Bolted Connections:
An Investigation of Effects of Torque, Preload, and Gaskets on Bolted Connections

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Abstract

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1 Nomenclature

Symbol	Description	Units
A_s	Cross sectional area of the bolt	mm^2
d	Nominal bolt diameter	mm
d_m	Mean bolt diameter	mm
E_b	Young's modulus	GPa
E_{in}	Bridge excitation voltage	V
E_o	Bridge output voltage	V
F_b	Total bolt load	N
F_i	Preload	N
F_m	Total member load	N
G	Gain	-
K	Torque coefficient	-
K_g	Gauge factor	-
k_b	Bolt stiffness	kN/mm
k_g	Gasket stiffness	kN/mm
k_m	Member stiffness	kN/mm
L	Bolt/ Grip length	mm
P	Total external load	N
P_b	Load carried by bolt	N
P_m	Load carried by member	N
T	Torque	Nm
α	Thread half angle	$^\circ$

δ_b	Bolt deflection	mm
δ_m	Member deflection	mm
ε	Strain	mm/mm
σ_a	Alternating stress	MPa
σ_m	Mean stress	MPa
$\sigma_{\max}$	Maximum stress	MPa
$\sigma_{\min}$	Minimum stress	MPa
σ_y	Yield stress	MPa

2 Introduction

The purpose of this lab is to determine the stiffness of a bolted connection, the modulus of elasticity of the bolt, and the separation point of the joint. The stiffness of the bolted connection is important in design and analysis of bolted connections in critical applications. The results of this lab will be useful in future work to ensure safe and reliable operation of bolted connections in many engineering applications.

3 Procedure

3.1 Equipment

The experimental setup is shown in Figure 1. The following equipment will be used to conduct the experiment:

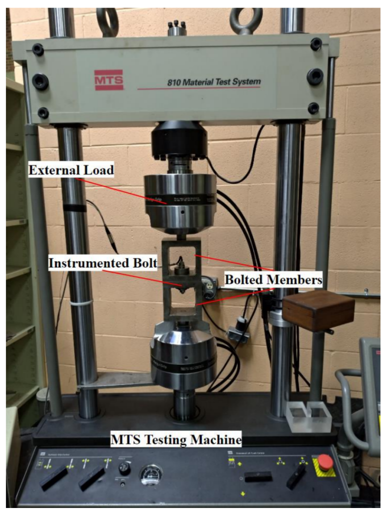

Figure 1: Experimental setup for bolted connection testing

- MTS testing machine, for applying controlled external loads to the bolted connection and measuring its response. The machine can also apply dynamic, or cyclic loads.
- Instrumented bolt (Strainsert Type W), to output the bolt's strain as a voltage.
- Instrumented washer (Lebow Model 3711-375), to output the washer's strain as a voltage.
- Vishay strain gauge conditioner, to condition and amplify the signals from the strain gauges.
- Torque wrench, for applying specific torque values to the bolt during preload and torque tests.
- Gasket of unknown material, the material will be analyzed during the experiment to determine its properties.

3.2 Procedure

3.2.1 Zero Preload

1. Attach the bolted connection to the MTS machine with the nut attached ”finger tight” (without a gasket).
2. Load the bolt 8 times with a range of 0 - 7.5 kN.
3. Record the external load and bridge imbalance at each load (0, 1, 2, 3, 4, 5, 6, 7, 7.5).

3.2.2 Repeatability Test

1. Attach the bolt to the MTS machine.
2. Use the torque wrench to apply a preload of 50 in-lb.
3. Record the voltage readings from the bolt and washer gauges.
4. Loosen the bolt to remove any preload.
5. Repeat steps 2-4 four more times.

3.2.3 Zero Loading (Torque Test)

1. Attach the bolt to the MTS machine, without a gasket.
2. Set the external load to 0 kN.
3. Record the voltage readings from the bolt and washer gauges.
4. Increase the torque by 25 in-lb.
5. Record the voltage readings from the bolt and washer gauges.
6. Repeat steps 4 and 5 four more times, obtaining readings from 0 to 125 in-lb of torque (0, 25, 50, 75, 100, 125).

3.2.4 Static Loading

1. Attach the bolt to the MTS machine (without a gasket).
2. Tighten the bolt to 60 in-lb of torque.
3. Set the external load on the MTS machine to 0 kN.
4. Set the external load on the MTS machine to 7.5 kN.
5. Record the readings from the bolt and washer.
6. Repeat steps 3-5 two more times, totaling three readings (shakedown test).
7. Leave the bolt assembled, and apply loads ranging from 0-7.5 kN (0, 1, 2, 3, 4, 5, 6, 7, 7.5). Record the output readings from the bolt and washer at each load.
8. Set the load back to 0 kN.
9. Disassemble and reassemble the joint with the gasket in place.
10. Repeat steps 2-9 with a gasket.

3.2.5 Dynamic Loading

1. Attach the bolt to the MTS machine (without a gasket).
2. Set the bolt to the ”finger tight” torque setting.
3. Apply an external load of 5 kN and an alternating load of 1.25 kN at 0.3 Hz.
4. Record data for at least 10 cycles.
5. Repeat steps 3-4 at different torque settings of 60, 75, and 125 in-lb.
6. Disassemble and reassemble the joint with the gasket in place.
7. Repeat steps 2-5 with a gasket.

4 Theory

4.1 Mechanics of Bolted Connections Loading

The typical bolted connection is shown in Figure 2a. The key forces in the above diagram are the preload, F_i, and the external load, P. This connection can be viewed as an analogy to the spring system seen in Figure 2b. By Hooke’s law, the deflection of the bolt and the member are given by

(a) (b)

Figure 2: a) Bolted Joint Diagram with Preload and External Load, b) Spring Analogy

the equations below:

$$\delta_b = \frac{F_i}{k_b} \tag{1}$$

$$\delta_m = \frac{F_i}{k_m} \tag{2}$$

When the external load, P, is applied to the joint, a change in the deformation of the bolt and the reduction of compression in the joined members occurs. Similar to deflection, the change in deformation can be calculated using the equations:

$$\Delta\delta_b = \frac{P_b}{k_b} \tag{3}$$

$$\Delta\delta_m = \frac{P_m}{k_m} \tag{4}$$

If the members are not separated, the deformation in the member and the bolt are equivalent, shown by the relation below:

$$\frac{P_b}{k_b} = \frac{P_m}{k_m} \tag{5}$$

The total load on the bolt and the member must equal the sum of the change in load of the bolt, P_b, and the member, P_m,

$$P = P_m + P_b$$

Using (5), the change in load of the bolt and the member can be expressed as:

$$P_b = \frac{k_b P}{k_b + k_m} \tag{6}$$

$$P_m = \frac{k_m P}{k_b + k_m} \tag{7}$$

Similarly, the total loads on the bolt and the member are given by:

$$F_b = F_i + P_b \tag{8}$$

$$F_m = F_i + P_m \tag{9}$$

4.2 Quarters Bridge Equations

The instrumented bolt uses a Wheatstone quarter bridge to measure strain. The voltage reading from the bridge, V_o, can be expressed using the input voltage, V_{in}, gauge factor, K_g, gain, G, and strain, ε.

$$\varepsilon = \frac{4V_o}{K_g V_{in} G} \tag{10}$$

4.3 Stress-Strain Relationship

By Hooke's law, the stress-strain relationship is given by:

$$\sigma = E_b \varepsilon \tag{11}$$

Since stress is force per unit area,

$$F_b = A_s \sigma \tag{12}$$

4.4 Member Stiffness

For a given member, the stiffness can be calculated as

$$k_i = \frac{A_i E_i}{L_i} \tag{13}$$

Members in a bolted connection can be viewed as a series of springs. Equivalent stiffness for this system is given by:

$$\frac{1}{k_m} = \sum_{i=1}^{n} \frac{1}{k_i} \tag{14}$$

where k_i is the stiffness of the ith section of the member.

