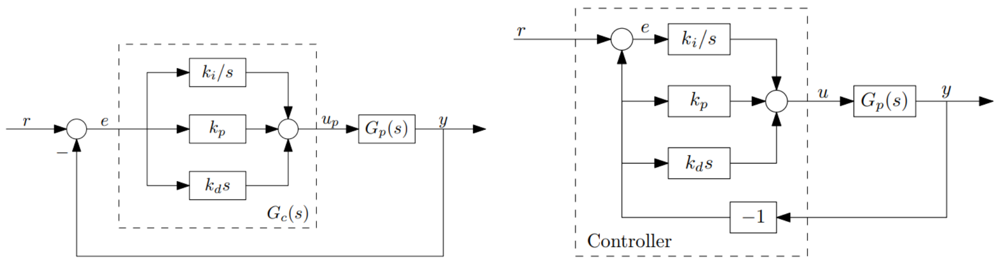
Question 1

Consider the PID and I-PD controllers shown below, each connected to a standard first-order system plant $G_p(s) = \frac{1}{\tau s+1}$:



(a) Taking the parameter values as $\tau = 0.1, k_p = 0.5, k_i = 1, k_d = 0.1$, set up a Simulink diagram for each design, using a Step block for the reference r input, and two Scope blocks to record u and y. Include a sketch or screenshot of both designs.

(b) Now run both simulations for the reference step input $r(t) = 1+(t)$, and provide plots for y and u for each case. How does the performance of the two designs compare?

(a)

The Simulink diagram for the PID controller is shown in Figure 1. The Simulink diagram

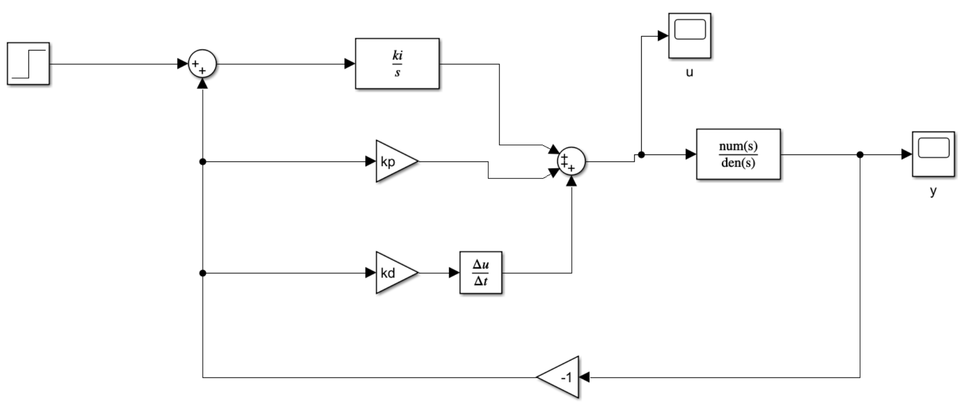


Figure 1: Simulink diagram for the PID controller

for the I-PD controller is shown in Figure 2.

(b)

The scopes for the PID controller are shown in Figure 3.

The scopes for the I-PD controller are shown in Figure 4.

