MEC E 403
Lab 2: Bolted Connections

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Abstract

to be done

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1 Nomenclature

Symbol	Description	Units
b	Blade height at the exit	m
β_2	Blade angle	rad
D_1	Pump suction line diameter	m
D_2	Pump discharge line diameter	m
g	Gravitational acceleration	m/s^2
H	Stagnation head	m
H'_{ideal}	Ideal shutoff head	m
H_t	Transducer head	m
H'_{thumb}	Rule of thumb shutoff head	m
$\dot{m}$	Mass flow rate of water	kg/s
N	Number of pump impeller blades	-
ΔP	Pressure change	Pa
Q	Volumetric flow rate	m^3/s
r_2	Impeller radius	m
t	Time	s
T	Torque exerted by the pump impeller	Nm
U	Impeller tip speed	m/s
v_{2r}	Radial component of the exit velocity	m/s
$v_{2\theta}$	Tangential component of the exit velocity	m/s
w	Blade width	m
η	Pump efficiency	%

ρ	Density of water	kg/m^3
Ω	Pump impeller angular speed	rad/s
Φ	Flow coefficient	-
Ψ	Head coefficient	-

2 Introduction

The focus of this lab was to analyze the performance of centrifugal pumps at different operating speeds, valve orientations, along with both parallel and series configurations. The data obtained from the lab testing is used to determine head, flow, and efficiency. The lab results are compared with the manufacturer provided data sheets and theoretical calculations. The parameters measured during each test are the moment arm, time to collect water (200lbs), and the transducer reading. The goal of determining the performance of different pumping configurations is useful for understanding how to achieve optimal efficiency in a piping system, and the effect certain factors (i.e., pump speed) have.

3 Procedure

3.1 Equipment

3.2 Procedure

3.2.1 Zero Preload

1. Attach the bolted connection to the MTS machine with the nut attached ”finger tight” (without a gasket).
2. Load the bolt 8 times with a range of 0 - 7.5 kN.
3. Record the external load and bridge imbalance at each load (0, 1, 2, 3, 4, 5, 6, 7, 7.5).

3.2.2 Repeatability Test

1. Attach the bolt to the MTS machine.
2. Use the torque wrench to apply a preload of 50 in-lb.
3. Record the voltage readings from the bolt and washer gauges.
4. Loosen the bolt to remove any preload.
5. Repeat steps 2-4 four more times.

3.2.3 Zero Loading (Torque Test)

1. Attach the bolt to the MTS machine, without a gasket.
2. Set the external load to 0 kN.
3. Record the voltage readings from the bolt and washer gauges.

4. Increase the torque by 25 in-lb.

5. Record the voltage readings from the bolt and washer gauges.

6. Repeat steps 4 and 5 four more times, obtaining readings from 0 to 125 in-lb of torque (0, 25, 50, 75, 100, 125).

3.2.4 Static Loading

1. Attach the bolt to the MTS machine (without a gasket).

2. Tighten the bolt to 60 in-lb of torque.

3. Set the external load on the MTS machine to 0 kN.

4. Set the external load on the MTS machine to 7.5 kN.

5. Record the readings from the bolt and washer.

6. Repeat steps 3-5 two more times, totaling three readings (shakedown test).

7. Leave the bolt assembled, and apply loads ranging from 0-7.5 kN (0, 1, 2, 3, 4, 5, 6, 7, 7.5). Record the output readings from the bolt and washer at each load.

8. Set the load back to 0 kN.

9. Disassemble and reassemble the joint with the gasket in place.

10. Repeat steps 2-9 with a gasket.

3.2.5 Dynamic Loading

1. Attach the bolt to the MTS machine (without a gasket).

2. Set the bolt to the ”finger tight” torque setting.

3. Apply an external load of 5 kN and an alternating load of 1.25 kN at 0.3 Hz.

4. Record data for at least 10 cycles.

5. Repeat steps 3-4 at different torque settings of 60, 75, and 125 in-lb.

6. Disassemble and reassemble the joint with the gasket in place.

7. Repeat steps 2-5 with a gasket.

4 Theory

4.1 Mechanics of Bolted Connections Loading

The typical bolted connection is shown in Figure 1a. The key forces in the above diagram are the preload, F_i, and the external load, P. This connection can be viewed as an analogy to the spring system seen in Figure 1b. By Hooke's law, the deflection of the bolt and the member are given by

(a) (b)

Figure 1: a) Bolted Joint Diagram with Preload and External Load, b) Spring Analogy

the equations below:

$$\delta_b = \frac{F_i}{k_b} \tag{1}$$

$$\delta_m = \frac{F_i}{k_m} \tag{2}$$

When the external load, P, is applied to the joint, a change in the deformation of the bolt and the reduction of compression in the joined members occurs. Similar to deflection, the change in deformation can be calculated using the equations:

$$\Delta\delta_b = \frac{P_b}{k_b} \tag{3}$$

$$\Delta\delta_m = \frac{P_m}{k_m} \tag{4}$$

If the members are not separated, the deformation in the member and the bolt are equivalent, shown by the relation below:

$$\frac{P_b}{k_b} = \frac{P_m}{k_m} \tag{5}$$

The total load on the bolt and the member must equal the sum of the change in load of the bolt, P_b, and the member, P_m,

$$P = P_m + P_b$$

Using (5), the change in load of the bolt and the member can be expressed as:

$$P_b = \frac{k_b P}{k_b + k_m} \tag{6}$$

$$P_m = \frac{k_m P}{k_b + k_m} \tag{7}$$

Similarly, the total loads on the bolt and the member are given by:

$$F_b = F_i + P_b \tag{8}$$

$$F_m = F_i + P_m \tag{9}$$

4.2 Quarters Bridge Equations

The instrumented bolt uses a Wheatstone quarter bridge to measure strain. The voltage reading from the bridge, V_o, can be expressed using the input voltage, V_{in}, gauge factor, K_g, gain, G, and strain, ε.

