MEC E 403
Lab 1: Centrifugal Pumps

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Abstract

This will be a summary of the lab. It will include the purpose of the lab, the methods used, and the results obtained. This will be completed after the lab is finished.

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1 Nomenclature

will do this after the lab is done because I don't know what all the variables used will be yet.

asd

2 Introduction

2.1 Background

TO DO: Reword and find sources for information. Pumps, turbines, and fans are turbo machines whose function is to change the energy level of a fluid. A pump or compressor increases the total head or pressure of the fluid while a turbine decreases it and extracts energy from the flow. Analysis of turbo machines is an important piece of technology, and a knowledge of their general characteristics is essential for many branches of engineering.

The geometry of turbo machines varies appreciably for the differing types that have been developed. Broadly there are two classes. In the first class there is a pronounced change in radius from the inlet to the discharge; these may be said to be centrifugal turbo machines. This is an important type of turbo machine as there are a great number of pumps, turbines and compressors that fall into this category. The other class consists of axial machines in which the flow is largely parallel to the axis of rotation. Between these extremes are examples in which the flow may proceed along conical surfaces of revolution, and these are sometimes called mixed-flow turbo machines. In all these varying types, however, there must be a rotating member, usually called a rotor, or impeller, to do work on the fluid.

2.2 Objectives

1. To measure the performance of a centrifugal pump and compare the results with the manufacturer's specification and theoretical predictions.

2. To compare the performance of parallel and series pump system configurations.

Pump	Flow Rate (L/min)	Head (m)	Power (W)	Efficiency (%)
1	0asdasdasd	0	0	0
2	0	0	0	0
3	0	0	0	0

Table 1: Pump Performance

3 Procedure

3.1 Equipment

- Pump system that can be configured as a single pump or as two pumps in series or parallel operation. This is the system being studied.

- Stopwatch to measure time to fill a container

- Strobotach to verify pump rotational speed
- Mass scale to measure 200 lbs of water
- 1.401 kg mass to measure moment arm (torque) of motor
- Pressure transducers to measure pump pressure differential

3.2 Procedure

1. The pump system will be tested individually at 1800, 2700, and 3600 RPM. The speed of the pump will be verified using the strobotach.
2. At each pump speed, the flow rates at 4 different pressures will be recorded (shutoff, full open, and two other equally spaced intermediate settings). The speed of the pump will be set to the correct value when changing the pressures before taking any other readings.
3. At each operating condition, the pump speed, pressure transducer output, the moment arm and dyno mass (to determine the torque produced by the motor), and time required to collect a known quantity of water will be recorded.
4. Items (2)-(4) will be repeated for parallel and series system configurations with both pumps set at 2700 RPM.

4 Theory

4.1 Euler's Turbomachinery Equations

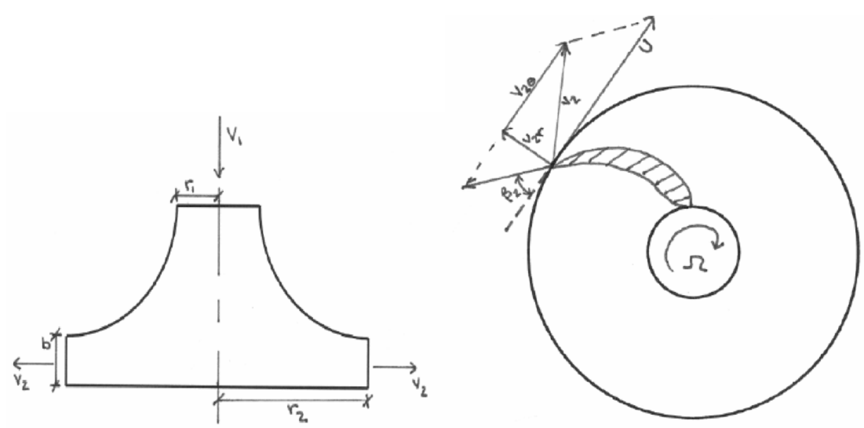

Figure 1: Impeller Diagram

Using the following assumptions:

1. Viscous effects are negligible

2. Velocity profile is uniform at the exit
3. All work done by the pump is transferred to the fluid

Then by conservation of angular momentum,

$$T = \dot{m} r_2 v_{2\theta} \tag{1}$$

where T is the torque, $\dot{m}$ is the mass flow rate, r_2 is the radius of the impeller, and $v_{2\theta}$ is the tangential velocity at the exit. From Figure 1, we can see that

$$v_{2\theta} = U - v_{2r} \cot \beta_2 \tag{2}$$

where U is the tip speed, v_{2r} is the radial velocity at the exit, and β_2 is the blade angle. Combining (1) and (2), we get

$$T = \dot{m} r_2 (U - v_{2r} \cot \beta_2) \tag{3}$$

By Assumption 3, we can say that

$$T\Omega = \dot{m} g H \tag{4}$$

where Ω is the impeller angular velocity, g is the acceleration due to gravity, and H is the total head rise across the pump. By combining (3), (4), and the kinematic relationship $U = r_2\Omega$, we get

$$\frac{Hg}{U^2} = 1 - \frac{v_{2r}}{U} \cot \beta_2 \tag{5}$$

Defining the head coefficient Ψ and the flow coefficient Φ as

$$\Psi = \frac{Hg}{U^2} \quad \Phi = \frac{v_{2r}}{U} \tag{6}$$

then (5) becomes

$$\Psi = 1 - \Phi \cot \beta_2 \tag{7}$$

also, v_{2r} can be expressed by the continuity equation as

$$v_{2r} = \frac{Q}{A_2} = \frac{Q}{b(2\pi r_2 - Nw)} \tag{8}$$

where Q is the flow rate, A_2 is the area of the exit, b is the blade height at the exit, N is the number of blades, and w is the width of the blade.

4.2 Shutoff Head

The ideal shutoff head can be obtained as

$$H'_{\text{ideal}} = \frac{U^2}{g} \tag{9}$$

by setting $\Phi = 0$ in (7). If it is assumed that all kinetic energy is lost due to friction, then

$$\frac{KE}{\text{unit weight}} = \frac{U^2}{2g}$$

where KE is the kinetic energy. Therefore, the rule of thumb for the shutoff head is

$$\begin{aligned} H'_{\text{thumb}} &= \frac{U^2}{g} - \frac{U^2}{2g} \\ H'_{\text{thumb}} &= \frac{U^2}{2g} = \frac{1}{2} H'_{\text{ideal}} \end{aligned} \tag{10}$$

4.3 Affinity Laws

For large Reynolds numbers, the flow is dynamically similar in geometrically similar machines when the flow and head coefficients are the same. For geometrically similar machines operating at different conditions (i) and (ii) such that the head and flow coefficients are the same, the following relationships hold:

$$\begin{aligned} \Psi_i &= \Psi_{ii} \\ \frac{H_i}{U_i^2} &= \frac{H_{ii}}{U_{ii}^2} \end{aligned}$$

so,

$$\frac{H_i}{H_{ii}} = \left(\frac{U_i}{U_{ii}}\right)^2 \approx \left(\frac{D_i \Omega_i}{D_{ii} \Omega_{ii}}\right)^2 \tag{11}$$

where D is the diameter of the impeller. Also,

$$\begin{aligned} \Phi_i &= \Phi_{ii} \\ \frac{v_{2ri}}{U_i} &= \frac{v_{2rii}}{U_{ii}} \end{aligned} \tag{12}$$

Assuming the blade width is negligible, from (8), we get

$$v_{2r} = \frac{Q}{\pi D b} \tag{13}$$

so (12) becomes

$$\begin{aligned}\frac{Q_i D_{ii} b_{ii}}{Q_{ii} D_i b_i} &= \frac{U_i}{U_{ii}} \\ \frac{Q_i}{Q_{ii}} &= \frac{\Omega_i b_i}{\Omega_{ii} b_{ii}} \left(\frac{D_i}{D_{ii}}\right)^2\end{aligned} \tag{14}$$

For geometrically similar machines, the ratios of b/D are the same, so (14) becomes

$$\frac{Q_i}{Q_{ii}} = \frac{\Omega_i}{\Omega_{ii}} \left(\frac{D_i}{D_{ii}}\right)^3 \tag{15}$$

4.4 Transducer Head Adjustment

In this experiment, since the inlet and outlet pipe diameters are different, the transducer head, H_t, must be corrected for flow kinetic energy to give the pump stagnation head, H, as

$$\begin{aligned}H &= H_t + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \\ H &= H_t + \frac{v_2^2}{2g}\left(1 - \left(\frac{v_1}{v_2}\right)^2\right)\end{aligned} \tag{16}$$

where H_t is the transducer head, v_1 is the velocity at the inlet, and v_2 is the velocity at the outlet. The volume flowrate through the system can be written as

$$Q = \frac{v_2 \pi D_2^2}{4} \tag{17}$$

and with the pipe diameters D_1 at the inlet and D_2 at the outlet, mass conservation requires that

$$v_2 \pi D_2^2 = v_1 \pi D_1^2 \tag{18}$$

With (17) and (18), (16) becomes

$$H = H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) \tag{19}$$

4.5 Pumps in Series and Parallel

When pumps are connected in series, the total head is the sum of the individual heads, and the flow rate is the same as the individual flow rates. **CITE A FIGURE**

$$H_{\text{series},t} = H_{1,t} + H_{2,t} \tag{20}$$

$$Q_{\text{series}} = Q_1 = Q_2 \tag{21}$$

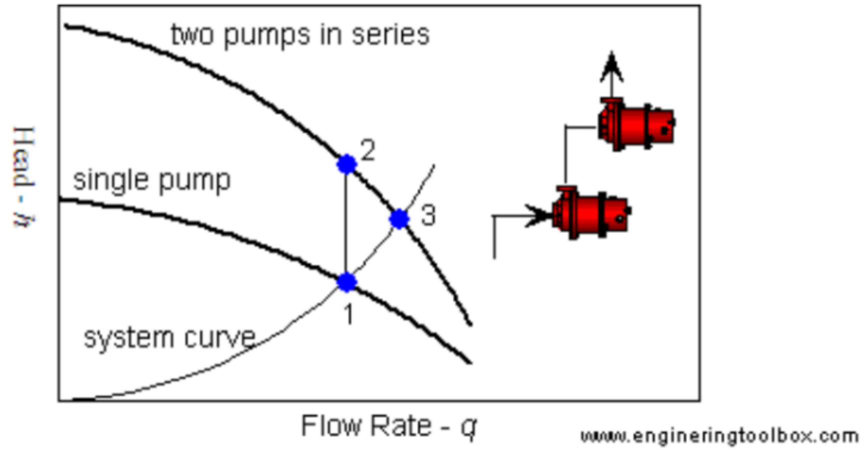

Figure 2: Pumps in Series

When pumps are connected in parallel, the total head is the average of the individual heads, and the total flow rate is the sum of the individual flow rates.