For members with a gasket, loading can be estimated by assuming the load spreads at a fixed 45° angle. The compression of each element is then divided into infinitesimally small annular elements. It can be shown that the stiffness for two identical members bolted together is given by:

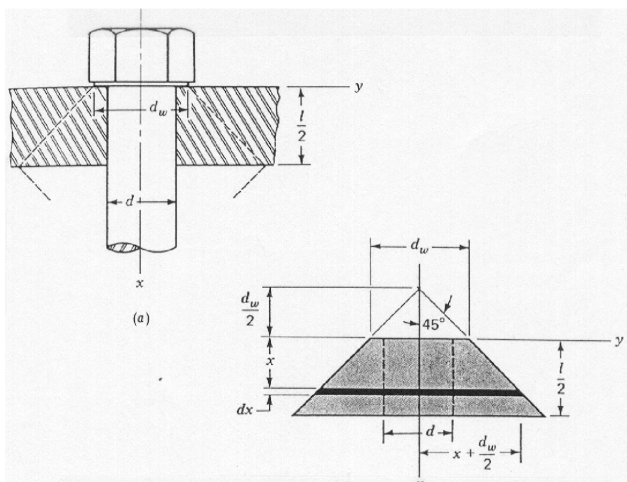

Figure 3: Analysis of the compression of members in a bolted connection

$$k_m = \frac{\pi E_b d}{2 \ln\left(\frac{5(L+0.5d)}{L+2.5d}\right)} \tag{15}$$

4.5 Torque Requirement for Preloading

From basic screw-thread theory, the torque required to preload a bolt is given by:

$$T = \frac{F_i d}{2}\left[\frac{L + \pi \mu d_m \sec\alpha}{\pi d_m - \mu L \sec\alpha}\right]$$

it can be shown that

$$\begin{aligned} T &= F_i d \left[\left(\frac{d_m}{2d} \right) \left(\frac{\tan\lambda + \mu \sec\alpha}{1 - \mu \tan\lambda \sec\alpha} \right) + 0.625\mu_c \right] \\ &= K d F_i \end{aligned} \tag{16}$$

4.6 Bolt Preload for Static Loading

Preloading the bolt is meant to prevent the jointed member from separating and the bolt from yielding. Using Equations (6) and (8), the total load on the bolt can be given by:

$$F_b = F_i + CP \tag{17}$$

where the constant C is defined as:

$$C = \frac{k_b}{k_b + k_m} \tag{18}$$

at the point of joint separation, $F_b = P$. Rearranging (17) gives:

$$F_i = P(1 - C) \tag{19}$$

To avoid yielding, a safety factor is introduced, N. Rearranging (17) gives:

$$F_i = \frac{A_t \sigma_y}{N} - CP \tag{20}$$

4.7 Bolt Preload for Dynamic Loading

Cyclic loading cycles are used to vary the load on a bolt over time. The two parameters often analyzed from these trials are the mean and alternating stresses.

$$\sigma_m = \frac{F_{\text{max}} + F_{\text{min}}}{2A_s} \tag{21}$$

$$\sigma_a = \frac{F_{\text{max}} - F_{\text{min}}}{2A_s} \tag{22}$$

The modified Goodman criteria states:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ut}} = 1$$

where σ_e is the endurance limit, and σ_{ut} is the ultimate tensile strength. It can be shown the follow holds,

$$F_i = A_s \sigma_{ut} - \frac{NCP}{2}\left[\frac{\sigma_{ut}}{\sigma_e} - 1\right]$$

5 Results and Discussion

5.1 Zero Preload — Young's Modulus of Bolt

The Young's Modulus for the bolt, E_b, was determined to be 205 ± 4.70 GPa. The zero-preload trial was used to determine Young's Modulus for the bolt. The nut was tightened to finger tight, ensuring the preload force is negligible. The external load in this case is then equivalent to the total load on the bolt. The output voltage of the strain gauge was used to determine the strain in the bolt. Figure 4 shows the plot of the bolt strain against the external load and its linear regression through the origin.

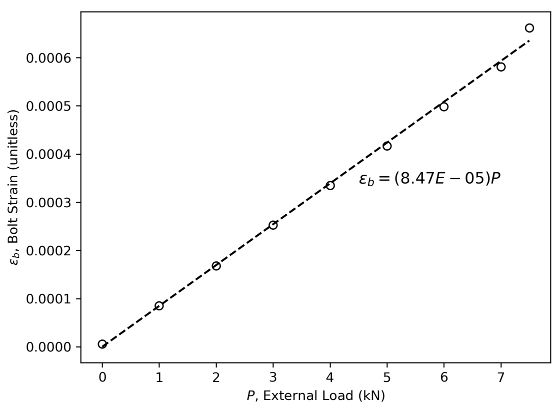

Figure 4: External Load vs. Bolt Strain

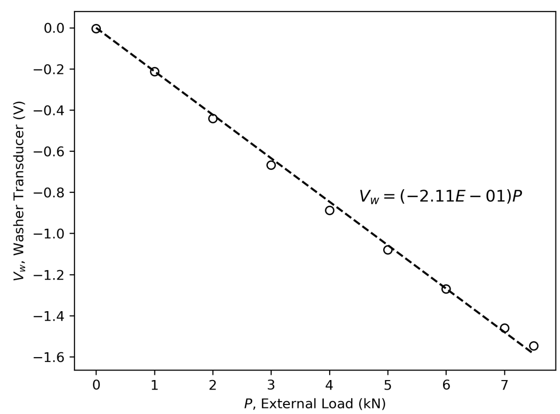

Figure 5: External Load vs. Washer Transducer

Using the slope from Figure 4 coupled with Eqs. (11) and (12), the Young's Modulus for the bolt, E_b, was determined to be 205 ± 4.70 GPa. The linear regression had an $R^2 = 0.9992$, indicating a strong linear relationship between the external load and the bolt strain, agreeing with the stress-strain theory. The uncertainty was determined using a 95% confidence interval. The error of the modulus was relatively small, adding confidence to the results. Sample calculations and error analysis can be found in Appendix A.

5.2 Zero Preload — Washer Transducer Calibration

The relationship between the washer output, $V_{o,w}$, and the external load (kN), P, was determined to be $V_{o,w} = -2.11 \times 10^{-4} P$. Again, the zero-preload trial was used to determine the washer calibration. The nut was tightened to finger tight, ensuring the preload force is negligible. The voltage output from the washer during the zero-preload trial was used to determine the washer calibration. Figure 5 shows the plot of the experimental data and its linear regression through the origin. The regression had an $R^2 = 0.9994$, indicating a strong linear relationship between the external load and the washer transducer.

Using the regression and Eq. (10), the relationship between the washer strain, $V_{o,w}$, and the external load (kN), P, was determined to be $V_{o,w} = -2.11 \times 10^{-4} P$. The negative sign in the equation indicates that the washer compresses as the external load increases, consistent with expectations. The high R^2 value indicates a strong linear relationship between the washer strain and the external load, adding confidence to the results. Sample calculations can be found in Appendix A.

