

MEC E 403

Lab 1: Centrifugal Pumps

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Abstract

Centrifugal pumps are used in many applications, including water supply, irrigation, and sewage systems. Understanding pump performance and key parameters is important for the design and operation of these systems.

This lab experiment was conducted to investigate the performance of a centrifugal pump. The pump was tested in single, parallel, and series configurations. The pump efficiency was also calculated.

Generally, the manufacturer's performance was higher than the experimental performance. This is illustrated in multiple plots, where the manufacturer specifications had higher head for a given flow rate. In addition, in the efficiency plot, the manufacturer's efficiency was higher than the experimental efficiency for all pump speeds. The history of the experimental pump was not known, and it is possible that the pump was not operating at its peak performance.

Shut off head was also analyzed. Two scenarios were considered: the rule of thumb shutoff head and the ideal shutoff head. The ideal case assumes that all kinetic energy is lost due to friction. The rule of thumb case assumes that half of the kinetic energy is lost due to friction. The ideal shutoff head was 48.3% higher than the experimental shutoff head, while the rule of thumb shutoff head was within 3.3% of the experimental shutoff head. This suggests good agreement between the experimental and rule of thumb shutoff head.

Uncertainties were calculated for all parameters. The largest source of error for all calculated parameters was the precision error from time. This was due to the reaction time of the stopwatch operator. To reduce this uncertainty, a calibrated digital flow meter should be used to measure the flow rate.

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1 Nomenclature

Symbol	Description	Units
b	Blade height at the exit	m
β_2	Blade angle	rad
D_1	Pump suction line diameter	m
D_2	Pump discharge line diameter	m
g	Gravitational acceleration	m/s ²
H	Stagnation head	m
H'_{ideal}	Ideal shutoff head	m
H_t	Transducer head	m
H'_{thumb}	Rule of thumb shutoff head	m
\dot{m}	Mass flow rate of water	kg/s
N	Number of pump impeller blades	-
ΔP	Pressure change	Pa
Q	Volumetric flow rate	m ³ /s
r_2	Impeller radius	m
t	Time	s
T	Torque exerted by the pump impeller	Nm
U	Impeller tip speed	m/s
v_{2r}	Radial component of the exit velocity	m/s
$v_{2\theta}$	Tangential component of the exit velocity	m/s
w	Blade width	m
η	Pump efficiency	%

ρ	Density of water	kg/m ³
Ω	Pump impeller angular speed	rad/s
Φ	Flow coefficient	-
Ψ	Head coefficient	-

2 Introduction

The focus of this lab was to analyze the performance of centrifugal pumps at different operating speeds, valve orientations, along with both parallel and series configurations. The data obtained from the lab testing is used to determine head, flow, and efficiency. The lab results are compared with the manufacturer provided data sheets and theoretical calculations. The parameters measured during each test are the moment arm, time to collect water (200lbs), and the transducer reading. The goal of determining the performance of different pumping configurations is useful for understanding how to achieve optimal efficiency in a piping system, and the effect certain factors (i.e., pump speed) have.

3 Procedure

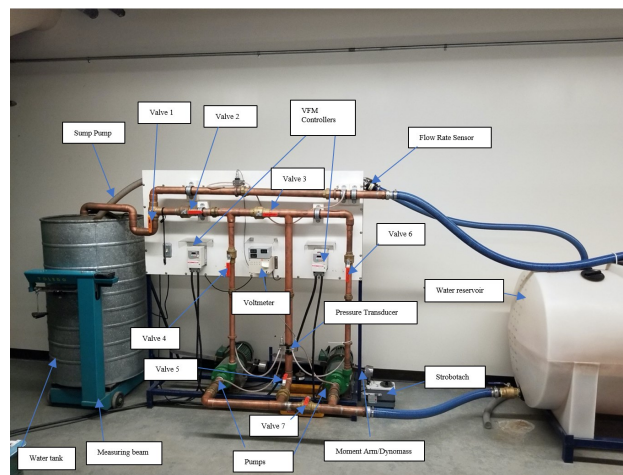


Figure 1: Experimental setup for the pump system

3.1 Equipment

- Pump system that can be configured as a single pump or as two pumps in series or parallel operation. This is the system being studied.
- Stopwatch to measure time to fill a container
- Stroboscope to verify pump rotational speed
- Mass scale to measure 200 lbs of water
- 1.401 kg mass to measure moment arm (torque) of motor
- Pressure transducers to measure pump pressure differential

3.2 Procedure

1. The pump system will be tested in single pump configuration at 1800, 2700, and 3600 RPM. The speed of the pump will be verified using the strobotach.
2. At each pump speed, the following measurements at four different pressures will be recorded (shutoff, full open, and two other equally spaced intermediate settings):
 - i. Time to fill a container to 200 lbs of water will be recorded three times.
 - ii. Pressure transducer(s) voltage output will be recorded.
 - iii. The moment arm and dyno mass will be recorded to determine the torque produced by the motor.

Make sure to clear the water container after recording a time with the sump pump. A total of 36 time measurements will be recorded for the single pump configuration.

3. Item (2) will be repeated for parallel and series system configurations with both pumps set at 2700 RPM (neglect the 1800 and 3600 RPM configurations for parallel and series systems). Neglect recording the moment arm and ensure the output of both pressure transducers are recorded. A total of 24 time measurements will be recorded for the parallel and series configurations.

4 Theory

4.1 Euler's Turbomachinery Equations

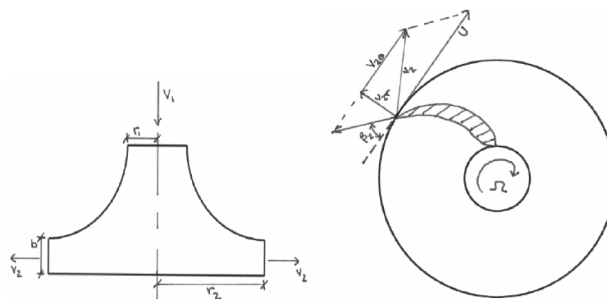


Figure 2: Impeller Diagram

Using the following assumptions:

1. Viscous effects are negligible
2. Velocity profile is uniform at the exit

3. All work done by the pump is transferred to the fluid

Then by conservation of angular momentum,

$$T = \dot{m} r_2 v_{2\theta} \quad (1)$$

where T is the torque, \dot{m} is the mass flow rate, r_2 is the radius of the impeller, and $v_{2\theta}$ is the tangential velocity at the exit. From Figure 2, we can see that

$$v_{2\theta} = U - v_{2r} \cot \beta_2 \quad (2)$$

where U is the tip speed, v_{2r} is the radial velocity at the exit, and β_2 is the blade angle. Combining (1) and (2), we get

$$T = \dot{m} r_2 (U - v_{2r} \cot \beta_2) \quad (3)$$

By Assumption 3, we can say that

$$T\Omega = \dot{m}gH \quad (4)$$

where Ω is the impeller angular velocity, g is the acceleration due to gravity, and H is the total head rise across the pump. By combining (3), (4), and the kinematic relationship $U = r_2\Omega$, we get

$$\frac{Hg}{U^2} = 1 - \frac{v_{2r}}{U} \cot \beta_2 \quad (5)$$

Defining the head coefficient Ψ and the flow coefficient Φ as

$$\Psi = \frac{Hg}{U^2}, \quad \Phi = \frac{v_{2r}}{U} \quad (6)$$

then (5) becomes

$$\Psi = 1 - \Phi \cot \beta_2 \quad (7)$$

also, v_{2r} can be expressed by the continuity equation as

$$v_{2r} = \frac{Q}{A_2} = \frac{Q}{b(2\pi r_2 - Nw)} \quad (8)$$

where Q is the flow rate, A_2 is the area of the exit, b is the blade height at the exit, N is the number of blades, and w is the width of the blade.

4.2 Shutoff Head

The ideal shutoff head can be obtained as

$$H'_{\text{ideal}} = \frac{U^2}{g} \quad (9)$$

by setting $\Phi = 0$ in (7). If it is assumed that all kinetic energy is lost due to friction, then

$$\frac{KE}{\text{unit weight}} = \frac{U^2}{2g}$$

where KE is the kinetic energy. Therefore, the rule of thumb for the shutoff head is

$$\begin{aligned} H'_{\text{thumb}} &= \frac{U^2}{g} - \frac{U^2}{2g} \\ H'_{\text{thumb}} &= \frac{U^2}{2g} = \frac{1}{2} H'_{\text{ideal}} \end{aligned} \quad (10)$$

4.3 Affinity Laws

For large Reynolds numbers, the flow is dynamically similar in geometrically similar machines when the flow and head coefficients are the same. For geometrically similar machines operating at different conditions (i) and (ii) such that the head and flow coefficients are the same, the following relationships hold:

$$\begin{aligned} \Psi_i &= \Psi_{ii} \\ \frac{H_i}{U_i^2} &= \frac{H_{ii}}{U_{ii}^2} \end{aligned}$$

so,

$$\frac{H_i}{H_{ii}} = \left(\frac{U_i}{U_{ii}} \right)^2 \approx \left(\frac{D_i \Omega_i}{D_{ii} \Omega_{ii}} \right)^2 \quad (11)$$

where D is the diameter of the impeller. Also,

$$\begin{aligned} \Phi_i &= \Phi_{ii} \\ \frac{v_{2ri}}{U_i} &= \frac{v_{2rii}}{U_{ii}} \end{aligned} \quad (12)$$

$$\frac{b_i}{D_i} = \frac{b_{ii}}{D_{ii}} \quad (13)$$

For geometrically similar machines, the ratios of b/D are the same. It can be shown that

$$\frac{Q_i}{Q_{ii}} = \frac{\Omega_i}{\Omega_{ii}} \left(\frac{D_i}{D_{ii}} \right)^3 \quad (14)$$

4.4 Transducer Head Adjustment

In this experiment, since the inlet and outlet pipe diameters are different, the transducer head, H_t , must be corrected for flow kinetic energy to give the pump stagnation head, H , as

$$\begin{aligned} H &= H_t + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \\ H &= H_t + \frac{v_2^2}{2g} \left(1 - \left(\frac{v_1}{v_2} \right)^2 \right) \end{aligned} \quad (15)$$

where H_t is the transducer head, v_1 is the velocity at the inlet, and v_2 is the velocity at the outlet. The volume flowrate through the system can be written as

$$Q = \frac{v_2 \pi D_2^2}{4} \quad (16)$$

and with the pipe diameters D_1 at the inlet and D_2 at the outlet, mass conservation requires that

$$v_2 \pi D_2^2 = v_1 \pi D_1^2 \quad (17)$$

With (16) and (17), (15) becomes

$$H = H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left(1 - \left(\frac{D_2}{D_1} \right)^4 \right) \quad (18)$$

4.5 Pumps in Series and Parallel

When pumps are connected in series, the total head is the sum of the individual heads, and the flow rate is the same as the individual flow rates.

$$H_{\text{series},t} = H_{1,t} + H_{2,t} \quad (19)$$

$$Q_{\text{series}} = Q_1 = Q_2 \quad (20)$$

When pumps are connected in parallel, the total head is the average of the individual heads, and the total flow rate is the sum of the individual flow rates.

$$H_{\text{parallel},t} = \frac{H_{1,t} + H_{2,t}}{2} \quad (21)$$

$$Q_{\text{parallel}} = Q_1 + Q_2 \quad (22)$$

4.6 Pump Efficiency

The pump efficiency is defined as

$$\begin{aligned} \eta &= \frac{\text{useful work done}}{\text{total work done}} \\ &= \frac{\rho g Q H}{T \Omega} \end{aligned} \quad (23)$$

5 Results and Discussion

5.1 Main Results

Table 2 shows the experimental and manufacturer pump performance summary. The pump configuration, valve configuration, pump speed, volumetric flow rate, corrected head, head coefficient, and flow coefficient are shown. Sample calculations for the single pump in Appendix A. Parallel and series pump performance analysis is shown in Appendix B.