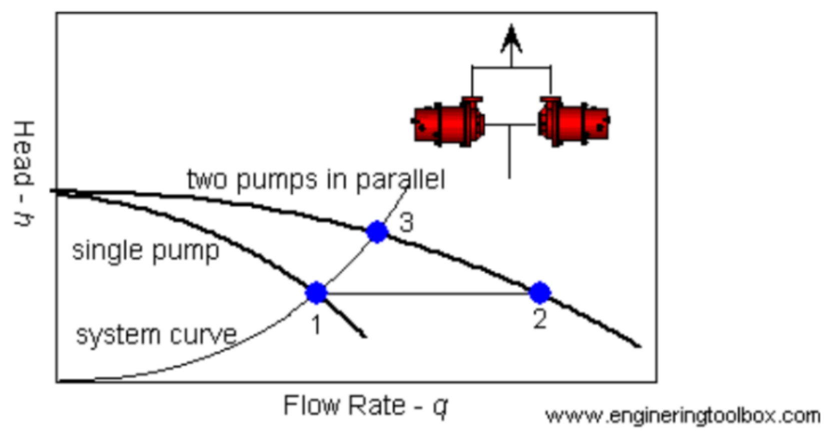

Figure 3: Pumps in Parallel

$$H_{\text{parallel},t} = \frac{H_{1,t} + H_{2,t}}{2} \tag{22}$$

$$Q_{\text{parallel}} = Q_1 + Q_2 \tag{23}$$

5 Results and Discussion

5.1 Main Results

Table 2 shows the experimental and manufacturer pump performance summary. The pump configuration, valve configuration, pump speed, volumetric flow rate, corrected head, head coefficient, and flow coefficient are shown. Sample calculations for the single pump in Appendix A. Parallel and series pump performance analysis is shown in Appendix B.

Table 2: Experimental and Manufacturer Pump Performance Summary

Pump Config.	Valve Config.	Pump Speed, Ω (RPM)	Volumetric Flow Rate, Q ($\mathrm{m^3\,s^{-1}}$)	Corrected Head, H (m)	Head Coefficient, Ψ	Flow Coefficient, Φ
Single	Fully open	1800	0.003159	2.91	0.275	0.12
Single	Partial 1	1800	0.003003	3.16	0.300	0.11
Single	Partial 2	1800	0.001492	5.09	0.482	0.05
Single	Closed	1800	-	5.49	0.520	0.00
Single	Fully open	2700	0.004858	5.91	0.249	0.12
Single	Partial 1	2700	0.004749	6.31	0.265	0.12
Single	Partial 2	2700	0.002911	10.6	0.445	0.07
Single	Closed	2700	-	12.4	0.522	0.00
Single	Fully open	3600	0.006474	9.72	0.230	0.12
Single	Partial 1	3600	0.006240	11.2	0.266	0.11
Single	Partial 2	3600	0.002454	20.8	0.491	0.05
Single	Closed	3600	-	21.8	0.515	0.00
Single	Manufacturer	1800	0.0040	2.58	0.244	0.15
Single	Manufacturer	1800	0.0033	4.14	0.392	0.12
Single	Manufacturer	1800	0.0026	4.96	0.470	0.10
Single	Manufacturer	1800	0.0020	5.40	0.511	0.07
Single	Manufacturer	2700	0.0060	5.77	0.243	0.15
Single	Manufacturer	2700	0.0050	9.12	0.384	0.12
Single	Manufacturer	2700	0.0040	11.0	0.463	0.10
Single	Manufacturer	2700	0.0030	12.1	0.509	0.07

Single	Manufacturer	2700	0.0020	12.8	0.539	0.05
Single	Manufacturer	3600	0.0080	10.3	0.244	0.15
Single	Manufacturer	3600	0.0067	16.1	0.381	0.12
Single	Manufacturer	3600	0.0054	19.2	0.454	0.10
Single	Manufacturer	3600	0.0040	21.6	0.511	0.07
Single	Manufacturer	3600	0.0027	22.8	0.540	0.05
Parallel	Fully open	2700	0.007266	11.3	0.474	0.18
Parallel	Partial 1	2700	0.006283	11.7	0.492	0.15
Parallel	Partial 2	2700	0.002217	12.5	0.528	0.05
Parallel	Closed	2700	-	12.5	0.526	0.00
Series	Fully open	2700	0.005584	9.15	0.385	0.14
Series	Partial 1	2700	0.005449	10.4	0.439	0.13
Series	Partial 2	2700	0.002983	21.9	0.921	0.07
Series	Closed	2700	-	24.9	1.048	0.00

5.2 Single Pump Performance

5.2.1 Head vs. Flow Rate

Using the data from Table 2, the head vs. flow rate for the experimental single pump data was plotted in Figure 4. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix A.

5.2.2 Head Coefficient vs. Flow Coefficient

The head coefficient and flow coefficient for the experimental, manufacturer, and ideal pump data are shown in Figure 5. The ideal pump data was calculated from ideal pump equation (Eq. 7) using the impeller angle. Impeller angle was determined visually, shown in Appendix C. Sample calculations for the experimental and manufacturer head and flow coefficients are shown in Appendix A.

For the single experimental pump, the head and flow coefficients appear to fall onto the same curve. A non-linear relationship is observed, where the head coefficient decreases as the flow coefficient increases. Most points of different speeds are within error bars of each other. This suggests that the head and flow

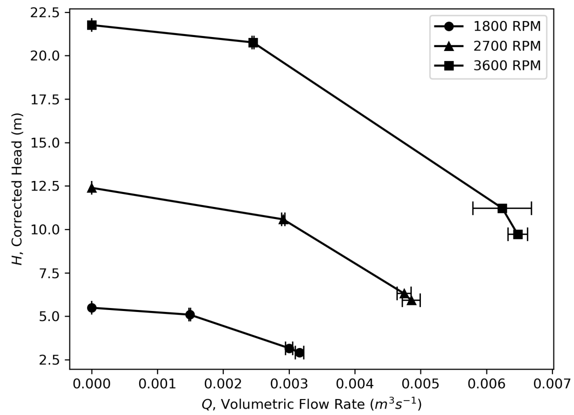

Figure 4: Single pump experimental head vs. flow rate plot.

coefficients are independent of pump speed. The largest source of error was the precision error caused by the reaction time of the stopwatch operator.

The manufacturer head and flow coefficients also appear to fall onto the same, but different than the experimental, curve. The manufacturer head coefficient is higher than the experimental head coefficient for the same flow coefficient. This suggests that the manufacturer pump is more efficient than the experimental pump.

The ideal head and flow coefficients are shown as a straight line. The ideal head coefficient is much higher than the experimental and manufacturer head coefficient for the same flow coefficient. The ideal pump neglects all losses and is overly optimistic. The linear trend does not follow the experimental and manufacturer data.

5.2.3 Ideal and Rule of Thumb Shutoff Head

Table 3: Ideal, rule of thumb, and experimental shutoff head for the single pump at 3600 RPM.

Ideal Shutoff Head, H'_{ideal} (m)	Rule of Thumb Shutoff Head, H'_{thumb} (m)	Experimental Shutoff Head, H'_{exp} (m)
42.2	21.1	21.8

The ideal and rule of thumb shutoff head for the single pump at 3600 RPM was calculated and compared to the experimental shutoff head (single closed valve configuration). The ideal and rule of thumb shutoff head was calculated using Eq. 9 and 10. The experimental shutoff head was taken from Table 2. Sample calculations for the ideal and rule of thumb shutoff head are shown in Appendix D and the experimental

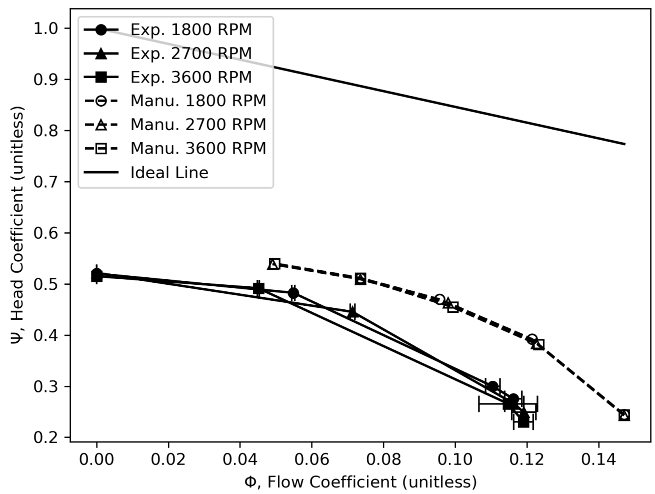

Figure 5: Single pump experimental, manufacturer, and ideal head coefficient vs. flow coefficient plot.

shutoff head in Appendix A.