$$\varepsilon = \frac{4V_o}{K_g V_{in} G} \tag{10}$$

4.3 Stress-Strain Relationship

By Hooke's law, the stress-strain relationship is given by:

$$\sigma = E_b \varepsilon \tag{11}$$

Since stress is force per unit area,

$$F_b = A_s \sigma \tag{12}$$

4.4 Member Stiffness

Members in a bolted connection can be viewed as a series of springs. Equivalent stiffness for this system is given by:

$$\frac{1}{k_m} = \sum_{i=1}^{n} \frac{1}{k_i} \tag{13}$$

$$\tag{14}$$

where k_i is the stiffness of the ith section of the member.

For members with a gasket, loading can be estimated by assuming the load spreads at a fixed 45° angle. The compression of each element is then divided into infinitesimally small annular elements. It can be shown that the stiffness for two identical members bolted together is given by:

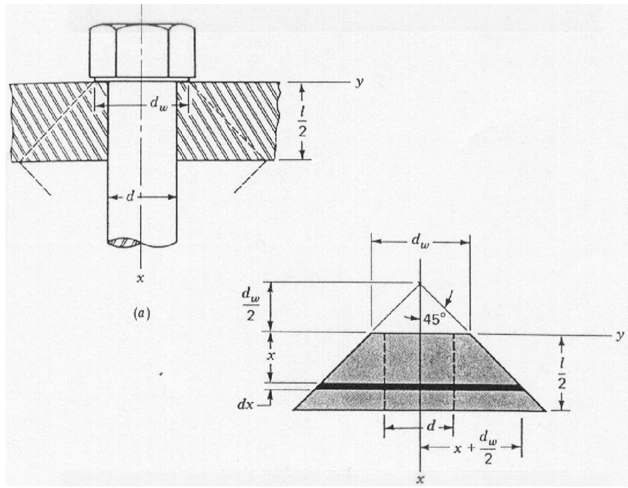

Figure 2: Analysis of the compression of members in a bolted connection

$$k_m = \frac{\pi E_b d}{2 \ln \left(\frac{5(L+0.5d)}{L+2.5d} \right)} \tag{15}$$

4.5 Torque Requirement for Preloading

From basic screw-thread theory, the torque required to preload a bolt is given by:

$$T = \frac{F_i d}{2} \left[\frac{L + \pi \mu d_m \sec \alpha}{\pi d_m - \mu L \sec \alpha} \right]$$

it can be shown that

$$\begin{aligned} T &= F_i d \left[\left(\frac{d_m}{2d} \right) \left(\frac{\tan \lambda + \mu \sec \alpha}{1 - \mu \tan \lambda \sec \alpha} \right) + 0.625 \mu_c \right] \\ &= K d F_i \end{aligned} \tag{16}$$

4.6 Bolt Preload for Static Loading

Preloading the bolt is meant to prevent the jointed member from separating and the bolt from yielding. Using Equations (6) and (8), the total load on the bolt can be given by:

$$F_b = F_i + CP \tag{17}$$

where the constant C is defined as:

$$C = \frac{k_b}{k_b + k_m} \tag{18}$$

at the point of joint separation, $F_b = P$. Rearranging (17) gives:

$$F_i = P(1 - C) \tag{19}$$

To avoid yielding, a safety factor is introduced, N. Rearranging (17) gives:

$$F_i = \frac{A_t \sigma_y}{N} - CP \tag{20}$$

4.7 Bolt Preload for Dynamic Loading

Cyclic loading cycles are used to vary the load on a bolt over time. The two parameters often analyzed from these trials are the mean and alternating stresses.

$$\sigma_m = \frac{F_{\text{max}} + F_{\text{min}}}{2A_s} \tag{21}$$

$$\sigma_a = \frac{F_{\text{max}} - F_{\text{min}}}{2A_s} \tag{22}$$

The modified Goodman criteria states:

$$\frac{\sigma_a}{\sigma_e} + \frac{\sigma_m}{\sigma_{ut}} = 1$$

where σ_e is the endurance limit, and σ_{ut} is the ultimate tensile strength. It can be shown the follow holds,

$$F_i = A_s \sigma_{ut} - \frac{NCP}{2}\left[\frac{\sigma_{ut}}{\sigma_e} - 1\right]$$