5.3 Zero Loading — Torque and Preload

The relationship between torque and preload was $F_i = 0.636T$. The nut was tightened to various torques using a torque wrench, and the strain gauge output was used to determine the preload. The output voltage from the bolt transducer, coupled with Eq. (10) and (11) was used to determine the preload on the bolt. Figure 6 shows the plot of the experimental data and its linear regression. The regression had an $R^2 = 0.9995$, indicating a strong linear relationship between the torque and the preload.

To calculate the preload, the modulus of elasticity, E_b, and strain transducer reading V_b were used. The error for the preload by propagation of uncertainty. The uncertainty for the transducer reading was determined from a repeatability test. The major source of error was the strain transducer reading, which dominated the uncertainty. Sample calculations and error analysis can be found in Appendix B.

The results are expected as a linear relationship between torque and preload was discussed in Section 4.5 by Eq. (16). The uncertainty of the 125 in-lb torque wrench had the highest uncertainty of ± 0.422 kN, which was 4.60% relative uncertainty. The absolute uncertainty was inversely proportional to the strain transducer reading. This means that the uncertainty of the first measurement was relatively large, and decreases as the strain transducer reading increases. The main source of error was the transducer reading. The relative error of 4.60%, along with the $R^2 = 0.9995$, adds confidence to the results.

5.4 Zero Loading — Torque Coefficient

The torque coefficient, K, was determined to be 0.167. This was obtained using the results from Section 5.3 combined with Eq. (16). This value was within the expected range of $0.1-0.2$, agreeing with theory. The confidence is high due to the $R^2 = 0.9995$ and the low relative uncertainty of 4.60%. This result will be used later to determine the preload in later sections. While uncertainty was not calculated, future work utilizing the standard error of the slope along with a confidence

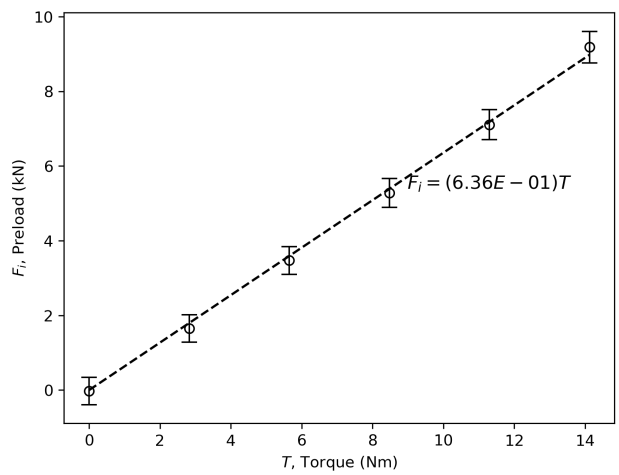

Figure 6: Torque vs. Preload

of 95% could be used to determine the torque coefficient uncertainty. Sample calculations can be found in Appendix A.

5.5 No Gasket, Static Loading — Torsional Loading

Negligible evidence of torsional loading was found during the shakedown test. A shakedown test was performed by ramping the external load from 0 kN to 7.5 kN back down to 0 kN three times in succession. The voltage at the end of each ramp was recorded. The strain transducer reading varied by ± 0.005 V, which was the same magnitude as the resolution of the strain transducer. This indicates that the bolt was not subjected to any torsional loading. The results were consistent with expectations, as the bolt was not subjected to any torsional loading. Sample calculations can be found in Appendix C.

5.6 Bolt Stiffness

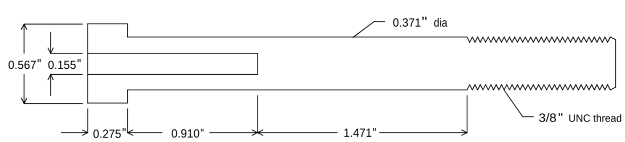

Figure 7: Cross Section of the Strainsert Type W Bolt Transducer

The stiffness of the bolt was determined to be $k_b = 209$ MN/m. The bolt was divided into three sections, as shown in Figure 7. The stiffness of each section was determined using Eq. (13). The stiffness of sections 1, 2, and 3 were determined to be 510 MN/m, 383 MN/m, and 4834 MN/m, respectively. The total bolt stiffness was then determined using Eq. (14). This value will be used later to determine the experimental and theoretical stiffness of the member. While uncertainty analysis was not conducted, the largest contributor to error was the uncertainty from modulus of elasticity, E_b. Sample calculations can be found in Appendix D.

5.7 Theoretical Joined Member Stiffness

The theoretical stiffness of the joined members was determined to be $k_m = 2222$ MN/m. The stiffness of the member was determined using Eq. (15). This assumes the angle of the stress distribution was 45°. This will be used as the expected value for the member stiffness, and will be compared to the experimental value later. Sample calculations can be found in Appendix E.

5.8 Static Loading — With and Without Gasket

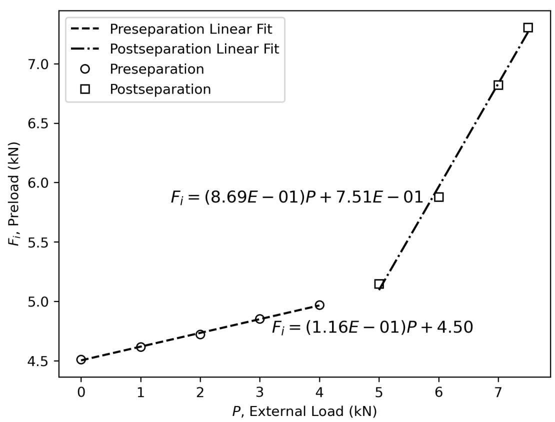

Figure 8: Static loading of bolted connection without gasket

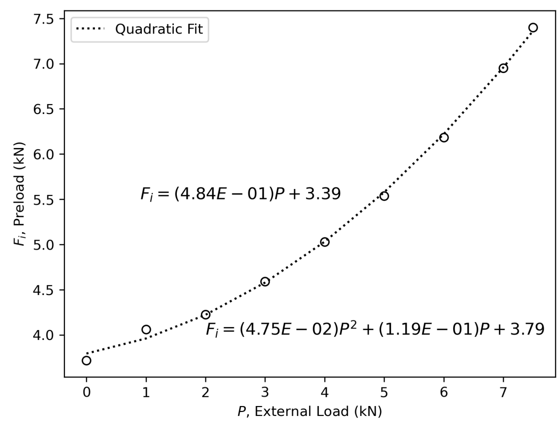

Figure 9: Static loading of bolted connection with gasket

The no gasket trial shown in Figure 8 had a linear relationship between the external load and the bolt strain. Two regressions of $F_{b,\text{pre}} = 0.1157P + 4.5022$ and $F_{b,\text{post}} = 0.8687P + 0.7507$ were used to fit the data. Two regressions were required due to the separation point at 4.98 kN. The pre and post seperation R^2 values were 0.9983 and 0.9958, respectively, indicating a strong linear relationship between the external load and the bolt strain. An increase in the bolt force for an external load was observed in the separation point trial. This is expected as the bolted connection was no longer in contact with the members. The quadratic fit was good, with an $R^2 = 0.9985$, indicating a strong quadratic relationship between the external load and the bolt strain.

