MEC E 301
Lab 1: Dimensional Measurement

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Question 1

(a)

[testetsetset] The following table is a summary of the accuracy, repeatability, resolution, and range of various measurement devices.

Table 1: Accuracy, Repeatability, Resolution, and Range of Various Measurement Devices

Device	Accuracy (mm)	Repeatability (mm)	Resolution (mm)	Range (mm)
Digital Caliper	±0.04	0.03	0.01	(0.00, 153.90)
Micrometer	±0.013	0.005	0.001	(0.000, 26.886)
Vernier Caliper	±0.02	0.02	0.02	(0.00, 155.30)

(b)

Below is a deviation table for the micrometer. The deviation from standard is calculated for each gauge block length the maximum deviation is highlighted in bold and is the accuracy.

Table 2: Gauge Block Deviation Table for Micrometer

	Deviation From Standard					
Gauge length (mm)	Reading 1 (mm)	Reading 2 (mm)	Reading 3 (mm)	Reading 4 (mm)	Reading 5 (mm)	Max Dev. (mm)
0.000	0.000	0.000	0.000	0.000	0.000	0.000
2.008	0.003	0.002	0.001	0.002	0.002	0.002
10.000	0.003	0.003	0.004	0.003	0.003	0.001
14.000	0.011	0.009	0.010	0.010	0.010	0.002
20.000	0.007	0.006	0.005	0.004	0.005	0.003
25.000	0.011	0.010	**0.013**	0.011	0.008	**0.005**

The repeatability is simply the maximum deviation minus the minimum deviation. The maximum deviation is highlighted in bold and is the repeatability.

Sample calculation for the 2.008mm gauge block length deviation from standard:

$$\begin{aligned}\text{Deviation from Standard} &= \text{Reading} - \text{Standard}\\ &= 2.008 - 2.005\\ &= \boxed{0.003}\end{aligned}$$

Sample calculation for the maximum deviation for the 25.000mm gauge block length:

$$\begin{aligned}\text{Max Deviation} &= \text{Max Reading} - \text{Min Reading}\\ &= 0.013 - 0.008\\ &= \boxed{0.005}\end{aligned}$$

To obtain the accuracy, the maximum deviation is taken from the deviation table. That is, accuracy = max(all deviations) = 0.013 mm.

Similarly, the repeatability is obtained by taking the maximum deviation minus the minimum deviation. That is, repeatability = max(max deviation per gauge block) = 0.005 mm.

Question 2

(a)

The sample standard deviation for the sample measurements for F can simply be calculated using the following equation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

Allowing Excel to handle the calculation, the standard dev is 0.001906 mm. The next value of interest is the one-sided inverse t-distribution value. This was calculated using Excel where a confidence level of 95% was used, with $v = n - 1$ degrees of freedom which corresponds to $\alpha = 1 - 0.95 = 0.05$ and $v = 20 - 1 = 19$. The value is $t_{0.05/2,4} = 2.0930$. The precision can now be calculated using the following equation:

$$P_x = \frac{t_{\alpha/2,v}\sigma}{\sqrt{n}} = \frac{2.0930 \times 0.001906}{\sqrt{19}} = \pm 0.0008919 \text{ mm} = \boxed{\pm 0.001 \text{ mm}}$$

Using the results from Table 1, the accuracy of the micrometer is $B_x = 0.013$ mm. The total uncertainty is calculated using the following equation:

$$\delta s_{\mathrm{F}} = \sqrt{P_x^2 + B_x^2} = \sqrt{(0.0008919)^2 + (0.013)^2} = \pm 0.01303 \text{ mm} = \boxed{\pm 0.013 \text{ mm}}$$

A table summary of all the uncertainties for the measured dimensions is shown below:

Table 3: Uncertainty of Measured Dimensions of Block 22

Dim.	Device (mm)	Acc. (mm)	STDEV (mm)	T-Dist Inv. (mm)	P (mm)	U (mm)	Value (mm)
A	Digital Caliper	0.04	3.468E-02	2.0930	0.02	0.04	66.42
B	Digital Caliper	0.04	8.367E-03	2.7764	0.01	0.04	12.60
C	Digital Caliper	0.04	8.944E-03	2.7764	0.01	0.04	6.51
C	Calculated	N/A	N/A	N/A	N/A	0.223	6.417
D	Calculated	N/A	N/A	N/A	N/A	0.05	18.05
E	Calculated	N/A	N/A	N/A	N/A	0.06	16.10
F	Micrometer	0.013	1.906E-03	2.0930	0.001	0.013	16.087
G	Vernier Caliper	0.02	7.327E-03	2.0930	0.00	0.02	43.80
H	Micrometer	0.013	1.788E-01	2.7764	0.222	0.222	9.669
J	Digital Caliper	0.04	2.881E-02	2.7764	0.04	0.05	9.80
K	Digital Caliper	0.04	5.477E-03	2.7764	0.01	0.04	11.75

