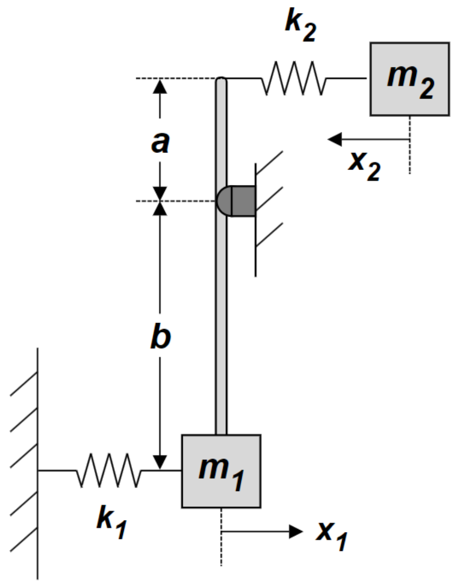
Instructions:

- Submit your assignment as a single PDF file through eClass.
- Show all your steps and solution procedures including clear and well labelled FBD/MAD diagrams when needed.
- Make sure that your solution is well organised and that you are using appropriate headers for each question and sub-question.
- Scanned photos of your handwritten solution are acceptable as long as they are **legible**

Question 1 (10 points)



For the two degree of freedom system shown, determine:

a) **(3 pts)** The flexibility influence coefficients, starting from the definition.

b) **(3 pts)** The stiffness influence coefficients, starting from the definition.

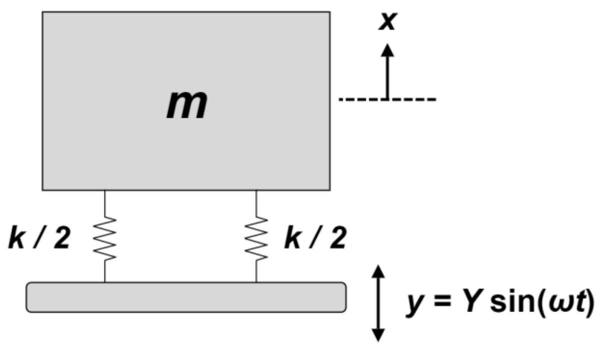
If $k_1 = k,\ \ k_2 = 2k,\ \ m_2 = m_1 = m,$ and $b = 2a,$ the equations of motion are:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 3k/2 & -k \\ -k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \{0\}$$

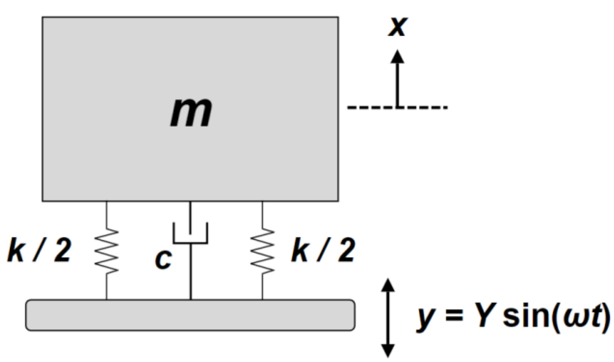
c) **(4 pts)** Determine the natural frequencies and mode shapes.

Question 2 (10 points)

The installation of a magnetic resonance imaging (MRI) system requires the machine to be well isolated from vibration. The original isolation is shown where the support motion $y = Ysin(\omega t)$ is due to the floor vibrating from the equipment in the room below. This equipment is assumed to run at 2400 rpm, and the natural frequency of the installation is estimated to be 15 Hz.

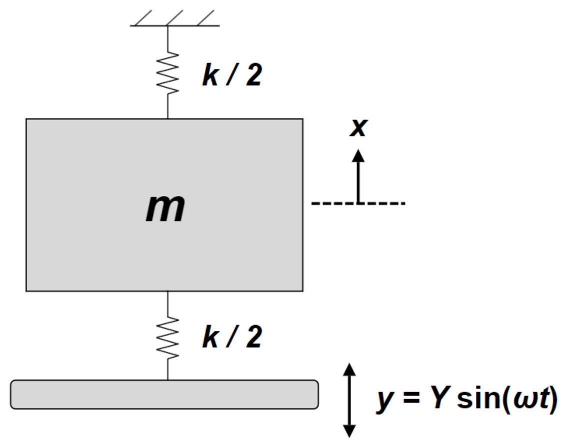


a) **(1 pt)** Estimate the amplitude of vibration of m compared to the amplitude of the floor motion.

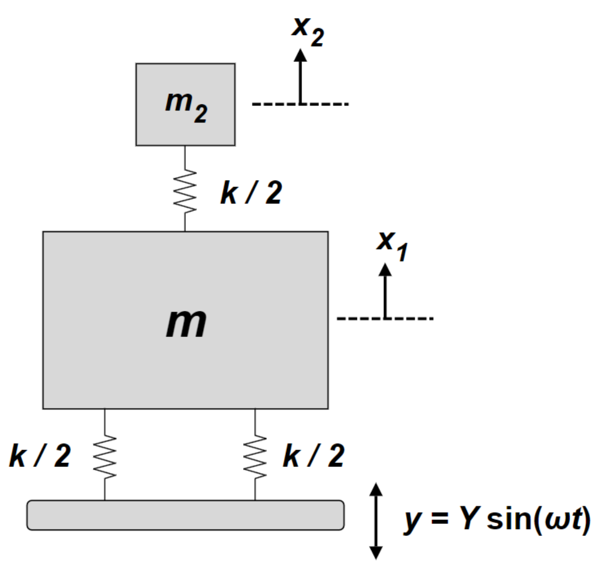


b) **(2 pts)** As the calculation in (a) indicated that the motion of m was too large, the supplier suggests the installation of a damper to give a damping ratio of $\zeta = 0.2$. Will the damper reduce of increase the amplitude of motion? Calculate the difference the damper will make.