The gasket trial shown in Figure 9 had a quadratic relationship between the external load and the bolt strain. The regression was found to be $F_{b,\text{gasket}} = 0.0475P^2 + 0.119P + 3.794$. The quadratic fit was good, with an $R^2 = 0.9985$, indicating a strong quadratic relationship between the external load and the bolt strain. In general, the gasket trial had a lower bolt force for a given external load, indicating the gasket was effective in reducing the bolt force. The separation point was not observed in the gasket trial, as the effect of the gasket prevented the members from separating within the tested loads. Future work in testing values beyond 7.5 kN could be used to determine the separation point of the gasket trial.

The results were consistent with expectations. The no gasket had a steeper slope than the gasket trial, indicating a higher stiffness. The separation point was observed in the no gasket trial, but not in the gasket trial. The R^2 values were high, indicating a strong linear and quadratic relationship between the external load and the bolt strain.

The effect of the gasket reduces bolt force for a given external load. The gasket also helps to prevent the members from separating. The separation point was observed in the no gasket trial, but not in the gasket trial. The R^2 values were high, indicating a strong linear and quadratic relationship between the external load and the bolt strain.

5.9 Experimental Joined Member Stiffness Without Gasket

The stiffness of the joined members was experimentally determined to be $k_m = 1600$ MN/m. The stiffness of the member was determined using the preseparation regression from Section 5.8. The constant C was determined to be 0.1157 from the preseparation regression. This value was then used to determine the experimental stiffness of the member using Eq. (18).

The relative error between the theoretical value of $k_m = 2222$ MN/m and the experimental value of $k_m = 1600$ MN/m was 28.0%. The error was relatively large, indicating a large discrepancy between the theoretical and experimental values. While a portion of the error was due to uncertainty from the modulus of elasticity, E_m, and strain transducer uncertainty, V_b, this does not account for the large discrepancy. A key assumption made in the theoretical calculation was that the stress distribution was 45°. This was never verified, and could be a source of error. Future work could be done to verify the stress distribution angle. Sample calculations can be found in Appendix F.

5.10 Experimental Joined Member Stiffness With Gasket

Determining the joined member stiffness with gasket is difficult. The lack of a separation point makes it difficult to calculate the stiffness of the member. In addition, the theory was developed for a linear relationship between bolt force and external load. Since the gasket is made of a softer material, it could be expected that the stiffness would decrease. This is beneficial as the gasket would help to reduce the bolt force for a given external load. Future work in developing a model that accounts for quadratic fits as well as increasing the range of loads tested could be used to determine the stiffness of the member with gasket.

5.11 Static Loading — Separation Point

The external load at the separation point for the experimental and theoretical values were determined to be 4.98 kN and 4.67 kN, respectively. The experimental separation point was determined by equating the preseparation and postseparation regressions from Section 5.8. The theoretical separation point was determined using Eq. (19) using the experimental k_b and theoretical k_m. The relative error between the experimental and theoretical separation points was 6.78%. This error was modestly small. The main discrepancy came from the difference in the experimental and theoretical stiffness of the member, with a relative error of 28.0%. Despite the large difference in the stiffness of the member, the separation point was relatively close. Future work in determining the stress distribution angle could be used to determine the separation point with higher accuracy and confidence. Sample calculations can be found in Appendix G.

Table 2: Dynamic Loading Summary for Various Torques and Gasket Conditions

	Torque, T (in-lb)	Max Stress, σ_{max} (MPa)	Min Stress, σ_{min} (MPa)	Mean Stress, σ_{mean} (MPa)	Alternating Stress, σ_{a} (MPa)
With Gasket	0	105.035	61.804	83.420	21.615
	60	110.300	88.864	99.582	10.718
	75	124.101	110.488	117.295	6.806
	125	174.584	167.250	170.917	3.667
No Gasket	0	105.330	62.098	83.714	21.616
	60	106.347	84.153	95.250	11.097
	75	108.671	101.337	105.004	3.667
	125	166.652	163.258	164.955	1.697

5.12 Dynamic Loading — Mean and Alternating Stresses

The results are summarized in Table 2. The mean stress increases as torque increases and the alternating stress decreases as torque increases. The gasket generally increased the mean stress and alternating stress. In this test, the MTS testing machine was utilized to apply an alternating load of 1.25 kN at 0.3 Hz frequency. The alternating load was applied to the bolted connection, and the voltage output from the bolt transducer was used to determine the stress. The preload from torque had a larger effect on the gasket than the no gasket trial. If fatigue is a design factor, torquing the bolt without a gasket will lower the alternating stress. If mean stress is more important, minimizing the torque and not using a gasket will lower the mean stress. Sample calculations can be found in Appendix H.

6 Conclusion

The modulus of elasticity was determined to be $E = 205 \pm 4.70$ GPa. This quantity was determined from the zero preload trial. The regression used to determine the modulus had an $R^2 = 0.9992$, indicating a strong linear relationship between the external load and the bolt strain. The preload uncertainty was determined to be $\pm 5\%$ kN from the zero loading trial. The uncertainty is relatively small, which was calculated from the standard error S_a of the regression.

The washer calibration was found to be $V_{o,w} = -2.11 \times 10^{-4} P$. This was also calculated from the zero preload trial. This allows prediction of the washer voltage output for a given external load. The regression had an $R^2 = 0.9994$, indicating a strong linear relationship between the external load and the washer transducer.

The torque coefficient was determined to be 0.167. This was from the zero loading trial, where the torque was varied and the bolt strain was measured. The regression had an $R^2 = 0.9995$, indicating a strong linear relationship between the torque and the preload. The slope value was then used to determine the torque coefficient. This value was within the expected range of $0.1 - 0.2$, adding confidence to the results.

A shakedown test was performed to determine if the bolt was subjected to any torsional loading. The voltage output varied by ± 0.005 V, which was small, the same magnitude as the resolution of the strain transducer. This suggest that the bolt was not subjected to any torsional loading.

The stiffness of the bolt was determined to be $k_b = 209$ MN/m. The bolt was divided into three sections, and the stiffness of each section was determined by modelling the bolt as three distinct sections which act as three springs in series, with stiffness $k_1 = 510$ MN/m, $k_2 = 383$ MN/m, and $k_3 = 4834$ MN/m.

The theoretical and experimental stiffness of the joined members was determined to be $k_m = 2222$ MN/m and $k_m = 1600$ MN/m, respectively. The theoretical stiffness of the joined members was determined by modelling the angle of stress distribution on the member to be 45°. The experimental stiffness of the member was determined using the preseparation regression from the static loading no gasket trial. The constant C was determined to be 0.1157 from the preseparation regression ($R^2 = 0.9983$). The relative error between the theoretical and experimental stiffness of the member was 28.0%. The assumption of the stress distribution angle was likely responsible for discrepancies between the theoretical and experimental stiffness of the member.

During the static loading trials, the gasket decreased the bolt force for a given external load. A separation was observed in the no gasket trial, but not in the gasket trial. This suggest the gasket helped prevent the members from separating. Further testing could be done to determine the separation point of the gasket. The R^2 values for the no gasket trial and gasket trial were 0.9983 and 0.9985, respectively, indicating a strong linear and quadratic relationship between the external load and the bolt strain.

The separation point was observed to be $P = 4.98$ kN and $P = 4.67$ kN for the experimental and theoretical values, respectively. The theoretical separation point was determined using the experimental k_b and theoretical k_m, whereas the experimental separation point was determined by equating the preseparation and postseparation regressions from the no gasket trial. The relative error between the experimental and theoretical separation points was 6.78%. The main discrepancy came from the difference in the experimental and theoretical stiffness of the member, which had a relative error of 28.0%.