(b)

From geometry, it can be observed that measurement D, s_{D}, can be calculated using the following equation:

$$\begin{aligned} s_{\mathrm{D}} &= s_{\mathrm{K}} + \frac{s_{\mathrm{B}}}{2} = f(s_{\mathrm{K}}, s_{\mathrm{B}}) \\ &= 11.75 + \frac{12.60}{2} \\ &= \boxed{18.05 \text{ mm}} \end{aligned}$$

Simply plugging in the numbers yields us the measurement D, $s_{\mathrm{D}} = 18.05$ mm. The uncertainty of D, δs_{D}, can be calculated using the following equation:

$$\begin{aligned} \delta s_{\mathrm{D}} &= \sqrt{\left(\frac{\partial f}{\partial s_{\mathrm{K}}}\right)^2 (\delta s_{\mathrm{K}})^2 + \left(\frac{\partial f}{\partial s_{\mathrm{B}}}\right)^2 (\delta s_{\mathrm{B}})^2} \\ &= \sqrt{(1)^2(\delta s_{\mathrm{K}})^2 + \left(\frac{1}{2}\right)^2 (\delta s_{\mathrm{B}})^2} \\ &= \sqrt{(\delta s_{\mathrm{K}})^2 + \frac{1}{4}(\delta s_{\mathrm{B}})^2} \\ &= \sqrt{(0.04)^2 + \frac{1}{4}(0.04)^2} \\ &= 0.046 \text{ mm} \\ &= \boxed{0.05 \text{ mm}} \end{aligned}$$

Plugging in the values from Table 3 yields us the uncertainty of D, $\delta s_{\mathrm{D}} = 0.046$ mm. That means the measurement for D is:

$$\boxed{s_{\mathrm{D}} = 18.05 \pm 0.05 \text{ mm}}$$

(c)

Let us calculate measurement C'. From geometry, it can be observed that measurement C', can be calculated using the following equation:

$$\begin{aligned} s_{\text{C'}} &= s_{\text{F}} - s_{\text{H}} = f(s_{\text{F}}, s_{\text{H}}) \\ &= 16.087 - 9.669 \\ &= \boxed{6.417 \text{ mm}} \end{aligned}$$

Plugging in the numbers yields us the measurement C, $s_{\text{C'}} = 6.417$ mm. The uncertainty of C, δs_{C}, can be calculated using the following equation:

$$\begin{aligned} \delta s_{\text{C'}} &= \sqrt{\left(\frac{\partial f}{\partial s_{\text{F}}}\right)^2 (\delta s_{\text{F}})^2 + \left(\frac{\partial f}{\partial s_{\text{H}}}\right)^2 (\delta s_{\text{H}})^2} \\ &= \sqrt{(1)^2(\delta s_{\text{F}})^2 + (-1)^2(\delta s_{\text{H}})^2} \\ &= \sqrt{(\delta s_{\text{F}})^2 + (\delta s_{\text{H}})^2} \\ &= \sqrt{(0.013)^2 + (0.222)^2} \\ &= \boxed{0.226 \text{ mm}} \end{aligned}$$

Plugging in the values from Table 3 yields us the uncertainty of C, $\delta s_{\text{C'}} = 0.226$ mm. That means the measurement for C is:

$$\boxed{s_{\text{C'}} = 6.417 \pm 0.226 \text{ mm}}$$

The measured value for C was 6.51 ± 0.04 mm. The measured value is not within the uncertainty range of the calculated value. This is likely due to the bore hole having a taper, which would make measuring the bottom of the bore hole difficult. The measured value is likely smaller as a result and agrees with experimental data.

