MEC E 451 Winter 2024 **Assignment #3** **Due: March 1, 11:59 PM**

Instructions:

- Submit your assignment as a single PDF file through eClass.
- Show all your steps and solution procedures including clear and well labelled FBD/MAD diagrams when needed.
- Make sure that your solution is well organised and that you are using appropriate headers for each question and sub-question.
- Scanned photos of your handwritten solution are acceptable as long as they are **legible**

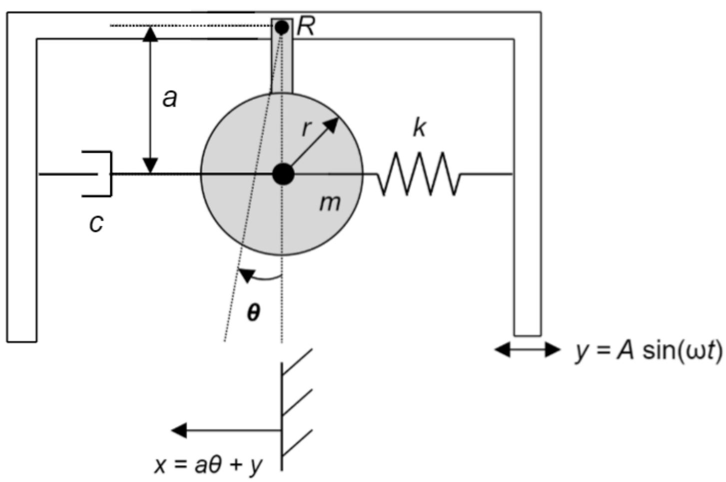
Question 1 (10 points)

The diagram models a gate valve which regulates the fluid flow in an engine. The gate valve is modelled as a pendulum (with a **massless** rod and uniform disk of mass m and radius r) pinned to the engine at R, and is controlled by a spring/damper mechanism connecting the centre of the disk to the engine.

The engine's motion during operation is denoted as $y = A \sin sin\,(\omega t)$, and the distance from R to the centre of the disk is a. Assuming small oscillations, the total horizontal displacement of the disk is denoted as x. The relative motion $z(t)$ between the gate and engine is given as:

$$z = x - y = a\theta$$

Note: the disk is fixed to the rod, so it does not spin about the point of connection with the rod, but the rod itself is pinned at R. For a uniform disk, $J_G = \frac{1}{2}mr^2$.



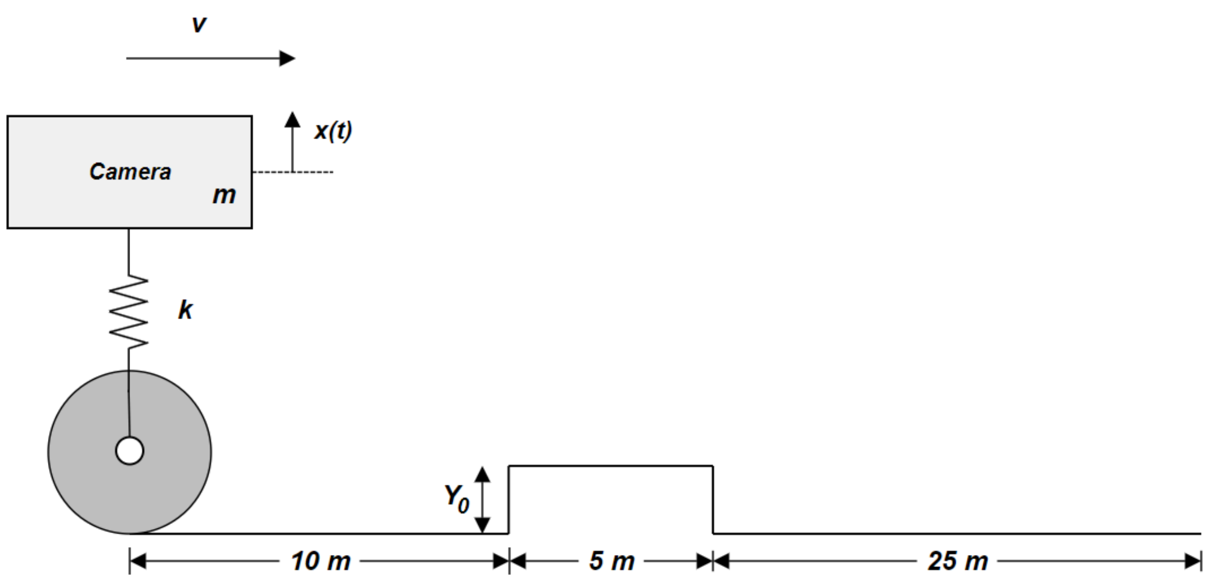
a) **(5 pts)** Determine the equation of motion for the relative motion of the gate with respect to the engine z, in terms of $m, k, a, g,$ and r. Assume small oscillations.

