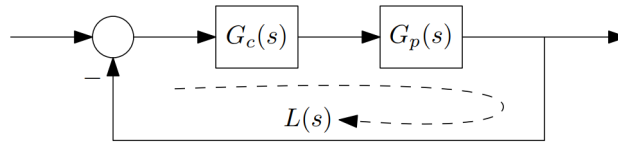


Question 1

Consider the simplified closed-loop diagram where



$$G_c(s) = 1, \quad G_p(s) = \frac{1}{s+1}$$

- By hand, sketch the Nyquist plot of $L(s) = G_p(s)G_c(s)$. Provide both the Nyquist contour \mathcal{D} and the Nyquist plot \mathcal{G} , and label the segment endpoints A , B and C .
- Use MATLAB's `nyquist` command to check your answer in (a). Provide the MATLAB commands you used to obtain this (you do not need to include the plot)
- Apply Theorem 5.3.1 (Nyquist Stability Criterion) to the above to check the stability of the closed-loop system.
- Now form the characteristic polynomial of the above system, and use Theorem 4.4.1 to check closed-loop stability. Does your answer match with (c)?

(a)

The loop transfer function $L(s)$ is given by

$$L(s) = G_p(s)G_c(s) = \frac{1}{s+1}$$

Desmos will be used to "sketch" the Nyquist plot since I'm bad at drawing with a mouse. Since $L(s)$ has real constant coefficients, the curve will be symmetric about the real axis.

From a to b , $s = j\omega$, $\omega \in [0, \infty)$,

$$\begin{aligned} L(j\omega) &= \frac{1}{j\omega + 1} \\ &= \frac{1 - j\omega}{1 + \omega^2} \end{aligned}$$

which results in

$$\operatorname{Re}\{L(j\omega)\} = \frac{1}{1 + \omega^2}, \quad \operatorname{Im}\{L(j\omega)\} = -\frac{\omega}{1 + \omega^2}$$

Observe that at $\omega = 0$, $L(0) = 1$, and as $\omega \rightarrow \infty$, $L(\infty) = 0$. Sketch is shown in Figure 1

(b)

The following MATLAB commands were used to generate the Nyquist plot:

```
nyquist(tf([1], [1 1]))
```

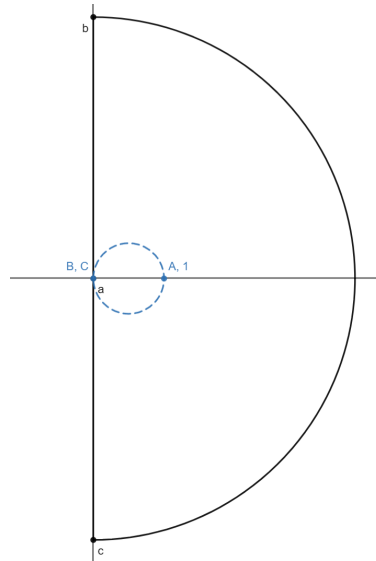


Figure 1: 1(a) Nyquist sketch

(c)

$L(s)$ has no poles in the right half plane, so $n = 0$. Since the Nyquist plot does not pass through -1 and encircles the origin $n = 0$ times, the closed-loop system is stable.

(d)

The characteristic polynomial of the closed-loop system is given by

$$p(s) = n_p n_c + d_p d_c = 1 + (s + 1) = s + 2$$

By Theorem 4.4.1, the closed-loop system is stable if and only if $p(s)$ has negative real roots. Since $p(s)$ has a single at $s = -2$, the closed-loop system is stable. This matches with the result from (c).

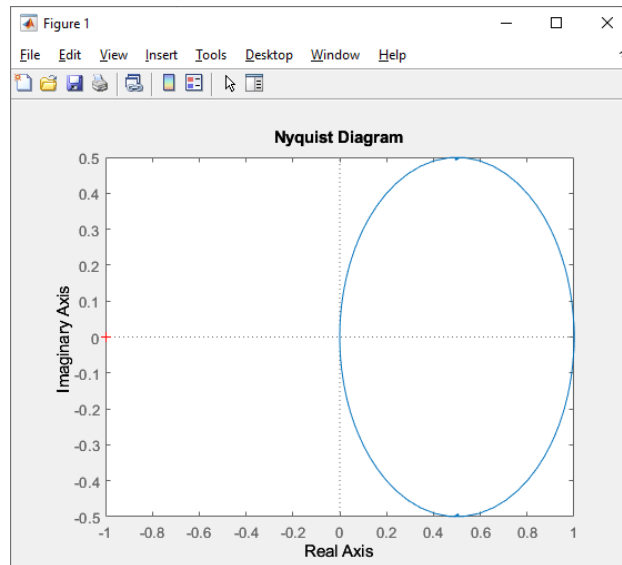


Figure 2: 1(b) Nyquist plot generated by MATLAB

Question 2

Reconsider the closed-loop diagram from Question 1, now with

$$G_c(s) = 2 + \frac{2}{s} = \frac{2s + 2}{s}, \quad G_p(s) = \frac{1}{s - 1}$$

i.e. a BIBO unstable plant connected to a PI controller.

