MEC E 301
Lab 6: Temperature Measurement Devices

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1 Theory

1.1 Euler's Turbomachinery Equations

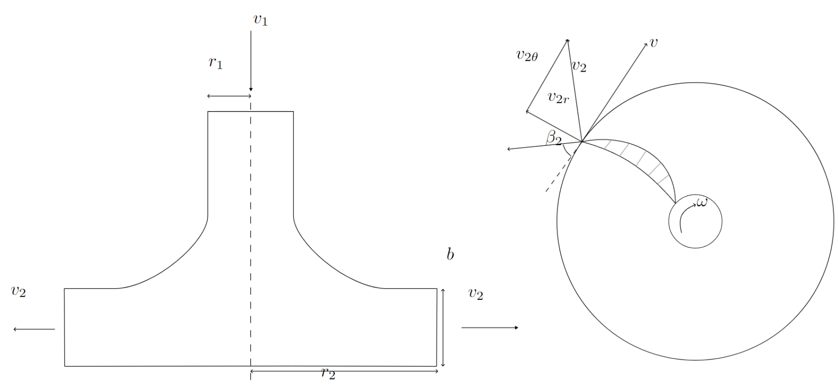

Figure 1: Impeller Diagram

Using the following assumptions:

1. Viscous effects are negligible
2. Velocity profile is uniform at the exit
3. All work done by the pump is transferred to the fluid

Then by conservation of angular momentum,

$$\mathrm{T} = \dot{\mathrm{m}}\mathrm{r}_2\mathrm{v}_{2\theta} \tag{1}$$

where T is the torque, $\dot{\mathrm{m}}$ is the mass flow rate, r_2 is the radius of the impeller, and $\mathrm{v}_{2\theta}$ is the tangential velocity at the exit. From Figure 1, we can see that

$$\mathrm{v}_{2\theta} = \mathrm{U} - \mathrm{v}_{2\mathrm{r}} \cot \beta_2 \tag{2}$$

where U is the tip speed, $\mathrm{v}_{2\mathrm{r}}$ is the radial velocity at the exit, and β_2 is the blade angle. Combining (1) and (2), we get

$$\mathrm{T} = \dot{\mathrm{m}}\mathrm{r}_2(\mathrm{U} - \mathrm{v}_{2\mathrm{r}} \cot \beta_2) \tag{3}$$

By Assumption 3, we can say that

$$\mathrm{T}\Omega = \dot{\mathrm{m}}\mathrm{g}\mathrm{H} \tag{4}$$

where Ω is the impeller angular velocity, g is the acceleration due to gravity, and H is the total head rise across the pump. By combining (3), (4), and the kinematic relationship $U = r_2\Omega$, we get

$$\frac{Hg}{U^2} = 1 - \frac{v_{2r}}{U}\cot\beta_2 \tag{5}$$

Defining the head coefficient Ψ and the flow coefficient Φ as

$$\Psi = \frac{Hg}{U^2} \qquad \Phi = \frac{v_{2r}}{U} \tag{6}$$

then (5) becomes

$$\Psi = 1 - \Phi\cot\beta_2 \tag{7}$$

also, v_{2r} can be expressed by the continuity equation as

$$v_{2r} = \frac{Q}{A_2} = \frac{Q}{b(2\pi r_2 - Nw)} \tag{8}$$

where Q is the flow rate, A_2 is the area of the exit, b is the blade height at the exit, N is the number of blades, and w is the width of the blade.

1.2 Shutoff Head

The ideal shutoff head can be obtained as

$$H'_{\text{ideal}} = \frac{U^2}{g}$$

by setting $\Phi = 0$ in (7). If it is assumed that all kinetic energy is lost due to friction, then

$$\frac{KE}{\text{unit weight}} = \frac{U^2}{2g}$$

where KE is the kinetic energy. Therefore, the rule of thumb for the shutoff head is

$$\begin{aligned} H'_{\text{thumb}} &= \frac{U^2}{g} - \frac{U^2}{2g} \\ H'_{\text{thumb}} &= \frac{U^2}{2g} = \frac{1}{2}H'_{\text{ideal}} \end{aligned} \tag{9}$$

1.3 Affinity Laws

For large Reynolds numbers, the flow is dynamically similar in geometrically similar machines when the flow and head coefficients are the same. For geometrically similar machines operating at different conditions (i) and (ii) such that the head and flow coefficients are the same, the following relationships hold:

$$\begin{aligned}\Psi_\mathrm{i} &= \Psi_\mathrm{ii}\\ \frac{\mathrm{H_i}}{\mathrm{U_i^2}} &= \frac{\mathrm{H_{ii}}}{\mathrm{U_{ii}^2}}\end{aligned}$$

so,

$$\frac{\mathrm{H_i}}{\mathrm{H_{ii}}} = \left(\frac{\mathrm{U_i}}{\mathrm{U_{ii}}}\right)^2 \approx \left(\frac{\mathrm{D_i}\Omega_\mathrm{i}}{\mathrm{D_{ii}}\Omega_\mathrm{ii}}\right)^2 \tag{10}$$

where D is the diameter of the impeller. Also,

$$\begin{aligned}\Phi_\mathrm{i} &= \Phi_\mathrm{ii}\\ \frac{\mathrm{v_{2ri}}}{\mathrm{U_i}} &= \frac{\mathrm{v_{2rii}}}{\mathrm{U_{ii}}}\end{aligned} \tag{11}$$

Assuming the blade width is negligible, from (8), we get

$$\mathrm{v_{2r}} = \frac{\mathrm{Q}}{\pi \mathrm{Db}} \tag{12}$$

so (11) becomes

$$\begin{aligned}\frac{\mathrm{Q_i D_{ii} b_{ii}}}{\mathrm{Q_{ii} D_i b_i}} &= \frac{\mathrm{U_i}}{\mathrm{U_{ii}}}\\ \frac{\mathrm{Q_i}}{\mathrm{Q_{ii}}} &= \frac{\Omega_\mathrm{i}\mathrm{b_i}}{\Omega_\mathrm{ii}\mathrm{b_{ii}}}\left(\frac{\mathrm{D_i}}{\mathrm{D_{ii}}}\right)^2\end{aligned} \tag{13}$$

For geometrically similar machines, the ratios of b/D are the same, so (13) becomes

$$\frac{\mathrm{Q_i}}{\mathrm{Q_{ii}}} = \frac{\Omega_\mathrm{i}}{\Omega_\mathrm{ii}}\left(\frac{\mathrm{D_i}}{\mathrm{D_{ii}}}\right)^3 \tag{14}$$

1.4 Transducer Head Adjustment

In this experiment, since the inlet and outlet pipe diameters are different, the transducer head, H_t, must be corrected for flow kinetic energy to give the pump stagnation head, H, as

$$\begin{aligned} H &= H_t + \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \\ H &= H_t + \frac{v_2^2}{2g}\left(1 - \left(\frac{v_1}{v_2}\right)^2\right) \end{aligned} \tag{15}$$

The volume flowrate through the system can be written as

$$Q = \frac{v_2 \pi D_2^2}{4} \tag{16}$$

and with the pipe diameters D_1 at the inlet and D_2 at the outlet, mass conservation requires that

$$v_2 \pi D_2^2 = v_1 \pi D_1^2 \tag{17}$$

With (16) and (17), (15) becomes

$$H = H_t + \frac{8Q^2}{\pi^2 D_2^4 g}\left(1 - \left(\frac{D_2}{D_1}\right)^4\right) \tag{18}$$