.01	0.08
Closed	3600	-	-	-	-

Table A.7: Single pump experimental discharge and head uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed (RPM)	Flow Uncertainty, δ_Q ($\pm\mathrm{m^3\,s^{-1}}$)	Transducer Uncertainty, $\delta_{V_{\mathrm{tr}}}$ ($\pm$V)	Head Uncertainty, δ_{H_t} ($\pm$m)	Corrected Head Uncertainty, δ_H ($\pm$m)
Fully open	1800	6.01E-05	0.01	0.04	0.04
Partial 1	1800	5.13E-05	0.01	0.04	0.04
Partial 2	1800	1.04E-05	0.01	0.04	0.04
Closed	1800	-	0.01	0.04	0.04
Fully open	2700	1.26E-04	0.01	0.04	0.05
Partial 1	2700	9.61E-05	0.01	0.04	0.04
Partial 2	2700	1.74E-05	0.01	0.04	0.04
Closed	2700	-	0.01	0.04	0.04
Fully open	3600	1.16E-04	0.01	0.04	0.05
Partial 1	3600	4.37E-04	0.01	0.04	0.2
Partial 2	3600	4.98E-06	0.01	0.04	0.04
Closed	3600	-	0.01	0.04	0.04

Sample calculations are evaluated for the fully open configuration at 1800 RPM.

First, time standard deviation was calculated with `STDEV.S` in Excel. A confidence of 95% ($\alpha/2 = 0.025$) was used to calculate the t-distribution value ($\nu = 3 - 1 = 2$) with `T.INV.T` in Excel. This gave a value of $t_{\alpha/2,\nu} = 4.3027$. The time precision was calculated by

$$
\begin{aligned}
P_t &= t_{\alpha/2,\nu} \times \frac{S_t}{\sqrt{n}} \\
&= 4.3027 \times \frac{0.2203\,\mathrm{s}}{\sqrt{3}} \\
&= 0.5473\,\mathrm{s}
\end{aligned}
$$

The time bias was assumed to be the resolution of the device, which was estimated to be $0.01\,\mathrm{s}$.

The time total uncertainty was calculated by

$$\begin{aligned}\delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.5473\,\mathrm{s})^2 + (0.01\,\mathrm{s})^2} \\ &= 0.55\,\mathrm{s}\end{aligned}$$

The flow uncertainty was calculated by propagation of error. The function for flow is

$$Q = \frac{m}{\rho t}$$

This is the special purely multiplicative case of the general formula for error propagation. Assuming the mass and density errors are negligible, the flow uncertainty was calculated by

$$\begin{aligned}\delta_Q &= Q\sqrt{\left(\frac{\delta_t}{t}\right)^2} \\ &= Q\left|\frac{\delta_t}{t}\right| \\ &= 0.003159\,\mathrm{m}^3\,\mathrm{s}^{-1}\left|\frac{0.55\,\mathrm{s}}{28.78\,\mathrm{s}}\right| \\ &= 6.01\times10^{-5}\,\mathrm{m}^3\,\mathrm{s}^{-1}\end{aligned}$$

Transducer bias was assumed to be $0.01\,\mathrm{V}$. Transducer precision was not considered since calibration was not performed. So

$$\delta_{V_{\mathrm{tr}}} = 0.01\,\mathrm{V}$$

Next, the head uncertainty was calculated by propagation of error. The function for head is

$$H_t = \frac{\Delta P}{\rho g} = \frac{V_{\mathrm{tr}} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, we have the special purely multiplicative case for error propagation. The head uncertainty was calculated by

$$\begin{aligned}
\delta_{H_t} &= H_t\sqrt{\left(\frac{\delta_{V_{\text{tr}}}}{V_{\text{tr}}}\right)^2} \\
&= H_t\left|\frac{\delta_{V_{\text{tr}}}}{V_{\text{tr}}}\right| \\
&= 2.64\,\text{m}\left|\frac{0.01\,\text{V}}{0.75\,\text{V}}\right| \\
&= 0.04\,\text{m}
\end{aligned}$$

Lastly, the function for corrected head is

$$\begin{aligned}
H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\
\frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{2(H - H_t)}{Q} \\
\frac{\partial H}{\partial H_t} &= 1
\end{aligned}$$

Then, the corrected head uncertainty was determined by

$$\begin{aligned}
\delta_H &= \sqrt{\left(\frac{\partial H}{\partial Q}\right)^2 \delta_Q^2 + \left(\frac{\partial H}{\partial H_t}\right)^2 \delta_{H_t}^2} \\
&= \sqrt{\left(\frac{2(H - H_t)}{Q}\right)^2 \delta_Q^2 + \delta_{H_t}^2} \\
&= \sqrt{\left(\frac{2(2.91\,\text{m} - 2.64\,\text{m})}{0.003159\,\text{m}^3\,\text{s}^{-1}}\right)^2 (6.01\times 10^{-5}\,\text{m}^3\,\text{s}^{-1})^2 + (0.04\,\text{m})^2} \\
&= 0.04\,\text{m}
\end{aligned}$$

A.2 Single Pump Head and Flow Coefficients

Table A.8: Single pump experimental tip and radial exit velocities for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Tip Speed, U ($\mathrm{m\,s^{-1}}$)	Radial Exit Velocity, v_{2r} ($\mathrm{m\,s^{-1}}$)
Fully open	1800	0.003159	10.2	1.2
Partial 1	1800	0.003003	10.2	1.1
Partial 2	1800	0.001492	10.2	0.6
Closed	1800	-	10.2	0.0
Fully open	2700	0.004858	15.3	1.8
Partial 1	2700	0.004749	15.3	1.8
Partial 2	2700	0.002911	15.3	1.1
Closed	2700	-	15.3	0.0
Fully open	3600	0.006474	20.4	2.4
Partial 1	3600	0.006240	20.4	2.3
Partial 2	3600	0.002454	20.4	0.9
Closed	3600	-	20.4	0.0

Table A.9: Single pump experimental head and flow coefficients for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Head Coefficient, Ψ	Flow Coefficient, Φ
Fully open	1800	0.275	0.12
Partial 1	1800	0.300	0.11
Partial 2	1800	0.482	0.05
Closed	1800	0.520	0.00
Fully open	2700	0.249	0.12
Partial 1	2700	0.265	0.12
Partial 2	2700	0.445	0.07
Closed	2700	0.522	0.00
Fully open	3600	0.230	0.12
Partial 1	3600	0.266	0.11
Partial 2	3600	0.491	0.05
Closed	3600	0.515	0.00

Sample calculations are evaluated for the fully open configuration at 1800 RPM. The tip speed was calculated using the impeller radius, $r_2 = 0.108/2 = 0.054\,\mathrm{m}$, by

$$\begin{aligned} U &= r_2\Omega \\ &= 0.054\,\mathrm{m} \times 1800\,\mathrm{RPM} \times \left(\frac{2\pi}{60}\,\mathrm{RPM}^{-1}\right) \\ &= 10.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