5 Results and Discussion

5.1 Main Results

6 Conclusion

6.1 Technical Recommendations

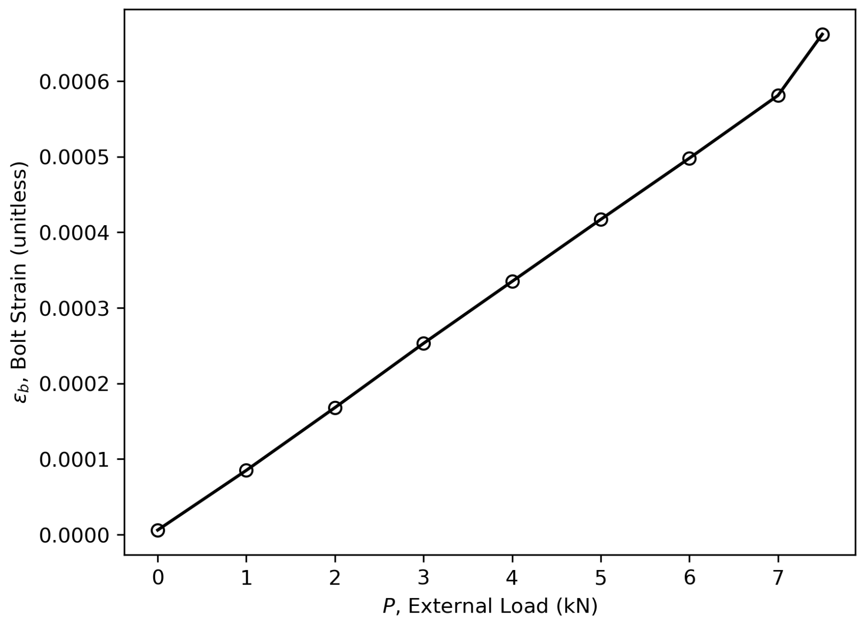

Figure 3: External Load vs. Bolt Strain

7 References

[1] A. J. Wheeler and A. R. Ganji, *Introduction to engineering experimentation*, 3rd ed. Upper Saddle River, N.J: Pearson Higher Education, 2010, oCLC: ocn459211853.

[2] T. pandas development team, "pandas-dev/pandas: Pandas," Feb. 2024. [Online]. Available: https://doi.org/10.5281/zenodo.10697587

