MEC E 430 Formula Sheet

Non-Dim of NS

$$t^* = ft,\ \vec{x}^* = \frac{\vec{x}}{L},\ v^* = \frac{\vec{v}}{V}$$

$$P^* \frac{P - P_\infty}{P_0 - P_\infty},\ g^* = \frac{\vec{g}}{g},\ \vec{\nabla}^* = L\vec{\nabla}$$

Non-dimensionalized NS:

$$[\mathrm{St}]\frac{\partial v^*}{\partial t^*} + (\vec{v}\cdot\vec{\nabla})\vec{v}^* = -[\mathrm{Eu}]\vec{\nabla}P^* + [\mathrm{Fr}^{-2}]\vec{g}^* + [\mathrm{Re}^{-1}]\vec{\nabla}^2\vec{v}^*$$

where

$$\mathrm{St} = \frac{fL}{V},\ \mathrm{Eu} = \frac{P_0 - P_\infty}{\rho V^2}$$

$$\mathrm{Fr} = \frac{V^2}{gL},\ \mathrm{Re} = \frac{\rho V L}{\mu}$$

Creeping Flow (Stokes Flow)

$\mathrm{Re} \ll 1$, viscous forces dominate, inertia negligible. NS becomes:

$$\vec{\nabla}P = \mu\vec{\nabla}^2\vec{v}$$

Drag on creeping flow,

$$F_D = cVL\mu,\ F_{D,\mathrm{sphere}} = 3\pi\mu V D$$

Typical balance for falling sphere:

$$W = F_D + F_{\mathrm{buoyancy}}$$

$$F_{\mathrm{buoyancy}} = \frac{\pi D^3}{6}\rho_{\mathrm{fluid}}g,\ W = \rho_{\mathrm{particle}}\frac{\pi D^3}{6}g$$

Inviscid Flow (Euler Flow)

Viscous stresses are negligible, $\tau \approx 0$. Typically high Re, away from wall. Can't specify no-slip at wall.

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}P + \rho\vec{g}$$

a consequence,

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{constant along streamline}$$

Check shear to make sure 0:

$$\tau = \mu\frac{\partial U}{\partial y}$$

$$\tau_{r\theta} = \mu\left(r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)$$

Irrotational Flow

$\vec{\nabla}\times\vec{v} = 0$. Velocity potential, ϕ,

$$u = \frac{\partial\phi}{\partial x},\ v = \frac{\partial\phi}{\partial y},\ w = \frac{\partial\phi}{\partial z}$$

$$u_r = \frac{\partial\phi}{\partial r},\ u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta},\ u_z = \frac{\partial\phi}{\partial z}$$

Solve cont. ($\nabla^2\phi = 0$), calculate $\vec{v}$, and calculate P from Bernoulli. Bernoulli is **constant everywhere** for irrotational flow.

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

$$\nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

Boundary Layer

Laminar BL assumptions are: steady, incomp, 2D, Re is large such that $\delta/x \ll 1$, BL stays laminar:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Blasius similarity variable

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}},\ f' = \frac{f}{U_\infty}$$

Blasius solution for flat plate:

	Laminar	Turbulent
BL Thick.	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} = \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$
Displ. Thick.	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} = \frac{0.20}{(\mathrm{Re}_x)^{1/7}}$
Momen. Thick.	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} = \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$
Local Skin Fric. Coeff.	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} = \frac{0.027}{\mathrm{Re}_x^{1/7}}$

Displacement thickness

$$\delta^* = \int_0^\delta (1 - \frac{u}{U_\infty})\,dy$$

Momentum thickness

$$\theta = \int_0^\delta \frac{u}{U_\infty}(1 - \frac{u}{U_\infty})\,dy$$

Drag Coefficient

$$C_D = \frac{2F_D}{\rho V^2 A}$$

1D Isentropic Nozzle

Assumptions: 1D (negligible boundary layer effects); steady; adiabatic; ideal gas; isentropic; inlet conditions do not change appreciably. Stagnation properties:

$$h_0 = h + \frac{V^2}{2}$$

$$T_0 = T + \frac{V^2}{2c_p}$$

If isentropic:

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}}$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$$

