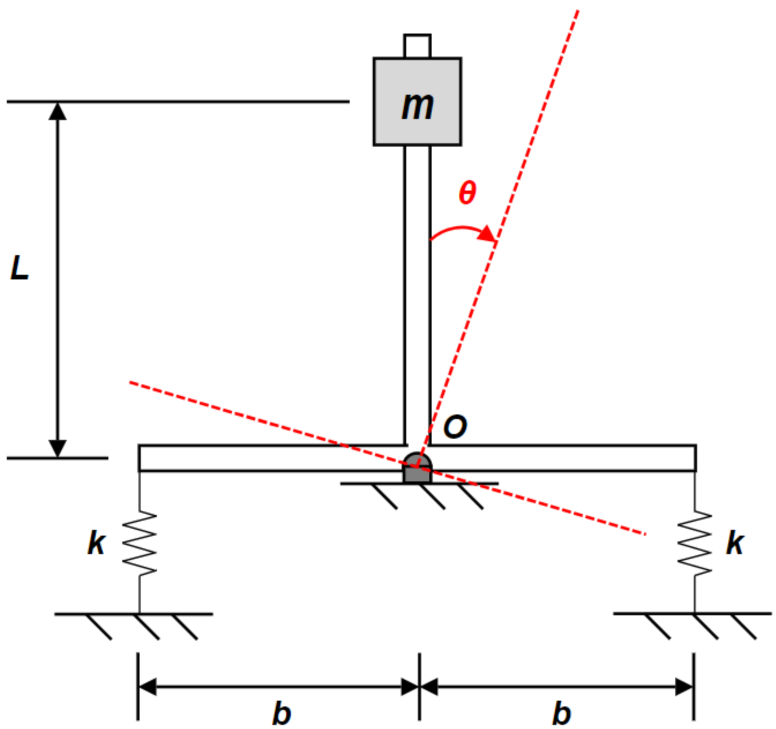
MEC E 451 Winter 2024 **Assignment #1** **Due: Jan 26, 11:59 PM**

Instructions:

- Submit your assignment as a single PDF file through eClass.
- Show all your steps and solution procedures including clear and well labelled FBD/MAD diagrams when needed.
- Make sure that your solution is well organised and that you are using appropriate headers for each question and sub-question.
- Scanned photos of your handwritten solution are acceptable as long as they are **legible**

Question 1 (10 points)

The metronome shown is used as a timing device for musicians. The essence of the device is an inverted 'T' mounted on springs mounted on springs. The natural frequency of the device is varied by moving the mass m to different heights L.



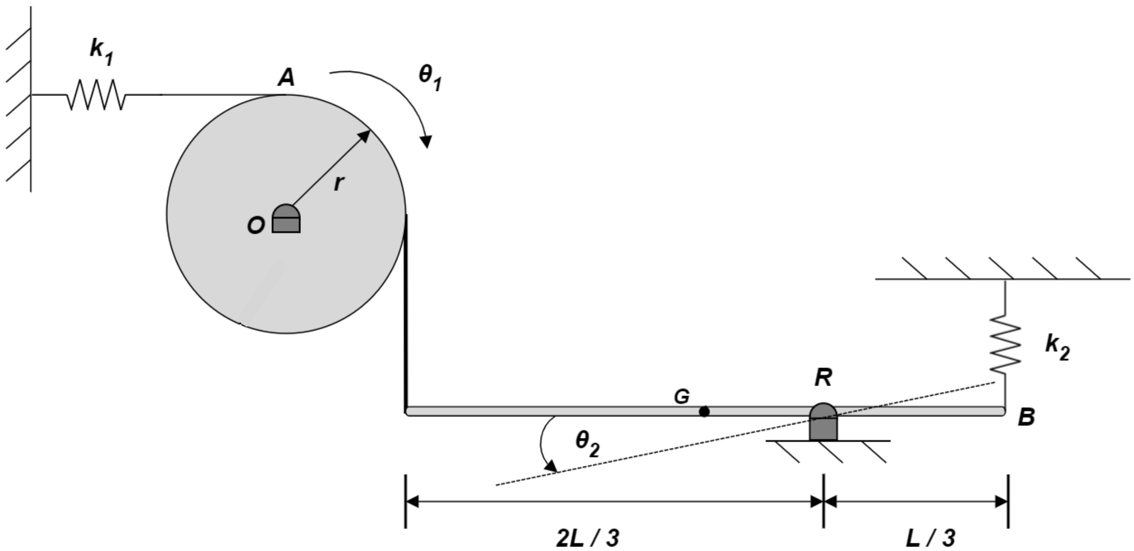
Assuming that the system undergoes only small oscillations about the axis through point O (the static equilibrium configuration is at $\theta = 0$):