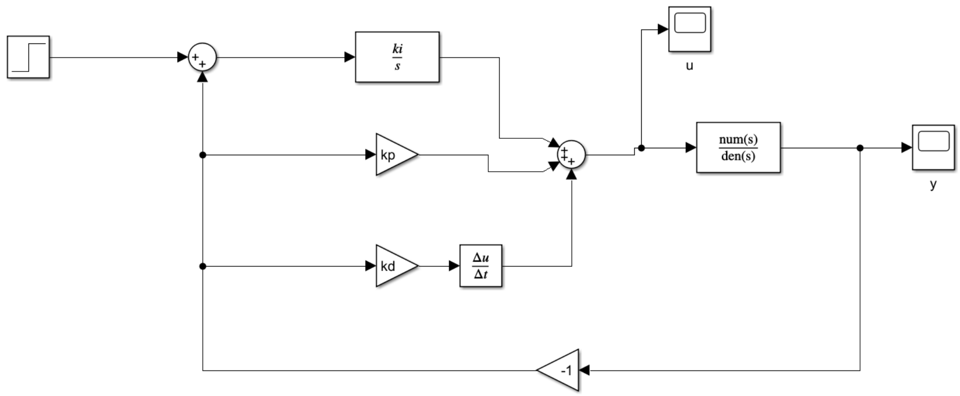


Figure 2: Simulink diagram for the I-PD controller

The I-PD controller is smoother than the PID controller. If a step response of $r = 1_+(t-1)$ is used the PID has a large spike as shown in Figure 5.

Question 2

Consider the closed-loop feedback system with a plant model

$$G_p(s) = \frac{1}{s(s+1)(s+5)}$$

and a PID controller of the form

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s$$

with gain values of $k_p = 39.42, k_i = 12.81, k_d = 30.32$ obtained by tuning, providing a stable closed-loop system.

(a) Will this system be able to asymptotically track a reference step $r(t) = 1_+(t)$? A ramp $r(t) = t$? A sine wave $r(t) = \sin t$? Explain why or why not in each case.

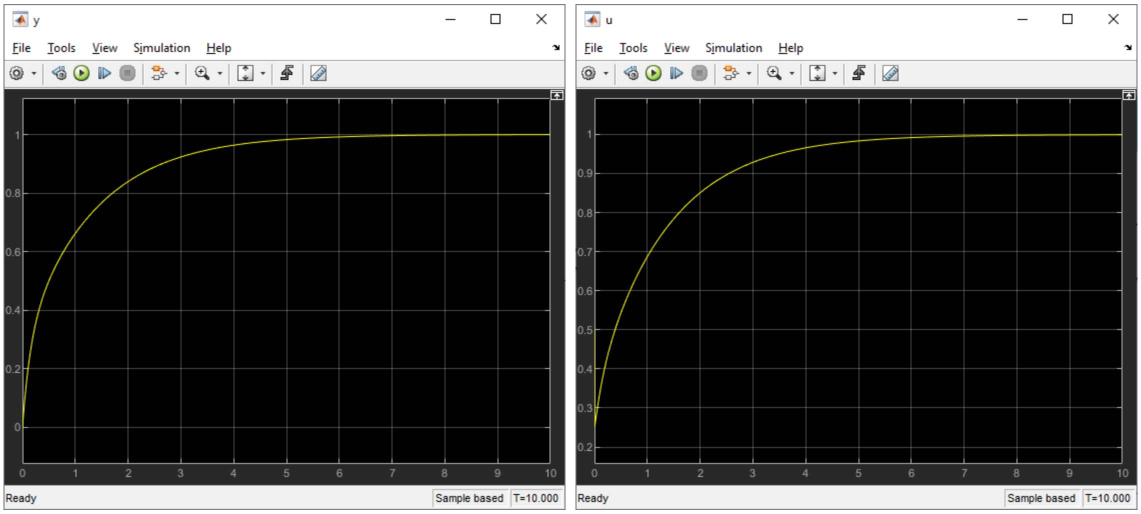
(b) Using Simulink's PID Controller block, simulate the response of the system for each type of reference input given in (b). Show the plot of y in each case. Note: in the parameters of this block, leave the Filter coefficient (N) at its default value of 100.

(a)

