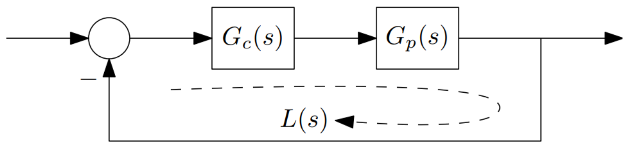
Question 1

Consider the simplified closed-loop diagram where



$$G_c(s) = 1, \quad G_p(s) = \frac{1}{s+1}$$

(a) By hand, sketch the Nyquist plot of $L(s) = G_p(s)G_c(s)$. Provide both the Nyquist contour $\mathcal{D}$ and the Nyquist plot $\mathcal{G}$, and label the segment endpoints A, B and C.

(b) Use MATLAB's `nyquist` command to check your answer in (a). Provide the MATLAB commands you used to obtain this (you do not need to include the plot)

(c) Apply Theorem 5.3.1 (Nyquist Stability Criterion) to the above to check the stability of the closed-loop system.

(d) Now form the characteristic polynomial of the above system, and use Theorem 4.4.1 to check closed-loop stability. Does your answer match with (c)?

(a)

The loop transfer function $L(s)$ is given by

$$L(s) = G_p(s)G_c(s) = \frac{1}{s+1}$$

Desmos will be used to "sketch" the Nyquist plot since I'm bad at drawing with a mouse. Since $L(s)$ has real constant coefficients, the curve will be symmetric about the real axis.

From a to b, $s = j\omega$, $\omega \in [0, \infty)$,

$$\begin{aligned} L(j\omega) &= \frac{1}{j\omega + 1} \\ &= \frac{1 - j\omega}{1 + \omega^2} \end{aligned}$$

which results in

$$\text{Re}\left\{L(j\omega)\right\} = \frac{1}{1+\omega^2}, \quad \text{Im}\left\{L(j\omega)\right\} = -\frac{\omega}{1+\omega^2}$$

Observe that at $\omega = 0$, $L(0) = 1$, and as $\omega \to \infty$, $L(\infty) = 0$. Sketch is shown in Figure 1

(b)

The following MATLAB commands were used to generate the Nyquist plot:

```
nyquist(tf([1], [1 1]))
```

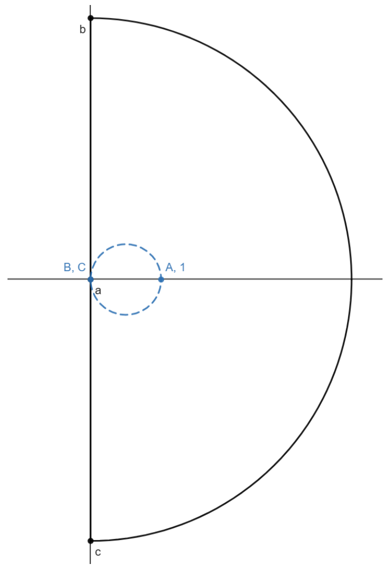


Figure 1: 1(a) Nyquist sketch

(c)

$L(s)$ has no poles in the right half plane, so $n = 0$. Since the Nyquist plot does not pass through -1 and encircles the origin $n = 0$ times, the closed-loop system is stable.

(d)

The characteristic polynomial of the closed-loop system is given by

$$p(s) = n_p n_c + d_p d_c = 1 + (s + 1) = s + 2$$

By Theorem 4.4.1, the closed-loop system is stable if and only if $p(s)$ has negative real roots. Since $p(s)$ has a single at $s = -2$, the closed-loop system is stable. This matches with the result from (c).