The radial exit velocity was calculated using the impeller diameter, the blade height at exit, $b = 0.009\,\mathrm{m}$, blade width at exit, $w = 0.0085\,\mathrm{m}$, and the number of blades $N = 5$, by

$$\begin{aligned} v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.003159\,\mathrm{m^3\,s^{-1}}}{0.009\,\mathrm{m} \times (2\pi \times 0.054\,\mathrm{m} - 5 \times 0.0085\,\mathrm{m})} \\ &= 1.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

The head coefficient was found by

$$\begin{aligned} \Psi &= \frac{Hg}{U^2} \\ &= \frac{2.91\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{(10.2\,\mathrm{m\,s^{-1}})^2} \\ &= 0.275 \end{aligned}$$

The flow coefficient was found by

$$\begin{aligned} \Phi &= \frac{v_{2r}}{U} \\ &= \frac{1.2\,\mathrm{m\,s^{-1}}}{10.2\,\mathrm{m\,s^{-1}}} \\ &= 0.12 \end{aligned}$$

A.2.1 Single Pump Head and Flow Coefficient Uncertainty

Table A.10: Single pump experimental head and flow coefficient uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed (RPM)	Flow Uncertainty, δ_Q ($\pm\mathrm{m^3\,s^{-1}}$)	Head Uncertainty, δ_H ($\pm$m)	Radial Exit Velocity Uncertainty, $\delta_{v_{2r}}$ ($\pm\mathrm{m\,s^{-1}}$)	Head Coefficient Uncertainty, δ_Ψ ($\pm$)	Flow Coefficient Uncertainty, δ_Φ ($\pm$)
Fully open	1800	6.01E-05	0.04	0.024	0.003	0.0024
Partial 1	1800	5.13E-05	0.04	0.022	0.003	0.0019
Partial 2	1800	1.04E-05	0.04	0.0039	0.003	0.0004
Closed	1800	-	0.04	-	0.003	-
Fully open	2700	1.26E-04	0.05	0.047	0.002	0.0031
Partial 1	2700	9.61E-05	0.04	0.036	0.002	0.0024
Partial 2	2700	1.74E-05	0.04	0.007	0.001	0.00043
Closed	2700	-	0.04	-	0.001	-
Fully open	3600	1.16E-04	0.05	0.044	0.001	0.0021
Partial 1	3600	4.37E-04	0.2	0.16	0.004	0.0080
Partial 2	3600	4.98E-06	0.04	0.0019	0.001	0.00009
Closed	3600	-	0.04	-	0.001	-

Sample calculations are evaluated for the fully open configuration at 1800 RPM. The RPM was measured by the stroboscope. It was assumed that pump speed uncertainty was negligible. The radial exit velocity is a function of

$$v_{2r} = \frac{Q}{b(2\pi r_2 - Nw)}$$

Assuming the blade height and width errors are negligible, the radial exit velocity uncertainty was calculated by

$$\begin{aligned}\delta_{v_{2r}} &= v_{2r}\sqrt{\left(\frac{\delta_Q}{Q}\right)^2} \\ &= v_{2r}\left|\frac{\delta_Q}{Q}\right| \\ &= 1.2\,\mathrm{m\,s^{-1}}\left|\frac{6.40\times10^{-5}\,\mathrm{m^3\,s^{-1}}}{0.003159\,\mathrm{m^3\,s^{-1}}}\right| \\ &= 0.024\,\mathrm{m\,s^{-1}}\end{aligned}$$

The function for head coefficient is

$$\Psi = \frac{Hg}{U^2}$$

Assuming the gravity and tip speed errors are negligible, the head coefficient uncertainty was calculated by

$$\begin{aligned}\delta_\Psi &= \Psi\sqrt{\left(\frac{\delta_H}{H}\right)^2} \\ &= \Psi\left|\frac{\delta_H}{H}\right| \\ &= 0.275\left|\frac{0.04\,\mathrm{m}}{2.91\,\mathrm{m}}\right| \\ &= 0.003\end{aligned}$$

The function for flow coefficient is

$$\Phi = \frac{v_{2r}}{U}$$

since the tip speed error is negligible, this is a special purely multiplicative case of the general formula for error propagation. The flow coefficient uncertainty was calculated by

$$\begin{aligned}\delta_\Phi &= \Phi\sqrt{\left(\frac{\delta_{v_{2r}}}{v_{2r}}\right)^2} \\ &= \Phi\left|\frac{\delta_{v_{2r}}}{v_{2r}}\right| \\ &= 0.12\left|\frac{0.024\,\mathrm{m\,s^{-1}}}{1.2\,\mathrm{m\,s^{-1}}}\right| \\ &= 0.0024\end{aligned}$$

A.3 Single Pump Manufacturer's Data

Table A.11: Single pump manufacturer's data for 1800 RPM, 2700 RPM, and 3600 RPM

Pump Speed (RPM)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Head, H (m)	Tip Speed, U ($\mathrm{m\,s^{-1}}$)	Radial Exit Velocity, v_{2r} ($\mathrm{m\,s^{-1}}$)	Head Coefficient, Ψ	Flow Coefficient, Φ
1800	0.0040	2.58	10.2	1.5	0.244	0.15
1800	0.0033	4.14	10.2	1.2	0.392	0.12
1800	0.0026	4.96	10.2	1.0	0.470	0.10
1800	0.0020	5.40	10.2	0.75	0.511	0.07
2700	0.0060	5.77	15.3	2.2	0.243	0.15
2700	0.0050	9.12	15.3	1.9	0.384	0.12
2700	0.0040	11.0	15.3	1.5	0.463	0.10
2700	0.0030	12.1	15.3	1.1	0.509	0.07
2700	0.0020	12.8	15.3	0.75	0.539	0.05
3600	0.0080	10.3	20.4	3.0	0.244	0.15
3600	0.0067	16.1	20.4	2.5	0.381	0.12
3600	0.0054	19.2	20.4	2.0	0.454	0.10
3600	0.0040	21.6	20.4	1.5	0.511	0.07
3600	0.0027	22.8	20.4	1.0	0.540	0.05

Sample calculations are evaluated 1800 RPM with a volumetric flow rate of $Q = 0.0040\,\mathrm{m^3\,s^{-1}}$.