From Section 4.4,

$$Y = \frac{G_p G_c}{1 + G_p G_c} R = G_{yr} R$$

For $R_1 = 1_+(t)$, $R_t = t$, and $R_{\text{wave}} = \sin t$, Y is, by Matlab,



(a) Response of y to a step input $r(t) = 1_+(t-1)$

(b) Response of u to a step input $r(t) = 1_+(t-1)$

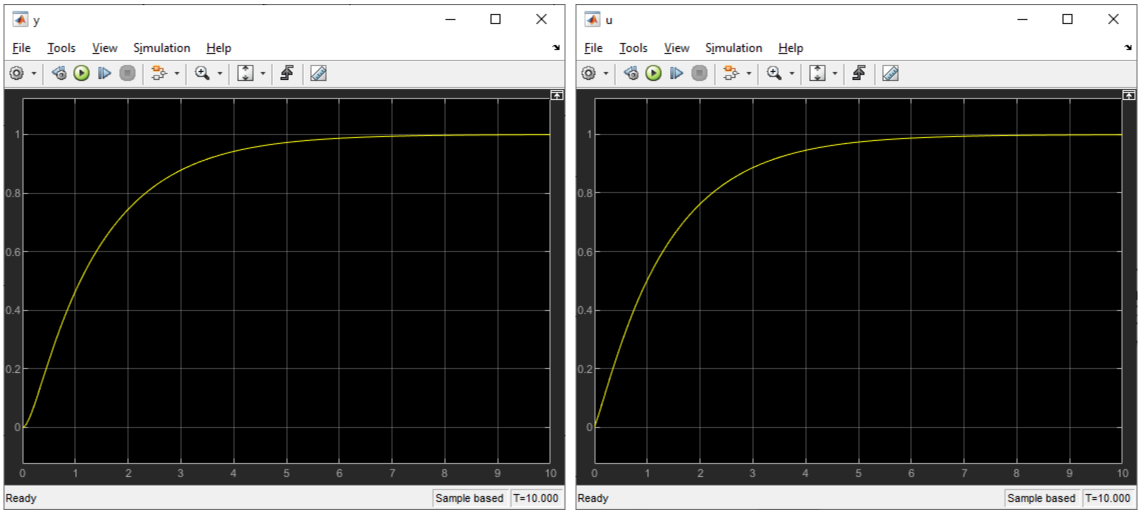
Figure 3: Scopes for the PID controller

```
clc; clear all; close all;
syms s t;
Gc = 39.42 + 12.81/s + 30.32*s;
Gp = 1/(s*(s+1)*(s+5));
G_yr = Gc*Gp/(1+Gc*Gp);

R1 = laplace(1 + 0*t);
Rt = laplace(t);
Rsin = laplace(sin(t));

Y1 = simplify(G_yr*R1)
Yt = simplify(G_yr*Rt)
Ysin = simplify(G_yr*Rsin)
```

$$Y_1 = \frac{3032s^2 + 3942s + 1281}{s(100s^4 + 600s^3 + 3532s^2 + 3942s + 1281)}$$
$$Y_t = \frac{3032s^2 + 3942s + 1281}{s^2(100s^4 + 600s^3 + 3532s^2 + 3942s + 1281)}$$
$$Y_{\text{wave}} = \frac{3032s^2 + 3942s + 1281}{100s^6 + 600s^5 + 3632s^4 + 4542s^3 + 4813s^2 + 3942s + 1281}$$

Looking for the roots for Y_1,

```
>> vpa(root((s*(100*s^4 + 600*s^3 + 3532*s^2 + 3942*s + 1281)), s))

ans =
```



(a) Response of y to a step input $r(t) = 1_+(t)$ (b) Response of u to a step input $r(t) = 1_+(t)$

Figure 4: Scopes for the I-PD controller

```
                                                                       0
- 0.64860612459783461771340131819613 - 0.15652180448584118882475979750655i
- 0.64860612459783461771340131819613 + 0.15652180448584118882475979750655i
  - 2.3513938754021653822865986818039 - 4.8213321796848982440561485616617i
  - 2.3513938754021653822865986818039 + 4.8213321796848982440561485616617i
```

Since all the roots have negative real parts, the system is stable and will be able to asymptotically track a reference step $r(t) = 1_+(t)$.