a) **(5 pts)** Determine the equation of motion of the system **using θ as your coordinate**. Neglect the mass of the 'T' bar. Be sure to include the force of gravity as there is no static deflection of the springs in the static equilibrium configuration.

b) **(5 pts)** What is the limiting value of L so that the system is **stable**?

Question 2 (10 points)

The system shown below consists of a pulley (radius r) pinned about point O, which is connected to a uniform rod (length L) that is pinned about point R. The system is supported by two springs k_1 and k_2, connected to points A and B, respectively. The cable connecting the pulley and rod can be considered approximately inextensible (ie. the pulley and rod are rigidly connected). Assume small oscillations.



a) **(5 pts)** Determine the effective stiffness of the system with respect to θ_2 **using the stiffness approach,** by setting $\theta_2 = 1$.

b) **(5 pts)** Assume that a downward force of 5 N is applied at the rod's centre of mass (point G) while the system is initially as shown above. Using your answer from part a), determine the resulting compression of spring k_2 if $r = 0.5$ m, $L = 3$ m, and $k_1 = k_2 = 100$ N/m.

Question 3 (10 points)

A machine component is modeled as a system comprising a pendulum and a spring, illustrated in Fig 1. This setup combines gravitational and spring forces, with the pendulum contributing rotational dynamics influenced by gravity, and the spring providing a proportional restoring force. The geometric arrangement and interconnection of these components are depicted in Fig 1. (**Assumption:** the rotation is small enough so that the spring only deflects horizontally.)

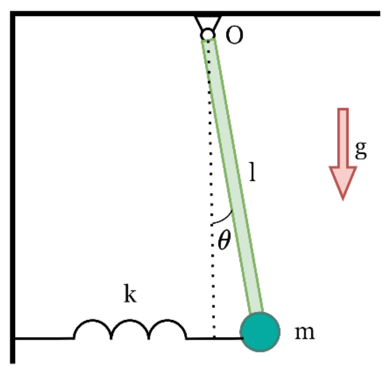


Fig 1. The pendulum system connected to a spring

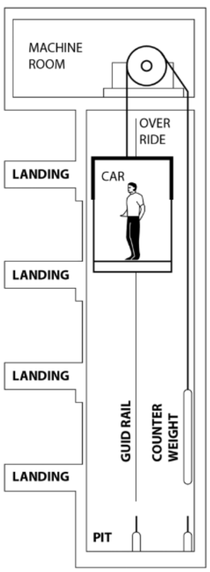
a) (5 pts) Derive the equation of motion for this system using the energy method.

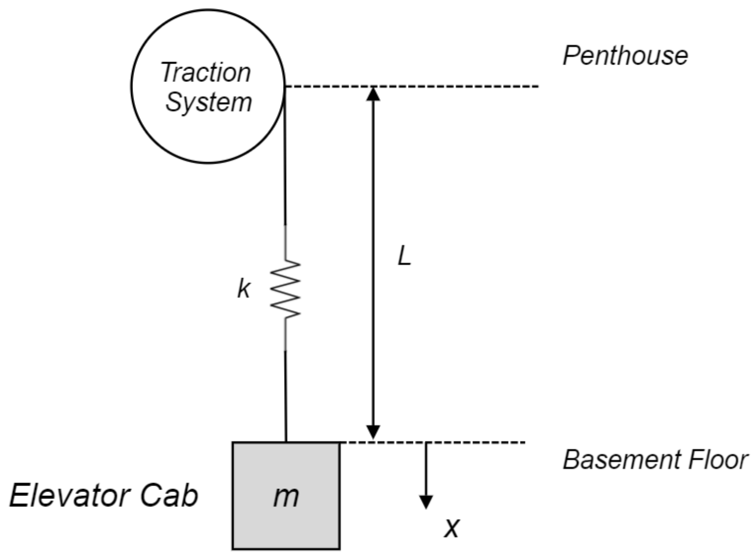
b) (3 pts) Linearize the equation of motion and derive a formula for the natural frequency of the system.