The tip speed was calculated using the impeller radius, $r_2 = 0.108/2 = 0.054\,\mathrm{m}$, by

$$\begin{aligned} U &= r_2 \Omega \\ &= 0.054\,\mathrm{m} \times 1800\,\mathrm{RPM} \times \left(\frac{2\pi}{60}\,\mathrm{RPM}^{-1}\right) \\ &= 10.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

The radial exit velocity was calculated using the impeller diameter, the blade height at exit, $b = 0.009\,\mathrm{m}$, blade width at exit, $w = 0.0085\,\mathrm{m}$, and the number of blades $N = 5$, by

$$\begin{aligned} v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.0040\,\mathrm{m^3\,s^{-1}}}{0.009\,\mathrm{m} \times (2\pi \times 0.054\,\mathrm{m} - 5 \times 0.0085\,\mathrm{m})} \\ &= 1.5\,\mathrm{m\,s^{-1}} \end{aligned}$$

The head coefficient was found by

$$\begin{aligned} \Psi &= \frac{Hg}{U^2} \\ &= \frac{2.58\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{(10.2\,\mathrm{m\,s^{-1}})^2} \\ &= 0.244 \end{aligned}$$

The flow coefficient was found by

$$\begin{aligned} \Phi &= \frac{v_{2r}}{U} \\ &= \frac{1.5\,\mathrm{m\,s^{-1}}}{10.2\,\mathrm{m\,s^{-1}}} \\ &= 0.15 \end{aligned}$$

B Appendix: Parallel and Series Pump Analysis

This section will discuss the parallel and series pump analysis. The pump discharge and head for both the parallel and series configurations will be calculated and compared to the theoretical values based on extending the results from the single pump configuration. The uncertainty in the pump discharge and head for both the parallel and series configurations will also be calculated.

B.1 Parallel Experimental Pump Discharge and Head

Table B.12: Summary of recorded experimental parallel pump time for 2700 RPM

Valve Configuration	Transducer Output		Time to collect water			Nominal Time, t
	Pump 1	Pump 2	Trial 1	Trial 2	Trial 3	
	(V)	(V)	(s)	(s)	(s)	(s)
Fully open	2.87	2.72	12.46	12.28	12.79	12.51
Partial 1	3.08	2.96	14.49	14.49	14.42	14.47
Partial 2	3.52	3.53	41.05	40.99	40.99	41.01
Closed	3.55	3.55	-	-	-	-

Table B.13: Summary of recorded experimental parallel pump discharge and heads for 2700 RPM

Valve Configuration	Volumetric Flow, Q	Transducer Head, H_t			Corrected Head, H
		Pump 1	Pump 2	Nominal	
	($\mathrm{m^3\,s^{-1}}$)	(m)	(m)	(m)	(m)
Fully open	0.007266	10.1	9.58	9.84	11.3
Partial 1	0.006283	10.8	10.4	10.6	11.7
Partial 2	0.002217	12.4	12.4	12.4	12.5
Closed	-	12.5	12.5	12.5	12.5

Sample calculations will be shown for the fully open valve configuration for Tables B.12 and B.13. The same calculations were done for the other valve configurations. The nominal time was calculated as the average of the three trials by

$$\begin{aligned} t &= \frac{\sum t_i}{n} \\ &= \frac{12.46 + 12.28 + 12.79}{3} \\ &= 12.51\,\mathrm{s} \end{aligned}$$

The volumetric flow rate was calculated using $m = 200\,\mathrm{lb} = 90.7\,\mathrm{kg}$ and $\rho = 998\,\mathrm{kg\,m^{-3}}$ by

$$\begin{aligned} Q &= \frac{m}{\rho t} \\ &= \frac{90.7\,\mathrm{kg}}{998\,\mathrm{kg\,m^{-3}} \times 12.51\,\mathrm{s}} \\ &= 0.007266\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

Next, the transducer head for pumps 1 and pump 2 were calculated by

$$\begin{aligned} H_t &= \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g} \\ \implies H_{t1} &= \frac{2.87\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 10.11\,\mathrm{m} \\ \implies H_{t2} &= \frac{2.72\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 9.58\,\mathrm{m} \end{aligned}$$

The nominal head was calculated by averaging the transducer heads by

$$\begin{aligned} H &= \frac{\sum H_{t_i}}{n} \\ &= \frac{10.11 + 9.58}{2} \\ &= 9.84\,\mathrm{m} \end{aligned}$$

The corrected head was calculated by

$$
\begin{aligned}
H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\
&= 9.84\,\mathrm{m} + \frac{8 \times (0.007266\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.0381\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.0381\,\mathrm{m}}{0.0508\,\mathrm{m}}\right)^4\right] \\
&= 11.3\,\mathrm{m}
\end{aligned}
$$

B.1.1 Parallel Experimental Pump Discharge and Head Uncertainty

Table B.14: Parallel experimental pump time and discharge uncertainties for 2700 RPM

Config.	Time STDEV, S_t (s)	Time Precision, P_t (s)	Time Bias, B_t (s)	Time Uncertainty, δ_t (s)	Flow Uncertainty, δ_Q ($\mathrm{m^3\,s^{-1}}$)
Fully open	0.2587	0.6425	0.01	0.64	3.73E-04
Partial 1	0.0404	0.1004	0.01	0.10	4.38E-05
Partial 2	0.0346	0.0861	0.01	0.09	4.68E-06
Closed	-	-	-	-	-

Sample calculations are evaluated for the fully open configuration at 2700 RPM. A 95% confidence interval was used. The standard deviation was calculated by Excel using `STDEV.S`. The t-distribution value was found using $\alpha/2 = 0.025$ and $\nu = 3 - 1 = 2$. Then,

$$
\begin{aligned}
P_t &= t_{\alpha/2,\nu} \times \frac{S_t}{\sqrt{n}} \\
&= 4.303 \times \frac{0.2587\,\mathrm{s}}{\sqrt{3}} \\
&= 0.6425\,\mathrm{s}
\end{aligned}
$$

The time bias was approximated to be the resolution of the stopwatch,

$$B_t = 0.01\,\mathrm{s}$$

Table B.15: Parallel experimental pump head uncertainties for 2700 RPM

Config.	Transducer Uncertainty		Transducer Head Uncertainty			Corrected Head Uncertainty, δ_H
	Pump 1, $\delta_{V_{\text{tr1}}}$ (V)	Pump 2, $\delta_{V_{\text{tr2}}}$ (V)	Pump 1, $\delta_{H_{t1}}$ (m)	Pump 2, $\delta_{H_{t2}}$ (m)	Nominal, δ_{H_t} (m)	(m)
Fully open	0.01	0.01	0.04	0.04	0.02	0.1
Partial 1	0.01	0.01	0.04	0.04	0.02	0.03
Partial 2	0.01	0.01	0.04	0.04	0.02	0.02
Closed	0.01	0.01	0.04	0.04	0.02	0.02

The time uncertainty was calculated by

$$\begin{aligned}\delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.6425\,\mathrm{s})^2 + (0.01\,\mathrm{s})^2} \\ &= 0.64\,\mathrm{s}\end{aligned}$$