- By hand, sketch the Nyquist contour \mathcal{D} and Nyquist plot \mathcal{G} , then apply Theorem 5.3.1 to check the stability of the closed-loop system
- Give the plot obtained with MATLAB's `nyquist` command. What's the major difference between this result and the sketch from (a)?

(a)

The loop transfer function $L(s)$ is given by

$$L(s) = G_p(s)G_c(s) = \frac{2s + 2}{s(s - 1)}$$

The poles are at $s = 0$ and $s = 1$. Since there is a pole at $s = 0$, a small semi-circle of radius ϵ is drawn around the origin.

From a to b , $s = j\omega$, $\omega \in [0, \infty)$,

$$L(j\omega) = \frac{2j\omega + 2}{j\omega(j\omega - 1)}$$

With help from MATLAB,

```
syms s omega real
L(s) = (2*s + 2)/(s*(s-1));
simplify(real(L(j*omega)))
simplify(imag(L(j*omega)))
```

which gives

$$\operatorname{Re}\{L(j\omega)\} = \frac{-4}{\omega^2 + 1}, \quad \operatorname{Im}\{L(j\omega)\} = \frac{2(1 - \omega^2)}{\omega(\omega^2 + 1)}$$

Starting at point a the point $s = j\epsilon$,

$$\operatorname{Re}\{L(j\epsilon)\} = \frac{-4}{\epsilon^2 + 1} \approx -4, \quad \operatorname{Im}\{L(j\epsilon)\} = \frac{2(1 - \epsilon^2)}{\epsilon(\epsilon^2 + 1)} \rightarrow \infty$$

to point b , $s = j\omega$, $\omega \rightarrow \infty$,

$$\operatorname{Re}\{L(j\omega)\} = \frac{-4}{\omega^2 + 1} \rightarrow 0, \quad \operatorname{Im}\{L(j\omega)\} = \frac{2(1 - \omega^2)}{\omega(\omega^2 + 1)} \rightarrow 0$$

By symmetry, c to d is the same as a to b reflected about the real axis. Note the curve intersects the real axis at $s = -2$ on its way to b and c .

The last section a to d is derived in the notes as

$$L(\epsilon e^{j\theta}) \approx \frac{e^{-j\theta}}{\epsilon} \frac{2s + 2}{s - 1}$$

which sweeps out a semi-circle of infinite radius of angular distance π .

The Nyquist plot is shown in Figure 3

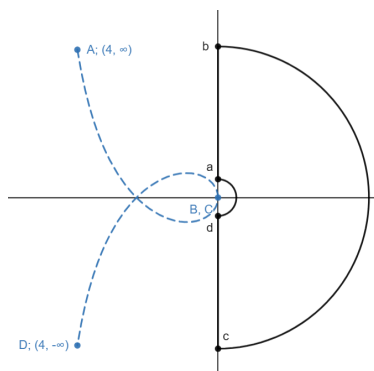


Figure 3: 2(a) Nyquist sketch

Since there is one pole $\operatorname{Re}(s) > 0$, $n = 1$. Since the Nyquist plot does not pass through -1 and encircles it once in the counter-clockwise direction, the closed-loop system is stable.

(b)

```
nyquist(tf([2 2], [1 -1 0]))
```

The major difference is the lack of the semi-circle of infinite radius. The sketch approximated the infinite radius semi-circle as a semi-circle of finite radius.

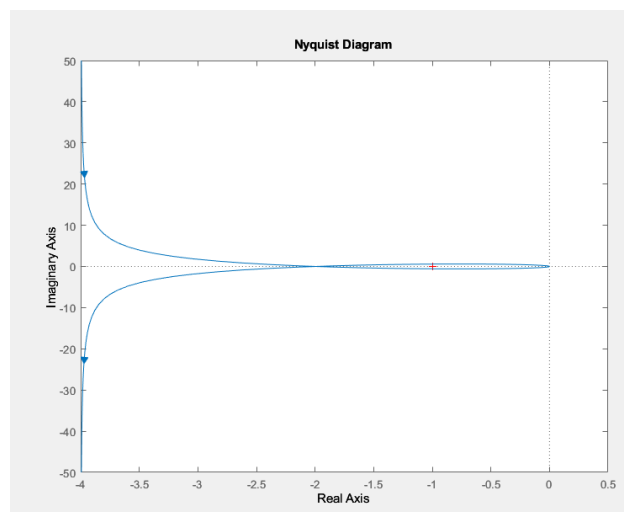


Figure 4: 2(b) Nyquist plot generated by MATLAB