1   System Response for Unforced SDOF Systems

Table 1: Steady State Solutions for Unforced SDOF Systems

System	Response
Undamped Spring Mass	$\dfrac{v_0}{p}\sin(pt) + x_0\cos(pt)$
Damped Spring Mass	$e^{-\zeta pt}\left[\dfrac{v_0 + \zeta p x_0}{\sqrt{1-\zeta^2}\,p}\sin(\sqrt{1-\zeta^2}\,pt) + x_0\cos(\sqrt{1-\zeta^2}\,pt)\right]$

2   Steady State Solutions for Forced SDOF Systems

Table 2: Steady State Solutions for Forced SDOF Systems

System	Steady State	DMF (or Amplitude Response)	Transmissibility
Undamped Forced Spring Mass			
Forced Spring Mass	$\left(\dfrac{F_0}{k}\right)\left(\dfrac{1}{1-\left(\frac{\omega}{p}\right)^2}\right)\sin(\omega t)$	$\dfrac{\mathbb{X}}{\delta_{\text{ST}}} = \dfrac{1}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}$	$\dfrac{F_{T_{\max}}}{F_0} = \dfrac{1}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}$
Rotating Imbalance	$\dfrac{F_0/k}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}\sin(\omega t - \phi)$	$\dfrac{M\mathbb{X}}{\tilde{m}e} = \dfrac{\left(\frac{\omega}{p}\right)^2}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}$	$\dfrac{F_{T_{\max}}}{\tilde{m}e\omega^2} = \dfrac{1}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}$
Base Excitation	$a\left(\dfrac{1}{1-\left(\frac{\omega}{p}\right)^2}\right)\sin(\omega t)$	$\dfrac{\mathbb{X}}{a} = \dfrac{1}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}$	$\dfrac{F_{T_{\max}}}{ka} = \dfrac{\left(\frac{\omega}{p}\right)^2}{\left|1-\left(\frac{\omega}{p}\right)^2\right|}$
Damped Forced Spring Mass			
Forced Spring Mass	$\dfrac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\dfrac{\mathbb{X}}{\delta_{\text{ST}}} = \dfrac{1}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\dfrac{F_{T_{\max}}}{F_0} = \dfrac{\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$
Rotating Imbalance	$\dfrac{\tilde{m}e\omega^2/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\dfrac{M\mathbb{X}}{\tilde{m}e} = \dfrac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\dfrac{F_{T_{\max}}}{\tilde{m}e\omega^2} = \dfrac{\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$
Base Excitation	$\dfrac{a\sqrt{k^2+(c\omega)^2}/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\dfrac{\mathbb{X}}{a} = \dfrac{\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$ $\dfrac{\mathbb{Z}}{a} = \dfrac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\dfrac{F_{T_{\max}}}{ka} = \dfrac{\left(\frac{\omega}{p}\right)^2\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$

General solution for forced damped SDOF system [1]

$$m_{\text{eff}}\ddot{x} + c_{\text{eff}}\dot{x} + k_{\text{eff}}x = F_0\sin(\omega t - \alpha)$$

is given by

$$x(t) = \mathbb{X}\sin(\omega t - \alpha - \phi)$$

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[1] F_0 for spring, imbalance, and excitation is F_0, $me\omega^2$, and $a\sqrt{k^2+(c\omega)^2}$, respectively.

where

$$\mathbb{X} = \frac{F_0}{\sqrt{(k_{\text{eff}} - m_{\text{eff}}\omega^2)^2 + (c_{\text{eff}}\omega)^2}} = \frac{F_0/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$$

3 Steady State Solutions for Forced Non-Harmonic SDOF Systems

The general steady state solution for a periodic ($\tau = 2\pi/\omega$) forced damped SDOF system is given by:

$$x(t) = \frac{a_0}{2k} + \sum_{j=1}^{\infty} \frac{a_j/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}} \cos(j\omega t - \phi_j)$$

$$+ \sum_{j=1}^{\infty} \frac{b_j/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}} \sin(j\omega t - \phi_j)$$

where

$$\phi_j = \tan^{-1}\left[\frac{2\zeta\frac{\omega}{p}}{1 - \left(\frac{j\omega}{p}\right)^2}\right]$$

$$a_0 = \frac{2}{\tau}\int_0^\tau F(t)dt = 2F_{\text{avg}}$$

$$a_j = \frac{2}{\tau}\int_0^\tau F(t)\cos(j\omega t)dt, \quad j = 1, 2, 3, \dots$$

$$b_j = \frac{2}{\tau}\int_0^\tau F(t)\sin(j\omega t)dt, \quad j = 1, 2, 3, \dots$$

Todo: add common solutions such as step, ramp. Ask TA if you can infer some coefficients are zero based on some symmetry.

4 Transient Response of Spring-Mass Systems

Table 3: Particular Response of Spring-Mass Systems

Input	F_0	Particular Response
Step	$F_0 = F_1$	$x_p(t) = \dfrac{F_1}{k}$
Ramp	$F_0 = \beta t$	$x_p(t) = \dfrac{\beta}{k}t$
Exponential	$F_0 = F_1 e^{-at}$	$x_p(t) = \dfrac{F_1}{ma^2 + k}e^{-at}$