The ideal shutoff head assumes all kinetic energy is lost too friction and is overly optimistic. The error was calculated to be 48.3%. This suggests poor agreement between the experimental and ideal shutoff head.

The rule of thumb shutoff head is a more realistic estimate and is within 3.3% of the experimental shutoff head. This suggests good agreement between the experimental and rule of thumb shutoff head.

5.3 Parallel Pump Performance

The head vs. flow rate for the experimental parallel pump data was plotted in Figure 6. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix B.

The theoretical curve was calculated using the single pump data. The theoretical head and flow was calculated using Eq. 22 and 23. The theoretical head and flow was then plotted against the experimental head and flow.

The curves have poor agreement. The theoretical head was higher than the experimental head for the same flow. In addition, a higher flow was observed at the full open valve configuration. This suggests the theoretical model does not accurately predict the parallel pump performance.

5.4 Series Pump Performance

The head vs. flow rate for the experimental series pump data was plotted in Figure 7. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix B.

The theoretical curve was calculated using the single pump data. The theoretical head and flow was

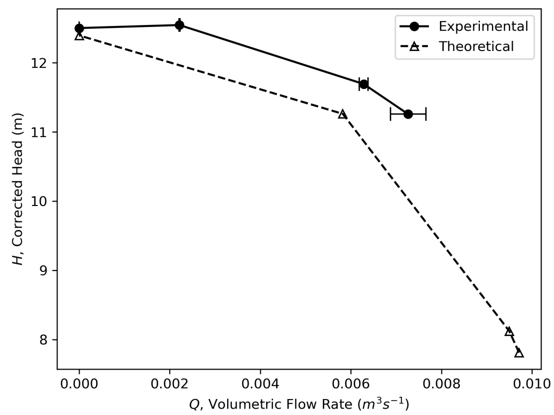

Figure 6: Parallel pump experimental and theoretical head vs. flow rate plot.

calculated using Eq. 20 and 21. The theoretical head and flow was then plotted against the experimental head and flow.

The curves have some agreement. The theoretical head was lower than the experimental head for the same flow. In addition, a higher flow was observed at the full open valve configuration. These deviations were smaller than the parallel pump. This suggests the theoretical model does somewhat accurately predicts the series pump performance.

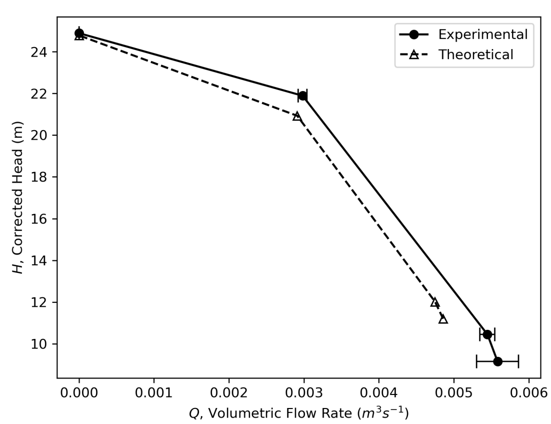

Figure 7: Series pump experimental and theoretical head vs. flow rate plot.

5.5 Geometric Similarity in Manufacturer's Specifications

The manufacturer's specifications are for geometrically dissimilar pumps. To investigate the effects of assuming similar geometry, two plots were produced. Sample calculations for the head and flow coefficients are shown in Appendix E.

The first plot, Figure 8, shows the head coefficient vs. flow coefficient for geometrically similar pumps where impeller blade height, b, and impeller width, w, were scaled by the impeller diameter, D. An observation is that the head coefficient decreases for a given flow coefficient as the impeller diameter decreases.

The second plot, Figure 9, shows the head coefficient vs. flow coefficient for geometrically dissimilar pumps where impeller blade height, b, and impeller width, w, were not scaled by the impeller diameter, D. An observation is that the head coefficient decreases for a given flow coefficient as the impeller diameter decreases.

The head and flow coefficients for the geometrically similar pumps fall only somewhat collapse onto the same curve. The head and flow coefficients for the geometrically dissimilar pumps fall more so onto the same curve. This suggests that the pumps are geometrically dissimilar, which is consistent with the manufacturer's specifications.

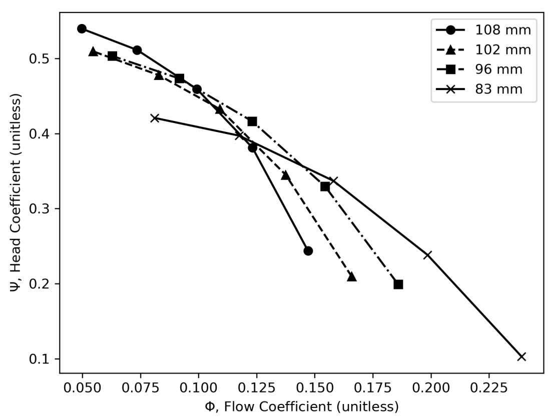

Figure 8: Head coefficient vs. flow coefficient for geometrically similar pumps.

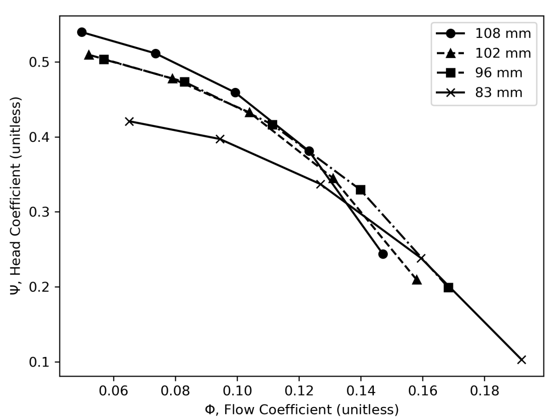

Figure 9: Head coefficient vs. flow coefficient for geometrically dissimilar pumps.

5.6 Pump Efficiency

The pump efficiencies for the experimental data was calculated in Appendix **??**. The pump effiecncies for the manufacturer data was given in the manufacturer's specifications. The plot of the experimental and manufacturer pump efficiencies is shown in Figure 10.

The pump was most effiicent when operating at $2700\ \mathrm{RPM}$ in a partially closed valve configuration. The pump was least efficient when operating at $1800\ \mathrm{RPM}$ in a partially closed valve configuration.

The actual pump efficiency was lower than the manufacturer pump efficiency for all pump speeds. This could be due to degradation of the pump over time as the setup has been used for many years.

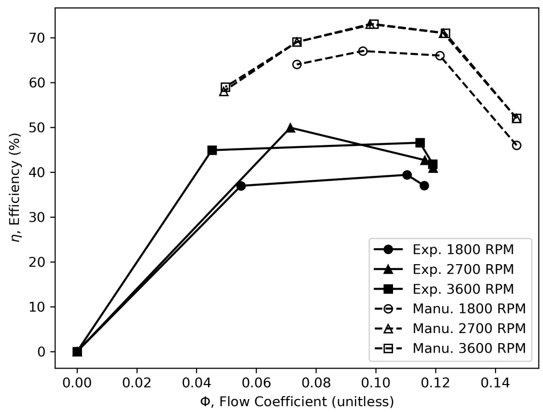

Figure 10: Single pump experimental and manufacturer efficiency plot.

5.7 Elevation Effects

The vertical elevation of the pump lines have an effect on the head of the pump. This variation was not accounted for in the modelling. This was because the vertical displacements between where the pressure was measured was small $(< 0.5\,\mathrm{m})$. It was assumed that this variation was negligible.

6 Conclusion

- Summarize the results of the lab
- Discuss the significance of the results
- Discuss the sources of error
- Discuss the limitations of the experiment
- Discuss the implications of the results
- Discuss the future work

6.1 Technical Recommendations

7 References

The references section will be auto populated with Bibtex.

A Appendix: Single Pump Analysis

A.1 Single Pump Discharge and Heads

Table A.4: Summary of single pump experimental discharge and heads for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed	Transducer Output, V_t	Time to collect water (s)			Nominal Time, t	Volumetric Flow, Q	Transducer Head, H_t	Corrected Head, H
	(RPM)	(V)	Trial 1	Trial 2	Trial 3	(s)	($\mathrm{m^3\,s^{-1}}$)	(m)	(m)
Fully open	1800	0.75	28.55	28.99	28.79	28.78	0.003159	2.64	2.91
Partial 1	1800	0.83	30.14	30.51	30.16	30.27	0.003003	2.92	3.16
Partial 2	1800	1.43	61.12	60.80	60.86	60.93	0.001492	5.04	5.09
Closed	1800	1.56	-	-	-	-	-	5.49	5.49
Fully open	2700	1.50	18.66	18.93	18.55	18.71	0.004858	5.28	5.91
Partial 1	2700	1.62	19.23	19.23	18.96	19.14	0.004749	5.70	6.31
Partial 2	2700	2.94	31.15	31.30	31.22	31.22	0.002911	10.4	10.6
Closed	2700	3.52	-	-	-	-	-	12.4	12.4
Fully open	3600	2.44	14.13	13.93	14.06	14.04	0.006474	8.59	9.72
Partial 1	3600	2.89	15.04	14.35	14.31	14.57	0.006240	10.2	11.2
Partial 2	3600	5.85	37.01	37.04	37.07	37.04	0.002454	20.6	20.8
Closed	3600	6.18	-	-	-	-	-	21.8	21.8

Sample calculations are evaluated for the fully open configuration at 1800 RPM. Starting with time,

$$t = \frac{\sum t_i}{n} = \frac{28.55\,\mathrm{s} + 28.99\,\mathrm{s} + 28.79\,\mathrm{s}}{3} = 28.78\,\mathrm{s}$$

The volumetric flow rate was calculated using water which had a mass of $m = 200\,\mathrm{lb} = 90.7\,\mathrm{kg}$ and a density of $\rho = 998\,\mathrm{kg\,m^{-3}}$. Then,

$$Q = \frac{m}{\rho t} = \frac{90.7\,\mathrm{kg}}{998\,\mathrm{kg\,m^{-3}} \times 28.78\,\mathrm{s}} = 0.003159\,\mathrm{m^3\,s^{-1}}$$