Both the mean and alternating stresses increased as torque increased. The gasket generally increased the mean stress and alternating stress. The alternating stress decreased as torque increased, and the mean stress increased as torque increased.

The objectives of the lab were achieved: the modulus of elasticity was obtained, the stiffness of the member and bolts were determined, and insights into the effects of torque, preload, external loads, gaskets, and dynamic loading were obtained. The results were consistent with expectations, and the uncertainty was relatively small. The largest sources of error was the uncertainty from the strain transducer, $V_{o,b}$ and the modulus, E_b. Future work could be done to verify the stress distribution angle, and to determine the separation point of the gasket.

Understanding these parameters will aid in design and analysis of bolted connections in critical applications. Ensuring safe and reliable operation of bolted connections is essential in many engineering applications, and the results of this lab will be useful in future work.

7 Technical Recommendations

One transducer reading for the washer and bolt was measured for a given external load for the zero preload trial. This totaled to nine measurements for the linear regression. Future work could expand by taking three measurements for the washer and bolt transducer for a given external load, which should reduce the standard error of the regression. More thorough calibration of the washer and bolt transducer could be done to reduce the bias uncertainty.

In addition, hysteresis was not accounted for in the zero preload trial, as measurements were only taken upwards. Performing the measurements in both directions could be used to determine the effect of hysteresis on the modulus of elasticity. These recommendations could be used to increase the accuracy and confidence of the modulus of elasticity.

The theoretical value for the stiffness of the member did not match the experimental value, and the effects rippled through the theoretical separation point calculations. The stress distribution angle was assumed to be $45°$, and was never verified. Future work could be done to verify the stress distribution angle, which could be used to determine the stiffness of the member with higher accuracy and confidence.

The torque wrench was not calibrated, and the uncertainty of the torque wrench was not determined. The torque wrench may be inaccurate, as the operator can go past the click point. Utilizing a digital solution could be more accurate to apply the torque on the bolt.

8 References

[1] A. J. Wheeler and A. R. Ganji, *Introduction to engineering experimentation*, 3rd ed. Upper Saddle River, N.J: Pearson Higher Education, 2010, oCLC: ocn459211853.

[2] T. pandas development team, "pandas-dev/pandas: Pandas," Feb. 2024. [Online]. Available: https://doi.org/10.5281/zenodo.10697587

A Appendix: Zero Preload Data Analysis

This Appendix provides the analysis of the experimental data "bolt stiffness and washer calibration (finger tight)" to determine the modulus of elasticity of the bolt. In addition, error analysis was performed with a confidence of 95% to determine the corresponding uncertainty. In addition, the washer calibration was also performed to determine the relationship between the external load, washer transducer reading, and washer strain.

A.1 Modulus of Elasticity Analysis

Table A.3: Bolt Stiffness and Washer Calibration data

External Load, P (kN)	Bolt Out, V_b (V)	Washer Out, V_w (V)	Bolt Strain, ε_b
0	0.006	-0.003	6.00E-06
1	0.085	-0.212	8.50E-05
2	0.168	-0.441	1.68E-04
3	0.253	-0.667	2.53E-04
4	0.335	-0.888	3.35E-04
5	0.417	-1.08	4.17E-04
6	0.498	-1.27	4.98E-04
7	0.581	-1.46	5.81E-04
7.5	0.662	-1.547	6.62E-04

The experimental data was collected and shown in Table A.3. Sample calculations will be shown for external load of 0 kN. The bolt strain was calculated from Eq. (10),

$$\begin{aligned}\varepsilon &= \frac{4V_b}{K_g E_{\text{in}} G}\\ \varepsilon &= \frac{40.006\,\text{V}}{2 \cdot 5\,\text{V} \cdot 400}\\ &= 6.00 \times 10^{-6}\end{aligned}$$

where E_o is transducer reading, K_g is the gauge factor, E_{in} is the voltage input, and G is the gain set. From the experimental setup, $K_g = 2$, $E_{\text{in}} = 5$, and $G = 400$.

Next, a linear regression of the external load (P) and bolt strain (ε_b), forced through the origin, was performed on the data in Table A.3 to determine the modulus of elasticity. The linear regression equation was determined using `=LINEST()` from Excel. The results are shown in Table A.4. The equation is then

$$\varepsilon_b = 8.47134 \times 10^{-5} P$$

or in another form,

$$\frac{P}{\varepsilon_b} = \frac{1}{8.47134 \times 10^{-5}}$$

The area where the force was applied is the outer diameter, d_o, minus the inner diameter, d_i, of the

Table A.4: Linear Regression Results

Parameter	Value
Slope (mm/kN)	8.47134E-05
Slope Standard Error, S_a	8.20567E-07
R^2	0.999249954

bolt. From the experimental setup, $d_o = 0.371\,\mathrm{in}$ and $d_i = 0.155\,\mathrm{in}$. The area is then

$$\begin{aligned} A_1 &= \frac{\pi}{4}(d_o^2 - d_i^2) \\ &= \frac{\pi}{4}((0.371\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 - (0.155\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2) \\ &= 57.570\,\mathrm{mm^2} \end{aligned}$$

The modulus of elasticity is then

$$\begin{aligned} E &= \frac{P}{\varepsilon_b A_1} \\ &= \frac{1\,\mathrm{kN}}{8.47134 \times 10^{-5} \times 57.570\,\mathrm{mm^2}} \\ &= \boxed{205\,\mathrm{GPa}} \end{aligned}$$

A.2 Modulus of Elasticity Error Analysis

A.2.1 Error Propagation Derivation

Because I got dinged for not showing "full" error propagation, I will rewrite the derivation here because I am upset.

If we know how a quantity of interest depends on other, directly measurable quantizes, it is posable to estimate the uncertainty of this "output" quantity based on the uncertainties in the measured quantizes. For example, we can calculate the uncertainty associated to a volume based on the uncertainty of the measurement of the individual dimensions.

Consider as results, R, which is a function of n variables, $x_1, \ldots, x_n$:

$$R = f(x_1, \ldots, x_n)$$

If the individual measurands, x_i, have an associated uncertainty w_{x_i}, what is teh uncertainty of w_R of the result R?

Defining $x := (x_1, \ldots, x_n)$, and $x_m := (x_{m,1}, \ldots, x_{m,n})$, perform the Taylor series expansion of $R = f(x)$ about the point $x = x_m$, taking $x_i - x_{m,i} = w_{x_i}$:

$$R = f(x_m) + \left.\frac{\partial f}{\partial x_1}\right|_{x=x_m} \underbrace{(x_1 - x_{m,1})}_{w_{x_1}} + \ldots + \left.\frac{\partial f}{\partial x_n}\right|_{x=x_m} \underbrace{(x_n - x_{m,n})}_{w_{x_n}} + \text{H.O.T.}$$

$$\underbrace{R - f(x_m)}_{w_R} = \left.\frac{\partial f}{\partial x_1}\right|_{x=x_m} w_{x_1} + \ldots + \left.\frac{\partial f}{\partial x_n}\right|_{x=x_m} w_{x_n} + \text{H.O.T.}$$

The higher-order terms contain quadratic terms $w_{x_i} w_{x_j}$, cubic terms $w_{x_i} w_{x_j} w_{x_k}$, and so on. Assuming the individual uncertainties w_{x_i} are small, we can take these higher-order terms as zero, giving

$$w_R = \sum_{i=1}^{n} \left| \left.\frac{\partial f}{\partial x_i}\right|_{x=x_m} w_{x_i} \right|$$

However, this is the worst-case uncertainty, and is an overly conservative estimate. A better estimate is to use the root of sum of squares

$$w_R = \sqrt{\sum_{i=1}^{n} \left[\left.\frac{\partial f}{\partial x_i}\right|_{x=x_m} w_{x_i} \right]^2} \tag{23}$$

If the confidence levels associated to the individual uncertainties w_{x_i} are all identical (for instance

95%), the confidence level of the result w_R will be the same.