Table 2: Experimental and Manufacturer Pump Performance Summary

Pump Config.	Valve Config.	Pump Speed, Ω (RPM)	Volumetric Flow Rate, Q ($\text{m}^3 \text{s}^{-1}$)	Corrected Head, H (m)	Head Coefficient, Ψ	Flow Coefficient, Φ
Single	Fully open	1800	0.003159	2.91	0.275	0.12
Single	Partial 1	1800	0.003003	3.16	0.300	0.11
Single	Partial 2	1800	0.001492	5.09	0.482	0.05
Single	Closed	1800	-	5.49	0.520	0.00
Single	Fully open	2700	0.004858	5.91	0.249	0.12

Single	Partial 1	2700	0.004749	6.31	0.265	0.12
Single	Partial 2	2700	0.002911	10.6	0.445	0.07
Single	Closed	2700	-	12.4	0.522	0.00
Single	Fully open	3600	0.006474	9.72	0.230	0.12
Single	Partial 1	3600	0.006240	11.2	0.266	0.11
Single	Partial 2	3600	0.002454	20.8	0.491	0.05
Single	Closed	3600	-	21.8	0.515	0.00
Single	Manufacturer	1800	0.0040	2.58	0.244	0.15
Single	Manufacturer	1800	0.0033	4.14	0.392	0.12
Single	Manufacturer	1800	0.0026	4.96	0.470	0.10
Single	Manufacturer	1800	0.0020	5.40	0.511	0.07
Single	Manufacturer	2700	0.0060	5.77	0.243	0.15
Single	Manufacturer	2700	0.0050	9.12	0.384	0.12
Single	Manufacturer	2700	0.0040	11.0	0.463	0.10
Single	Manufacturer	2700	0.0030	12.1	0.509	0.07
Single	Manufacturer	2700	0.0020	12.8	0.539	0.05
Single	Manufacturer	3600	0.0080	10.3	0.244	0.15
Single	Manufacturer	3600	0.0067	16.1	0.381	0.12
Single	Manufacturer	3600	0.0054	19.2	0.454	0.10
Single	Manufacturer	3600	0.0040	21.6	0.511	0.07
Single	Manufacturer	3600	0.0027	22.8	0.540	0.05
Parallel	Fully open	2700	0.007266	11.3	0.474	0.18
Parallel	Partial 1	2700	0.006283	11.7	0.492	0.15
Parallel	Partial 2	2700	0.002217	12.5	0.528	0.05

Parallel	Closed	2700	-	12.5	0.526	0.00
Series	Fully open	2700	0.005584	9.15	0.385	0.14
Series	Partial 1	2700	0.005449	10.4	0.439	0.13
Series	Partial 2	2700	0.002983	21.9	0.921	0.07
Series	Closed	2700	-	24.9	1.048	0.00

5.2 Single Pump Performance

5.2.1 Head vs. Flow Rate

Using the data from Table 2, the head vs. flow rate for the experimental single pump data was plotted in Figure 3. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix A.

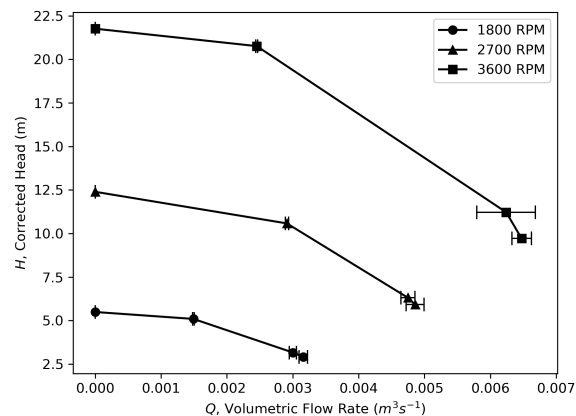


Figure 3: Single pump experimental head vs. flow rate plot.

5.2.2 Head Coefficient vs. Flow Coefficient

The head coefficient and flow coefficient for the experimental, manufacturer, and ideal pump data are shown in Figure 4. The ideal pump data was calculated from ideal pump equation (Eq. 7) using the impeller angle. Impeller angle was determined visually, shown in Appendix C. Sample calculations for the experimental and manufacturer head and flow coefficients are shown in Appendix A.

For the single experimental pump, the head and flow coefficients appear to fall onto the same curve. A non-linear relationship is observed, where the head coefficient decreases as the flow coefficient increases. Most points of different speeds are within error bars of each other. This suggests that the head and flow coefficients are independent of pump speed. The largest source of error was the precision error caused by the reaction time of the stopwatch operator.

The manufacturer head and flow coefficients also appear to fall onto the same, but different than the experimental, curve. The manufacturer head coefficient is higher than the experimental head coefficient for the same flow coefficient. This suggests that the manufacturer pump is more efficient than the experimental pump.

The ideal head and flow coefficients are shown as a straight line. The ideal head coefficient is much higher than the experimental and manufacturer head coefficient for the same flow coefficient. The ideal pump neglects all losses and is overly optimistic. The linear trend does not follow the experimental and manufacturer data.

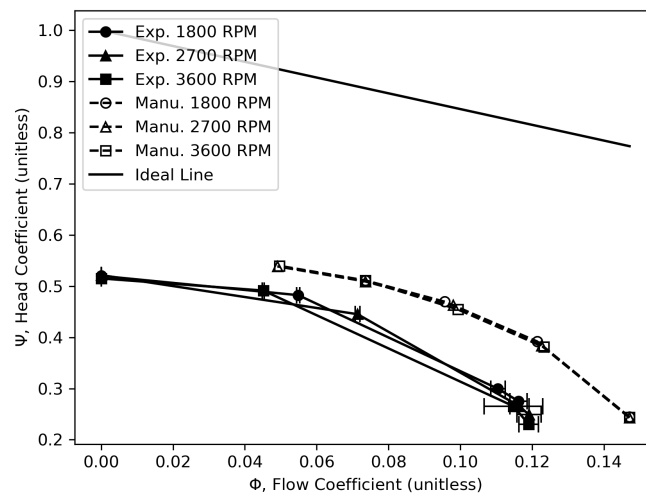


Figure 4: Single pump experimental, manufacturer, and ideal head coefficient vs. flow coefficient plot.

5.2.3 Ideal and Rule of Thumb Shutoff Head

Table 3: Ideal, rule of thumb, and experimental shutoff head for the single pump at 3600 RPM.

Ideal Shutoff Head, H'_{ideal}	Rule of Thumb Shutoff Head, H'_{thumb}	Experimental Shutoff Head, H'_{exp}
(m)	(m)	(m)
42.2	21.1	21.8

The ideal and rule of thumb shutoff head for the single pump at 3600 RPM was calculated and compared to the experimental shutoff head (single closed valve configuration). The ideal and rule of thumb shutoff head was calculated using Eq. 9 and 10. The experimental shutoff head was taken from Table 2. Sample calculations for the ideal and rule of thumb shutoff head are shown in Appendix D and the experimental shutoff head in Appendix A.

The ideal shutoff head assumes all kinetic energy is lost too friction and is overly optimistic. The error was calculated to be 48.3%. This suggests poor agreement between the experimental and ideal shutoff head.

The rule of thumb shutoff head is a more realistic estimate and is within 3.3% of the experimental shutoff head. This suggests good agreement between the experimental and rule of thumb shutoff head.

5.3 Parallel Pump Performance

The head vs. flow rate for the experimental parallel pump data was plotted in Figure 5. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix B.

The theoretical curve was calculated using the single pump data. The theoretical head and flow was calculated using Eq. 21 and 22. The theoretical head and flow was then plotted against the experimental head and flow.

The curves have poor agreement. The theoretical head was higher than the experimental head for the same flow. In addition, a higher flow was observed at the full open valve configuration. This suggests the theoretical model does not accurately predict the parallel pump performance.

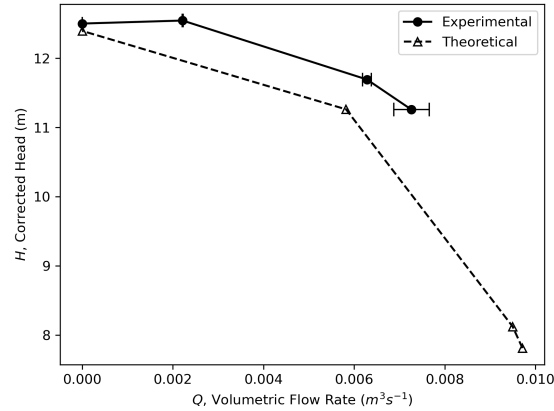


Figure 5: Parallel pump experimental and theoretical head vs. flow rate plot.

5.4 Series Pump Performance

The head vs. flow rate for the experimental series pump data was plotted in Figure 6. Error bars were shown for the volumetric flow, where time was the biggest contributor to error. This is likely due to the reaction time of the stopwatch operator causing a high precision error. The error bars for the head were much smaller and omitted for visual clarity. Calculations for the error bars are shown in Appendix B.

The theoretical curve was calculated using the single pump data. The theoretical head and flow was calculated using Eq. 19 and 20. The theoretical head and flow was then plotted against the experimental head and flow.

The curves have some agreement. The theoretical head was lower than the experimental head for the same flow. In addition, a higher flow was observed at the full open valve configuration. These deviations were smaller than the parallel pump. This suggests the theoretical model does somewhat accurately predicts the series pump performance.