The transducer head was found by

$$H_t = \frac{\Delta P}{\rho g} = \frac{0.75\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} = 2.64\,\mathrm{m}$$

The corrected head was found using inlet diameter, $D_1 = 0.0508\,\mathrm{m}$, and outlet diameter, $D_2 = 0.0381\,\mathrm{m}$, by

$$
\begin{aligned}
H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1-\left(\frac{D_2}{D_1}\right)^4\right] \\
&= 2.64\,\mathrm{m} + \frac{8\times(0.003159\,\mathrm{m^3\,s^{-1}})^2}{\pi^2\times(0.0381\,\mathrm{m})^4\times 9.81\,\mathrm{m\,s^{-2}}}\left[1-\left(\frac{0.0381\,\mathrm{m}}{0.0508\,\mathrm{m}}\right)^4\right] \\
&= 2.91\,\mathrm{m}
\end{aligned}
$$

A.1.1 Single Pump Discharge and Head Uncertainty

Table A.5: Single pump experimental discharge and head uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed (RPM)	Time STDEV, S_t (s)	Time Precision, P_t ($\pm$s)	Time Bias, B_t ($\pm$s)	Time Uncertainty, δ_t ($\pm$s)	Flow Uncertainty, δ_Q ($\pm\mathrm{m^3\,s^{-1}}$)	Transducer Uncertainty, $\delta_{V_{tr}}$ ($\pm$V)	Head Uncertainty, δ_{H_t} ($\pm$m)	Corrected Head Uncertainty, δ_H ($\pm$m)
Fully open	1800	0.2203	0.5473	0.20	0.58	6.40E-05	0.01	0.04	0.04
Partial 1	1800	0.2081	0.5169	0.20	0.55	5.50E-05	0.01	0.04	0.04
Partial 2	1800	0.1701	0.4225	0.20	0.47	1.14E-05	0.01	0.04	0.04
Closed	1800	-	-	-	-	-	0.01	0.04	0.04
Fully open	2700	0.1955	0.4857	0.20	0.53	1.36E-04	0.01	0.04	0.05
Partial 1	2700	0.1559	0.3872	0.20	0.44	1.08E-04	0.01	0.04	0.04
Partial 2	2700	0.0751	0.1864	0.20	0.27	2.55E-05	0.01	0.04	0.04
Closed	2700	-	-	-	-	-	0.01	0.04	0.04
Fully open	3600	0.1015	0.2521	0.20	0.32	1.48E-04	0.01	0.04	0.06
Partial 1	3600	0.4104	1.0195	0.20	1.04	4.45E-04	0.01	0.04	0.2
Partial 2	3600	0.0300	0.0745	0.20	0.21	1.41E-05	0.01	0.04	0.04
Closed	3600	-	-	-	-	-	0.01	0.04	0.04

Sample calculations are evaluated for the fully open configuration at 1800 RPM.

First, time standard deviation was calculated with `STDEV.S` in Excel. A confidence of 95% ($\alpha/2 = 0.025$) was used to calculate the t-distribution value ($\nu = 3 - 1 = 2$) with `T.INV.T` in Excel. This gave a value of $t_{\alpha/2,\nu} = 4.3027$. The time precision was calculated by

$$
\begin{aligned}
P_t &= t_{\alpha/2,\nu} \times \frac{S_t}{\sqrt{n}} \\
&= 4.3027 \times \frac{0.2203\,\mathrm{s}}{\sqrt{3}} \\
&= 0.5473\,\mathrm{s}
\end{aligned}
$$

The time bias was assumed to be the reaction time of the operator, which was estimated to be $0.20\,\mathrm{s}$. The time total uncertainty was calculated by

$$\begin{aligned}\delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.5473\,\mathrm{s})^2 + (0.20\,\mathrm{s})^2} \\ &= 0.58\,\mathrm{s}\end{aligned}$$

The flow uncertainty was calculated by propagation of error. The function for flow is

$$Q = \frac{m}{\rho t}$$

This is the special purely multiplicative case of the general formula for error propagation. Assuming the mass and density errors are negligible, the flow uncertainty was calculated by

$$\begin{aligned}\delta_Q &= Q\sqrt{\left(\frac{\delta_t}{t}\right)^2} \\ &= Q\left|\frac{\delta_t}{t}\right| \\ &= 0.003159\,\mathrm{m^3\,s^{-1}}\left|\frac{0.58\,\mathrm{s}}{28.78\,\mathrm{s}}\right| \\ &= 6.40 \times 10^{-5}\,\mathrm{m^3\,s^{-1}}\end{aligned}$$

Transducer bias was assumed to be $0.01\,\mathrm{V}$. Transducer precision was not considered since calibration was not performed. So

$$\delta_{V_{\mathrm{tr}}} = 0.01\,\mathrm{V}$$

Next, the head uncertainty was calculated by propagation of error. The function for head is

$$H_t = \frac{\Delta P}{\rho g} = \frac{V_{\mathrm{tr}} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, we have the special purely multiplicative case for error propagation. The head uncertainty was calculated by

$$\begin{aligned}\delta_{H_t} &= H_t\sqrt{\left(\frac{\delta_{V_{\mathrm{tr}}}}{V_{\mathrm{tr}}}\right)^2} \\ &= H_t\left|\frac{\delta_{V_{\mathrm{tr}}}}{V_{\mathrm{tr}}}\right| \\ &= 2.64\,\mathrm{m}\left|\frac{0.01\,\mathrm{V}}{0.75\,\mathrm{V}}\right| \\ &= 0.04\,\mathrm{m}\end{aligned}$$

Lastly, the function for corrected head is

$$
\begin{aligned}
H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\
\frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{2(H - H_t)}{Q} \\
\frac{\partial H}{\partial H_t} &= 1
\end{aligned}
$$

Then, the corrected head uncertainty was determined by

$$
\begin{aligned}
\delta_H &= \sqrt{\left(\frac{\partial H}{\partial Q}\right)^2 \delta_Q^2 + \left(\frac{\partial H}{\partial H_t}\right)^2 \delta_{H_t}^2} \\
&= \sqrt{\left(\frac{2(H - H_t)}{Q}\right)^2 \delta_Q^2 + \delta_{H_t}^2} \\
&= \sqrt{\left(\frac{2(2.91\,\mathrm{m} - 2.64\,\mathrm{m})}{0.003159\,\mathrm{m^3\,s^{-1}}}\right)^2 (6.40 \times 10^{-5}\,\mathrm{m^3\,s^{-1}})^2 + (0.04\,\mathrm{m})^2} \\
&= 0.04\,\mathrm{m}
\end{aligned}
$$

A.2 Single Pump Head and Flow Coefficients

Table A.6: Single pump experimental head and flow coefficients for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Corrected Head, H (m)	Tip Speed, U ($\mathrm{m\,s^{-1}}$)	Radial Exit Velocity, v_{2r} ($\mathrm{m\,s^{-1}}$)	Head Coefficient, Ψ	Flow Coefficient, Φ
Fully open	1800	0.003159	2.91	10.2	1.2	0.275	0.12
Partial 1	1800	0.003003	3.16	10.2	1.1	0.300	0.11
Partial 2	1800	0.001492	5.09	10.2	0.6	0.482	0.05
Closed	1800	-	5.49	10.2	0.0	0.520	0.00
Fully open	2700	0.004858	5.91	15.3	1.8	0.249	0.12
Partial 1	2700	0.004749	6.31	15.3	1.8	0.265	0.12
Partial 2	2700	0.002911	10.6	15.3	1.1	0.445	0.07
Closed	2700	-	12.4	15.3	0.0	0.522	0.00
Fully open	3600	0.006474	9.72	20.4	2.4	0.230	0.12
Partial 1	3600	0.006240	11.2	20.4	2.3	0.266	0.11
Partial 2	3600	0.002454	20.8	20.4	0.9	0.491	0.05
Closed	3600	-	21.8	20.4	0.0	0.515	0.00

Sample calculations are evaluated for the fully open configuration at 1800 RPM. The tip speed was calculated using the impeller radius, $r_2 = 0.108/2 = 0.054\,\mathrm{m}$, by

$$\begin{aligned} U &= r_2 \Omega \\ &= 0.054\,\mathrm{m} \times 1800\,\mathrm{RPM} \times \left(\frac{2\pi}{60}\,\mathrm{RPM}^{-1} \right) \\ &= 10.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

The radial exit velocity was calculated using the impeller diameter, the blade height at exit, $b = 0.009\,\mathrm{m}$, blade width at exit, $w = 0.0085\,\mathrm{m}$, and the number of blades $N = 5$, by

$$\begin{aligned} v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.003159\,\mathrm{m^3\,s^{-1}}}{0.009\,\mathrm{m} \times (2\pi \times 0.054\,\mathrm{m} - 5 \times 0.0085\,\mathrm{m})} \\ &= 1.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

The head coefficient was found by

$$\begin{aligned} \Psi &= \frac{Hg}{U^2} \\ &= \frac{2.91\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{(10.2\,\mathrm{m\,s^{-1}})^2} \\ &= 0.275 \end{aligned}$$

The flow coefficient was found by

$$\begin{aligned} \Phi &= \frac{v_{2r}}{U} \\ &= \frac{1.2\,\mathrm{m\,s^{-1}}}{10.2\,\mathrm{m\,s^{-1}}} \\ &= 0.12 \end{aligned}$$