a) **(5 pts)** If $\omega = 2400$ rpm, $a = 70$ mm, $r = 35$ mm, $m = 2.25$ kg, $k = 8.75$ kN/m, and $c = 30$ Ns/m, determine the amplitude of the **relative motion** Z between the gate valve and engine in terms of A.

Question 2 (10 points)

A film crew is shooting a movie and performs a tracking shot, which involves a moving camera that follows a subject. In this case the camera is situated on a wheeled cart, and the crew encounters a small bump on the road which causes the camera to oscillate vertically.

The total distance to be travelled by the camera is 40 m, with the length of each section shown below, and the combined mass of the camera and the cart is 210 kg. Assume the cart moves at a constant speed v for the entire duration, and the height of the bump Y_0 is 5 cm.

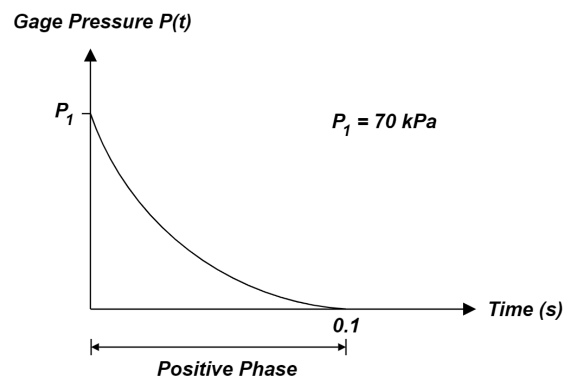


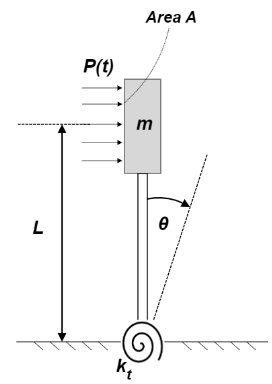
a) **(5 pts)** Fortunately, the footage recorded while the camera moves on the bump will not be used in the final production, but the crew does not want any oscillations to occur after the camera cart has cleared the bump. If the entire tracking shot is to be filmed over 10 seconds, determine the required wheel suspension stiffness k.

b) **(5 pts)** The production crew tries refilming the shot with a camera speed of $v/2$. In reference to the indicated coordinate x, determine the displacement (x) and velocity $(\dot{x})$ of the camera at the end of the shot (ie. at 40 m). Assume that the stiffness k corresponds to the answer in part a).

Question 3 (10 points)

Explosions send blast waves from the centre, which consist of an initial positive pressure wave front followed by a phase of negative pressure which pulls objects back towards the detonation centre. For this question, consider the positive pressure phase of a blast wave as shown below. Typically, the blast wave last 100 ms, and reaches a max pressure (gage) of around 70 kPa.