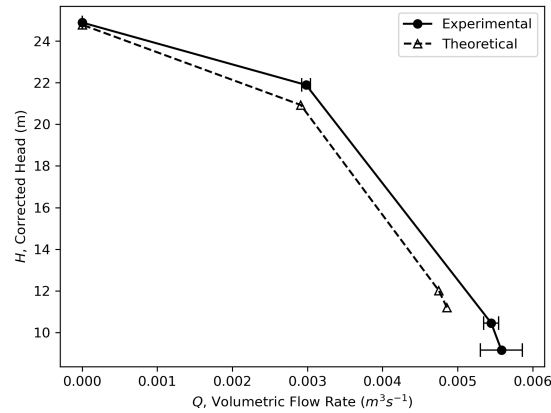


Figure 6: Series pump experimental and theoretical head vs. flow rate plot.

5.5 Geometric Similarity in Manufacturer's Specifications

The manufacturer's specifications are for geometrically dissimilar pumps. To investigate the effects of assuming similar geometry, two plots were produced. Sample calculations for the head and flow coefficients are shown in Appendix E.

The first plot, Figure 7, shows the head coefficient vs. flow coefficient for geometrically similar pumps where impeller blade height, b , and impeller width, w , were scaled by the impeller diameter, D . An observation is that the head coefficient decreases for a given flow coefficient as the impeller diameter decreases.

The second plot, Figure 8, shows the head coefficient vs. flow coefficient for geometrically dissimilar pumps where impeller blade height, b , and impeller width, w , were not scaled by the impeller diameter, D . An observation is that the head coefficient decreases for a given flow coefficient as the impeller diameter decreases.

The head and flow coefficients for the geometrically similar pumps fall only somewhat collapse onto the same curve. The head and flow coefficients for the geometrically dissimilar pumps fall more so onto the same curve. This suggests that the pumps are geometrically dissimilar, which is consistent with the manufacturer's specifications.

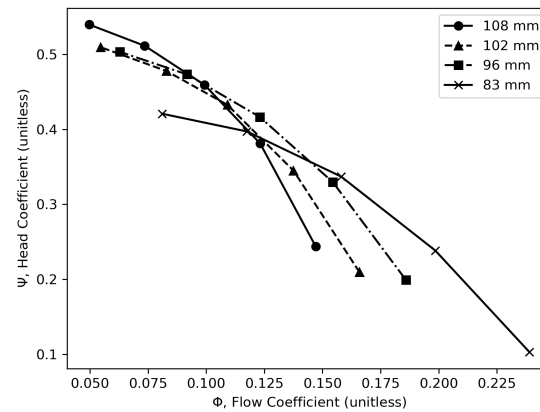


Figure 7: Head coefficient vs. flow coefficient for geometrically similar pumps.

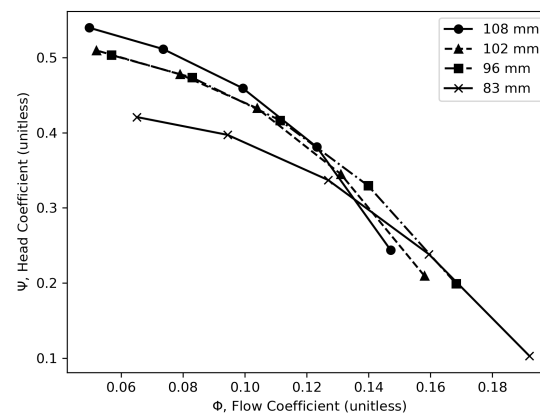


Figure 8: Head coefficient vs. flow coefficient for geometrically dissimilar pumps.

5.6 Pump Efficiency

The pump efficiencies for the experimental data was calculated in Appendix F. The pump efficiencies for the manufacturer data was given in the manufacturer's specifications. The plot of the experimental and manufacturer pump efficiencies is shown in Figure 9.

The pump was most efficient when operating at 2700 RPM in a partially closed valve configuration. The pump was least efficient when operating at 1800 RPM in a partially closed valve configuration.

The actual pump efficiency was lower than the manufacturer pump efficiency for all pump speeds. This could be due to degradation of the pump over time as the setup has been used for many years.

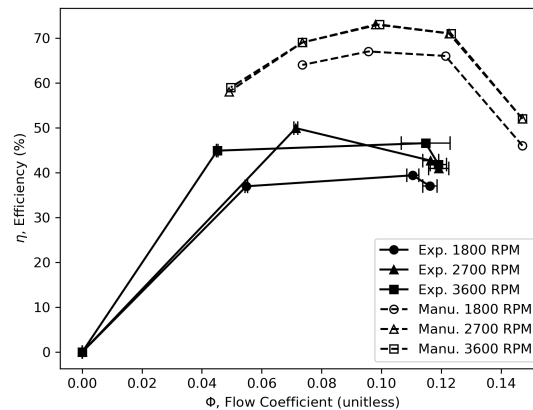


Figure 9: Single pump experimental and manufacturer efficiency plot.

5.7 Elevation Effects

The vertical elevation of the pump lines have an effect on the head of the pump. This variation was not accounted for in the modelling. This was because the vertical displacements between where the pressure was measured was small (< 0.5 m). It was assumed that this variation was negligible.

6 Conclusion

The focus of this lab was to investigate and understand the performance of centrifugal pumps. The pump performance was studied in single, parallel, and series configurations. The pump efficiency was also calculated.

The head and flow coefficients were plotted for the experimental data. The head and flow coefficients for the single pump fell onto the same curve. The manufacturer's head and flow coefficients also fell onto a single, but different, curve. The manufacturer head coefficient was higher than the experimental head coefficient for the same flow coefficient. The ideal head and flow coefficients were linear and did not agree with the experimental and manufacturer data.

The rule of thumb shutoff head was within 3.3% of the experimental shutoff head. This suggests good agreement between the experimental and rule of thumb shutoff head. The ideal shutoff head was 48.3% higher than the experimental shutoff head. This suggests poor agreement between the experimental and ideal shutoff head.

The parallel pump performance had poor agreement between the theoretical and experimental data. The theoretical head was higher than the experimental head for the same flow. The series pump performance had some agreement between the theoretical and experimental data. The theoretical head was lower than the experimental head for the same flow.

The head and flow coefficients for the geometrically similar pumps fell only somewhat collapse

onto the same curve. The head and flow coefficients for the geometrically dissimilar pumps fell more so onto the same curve. This suggests that the pumps are geometrically dissimilar, which is consistent with the manufacturer's specifications.

The pump efficiencies were calculated for the experimental and manufacturer data. The pump was most efficient when operating at 2700 RPM in a partially closed valve configuration. The pump was least efficient when operating at 1800 RPM in a partially closed valve configuration. The actual pump efficiency was lower than the manufacturer pump efficiency for all pump speeds.

The vertical elevation of the pump lines have an effect on the head of the pump. This variation was not accounted for in the modelling. It was assumed that this variation was negligible.

6.1 Technical Recommendations

The largest source of error for all calculated parameters was the precision error from time. This was due to the reaction time of the stopwatch operator. To reduce this error, a digital flow meter should be used to measure the flow rate. This would reduce the error in the flow rate and the head.

The pressure transducer was assumed to have no precision error since calibration was not performed and only one measurement was taken. To account for this error, the pressure transducer should be calibrated and multiple measurements should be taken.

A Appendix: Single Pump Analysis

A.1 Single Pump Discharge and Heads

Table A.4: Summary of single pump experimental discharge and heads for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Transducer Output, V_t (V)	Time to collect water (s)			Nominal Time, t (s)	Volumetric Flow, Q ($\text{m}^3 \text{s}^{-1}$)	Transducer Head, H_t (m)	Corrected Head, H (m)
			Trial 1	Trial 2	Trial 3				
Fully open	1800	0.75	28.55	28.99	28.79	28.78	0.003159	2.64	2.91
Partial 1	1800	0.83	30.14	30.51	30.16	30.27	0.003003	2.92	3.16
Partial 2	1800	1.43	61.12	60.80	60.86	60.93	0.001492	5.04	5.09
Closed	1800	1.56	-	-	-	-	-	5.49	5.49
Fully open	2700	1.50	18.66	18.93	18.55	18.71	0.004858	5.28	5.91
Partial 1	2700	1.62	19.23	19.23	18.96	19.14	0.004749	5.70	6.31
Partial 2	2700	2.94	31.15	31.30	31.22	31.22	0.002911	10.4	10.6
Closed	2700	3.52	-	-	-	-	-	12.4	12.4
Fully open	3600	2.44	14.13	13.93	14.06	14.04	0.006474	8.59	9.72
Partial 1	3600	2.89	15.04	14.35	14.31	14.57	0.006240	10.2	11.2
Partial 2	3600	5.85	37.01	37.04	37.07	37.04	0.002454	20.6	20.8
Closed	3600	6.18	-	-	-	-	-	21.8	21.8

Sample calculations are evaluated for the fully open configuration at 1800 RPM. Starting with time,

$$t = \frac{\sum t_i}{n} = \frac{28.55 \text{ s} + 28.99 \text{ s} + 28.79 \text{ s}}{3} = 28.78 \text{ s}$$

The volumetric flow rate was calculated using water which had a mass of $m = 200 \text{ lb} = 90.7 \text{ kg}$ and a density of $\rho = 998 \text{ kg m}^{-3}$. Then,

$$Q = \frac{m}{\rho t} = \frac{90.7 \text{ kg}}{998 \text{ kg m}^{-3} \times 28.78 \text{ s}} = 0.003159 \text{ m}^3 \text{ s}^{-1}$$

The transducer head was found by

$$H_t = \frac{\Delta P}{\rho g} = \frac{0.75 \text{ V} \times 5 \text{ psi V}^{-1} \times 6894.76 \text{ Pa psi}^{-1}}{998 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2}} = 2.64 \text{ m}$$

The corrected head was found using inlet diameter, $D_1 = 0.0508 \text{ m}$, and outlet diameter, $D_2 =$

0.0381 m, by

$$\begin{aligned}
 H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\
 &= 2.64 \text{ m} + \frac{8 \times (0.003159 \text{ m}^3 \text{ s}^{-1})^2}{\pi^2 \times (0.0381 \text{ m})^4 \times 9.81 \text{ m s}^{-2}} \left[1 - \left(\frac{0.0381 \text{ m}}{0.0508 \text{ m}} \right)^4 \right] \\
 &= 2.91 \text{ m}
 \end{aligned}$$

A.1.1 Single Pump Discharge and Head Uncertainty

Table A.5: Single pump experimental discharge and head uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed	Time STDEV, S_t	Time Precision, P_t	Time Bias, B_t	Time Un- certainty, δ_t	Flow Un- certainty, δ_Q	Transducer Uncer- tainty, $\delta_{V_{tr}}$	Head Uncertainty, δ_{H_t}	Corrected Head Un- certainty, δ_H
	(RPM)	(s)	(\pm s)	(\pm s)	(\pm s)	($\pm \text{m}^3 \text{s}^{-1}$)	($\pm \text{V}$)	($\pm \text{m}$)	($\pm \text{m}$)
Fully open	1800	0.2203	0.5473	0.20	0.58	6.40E-05	0.01	0.04	0.04
Partial 1	1800	0.2081	0.5169	0.20	0.55	5.50E-05	0.01	0.04	0.04
Partial 2	1800	0.1701	0.4225	0.20	0.47	1.14E-05	0.01	0.04	0.04
Closed	1800	-	-	-	-	-	0.01	0.04	0.04
Fully open	2700	0.1955	0.4857	0.20	0.53	1.36E-04	0.01	0.04	0.05
Partial 1	2700	0.1559	0.3872	0.20	0.44	1.08E-04	0.01	0.04	0.04
Partial 2	2700	0.0751	0.1864	0.20	0.27	2.55E-05	0.01	0.04	0.04
Closed	2700	-	-	-	-	-	0.01	0.04	0.04
Fully open	3600	0.1015	0.2521	0.20	0.32	1.48E-04	0.01	0.04	0.06
Partial 1	3600	0.4104	1.0195	0.20	1.04	4.45E-04	0.01	0.04	0.2
Partial 2	3600	0.0300	0.0745	0.20	0.21	1.41E-05	0.01	0.04	0.04
Closed	3600	-	-	-	-	-	0.01	0.04	0.04

Sample calculations are evaluated for the fully open configuration at 1800 RPM.