A.2.1 Single Pump Head and Flow Coefficient Uncertainty

Table A.7: Single pump experimental head and flow coefficient uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed (RPM)	Flow Uncertainty, δ_Q ($\pm\mathrm{m^3\,s^{-1}}$)	Head Uncertainty, δ_H ($\pm$m)	Radial Exit Velocity Uncertainty, $\delta_{v_{2r}}$ ($\pm\mathrm{m\,s^{-1}}$)	Head Coefficient Uncertainty, δ_Ψ ($\pm$)	Flow Coefficient Uncertainty, δ_Φ ($\pm$)
Fully open	1800	6.40E-05	0.04	0.024	0.003	0.0024
Partial 1	1800	5.50E-05	0.04	0.021	0.003	0.0020
Partial 2	1800	1.14E-05	0.04	0.0043	0.003	0.0004
Closed	1800	-	0.04	-	0.003	-
Fully open	2700	1.36E-04	0.05	0.051	0.002	0.0033
Partial 1	2700	1.08E-04	0.04	0.040	0.002	0.0027
Partial 2	2700	2.55E-05	0.04	0.010	0.001	0.00063
Closed	2700	-	0.04	-	0.001	-
Fully open	3600	1.48E-04	0.06	0.056	0.001	0.0027
Partial 1	3600	4.45E-04	0.2	0.17	0.004	0.0082
Partial 2	3600	1.41E-05	0.04	0.0053	0.001	0.00026
Closed	3600	-	0.04	-	0.001	-

Sample calculations are evaluated for the fully open configuration at 1800 RPM. The RPM was measured by the stroboscope. It was assumed that pump speed uncertainty was negligible. The radial exit velocity is a function of

$$v_{2r} = \frac{Q}{b(2\pi r_2 - Nw)}$$

Assuming the blade height and width errors are negligible, the radial exit velocity uncertainty was calculated by

$$\begin{aligned}
\delta_{v_{2r}} &= v_{2r}\sqrt{\left(\frac{\delta_Q}{Q}\right)^2} \\
&= v_{2r}\left|\frac{\delta_Q}{Q}\right| \\
&= 1.2\,\mathrm{m\,s^{-1}}\left|\frac{6.40\times10^{-5}\,\mathrm{m^3\,s^{-1}}}{0.003159\,\mathrm{m^3\,s^{-1}}}\right| \\
&= 0.024\,\mathrm{m\,s^{-1}}
\end{aligned}$$

The function for head coefficient is

$$\Psi = \frac{Hg}{U^2}$$

Assuming the gravity and tip speed errors are negligible, the head coefficient uncertainty was calculated by

$$\begin{aligned}\delta_\Psi &= \Psi\sqrt{\left(\frac{\delta_H}{H}\right)^2}\\ &= \Psi\left|\frac{\delta_H}{H}\right|\\ &= 0.275\left|\frac{0.04\,\mathrm{m}}{2.91\,\mathrm{m}}\right|\\ &= 0.003\end{aligned}$$

The function for flow coefficient is

$$\Phi = \frac{v_{2r}}{U}$$

since the tip speed error is negligible, this is a special purely multiplicative case of the general formula for error propagation. The flow coefficient uncertainty was calculated by

$$\begin{aligned}\delta_\Phi &= \Phi\sqrt{\left(\frac{\delta_{v_{2r}}}{v_{2r}}\right)^2}\\ &= \Phi\left|\frac{\delta_{v_{2r}}}{v_{2r}}\right|\\ &= 0.12\left|\frac{0.024\,\mathrm{m\,s^{-1}}}{1.2\,\mathrm{m\,s^{-1}}}\right|\\ &= 0.0024\end{aligned}$$

A.3 Single Pump Manufaturer's Data

Sample calculations are evaluated 1800 RPM with a volumetric flow rate of $Q = 0.0040\,\mathrm{m^3\,s^{-1}}$. The tip speed was calculated using the impeller radius, $r_2 = 0.108/2 = 0.054\,\mathrm{m}$, by

$$\begin{aligned}U &= r_2\Omega\\ &= 0.054\,\mathrm{m} \times 1800\,\mathrm{RPM} \times \left(\frac{2\pi}{60}\,\mathrm{RPM^{-1}}\right)\\ &= 10.2\,\mathrm{m\,s^{-1}}\end{aligned}$$

Table A.8: Single pump manufacturer's data for 1800 RPM, 2700 RPM, and 3600 RPM

Pump Speed (RPM)	Volumetric Flow, Q ($\mathrm{m^3\,s^{-1}}$)	Head, H (m)	Tip Speed, U ($\mathrm{m\,s^{-1}}$)	Radial Exit Velocity, v_{2r} ($\mathrm{m\,s^{-1}}$)	Head Coefficient, Ψ	Flow Coefficient, Φ
1800	0.0040	2.58	10.2	1.5	0.244	0.15
1800	0.0033	4.14	10.2	1.2	0.392	0.12
1800	0.0026	4.96	10.2	1.0	0.470	0.10
1800	0.0020	5.40	10.2	0.75	0.511	0.07
2700	0.0060	5.77	15.3	2.2	0.243	0.15
2700	0.0050	9.12	15.3	1.9	0.384	0.12
2700	0.0040	11.0	15.3	1.5	0.463	0.10
2700	0.0030	12.1	15.3	1.1	0.509	0.07
2700	0.0020	12.8	15.3	0.75	0.539	0.05
3600	0.0080	10.3	20.4	3.0	0.244	0.15
3600	0.0067	16.1	20.4	2.5	0.381	0.12
3600	0.0054	19.2	20.4	2.0	0.454	0.10
3600	0.0040	21.6	20.4	1.5	0.511	0.07
3600	0.0027	22.8	20.4	1.0	0.540	0.05

The radial exit velocity was calculated using the impeller diameter, the blade height at exit, $b = 0.009\,\mathrm{m}$, blade width at exit, $w = 0.0085\,\mathrm{m}$, and the number of blades $N = 5$, by

$$\begin{aligned} v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.0040\,\mathrm{m^3\,s^{-1}}}{0.009\,\mathrm{m} \times (2\pi \times 0.054\,\mathrm{m} - 5 \times 0.0085\,\mathrm{m})} \\ &= 1.5\,\mathrm{m\,s^{-1}} \end{aligned}$$

The head coefficient was found by

$$\begin{aligned} \Psi &= \frac{Hg}{U^2} \\ &= \frac{2.58\,\mathrm{m} \times 9.81\,\mathrm{m\,s^{-2}}}{(10.2\,\mathrm{m\,s^{-1}})^2} \\ &= 0.244 \end{aligned}$$

The flow coefficient was found by

$$\begin{aligned}\Phi &= \frac{v_{2r}}{U} \\ &= \frac{1.5\,\mathrm{m\,s^{-1}}}{10.2\,\mathrm{m\,s^{-1}}} \\ &= 0.15\end{aligned}$$

B Appendix: Parallel and Series Pump Analysis

B.1 Parallel Experimental Pump Discharge and Head

Table B.9: Summary of experimental parallel pump discharge and heads for 2700 RPM

Valve Configuration	Transducer Output		Time to collect water			Nominal Time, t	Volumetric Flow, Q	Transducer Head, H_t			Corrected Head, H
	Pump 1	Pump 2	Trial 1	Trial 2	Trial 3			Pump 1	Pump 2	Nominal	
	(V)	(V)	(s)	(s)	(s)	(s)	($\mathrm{m^3\,s^{-1}}$)	(m)	(m)	(m)	(m)
Fully open	2.87	2.72	12.46	12.28	12.79	12.51	0.007266	10.1	9.58	9.84	11.3
Partial 1	3.08	2.96	14.49	14.49	14.42	14.47	0.006283	10.8	10.4	10.6	11.7
Partial 2	3.52	3.53	41.05	40.99	40.99	41.01	0.002217	12.4	12.4	12.4	12.5
Closed	3.55	3.55	-	-	-	-	-	12.5	12.5	12.5	12.5

Sample calculations will be shown for the fully open valve configuration. The same calculations were done for the other valve configurations. The nominal time was calculated as the average of the three trials by

$$\begin{aligned} t &= \frac{\sum t_i}{n} \\ &= \frac{12.46 + 12.28 + 12.79}{3} \\ &= 12.51\,\mathrm{s} \end{aligned}$$

The volumetric flow rate was calculated using $m = 200\,\mathrm{lb} = 90.7\,\mathrm{kg}$ and $\rho = 998\,\mathrm{kg\,m^{-3}}$ by

$$\begin{aligned} Q &= \frac{m}{\rho t} \\ &= \frac{90.7\,\mathrm{kg}}{998\,\mathrm{kg\,m^{-3}} \times 12.51\,\mathrm{s}} \\ &= 0.007266\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

Next, the transducer head for pumps 1 and pump 2 were calculated by

$$\begin{aligned} H_t &= \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g} \\ \implies H_{t1} &= \frac{2.87\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 10.11\,\mathrm{m} \\ \implies H_{t2} &= \frac{2.72\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 9.58\,\mathrm{m} \end{aligned}$$

The nominal head was calculated by averaging the transducer heads by

$$\begin{aligned}H &= \frac{\sum H_{t_i}}{n} \\ &= \frac{10.11 + 9.58}{2} \\ &= 9.84\,\mathrm{m}\end{aligned}$$

The corrected head was calculated by

$$\begin{aligned}H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 9.84\,\mathrm{m} + \frac{8 \times (0.007266\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.108\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.108\,\mathrm{m}}{0.108\,\mathrm{m}}\right)^4\right] \\ &= 11.3\,\mathrm{m}\end{aligned}$$

B.1.1 Parallel Experimental Pump Discharge and Head Uncertainty

Table B.10: Parallel experimental pump time and discharge uncertainties for 2700 RPM

Valve Configuration	Time STDEV, S_t (s)	Time Precision, P_t (s)	Time Bias, B_t (s)	Time Uncertainty, δ_t (s)	Flow Uncertainty, δ_Q ($\mathrm{m^3\,s^{-1}}$)
Fully open	0.2587	0.6425	0.2	0.67	3.91E-04
Partial 1	0.0404	0.1004	0.2	0.22	9.72E-05
Partial 2	0.0346	0.0861	0.2	0.22	1.18E-05
Closed	-	-	-	-	-

