

1 Steady State Solutions for unforced SDOF Systems

Table 1: Steady State Solutions for unforced SDOF Systems

System	Steady State Solution
Undamped Spring Mass	$\frac{v_0}{p} \sin(pt) + x_0 \cos(pt)$
Damped Spring Mass	$e^{-\zeta pt} \left[\frac{v_0 + \zeta p x_0}{\sqrt{1 - \zeta^2 p}} \sin(\sqrt{1 - \zeta^2} pt) + x_0 \cos(\sqrt{1 - \zeta^2} pt) \right]$

2 Steady State Solutions for forced SDOF Systems

Table 2: Steady State Solutions for forced SDOF Systems

System	Steady State	DMF (or Amplitude Response)	Transmissibility
Undamped Forced Spring Mass			
Forced Spring Mass	$\left(\frac{F_0}{k}\right) \left(\frac{1}{1 - \left(\frac{\omega}{p}\right)^2}\right) \sin(\omega t)$	$\frac{\mathbb{X}}{\delta_{ST}} = \frac{1}{\left 1 - \left(\frac{\omega}{p}\right)^2\right }$	$\frac{1}{\left 1 - \left(\frac{\omega}{p}\right)^2\right }$
Rotating Imbalance	$\frac{\tilde{m}e\omega^2/k}{\left 1 - \left(\frac{\omega}{p}\right)^2\right } \sin(\omega t - \phi)$	$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\left 1 - \left(\frac{\omega}{p}\right)^2\right }$	$\frac{1}{\left 1 - \left(\frac{\omega}{p}\right)^2\right }$
Base Excitation	$a \left(\frac{1}{1 - \left(\frac{\omega}{p}\right)^2}\right) \sin(\omega t)$	$\frac{\mathbb{X}}{\delta_{ST}} = \frac{1}{\left 1 - \left(\frac{\omega}{p}\right)^2\right }$	$\frac{\left(\frac{\omega}{p}\right)^2}{\left 1 - \left(\frac{\omega}{p}\right)^2\right }$
Damped Forced Spring Mass			
Forced Spring Mass	$\frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$	$\frac{\mathbb{X}}{\delta_{ST}} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{\sqrt{1 + \left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$
Rotating Imbalance	$\frac{\tilde{m}e\omega^2/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{\sqrt{1 + \left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$
Base Excitation	$\frac{a\sqrt{k^2 + (c\omega)^2}}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$	$\frac{\mathbb{X}}{a} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$ $\frac{\mathbb{Z}}{a} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$	$\frac{\left(\frac{\omega}{p}\right)^2 \sqrt{1 + \left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$

General solution for

$$m_{\text{eff}}\ddot{x} + c_{\text{eff}}\dot{x} + k_{\text{eff}}x = F_0 \sin(\omega t - \alpha)$$

is given by

$$x(t) = \mathbb{X} \sin(\omega t - \alpha - \phi)$$

where

$$\mathbb{X} = \frac{F_0}{\sqrt{(k_{\text{eff}} - m_{\text{eff}}\omega^2)^2 + (c_{\text{eff}}\omega)^2}}$$

3 Steady State Solutions for Forced Non-Harmonic SDOF Systems

The general steady state solution for a periodic ($\tau = 2\pi/\omega$) forced damped SDOF system is given by:

$$\begin{aligned} x(t) = & \frac{a_0}{2k} + \sum_{j=1}^{\infty} \frac{a_j/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}} \cos(j\omega t - \phi_j) \\ & + \sum_{j=1}^{\infty} \frac{b_j/k}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}} \sin(j\omega t - \phi_j) \end{aligned}$$

where

$$\begin{aligned} \phi_j &= \tan^{-1} \left[\frac{2\zeta\frac{\omega}{p}}{1 - \left(\frac{j\omega}{p}\right)^2} \right] \\ a_0 &= \frac{2}{\tau} \int_0^{\tau} F(t) dt = 2F_{\text{avg}} \\ a_j &= \frac{2}{\tau} \int_0^{\tau} F(t) \cos(j\omega t) dt, \quad j = 1, 2, 3, \dots \\ b_j &= \frac{2}{\tau} \int_0^{\tau} F(t) \sin(j\omega t) dt, \quad j = 1, 2, 3, \dots \end{aligned}$$

Todo: add common solutions such as step, ramp,

4 Transient Response of SDOF Systems