First, time standard deviation was calculated with STDEV.S in Excel. A confidence of 95% ($\alpha/2 = 0.025$) was used to calculate the t-distribution value ($\nu = 3 - 1 = 2$) with T.INV.T in Excel. This gave a value of $t_{\alpha/2, \nu} = 4.3027$. The time precision was calculated by

$$\begin{aligned}
 P_t &= t_{\alpha/2, \nu} \times \frac{S_t}{\sqrt{n}} \\
 &= 4.3027 \times \frac{0.2203 \text{ s}}{\sqrt{3}} \\
 &= 0.5473 \text{ s}
 \end{aligned}$$

The time bias was assumed to be the reaction time of the operator, which was estimated to be 0.20 s. The time total uncertainty was calculated by

$$\begin{aligned}\delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.5473 \text{ s})^2 + (0.20 \text{ s})^2} \\ &= 0.58 \text{ s}\end{aligned}$$

The flow uncertainty was calculated by propagation of error. The function for flow is

$$Q = \frac{m}{\rho t}$$

This is the special purely multiplicative case of the general formula for error propagation. Assuming the mass and density errors are negligible, the flow uncertainty was calculated by

$$\begin{aligned}\delta_Q &= Q \sqrt{\left(\frac{\delta_t}{t}\right)^2} \\ &= Q \left| \frac{\delta_t}{t} \right| \\ &= 0.003159 \text{ m}^3 \text{ s}^{-1} \left| \frac{0.58 \text{ s}}{28.78 \text{ s}} \right| \\ &= 6.40 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}\end{aligned}$$

Transducer bias was assumed to be 0.01 V. Transducer precision was not considered since calibration was not performed. So

$$\delta_{V_{tr}} = 0.01 \text{ V}$$

Next, the head uncertainty was calculated by propagation of error. The function for head is

$$H_t = \frac{\Delta P}{\rho g} = \frac{V_{tr} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, we have the special purely multiplicative case

for error propagation. The head uncertainty was calculated by

$$\begin{aligned}
 \delta_{H_t} &= H_t \sqrt{\left(\frac{\delta_{V_{tr}}}{V_{tr}}\right)^2} \\
 &= H_t \left| \frac{\delta_{V_{tr}}}{V_{tr}} \right| \\
 &= 2.64 \text{ m} \left| \frac{0.01 \text{ V}}{0.75 \text{ V}} \right| \\
 &= 0.04 \text{ m}
 \end{aligned}$$

Lastly, the function for corrected head is

$$\begin{aligned}
 H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\
 \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] = \frac{2(H - H_t)}{Q} \\
 \frac{\partial H}{\partial H_t} &= 1
 \end{aligned}$$

Then, the corrected head uncertainty was determined by

$$\begin{aligned}
 \delta_H &= \sqrt{\left(\frac{\partial H}{\partial Q}\right)^2 \delta_Q^2 + \left(\frac{\partial H}{\partial H_t}\right)^2 \delta_{H_t}^2} \\
 &= \sqrt{\left(\frac{2(H - H_t)}{Q}\right)^2 \delta_Q^2 + \delta_{H_t}^2} \\
 &= \sqrt{\left(\frac{2(2.91 \text{ m} - 2.64 \text{ m})}{0.003159 \text{ m}^3 \text{ s}^{-1}}\right)^2 (6.40 \times 10^{-5} \text{ m}^3 \text{ s}^{-1})^2 + (0.04 \text{ m})^2} \\
 &= 0.04 \text{ m}
 \end{aligned}$$

Table A.6: Single pump experimental head and flow coefficients for 1800 RPM, 2700 RPM, and 3600 RPM

Configuration	Pump Speed (RPM)	Volumetric Flow, Q ($\text{m}^3 \text{s}^{-1}$)	Corrected Head, H (m)	Tip Speed, U (m s^{-1})	Radial Exit Velocity, v_{2r} (m s^{-1})	Head Coefficient, Ψ	Flow Coefficient, Φ
Fully open	1800	0.003159	2.91	10.2	1.2	0.275	0.12
Partial 1	1800	0.003003	3.16	10.2	1.1	0.300	0.11
Partial 2	1800	0.001492	5.09	10.2	0.6	0.482	0.05
Closed	1800	-	5.49	10.2	0.0	0.520	0.00
Fully open	2700	0.004858	5.91	15.3	1.8	0.249	0.12
Partial 1	2700	0.004749	6.31	15.3	1.8	0.265	0.12
Partial 2	2700	0.002911	10.6	15.3	1.1	0.445	0.07
Closed	2700	-	12.4	15.3	0.0	0.522	0.00
Fully open	3600	0.006474	9.72	20.4	2.4	0.230	0.12
Partial 1	3600	0.006240	11.2	20.4	2.3	0.266	0.11
Partial 2	3600	0.002454	20.8	20.4	0.9	0.491	0.05
Closed	3600	-	21.8	20.4	0.0	0.515	0.00

A.2 Single Pump Head and Flow Coefficients

Sample calculations are evaluated for the fully open configuration at 1800 RPM. The tip speed was calculated using the impeller radius, $r_2 = 0.108/2 = 0.054 \text{ m}$, by

$$\begin{aligned}
 U &= r_2 \Omega \\
 &= 0.054 \text{ m} \times 1800 \text{ RPM} \times \left(\frac{2\pi}{60} \text{ RPM}^{-1} \right) \\
 &= 10.2 \text{ m s}^{-1}
 \end{aligned}$$

The radial exit velocity was calculated using the impeller diameter, the blade height at exit, $b = 0.009 \text{ m}$, blade width at exit, $w = 0.0085 \text{ m}$, and the number of blades $N = 5$, by

$$\begin{aligned}
 v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\
 &= \frac{0.003159 \text{ m}^3 \text{s}^{-1}}{0.009 \text{ m} \times (2\pi \times 0.054 \text{ m} - 5 \times 0.0085 \text{ m})} \\
 &= 1.2 \text{ m s}^{-1}
 \end{aligned}$$

The head coefficient was found by

$$\begin{aligned}\Psi &= \frac{Hg}{U^2} \\ &= \frac{2.91 \text{ m} \times 9.81 \text{ m s}^{-2}}{(10.2 \text{ m s}^{-1})^2} \\ &= 0.275\end{aligned}$$

The flow coefficient was found by

$$\begin{aligned}\Phi &= \frac{v_{2r}}{U} \\ &= \frac{1.2 \text{ m s}^{-1}}{10.2 \text{ m s}^{-1}} \\ &= 0.12\end{aligned}$$

A.2.1 Single Pump Head and Flow Coefficient Uncertainty

Table A.7: Single pump experimental head and flow coefficient uncertainties for 1800 RPM, 2700 RPM, and 3600 RPM

Valve Configuration	Pump Speed	Flow Uncertainty, δ_Q	Head Uncertainty, δ_H	Radial Exit Velocity Uncertainty, $\delta_{v_{2r}}$	Head Coefficient Uncertainty, δ_Ψ	Flow Coefficient Uncertainty, δ_Φ
	(RPM)	($\pm \text{m}^3 \text{s}^{-1}$)	($\pm \text{m}$)	($\pm \text{m s}^{-1}$)	(\pm)	(\pm)
Fully open	1800	6.40E-05	0.04	0.024	0.003	0.0024
Partial 1	1800	5.50E-05	0.04	0.021	0.003	0.0020
Partial 2	1800	1.14E-05	0.04	0.0043	0.003	0.0004
Closed	1800	-	0.04	-	0.003	-
Fully open	2700	1.36E-04	0.05	0.051	0.002	0.0033
Partial 1	2700	1.08E-04	0.04	0.040	0.002	0.0027
Partial 2	2700	2.55E-05	0.04	0.010	0.001	0.00063
Closed	2700	-	0.04	-	0.001	-
Fully open	3600	1.48E-04	0.06	0.056	0.001	0.0027
Partial 1	3600	4.45E-04	0.2	0.17	0.004	0.0082
Partial 2	3600	1.41E-05	0.04	0.0053	0.001	0.00026
Closed	3600	-	0.04	-	0.001	-

Sample calculations are evaluated for the fully open configuration at 1800 RPM. The RPM was measured by the stroboscope. It was assumed that pump speed uncertainty was negligible. The radial exit velocity is a function of

$$v_{2r} = \frac{Q}{b(2\pi r_2 - Nw)}$$

Assuming the blade height and width errors are negligible, the radial exit velocity uncertainty was

calculated by

$$\begin{aligned}
 \delta_{v_{2r}} &= v_{2r} \sqrt{\left(\frac{\delta_Q}{Q}\right)^2} \\
 &= v_{2r} \left| \frac{\delta_Q}{Q} \right| \\
 &= 1.2 \text{ m s}^{-1} \left| \frac{6.40 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}}{0.003159 \text{ m}^3 \text{ s}^{-1}} \right| \\
 &= 0.024 \text{ m s}^{-1}
 \end{aligned}$$

The function for head coefficient is

$$\Psi = \frac{Hg}{U^2}$$

Assuming the gravity and tip speed errors are negligible, the head coefficient uncertainty was calculated by

$$\begin{aligned}
 \delta_\Psi &= \Psi \sqrt{\left(\frac{\delta_H}{H}\right)^2} \\
 &= \Psi \left| \frac{\delta_H}{H} \right| \\
 &= 0.275 \left| \frac{0.04 \text{ m}}{2.91 \text{ m}} \right| \\
 &= 0.003
 \end{aligned}$$