The flow uncertainty was calculated by

$$\begin{aligned}\delta_Q &= Q\left|\frac{\delta_t}{t}\right| \\ &= 0.007266\,\mathrm{m^3\,s^{-1}}\frac{0.64\,\mathrm{s}}{12.51\,\mathrm{s}} \\ &= 3.73E-04\end{aligned}$$

The transducer uncertainty was assumed to be the resolution of the device, $\delta_{V_{\text{tr}}} = 0.01\,\mathrm{V}$. Transducer precision error was not considered since calibration was not performed. The transducer heads were found by

$$H_t = \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, this is the special purely multiplicative case of the general formula for error propagation. The transducer head uncertainty was calculated by

$$\begin{aligned}
\delta_{H_{t1}} &= H_{t1}\sqrt{\left(\frac{\delta_{V_{\text{tr1}}}}{V_{\text{tr1}}}\right)^2} \\
&= H_{t1}\left|\frac{\delta_{V_{\text{tr1}}}}{V_{\text{tr1}}}\right| \\
&= 10.11\,\text{m}\left|\frac{0.01\,\text{V}}{2.87\,\text{V}}\right| \\
&= 0.04\,\text{m}
\end{aligned}$$

The nominal transducer head uncertainty was calculated by

$$\begin{aligned}
H_t &= \frac{H_{t1} + H_{t2}}{2} \\
\implies \frac{\partial H_t}{\partial H_{t1}} &= \frac{1}{2} \\
\implies \frac{\partial H_t}{\partial H_{t2}} &= \frac{1}{2}
\end{aligned}$$

so the uncertainty was

$$\begin{aligned}
\delta_{H_t} &= \frac{1}{2}\sqrt{(\delta_{H_{t1}})^2 + (\delta_{H_{t2}})^2} \\
&= \frac{1}{2}\sqrt{(0.04\,\text{m})^2 + (0.04\,\text{m})^2} \\
&= 0.02\,\text{m}
\end{aligned}$$

The corrected head is found by

$$\begin{aligned}
H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\
\implies \frac{\partial H}{\partial H_t} &= 1 \\
\implies \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{2(H - H_t)}{Q}
\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_H &= \sqrt{(\delta_{H_t})^2 + \left(\frac{2(H-H_t)}{Q}\delta_Q\right)^2} \\ &= \sqrt{(0.02\,\mathrm{m})^2 + \left(\frac{2(11.3\,\mathrm{m} - 9.84\,\mathrm{m})}{0.007266\,\mathrm{m}^3\,\mathrm{s}^{-1}} \times 3.73 \times 10^{-4}\,\mathrm{m}^3\,\mathrm{s}^{-1}\right)^2} \\ &= 0.2\,\mathrm{m}\end{aligned}$$

B.2 Series Experimental Pump Discharge and Head

Table B.16: Summary of recorded experimental series pump time for 2700 RPM

Valve Configuration	Transducer Output		Time to collect water			Nominal Time, t
	Pump 1	Pump 2	Trial 1	Trial 2	Trial 3	
	(V)	(V)	(s)	(s)	(s)	(s)
Fully open	0.68	1.68	16.64	16.17	16.03	16.28
Partial 1	0.9	1.84	16.71	16.58	16.76	16.68
Partial 2	2.89	3.26	30.52	30.23	30.67	30.47
Closed	3.52	3.55	-	-	-	-

Table B.17: Summary of recorded experimental series pump discharge and heads for 2700 RPM

Valve Configuration	Volumetric Flow, Q	Transducer Head, H_t			Corrected Head, H
		Pump 1	Pump 2	Nominal	
	($\mathrm{m}^3\,\mathrm{s}^{-1}$)	(m)	(m)	(m)	(m)
Fully open	0.005584	2.39	5.92	8.31	9.1
Partial 1	0.005449	3.17	6.48	9.65	10.4
Partial 2	0.002983	10.2	11.5	21.7	21.9
Closed	-	12.4	12.5	24.9	24.9

Sample calculations will be shown for the fully open valve configuration. The same calculations were done for the other valve configurations. The nominal time was calculated as the average of the three trials by

$$\begin{aligned} t &= \frac{\sum t_i}{n} \\ &= \frac{16.64 + 16.17 + 16.03}{3} \\ &= 16.28\,\mathrm{s} \end{aligned}$$

The volumetric flow rate was calculated using $m = 200\,\mathrm{lb} = 90.7\,\mathrm{kg}$ and $\rho = 998\,\mathrm{kg\,m^{-3}}$ by

$$\begin{aligned} Q &= \frac{m}{\rho t} \\ &= \frac{90.7\,\mathrm{kg}}{998\,\mathrm{kg\,m^{-3}} \times 16.28\,\mathrm{s}} \\ &= 0.005584\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

Next, the transducer head for pumps 1 and pump 2 were calculated by

$$\begin{aligned} H_t &= \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g} \\ \implies H_{t1} &= \frac{0.68\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 2.39\,\mathrm{m} \\ \implies H_{t2} &= \frac{1.68\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 5.92\,\mathrm{m} \end{aligned}$$

The nominal head was calculated by summing the transducer heads by

$$\begin{aligned} H_t &= \sum H_{t_i} \\ &= 2.39\,\mathrm{m} + 5.92\,\mathrm{m} \\ &= 8.31\,\mathrm{m} \end{aligned}$$

The corrected head was calculated by

$$\begin{aligned} H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 8.31\,\mathrm{m} + \frac{8 \times (0.005584\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.108\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.108\,\mathrm{m}}{0.108\,\mathrm{m}}\right)^4\right] \\ &= 9.1\,\mathrm{m} \end{aligned}$$

B.2.1 Series Experimental Pump Discharge and Head Uncertainty

Table B.18: Series experimental pump time and discharge uncertainties for 2700 RPM

Valve Configuration	Time STDEV, S_t (s)	Time Precision, P_t (s)	Time Bias, B_t (s)	Time Uncertainty, δ_t (s)	Flow Uncertainty, δ_Q ($\mathrm{m^3\,s^{-1}}$)
Fully open	0.3195	0.7938	0.01	0.79	2.72E-04
Partial 1	0.0929	0.2308	0.01	0.23	7.55E-05
Partial 2	0.2237	0.5557	0.01	0.56	5.44E-05
Closed	-	-	-	-	-

Sample calculations are evaluated for the fully open configuration at 2700 RPM. A 95% confidence interval was used. The standard deviation was calculated by Excel using `STDEV.S`. The t-distribution value was found using $\alpha/2 = 0.025$ and $\nu = 3 - 1 = 2$. Then,

$$\begin{aligned} P_t &= t_{\alpha/2,\nu} \times \frac{S_t}{\sqrt{n}} \\ &= 4.303 \times \frac{0.3195\,\mathrm{s}}{\sqrt{3}} \\ &= 0.7938\,\mathrm{s} \end{aligned}$$

