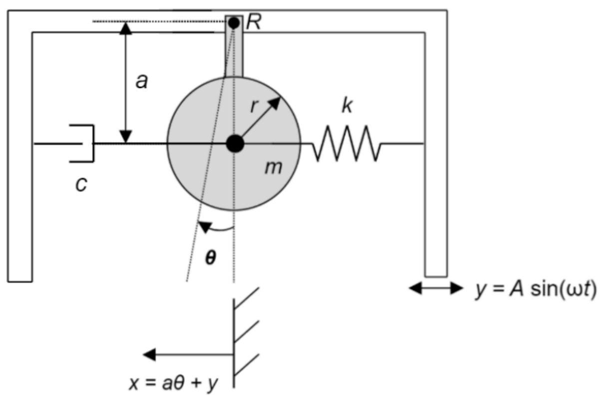
Question 1

The diagram models a gate valve which regulates the fluid flow in an engine. The gate valve is modelled as a pendulum (with a massless rod and uniform disk of mass m and radius r) pinned to the engine at R, and is controlled by a spring/damper mechanism connecting the centre of the disk to the engine.

The engine's motion during operation is denoted as $y = A\sin(\omega t)$, and the distance from R to the centre of the disk is a. Assuming small oscillations, the total horizontal displacement of the disk is denoted as x. The relative motion $z(t)$ between the gate and engine is given as:

$$z = x - y = a\theta$$

Note: the disk is fixed to the rod, so it does not spin about the point of connection with the rod, but the rod itself is pinned at R. For a uniform disk, $J = \frac{1}{2}mr^2$.



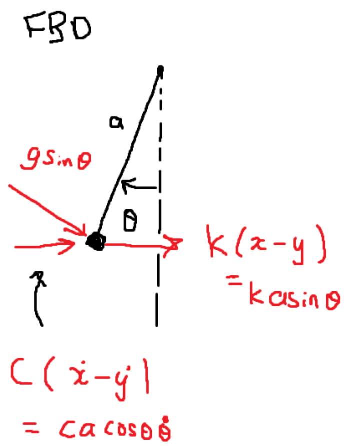
(a) *Determine the equation of motion for the relative motion of the gate with respect to the engine z, in terms of m, k, a, g, and r. Assume small oscillations.*

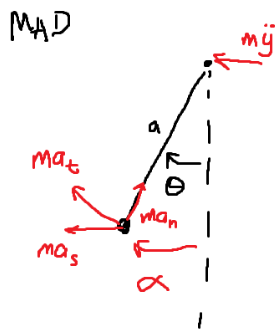
(b) *If $\omega = 2400$ rpm, $a = 70$ mm, $r = 35$ mm, $m = 2.25$ kg, $k = 8.75$ kN/m, and $c = 30$ Ns/m, determine the amplitude of the relative motion Z between the gate valve and engine in terms of A.*

Solution

(a)

Here is the FBD and MAD for the problem:





The equation of motion for the relative motion of the gate can be found by first taking the moment about the pin, R. Since the disk is undergoing rotation and translation, we consider general plane motion. First simplify the kinetic moment about the pin using the MAD diagram,

$$\begin{aligned} \circlearrowright \sum M_R &= (\mathcal{M}_k)_R \\ &= J_G\ddot{\theta} + (a)m(a_G)_t + (a\cos\theta)m(a_G)_s \\ &= J_G\ddot{\theta} + ma(a\ddot{\theta}) + m(a\cos\theta)(\ddot{y}) \\ &= \underbrace{\left(J_G + ma^2\right)}_{J_R}\ddot{\theta} + ma\ddot{y}\cos\theta \end{aligned} \tag{1}$$

Next, summing the moments in the FBD,

$$\circlearrowright \sum M_R = -ka^2\sin\theta - ca^2\cos\theta\dot{\theta} - mga\sin\theta \tag{2}$$

Equating (1) and (2),

$$J_R\ddot{\theta} + ma\ddot{y}\cos\theta = -ka^2\sin\theta - ca^2\cos\theta\dot{\theta} - mga\sin\theta$$