The function for flow coefficient is

$$\Phi = \frac{v_{2r}}{U}$$

since the tip speed error is negligible, this is a special purely multiplicative case of the general

formula for error propagation. The flow coefficient uncertainty was calculated by

$$\begin{aligned}
 \delta_{\Phi} &= \Phi \sqrt{\left(\frac{\delta_{v_{2r}}}{v_{2r}}\right)^2} \\
 &= \Phi \left| \frac{\delta_{v_{2r}}}{v_{2r}} \right| \\
 &= 0.12 \left| \frac{0.024 \text{ m s}^{-1}}{1.2 \text{ m s}^{-1}} \right| \\
 &= 0.0024
 \end{aligned}$$

A.3 Single Pump Manufacturer's Data

Table A.8: Single pump manufacturer's data for 1800 RPM, 2700 RPM, and 3600 RPM

Pump Speed (RPM)	Volumetric Flow, Q ($\text{m}^3 \text{s}^{-1}$)	Head, H (m)	Tip Speed, U (m s^{-1})	Radial Exit Velocity, v_{2r} (m s^{-1})	Head Coefficient, Ψ	Flow Coefficient, Φ
1800	0.0040	2.58	10.2	1.5	0.244	0.15
1800	0.0033	4.14	10.2	1.2	0.392	0.12
1800	0.0026	4.96	10.2	1.0	0.470	0.10
1800	0.0020	5.40	10.2	0.75	0.511	0.07
2700	0.0060	5.77	15.3	2.2	0.243	0.15
2700	0.0050	9.12	15.3	1.9	0.384	0.12
2700	0.0040	11.0	15.3	1.5	0.463	0.10
2700	0.0030	12.1	15.3	1.1	0.509	0.07
2700	0.0020	12.8	15.3	0.75	0.539	0.05
3600	0.0080	10.3	20.4	3.0	0.244	0.15
3600	0.0067	16.1	20.4	2.5	0.381	0.12
3600	0.0054	19.2	20.4	2.0	0.454	0.10
3600	0.0040	21.6	20.4	1.5	0.511	0.07
3600	0.0027	22.8	20.4	1.0	0.540	0.05

Sample calculations are evaluated 1800 RPM with a volumetric flow rate of $Q = 0.0040 \text{ m}^3 \text{s}^{-1}$.

The tip speed was calculated using the impeller radius, $r_2 = 0.108/2 = 0.054$ m, by

$$\begin{aligned}U &= r_2 \Omega \\&= 0.054 \text{ m} \times 1800 \text{ RPM} \times \left(\frac{2\pi}{60} \text{ RPM}^{-1} \right) \\&= 10.2 \text{ m s}^{-1}\end{aligned}$$

The radial exit velocity was calculated using the impeller diameter, the blade height at exit, $b = 0.009$ m, blade width at exit, $w = 0.0085$ m, and the number of blades $N = 5$, by

$$\begin{aligned}v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\&= \frac{0.0040 \text{ m}^3 \text{ s}^{-1}}{0.009 \text{ m} \times (2\pi \times 0.054 \text{ m} - 5 \times 0.0085 \text{ m})} \\&= 1.5 \text{ m s}^{-1}\end{aligned}$$

The head coefficient was found by

$$\begin{aligned}\Psi &= \frac{Hg}{U^2} \\&= \frac{2.58 \text{ m} \times 9.81 \text{ m s}^{-2}}{(10.2 \text{ m s}^{-1})^2} \\&= 0.244\end{aligned}$$

The flow coefficient was found by

$$\begin{aligned}\Phi &= \frac{v_{2r}}{U} \\&= \frac{1.5 \text{ m s}^{-1}}{10.2 \text{ m s}^{-1}} \\&= 0.15\end{aligned}$$

B Appendix: Parallel and Series Pump Analysis

B.1 Parallel Experimental Pump Discharge and Head

Table B.9: Summary of experimental parallel pump discharge and heads for 2700 RPM

Valve Configuration	Transducer Output		Time to collect water			Nominal Time, t	Volumetric Flow, Q	Transducer Head, H_t			Corrected Head, H
	Pump 1	Pump 2	Trial 1	Trial 2	Trial 3			Pump 1	Pump 2	Nominal	
	(V)	(V)	(s)	(s)	(s)	(s)	(m ³ s ⁻¹)	(m)	(m)	(m)	(m)
Fully open	2.87	2.72	12.46	12.28	12.79	12.51	0.007266	10.1	9.58	9.84	11.3
Partial 1	3.08	2.96	14.49	14.49	14.42	14.47	0.006283	10.8	10.4	10.6	11.7
Partial 2	3.52	3.53	41.05	40.99	40.99	41.01	0.002217	12.4	12.4	12.4	12.5
Closed	3.55	3.55	-	-	-	-	-	12.5	12.5	12.5	12.5

Sample calculations will be shown for the fully open valve configuration. The same calculations were done for the other valve configurations. The nominal time was calculated as the average of the three trials by

$$\begin{aligned}
 t &= \frac{\sum t_i}{n} \\
 &= \frac{12.46 + 12.28 + 12.79}{3} \\
 &= 12.51 \text{ s}
 \end{aligned}$$

The volumetric flow rate was calculated using $m = 200 \text{ lb} = 90.7 \text{ kg}$ and $\rho = 998 \text{ kg m}^{-3}$ by

$$\begin{aligned}
 Q &= \frac{m}{\rho t} \\
 &= \frac{90.7 \text{ kg}}{998 \text{ kg m}^{-3} \times 12.51 \text{ s}} \\
 &= 0.007266 \text{ m}^3 \text{ s}^{-1}
 \end{aligned}$$

Next, the transducer head for pumps 1 and pump 2 were calculated by

$$\begin{aligned}
 H_t &= \frac{V_{tr} \times \text{Conversion}}{\rho g} \\
 \Rightarrow H_{t1} &= \frac{2.87 \text{ V} \times 5 \text{ psi V}^{-1} \times 6894.76 \text{ Pa psi}^{-1}}{998 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2}} \\
 &= 10.11 \text{ m} \\
 \Rightarrow H_{t2} &= \frac{2.72 \text{ V} \times 5 \text{ psi V}^{-1} \times 6894.76 \text{ Pa psi}^{-1}}{998 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2}} \\
 &= 9.58 \text{ m}
 \end{aligned}$$

The nominal head was calculated by averaging the transducer heads by

$$\begin{aligned}
 H &= \frac{\sum H_{t_i}}{n} \\
 &= \frac{10.11 + 9.58}{2} \\
 &= 9.84 \text{ m}
 \end{aligned}$$

The corrected head was calculated by

$$\begin{aligned}
 H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\
 &= 9.84 \text{ m} + \frac{8 \times (0.007266 \text{ m}^3 \text{ s}^{-1})^2}{\pi^2 \times (0.108 \text{ m})^4 \times 9.81 \text{ m s}^{-2}} \left[1 - \left(\frac{0.108 \text{ m}}{0.108 \text{ m}} \right)^4 \right] \\
 &= 11.3 \text{ m}
 \end{aligned}$$

B.1.1 Parallel Experimental Pump Discharge and Head Uncertainty

Table B.10: Parallel experimental pump time and discharge uncertainties for 2700 RPM

Valve Configu- ration	Time STDEV, S_t (s)	Time Precision, P_t (s)	Time Bias, B_t (s)	Time Uncertainty, δ_t (s)	Flow Uncertainty, δ_Q (m ³ s ⁻¹)
Fully open	0.2587	0.6425	0.2	0.67	3.91E-04
Partial 1	0.0404	0.1004	0.2	0.22	9.72E-05
Partial 2	0.0346	0.0861	0.2	0.22	1.18E-05
Closed	-	-	-	-	-

Table B.11: Parallel experimental pump head uncertainties for 2700 RPM

Valve Configuration	Transducer Uncertainty		Transducer Head Uncertainty			Corrected Head Uncertainty, δ_H (m)
	Pump 1, $\delta_{V_{t1}}$	Pump 2, $\delta_{V_{t2}}$	Pump 1, $\delta_{H_{t1}}$	Pump 2, $\delta_{H_{t2}}$	Nominal, δ_{H_t}	
	(V)	(V)	(m)	(m)	(m)	
Fully open	0.01	0.01	0.04	0.04	0.02	0.2
Partial 1	0.01	0.01	0.04	0.04	0.02	0.04
Partial 2	0.01	0.01	0.04	0.04	0.02	0.02
Closed	0.01	0.01	0.04	0.04	0.02	0.02

Sample calculations are evaluated for the fully open configuration at 2700 RPM. A 95% confidence interval was used. The standard deviation was calculated by Excel using STDEV.S. The t-distribution value was found using $\alpha/2 = 0.025$ and $\nu = 3 - 1 = 2$. Then,

$$\begin{aligned}
 P_t &= t_{\alpha/2, \nu} \times \frac{S_t}{\sqrt{n}} \\
 &= 4.303 \times \frac{0.2587 \text{ s}}{\sqrt{3}} \\
 &= 0.6425 \text{ s}
 \end{aligned}$$

The time bias was approximated to be the reaction time of the operator by

$$B_t = 0.2 \text{ s}$$

The time uncertainty was calculated by

$$\begin{aligned}
 \delta_t &= \sqrt{P_t^2 + B_t^2} \\
 &= \sqrt{(0.6425 \text{ s})^2 + (0.2 \text{ s})^2} \\
 &= 0.67 \text{ s}
 \end{aligned}$$

The flow uncertainty was calculated by

$$\begin{aligned}\delta_Q &= Q \left| \frac{\delta_t}{t} \right| \\ &= 0.007266 \text{ m}^3 \text{ s}^{-1} \frac{0.67 \text{ s}}{12.51 \text{ s}} \\ &= 3.91E-04\end{aligned}$$

The transducer uncertainty was assumed to be the resolution of the device, $\delta_{V_{tr}} = 0.01 \text{ V}$. Transducer precision error was not considered since calibration was not performed. The transducer heads were found by

$$H_t = \frac{V_{tr} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, this is the special purely multiplicative case of the general formula for error propagation. The transducer head uncertainty was calculated by

$$\begin{aligned}\delta_{H_{t1}} &= H_{t1} \sqrt{\left(\frac{\delta_{V_{tr1}}}{V_{tr1}} \right)^2} \\ &= H_{t1} \left| \frac{\delta_{V_{tr1}}}{V_{tr1}} \right| \\ &= 10.11 \text{ m} \left| \frac{0.01 \text{ V}}{2.87 \text{ V}} \right| \\ &= 0.04 \text{ m}\end{aligned}$$

