Question 1

Watch the BBQ Temperature Control video posted on eClass.

(a)

What are the reference signal and plant output for this system?

For this system, the reference signal is the desired temperature of the BBQ, and the plant output is the actual temperature of the BBQ.

(b)

What is a disturbance for this system?

For this system, a disturbance are environmental factors that affect the temperature of the BBQ, such as wind, rain, ambient temperature, etc.

(c)

Draw a block diagram of the overall closed-loop system. Label the signal arrows and name the controller, actuator, plant and sensor in your diagram.

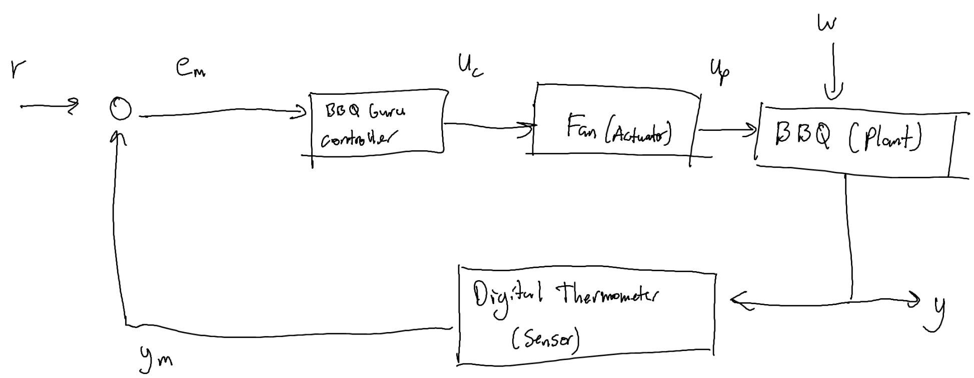

Figure 1: Block diagram of the overall closed-loop system for BBQ Guru.

Where

- r is the reference signal, the desired temperature of the BBQ
- e_m is the measured tracking error, the difference between the desired temperature and the measured temperature of the BBQ
- u_c is the control input, which is the output of the controller to the fan
- u_p is the plant input, which is the fan speed which controls the amount of oxygen supplied to the charcoal, affecting the temperature of the BBQ
- y is the plant output, the actual temperature of the BBQ

- y_m is the measured plant output, the measured temperature of the BBQ from the temperature sensor as an electronic signal
- w is the disturbance, which is the environmental factors that affect the temperature of the BBQ
- v is the sensor noise, which is the noise from the temperature sensor

Question 2

Consider the following system consisting of two carts connected by a damper:

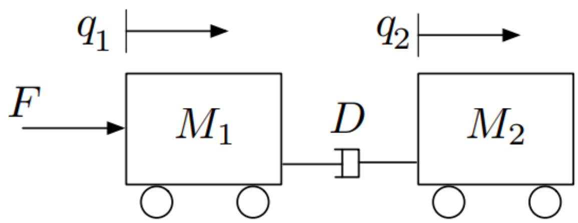

Figure 2: System consisting of two carts connected by a damper.

The input is the force F applied to the first cart, while q_1 and q_2 are the positions of the first and second cart, respectively. The output is the position of the second cart q_2.

(a)

Using Newton's Second Law, obtain the equations of motion of the system.

Since both carts are moving, the damper is applying a force based on the relative velocity of the two carts. The equation of motion for the first cart is:

$$M_1\ddot{q}_1 = F - D(\dot{q}_2 - \dot{q}_1)$$

By Newton's Third Law, the damping force on the second cart is equal and opposite to the damping force on the first cart. The equation of motion for the second cart is:

$$M_2\ddot{q}_2 = D(\dot{q}_2 - \dot{q}_1)$$

(b)

Introduce the state variables $x_1 = q_1$, $x_2 = q_2$, $x_3 = \dot{q}_1$, $x_4 = \dot{q}_2$. Obtain the state model of this system.

In addition to the above state variables, let the system input be $u = F$. The state model of this system is:

$$
\begin{aligned}
\dot{x_1} &= x_3 \\
\dot{x_2} &= x_4 \\
\dot{x_3} &= \frac{1}{M_1}u - \frac{D}{M_1}(x_4 - x_3) \\
\dot{x_4} &= \frac{D}{M_2}(x_4 - x_3) \\
y &= x_2
\end{aligned}
$$

(c)

Obtain the state-space form of this system.

By inspection, the state dynamics are linear, and the output is linear. Rewriting the state model in matrix form:

$$
x = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{D}{M_1} & \frac{D}{M_1} \\ 0 & 0 & \frac{D}{M_2} & -\frac{D}{M_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_1} \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}
$$

$$
y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}
$$

Question 3

Consider the electronic circuit illustrated below:

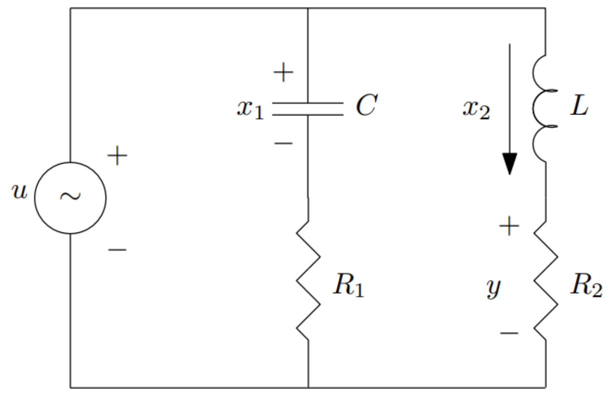

Figure 3: Electronic RLC circuit.