The time bias was approximated to be the resolution of the stopwatch,

$$B_t = 0.01\,\mathrm{s}$$

Table B.19: Series experimental pump head uncertainties for 2700 RPM

Valve Configuration	Transducer Uncertainty		Transducer Head Uncertainty			Corrected Head Uncertainty, δ_H
	Pump 1, $\delta_{V_{\text{tr1}}}$	Pump 2, $\delta_{V_{\text{tr2}}}$	Pump 1, $\delta_{H_{t1}}$	Pump 2, $\delta_{H_{t2}}$	Nominal, δ_{H_t}	
	(V)	(V)	(m)	(m)	(m)	(m)
Fully open	0.01	0.01	0.04	0.04	0.05	0.1
Partial 1	0.01	0.01	0.04	0.04	0.05	0.05
Partial 2	0.01	0.01	0.04	0.04	0.05	0.05
Closed	0.01	0.01	0.04	0.04	0.05	0.05

The time uncertainty was calculated by

$$\begin{aligned}\delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.7938\,\mathrm{s})^2 + (0.01\,\mathrm{s})^2} \\ &= 0.79\,\mathrm{s}\end{aligned}$$

The flow uncertainty was calculated by

$$\begin{aligned}\delta_Q &= Q\left|\frac{\delta_t}{t}\right| \\ &= 0.005584\,\mathrm{m^3\,s^{-1}}\frac{0.79\,\mathrm{s}}{16.28\,\mathrm{s}} \\ &= 2.72E-04\end{aligned}$$

The transducer uncertainty was assumed to be the resolution of the device, $\delta_{V_{\text{tr}}} = 0.01\,\mathrm{V}$. Transducer precision error was not considered since calibration was not performed. The transducer heads were found by

$$H_t = \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, this is the special purely multiplicative case of the general formula for error propagation. The transducer head uncertainty was calculated by

$$\begin{aligned}
\delta_{H_{t1}} &= H_{t1}\sqrt{\left(\frac{\delta_{V_{\mathrm{tr1}}}}{V_{\mathrm{tr1}}}\right)^2} \\
&= H_{t1}\left|\frac{\delta_{V_{\mathrm{tr1}}}}{V_{\mathrm{tr1}}}\right| \\
&= 2.39\,\mathrm{m}\left|\frac{0.01\,\mathrm{V}}{0.68\,\mathrm{V}}\right| \\
&= 0.04\,\mathrm{m}
\end{aligned}$$

The nominal transducer head uncertainty was calculated by

$$\begin{aligned}
H_t &= H_{t1} + H_{t2} \\
\implies \frac{\partial H_t}{\partial H_{t1}} &= 1 \\
\implies \frac{\partial H_t}{\partial H_{t2}} &= 1
\end{aligned}$$

so the uncertainty was

$$\begin{aligned}
\delta_{H_t} &= \sqrt{(\delta_{H_{t1}})^2 + (\delta_{H_{t2}})^2} \\
&= \sqrt{(0.04\,\mathrm{m})^2 + (0.04\,\mathrm{m})^2} \\
&= 0.05\,\mathrm{m}
\end{aligned}$$

The corrected head is found by

$$\begin{aligned}
H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\
\implies \frac{\partial H}{\partial H_t} &= 1 \\
\implies \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{2(H - H_t)}{Q}
\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_H &= \sqrt{(\delta_{H_t})^2 + \left(\frac{2(H-H_t)}{Q}\delta_Q\right)^2} \\ &= \sqrt{(0.05\,\mathrm{m})^2 + \left(\frac{2(9.1\,\mathrm{m} - 8.31\,\mathrm{m})}{0.005584\,\mathrm{m^3\,s^{-1}}} \times 2.72\times10^{-4}\,\mathrm{m^3\,s^{-1}}\right)^2} \\ &= 0.1\,\mathrm{m}\end{aligned}$$

B.3 Parallel and Series Experimental vs. Theoretical Pump Discharge and Head

Table B.20: Parallel and series experimental vs. theoretical pump discharge and heads for 2700 RPM

		Actual			Theoretical		
		Volumetric Flow, Q_{act} ($\mathrm{m^3\,s^{-1}}$)	Transducer Head, $H_{t_{\text{act}}}$ (m)	Corrected Head, H_{act} (m)	Volumetric Flow, Q_{th} ($\mathrm{m^3\,s^{-1}}$)	Transducer Head, $H_{t_{\text{th}}}$ (m)	Corrected Head, H_{th} (m)
Parallel	Fully open	0.007266	9.84	11.3	0.009715	5.28	7.81
Parallel	Partial 1	0.006283	10.6	11.7	0.009498	5.70	8.12
Parallel	Partial 2	0.002217	12.4	12.5	0.005823	10.4	11.3
Parallel	Closed	-	12.5	12.5	-	12.4	12.4
Series	Fully open	0.005584	8.31	9.1	0.004858	10.6	11.2
Series	Partial 1	0.005449	9.65	10.4	0.004749	11.4	12.0
Series	Partial 2	0.002983	21.7	21.9	0.002911	20.7	20.9
Series	Closed	-	24.9	24.9	-	24.8	24.8

The actual results were pulled directly from Table B.13 and Table B.17. The theoretical results were calculated using the results from the single pump analysis Table A.5.

B.3.1 Parallel Experimental vs. Theoretical Pump Discharge and Head

Sample calculations are evaluated for the fully open configuration. The theoretical volumetric flow was found by

$$\begin{aligned} Q_{\text{th}} &= 2Q_{\text{single}} \\ &= 2 \times 0.004858\,\mathrm{m^3\,s^{-1}} \\ &= 0.00971503\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

The theoretical transducer head was found by

$$\begin{aligned} H_{t_{\text{th}}} &= H_{t_{\text{single}}} \\ &= 5.28\,\mathrm{m} \end{aligned}$$

The theoretical corrected head was found by

$$\begin{aligned} H_{\text{th}} &= H_{t_{\text{th}}} + \frac{8Q_{\text{th}}^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 5.28\,\mathrm{m} + \frac{8 \times (0.00971503\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.108\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.108\,\mathrm{m}}{0.108\,\mathrm{m}}\right)^4\right] \\ &= 7.81\,\mathrm{m} \end{aligned}$$

B.3.2 Series Experimental vs. Theoretical Pump Discharge and Head

Sample calculations are evaluated for the fully open configuration. The theoretical volumetric flow was found by

$$\begin{aligned} Q_{\text{th}} &= Q_{\text{single}} \\ &= 0.004858\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

The theoretical transducer head was found by

$$\begin{aligned} H_{t_{\text{th}}} &= 2H_{t_{\text{single}}} \\ &= 2 \times 5.28\,\mathrm{m} \\ &= 10.6\,\mathrm{m} \end{aligned}$$

The theoretical corrected head was found by

$$\begin{aligned}
H_{\text{th}} &= H_{t_{\text{th}}} + \frac{8Q_{\text{th}}^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\
&= 10.6\,\mathrm{m} + \frac{8 \times (0.004858\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.108\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.108\,\mathrm{m}}{0.108\,\mathrm{m}}\right)^4\right] \\
&= 11.2\,\mathrm{m}
\end{aligned}$$

C Appendix: Impeller Angle

This section will discuss the estimation of the impeller angle. The impeller angle was estimated using a digital protractor.