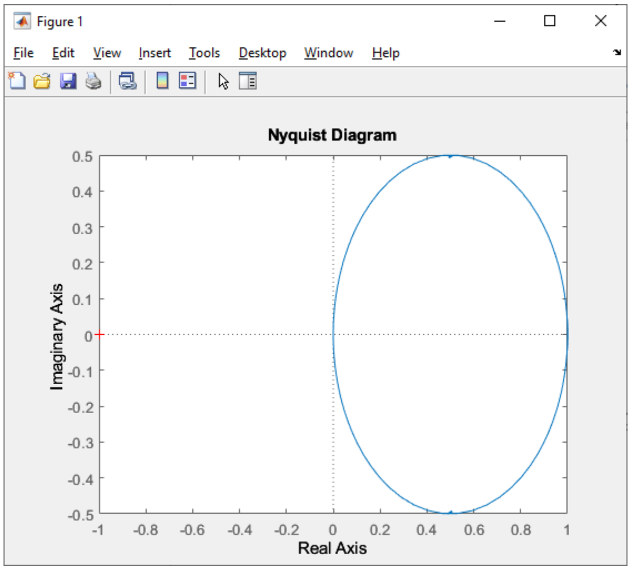


Figure 2: 1(b) Nyquist plot generated by MATLAB

Question 2

Reconsider the closed-loop diagram from Question 1, now with

$$G_c(s) = 2 + \frac{2}{s} = \frac{2s+2}{s}, \quad G_p(s) = \frac{1}{s-1}$$

i.e. a BIBO unstable plant connected to a PI controller.

(a) By hand, sketch the Nyquist contour $\mathcal{D}$ and Nyquist plot $\mathcal{G}$, then apply Theorem 5.3.1 to check the stability of the closed-loop system

(b) Give the plot obtained with MATLAB's `nyquist` command. What's the major difference between this result and the sketch from (a)?

(a)

The loop transfer function $L(s)$ is given by

$$L(s) = G_p(s)G_c(s) = \frac{2s+2}{s(s-1)}$$

The poles are at $s = 0$ and $s = 1$. Since there is a pole at $s = 0$, a small semi-circle of radius ϵ is drawn around the origin.

From a to b, $s = j\omega$, $\omega \in [0, \infty)$,

$$L(j\omega) = \frac{2j\omega + 2}{j\omega(j\omega - 1)}$$

With help from MATLAB,

```
syms s
syms omega real
L(s) = (2*s + 2)/(s*(s-1));
simplify(real(L(j*omega)))
simplify(imag(L(j*omega)))
```

which gives

$$\text{Re}\left\{L(j\omega)\right\} = \frac{-4}{\omega^2+1}, \quad \text{Im}\left\{L(j\omega)\right\} = \frac{2(1-\omega^2)}{\omega(\omega^2+1)}$$

Starting at point a the point $s = j\epsilon$,

$$\text{Re}\left\{L(j\epsilon)\right\} = \frac{-4}{\epsilon^2+1} \approx -4, \quad \text{Im}\left\{L(j\epsilon)\right\} = \frac{2(1-\epsilon^2)}{\epsilon(\epsilon^2+1)} \to \infty$$

to point b, $s = j\omega$, $\omega \to \infty$,

$$\text{Re}\left\{L(j\omega)\right\} = \frac{-4}{\omega^2+1} \to 0, \quad \text{Im}\left\{L(j\omega)\right\} = \frac{2(1-\omega^2)}{\omega(\omega^2+1)} \to 0$$

By symmetry, c to d is the same as a to b reflected about the real axis. Note the curve intersects the real axis at $s = -2$ on its way to b and c.

The last section a to d is derived in the notes as

$$L(\epsilon e^{j\theta}) \approx \frac{e^{-j\theta}}{\epsilon}\frac{2s+2}{s-1}$$

which sweeps out a semi-circle of infinite radius of angular distance π.

The Nyquist plot is shown in Figure 3

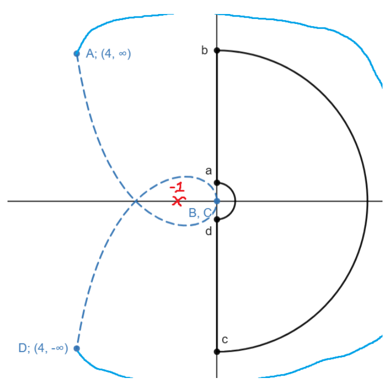


Figure 3: 2(a) Nyquist sketch

Since there is one pole $\text{Re}(s) > 0$, $n = 1$. Since the Nyquist plot does not pass through -1 and encircles it once in the counter-clockwise direction, the closed-loop system is stable.