The states of the system are chosen as the voltage drop across the capacitor x_1 and the current flow through the inductor x_2. The input of the system is the voltage u and the output is the voltage drop y across resistor R_2. Derive the (linear) state-space form of this system.

In addition, let

- V_R be the voltage drop across resistor R
- V_C be the voltage drop across capacitor C
- V_L be the voltage drop across inductor L

The governing equations for the for components of the circuit are:

$$V_R = Ri_R \tag{1}$$

$$i_C = C\frac{dV_C}{dt} \tag{2}$$

$$V_L = L\frac{di_L}{dt} \tag{3}$$

Performing KVL on the circuit, we get two equations:

$$u = x_1 + V_{R1} \tag{4}$$

$$u = V_L + V_{R2} \tag{5}$$

By KCL, $i_{R1} = i_C$. Further manipulation of (4) yields:

$$\begin{aligned} u &= x_1 + R_1 i_C && (6)\\ &= x_1 + R_1 C\frac{dV_C}{dt} && (7)\\ &= x_1 + R_1 C\dot{x}_1 && (8) \end{aligned}$$

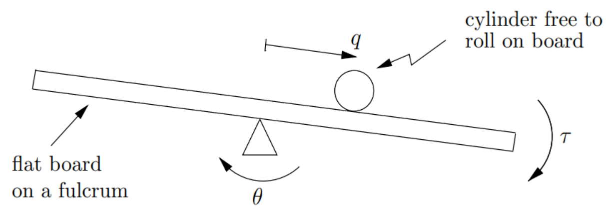

Figure 4: Cylinder rolling on a board.

By KCL, $x_2 = i_L = i_{R2}$. Further manipulation of (5) yields:

$$\begin{aligned} u &= V_L + R_2 i_L && (9)\\ &= L\frac{di_L}{dt} + R_2 i_L && (10)\\ &= L\dot{x_2} + R_2 x_2 && (11) \end{aligned}$$

The state space form is therefore:

$$x = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -\frac{R_1}{C} & 0 \\ 0 & -\frac{R_2}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ \frac{1}{L} \end{bmatrix} \begin{bmatrix} u \end{bmatrix} \quad (12)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (13)$$

Question 4

Consider a cylinder rolling on a board, with a torque applied to the latter:

As shown, the angle of tilt is denoted by θ, the torque applied to the board is τ, and q measures the distance the cylinder rolls down the board. Let J denote the mass moment of inertia of the board about its pivot and J_c, R_c and M_c the mass moment of inertia, radius and mass of the cylinder, respectively. Assuming roll without slip, the equations of motion can be shown to be:

$$\left(\frac{J_c}{R_c^2} + M_c\right)\ddot{q} + M_c g \sin\theta - M_c q \dot{\theta}^2 = 0 \quad (14)$$

$$(M_c q^2 + J + J_c)\ddot{\theta} + 2M_c q \dot{q}\dot{\theta} + M_c g q \cos\theta = \tau \quad (15)$$

(a)

What is the order of this system?

The system consists of two 2nd order differential equations, so the order of the system is 2.

(b)

Defining $x = (q, \theta, \dot{q}, \dot{\theta})$, $u = \tau$ and $y = q$, write the state model of this system.

Isolating the $\ddot{q}$ and $\ddot{\theta}$ terms in (14) and (15) respectively, we get:

$$\begin{aligned}
\ddot{q} &= \frac{1}{\frac{J_c}{R_c^2} + M_c}\left(M_c q\dot{\theta}^2 - M_c g \sin\theta\right) \\
\ddot{\theta} &= \frac{1}{M_c q^2 + J + J_c}\left(u - 2M_c q\dot{q}\dot{\theta} - M_c g q \cos\theta\right)
\end{aligned}$$

The state model is therefore:

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \frac{1}{\frac{J_c}{R_c^2}+M_c}\left(M_c q\dot{\theta}^2 - M_c g \sin\theta\right) \\ \frac{1}{M_c q^2+J+J_c}\left(u - 2M_c q\dot{q}\dot{\theta} - M_c g q \cos\theta\right) \end{bmatrix} \\
y &= q
\end{aligned}$$

(c)

Does the system in (b) have a state-space form? Why or why not?

The system in (b) does not have a state-space form because the state dynamics are not linear due to the presence of the q^2, $\sin\theta$, $\cos\theta$, θ^2, $2M_c q\dot{q}\dot{\theta}$, and various other non-linear terms.