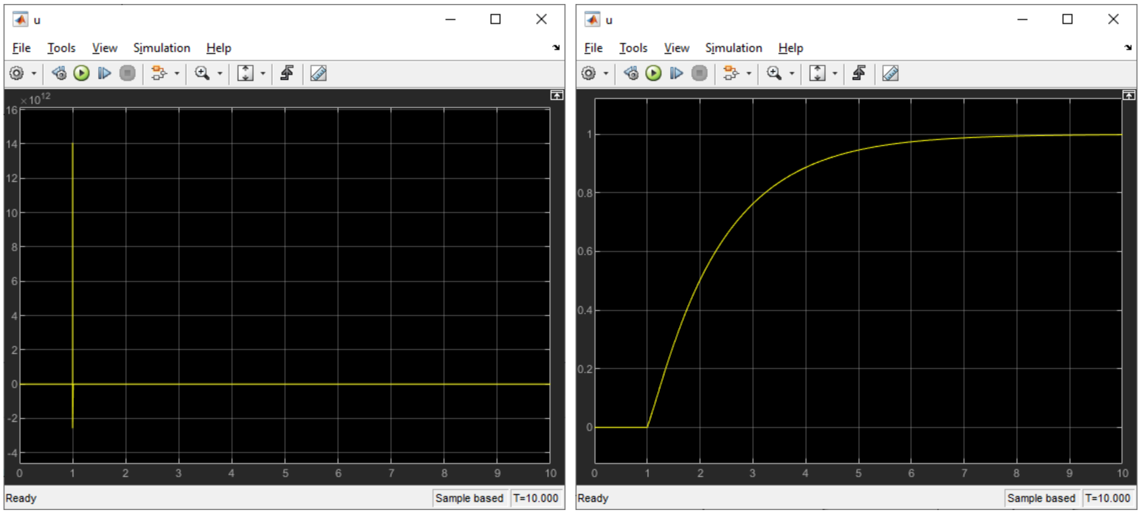
Looking for the roots for Y_t,

```
>> vpa(root((s^2*(100*s^4 + 600*s^3 + 3532*s^2 + 3942*s + 1281)), s))

ans =

                                                                       0
                                                              1.0e-1032
- 0.64860612459783461771340131819613 - 0.15652180448584118882475979750655i
- 0.64860612459783461771340131819613 + 0.15652180448584118882475979750655i
  - 2.3513938754021653822865986818039 + 4.8213321796848982440561485616617i
  - 2.3513938754021653822865986818039 - 4.8213321796848982440561485616617i
```

Since there is a root with $\mathrm{Re}(s) > 0$, the system is unstable and will not be able to asymptotically track a ramp $r(t) = t$.

Looking for the roots for Y_{wave},

```
>> vpa(root((100*s^6 + 600*s^5 + 3632*s^4 + 4542*s^3 + 4813*s^2 + 3942*s + 1281), s))

ans =
```



(a) Response of u to a step input $r(t) = 1_+(t-1)$

(b) Response of u to a step input $r(t) = 1_+(t-1)$

Figure 5: Scopes for the PID and I-PD controller with a step input $r(t) = 1_+(t-1)$

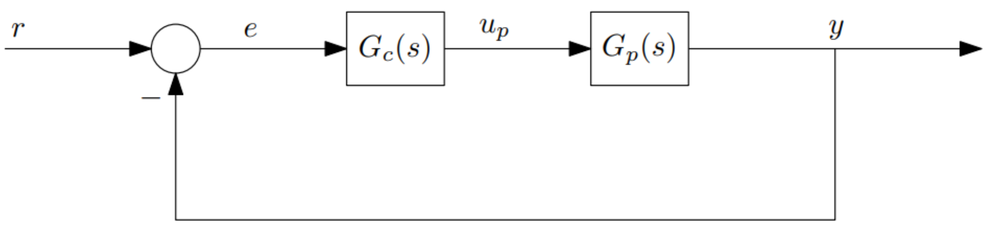


Figure 6: Closed-loop feedback system

```
- 0.64860612459783461771340131819613 - 0.15652180448584118882475979750655i
- 0.64860612459783461771340131819613 + 0.15652180448584118882475979750655i
                                                                      -1.0i
                                                                       1.0i
    - 2.3513938754021653822865986818039 - 4.8213321796848982440561485616617i
    - 2.3513938754021653822865986818039 + 4.8213321796848982440561485616617i
```

Since the system has a root at $\mathrm{Re}(s) = 0$, the system is unstable and will not be able to asymptotically track a sine wave $r(t) = \sin t$.