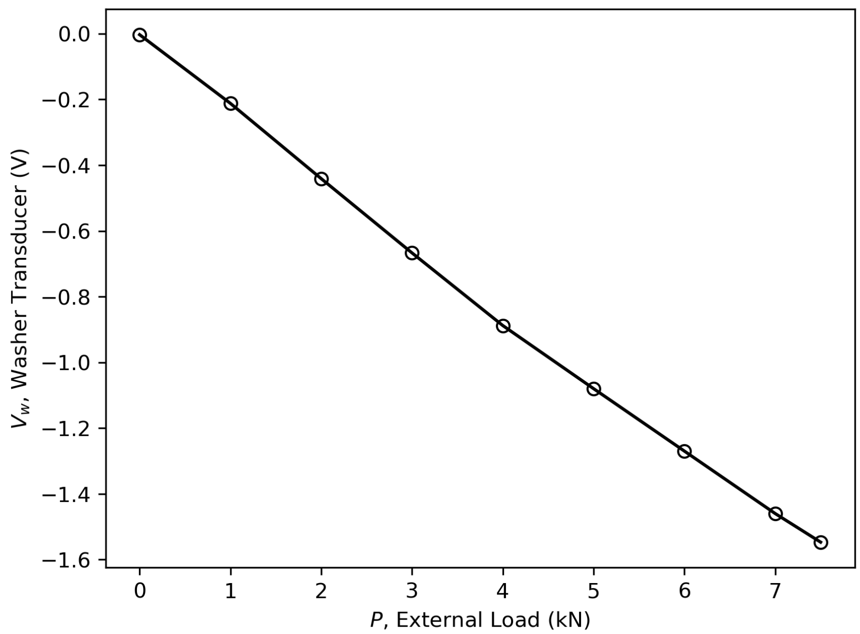

Figure 4: External Load vs. Washer Transducer

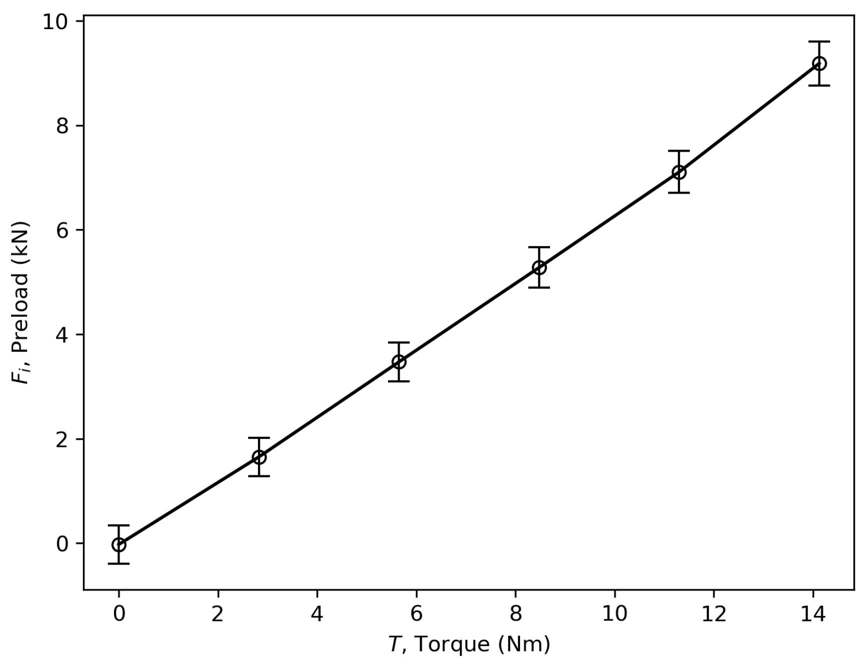

Figure 5: Torque vs. Preload

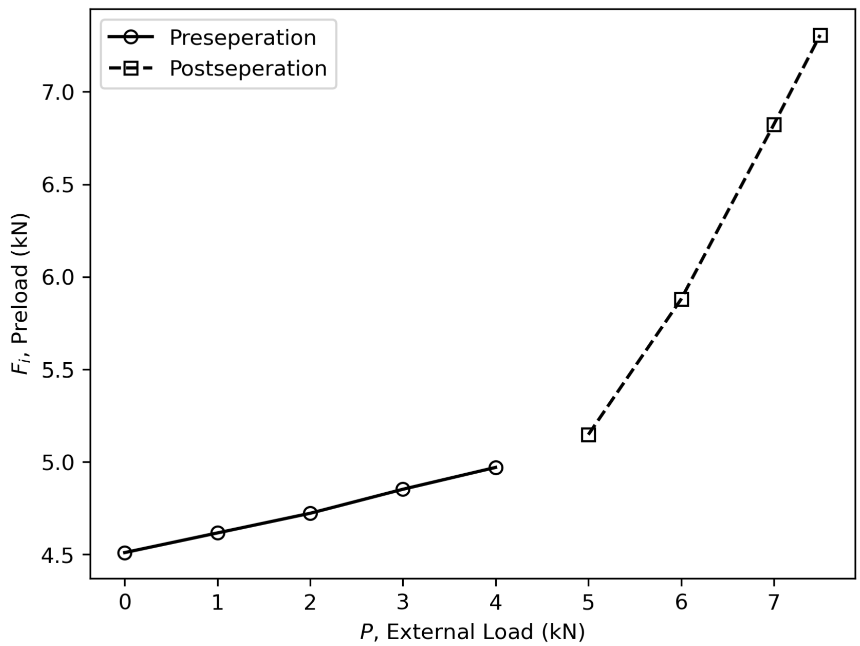

Figure 6: Static loading of bolted connection without gasket

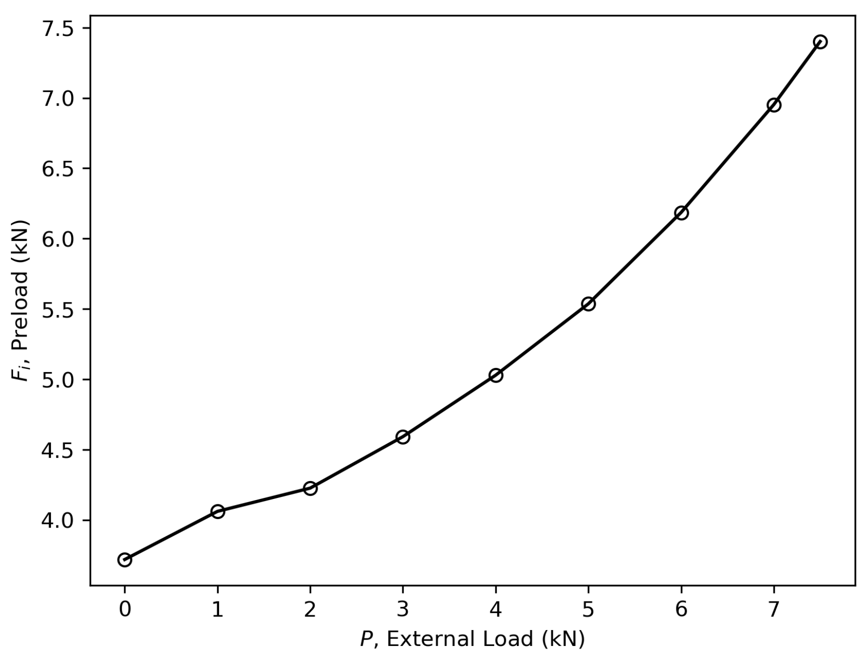

Figure 7: Static loading of bolted connection with gasket

A Appendix: Zero Preload Data Analysis

This Appendix provides the analysis of the experimental data ”bolt stiffness and washer calibration (finger tight)” to determine the modulus of elasticity of the bolt. In addition, error analysis was performed with a confidenc e of 95% to determine the corresponding uncertainty. In addition, the washer calibration was also performed to determine the relationship between the external load, washer transducer reading, and washer strain.