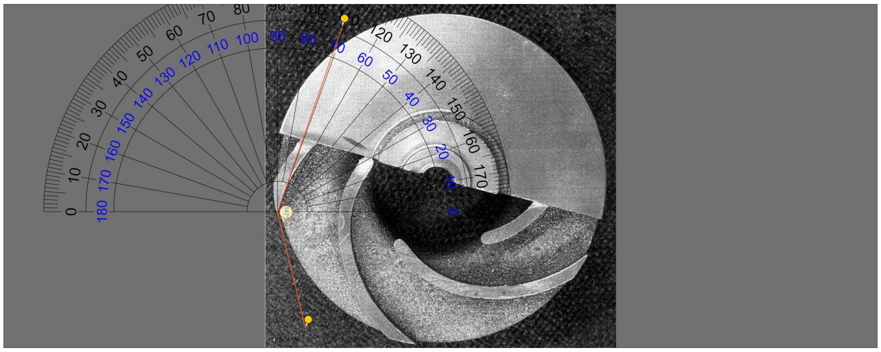

Figure C.10: Estimation of the impeller angle using digital protractor

The impeller angle was estimated using a digital protractor, as shown in Figure C.10. The impeller angle was measured to be

$$\beta_2 = 180^\circ - 147^\circ = 33^\circ$$

From equation (7), the ideal operating curve is

$$\begin{aligned}\Psi &= 1 - \Phi \cot \beta_2 \\ &= 1 - \Phi \cot 33^\circ \\ &= 1 - 1.540\Phi\end{aligned}$$

D Appendix: Shut Off Head

This section will discuss the shut off head of the pump. The ideal and ”rule of thumb” shut off head will be calculated and compared to the experimental shut off head. The ideal and ”rule of thumb” shut off head will be calculated using the highest experimental speed, $\Omega = 3600\,\mathrm{RPM}$, and the tip speed, $U = 20.4\,\mathrm{m\,s^{-1}}$ from Table A.9. The actual shutoff head was taken from Table A.5 and is $21.8\,\mathrm{m}$. The error for the rule of thumb shutoff head can be calculated as

Shut off head was discussed in the theory section. From Eq. 9 and 10, the ideal and ”rule of thumb” shut off head can be calculated. The calculation will be performed on the highest experimental speed, $\Omega = 3600\,\mathrm{RPM}$ which, from Table A.9.

$$\begin{aligned} H'_{\text{ideal}} &= \frac{U^2}{g} \\ &= \frac{(20.4\,\mathrm{m\,s^{-1}})^2}{9.81} \\ &= 42.2\,\mathrm{m} \\ H'_{\text{thumb}} &= \frac{1}{2}H'_{\text{ideal}} \\ &= \frac{1}{2} \times 42.2\,\mathrm{m} \\ &= 21.1\,\mathrm{m} \end{aligned}$$

The actual shutoff head was taken from Table A.5 and is $21.8\,\mathrm{m}$. The error for the rule of thumb shutoff head can be calculated as

$$\begin{aligned} \%\ \text{Error} &= \left|\frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}}\right| \times 100\% \\ &= \left|\frac{H'_{\text{exp}} - H'_{\text{thumb}}}{H'_{\text{thumb}}}\right| \times 100\% \\ &= \left|\frac{21.8 - 21.1}{21.1}\right| \times 100\% \\ &= 3.3\% \end{aligned}$$

The error for the ideal shutoff head can be calculated as

$$\begin{aligned}
\%\text{ Error} &= \left|\frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}}\right| \times 100\% \\
&= \left|\frac{H'_{\text{exp}} - H'_{\text{ideal}}}{H'_{\text{ideal}}}\right| \times 100\% \\
&= \left|\frac{21.8 - 42.2}{42.2}\right| \times 100\% \\
&= 48.3\%
\end{aligned}$$

E Manufacturer Geometrically Similar and Dissimilar Pumps

This section will show calculations for whether the manufacturer's specification sheet are for geometrically similar or dissimilar pumps. Geometrically similar is defined here as whether impeller height, b, and width, w, are scaled proportionally to the impeller diameter, D. Head and flow coefficients will be calculated for each pump for both assumptions.

Table E.21: Geometrically Similar and Dissimilar Pump Dimensions

Geometrically Similar	Impeller Diameter, D (m)	Blade Height, b (m)	Blade Width, w (m)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Pump Speed, N (RPM)	Head, H (m)
No	0.108	0.009	0.0085	0.0080	3600	10.3
No	0.108	0.009	0.0085	0.0067	3600	16.1
No	0.108	0.009	0.0085	0.0054	3600	19.4
No	0.108	0.009	0.0085	0.0040	3600	21.6
No	0.108	0.009	0.0085	0.0027	3600	22.8
No	0.102	0.009	0.0085	0.0076	3600	7.9
No	0.102	0.009	0.0085	0.0063	3600	13.0
No	0.102	0.009	0.0085	0.0050	3600	16.3
No	0.102	0.009	0.0085	0.0038	3600	18.0
No	0.102	0.009	0.0085	0.0025	3600	19.2
No	0.096	0.009	0.0085	0.0071	3600	6.7
No	0.096	0.009	0.0085	0.0059	3600	11.0
No	0.096	0.009	0.0085	0.0047	3600	13.9
No	0.096	0.009	0.0085	0.0035	3600	15.8
No	0.096	0.009	0.0085	0.0024	3600	16.8

No	0.083	0.009	0.0085	0.0059	3600	2.6
No	0.083	0.009	0.0085	0.0049	3600	6.0
No	0.083	0.009	0.0085	0.0039	3600	8.4
No	0.083	0.009	0.0085	0.0029	3600	9.9
No	0.083	0.009	0.0085	0.0020	3600	10.5
Yes	0.108	0.009	0.0085	0.0080	3600	10.3
Yes	0.108	0.009	0.0085	0.0067	3600	16.1
Yes	0.108	0.009	0.0085	0.0054	3600	19.4
Yes	0.108	0.009	0.0085	0.0040	3600	21.6
Yes	0.108	0.009	0.0085	0.0027	3600	22.8
Yes	0.102	0.009	0.00803	0.0076	3600	7.9
Yes	0.102	0.009	0.00803	0.0063	3600	13.0
Yes	0.102	0.009	0.00803	0.0050	3600	16.3
Yes	0.102	0.009	0.00803	0.0038	3600	18.0
Yes	0.102	0.009	0.00803	0.0025	3600	19.2
Yes	0.096	0.008	0.00756	0.0071	3600	6.7
Yes	0.096	0.008	0.00756	0.0059	3600	11.0
Yes	0.096	0.008	0.00756	0.0047	3600	13.9
Yes	0.096	0.008	0.00756	0.0035	3600	15.8
Yes	0.096	0.008	0.00756	0.0024	3600	16.8
Yes	0.083	0.007	0.00653	0.0059	3600	2.6
Yes	0.083	0.007	0.00653	0.0049	3600	6.0
Yes	0.083	0.007	0.00653	0.0039	3600	8.4
Yes	0.083	0.007	0.00653	0.0029	3600	9.9