(d)

From geometry, it can be observed that measurement E, can be calculated using the following equation:

$$\begin{aligned} s_{\text{E}} &= s_{\text{j}} + \frac{s_{\text{B}}}{2} = f(s_{\text{j}}, s_{\text{B}}) \\ &= 9.80 + \frac{12.60}{2} \\ &= \boxed{16.10 \text{ mm}} \end{aligned}$$

From differential calculus, the uncertainty of E, δs_{E}, can be calculated using the following equation:

$$\begin{aligned}\delta s_{\mathrm{E}} &= \sqrt{\left(\frac{\partial f}{\partial s_{\mathrm{j}}}\right)^2 (\delta s_{\mathrm{j}})^2 + \left(\frac{\partial f}{\partial s_{\mathrm{B}}}\right)^2 (\delta s_{\mathrm{B}})^2} \\ &= \sqrt{(1)^2(\delta s_{\mathrm{j}})^2 + \left(\frac{1}{2}\right)^2 (\delta s_{\mathrm{B}})^2} \\ &= \sqrt{(\delta s_{\mathrm{j}})^2 + \frac{1}{4}(\delta s_{\mathrm{B}})^2} \\ &= \sqrt{(0.05)^2 + \frac{1}{4}(0.04)^2} \\ &= \boxed{0.06\ \mathrm{mm}}\end{aligned}$$

Plugging everything in,

$$\boxed{s_{\mathrm{E}} = 16.10 \pm 0.06\ \mathrm{mm}}$$

The summary table was shown in Table 3.

Question 3

(a)

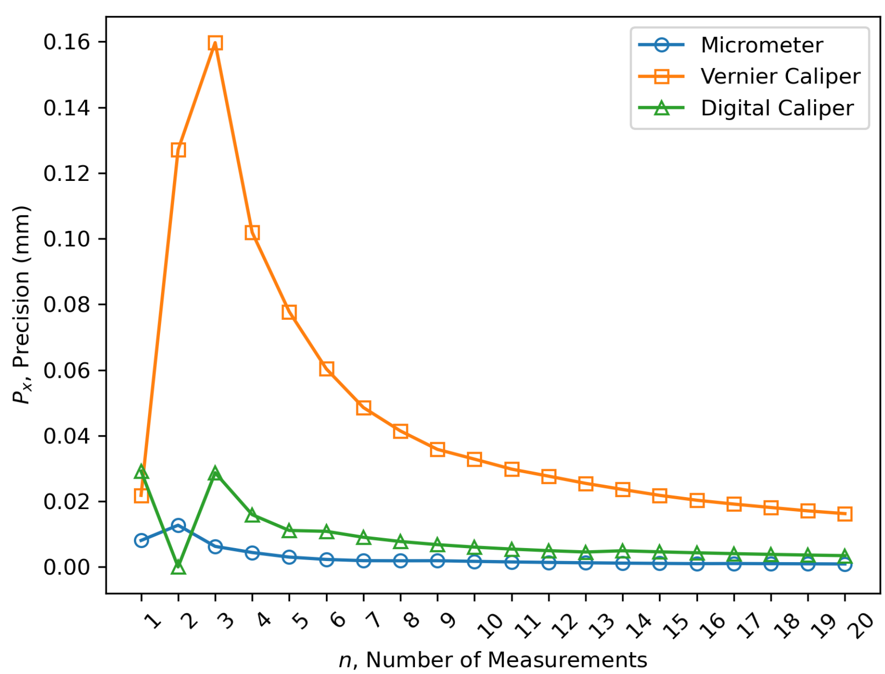

Figure 1: Precision of various measurement devices as measurement count increases