A.1 Modulus of Elasticity Analysis

Table A.2: Bolt Stiffness and Washer Calibration data

External Load, P (kN)	Bolt Out, V_b (V)	Washer Out, V_w (V)	Bolt Strain, ε_b
0	0.006	-0.003	6.00E-06
1	0.085	-0.212	8.50E-05
2	0.168	-0.441	1.68E-04
3	0.253	-0.667	2.53E-04
4	0.335	-0.888	3.35E-04
5	0.417	-1.08	4.17E-04
6	0.498	-1.27	4.98E-04
7	0.581	-1.46	5.81E-04
7.5	0.662	-1.547	6.62E-04

The experimental data was collected and shown in Table A.2. Sample calculations will be shown for external load of 0 kN. The bolt strain was calculated from Eq. (10),

$$
\begin{aligned}
\varepsilon &= \frac{4V_b}{K_g E_{\text{in}} G} \\
\varepsilon &= \frac{40.006\,\text{V}}{2 \cdot 5\,\text{V} \cdot 400} \\
&= 6.00 \times 10^{-6}
\end{aligned}
$$

where E_o is transducer reading, K_g is the gauge factor, E_{in} is the voltage input, and G is the gain set. From the experimental setup, $K_g = 2$, $E_{\text{in}} = 5$, and $G = 400$.

Next, a linear regression of the external load (P) and bolt strain (ε_b), forced through the origin, was performed on the data in Table A.2 to determine the modulus of elasticity. The linear regression equation was determined using `=LINEST()` from Excel. The results are shown in Table A.3. The equation is then

$$\varepsilon_b = 8.47134 \times 10^{-5} P$$

or in another form,

$$\frac{P}{\varepsilon_b} = \frac{1}{8.47134 \times 10^{-5}}$$

The area where the force was applied is the outer diameter, d_o, minus the inner diameter, d_i, of the bolt. From the experimental setup, $d_o = 0.371\,\mathrm{in}$ and $d_i = 0.155\,\mathrm{in}$. The area is then

Table A.3: Linear Regression Results

Parameter	Value
Slope (mm/kN)	8.47134E-05
Slope Standard Error, S_a	8.20567E-07
R^2	0.999249954

$$\begin{aligned} A_1 &= \frac{\pi}{4}(d_o^2 - d_i^2) \\ &= \frac{\pi}{4}((0.371\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 - (0.155\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2) \\ &= 57.570\,\mathrm{mm}^2 \end{aligned}$$

The modulus of elasticity is then

$$\begin{aligned} E &= \frac{P}{\varepsilon_b A_1} \\ &= \frac{1\,\mathrm{kN}}{8.47134 \times 10^{-5} \times 57.570\,\mathrm{mm}^2} \\ &= \boxed{205\,\mathrm{GPa}} \end{aligned}$$

A.2 Modulus of Elasticity Error Analysis

The uncertainty of slope was determined using the standard error of the slope, S_a, from the linear regression in Table A.3 at a confidence level of 95%. The t-distribution value was determined by

$$\begin{aligned}\alpha/2 &= \frac{1-0.95}{2} = 0.025\\ n-2 &= 9-2 = 7\\ t_{\alpha/2,n-2} &= 2.3646\end{aligned}$$

The uncertainty of the slope is then [1]

$$\begin{aligned}\delta\text{slope} &= t_{\alpha/2,n-2}\cdot S_a\\ &= 2.3646\cdot 8.20567\times 10^{-7}\\ &= 1.94\times 10^{-6}\,\text{kN}^{-1}\end{aligned}$$

The function for modulus of elasticity is

$$\begin{aligned}E &= P^1\varepsilon_b^{-1}A_1^{-1}\\ &= (\text{slope})^{-1}A_1^{-1}\end{aligned}$$

Assuming the error for A_1 is negligible, the uncertainty of the modulus of elasticity is then

$$\begin{aligned}\delta E &= E\left|\frac{\delta\text{slope}}{\text{slope}}\right|\\ &= 205\,\text{GPa}\frac{1.94\times 10^{-6}\,\text{kN}^{-1}}{8.47134\times 10^{-5}\,\text{kN}^{-1}}\\ &= \boxed{\pm 4.70\,\text{GPa}}\end{aligned}$$

A.3 Washer Calibration Analysis

The external load and washer transducer readings from Table A.2 were fitted with a linear regression through the origin. The linear regression equation was determined using `=LINEST()` from Excel. The equation was

$$E_{o,w} = -0.211P$$

Converting to strain using Eq. (10), where $K_g = 2$, $E_{\text{in}} = 5$, and $G = 400$:

$$\begin{aligned} \varepsilon_w &= \frac{4V_w}{K_g E_{\text{in}} G} \\ &= \frac{4 - 0.211\,\mathrm{V}P}{2 \cdot 5\,\mathrm{V} \cdot 400} \\ &= -2.11 \times 10^{-4} P \end{aligned}$$

B Appendix: Preload-Torque Test Data Analysis

The following is the analysis of the preload-torque test data. The data was collected from the experiment and is shown in Table B.4. The data was then analyzed to determine the preload, preload uncertainty, and torque coefficient. The following sections will detail the analysis of the data and the results of the analysis.

B.1 Preload vs Torque Analysis

The results from the experiment are shown in Table B.4. Sample calculations will be shown for the second row of the table. First, the torque was converted to metric units.