The nominal transducer head uncertainty was calculated by

$$\begin{aligned}H_t &= \frac{H_{t1} + H_{t2}}{2} \\ \Rightarrow \frac{\partial H_t}{\partial H_{t1}} &= \frac{1}{2} \\ \Rightarrow \frac{\partial H_t}{\partial H_{t2}} &= \frac{1}{2}\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_{H_t} &= \frac{1}{2} \sqrt{(\delta_{H_{t1}})^2 + (\delta_{H_{t2}})^2} \\ &= \frac{1}{2} \sqrt{(0.04 \text{ m})^2 + (0.04 \text{ m})^2} \\ &= 0.02 \text{ m}\end{aligned}$$

The corrected head is found by

$$\begin{aligned}H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\ \Rightarrow \frac{\partial H}{\partial H_t} &= 1 \\ \Rightarrow \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] = \frac{2(H - H_t)}{Q}\end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_H &= \sqrt{(\delta_{H_t})^2 + \left(\frac{2(H - H_t)}{Q} \delta_Q \right)^2} \\ &= \sqrt{(0.02 \text{ m})^2 + \left(\frac{2(11.3 \text{ m} - 9.84 \text{ m})}{0.007266 \text{ m}^3 \text{ s}^{-1}} \times 3.91 \times 10^{-4} \text{ m}^3 \text{ s}^{-1} \right)^2} \\ &= 0.2 \text{ m}\end{aligned}$$

B.2 Series Experimental Pump Discharge and Head

Table B.12: Summary of experimental series pump discharge and heads for 2700 RPM

Valve Con- figuration	Transducer Output		Time to collect water			Nominal Time, t	Volumetric Flow, Q	Transducer Head, H_t			Corrected Head, H
	Pump 1	Pump 2	Trial 1	Trial 2	Trial 3			Pump 1	Pump 2	Nominal	
	(V)	(V)	(s)	(s)	(s)	(s)	(m ³ s ⁻¹)	(m)	(m)	(m)	(m)
Fully open	0.68	1.68	16.64	16.17	16.03	16.28	0.005584	2.39	5.92	8.31	9.1
Partial 1	0.9	1.84	16.71	16.58	16.76	16.68	0.005449	3.17	6.48	9.65	10.4
Partial 2	2.89	3.26	30.52	30.23	30.67	30.47	0.002983	10.2	11.5	21.7	21.9
Closed	3.52	3.55	-	-	-	-	-	12.4	12.5	24.9	24.9

Sample calculations will be shown for the fully open valve configuration. The same calculations were done for the other valve configurations. The nominal time was calculated as the average

of the three trials by

$$\begin{aligned}
 t &= \frac{\sum t_i}{n} \\
 &= \frac{16.64 + 16.17 + 16.03}{3} \\
 &= 16.28 \text{ s}
 \end{aligned}$$

The volumetric flow rate was calculated using $m = 200 \text{ lb} = 90.7 \text{ kg}$ and $\rho = 998 \text{ kg m}^{-3}$ by

$$\begin{aligned}
 Q &= \frac{m}{\rho t} \\
 &= \frac{90.7 \text{ kg}}{998 \text{ kg m}^{-3} \times 16.28 \text{ s}} \\
 &= 0.005584 \text{ m}^3 \text{ s}^{-1}
 \end{aligned}$$

Next, the transducer head for pumps 1 and pump 2 were calculated by

$$\begin{aligned}
 H_t &= \frac{V_{\text{tr}} \times \text{Conversion}}{\rho g} \\
 \Rightarrow H_{t1} &= \frac{0.68 \text{ V} \times 5 \text{ psi V}^{-1} \times 6894.76 \text{ Pa psi}^{-1}}{998 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2}} \\
 &= 2.39 \text{ m} \\
 \Rightarrow H_{t2} &= \frac{1.68 \text{ V} \times 5 \text{ psi V}^{-1} \times 6894.76 \text{ Pa psi}^{-1}}{998 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2}} \\
 &= 5.92 \text{ m}
 \end{aligned}$$

The nominal head was calculated by summing the transducer heads by

$$\begin{aligned}
 H_t &= \sum H_{t_i} \\
 &= 2.39 \text{ m} + 5.92 \text{ m} \\
 &= 8.31 \text{ m}
 \end{aligned}$$

The corrected head was calculated by

$$\begin{aligned}
 H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\
 &= 8.31 \text{ m} + \frac{8 \times (0.005584 \text{ m}^3 \text{ s}^{-1})^2}{\pi^2 \times (0.108 \text{ m})^4 \times 9.81 \text{ m s}^{-2}} \left[1 - \left(\frac{0.108 \text{ m}}{0.108 \text{ m}} \right)^4 \right] \\
 &= 9.1 \text{ m}
 \end{aligned}$$

B.2.1 Series Experimental Pump Discharge and Head Uncertainty

Table B.13: Series experimental pump time and discharge uncertainties for 2700 RPM

Valve Configuration	Time STDEV, S_t	Time Precision, P_t	Time Bias, B_t	Time Uncertainty, δ_t	Flow Uncertainty, δ_Q
	(s)	(s)	(s)	(s)	(m ³ s ⁻¹)
Fully open	0.3195	0.7938	0.2	0.82	2.81E-04
Partial 1	0.0929	0.2308	0.2	0.31	9.97E-05
Partial 2	0.2237	0.5557	0.2	0.59	5.78E-05
Closed	-	-	-	-	-

Table B.14: Series experimental pump head uncertainties for 2700 RPM

Valve Configuration	Transducer Uncertainty		Transducer Head Uncertainty			Corrected Head Uncertainty, δ_H
	Pump 1, $\delta_{V_{t1}}$	Pump 2, $\delta_{V_{t2}}$	Pump 1, $\delta_{H_{t1}}$	Pump 2, $\delta_{H_{t2}}$	Nominal, δ_{H_t}	(m)
	(V)	(V)	(m)	(m)	(m)	
Fully open	0.01	0.01	0.04	0.04	0.05	0.1
Partial 1	0.01	0.01	0.04	0.04	0.05	0.06
Partial 2	0.01	0.01	0.04	0.04	0.05	0.05
Closed	0.01	0.01	0.04	0.04	0.05	0.05

Sample calculations are evaluated for the fully open configuration at 2700 RPM. A 95% confidence interval was used. The standard deviation was calculated by Excel using STDEV.S. The

t-distribution value was found using $\alpha/2 = 0.025$ and $\nu = 3 - 1 = 2$. Then,

$$\begin{aligned} P_t &= t_{\alpha/2, \nu} \times \frac{S_t}{\sqrt{n}} \\ &= 4.303 \times \frac{0.3195 \text{ s}}{\sqrt{3}} \\ &= 0.7938 \text{ s} \end{aligned}$$

The time bias was approximated to be the reaction time of the operator by

$$B_t = 0.2 \text{ s}$$

The time uncertainty was calculated by

$$\begin{aligned} \delta_t &= \sqrt{P_t^2 + B_t^2} \\ &= \sqrt{(0.7938 \text{ s})^2 + (0.2 \text{ s})^2} \\ &= 0.82 \text{ s} \end{aligned}$$

The flow uncertainty was calculated by

$$\begin{aligned} \delta_Q &= Q \left| \frac{\delta_t}{t} \right| \\ &= 0.005584 \text{ m}^3 \text{ s}^{-1} \frac{0.82 \text{ s}}{16.28 \text{ s}} \\ &= 2.81E-04 \end{aligned}$$

The transducer uncertainty was assumed to be the resolution of the device, $\delta_{V_{tr}} = 0.01 \text{ V}$. Transducer precision error was not considered since calibration was not performed. The transducer heads were found by

$$H_t = \frac{V_{tr} \times \text{Conversion}}{\rho g}$$

Assuming density and gravity errors are negligible, this is the special purely multiplicative case of

the general formula for error propagation. The transducer head uncertainty was calculated by

$$\begin{aligned}
 \delta_{H_{t1}} &= H_{t1} \sqrt{\left(\frac{\delta_{V_{tr1}}}{V_{tr1}}\right)^2} \\
 &= H_{t1} \left| \frac{\delta_{V_{tr1}}}{V_{tr1}} \right| \\
 &= 2.39 \text{ m} \left| \frac{0.01 \text{ V}}{0.68 \text{ V}} \right| \\
 &= 0.04 \text{ m}
 \end{aligned}$$

The nominal transducer head uncertainty was calculated by

$$\begin{aligned}
 H_t &= H_{t1} + H_{t2} \\
 \Rightarrow \frac{\partial H_t}{\partial H_{t1}} &= 1 \\
 \Rightarrow \frac{\partial H_t}{\partial H_{t2}} &= 1
 \end{aligned}$$

so the uncertainty was

$$\begin{aligned}
 \delta_{H_t} &= \sqrt{(\delta_{H_{t1}})^2 + (\delta_{H_{t2}})^2} \\
 &= \sqrt{(0.04 \text{ m})^2 + (0.04 \text{ m})^2} \\
 &= 0.05 \text{ m}
 \end{aligned}$$

The corrected head is found by

$$\begin{aligned}
 H &= H_t + \frac{8Q^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\
 \Rightarrow \frac{\partial H}{\partial H_t} &= 1 \\
 \Rightarrow \frac{\partial H}{\partial Q} &= \frac{16Q}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] = \frac{2(H - H_t)}{Q}
 \end{aligned}$$

so the uncertainty was

$$\begin{aligned}\delta_H &= \sqrt{(\delta_{H_t})^2 + \left(\frac{2(H - H_t)}{Q} \delta_Q\right)^2} \\ &= \sqrt{(0.05 \text{ m})^2 + \left(\frac{2(9.1 \text{ m} - 8.31 \text{ m})}{0.005584 \text{ m}^3 \text{ s}^{-1}} \times 2.81 \times 10^{-4} \text{ m}^3 \text{ s}^{-1}\right)^2} \\ &= 0.1 \text{ m}\end{aligned}$$

B.3 Parallel and Series Experimental vs. Theoretical Pump Discharge and Head

Table B.15: Parallel and series experimental vs. theoretical pump discharge and heads for 2700 RPM

		Actual			Theoretical		
		Volumetric Flow, Q_{act} ($\text{m}^3 \text{ s}^{-1}$)	Transducer Head, $H_{t_{\text{act}}}$ (m)	Corrected Head, H_{act} (m)	Volumetric Flow, Q_{th} ($\text{m}^3 \text{ s}^{-1}$)	Transducer Head, $H_{t_{\text{th}}}$ (m)	Corrected Head, H_{th} (m)
Parallel	Fully open	0.007266	9.84	11.3	0.009715	5.28	7.81
Parallel	Partial 1	0.006283	10.6	11.7	0.009498	5.70	8.12
Parallel	Partial 2	0.002217	12.4	12.5	0.005823	10.4	11.3
Parallel	Closed	-	12.5	12.5	-	12.4	12.4
Series	Fully open	0.005584	8.31	9.1	0.004858	10.6	11.2
Series	Partial 1	0.005449	9.65	10.4	0.004749	11.4	12.0
Series	Partial 2	0.002983	21.7	21.9	0.002911	20.7	20.9
Series	Closed	-	24.9	24.9	-	24.8	24.8

The actual results were pulled directly from Table B.9 and Table B.12. The theoretical results were calculated using the results from the single pump analysis Table A.4.