Yes	0.083	0.007	0.00653	0.0020	3600	10.5

Table E.22: Geometrically Similar and Dissimilar Pump Coefficients

Geometrically Similar	Impeller Diameter, D (m)	Impeller Speed, U ($\mathrm{m\,s^{-1}}$)	Radial Exit Velocity, v_{2r} ($\mathrm{m\,s^{-1}}$)	Head Coefficient, Ψ	Flow Coefficient, Φ
No	0.108	20.4	3.0	0.24	0.15
No	0.108	20.4	2.5	0.38	0.12
No	0.108	20.4	2.0	0.46	0.10
No	0.108	20.4	1.5	0.51	0.07
No	0.108	20.4	1.0	0.54	0.05
No	0.102	19.2	3.0	0.21	0.16
No	0.102	19.2	2.5	0.34	0.13
No	0.102	19.2	2.0	0.43	0.10
No	0.102	19.2	1.5	0.48	0.08
No	0.102	19.2	1.0	0.51	0.05
No	0.096	18.1	3.0	0.20	0.17
No	0.096	18.1	2.5	0.33	0.14
No	0.096	18.1	2.0	0.42	0.11
No	0.096	18.1	1.5	0.47	0.08
No	0.096	18.1	1.0	0.50	0.06
No	0.083	15.6	3.0	0.10	0.19

No	0.083	15.6	2.5	0.24	0.16
No	0.083	15.6	2.0	0.34	0.13
No	0.083	15.6	1.5	0.40	0.09
No	0.083	15.6	1.0	0.42	0.07
Yes	0.108	20.4	3.0	0.24	0.15
Yes	0.108	20.4	2.5	0.38	0.12
Yes	0.108	20.4	2.0	0.46	0.10
Yes	0.108	20.4	1.5	0.51	0.07
Yes	0.108	20.4	1.0	0.54	0.05
Yes	0.102	19.2	3.2	0.21	0.17
Yes	0.102	19.2	2.6	0.34	0.14
Yes	0.102	19.2	2.1	0.43	0.11
Yes	0.102	19.2	1.6	0.48	0.08
Yes	0.102	19.2	1.0	0.51	0.05
Yes	0.096	18.1	3.4	0.20	0.19
Yes	0.096	18.1	2.8	0.33	0.15
Yes	0.096	18.1	2.2	0.42	0.12
Yes	0.096	18.1	1.7	0.47	0.09
Yes	0.096	18.1	1.1	0.50	0.06
Yes	0.083	15.6	3.7	0.10	0.24
Yes	0.083	15.6	3.1	0.24	0.20
Yes	0.083	15.6	2.5	0.34	0.16
Yes	0.083	15.6	1.8	0.40	0.12

Yes	0.083	15.6	1.3	0.42	0.08

E.1 Geometrically Similar Pumps Sample Calculations

Sample calculations for Table E.21 and Table E.22 will be shown for an impeller of $D = 102\,\mathrm{mm}$ and $Q = 0.0076\,\mathrm{m^3\,s^{-1}}$. Pump speed is given as $3600\,\mathrm{RPM}$ with number of blades, $N = 5$. Impeller speed is then

$$
\begin{aligned}
U &= \frac{D}{2} \times \Omega \\
&= \frac{102\,\mathrm{mm}}{2} \times 3600\,\mathrm{RPM} \times \frac{2\pi}{60}\frac{\mathrm{rad\,s^{-1}}}{\mathrm{RPM}} \\
&= 19.2\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Geometrically similar means that blade height and width are scaled proportionally to the impeller diameter. Blade height and width are then

$$
\begin{aligned}
b_{102} &= \frac{b_{108} \times D_{102}}{D_{108}} \\
&= \frac{9\,\mathrm{mm} \times 102\,\mathrm{mm}}{108\,\mathrm{mm}} \\
&= 8.5\,\mathrm{mm} \\
w_{102} &= \frac{w_{108} \times D_{102}}{D_{108}} \\
&= \frac{8.5\,\mathrm{mm} \times 102\,\mathrm{mm}}{108\,\mathrm{mm}} \\
&= 8.03\,\mathrm{mm}
\end{aligned}
$$

Radial exit velocity is then

$$
\begin{aligned}
v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\
&= \frac{0.0076\,\mathrm{m^3\,s^{-1}}}{8.5\,\mathrm{mm} \times (\pi \times 0.102\,\mathrm{m} - 5 \times 8.03\,\mathrm{mm})} \\
&= 3.2\,\mathrm{m\,s^{-1}}
\end{aligned}
$$

Head coefficient is then

$$\begin{aligned}\Psi &= \frac{Hg}{U^2} \\ &= \frac{7.9\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{(19.2\,\mathrm{m\,s^{-1}})^2} \\ &= 0.21\end{aligned}$$

Flow coefficient is then

$$\begin{aligned}\Phi &= \frac{v_{2r}}{U} \\ &= \frac{3.2\,\mathrm{m\,s^{-1}}}{19.2\,\mathrm{m\,s^{-1}}} \\ &= 0.17\end{aligned}$$

E.2 Geometrically Dissimilar Pumps Sample Calculations

Sample calculations for Table E.21 and Table E.22 will be shown for an impeller of $D = 102\,\mathrm{mm}$ and $Q = 0.0076\,\mathrm{m^3\,s^{-1}}$. Pump speed is given as $3600\,\mathrm{RPM}$ with number of blades, $N = 5$. Impeller speed is then

$$\begin{aligned}U &= \frac{D}{2} \times \Omega \\ &= \frac{102\,\mathrm{mm}}{2} \times 3600\,\mathrm{RPM} \times \frac{2\pi}{60}\frac{\mathrm{rad\,s^{-1}}}{\mathrm{RPM}} \\ &= 19.2\,\mathrm{m\,s^{-1}}\end{aligned}$$

Geometrically dissimilar means that blade height and width are not scaled proportionally to the impeller diameter. Blade height and width are unchanged between all pumps. That is,

$$\begin{aligned}b_{102} &= b_{108} \\ &= 9\,\mathrm{mm} \\ w_{102} &= w_{108} \\ &= 8.5\,\mathrm{mm}\end{aligned}$$