Table B.11: Parallel experimental pump head uncertainties for 2700 RPM

Valve Configuration	Transducer Uncertainty		Transducer Head Uncertainty			Corrected Head Uncertainty, δ_H
	Pump 1, $\delta_{V_{\mathrm{tr1}}}$ (V)	Pump 2, $\delta_{V_{\mathrm{tr2}}}$ (V)	Pump 1, $\delta_{H_{t1}}$ (m)	Pump 2, $\delta_{H_{t2}}$ (m)	Nominal, δ_{H_t} (m)	(m)
Fully open	0.01	0.01	0.04	0.04	0.02	0.2
Partial 1	0.01	0.01	0.04	0.04	0.02	0.04
Partial 2	0.01	0.01	0.04	0.04	0.02	0.02
Closed	0.01	0.01	0.04	0.04	0.02	0.02

Sample calculations are evaluated for the fully open configuration at 2700 RPM. A 95% confidence interval was used. The standard deviation was calculated by Excel using `STDEV.S`. The t-distribution value was found using $\alpha/2 = 0.025$ and $\nu = 3 - 1 = 2$. Then,

$$\begin{aligned} P_t &= t_{\alpha/2,\nu} \times \frac{S_t}{\sqrt{n}} \\ &= 4.303 \times \frac{0.2587\,\mathrm{s}}{\sqrt{3}} \\ &= 0.6425\,\mathrm{s} \end{aligned}$$

The time bias was approximated to be the reaction time of the operator by

$$B_t = 0.2\,\mathrm{s}$$

The time uncertainty was calculated by

$$\begin{aligned} \delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.6425\,\mathrm{s})^2 + (0.2\,\mathrm{s})^2} \\ &= 0.67\,\mathrm{s} \end{aligned}$$

The flow uncertainty was calculated by

$$\begin{aligned} \delta_Q &= Q\left|\frac{\delta_t}{t}\right| \\ &= 0.007266\,\mathrm{m}^3\,\mathrm{s}^{-1}\,\frac{0.67\,\mathrm{s}}{12.51\,\mathrm{s}} \\ &= 3.91E - 04 \end{aligned}$$

The transducer uncertainty was assumed to be the resolution of the device, $\delta_{V_\text{tr}} = 0.01\,\mathrm{V}$. Transducer precision error was not considered since calibration was not performed. The transducer heads were found by

$$H_t = \frac{V_\text{tr} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, this is the special purely multiplicative case of the general formula for error propagation. The transducer head uncertainty was calculated by

$$\begin{aligned}\delta_{H_{t1}} &= H_{t1}\sqrt{\left(\frac{\delta_{V_{\text{tr1}}}}{V_{\text{tr1}}}\right)^2} \\ &= H_{t1}\left|\frac{\delta_{V_{\text{tr1}}}}{V_{\text{tr1}}}\right| \\ &= 10.11\,\text{m}\left|\frac{0.01\,\text{V}}{2.87\,\text{V}}\right| \\ &= 0.04\,\text{m}\end{aligned}$$

The nominal transducer head uncertainty was calculated by

$$\begin{aligned}H_t &= \frac{H_{t1}+H_{t2}}{2} \\ \implies \frac{\partial H_t}{\partial H_{t1}} &= \frac{1}{2} \\ \implies \frac{\partial H_t}{\partial H_{t2}} &= \frac{1}{2}\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_{H_t} &= \frac{1}{2}\sqrt{(\delta_{H_{t1}})^2+(\delta_{H_{t2}})^2} \\ &= \frac{1}{2}\sqrt{(0.04\,\text{m})^2+(0.04\,\text{m})^2} \\ &= 0.02\,\text{m}\end{aligned}$$

The corrected head is found by

$$\begin{aligned}H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1-\left(\frac{D_2}{D_1}\right)^4\right] \\ \implies \frac{\partial H}{\partial H_t} &= 1 \\ \implies \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g}\left[1-\left(\frac{D_2}{D_1}\right)^4\right] = \frac{2(H-H_t)}{Q}\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_H &= \sqrt{(\delta_{H_t})^2+\left(\frac{2(H-H_t)}{Q}\delta_Q\right)^2} \\ &= \sqrt{(0.02\,\text{m})^2+\left(\frac{2(11.3\,\text{m}-9.84\,\text{m})}{0.007266\,\text{m}^3\,\text{s}^{-1}}\times 3.91\times 10^{-4}\,\text{m}^3\,\text{s}^{-1}\right)^2} \\ &= 0.2\,\text{m}\end{aligned}$$

B.2 Series Experimental Pump Discharge and Head

Table B.12: Summary of experimental series pump discharge and heads for 2700 RPM

Valve Configuration	Transducer Output		Time to collect water			Nominal Time, t	Volumetric Flow, Q	Transducer Head, H_t			Corrected Head, H
	Pump 1	Pump 2	Trial 1	Trial 2	Trial 3			Pump 1	Pump 2	Nominal	
	(V)	(V)	(s)	(s)	(s)	(s)	($\mathrm{m^3\,s^{-1}}$)	(m)	(m)	(m)	(m)
Fully open	0.68	1.68	16.64	16.17	16.03	16.28	0.005584	2.39	5.92	8.31	9.1
Partial 1	0.9	1.84	16.71	16.58	16.76	16.68	0.005449	3.17	6.48	9.65	10.4
Partial 2	2.89	3.26	30.52	30.23	30.67	30.47	0.002983	10.2	11.5	21.7	21.9
Closed	3.52	3.55	-	-	-	-	-	12.4	12.5	24.9	24.9

Sample calculations will be shown for the fully open valve configuration. The same calculations were done for the other valve configurations. The nominal time was calculated as the average of the three trials by

$$\begin{aligned} t &= \frac{\sum t_i}{n} \\ &= \frac{16.64 + 16.17 + 16.03}{3} \\ &= 16.28\,\mathrm{s} \end{aligned}$$

The volumetric flow rate was calculated using $m = 200\,\mathrm{lb} = 90.7\,\mathrm{kg}$ and $\rho = 998\,\mathrm{kg\,m^{-3}}$ by

$$\begin{aligned} Q &= \frac{m}{\rho t} \\ &= \frac{90.7\,\mathrm{kg}}{998\,\mathrm{kg\,m^{-3}} \times 16.28\,\mathrm{s}} \\ &= 0.005584\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

Next, the transducer head for pumps 1 and pump 2 were calculated by

$$\begin{aligned} H_t &= \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g} \\ \implies H_{t1} &= \frac{0.68\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 2.39\,\mathrm{m} \\ \implies H_{t2} &= \frac{1.68\,\mathrm{V} \times 5\,\mathrm{psi\,V^{-1}} \times 6894.76\,\mathrm{Pa\,psi^{-1}}}{998\,\mathrm{kg\,m^{-3}} \times 9.81\,\mathrm{m\,s^{-2}}} \\ &= 5.92\,\mathrm{m} \end{aligned}$$

The nominal head was calculated by summing the transducer heads by

$$\begin{aligned} H_t &= \sum H_{t_i} \\ &= 2.39\,\mathrm{m} + 5.92\,\mathrm{m} \\ &= 8.31\,\mathrm{m} \end{aligned}$$

The corrected head was calculated by

$$\begin{aligned} H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 8.31\,\mathrm{m} + \frac{8 \times (0.005584\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.108\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.108\,\mathrm{m}}{0.108\,\mathrm{m}}\right)^4\right] \\ &= 9.1\,\mathrm{m} \end{aligned}$$

B.2.1 Series Experimental Pump Discharge and Head Uncertainty

Table B.13: Series experimental pump time and discharge uncertainties for 2700 RPM

Valve Configuration	Time STDEV, S_t (s)	Time Precision, P_t (s)	Time Bias, B_t (s)	Time Uncertainty, δ_t (s)	Flow Uncertainty, δ_Q ($\mathrm{m^3\,s^{-1}}$)
Fully open	0.3195	0.7938	0.2	0.82	2.81E-04
Partial 1	0.0929	0.2308	0.2	0.31	9.97E-05
Partial 2	0.2237	0.5557	0.2	0.59	5.78E-05
Closed	-	-	-	-	-

Table B.14: Series experimental pump head uncertainties for 2700 RPM

Valve Configuration	Transducer Uncertainty		Transducer Head Uncertainty			Corrected Head Uncertainty, δ_H
	Pump 1, $\delta_{V_{\mathrm{tr1}}}$ (V)	Pump 2, $\delta_{V_{\mathrm{tr2}}}$ (V)	Pump 1, $\delta_{H_{t1}}$ (m)	Pump 2, $\delta_{H_{t2}}$ (m)	Nominal, δ_{H_t} (m)	(m)
Fully open	0.01	0.01	0.04	0.04	0.05	0.1
Partial 1	0.01	0.01	0.04	0.04	0.05	0.06
Partial 2	0.01	0.01	0.04	0.04	0.05	0.05
Closed	0.01	0.01	0.04	0.04	0.05	0.05

Sample calculations are evaluated for the fully open configuration at 2700 RPM. A 95% confidence interval was used. The standard deviation was calculated by Excel using `STDEV.S`. The t-distribution value was found using $\alpha/2 = 0.025$ and $\nu = 3 - 1 = 2$. Then,

$$\begin{aligned} P_t &= t_{\alpha/2,\nu} \times \frac{S_t}{\sqrt{n}} \\ &= 4.303 \times \frac{0.3195\,\mathrm{s}}{\sqrt{3}} \\ &= 0.7938\,\mathrm{s} \end{aligned}$$

The time bias was approximated to be the reaction time of the operator by

$$B_t = 0.2\,\mathrm{s}$$

The time uncertainty was calculated by

$$\begin{aligned} \delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.7938\,\mathrm{s})^2 + (0.2\,\mathrm{s})^2} \\ &= 0.82\,\mathrm{s} \end{aligned}$$