(b)

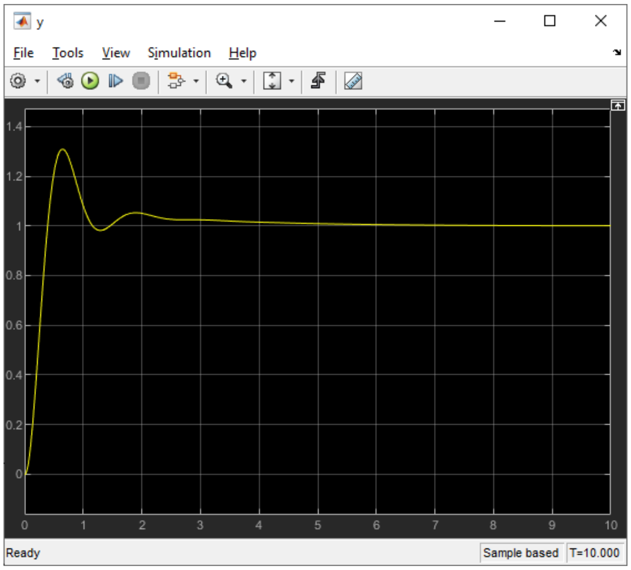


Figure 7: Response of y to a step input $r(t) = 1_+(t)$

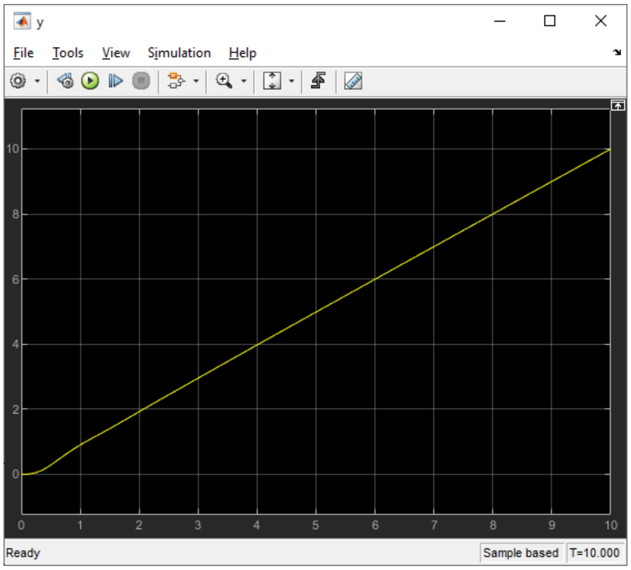


Figure 8: Response of y to a ramp input $r(t) = t$

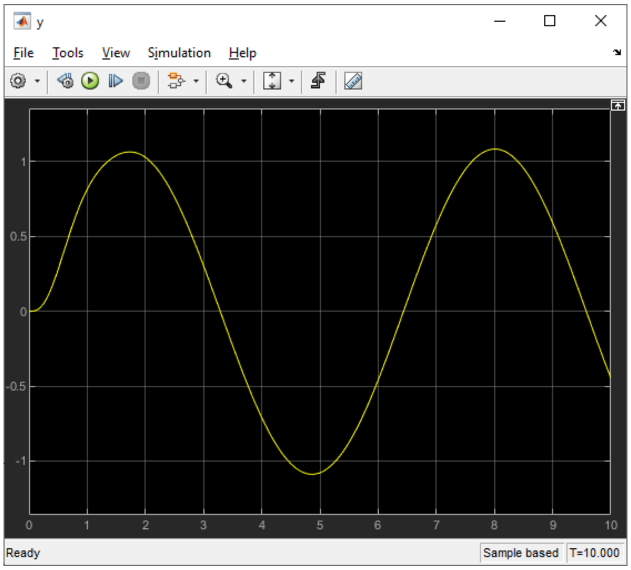


Figure 9: Response of y to a sine wave input $r(t) = \sin t$