Table B.4: Torque-Preload Test at Zero External Load

Torque, T (in-lb)	Torque, T (Nm)	Bolt Transducer, V_b (V)	Washer Transducer, V_w (V)	Bolt Strain, ε_b	Preload, F_i (kN)	Preload Uncertainty, δF_i (± kN)
0	0	-0.002	-0.001	-2.00E-06	-0.0236	0.366
25	2.825	0.140	-0.311	1.40E-04	1.65	0.368
50	5.649	0.294	-0.615	2.94E-04	3.47	0.375
75	8.474	0.447	-0.907	4.47E-04	5.28	0.386
100	11.298	0.602	-1.203	6.02E-04	7.11	0.401
125	14.123	0.778	-1.519	7.78E-04	9.18	0.422

$$\begin{aligned} T &= 25\,\mathrm{in-lb} \times 0.113\,\mathrm{N\,m^{-1}\,in-lb^{-1}} \\ &= 2.825\,\mathrm{N\,m} \end{aligned}$$

The bolt strain, ε_b, was then calculated by

$$\begin{aligned} \varepsilon_b &= \frac{4V_b}{K_g E_{\text{in}} G} \\ &= \frac{4 \times 0.140\,\mathrm{V}}{2 \times 5\,\mathrm{V} \times 400} \\ &= 1.40 \times 10^{-4} \end{aligned}$$

The preload, F_i, was then calculated by

$$\begin{aligned} F_i &= E_b \varepsilon_b A_1 \\ &= 205.046\,\mathrm{GPa} \times 1.40 \times 10^{-4} \times 57.570\,\mathrm{mm}^2 \\ &= \boxed{1.65\,\mathrm{kN}} \end{aligned}$$

B.2 Uncertainty Analysis of Preload

A repeatability test was performed at 50 lb-in of torque with no external load. The results of this test are shown in The standard deviation was determined with Excel to be $S_{V_b} = 0.0250\,\mathrm{V}$. Using

Table B.5: Repeatability Test at 50 lb-in of Torque and Zero External Load

Trial #	Bolt Transducer, V_b (V)	Washer Transducer (V)
1	0.372	-0.701
2	0.321	-0.684
3	0.354	-0.718
4	0.312	-0.654
5	0.327	-0.679

a confidence level of 95%, the t-distribution value was determined by

$$\begin{aligned} \alpha/2 &= \frac{1-0.95}{2} = 0.025 \\ n-1 &= 5-1 = 4 \\ t_{\alpha/2,n-1} &= 2.776 \end{aligned}$$

The precision uncertainty is then

$$\begin{aligned} P_{V_b} &= t_{\alpha/2,n-1} \cdot \frac{S_{V_b}}{\sqrt{n}} \\ &= 2.776 \cdot \frac{0.025\,\mathrm{V}}{\sqrt{5}} \\ &= 0.031\,\mathrm{V} \end{aligned}$$

Defining bias uncertainty as resolution, $B_{V_b} = 0.001$, the total uncertainty is then

$$\begin{aligned}\delta V_b &= \sqrt{P_{V_b}^2 + B_{V_b}^2} \\ &= \sqrt{(0.031\,\mathrm{V})^2 + (0.001\,\mathrm{V})^2} \\ &= 0.031\,\mathrm{V}\end{aligned}$$

The uncertainty of the preload for the second row of Table B.4 is then

$$\begin{aligned}\delta F_i &= F_i \sqrt{\left(\frac{\delta V_b}{V_b}\right)^2 + \left(\frac{\delta E_b}{E_b}\right)^2} \\ &= 1.65\,\mathrm{kN} \sqrt{\left(\frac{0.031\,\mathrm{V}}{0.140\,\mathrm{V}}\right)^2 + \left(\frac{4.70\,\mathrm{GPa}}{205.046\,\mathrm{GPa}}\right)^2} \\ &= 0.368\,\mathrm{kN}\end{aligned}$$

B.3 Torque Coefficient Analysis

Applyig linear regression, forced through the origin, to the data in Table B.4 using `=LINEST()` from Excel, the equation is,

$$\begin{aligned}F_i &= 0.636T \\ \implies \frac{T}{F_i} &= \frac{1}{0.636}\mathrm{mm}^{-1}\end{aligned}$$

where F_i is in kN and T is in Nm. From Eq. (16), the torque coefficient is then

$$\begin{aligned}K &= \frac{T}{F_i d} \\ &= \frac{1}{0.636\,\mathrm{mm}^{-1} \times 0.375\,\mathrm{in} \times 25.4\,\mathrm{mm\,in}^{-1}} \\ &= \boxed{0.167}\end{aligned}$$

C Appendix: Bolt Stiffness Calculations

C.1 Bolt Geometric Properties

The lengths of sections 1 and 2 were given as 0.91 in and 1.471 in, respectively. Section 3 is to be determined. The total length of the member was 63.5 mm. Then,

$$\begin{aligned} L_3 &= 63.5\,\mathrm{mm} - 0.91\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}} - 1.471\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}} \\ &= 3.0226\,\mathrm{mm} \end{aligned}$$

The cross-sectional area of each section was determined by,

$$\begin{aligned} A_1 &= \frac{\pi}{4}(d_o^2 - d_i^2) \\ &= \frac{\pi}{4}((0.371\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 - (0.155\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2) \\ &= 57.570\,\mathrm{mm^2} \end{aligned}$$

then,

$$\begin{aligned} A_2 &= \frac{\pi}{4}d_2^2 \\ &= \frac{\pi}{4}(3.71\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 \\ &= 69.744\,\mathrm{mm^2} \end{aligned}$$

lastly,

$$\begin{aligned} A_3 &= \frac{\pi}{4}d_3^2 \\ &= \frac{\pi}{4}(3.75\,\mathrm{in} \times 25.4\,\mathrm{mm\,in^{-1}})^2 \\ &= 71.256\,\mathrm{mm^2} \end{aligned}$$

The geometric properties of the bolt are summarized in Table C.6.