The key assumption behind RSS is that the set of measured variables $x_1, \ldots, x_n$ are **statistically indecent**.IIf this is not the case, a different formulation needs to be used. Also note that all uncertainties w_{x_i} need to be small such that the first-order Taylor series approximation holds.

Consider the case where the result R is dependent only on the product of the measured variables, $x_1, \ldots, x_n$ with associated uncertainties $w_{x_1}, \ldots, w_{x_n}$ as

$$R = Cx_1^{c_1} x_2^{c_2} \ldots x_n^{c_n}$$

where C and $c_1, \ldots, c_n$ are constants. In this case, the RSS formula gives

$$\begin{aligned} w_R &= \sqrt{\left(Cc_1 x_1^{c_1-1} w_{x_1}\right)^2 + \ldots + \left(Cc_n x_1^{c_1} x_2^{c_2} \ldots c_n x_n^{c_n-1} w_{x_n}\right)^2} \\ \implies \frac{w_R}{|R|} &= \sqrt{\left(\frac{c_1 w_{x_1}}{x_1}\right)^2 + \ldots + \left(\frac{c_n w_{x_n}}{x_n}\right)^2} \end{aligned} \tag{24}$$

A.2.2 Modulus of Elasticity Error Analysis

The uncertainty of slope was determined using the standard error of the slope, S_a, from the linear regression in Table A.4 at a confidence level of 95%. The t-distribution value was determined by

$$\begin{aligned} \alpha/2 &= \frac{1-0.95}{2} = 0.025 \\ n-2 &= 9-2 = 7 \\ t_{\alpha/2,n-2} &= 2.3646 \end{aligned}$$

The uncertainty of the slope is then [1]

$$\begin{aligned} \delta \text{slope} &= t_{\alpha/2,n-2} \cdot S_a \\ &= 2.3646 \cdot 8.20567 \times 10^{-7} \\ &= 1.94 \times 10^{-6}\,\text{kN}^{-1} \end{aligned}$$

The function for modulus of elasticity is

$$\begin{aligned} E &= P^1 \varepsilon_b^{-1} A_1^{-1} \\ &= (\text{slope})^{-1} A_1^{-1} \end{aligned}$$

This is the purely multiplicative case for error propagation [1]. Which is

$$\frac{\delta E}{|E|} = \sqrt{\left(\frac{1}{\text{slope}}\delta\text{slope}\right)^2 + \left(\frac{1}{A_1}\delta A_1\right)^2}$$

Assuming the error for A_1 is negligible, then by Eq. (24), the uncertainty of the modulus of elasticity is then

$$\begin{aligned}\delta E &= E\left|\frac{\delta\text{slope}}{\text{slope}}\right| \\ &= 205\,\text{GPa}\frac{1.94\times10^{-6}\,\text{kN}^{-1}}{8.47134\times10^{-5}\,\text{kN}^{-1}} \\ &= \boxed{\pm4.70\,\text{GPa}}\end{aligned}$$

This quantity is relatively small compared to the modulus of elasticity, and is expected. Please, let me know if I need to show an even more formal statistical derivation of error propagation to satisfy any pedantic needs.

A.3 Washer Calibration Analysis

The external load and washer transducer readings from Table A.3 were fitted with a linear regression through the origin. The linear regression equation was determined using `=LINEST()` from Excel. The equation was

$$E_{o,w} = -0.211P$$

Converting to strain using Eq. (10), where $K_g = 2$, $E_{\text{in}} = 5$, and $G = 400$:

$$\begin{aligned}\varepsilon_w &= \frac{4V_w}{K_g E_{\text{in}} G} \\ &= \frac{4 - 0.211\,\text{V}P}{2\cdot5\,\text{V}\cdot400} \\ &= -2.11\times10^{-4}P\end{aligned}$$

B Appendix: Preload-Torque Test Data Analysis

The following is the analysis of the preload-torque test data. The data was collected from the experiment and is shown in Table B.5. The data was then analyzed to determine the preload, preload uncertainty, and torque coefficient. The following sections will detail the analysis of the data and the results of the analysis.

B.1 Preload vs Torque Analysis

The results from the experiment are shown in Table B.5. Sample calculations will be shown for the second row of the table. First, the torque was converted to metric units.

Table B.5: Torque-Preload Test at Zero External Load

Torque, T (in-lb)	Torque, T (Nm)	Bolt Transducer, V_b (V)	Washer Transducer, V_w (V)	Bolt Strain, ε_b	Preload, F_i (kN)	Preload Uncertainty, δF_i (± kN)
0	0	-0.002	-0.001	-2.00E-06	-0.0236	0.366
25	2.825	0.140	-0.311	1.40E-04	1.65	0.368
50	5.649	0.294	-0.615	2.94E-04	3.47	0.375
75	8.474	0.447	-0.907	4.47E-04	5.28	0.386
100	11.298	0.602	-1.203	6.02E-04	7.11	0.401
125	14.123	0.778	-1.519	7.78E-04	9.18	0.422

$$\begin{aligned} T &= 25\,\mathrm{in}-\mathrm{lb} \times 0.113\,\mathrm{N\,m^{-1}\,in}-\mathrm{lb}^{-1} \\ &= 2.825\,\mathrm{N\,m} \end{aligned}$$

The bolt strain, ε_b, was then calculated by

$$\begin{aligned} \varepsilon_b &= \frac{4V_b}{K_g E_{\text{in}} G} \\ &= \frac{4 \times 0.140\,\mathrm{V}}{2 \times 5\,\mathrm{V} \times 400} \\ &= 1.40 \times 10^{-4} \end{aligned}$$

The preload, F_i, was then calculated by

$$\begin{aligned} F_i &= E_b \varepsilon_b A_1 \\ &= 205.046\,\text{GPa} \times 1.40 \times 10^{-4} \times 57.570\,\text{mm}^2 \\ &= \boxed{1.65\,\text{kN}} \end{aligned}$$

B.2 Uncertainty Analysis of Preload

A repeatability test was performed at 50 lb-in of torque with no external load. The results of this test are shown in The standard deviation was determined with Excel to be $S_{V_b} = 0.0250\,\text{V}$. Using

Table B.6: Repeatability Test at 50 lb-in of Torque and Zero External Load

Trial #	Bolt Transducer, V_b (V)	Washer Transducer (V)
1	0.372	-0.701
2	0.321	-0.684
3	0.354	-0.718
4	0.312	-0.654
5	0.327	-0.679

a confidence level of 95%, the t-distribution value was determined by

$$\begin{aligned} \alpha/2 &= \frac{1-0.95}{2} = 0.025 \\ n-1 &= 5-1 = 4 \\ t_{\alpha/2,n-1} &= 2.776 \end{aligned}$$

The precision uncertainty is then

$$\begin{aligned} P_{V_b} &= t_{\alpha/2,n-1} \cdot \frac{S_{V_b}}{\sqrt{n}} \\ &= 2.776 \cdot \frac{0.025\,\text{V}}{\sqrt{5}} \\ &= 0.031\,\text{V} \end{aligned}$$

