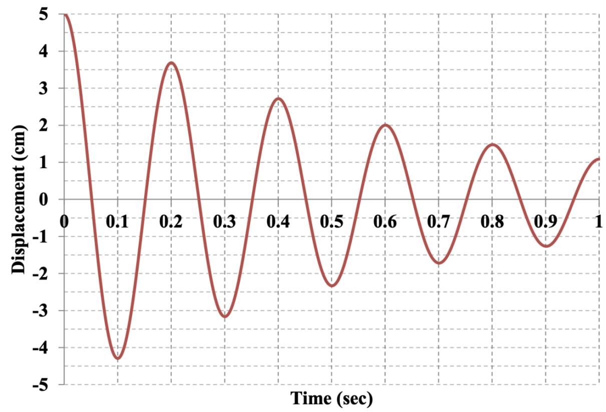
Question 1

The free vibration of a viscously damped SDOF system due to a non-zero initial displacement (zero initial velocity) is given in the graph shown below. Determine the following questions using this graph. Clearly indicate the values you obtain from the graph.



(a) (5 pts) Write a differential equation that governs the equation of motion of this system.

(b) (5 pts) If the same system was subjected only to a non-zero initial velocity of 100cm/sec (zero initial displacement), what would be the displacement response at t = 0.25 sec.

Notice that the graph exhibits the behaviour of a damped cosine wave. Using $x_0 = 5$ and $x_3 = 2$,

$$\begin{aligned}\delta &= \frac{1}{n}\ln\left(\frac{x_0}{x_3}\right)\\ &= \frac{1}{3}\ln\left(\frac{5}{2}\right)\\ &= 0.30543\end{aligned}$$

From this, we can find the damping ratio ζ,

$$\begin{aligned}\zeta &= \frac{\delta}{\sqrt{4\pi^2+\delta^2}}\\ &= \frac{0.30543}{\sqrt{4\pi^2+0.30543^2}}\\ &= 0.048553\end{aligned}$$

The period is $\tau = 0.1$ sec, so the natural frequency is

$$\begin{aligned}p &= \frac{2\pi}{\sqrt{1-\zeta^2\tau}}\\ &= \frac{2\pi}{\sqrt{1-0.048553^2}\cdot 0.1}\\ &= 62.91 \text{ rad/sec}\end{aligned}$$

Recall the equation of motion for a damped SDOF system,

$$m\ddot{x} + c\dot{x} + kx = 0$$

dividing by m and letting $p^2 = \frac{k}{m}$ and $2\zeta p = \frac{c}{m}$, we get

$$\ddot{x} + 2\zeta p\dot{x} + p^2 x = 0$$

substituting in the values we found, we get

$$\ddot{x} + 2(0.048553)(62.91)\dot{x} + (62.91)^2 x = 0$$
$$\boxed{\ddot{x} + 6.11\dot{x} + 3957.7x = 0}$$