Assuming small oscillations, we can use the small angle approximation, $\sin\theta \approx \theta$ and $\cos\theta \approx 1$ to simplify the equation.

$$J_R\ddot{\theta} + ma\ddot{y} = -ka^2\theta - ca^2\dot{\theta} - mga\theta$$

Rearranging,

$$[J_R]\,\ddot{\theta} + \left[ca^2\right]\dot{\theta} + \left[ka^2 + mga\right]\theta = -ma\ddot{y}$$

Since $z = a\theta$, multiplying both sides by a,

$$\begin{aligned} [J_R]\,a\ddot{\theta} + \left[ca^2\right]a\dot{\theta} + \left[ka^2 + mga\right]a\theta &= -ma^2\ddot{y} \\ [J_R]\,az + \left[ca^2\right]\dot{z} + \left[ka^2 + mga\right]z &= -ma^2\ddot{y} \end{aligned}$$

Finally,

$$\boxed{\underbrace{\left[\frac{1}{2}mr^2 + ma^2\right]}_{m_{\text{eff}}}\ddot{z} + \underbrace{\left[ca^2\right]}_{c_{\text{eff}}}\dot{z} + \underbrace{\left[ka^2 + mga\right]}_{k_{\text{eff}}}z = \underbrace{ma^2A\omega^2}_{F_o}\sin(\omega t)}$$

(b)

This is the standard form of a forced, damped harmonic oscillator. From Eq. (5.9) in the textbook, the amplitude is

$$\mathbb{Z} = \frac{F_o}{\sqrt{\left(k_{\text{eff}} - m_{\text{eff}}\omega^2\right)^2 + \left(c_{\text{eff}}\omega\right)^2}}$$

Determining the effective mass, damping, and stiffness,

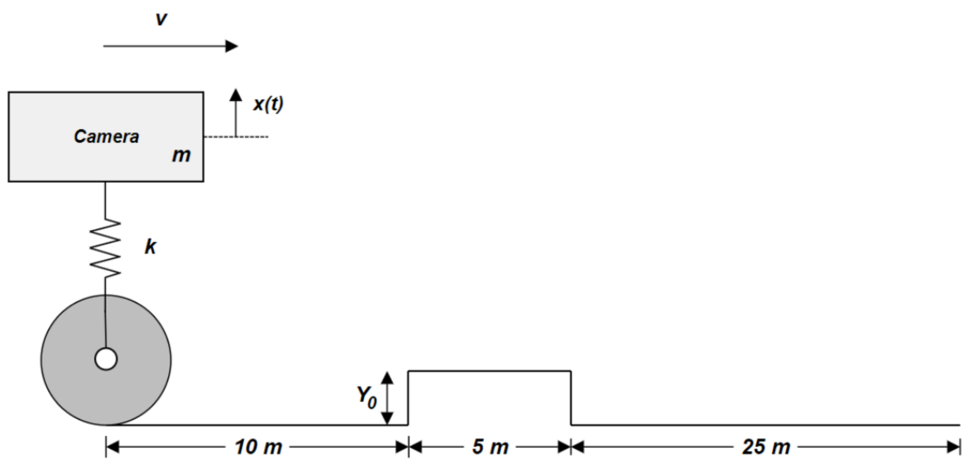
$$\begin{aligned}
m_{\text{eff}} &= \frac{1}{2}mr^2 + ma^2 = \frac{1}{2}(2.25)(0.035)^2 + (2.25)(0.070)^2 = 0.0124 \\
c_{\text{eff}} &= ca^2 = (30)(0.070)^2 = 0.147 \\
k_{\text{eff}} &= ka^2 + mga = (8.75 \times 10^3)(0.070)^2 + (2.25)(9.81)(0.070) = 44.420 \\
F_o &= ma^2A\omega^2 = (2.25)(0.070)^2(2400 \times 2\pi/60)^2A = 696.3993A
\end{aligned}$$

Substituting into the amplitude equation,

$$\begin{aligned}
\mathbb{Z} &= \frac{696.3993}{\sqrt{\left(44.420 - 0.0124(2400 \times 2\pi/60)^2\right)^2 + \left(0.147(2400 \times 2\pi/60)\right)^2}}A \\
&= \boxed{0.9414A}
\end{aligned}$$

Question 2

A film crew is shooting a movie and performs a tracking shot, which involves a moving camera that follows a subject. In this case the camera is situated on a wheeled cart, and the crew encounters a small bump on the road which causes the camera to oscillate vertically. The total distance to be travelled by the camera is 40 m, with the length of each section shown below, and the combined mass of the camera and the cart is 210 kg. Assume the cart moves at a constant speed v for the entire duration, and the height of the bump Y is 5 cm.