Table C.6: Bolt Stiffness Calculations

Length of section, L (in)	bolt stiffness (mm)	Cross Sectional Area, A_s (mm^2)	Stiffness, k (MN/m)	1/k (m/MN)
0.91	23.114	57.57	510.708	0.001958
1.471	37.3634	69.744	382.745	0.002613
0.119	3.0226	71.256	4833.819	0.000207

C.2 Bolt Stiffness

Sample calculations for Table C.6 will be shown for the stiffness of section 1. The stiffness of section 2 and 3 will be calculated in the same manner. The stiffness of section 1 was calculated by,

$$\begin{aligned} k_1 &= \frac{E_b A_1}{L_1} \\ &= \frac{205.046\,\mathrm{GPa} \times 57.570\,\mathrm{mm}^2}{23.114\,\mathrm{mm}} \\ &= 510.708\,\mathrm{MN\,m^{-1}} \end{aligned}$$

Where E_b was determined to be $205\,\mathrm{GPa}$ in Appendix **??**. Then,

$$\frac{1}{k_1} = 0.001958\,\mathrm{m\,MN^{-1}}$$

C.3 Total Bolt Stiffness

The total bolt stiffness was calculated by Eq. 13. The total bolt stiffness was then,

$$\begin{aligned} k_b &= \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} \\ &= \left(\frac{1}{0.001958\,\mathrm{m\,MN^{-1}}} + \frac{1}{0.002613\,\mathrm{m\,MN^{-1}}} + \frac{1}{0.000207\,\mathrm{m\,MN^{-1}}}\right)^{-1} \\ &= \boxed{209.308\,\mathrm{MN\,m^{-1}}} \end{aligned}$$

D Appendix: Theoretical Member Stiffness Calculations

From Eq. (15), the stiffness of the member was estimated to be,

$$k_{m,\text{th}} = \frac{\pi E_b d}{2\ln\left(\frac{5(L+0.5d)}{L+2.5d}\right)}$$

where E_b is the modulus of elasticity of the bolt, d is the diameter of the bolt, and L is the length of the member. The modulus of elasticity of the bolt was determined to be $205.046\,\text{GPa}$ in Appendix **??**. The diameter of the bolt was $0.371\,\text{in}$ and the length of the member was $63.5\,\text{mm}$. Then,

$$\begin{aligned} k_{m,\text{th}} &= \frac{\pi \times 205.046\,\text{GPa} \times 0.371\,\text{in} \times 25.4\,\text{mm}\,\text{in}^{-1}}{2\ln\left(\frac{5(63.5\,\text{mm}+0.5\times0.371\,\text{in}\times25.4\,\text{mm}\,\text{in}^{-1})}{63.5\,\text{mm}+2.5\times0.371\,\text{in}\times25.4\,\text{mm}\,\text{in}^{-1}}\right)} \\ &= \boxed{2222.774\,\text{MN}\,\text{m}^{-1}} \end{aligned}$$

E Appendix: Experimental Member Stiffness Calculations

E.1 Experimental Data

Table E.7: Various External Loads and Bolt Force at 60 in-lb Torque

	External Load, P (kN)	Bolt Out, V_b (V)	Washer Out, V_w (V)	Bolt Strain, ε_b	Bolt Force, F_i (kN)
Preseperation	0	0.382	-0.72	0.000382	4.509
Preseperation	1	0.391	-0.751	0.000391	4.616
Preseperation	2	0.4	-0.783	0.0004	4.722
Preseperation	3	0.411	-0.818	0.000411	4.852
Preseperation	4	0.421	-0.855	0.000421	4.970
Postseperation	5	0.436	-0.901	0.000436	5.147
Postseperation	6	0.498	-1.068	0.000498	5.879
Postseperation	7	0.578	-1.226	0.000578	6.823
Postseperation	7.5	0.619	-1.3	0.000619	7.307

Sample calculations will be shown for the first row of Table E.7. The bolt strain, ε_b, was calculated by

$$\begin{aligned}\varepsilon_b &= \frac{4V_b}{K_g E_{\text{in}} G} \\ &= \frac{4 \times 0.382\,\text{V}}{2 \times 5\,\text{V} \times 400} \\ &= 0.000382\end{aligned}$$

The force, F_i, was then calculated by

$$\begin{aligned}F_i &= E_b \varepsilon_b A_1 \\ &= 205.046\,\text{GPa} \times 0.000382 \times 57.570\,\text{mm}^2 \\ &= \boxed{4.509\,\text{kN}}\end{aligned}$$