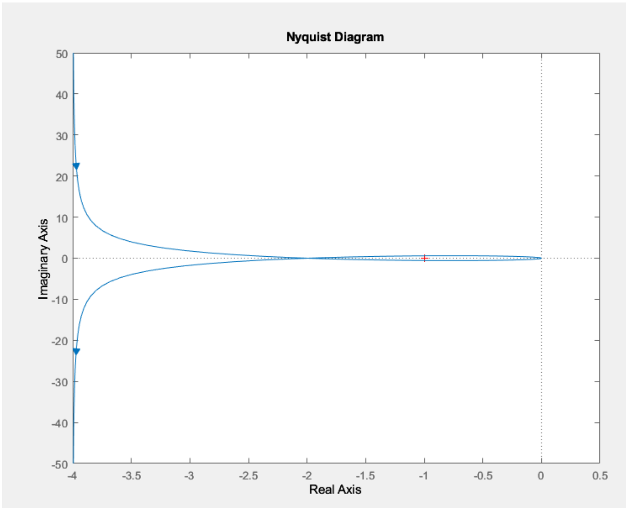


Figure 4: 2(b) Nyquist plot generated by MATLAB

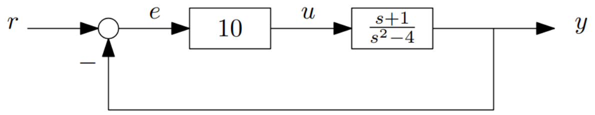
(b)

```
nyquist(tf([2 2], [1 -1 0]))
```

The major difference is the lack of the semi-circle of infinite radius. The sketch approximated the infinite radius semi-circle as a semi-circle of finite radius.

Question 3

Consider the following closed-loop system:



(a) Using MATLAB, produce the Nyquist plot of $L(s)$ for this system. Include a print-out of your plot.

(b) Based on the plot in (a), explain why the closed-loop system is stable

(c) Read off the GM, PM and t_{max} of the closed-loop system from the MATLAB-produced Nyquist plot

(d) Set up a Simulink diagram of the above diagram, and employ a reference step input $r(t) = 1 + (t)$. Provide a print-out of the response $y(t)$.

(e) Now add a time delay of $t_d = 0.1$ s into the feedback loop, using the Transport Delay block within the Continuous part of the Simulink library. Using the same reference as in (d), provide a print-out of the resulting response. How does this plot compare to the one in (d)?

(f) Now increase the time delay to $t_d = 0.5$ s. Comment on the step response of the closed-loop system in this case. Is this as expected?

(a)

The loop function for this system is:

$$L(s) = \frac{10s + 10}{s^2 - 4}$$

By Matlab,

```
nyquist(tf([10 10], [1 0 -4]))
```

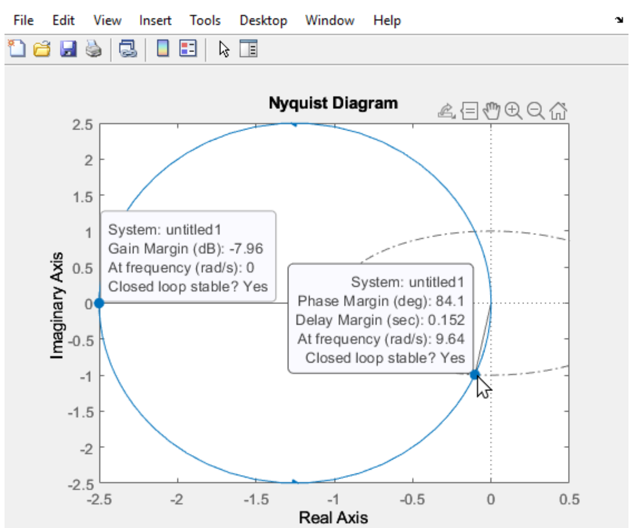


Figure 5: 3(a) Nyquist plot generated by MATLAB

(b)

The poles of $L(s)$ are at $s = \pm 2$. Then, $n = 1$. Since the Nyquist plot does not pass through -1 and encircles the origin once in the counter-clockwise direction, the closed-loop system is stable.

(c)

```
>> db2mag(-7.96)

ans =

    0.3999
```

then,

$$\boxed{\begin{aligned} GM &= 0.3999 \\ PM &= 84.1^\circ \\ t_d^{max} &= 0.152 \text{ s} \end{aligned}}$$

(d)

