Question 1

In this lab, the apparatus is a simple platform suspended by springs as shown above. When modelling a stiffness/elastic element as a spring, it is typically assumed that the spring provides a stiffness in only the axial direction (ie. in the z – direction).

(a) How much would the natural frequency of vibrations in the vertical direction increase if the stiffness of each spring is doubled?

(b) The springs are manufactured such that they are also able to resist lateral forces (ie. in the x and y directions). If the springs have both an axial and lateral stiffness, determine how many degrees of freedom the system has and state each degree of freedom.

(a)

First let the springs be k. Since they are in parallel,

$$k_{\text{eff}} = 4(k) = 4k$$
$$p_1 = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

Now, let all the springs be $2k$. Then,

$$k_{\text{eff}} = 4(2k) = 8k$$
$$p_2 = \sqrt{\frac{8k}{m}} = 2\sqrt{2}\sqrt{\frac{k}{m}}$$

So the natural frequency would increase by a factor of

$$\boxed{\sqrt{2}}$$

(b)

If the springs can move in the x, y, and z directions, then the system has 6 degrees of freedom:

1. Displacement in the x direction
2. Displacement in the y direction
3. Displacement in the z direction
4. Rotation about the x axis
5. Rotation about the y axis
6. Rotation about the z axis

Question 2 (4pts)

Plot the vertical acceleration versus time. Using your plot: Calculate the damping ratio using the logarithmic decrement. Use a set of peaks away from the beginning of the measured response due to the initial lateral motion of the platform when it is released.

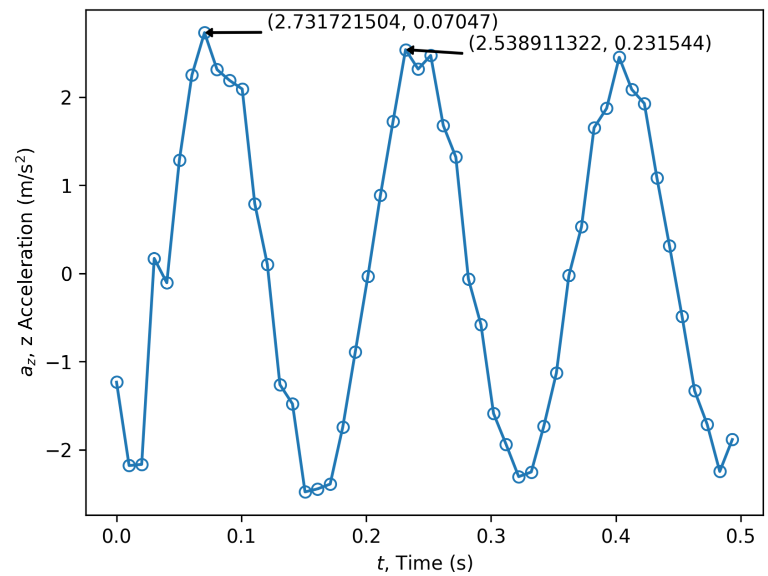

Figure 1: Vertical (z) acceleration of the platform after initial excitation.

Using the logarithmic decrement method,

$$\begin{aligned}\delta &= \frac{1}{n}\ln\left(\frac{z_0}{z_n}\right)\\ &= \frac{1}{1}\ln\left(\frac{2.731721504}{2.538911322}\right)\\ &= 0.07320\end{aligned}$$

then,

$$\begin{aligned}\zeta &= \frac{\delta}{\sqrt{4\pi^2+\delta^2}}\\ &= \frac{0.07320}{\sqrt{4\pi^2+0.07320^2}}\\ &= \boxed{0.01165}\end{aligned}$$

Question 3 (3pt)

If each spring has a stiffness of 2.8 kN/m, calculate the mass of the platform.

Experimentally, the period was determined to be

$$\tau = t_2 - t_1 = 0.231544 - 0.07047 = 0.161074$$

We can determine natural frequency from from Eq. 3.15,

$$\begin{aligned} \tau &= \frac{2\pi}{\sqrt{1-\zeta^2}p} \\ \implies p &= \frac{2\pi}{\tau\sqrt{1-\zeta^2}} \\ &= \frac{2\pi}{0.161074\sqrt{1-0.01165^2}} \\ &= 39.012 \text{ rad/s} \end{aligned}$$

Then, by definition of the natural frequency,

$$\begin{aligned} p &= \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} \\ \implies m_{\text{eff}} &= \frac{k_{\text{eff}}}{p^2} \\ \implies m_{\text{eff}} &= \frac{4k}{p^2} \\ &= \frac{4 \times 2.8 \times 10^3}{39.012^2} \\ &= \boxed{7.359 \text{ kg}} \end{aligned}$$

Since the typical mass of a smartphone is 0.2 kg, the mass of the platform is

$$\begin{aligned} m_{\text{platform}} &= m_{\text{eff}} - m_{\text{smartphone}} \\ &= 7.359 - 0.2 \\ &= \boxed{7.159 \text{ kg}} \end{aligned}$$

Question 4 (2 pts)

Determine the effective damping of the system.

From equation 3.6,

$$\begin{aligned} \zeta &= \frac{c}{C_c} = \frac{c}{2mp} \\ \implies c &= 2mp\zeta \\ &= 2(7.3590)(39.012)(0.01165) \\ &= \boxed{6.689 \text{ Ns/m}} \end{aligned}$$

Question 5 (2pts)

If the metal springs were replaced with rubber springs of the same stiffness, would you expect the oscillation period to increase or decrease? Why?

The equation of motion for a damped spring-mass system is given by

$$m\ddot{x} + c\dot{x} + kx = 0$$

If k does not change, then the period does not change. However, if the rubber springs have a weaker damping effect, then damping ratio ζ will decrease, and hence, period would increase.