B.3.1 Parallel Experimental vs. Theoretical Pump Discharge and Head

Sample calculations are evaluated for the fully open configuration. The theoretical volumetric flow was found by

$$\begin{aligned} Q_{\text{th}} &= 2Q_{\text{single}} \\ &= 2 \times 0.004858 \text{ m}^3 \text{ s}^{-1} \\ &= 0.00971503 \text{ m}^3 \text{ s}^{-1} \end{aligned}$$

The theoretical transducer head was found by

$$\begin{aligned} H_{t_{\text{th}}} &= H_{t_{\text{single}}} \\ &= 5.28 \text{ m} \end{aligned}$$

The theoretical corrected head was found by

$$\begin{aligned} H_{\text{th}} &= H_{t_{\text{th}}} + \frac{8Q_{\text{th}}^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\ &= 5.28 \text{ m} + \frac{8 \times (0.00971503 \text{ m}^3 \text{ s}^{-1})^2}{\pi^2 \times (0.108 \text{ m})^4 \times 9.81 \text{ m s}^{-2}} \left[1 - \left(\frac{0.108 \text{ m}}{0.108 \text{ m}} \right)^4 \right] \\ &= 7.81 \text{ m} \end{aligned}$$

B.3.2 Series Experimental vs. Theoretical Pump Discharge and Head

Sample calculations are evaluated for the fully open configuration. The theoretical volumetric flow was found by

$$\begin{aligned} Q_{\text{th}} &= Q_{\text{single}} \\ &= 0.004858 \text{ m}^3 \text{ s}^{-1} \end{aligned}$$

The theoretical transducer head was found by

$$\begin{aligned} H_{t_{\text{th}}} &= 2H_{t_{\text{single}}} \\ &= 2 \times 5.28 \text{ m} \\ &= 10.6 \text{ m} \end{aligned}$$

The theoretical corrected head was found by

$$\begin{aligned} H_{\text{th}} &= H_{t_{\text{th}}} + \frac{8Q_{\text{th}}^2}{\pi^2 D_2^4 g} \left[1 - \left(\frac{D_2}{D_1} \right)^4 \right] \\ &= 10.6 \text{ m} + \frac{8 \times (0.004858 \text{ m}^3 \text{ s}^{-1})^2}{\pi^2 \times (0.108 \text{ m})^4 \times 9.81 \text{ m s}^{-2}} \left[1 - \left(\frac{0.108 \text{ m}}{0.108 \text{ m}} \right)^4 \right] \\ &= 11.2 \text{ m} \end{aligned}$$

C Appendix: Impeller Angle

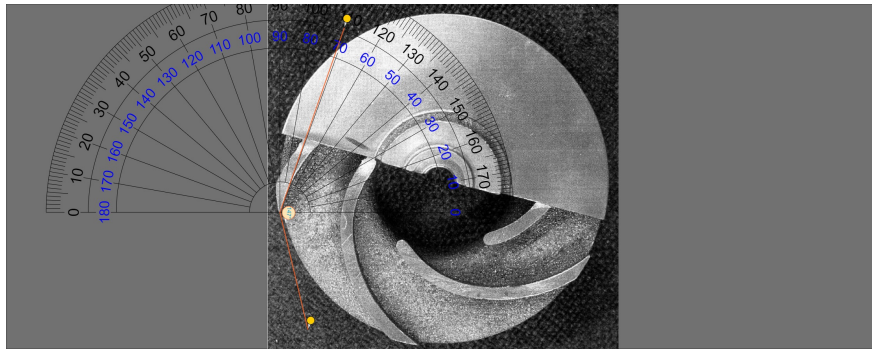


Figure C.10: Estimation of the impeller angle using digital protractor

The impeller angle was estimated using a digital protractor, as shown in Figure C.10. The impeller angle was measured to be

$$\beta_2 = 180^\circ - 147^\circ = 33^\circ$$

From equation (7), the ideal operating curve is

$$\begin{aligned}\Psi &= 1 - \Phi \cot \beta_2 \\ &= 1 - \Phi \cot 33^\circ \\ &= 1 - 1.540\Phi\end{aligned}$$

D Appendix: Shut Off Head

Shut off head was discussed in the theory section. From Eq. ?? and 10, the ideal and "rule of thumb" shut off head can be calculated. The calculation will be performed on the highest experimental speed, $\Omega = 3600$ RPM which, from Table A.6, the tip speed, $U = 20.4 \text{ m s}^{-1}$.

$$\begin{aligned}
 H'_{\text{ideal}} &= \frac{U^2}{g} \\
 &= \frac{(20.4 \text{ m s}^{-1})^2}{9.81} \\
 &= 42.2 \text{ m} \\
 H'_{\text{thumb}} &= \frac{1}{2} H'_{\text{ideal}} \\
 &= \frac{1}{2} \times 42.2 \text{ m} \\
 &= 21.1 \text{ m}
 \end{aligned}$$

The actual shutoff head was taken from Table ?? and is 21.8 m. The error for the rule of thumb shutoff head can be calculated as

$$\begin{aligned}
 \% \text{ Error} &= \left| \frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}} \right| \times 100\% \\
 &= \left| \frac{H'_{\text{exp}} - H'_{\text{thumb}}}{H'_{\text{thumb}}} \right| \times 100\% \\
 &= \left| \frac{21.8 - 21.1}{21.1} \right| \times 100\% \\
 &= 3.3\%
 \end{aligned}$$

The error for the ideal shutoff head can be calculated as

$$\begin{aligned}
 \% \text{ Error} &= \left| \frac{\text{Experimental} - \text{Theoretical}}{\text{Theoretical}} \right| \times 100\% \\
 &= \left| \frac{H'_{\text{exp}} - H'_{\text{ideal}}}{H'_{\text{ideal}}} \right| \times 100\% \\
 &= \left| \frac{21.8 - 42.2}{42.2} \right| \times 100\% \\
 &= 48.3\%
 \end{aligned}$$

E Manufacturer Geometrically Similar and Dissimilar Pumps

Table E.16: Geometrically Similar and Dissimilar Pump Dimensions

Geometrically Similar	Impeller Diameter, D	Blade Height, b	Blade Width, w	Volumetric Flow, Q	Pump Speed, N	Head, H
	(m)	(m)	(m)	(m ³ s ⁻¹)	(RPM)	(m)
No	0.108	0.009	0.0085	0.0080	3600	10.3
No	0.108	0.009	0.0085	0.0067	3600	16.1
No	0.108	0.009	0.0085	0.0054	3600	19.4
No	0.108	0.009	0.0085	0.0040	3600	21.6
No	0.108	0.009	0.0085	0.0027	3600	22.8
No	0.102	0.009	0.0085	0.0076	3600	7.9
No	0.102	0.009	0.0085	0.0063	3600	13.0
No	0.102	0.009	0.0085	0.0050	3600	16.3
No	0.102	0.009	0.0085	0.0038	3600	18.0
No	0.102	0.009	0.0085	0.0025	3600	19.2
No	0.096	0.009	0.0085	0.0071	3600	6.7
No	0.096	0.009	0.0085	0.0059	3600	11.0
No	0.096	0.009	0.0085	0.0047	3600	13.9
No	0.096	0.009	0.0085	0.0035	3600	15.8
No	0.096	0.009	0.0085	0.0024	3600	16.8
No	0.083	0.009	0.0085	0.0059	3600	2.6
No	0.083	0.009	0.0085	0.0049	3600	6.0

No	0.083	0.009	0.0085	0.0039	3600	8.4
No	0.083	0.009	0.0085	0.0029	3600	9.9
No	0.083	0.009	0.0085	0.0020	3600	10.5
Yes	0.108	0.009	0.0085	0.0080	3600	10.3
Yes	0.108	0.009	0.0085	0.0067	3600	16.1
Yes	0.108	0.009	0.0085	0.0054	3600	19.4
Yes	0.108	0.009	0.0085	0.0040	3600	21.6
Yes	0.108	0.009	0.0085	0.0027	3600	22.8
Yes	0.102	0.009	0.00803	0.0076	3600	7.9
Yes	0.102	0.009	0.00803	0.0063	3600	13.0
Yes	0.102	0.009	0.00803	0.0050	3600	16.3
Yes	0.102	0.009	0.00803	0.0038	3600	18.0
Yes	0.102	0.009	0.00803	0.0025	3600	19.2
Yes	0.096	0.008	0.00756	0.0071	3600	6.7
Yes	0.096	0.008	0.00756	0.0059	3600	11.0
Yes	0.096	0.008	0.00756	0.0047	3600	13.9
Yes	0.096	0.008	0.00756	0.0035	3600	15.8
Yes	0.096	0.008	0.00756	0.0024	3600	16.8
Yes	0.083	0.007	0.00653	0.0059	3600	2.6
Yes	0.083	0.007	0.00653	0.0049	3600	6.0
Yes	0.083	0.007	0.00653	0.0039	3600	8.4
Yes	0.083	0.007	0.00653	0.0029	3600	9.9
Yes	0.083	0.007	0.00653	0.0020	3600	10.5

Table E.17: Geometrically Similar and Dissimilar Pump Coefficients

Geometrically Similar	Impeller Diameter, D	Impeller Speed, U	Radial Exit Velocity, v_{2r}	Head Co- efficient, Ψ	Flow Co- efficient, Φ
	(m)	(m s ⁻¹)	(m s ⁻¹)		
No	0.108	20.4	3.0	0.24	0.15
No	0.108	20.4	2.5	0.38	0.12
No	0.108	20.4	2.0	0.46	0.10
No	0.108	20.4	1.5	0.51	0.07
No	0.108	20.4	1.0	0.54	0.05
No	0.102	19.2	3.0	0.21	0.16
No	0.102	19.2	2.5	0.34	0.13
No	0.102	19.2	2.0	0.43	0.10
No	0.102	19.2	1.5	0.48	0.08
No	0.102	19.2	1.0	0.51	0.05
No	0.096	18.1	3.0	0.20	0.17
No	0.096	18.1	2.5	0.33	0.14
No	0.096	18.1	2.0	0.42	0.11
No	0.096	18.1	1.5	0.47	0.08
No	0.096	18.1	1.0	0.50	0.06
No	0.083	15.6	3.0	0.10	0.19
No	0.083	15.6	2.5	0.24	0.16
No	0.083	15.6	2.0	0.34	0.13