Speed of sound:

$$c = \sqrt{\left(\frac{\partial P}{\partial\rho}\right)_s} = \sqrt{kRT}$$

Mach number:

$$\mathrm{Ma} = \frac{V}{c}$$

For an expansion with $\uparrow A$:

Subsonic	Supersonic
$\downarrow V, \mathrm{Ma}$ $\downarrow P, \rho$	$\uparrow V, \mathrm{Ma}$ $\uparrow P, \rho$

And if c_p is constant:

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}\mathrm{Ma}^2$$

$$\frac{P_0}{P} = \left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{k}{k-1}} = \frac{\rho_0}{\rho}$$

When Ma$=1$ (at throat), critical properties are

$$\frac{T^*}{T_0} = \frac{2}{k+1}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} = \frac{\rho^*}{\rho_0}$$

so exit pressure:

$$P_e = \begin{cases} P_b, & P_b > P^*(\mathrm{subsonic}, \mathrm{Ma} < 1) \\ P^*, & P_b \le P^*(\mathrm{sonic/chocked}, \mathrm{Ma} = 1) \end{cases}$$

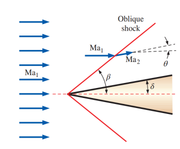
mass flow:

$$\dot{m} = \rho AV = P_0 A \mathrm{Ma}\sqrt{\frac{k}{RT_0}}\left(1 + \frac{k-1}{2}\mathrm{Ma}^2\right)^{\frac{-k-1}{2(k-1)}}$$

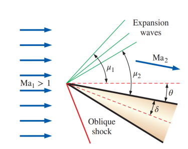
$$\dot{m}_{\max} = P_0 A^*\sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} = \rho^* A^* V^*$$

note, max mass flow is when Ma$=1$ and $A = A^*$. For converging-diverging nozzles,

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left(\frac{2}{k+1}\cdot\left(1 + \frac{k-1}{2}\right)\right)^{\frac{k+1}{2(k-1)}}$$



Oblique Shock



Expansion Fan

Normal Shock

Region 1 is upstream, Region 2 is downstream. The shock is not isentropic.

$$T_{01} = T_{02}$$

$$\mathrm{Ma}_2 = \sqrt{\frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - (k-1)}}$$

$$\frac{P_2}{P_1} = \frac{2k\mathrm{Ma}_1^2 - (k-1)}{k+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2/P_1}{T_2/T_1} = \frac{(k+1)\mathrm{Ma}_1^2}{2 + (k-1)\mathrm{Ma}_1^2} = \frac{V_1}{V_2}$$

$$\frac{T_2}{T_1} = \frac{2 + \mathrm{Ma}_1^2(k-1)}{2 + \mathrm{Ma}_2^2(k-1)}$$

$$\frac{P_{02}}{P_{01}} = \frac{\mathrm{Ma}_1}{\mathrm{Ma}_2}\left(\frac{1 + \mathrm{Ma}_1^2(k-1)/2}{1 + \mathrm{Ma}_2^2(k-1)/2}\right)^{\frac{k+1}{2(k-1)}}$$

$$s_2 - s_1 = c_p\ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

Oblique Shock

θ is the turning angle, β is the shock angle. **If thin boundary layer**, $\delta = \theta$. The **normal components** of the shock is the **same** as a **normal shock**. Use Ma_n and V_n for normal components. Pressure, temperature, and density are still P_0, T_0, ρ_0. The normal components:

$$\mathrm{Ma}_{1,n} = \mathrm{Ma}_1\sin\beta$$

$$\mathrm{Ma}_{2,n} = \mathrm{Ma}_2\sin(\beta - \theta)$$

$$\tan\theta = \frac{2\cot\beta(\mathrm{Ma}_1^2\sin^2\beta - 1)}{\mathrm{Ma}_1^2(k + \cos 2\beta) + 2}$$

Mach angle,

$$\mu = \sin^{-1}(1/\mathrm{Ma}_1)$$

Expansion Waves

$$\theta = \nu(\mathrm{Ma}_2) - \nu(\mathrm{Ma}_1)$$

$$\nu(\mathrm{Ma}) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\sqrt{\frac{k-1}{k+1}(\mathrm{Ma}^2 - 1)} - \tan^{-1}\sqrt{\mathrm{Ma}^2 - 1}$$

Use isentropic relations to find properties across the wave.