Continued on next page…



c) **(3 pts)** Another alternative is suggested by a trainee engineer who argued that the machine should be supported from above as well as from the vibration floor as illustrated (it is assumed that the upper support is fixed). Starting from a free body diagram, determine the equation of motion and an expression for the steady state amplitude of m. Compare the amplitude of motion of m for this case to the original design in (a) and the alternative design with the damper in (b).



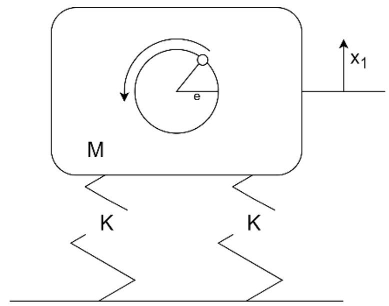
d) Another way to reduce the motion of m is to use a vibration absorber on the original design.

i. **(2 pts)** If the absorber has a mass m_2 equal to 15% of the main mass m, calculate the amplitude of motion of the absorber mass at the operating speed (2400 rpm).

ii. **(2 pts)** What are the two natural frequencies of the combined system?

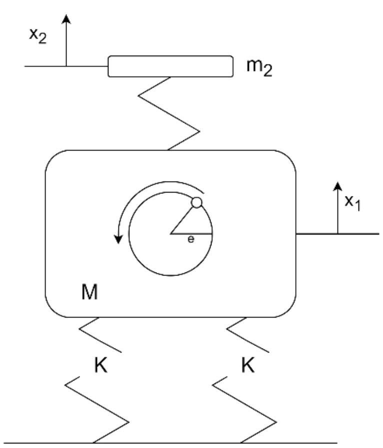
Question 3 (10 points)

The system shown is assumed to have a rotating imbalance and is operated at 100 rpm and 200 rpm. At 100 rpm the steady state amplitude of vibration is 1 mm while at 200 rpm it is 5 mm.



a) **(5 pts)** Since the system cannot be operated at other speeds it is not known where the maximum amplitude occurs. Assuming there is a negligible damping in the system, determine the <u>two</u> possible frequencies. Be sure to illustrate these possibilities on an appropriate frequency response curve.

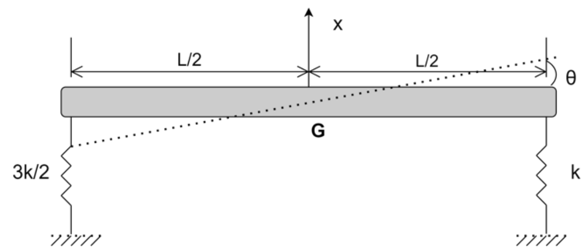
b) **(5 pts)** It is decided to operate the system shown at 200 rpm, however, the amplitude of vibration is too large, and a vibration absorber is to be added.



<u>Assuming</u> that the natural frequency of the original system is at <u>150 rpm</u>, what should the mass m_2 be as a fraction of m_1 to ensure that the lowest natural frequency of the combined system is 100 rpm?

Question 4 (10 points)

The system shown is a two-dimensional approximation to an automobile.



a) **(2.5 pts)** Using the coordinates shown determine the equations of motion.

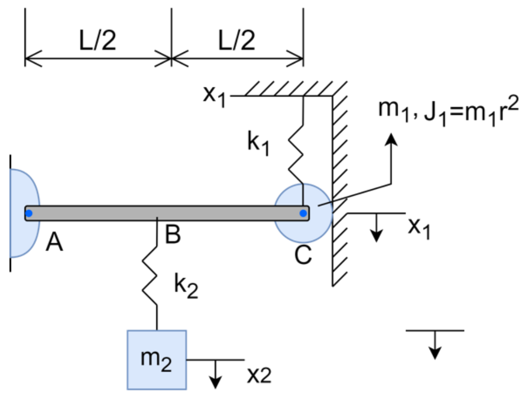
b) **(5 pts)** If $J_G = \frac{mL^2}{6}$, the equations of motions become as what follows:

$$\begin{bmatrix} m & 0 \\ 0 & \dfrac{mL^2}{6} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 5k/2 & -kL/4 \\ -kL/4 & 5kL^2/8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \{0\}$$

c) **(2.5 pts)** Sketch the mode shapes and clearly show the position on any nodes.

Question 5 (10 points)

Two degree of freedom system shown consists of a rigid bar AC of negligible mass, disc of mass m_1 which is pinned at C and rolls without slipping on the wall, and a mass m_2 suspended by a spring of stiffness k_2.



a) **(5 pts)** For the general case of m_1, m_2, k_1, k_2 determine the equations of motion. Be sure to include a detailed free-body diagram.

b) **(5 pts)** If $k_1 = k_2 = k$ and $m_1 = m_2 = m$, determine the natural frequencies and corresponding mode shapes.