E.2 Experimental Member Stiffness

Applying linear regression to the preseperation data in Table E.7 yields the following equation from `=LINEST()` in Excel,

$$F_i = \underbrace{0.1157}_{C} P + 4.5022$$

Comparing the form of the linear regression to Eq. (17), $C = 0.1157$. Then, by Eq. (18),

$$\begin{aligned} C &= \frac{k_b}{k_b + k_m} \\ \implies k_{m,\text{exp}} &= \frac{k_b}{\frac{1}{C} - 1} \\ &= \frac{209.308\,\text{kN}\,\text{m}^{-1}}{\frac{1}{0.1157} - 1} \\ &= \boxed{1599.998\,\text{kN}\,\text{m}^{-1}} \end{aligned}$$

Compared to the theoretical value of $2222.774\,\text{kN}\,\text{m}^{-1}$, the error is

$$\begin{aligned} \text{Error} &= \frac{k_{m,\text{th}} - k_{m,\text{exp}}}{k_{m,\text{th}}} \times 100\% \\ &= \frac{2222.774\,\text{kN}\,\text{m}^{-1} - 1599.998\,\text{kN}\,\text{m}^{-1}}{2222.774\,\text{kN}\,\text{m}^{-1}} \times 100\% \\ &= \boxed{28.0\%} \end{aligned}$$

F Appendix: Joint Separation

F.1 Experimental Seperation

The two regressions of the data from Table E.7,

$$F_{b,\text{pre}} = 0.1157P + 4.5022$$
$$F_{b,\text{post}} = 0.8687P + 0.7507$$

The separation point is when $F_{i,\text{pre}} = F_{i,\text{post}}$,

$$\begin{aligned} 0.1157P_{\text{exp}} + 4.5022 &= 0.8687P_{\text{exp}} + 0.7507 \\ P_{\text{exp}} &= \frac{3.7515}{0.753} \\ &= \boxed{4.98\,\text{kN}} \end{aligned}$$

Then,

$$\begin{aligned} F_{b,\text{sep}} &= 0.1157 \times 4.98\,\text{kN} + 4.5022 \\ &= \boxed{5.08\,\text{kN}} \end{aligned}$$

F.2 Theoretical Separation

The torque load was 60 in-lb for the data in Table E.7. From Eq. (16),

$$\begin{aligned} F_i &= \frac{T}{Kd} \\ &= \frac{60\,\text{in} - \text{lb} \times 0.112984\,\text{N}\,\text{m}\,\text{in} - \text{lb}^{-1}}{0.167 \times 0.375\,\text{in} \times 25.4\,\text{mm}\,\text{in}^{-1}} \\ &= \boxed{4.26\,\text{kN}} \end{aligned}$$

Then calculating C_{th} by Eq. (18),

$$\begin{aligned} C_{\text{th}} &= \frac{k_b}{k_b + k_{m,\text{th}}} \\ &= \frac{209.308\,\text{MN}\,\text{m}^{-1}}{209.308\,\text{MN}\,\text{m}^{-1} + 2222.774\,\text{MN}\,\text{m}^{-1}} \\ &= 0.116 \end{aligned}$$

Then by Eq. (19),

$$\begin{aligned} P &= \frac{F_i}{1 - C_{\text{th}}} \\ &= \frac{4.26\,\text{kN}}{1 - 0.116} \\ &= \boxed{4.82\,\text{kN}} \end{aligned}$$

F.3 Theoretical vs. Experimental Separation

The error is then,

$$\begin{aligned} \text{Error} &= \frac{P_{\text{th}} - P_{\text{exp}}}{P_{\text{th}}} \times 100\% \\ &= \frac{4.82\,\text{kN} - 4.98\,\text{kN}}{4.82\,\text{kN}} \times 100\% \\ &= \boxed{3.32\%} \end{aligned}$$

G Appendix: Dynamic Loading

Table G.8: Dynamic Loading Summary for Various Torques and Gasket Conditions

	Torque, T (in-lb)	Max Stress, σ_{max} (MPa)	Min Stress, σ_{min} (MPa)	Mean Stress, σ_{mean} (MPa)	Alternating Stress, σ_{a} (MPa)
With Gasket	0	105.035	61.804	83.420	21.615
	60	110.300	88.864	99.582	10.718
	75	124.101	110.488	117.295	6.806
	125	174.584	167.250	170.917	3.667
No Gasket	0	105.330	62.098	83.714	21.616
	60	106.347	84.153	95.250	11.097
	75	108.671	101.337	105.004	3.667
	125	166.652	163.258	164.955	1.697

The raw transducer data was converted using Eq. (10) in a similar fashion to Appendix A. Sample calculations will be shown for the first row of Table G.8. The min and max stress were calculated by `.max()` and `.min()` from Pandas [2]. The mean stress was calculated by

$$\begin{aligned}\sigma_{\text{mean}} &= \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} \\ &= \frac{105.035\,\text{MPa} + 61.804\,\text{MPa}}{2} \\ &= 83.420\,\text{MPa}\end{aligned}$$

The alternating stress was then calculated by

$$\begin{aligned}\sigma_{\text{a}} &= \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \\ &= \frac{105.035\,\text{MPa} - 61.804\,\text{MPa}}{2} \\ &= 21.615\,\text{MPa}\end{aligned}$$