The flow uncertainty was calculated by

$$\begin{aligned} \delta_Q &= Q\left|\frac{\delta_t}{t}\right| \\ &= 0.005584\,\mathrm{m^3\,s^{-1}}\,\frac{0.82\,\mathrm{s}}{16.28\,\mathrm{s}} \\ &= 2.81E-04 \end{aligned}$$

The transducer uncertainty was assumed to be the resolution of the device, $\delta_{V_{\text{tr}}} = 0.01\,\mathrm{V}$. Transducer precision error was not considered since calibration was not performed. The transducer heads were found by

$$H_t = \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, this is the special purely multiplicative case of the general formula for error propagation. The transducer head uncertainty was calculated by

$$\begin{aligned}\delta_{H_{t1}} &= H_{t1}\sqrt{\left(\frac{\delta_{V_{\text{tr1}}}}{V_{\text{tr1}}}\right)^2} \\ &= H_{t1}\left|\frac{\delta_{V_{\text{tr1}}}}{V_{\text{tr1}}}\right| \\ &= 2.39\,\text{m}\left|\frac{0.01\,\text{V}}{0.68\,\text{V}}\right| \\ &= 0.04\,\text{m}\end{aligned}$$

The nominal transducer head uncertainty was calculated by

$$\begin{aligned}H_t &= H_{t1} + H_{t2} \\ \implies \frac{\partial H_t}{\partial H_{t1}} &= 1 \\ \implies \frac{\partial H_t}{\partial H_{t2}} &= 1\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_{H_t} &= \sqrt{(\delta_{H_{t1}})^2 + (\delta_{H_{t2}})^2} \\ &= \sqrt{(0.04\,\text{m})^2 + (0.04\,\text{m})^2} \\ &= 0.05\,\text{m}\end{aligned}$$

The corrected head is found by

$$\begin{aligned}H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ \implies \frac{\partial H}{\partial H_t} &= 1 \\ \implies \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] = \frac{2(H - H_t)}{Q}\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_H &= \sqrt{(\delta_{H_t})^2 + \left(\frac{2(H - H_t)}{Q}\delta_Q\right)^2} \\ &= \sqrt{(0.05\,\text{m})^2 + \left(\frac{2(9.1\,\text{m} - 8.31\,\text{m})}{0.005584\,\text{m}^3\,\text{s}^{-1}} \times 2.81 \times 10^{-4}\,\text{m}^3\,\text{s}^{-1}\right)^2} \\ &= 0.1\,\text{m}\end{aligned}$$

		Actual			Theoretical		
		Volumetric Flow	Transducer Head	Corrected Head	Volumetric Flow	Transducer Head	Corrected Head
Parallel	Fully open	0.007266	9.84	11.3	0.00971503	5.28	7.81
Parallel	Partial 1	0.006283	10.6	11.7	0.009498	5.70	8.12
Parallel	Partial 2	0.002217	12.4	12.5	0.005823	10.4	11.3
Parallel	Closed	-	12.5	12.5	-	12.4	12.4
Series	Fully open	0.005584	8.31	9.1	0.004858	10.6	11.2
Series	Partial 1	0.005449	9.65	10.4	0.004749	11.4	12.0
Series	Partial 2	0.002983	21.7	21.9	0.002911	20.7	20.9
Series	Closed	-	24.9	24.9	-	24.8	24.8

B.3 Parallel and Series Experimental vs. Theoretical Pump Discharge and Head

The actual results were pulled directly from Table B.9 and Table B.12. The theoretical results were calculated using the results from the single pump analysis Table A.4.

B.3.1 Parallel Experimental vs. Theoretical Pump Discharge and Head

Sample calculations are evaluated for the fully open configuration. The theoretical volumetric flow was found by

$$\begin{aligned} Q_{\text{th}} &= 2Q_{\text{single}} \\ &= 2 \times 0.004858\,\mathrm{m^3\,s^{-1}} \\ &= 0.00971503\,\mathrm{m^3\,s^{-1}} \end{aligned}$$

The theoretical transducer head was found by

$$\begin{aligned} H_{t_{\text{th}}} &= H_{t_{\text{single}}} \\ &= 5.28\,\mathrm{m} \end{aligned}$$

The theoretical corrected head was found by

$$\begin{aligned} H_{\text{th}} &= H_{t_{\text{th}}} + \frac{8Q_{\text{th}}^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 5.28\,\mathrm{m} + \frac{8 \times (0.00971503\,\mathrm{m^3\,s^{-1}})^2}{\pi^2 \times (0.108\,\mathrm{m})^4 \times 9.81\,\mathrm{m\,s^{-2}}}\left[1 - \left(\frac{0.108\,\mathrm{m}}{0.108\,\mathrm{m}}\right)^4\right] \\ &= 7.81\,\mathrm{m} \end{aligned}$$

B.3.2 Series Experimental vs. Theoretical Pump Discharge and Head

Sample calculations are evaluated for the fully open configuration. The theoretical volumetric flow was found by

$$\begin{aligned} Q_{\text{th}} &= Q_{\text{single}} \\ &= 0.004858\,\text{m}^3\,\text{s}^{-1} \end{aligned}$$

The theoretical transducer head was found by

$$\begin{aligned} H_{t_{\text{th}}} &= 2H_{t_{\text{single}}} \\ &= 2 \times 5.28\,\text{m} \\ &= 10.6\,\text{m} \end{aligned}$$

The theoretical corrected head was found by

$$\begin{aligned} H_{\text{th}} &= H_{t_{\text{th}}} + \frac{8Q_{\text{th}}^2}{\pi^2 D_2^4 g}\left[1 - \left(\frac{D_2}{D_1}\right)^4\right] \\ &= 10.6\,\text{m} + \frac{8 \times (0.004858\,\text{m}^3\,\text{s}^{-1})^2}{\pi^2 \times (0.108\,\text{m})^4 \times 9.81\,\text{m}\,\text{s}^{-2}}\left[1 - \left(\frac{0.108\,\text{m}}{0.108\,\text{m}}\right)^4\right] \\ &= 11.2\,\text{m} \end{aligned}$$

C Appendix: Impeller Angle

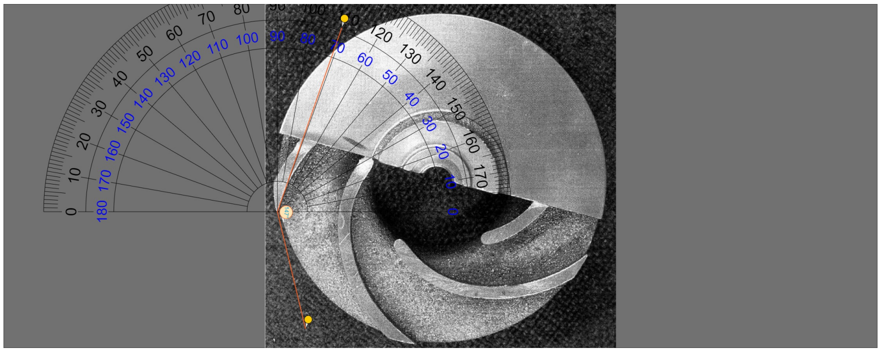

Figure C.11: Estimation of the impeller angle using digital protractor

The impeller angle was estimated using a digital protractor, as shown in Figure C.11. The impeller angle was measured to be

$$\beta_2 = 180^\circ - 147^\circ = 33^\circ$$

From equation (7), the ideal operating curve is

$$\begin{aligned} \Psi &= 1 - \Phi \cot \beta_2 \\ &= 1 - \Phi \cot 33^\circ & &= 1 - 1.540\Phi \end{aligned}$$

D Appendix: Shut Off Head

Shut off head was discussed in the theory section. From Eq. **??** and 10, the ideal and "rule of thumb" shut off head can be calculated. The calculation will be performed on the highest experimental speed, $\Omega = 3600\,\mathrm{RPM}$ which, from Table A.6, the tip speed, $U = 20.4\,\mathrm{m\,s^{-1}}$.

$$\begin{aligned}
H'_{\text{ideal}} &= \frac{U^2}{g} \\
&= \frac{(20.4\,\mathrm{m\,s^{-1}})^2}{9.81} \\
&= 42.2\,\mathrm{m} \\
H'_{\text{thumb}} &= \frac{1}{2} H'_{\text{ideal}} \\
&= \frac{1}{2} \times 42.2\,\mathrm{m} \\
&= 21.1\,\mathrm{m}
\end{aligned}$$

The actual shutoff head was taken from Table **??** and is $21.8\,\mathrm{m}$. The error for the rule of thumb shutoff head can be calculated as

$$\begin{aligned}
\%\ \text{Error} &= \left| \frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}} \right| \times 100\% \\
&= \left| \frac{H'_{\text{exp}} - H'_{\text{thumb}}}{H'_{\text{thumb}}} \right| \times 100\% \\
&= \left| \frac{21.8 - 21.1}{21.1} \right| \times 100\% \\
&= 3.3\%
\end{aligned}$$

The error for the ideal shutoff head can be calculated as

$$\begin{aligned}
\%\ \text{Error} &= \left| \frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}} \right| \times 100\% \\
&= \left| \frac{H'_{\text{exp}} - H'_{\text{ideal}}}{H'_{\text{ideal}}} \right| \times 100\% \\
&= \left| \frac{21.8 - 42.2}{42.2} \right| \times 100\% \\
&= 48.3\%
\end{aligned}$$