Defining bias uncertainty as resolution, $B_{V_b} = 0.001$, the total uncertainty is then

$$\begin{aligned}\delta V_b &= \sqrt{P_{V_b}^2 + B_{V_b}^2} \\ &= \sqrt{(0.031\,\mathrm{V})^2 + (0.001\,\mathrm{V})^2} \\ &= 0.031\,\mathrm{V}\end{aligned}$$

Since the equation for F_i is purely multiplicative, by Eq. (24), the uncertainty of the preload for the second row of Table B.5 is then

$$\begin{aligned}\delta F_i &= F_i\sqrt{\left(\frac{\delta V_b}{V_b}\right)^2 + \left(\frac{\delta E_b}{E_b}\right)^2} \\ &= 1.65\,\mathrm{kN}\sqrt{\left(\frac{0.031\,\mathrm{V}}{0.140\,\mathrm{V}}\right)^2 + \left(\frac{4.70\,\mathrm{GPa}}{205.046\,\mathrm{GPa}}\right)^2} \\ &= 0.368\,\mathrm{kN}\end{aligned}$$

If this is not ”complete” enough for error propagation, please, please refer to Section A.2.1 for the complete derivation of the error propagation formula.

B.3 Torque Coefficient Analysis

Applyig linear regression, forced through the origin, to the data in Table B.5 using `=LINEST()` from Excel, the equation is,

$$\begin{aligned} F_i &= 0.636T \\ \implies \frac{T}{F_i} &= \frac{1}{0.636}\mathrm{mm}^{-1}\end{aligned}$$

where F_i is in kN and T is in Nm. From Eq. (16), the torque coefficient is then

$$\begin{aligned}K &= \frac{T}{F_i d} \\ &= \frac{1}{0.636\,\mathrm{mm}^{-1} \times 0.375\,\mathrm{in} \times 25.4\,\mathrm{mm\,in}^{-1}} \\ &= \boxed{0.167}\end{aligned}$$

C Appendix: Shakedown Test Results

This section contains the results of the shakedown test. The shakedown test was performed to determine if the bolted connection was subjected to any torsional loading. The shakedown test was performed by ramping the external load from 0 kN to 7.5 kN back down to 0 kN three times in succession. The voltage at the end of each ramp was recorded. The strain transducer reading varied by ± 0.005 V, which was the same magnitude as the resolution of the strain transducer. This indicates that the bolt was not subjected to any torsional loading. The results were consistent with expectations, as the bolt was not subjected to any torsional loading.

Table C.7: Shakedown Test Results

	External Load (kN)	Bolt Out (V)	Washer Out (V)
Without Gasket	$0 \rightarrow 7.5 \rightarrow 0$	0.386	-0.731
	$0 \rightarrow 7.5 \rightarrow 0$	0.382	-0.720
	$0 \rightarrow 7.5 \rightarrow 0$	0.381	-0.715
With Gasket	$0 \rightarrow 7.5 \rightarrow 0$	0.327	-0.540
	$0 \rightarrow 7.5 \rightarrow 0$	0.323	-0.534
	$0 \rightarrow 7.5 \rightarrow 0$	0.321	-0.530

From the results in Table C.7, the bolt transducer reading varied by ± 0.005 V, which was the same magnitude as the resolution. This indicates that the bolt was not subjected to any torsional loading. The washer transducer reading varied by ± 0.010 V, which was a magnitude higher than the resolution, but still small relative to the measurement value. This indicates that the washer was not subjected to any torsional loading.

D Appendix: Bolt Stiffness Calculations

D.1 Bolt Geometric Properties

The lengths of sections 1 and 2 were given as 0.91 in and 1.471 in, respectively. Section 3 is to be determined. The total length of the member was 63.5 mm. Then,

$$\begin{aligned} L_3 &= 63.5\,\mathrm{mm} - 0.91\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}} - 1.471\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}} \\ &= 3.0226\,\mathrm{mm} \end{aligned}$$

The cross-sectional area of each section was determined by,

$$\begin{aligned} A_1 &= \frac{\pi}{4}(d_o^2 - d_i^2) \\ &= \frac{\pi}{4}((0.371\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 - (0.155\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2) \\ &= 57.570\,\mathrm{mm^2} \end{aligned}$$

then,

$$\begin{aligned} A_2 &= \frac{\pi}{4}d_2^2 \\ &= \frac{\pi}{4}(3.71\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 \\ &= 69.744\,\mathrm{mm^2} \end{aligned}$$

lastly,

$$\begin{aligned} A_3 &= \frac{\pi}{4}d_3^2 \\ &= \frac{\pi}{4}(3.75\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 \\ &= 71.256\,\mathrm{mm^2} \end{aligned}$$

The geometric properties of the bolt are summarized in Table D.8.

Table D.8: Bolt Stiffness Calculations

Length of section, L (in)	bolt stiffness (mm)	Cross Sectional Area, A_s (mm^2)	Stiffness, k (MN/m)	1/k (m/MN)
0.91	23.114	57.57	510.708	0.001958
1.471	37.3634	69.744	382.745	0.002613
0.119	3.0226	71.256	4833.819	0.000207

D.2 Bolt Stiffness

Sample calculations for Table D.8 will be shown for the stiffness of section 1. The stiffness of section 2 and 3 will be calculated in the same manner. The stiffness of section 1 was calculated by,

$$\begin{aligned} k_1 &= \frac{E_b A_1}{L_1} \\ &= \frac{205.046\,\mathrm{GPa} \times 57.570\,\mathrm{mm}^2}{23.114\,\mathrm{mm}} \\ &= 510.708\,\mathrm{MN\,m^{-1}} \end{aligned}$$

Where E_b was determined to be $205\,\mathrm{GPa}$ in Appendix A. Then,

$$\frac{1}{k_1} = 0.001958\,\mathrm{m\,MN^{-1}}$$

D.3 Total Bolt Stiffness

The total bolt stiffness was calculated by Eq. 14. The total bolt stiffness was then,

$$\begin{aligned} k_b &= \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} \\ &= \left(\frac{1}{0.001958\,\mathrm{m\,MN^{-1}}} + \frac{1}{0.002613\,\mathrm{m\,MN^{-1}}} + \frac{1}{0.000207\,\mathrm{m\,MN^{-1}}}\right)^{-1} \\ &= \boxed{209.308\,\mathrm{MN\,m^{-1}}} \end{aligned}$$

E Appendix: Theoretical Member Stiffness Calculations

From Eq. (13), the stiffness of the member was estimated to be,

$$k_{m,\text{th}} = \frac{\pi E_b d}{2\ln\left(\frac{5(L+0.5d)}{L+2.5d}\right)}$$

where E_b is the modulus of elasticity of the bolt, d is the diameter of the bolt, and L is the length of the member. The modulus of elasticity of the bolt was determined to be $205.046\,\mathrm{GPa}$ in Appendix **??**. The diameter of the bolt was $0.371\,\mathrm{in}$ and the length of the member was $63.5\,\mathrm{mm}$. Then,

$$\begin{aligned} k_{m,\text{th}} &= \frac{\pi \times 205.046\,\mathrm{GPa} \times 0.371\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}}}{2\ln\left(\frac{5(63.5\,\mathrm{mm}+0.5\times 0.371\,\mathrm{in}\times 25.4\,\mathrm{mm\,in^{-1}})}{63.5\,\mathrm{mm}+2.5\times 0.371\,\mathrm{in}\times 25.4\,\mathrm{mm\,in^{-1}}}\right)} \\ &= \boxed{2222.774\,\mathrm{MN\,m^{-1}}} \end{aligned}$$

