1 System Response for Unforced SDOF Systems

Table 1: Steady State Solutions for Unforced SDOF Systems

System	Response
Undamped Spring Mass	$\frac{v_0}{p}\sin(pt) + x_0\cos(pt)$
Damped Spring Mass	$e^{-\zeta pt}\left[\frac{v_0+\zeta p x_0}{\sqrt{1-\zeta^2}p}\sin(\sqrt{1-\zeta^2}pt) + x_0\cos(\sqrt{1-\zeta^2}pt)\right]$

2 Steady State Solutions for Forced SDOF Systems

Table 2: Steady State Solutions for Forced SDOF Systems

System	Steady State	DMF (or Amplitude Response)	Transmissibility
		Undamped Forced Spring Mass	
Forced Spring Mass	$\left(\frac{F_0}{k}\right)\left(\frac{1}{1-\left(\frac{\omega}{p}\right)^2}\right)\sin(\omega t)$	$\frac{\mathbb{X}}{\delta_{\mathrm{ST}}} = \frac{1}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}$	$\frac{F_{T_{\max}}}{F_0} = \frac{1}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}$
Rotating Imbalance	$\frac{F_0/k}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}\sin(\omega t-\phi)$	$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}$	$\frac{F_{T_{\max}}}{\tilde{m}e\omega^2} = \frac{1}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}$
Base Excitation	$a\left(\frac{1}{1-\left(\frac{\omega}{p}\right)^2}\right)\sin(\omega t)$	$\frac{\mathbb{X}}{a} = \frac{1}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}$	$\frac{F_{T_{\max}}}{ka} = \frac{\left(\frac{\omega}{p}\right)^2}{\left\lvert 1-\left(\frac{\omega}{p}\right)^2\right\rvert}$
		Damped Forced Spring Mass	
Forced Spring Mass	$\frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{\mathbb{X}}{\delta_{\mathrm{ST}}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{F_{T_{\max}}}{F_0} = \frac{\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$
Rotating Imbalance	$\frac{\tilde{m}e\omega^2/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{F_{T_{\max}}}{\tilde{m}e\omega^2} = \frac{\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$
Base Excitation	$\frac{a\sqrt{k^2+(c\omega)^2}/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{\mathbb{X}}{a} = \frac{\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$ $\frac{\mathbb{Z}}{a} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{F_{T_{\max}}}{ka} = \frac{\left(\frac{\omega}{p}\right)^2\sqrt{1+\left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}$

General solution for forced damped SDOF system [1]

$$m_{\text{eff}}\ddot{x} + c_{\text{eff}}\dot{x} + k_{\text{eff}}x = F_0\sin(\omega t-\alpha)$$

is given by

$$x(t) = \mathbb{X}\sin(\omega t-\alpha-\phi)$$

[1] F_0 for spring, imbalance, and excitation is F_0, $me\omega^2$, and $a\sqrt{k^2+(c\omega)^2}$, respectively.

where

$$\mathbb{X} = \frac{F_0}{\sqrt{(k_{\text{eff}} - m_{\text{eff}}\omega^2)^2 + (c_{\text{eff}}\omega)^2}} = \frac{F_0/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$$

3 Steady State Solutions for Forced Non-Harmonic SDOF Systems

The general steady state solution for a periodic ($\tau = 2\pi/\omega$) forced damped SDOF system is given by:

$$\begin{aligned} x(t) = \frac{a_0}{2k} &+ \sum_{j=1}^{\infty} \frac{a_j/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}} \cos(j\omega t - \phi_j) \\ &+ \sum_{j=1}^{\infty} \frac{b_j/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}} \sin(j\omega t - \phi_j) \end{aligned}$$

where

$$\begin{aligned} \phi_j &= \tan^{-1}\left[\frac{2\zeta\frac{\omega}{p}}{1 - \left(\frac{j\omega}{p}\right)^2}\right] \\ a_0 &= \frac{2}{\tau}\int_0^{\tau} F(t)dt = 2F_{\text{avg}} \\ a_j &= \frac{2}{\tau}\int_0^{\tau} F(t)\cos(j\omega t)dt, \quad j = 1, 2, 3, \ldots \\ b_j &= \frac{2}{\tau}\int_0^{\tau} F(t)\sin(j\omega t)dt, \quad j = 1, 2, 3, \ldots \end{aligned}$$

Todo: add common solutions such as step, ramp. Ask TA if you can infer some coefficients are zero based on some symmetry.

4 Transient Response of Spring-Mass Systems

Table 3: Particular Response of Spring-Mass Systems

Input	F_0	Particular Response
Step	$F_0 = F_1$	$x_p(t) = \frac{F_1}{k}$
Ramp	$F_0 = \beta t$	$x_p(t) = \frac{\beta}{k}t$
Exponential	$F_0 = F_1 e^{-at}$	$x_p(t) = \frac{F_1}{ma^2 + k}e^{-at}$