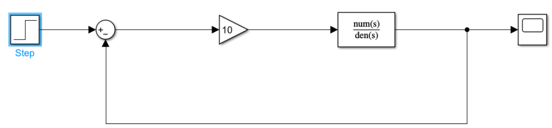


Figure 6: 3(d) Simulink diagram

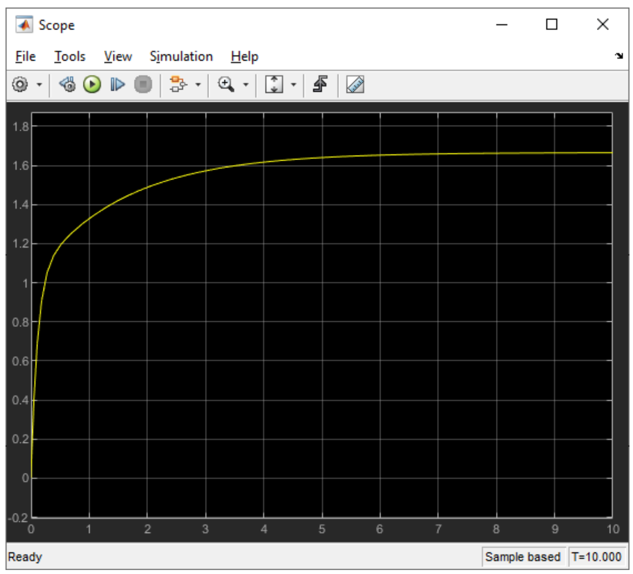


Figure 7: 3(d) $y(t)$ response

(e)

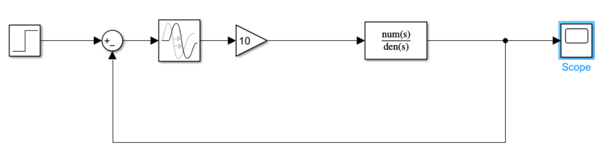


Figure 8: 3(e) Simulink diagram

The response is more oscillatory than in (d) initially. After some time they both converge to the same value.

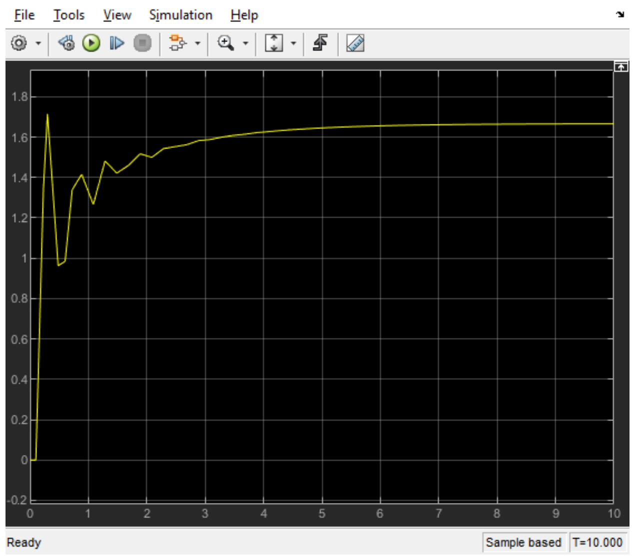


Figure 9: 3(e) $y(t)$ response

(f)

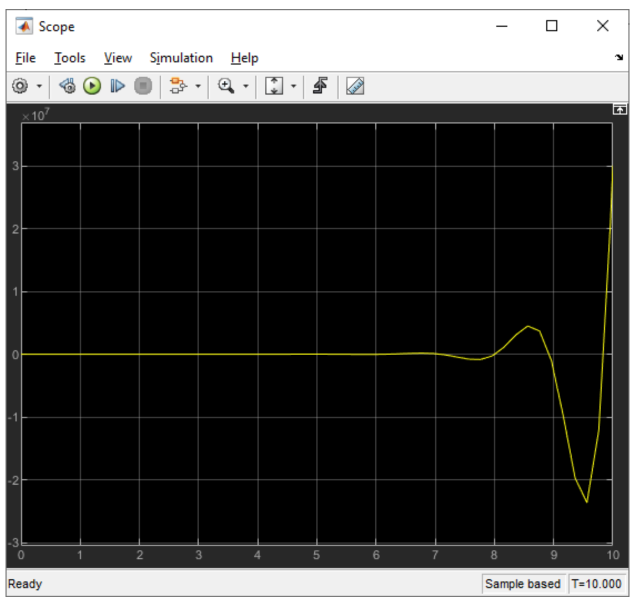


Figure 10: 3(f) $y(t)$ response

The response blows up to ∞ as $t \to \infty$. This is expected since the system is unstable at $t_d = 0.5$ s as discussed in (b).