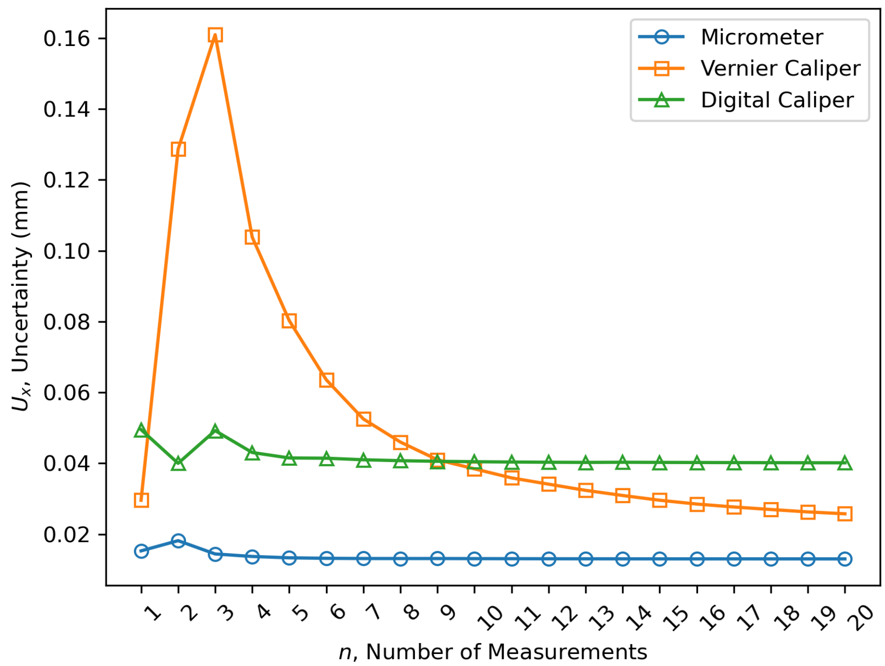

Figure 2: Uncertainty of various measurement devices as measurement count increases

(b)

Since the formula for the precision is a function of one-sided t-distribution inverse as well as standard deviation, both of which decrease as the number of measurements increases, the precision will generally decrease as the number of measurements increases. The $\sum_{i=1}^{n}(x_i - \bar{x})^2$ term contributes a lot while n is small so standard deviation may not decrease for the first couple of terms. A crude guess is that the precision will decrease $\propto \frac{1}{n}$.

The formula for uncertainty is a function of precision and bias/accuracy. Accuracy is also reliant on the deviation of the measurements and will often reach a stable value after a certain number of measurements. If accuracy is approximately constant, the only contributing term would be the precision. Therefore, the uncertainty will probably decrease around $\propto \frac{1}{n}$ (not too familiar with the behaviour of inverse one-sided t-distribution).

Question 4

From this investigation, the tool with the highest accuracy is the micrometer. The second best tool is the vernier caliper. With the caliper being nearly a magnitude more inaccurate than the micrometer.

This result is somewhat unexpected. One would assume that the digital caliper would be superior over the vernier. The issues lies with human error.

When using the digital caliper, determining when to stop applying pressure was difficult at the start. When a sufficient measurement was thought to be found, more pressure could be applied to further reduce the measurement distance. Fortunately, unlike the digital caliper, the micrometer has a built in system that stops the user from over tightening.

Investigating the data more, the major contributor to the high uncertainty of the digital caliper is the accuracy. The digital caliper was the first tool used during the lab session, and adjustment to the new tools was a major factor to the error. A rerun of this investigation would probably increase the accuracy of the digital caliper.

The vernier caliper has a resolution of 0.02. That meant that a different measurement reading would only be detected if there was a larger human error in measurement. In addition, at this point in the lab, familiarity with the tools made the measurements more consistent.

This results of this exercise suggest to use the micrometer and the vernier caliper for measurement. Human error, due to unfamiliarity with the tools, was a large contributing factor to why the vernier caliper was chosen over the digital caliper.
Despite vernier having a lower uncertainty, the digital caliper should be used. With proper usage, the uncertainty should be lower.

The table below summarizes the recommendations for each dimension.

Table 4: Summary table of all recommend tools for dimensions of block 22

Dim.	Vernier	Micrometer	Digital
A	×, Low Res.	×, Range Exceeded	✓ Compat. Geo., In Range, High Res.
B	×, Low Res.	×, Incompat. Geo.	✓ Compat. Geo., In Range, High Res.
C	×, Low Res.	✓ Compat. Geo., In Range, High Res.	×, Low Res.
F	×, Low Res.	✓ Compat. Geo., In Range, High Res.	×, Low Res.
G	×, Low Res.	×, Incompat. Geo.	✓ Compat. Geo., In Range, High Res.
H	×, Low Res.	✓ Compat. Geo., In Range, High Res.	×, Low Res.
J	×, Low Res.	×, Incompat. Geo.	✓ Compat. Geo., In Range, High Res.
K	×, Low Res.	×, Incompat. Geo.	✓ Compat. Geo., In Range, High Res.

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A Appendix: Matplotlib Python Code to Render Graphs

This is just some random code.

```
if transactions: Transaction.create_transactions() # if transactions
    = "true"
node.generate_emptyState() # empty state for all nodes
S.initial_events() # initiate initial events to start with

while not queue.isEmpty() and clock <= targetTime:
    next_e = queue.get_next_event()
    clock = next_e.time # move clock to the time of the event
    Event.execute_event(next_e)
    Queue.remove_event(next_e)

print results
```