No	0.083	15.6	1.5	0.40	0.09
No	0.083	15.6	1.0	0.42	0.07
Yes	0.108	20.4	3.0	0.24	0.15
Yes	0.108	20.4	2.5	0.38	0.12
Yes	0.108	20.4	2.0	0.46	0.10
Yes	0.108	20.4	1.5	0.51	0.07
Yes	0.108	20.4	1.0	0.54	0.05
Yes	0.102	19.2	3.2	0.21	0.17
Yes	0.102	19.2	2.6	0.34	0.14
Yes	0.102	19.2	2.1	0.43	0.11
Yes	0.102	19.2	1.6	0.48	0.08
Yes	0.102	19.2	1.0	0.51	0.05
Yes	0.096	18.1	3.4	0.20	0.19
Yes	0.096	18.1	2.8	0.33	0.15
Yes	0.096	18.1	2.2	0.42	0.12
Yes	0.096	18.1	1.7	0.47	0.09
Yes	0.096	18.1	1.1	0.50	0.06
Yes	0.083	15.6	3.7	0.10	0.24
Yes	0.083	15.6	3.1	0.24	0.20
Yes	0.083	15.6	2.5	0.34	0.16
Yes	0.083	15.6	1.8	0.40	0.12
Yes	0.083	15.6	1.3	0.42	0.08

E.1 Geometrically Similar Pumps Sample Calculations

Sample calculations for Table E.16 and Table E.17 will be shown for an impeller of $D = 102 \text{ mm}$ and $Q = 0.0076 \text{ m}^3 \text{ s}^{-1}$. Pump speed is given as 3600 RPM with number of blades, $N = 5$. Impeller speed is then

$$\begin{aligned} U &= \frac{D}{2} \times \Omega \\ &= \frac{102 \text{ mm}}{2} \times 3600 \text{ RPM} \times \frac{2\pi \text{ rad s}^{-1}}{60 \text{ RPM}} \\ &= 19.2 \text{ m s}^{-1} \end{aligned}$$

Geometrically similar means that blade height and width are scaled proportionally to the impeller diameter. Blade height and width are then

$$\begin{aligned} b_{102} &= \frac{b_{108} \times D_{102}}{D_{108}} \\ &= \frac{9 \text{ mm} \times 102 \text{ mm}}{108 \text{ mm}} \\ &= 8.5 \text{ mm} \\ w_{102} &= \frac{w_{108} \times D_{102}}{D_{108}} \\ &= \frac{8.5 \text{ mm} \times 102 \text{ mm}}{108 \text{ mm}} \\ &= 8.03 \text{ mm} \end{aligned}$$

Radial exit velocity is then

$$\begin{aligned} v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.0076 \text{ m}^3 \text{ s}^{-1}}{8.5 \text{ mm} \times (\pi \times 0.102 \text{ m} - 5 \times 8.03 \text{ mm})} \\ &= 3.0 \text{ m s}^{-1} \end{aligned}$$

Head coefficient is then

$$\begin{aligned} \Psi &= \frac{H}{U^2} \\ &= \frac{7.9 \text{ m}}{19.2 \text{ m s}^{-1}} \\ &= 0.41 \end{aligned}$$

Flow coefficient is then

$$\begin{aligned}\Phi &= \frac{Q}{U \times D^2} \\ &= \frac{0.0076 \text{ m}^3 \text{ s}^{-1}}{19.2 \text{ m s}^{-1} \times (0.102 \text{ m})^2} \\ &= 0.16\end{aligned}$$

E.2 Geometrically Dissimilar Pumps Sample Calculations

Sample calculations for Table E.16 and Table E.17 will be shown for an impeller of $D = 102 \text{ mm}$ and $Q = 0.0076 \text{ m}^3 \text{ s}^{-1}$. Pump speed is given as 3600 RPM with number of blades, $N = 5$. Impeller speed is then

$$\begin{aligned}U &= \frac{D}{2} \times \Omega \\ &= \frac{102 \text{ mm}}{2} \times 3600 \text{ RPM} \times \frac{2\pi \text{ rad s}^{-1}}{60 \text{ RPM}} \\ &= 19.2 \text{ m s}^{-1}\end{aligned}$$

Geometrically dissimilar means that blade height and width are not scaled proportionally to the impeller diameter. Blade height and width are unchanged between all pumps. That is,

$$\begin{aligned}b_{102} &= b_{108} \\ &= 9 \text{ mm} \\ w_{102} &= w_{108} \\ &= 8.5 \text{ mm}\end{aligned}$$

Radial exit velocity is then

$$\begin{aligned}v_{2r} &= \frac{Q}{b(2\pi r_2 - Nw)} \\ &= \frac{0.0076 \text{ m}^3 \text{ s}^{-1}}{9 \text{ mm} \times (\pi \times 0.102 \text{ m} - 5 \times 8.5 \text{ mm})} \\ &= 3.2 \text{ m s}^{-1}\end{aligned}$$

Head coefficient is then

$$\begin{aligned}\Psi &= \frac{H}{U^2} \\ &= \frac{7.9 \text{ m}}{19.2 \text{ m s}^{-1}} \\ &= 0.41\end{aligned}$$

Flow coefficient is then

$$\begin{aligned}\Phi &= \frac{Q}{U \times D^2} \\ &= \frac{0.0076 \text{ m}^3 \text{ s}^{-1}}{19.2 \text{ m s}^{-1} \times (0.102 \text{ m})^2} \\ &= 0.16\end{aligned}$$

F Pump Efficiency

Table F.18: Pump Efficiency for Single Pump Configuration

Valve Configuration	Pump Speed, Ω	Moment arm, d_m	Volumetric Flow, Q	Corrected Head, H	Torque, T	Efficiency, η
	(RPM)	(cm)	(m ³ s ⁻¹)	(m)	(N m)	(%)
Fully open	1800	8.75	0.003159	2.91	1.29	37.0
Partial 1	1800	8.50	0.003003	3.16	1.25	39.4
Partial 2	1800	7.25	0.001492	5.09	1.07	37.0
Closed	1800	4.50	-	5.49	0.663	0.00
Fully open	2700	16.50	0.004858	5.91	2.43	40.9
Partial 1	2700	16.50	0.004749	6.31	2.43	42.7
Partial 2	2700	14.50	0.002911	10.6	2.14	49.9
Closed	2700	8.50	-	12.4	1.25	0.00
Fully open	3600	26.50	0.006474	9.72	3.90	41.8
Partial 1	3600	26.50	0.006240	11.2	3.90	46.6
Partial 2	3600	20.00	0.002454	20.8	2.95	44.9
Closed	3600	13.00	-	21.8	1.92	0.00
Manufacturer	1800	-	0.0040	2.58	-	46
Manufacturer	1800	-	0.0033	4.14	-	66
Manufacturer	1800	-	0.0026	4.96	-	67
Manufacturer	1800	-	0.0020	5.40	-	64
Manufacturer	2700	-	0.0060	5.77	-	52
Manufacturer	2700	-	0.0050	9.12	-	71

Manufacturer	2700	-	0.0040	11.0	-	73
Manufacturer	2700	-	0.0030	12.1	-	69
Manufacturer	2700	-	0.0020	12.8	-	58
Manufacturer	3600	-	0.0080	10.3	-	52
Manufacturer	3600	-	0.0067	16.1	-	71
Manufacturer	3600	-	0.0054	19.2	-	73
Manufacturer	3600	-	0.0040	21.6	-	69
Manufacturer	3600	-	0.0027	22.8	-	59

Sample calculations will be shown for the single pump configuration at 1800 RPM, fully open valve. The torque was calculated from the dyno mass of $m = 1.502 \text{ kg}$ and the moment arm of $d_m = 8.75 \text{ cm}$ as

$$\begin{aligned}
 T &= mgd_m \\
 &= 1.502 \text{ kg} \times 9.81 \text{ m s}^{-2} \times 0.0875 \text{ m} \\
 &= 1.29 \text{ N m}
 \end{aligned}$$

The pump efficiency is calculated from Eq. (23) as

$$\begin{aligned}
 \eta &= \frac{\rho g Q H}{T \Omega} \\
 &= \frac{998 \text{ kg m}^{-3} \times 9.81 \text{ m s}^{-2} \times 0.003159 \text{ m}^3 \text{ s}^{-1} \times 2.91 \text{ m}}{1.29 \text{ N m} \times 1800 \text{ RPM} \times \frac{2\pi}{60} \times \frac{\text{rad s}^{-1}}{\text{RPM}}} \\
 &= 37.0 \%
 \end{aligned}$$

F.1 Pump Efficiency Uncertainty

Table F.19: Pump Efficiency Uncertainty

Valve Config- uration	Pump Speed, Ω (RPM)	Volumetric Flow, Q ($\text{m}^3 \text{s}^{-1}$)	Corrected Head, H (m)	Flow Un- certainty, δ_Q ($\text{m}^3 \text{s}^{-1}$)	Head Un- certainty, δ_H (m)	Efficiency, η (%)	Efficiency Uncer- tainty, δ_η (%)
Fully open	1800	0.003159	2.91	6.40E-05	0.04	37.0	0.9
Partial 1	1800	0.003003	3.16	5.50E-05	0.04	39.4	0.9
Partial 2	1800	0.001492	5.09	1.14E-05	0.04	37.0	0.4
Closed	1800	-	5.49	-	0.04	0.00	-
Fully open	2700	0.004858	5.91	1.36E-04	0.05	40.9	1
Partial 1	2700	0.004749	6.31	1.08E-04	0.04	42.7	1
Partial 2	2700	0.002911	10.6	2.55E-05	0.04	49.9	0.5
Closed	2700	-	12.4	-	0.04	0.00	-
Fully open	3600	0.006474	9.72	1.48E-04	0.06	41.8	1
Partial 1	3600	0.006240	11.2	4.45E-04	0.2	46.6	3
Partial 2	3600	0.002454	20.8	1.41E-05	0.04	44.9	0.3
Closed	3600	-	21.8	-	0.04	0	-

The uncertainty in the pump efficiency was calculated for the single pump in the fully open valve configuration at 1800 RPM. The pump efficiency equation is

$$\eta = \frac{\rho g Q H}{T \Omega}$$

This is the purely multiplicative case of uncertainty propagation. Assuming the errors of torque, density, gravity, and pump speeds are negligible, the uncertainty in the pump efficiency is

$$\begin{aligned}
 \delta_\eta &= \eta \sqrt{\left(\frac{\delta_Q}{Q}\right)^2 + \left(\frac{\delta_H}{H}\right)^2} \\
 &= 37.0\% \sqrt{\left(\frac{6.40 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}}{0.003159 \text{ m}^3 \text{ s}^{-1}}\right)^2 + \left(\frac{0.04 \text{ m}}{2.91 \text{ m}}\right)^2} \\
 &= 0.9\%
 \end{aligned}$$