(a) *Fortunately the footage recorded while the camera moves on the bump will not be used in the final production, but the crew does not want any oscillations to occur after the camera cart has cleared the bump. If the entire tracking shot is to be filmed over 10 seconds, determine the required wheel suspension stiffness k.*

(b) *The production crew tries refilming the shot with a camera speed of $v/2$. In reference to the indicated coordinate x, determine the displacement (x) and velocity $\dot{x}$ of the camera at the end of the shot (i.e. at 40 m). Assume that the stiffness k corresponds to the answer in part (a).*

Solution

(a)

Assume that the cart's vertical displacement follows the ground exactly. Then at 10m, the forces on the camera are:

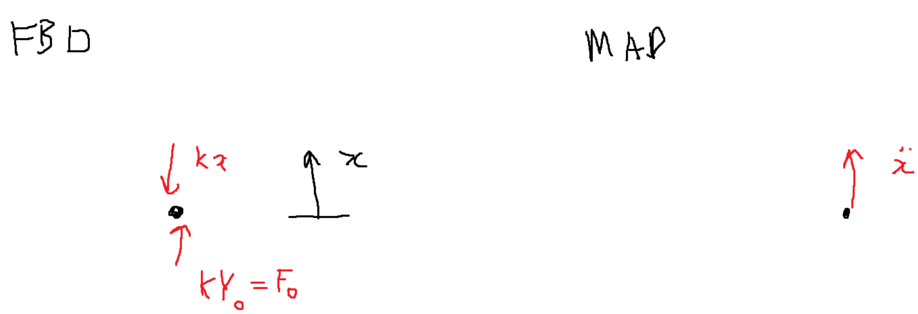


Figure 1: Free body diagram and mass acceleration diagram.

Define t as the time when the cart is at 10m. The initial conditions the time when the cart is at 10m, are:

$$x(0) = 0, \quad \dot{x}(0) = 0$$

From Eq. (7.3), the solution is given by:

$$\begin{aligned} x(t) &= \frac{F_0}{k}\left(1 - \cos(pt)\right) \\ &= Y_0\left(1 - \cos(pt)\right) \end{aligned}$$

From the textbook, we want to unload the cart at $t_2 = \frac{2\pi}{p}$ since this will be the time when the camera has zero displacement and zero velocity. We can solve for k using the following equation:

$$\begin{aligned} t_2 &= \frac{2\pi}{p} \\ t_2^2 &= \frac{4\pi^2}{k/m} \\ \implies k &= \frac{4\pi^2 m}{t_2^2} \end{aligned}$$

The time it takes for the cart to travel to the end of the bump is

$$t_2 = \frac{5}{v}$$

Then k is

$$\begin{aligned} k &= \frac{4\pi^2 m}{\left(\frac{5}{v}\right)^2} \\ &= \boxed{\frac{4\pi^2 m v^2}{25}} \end{aligned}$$

(b)

In part a), we found stiffness such that

$$\tau = \frac{2\pi}{p} = t_2 = \frac{5}{v}$$

Halving the speed of the cart will double the time it takes to travel the bump. The new time is then,

$$t_{2b} = \frac{5}{v/2} = \frac{10}{v} = \frac{4\pi}{p} = 2\tau$$

This means it will take 2 periods of oscillation for the cart to travel the bump. But at every integer multiple of τ, the displacement and velocity of the cart will be zero. Therefore, the displacement and velocity of the cart at the end of the shot will be zero.