c) (2 pts) Compare the natural frequency of this system with the case where there is no spring.

Question 4 (10 points)

The figure shows a traction elevator system used in high-rise residential buildings. These traction elevators consist of hoist cables connected to the top of the cab operated by a traction machine (electric motor) located in the penthouse. The system is modelled as a simple spring-mass system, where the spring represents the cable stiffness and the mass corresponds to the elevator cab and its occupants (counterweights are neglected).





The elevator provides a rapid ascent/descent, while not causing excessive acceleration to the passengers or stress in the cable system. The situation under consideration is the stop after descent to the basement floor level for a 10 floor apartment building (assume 3.5 m per story). Assume that the traction motor stops instantly when reaching the basement floor (acts as a fixed support). The velocity of the cab before stopping is 1.5 m/s. The cables have an equivalent stiffness of a single cable with a radius of 2 cm and an elastic modulus of 100 GPa $\left(k = \frac{EA}{L}\right)$.

a) Consider both cases with an unloaded cab (mass of 1.2 metric tonnes) and with a maximum capacity of 15 people with an average weight of 70 kg each. For each case, determine:
   i. **(1 pt)** The overshoot of the cab past the basement floor level after stopping.
   ii. **(1 pt)** The maximum acceleration felt by the occupants.
   iii. **(1 pt)** The maximum stress in the cables.

b) **(3 pts)** To reduce the maximum tension in the cables and acceleration of the cab, a coil spring ($k = 600$ kN/m) is inserted between the cable attachment and the cab. How does this change the maximum displacement, acceleration, and stress for both the loaded and unloaded cases?

The results for the vibration analysis of the original elevator system **(no coil spring)** was done under the assumption of no damping. However, the system components have an inherent damping. a test was done on an **UNLOADED cab** and it was found that the cab's oscillation amplitude decreased by 50% in two cycles.

c) **(2 pts)** Determine the damping ratio for the **LOADED case** assuming viscous damping.

d) **(2 pts)** For the **LOADED** case, estimate how much time is needed after reaching the ground floor so that the passengers feel virtually no vibration of the elevator cab. Assume that the vibrations essentially stop when the amplitude decreases to 8% of its maximum value. **Hint:** recall that the logarithmic decrement is measured between subsequent peaks, so you must account for the time from $t = 0$ to the first peak.

Question 5 (10 points)

The system shown, represents a floor of mass M supported by springs of stiffness k. A mass m is dropped on the floor from a height of h.

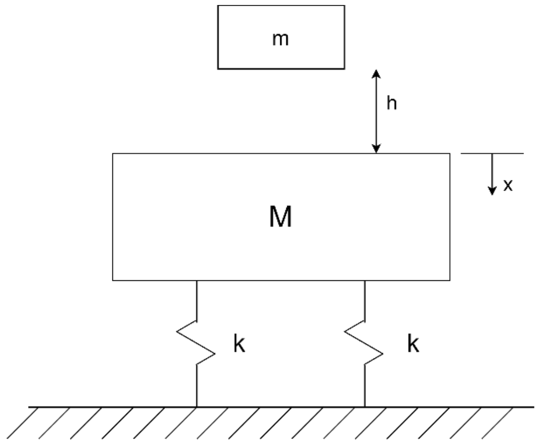


Fig 1. The floor and mass system

a) **(5 pts)** Determine the motion of the floor using x, the displacement from the static equilibrium configuration before impact.

b) **(5 pts)** Determine the maximum displacement for the case when $h = 0$. **NOTE**: Do not neglect the weight of the floor and the weight of mass m.