Radial exit velocity is then

$$\begin{aligned}
v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\
&= \frac{0.0076\,\mathrm{m^3\,s^{-1}}}{9\,\mathrm{mm} \times (\pi \times 0.102\,\mathrm{m} - 5 \times 8.5\,\mathrm{mm})} \\
&= 3.0\,\mathrm{m\,s^{-1}}
\end{aligned}$$

Head coefficient is then

$$\begin{aligned}
\Psi &= \frac{Hg}{U^2} \\
&= \frac{7.9\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{(19.2\,\mathrm{m\,s^{-1}})^2} \\
&= 0.21
\end{aligned}$$

Flow coefficient is then

$$\begin{aligned}
\Phi &= \frac{v_{2r}}{U} \\
&= \frac{3.0\,\mathrm{m\,s^{-1}}}{19.2\,\mathrm{m\,s^{-1}}} \\
&= 0.16
\end{aligned}$$

F Pump Efficiency

This section will discuss the pump efficiency of the single pump configuration. The pump efficiency will be calculated for the single pump configurations. Manufacturer's specifications for efficiency will be given as well. The uncertainty in the pump efficiency will also be calculated.

The pump efficiency was calculated using Eq. (24). The pump efficiency was calculated for the single pump configuration at 1800, 2700, and 3600 RPM, and for the fully open, partial 1, partial 2, and closed valve configurations. The pump efficiency was also calculated for the manufacturer's specifications.

Table F.23: Pump Efficiency for Single Pump Configuration

Valve Configuration	Pump Speed, Ω (RPM)	Moment arm, d_m (cm)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Corrected Head, H (m)	Torque, T (N m)	Efficiency, η (%)
Fully open	1800	8.75	0.003159	2.91	1.29	37.0
Partial 1	1800	8.50	0.003003	3.16	1.25	39.4
Partial 2	1800	7.25	0.001492	5.09	1.07	37.0
Closed	1800	4.50	-	5.49	0.663	0.00
Fully open	2700	16.50	0.004858	5.91	2.43	40.9
Partial 1	2700	16.50	0.004749	6.31	2.43	42.7
Partial 2	2700	14.50	0.002911	10.6	2.14	49.9
Closed	2700	8.50	-	12.4	1.25	0.00
Fully open	3600	26.50	0.006474	9.72	3.90	41.8
Partial 1	3600	26.50	0.006240	11.2	3.90	46.6
Partial 2	3600	20.00	0.002454	20.8	2.95	44.9
Closed	3600	13.00	-	21.8	1.92	0.00
Manufacturer	1800	-	0.0040	2.58	-	46
Manufacturer	1800	-	0.0033	4.14	-	66

Manufacturer	1800	-	0.0026	4.96	-	67
Manufacturer	1800	-	0.0020	5.40	-	64
Manufacturer	2700	-	0.0060	5.77	-	52
Manufacturer	2700	-	0.0050	9.12	-	71
Manufacturer	2700	-	0.0040	11.0	-	73
Manufacturer	2700	-	0.0030	12.1	-	69
Manufacturer	2700	-	0.0020	12.8	-	58
Manufacturer	3600	-	0.0080	10.3	-	52
Manufacturer	3600	-	0.0067	16.1	-	71
Manufacturer	3600	-	0.0054	19.2	-	73
Manufacturer	3600	-	0.0040	21.6	-	69
Manufacturer	3600	-	0.0027	22.8	-	59

Sample calculations will be shown for the single pump configuration at 1800 RPM, fully open valve. The torque was calculated from the dyno mass of $m = 1.502\,\mathrm{kg}$ and the moment arm of $d_m = 8.75\,\mathrm{cm}$ as

$$\begin{aligned} T &= mgd_m \\ &= 1.502\,\mathrm{kg} \times 9.81\,\mathrm{m\,s^{-2}} \times 0.0875\,\mathrm{m} \\ &= 1.29\,\mathrm{N\,m} \end{aligned}$$

The pump efficiency is calculated from Eq. (24) as

$$\begin{aligned} \eta &= \frac{\rho g Q H}{T\Omega} \\ &= \frac{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}} \times 0.003159\,\mathrm{m^3\,s^{-1}} \times 2.91\,\mathrm{m}}{1.29\,\mathrm{N\,m} \times 1800\,\mathrm{RPM} \times \frac{2\pi}{60} \times \frac{\mathrm{rad\,s^{-1}}}{\mathrm{RPM}}} \\ &= 37.0\,\% \end{aligned}$$

F.1 Pump Efficiency Uncertainty

Table F.24: Pump Efficiency Uncertainty

Valve Configuration	Pump Speed, Ω (RPM)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Corrected Head, H (m)	Flow Uncertainty, δ_Q ($\mathrm{m^3\,s^{-1}}$)	Head Uncertainty, δ_H (m)	Efficiency, η (%)	Efficiency Uncertainty, δ_η (%)
Fully open	1800	0.003159	2.91	6.40E-05	0.04	37.0	0.9
Partial 1	1800	0.003003	3.16	5.50E-05	0.04	39.4	0.9
Partial 2	1800	0.001492	5.09	1.14E-05	0.04	37.0	0.4
Closed	1800	-	5.49	-	0.04	0.00	-
Fully open	2700	0.004858	5.91	1.36E-04	0.05	40.9	1
Partial 1	2700	0.004749	6.31	1.08E-04	0.04	42.7	1
Partial 2	2700	0.002911	10.6	2.55E-05	0.04	49.9	0.5
Closed	2700	-	12.4	-	0.04	0.00	-
Fully open	3600	0.006474	9.72	1.48E-04	0.06	41.8	1
Partial 1	3600	0.006240	11.2	4.45E-04	0.2	46.6	3
Partial 2	3600	0.002454	20.8	1.41E-05	0.04	44.9	0.3
Closed	3600	-	21.8	-	0.04	0	-

The uncertainty in the pump efficiency was calculated for the single pump in the fully open valve configuration at 1800 RPM. The pump efficiency equation is

$$\eta = \frac{\rho g Q H}{T\Omega}$$

This is the purely multiplicative case of uncertainty propagation. Assuming the errors of torque, density, gravity, and pump speeds are negligible, the uncertainty in the pump efficiency is

$$\begin{aligned}
\delta_\eta &= \eta\sqrt{\left(\frac{\delta_Q}{Q}\right)^2 + \left(\frac{\delta_H}{H}\right)^2} \\
&= 37.0\,\%\sqrt{\left(\frac{6.40\times 10^{-5}\,\mathrm{m^3\,s^{-1}}}{0.003159\,\mathrm{m^3\,s^{-1}}}\right)^2 + \left(\frac{0.04\,\mathrm{m}}{2.91\,\mathrm{m}}\right)^2} \\
&= 0.9\,\%
\end{aligned}$$