$$\boxed{\begin{aligned} x(t_{40m}) &= 0 \\ \dot{x}(t_{40m}) &= 0 \end{aligned}}$$

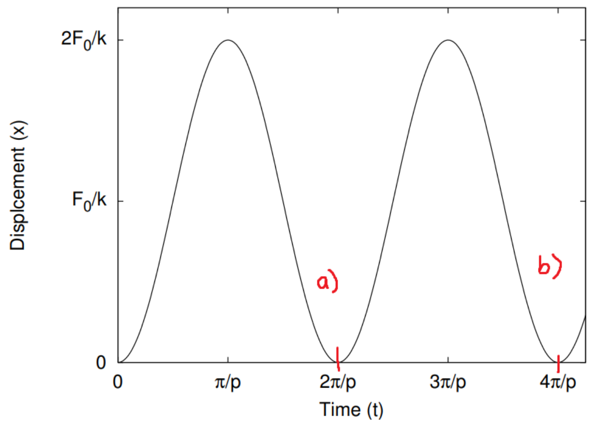
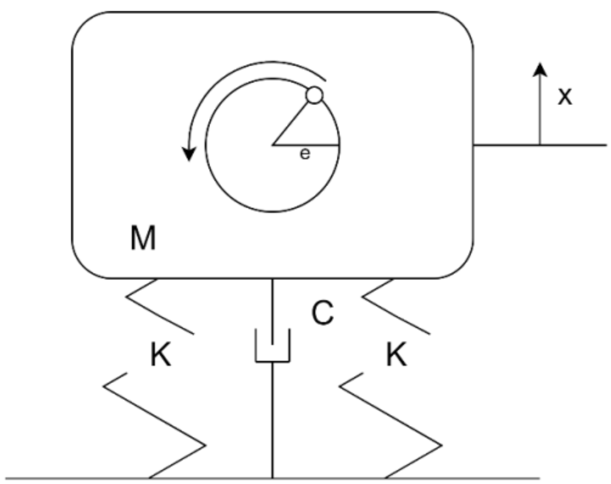


Figure 2: Displacement plot. Halving speed of the cart places it at $t = 4\pi/p$.

Question 3

Question 4

A machine of 100 kg mass is supported on springs of total stiffness 700 kN/m and has an unbalanced rotating element, which results in a disturbing force of 350 N, at the speed of 3000 rpm. Assuming a damping ratio of $\zeta = 0.2$*:*



(a) *The amplitude of motion of the machine due to the unbalance.*

(b) *The transmissibility and the transmitted force to the base.*

Solution

(a)

This is a forced damped system with rotating imbalance. The amplitude of vibration is given from Eq. (5.18),

$$\mathbb{X} = \frac{F_o/k}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{p}\right]^2}}$$

All parameters are known except p. The natural frequency is

$$p = \sqrt{\frac{k}{m}} = \sqrt{\frac{700\times10^3}{100}} \times \frac{60}{2\pi} = 798.95 \text{ RPM}$$

then,

$$\begin{aligned}\mathbb{X} &= \frac{350/(700\times10^3)}{\sqrt{\left[1-\left(\frac{3000}{798.95}\right)^2\right]^2 + \left[2\times0.2\times\frac{3000}{798.95}\right]^2}}\\ &= \boxed{3.792\times10^{-5}\text{ m}}\end{aligned}$$

(b)

Transmissibility is given in Eq. (5.19),

$$TR = \frac{F_{T_{\text{Max}}}}{F_o} = \frac{\sqrt{1+(2\zeta\frac{\omega}{p})^2}}{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{p}\right]^2}}$$

Substituting the known values,

$$\begin{aligned}TR &= \frac{\sqrt{1+(2\times0.2\times\frac{3000}{798.95})^2}}{\sqrt{\left[1-\left(\frac{3000}{798.95}\right)^2\right]^2 + \left[2\times0.2\times\frac{3000}{798.95}\right]^2}}\\ &= \boxed{0.1369}\end{aligned}$$

Then, the transmitted force to the base is,

$$F_{T_{\text{Max}}} = TR\times F_o = 0.1369\times350 = \boxed{47.92\text{ N}}$$

Question 5