E Manufacturer Geometrically Similar and Dissimilar Pumps

Table E.15: Geometrically Similar and Dissimilar Pump Dimensions

Geometrically Similar	Impeller Diameter, D (m)	Blade Height, b (m)	Blade Width, w (m)	Volumetric Flow, Q ($m^3\,s^{-1}$)	Pump Speed, N (RPM)	Head, H (m)
No	0.108	0.009	0.0085	0.0080	3600	10.3
No	0.108	0.009	0.0085	0.0067	3600	16.1
No	0.108	0.009	0.0085	0.0054	3600	19.4
No	0.108	0.009	0.0085	0.0040	3600	21.6
No	0.108	0.009	0.0085	0.0027	3600	22.8
No	0.102	0.009	0.0085	0.0076	3600	7.9
No	0.102	0.009	0.0085	0.0063	3600	13.0
No	0.102	0.009	0.0085	0.0050	3600	16.3
No	0.102	0.009	0.0085	0.0038	3600	18.0
No	0.102	0.009	0.0085	0.0025	3600	19.2
No	0.096	0.009	0.0085	0.0071	3600	6.7
No	0.096	0.009	0.0085	0.0059	3600	11.0
No	0.096	0.009	0.0085	0.0047	3600	13.9
No	0.096	0.009	0.0085	0.0035	3600	15.8
No	0.096	0.009	0.0085	0.0024	3600	16.8
No	0.083	0.009	0.0085	0.0059	3600	2.6
No	0.083	0.009	0.0085	0.0049	3600	6.0
No	0.083	0.009	0.0085	0.0039	3600	8.4
No	0.083	0.009	0.0085	0.0029	3600	9.9

No	0.083	0.009	0.0085	0.0020	3600	10.5
Yes	0.108	0.009	0.0085	0.0080	3600	10.3
Yes	0.108	0.009	0.0085	0.0067	3600	16.1
Yes	0.108	0.009	0.0085	0.0054	3600	19.4
Yes	0.108	0.009	0.0085	0.0040	3600	21.6
Yes	0.108	0.009	0.0085	0.0027	3600	22.8
Yes	0.102	0.009	0.00803	0.0076	3600	7.9
Yes	0.102	0.009	0.00803	0.0063	3600	13.0
Yes	0.102	0.009	0.00803	0.0050	3600	16.3
Yes	0.102	0.009	0.00803	0.0038	3600	18.0
Yes	0.102	0.009	0.00803	0.0025	3600	19.2
Yes	0.096	0.008	0.00756	0.0071	3600	6.7
Yes	0.096	0.008	0.00756	0.0059	3600	11.0
Yes	0.096	0.008	0.00756	0.0047	3600	13.9
Yes	0.096	0.008	0.00756	0.0035	3600	15.8
Yes	0.096	0.008	0.00756	0.0024	3600	16.8
Yes	0.083	0.007	0.00653	0.0059	3600	2.6
Yes	0.083	0.007	0.00653	0.0049	3600	6.0
Yes	0.083	0.007	0.00653	0.0039	3600	8.4
Yes	0.083	0.007	0.00653	0.0029	3600	9.9
Yes	0.083	0.007	0.00653	0.0020	3600	10.5

Table E.16: Geometrically Similar and Dissimilar Pump Coefficients

Geometrically Similar	Impeller Diameter, D (m)	Impeller Speed, U ($\mathrm{m\,s^{-1}}$)	Radial Exit Velocity, v_{2r} ($\mathrm{m\,s^{-1}}$)	Head Coefficient, Ψ	Flow Coefficient, Φ
No	0.108	20.4	3.0	0.24	0.15
No	0.108	20.4	2.5	0.38	0.12
No	0.108	20.4	2.0	0.46	0.10
No	0.108	20.4	1.5	0.51	0.07
No	0.108	20.4	1.0	0.54	0.05
No	0.102	19.2	3.0	0.21	0.16
No	0.102	19.2	2.5	0.34	0.13
No	0.102	19.2	2.0	0.43	0.10
No	0.102	19.2	1.5	0.48	0.08
No	0.102	19.2	1.0	0.51	0.05
No	0.096	18.1	3.0	0.20	0.17
No	0.096	18.1	2.5	0.33	0.14
No	0.096	18.1	2.0	0.42	0.11
No	0.096	18.1	1.5	0.47	0.08
No	0.096	18.1	1.0	0.50	0.06
No	0.083	15.6	3.0	0.10	0.19
No	0.083	15.6	2.5	0.24	0.16
No	0.083	15.6	2.0	0.34	0.13
No	0.083	15.6	1.5	0.40	0.09
No	0.083	15.6	1.0	0.42	0.07

Yes	0.108	20.4	3.0	0.24	0.15
Yes	0.108	20.4	2.5	0.38	0.12
Yes	0.108	20.4	2.0	0.46	0.10
Yes	0.108	20.4	1.5	0.51	0.07
Yes	0.108	20.4	1.0	0.54	0.05
Yes	0.102	19.2	3.2	0.21	0.17
Yes	0.102	19.2	2.6	0.34	0.14
Yes	0.102	19.2	2.1	0.43	0.11
Yes	0.102	19.2	1.6	0.48	0.08
Yes	0.102	19.2	1.0	0.51	0.05
Yes	0.096	18.1	3.4	0.20	0.19
Yes	0.096	18.1	2.8	0.33	0.15
Yes	0.096	18.1	2.2	0.42	0.12
Yes	0.096	18.1	1.7	0.47	0.09
Yes	0.096	18.1	1.1	0.50	0.06
Yes	0.083	15.6	3.7	0.10	0.24
Yes	0.083	15.6	3.1	0.24	0.20
Yes	0.083	15.6	2.5	0.34	0.16
Yes	0.083	15.6	1.8	0.40	0.12
Yes	0.083	15.6	1.3	0.42	0.08

E.1 Geometrically Similar Pumps Sample Calculations

Sample calculations for Table E.15 and Table E.16 will be shown for an impeller of $D = 102\,\mathrm{mm}$ and $Q = 0.0076\,\mathrm{m}^3\,\mathrm{s}^{-1}$. Pump speed is given as $3600\,\mathrm{RPM}$ with number of blades, $N = 5$. Impeller speed is then

$$\begin{aligned} U &= \frac{D}{2} \times \Omega \\ &= \frac{102\,\mathrm{mm}}{2} \times 3600\,\mathrm{RPM} \times \frac{2\pi}{60} \frac{\mathrm{rad\,s^{-1}}}{\mathrm{RPM}} \\ &= 19.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

Geometrically similar means that blade height and width are scaled proportionally to the impeller diameter. Blade height and width are then

$$\begin{aligned} b_{102} &= \frac{b_{108} \times D_{102}}{D_{108}} \\ &= \frac{9\,\mathrm{mm} \times 102\,\mathrm{mm}}{108\,\mathrm{mm}} \\ &= 8.5\,\mathrm{mm} \\ w_{102} &= \frac{w_{108} \times D_{102}}{D_{108}} \\ &= \frac{8.5\,\mathrm{mm} \times 102\,\mathrm{mm}}{108\,\mathrm{mm}} \\ &= 8.03\,\mathrm{mm} \end{aligned}$$

Radial exit velocity is then

$$\begin{aligned} v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.0076\,\mathrm{m^3\,s^{-1}}}{8.5\,\mathrm{mm} \times (\pi \times 0.102\,\mathrm{m} - 5 \times 8.03\,\mathrm{mm})} \\ &= 3.0\,\mathrm{m\,s^{-1}} \end{aligned}$$

Head coefficient is then

$$\begin{aligned} \Psi &= \frac{H}{U^2} \\ &= \frac{7.9\,\mathrm{m}}{19.2\,\mathrm{m\,s^{-1}}} \\ &= 0.41 \end{aligned}$$

Flow coefficient is then

$$\begin{aligned}\Phi &= \frac{Q}{U \times D^2} \\ &= \frac{0.0076\,\mathrm{m^3\,s^{-1}}}{19.2\,\mathrm{m\,s^{-1}} \times (0.102\,\mathrm{m})^2} \\ &= 0.16\end{aligned}$$

E.2 Geometrically Dissimilar Pumps Sample Calculations

Sample calculations for Table E.15 and Table E.16 will be shown for an impeller of $D = 102\,\mathrm{mm}$ and $Q = 0.0076\,\mathrm{m^3\,s^{-1}}$. Pump speed is given as $3600\,\mathrm{RPM}$ with number of blades, $N = 5$. Impeller speed is then

$$\begin{aligned}U &= \frac{D}{2} \times \Omega \\ &= \frac{102\,\mathrm{mm}}{2} \times 3600\,\mathrm{RPM} \times \frac{2\pi}{60}\frac{\mathrm{rad\,s^{-1}}}{\mathrm{RPM}} \\ &= 19.2\,\mathrm{m\,s^{-1}}\end{aligned}$$

Geometrically dissimilar means that blade height and width are not scaled proportionally to the impeller diameter. Blade height and width are unchanged between all pumps. That is,

$$\begin{aligned}b_{102} &= b_{108} \\ &= 9\,\mathrm{mm} \\ w_{102} &= w_{108} \\ &= 8.5\,\mathrm{mm}\end{aligned}$$

Radial exit velocity is then

$$\begin{aligned}v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.0076\,\mathrm{m^3\,s^{-1}}}{9\,\mathrm{mm} \times (\pi \times 0.102\,\mathrm{m} - 5 \times 8.5\,\mathrm{mm})} \\ &= 3.2\,\mathrm{m\,s^{-1}}\end{aligned}$$

Head coefficient is then

$$\begin{aligned}\Psi &= \frac{H}{U^2} \\ &= \frac{7.9\,\mathrm{m}}{19.2\,\mathrm{m\,s^{-1}}} \\ &= 0.41\end{aligned}$$

Flow coefficient is then

$$\begin{aligned}\Phi &= \frac{Q}{U \times D^2} \\ &= \frac{0.0076\,\mathrm{m}^3\,\mathrm{s}^{-1}}{19.2\,\mathrm{m}\,\mathrm{s}^{-1} \times (0.102\,\mathrm{m})^2} \\ &= 0.16\end{aligned}$$