A nearby road sign (shown above on the right), is approximated by a **massless** rod supporting a uniform thin disk (viewed from the side) of mass m, radius r, and frontal area A. The centre of the disk is at a height $L = 2r$ from the ground, and the ground connection is modelled as a torsional spring k_t. Assume that the pressure-time relation during the positive phase is approximated as:

$$P(t) = \frac{P_1}{4}\left(e^{-at} + 3e^{-bt} - 6t\right), \quad 0 \le t \le 0.1\ s$$

The moment of inertia of a thin disk about its central diameter is $J = \frac{1}{4}mr^2$ (see next page for clarification).

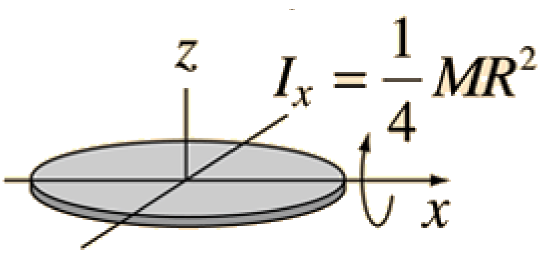
a. **(2 pts)** For the relation described above, determine the equation of motion for the sign during the positive phase in terms of $m,\ L,\ P(t),\ k_t, A$, and g. Use θ as your coordinate.

b. **(8 pts)** If the sign is initially at rest and $a = 10$, $b = 25$, $m = 1.5$ kg, $r = 0.25$ m, and $k_t = 1\times10^3$ Nm, calculate the response $\theta(t)$ of the sign during the positive phase of the blast wave. **You can neglect gravity in this case (k_t dominates the effective stiffness).**

Hint: To simplify calculations and algebra, work with dummy variables for the effective mass, stiffness, and coefficients of the particular solution rather than m, L, P_1, etc.

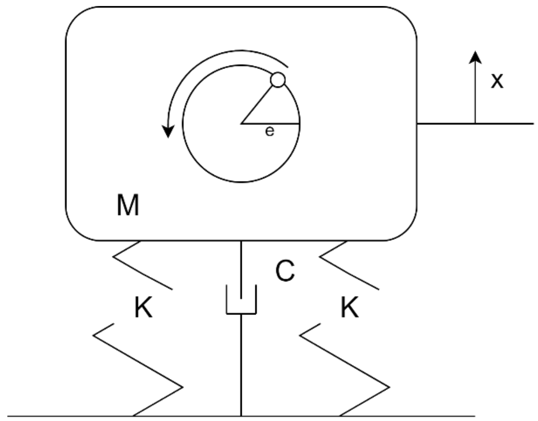
Moment of inertia about any diameter:

$$J = \frac{1}{4}mr^2$$



Question 4 (10 pts)

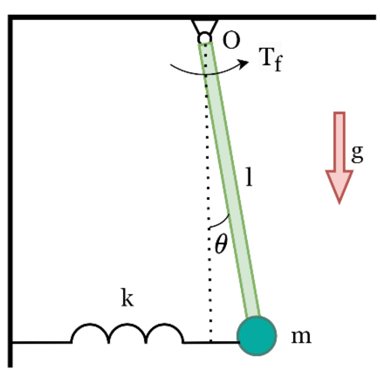
A machine of 100 kg mass is supported on springs of total stiffness 700 KN/m and has an unbalanced rotating element, which results in disturbing force of 350 N, at the speed of 3000 rpm. Assuming a damping ration of $\zeta = 0.2$:



a) **(5 pts.)** The amplitude of motion of the machine due to the unbalance.
b) **(5 pts.)** The transmissibility and the transmitted force to the base.

Question 5 (10 pts)

There is a pendulum with mass of M connected to a wall with a spring of stiffness k. Also, there is a constant Coulomb friction torque of T_f acting on the pendulum. (Note: Assume that the oscillations are small).



a) **(5 pts.)** Find the time response of the system and maximum angular acceleration of the pendulum. (Initial condition: $\theta = \frac{\pi}{12}$ *and* $\dot{\theta} = 0$).
b) **(5 pts.)** Compare the response of the system with the case where is no friction.