F Appendix: Experimental Member Stiffness Calculations

F.1 Experimental Data

Table F.9: Various External Loads and Bolt Force at 60 in-lb Torque Without Gasket

	External Load, P	Bolt Out, V_b	Washer Out, V_w	Bolt Strain, ε_b	Force, F_i
		(kN)	(V)	(V)	(kN)
Without Gasket	0	0.382	-0.720	0.000382	4.509
	1	0.391	-0.751	0.000391	4.616
	2	0.400	-0.783	0.000400	4.722
	3	0.411	-0.818	0.000411	4.852
	4	0.421	-0.855	0.000421	4.970
	5	0.436	-0.901	0.000436	5.147
	6	0.498	-1.068	0.000498	5.879
	7	0.578	-1.226	0.000578	6.823
	7.5	0.619	-1.300	0.000619	7.307
With Gasket	0	0.315	-0.528	0.000315	3.718
	1	0.344	-0.593	0.000344	4.061
	2	0.358	-0.674	0.000358	4.226
	3	0.389	-0.766	0.000389	4.592
	4	0.426	-0.856	0.000426	5.029
	5	0.469	-0.941	0.000469	5.536
	6	0.524	-1.036	0.000524	6.186
	7	0.589	-1.148	0.000589	6.953
	7.5	0.627	-1.212	0.000627	7.401

Sample calculations will be shown for the first row of Table F.9. The bolt strain, ε_b, was calculated by

$$\begin{aligned}\varepsilon_b &= \frac{4V_b}{K_g E_{\text{in}} G} \\ &= \frac{4 \times 0.382\,\text{V}}{2 \times 5\,\text{V} \times 400} \\ &= 0.000382\end{aligned}$$

The force, F_i, was then calculated by

$$\begin{aligned}F_i &= E_b \varepsilon_b A_1 \\ &= 205.046\,\text{GPa} \times 0.000382 \times 57.570\,\text{mm}^2 \\ &= \boxed{4.509\,\text{kN}}\end{aligned}$$

F.2 Experimental Member Stiffness Without Gasket

Applying linear regression to the preseperation data in Table F.9 yields the following equation from `=LINEST()` in Excel,

$$F_i = \underbrace{0.1157}_{C} P + 4.5022$$

Comparing the form of the linear regression to Eq. (17), $C = 0.1157$. Then, by Eq. (18),

$$\begin{aligned}C &= \frac{k_b}{k_b + k_m} \\ \implies k_{m,\text{exp}} &= \frac{k_b}{\frac{1}{C} - 1} \\ &= \frac{209.308\,\text{MN}\,\text{m}^{-1}}{\frac{1}{0.1157} - 1} \\ &= \boxed{1599.998\,\text{MN}\,\text{m}^{-1}}\end{aligned}$$

Compared to the theoretical value of $2222.774\,\text{kN}\,\text{m}^{-1}$, the error is

$$\begin{aligned}\text{Error} &= \frac{k_{m,\text{th}} - k_{m,\text{exp}}}{k_{m,\text{th}}} \times 100\% \\ &= \frac{2222.774\,\text{MN}\,\text{m}^{-1} - 1599.998\,\text{MN}\,\text{m}^{-1}}{2222.774\,\text{MN}\,\text{m}^{-1}} \times 100\% \\ &= \boxed{28.0\%}\end{aligned}$$

G Appendix: Joint Separation

G.1 Experimental Seperation

The two regressions of the data from Table F.9,

$$\begin{aligned} F_{b,\text{pre}} &= 0.1157P + 4.5022 \\ F_{b,\text{post}} &= 0.8687P + 0.7507 \end{aligned}$$

The separation point is when $F_{i,\text{pre}} = F_{i,\text{post}}$,

$$\begin{aligned} 0.1157P_{\text{exp}} + 4.5022 &= 0.8687P_{\text{exp}} + 0.7507 \\ P_{\text{exp}} &= \frac{3.7515}{0.753} \\ &= \boxed{4.98\,\text{kN}} \end{aligned}$$

Then,

$$\begin{aligned} F_{b,\text{sep}} &= 0.1157 \times 4.98\,\text{kN} + 4.5022 \\ &= \boxed{5.08\,\text{kN}} \end{aligned}$$

G.2 Theoretical Separation

The torque load was 60 in-lb for the data in Table F.9. From Eq. (16),

$$\begin{aligned} F_i &= \frac{T}{Kd} \\ &= \frac{60\,\text{in} - \text{lb} \times 0.112984\,\text{N}\,\text{m}\,\text{in} - \text{lb}^{-1}}{0.167 \times 0.375\,\text{in} \times 25.4\,\text{mm}\,\text{in}^{-1}} \\ &= \boxed{4.26\,\text{kN}} \end{aligned}$$

Then calculating C_{th} by Eq. (18),

$$\begin{aligned} C_{\text{th}} &= \frac{k_b}{k_b + k_{m,\text{th}}} \\ &= \frac{209.308\,\text{MN}\,\text{m}^{-1}}{209.308\,\text{MN}\,\text{m}^{-1} + 2222.774\,\text{MN}\,\text{m}^{-1}} \\ &= 0.116 \end{aligned}$$

Then by Eq. (19),

$$\begin{aligned} P &= \frac{F_i}{1 - C_{\text{th}}} \\ &= \frac{4.26\,\text{kN}}{1 - 0.116} \\ &= \boxed{4.67\,\text{kN}} \end{aligned}$$

G.3 Theoretical vs. Experimental Separation

The error is then,

$$\begin{aligned} \text{Error} &= \frac{P_{\text{th}} - P_{\text{exp}}}{P_{\text{th}}} \times 100\% \\ &= \frac{4.82\,\text{kN} - 4.67\,\text{kN}}{4.82\,\text{kN}} \times 100\% \\ &= \boxed{6.78\%} \end{aligned}$$

H Appendix: Dynamic Loading

Table H.10: Dynamic Loading Summary for Various Torques and Gasket Conditions

	Torque, T (in-lb)	Max Stress, σ_{max} (MPa)	Min Stress, σ_{min} (MPa)	Mean Stress, σ_{mean} (MPa)	Alternating Stress, σ_{a} (MPa)
With Gasket	0	105.035	61.804	83.420	21.615
	60	110.300	88.864	99.582	10.718
	75	124.101	110.488	117.295	6.806
	125	174.584	167.250	170.917	3.667
No Gasket	0	105.330	62.098	83.714	21.616
	60	106.347	84.153	95.250	11.097
	75	108.671	101.337	105.004	3.667
	125	166.652	163.258	164.955	1.697

The raw transducer data was converted using Eq. (10) in a similar fashion to Appendix A. Sample calculations will be shown for the first row of Table H.10. The min and max stress were calculated by `.max()` and `.min()` from Pandas [2]. The mean stress was calculated by

$$\begin{aligned}\sigma_{\text{mean}} &= \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \\ &= \frac{105.035\,\text{MPa} + 61.804\,\text{MPa}}{2} \\ &= 83.420\,\text{MPa}\end{aligned}$$

The alternating stress was then calculated by

$$\begin{aligned}\sigma_{\text{a}} &= \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \\ &= \frac{105.035\,\text{MPa} - 61.804\,\text{MPa}}{2} \\ &= 21.615\,\text{MPa}\end{aligned}$$