Question 1

In this lab, the apparatus is a simple platform suspended by springs as shown above. When modelling a stiffness/elastic element as a spring, it is typically assumed that the spring provides a stiffness in only the axial direction (ie. in the $z-$ direction).

(a) How much would the natural frequency of vibrations in the vertical direction increase if the stiffness of each spring is doubled?

(b) The springs are manufactured such that they are also able to resist lateral forces (ie. in the x and y directions). If the springs have both an axial and lateral stiffness, determine how many degrees of freedom the system has and state each degree of freedom.

First let the springs be k. Since they are in parallel,

$$k_{\text{eff}} = 4(k) = 4k$$

$$p_1 = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

Now, let all the springs be $2k$. Then,

$$k_{\text{eff}} = 4(2k) = 8k$$

$$p = \sqrt{\frac{8k}{m}} = 2\sqrt{2}\sqrt{\frac{k}{m}}$$

So the natural frequency would increase by a factor of

$$\boxed{\sqrt{2}}$$

(a)

If the springs can move in the x, y, and z directions, then the system has 3 degrees of freedom. The degrees of freedom are the displacements in the x, y, and z directions.

Question 2 (4pts)

Plot the vertical acceleration versus time. Using your plot: Calculate the damping ratio using the logarithmic decrement. Use a set of peaks away from the beginning of the measured response due to the initial lateral motion of the platform when it is released.

Using the logarithmic decrement method,

$$\begin{aligned}\delta &= \frac{1}{n}\ln\left(\frac{z_0}{z_n}\right) \\ &= \frac{1}{1}\ln\left(\frac{2.731721504}{2.538911322}\right) \\ &= 0.07320\end{aligned}$$

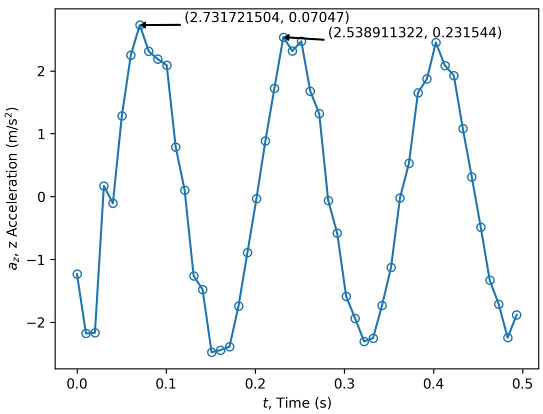

Figure 1: Vertical (z) acceleration of the platform after initial excitation.

then,

$$\begin{aligned}\zeta &= \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \\ &= \frac{0.07320}{\sqrt{4\pi^2 + 0.07320^2}} \\ &= 0.01165\end{aligned}$$

Question 3 (3pt)

If each spring has a stiffness of 2.8 kN/m, calculate the mass of the platform.

Experimentally, the period was determined to be

$$\tau = t_2 - t_1 = 0.231544 - 0.07047 = 0.161074$$

We can determine natural frequency from from Eq. 3.15,

$$\begin{aligned}\tau &= \frac{2\pi}{\sqrt{1-\zeta^2}p} \\ \implies p &= \frac{2\pi}{\tau\sqrt{1-\zeta^2}} \\ &= \frac{2\pi}{0.161074\sqrt{1-0.01165^2}} \\ &= 39.012 \text{ rad/s}\end{aligned}$$

Then, by definition of the natural frequency,

$$\begin{aligned} p &= \sqrt{\frac{k}{m}} \\ \implies m &= \frac{k}{p^2} \\ &= \frac{2.8 \times 10^3}{39.012^2} \\ &= 1.8398 \text{ kg} \end{aligned}$$

Since the typical mass of a smartphone is 0.2 kg, the mass of the platform is

$$\begin{aligned} m_{\text{platform}} &= m_{\text{total}} - m_{\text{smartphone}} \\ &= 1.8398 - 0.2 \\ &= \boxed{1.6398 \text{ kg}} \end{aligned}$$

Question 4

The figure shows a traction elevator system used in high-rise residential buildings. These traction elevators consist of hoist cables connected to the top of the cab operated by a traction machine (electric motor) located in the penthouse. The system is modelled as a simple spring-mass system, where the spring represents the cable stiffness and the mass corresponds to the elevator cab and its occupants (counterweights are neglected). The elevator provides a rapid

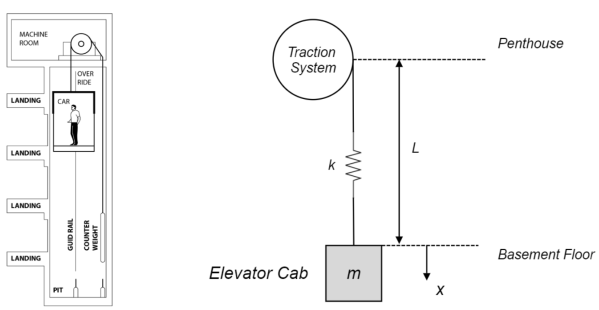

Figure 2: The traction elevator system

ascent/descent, while not causing excessive acceleration to the passengers or stress in the cable system. The situation under consideration is the stop after descent to the basement floor level for a 10 floor apartment building (assume 3.5 m per story). Assume that the traction motor stops instantly when reaching the basement floor (acts as a fixed support). The velocity of the cab before stopping is 1.5 m/s. The cables have an equivalent stiffness of a single cable with a radius of 2 cm and an elastic modulus of 100 GPa ($k = \frac{EA}{L}$).

(a) Consider both cases with an unloaded cab (mass of 1.2 metric tonnes) and with a maximum capacity of 15 people with an average weight of 70 kg each. For each case, determine:

(i) (1 pt) The overshoot of the cab past the basement floor level after stopping.

(ii) (1 pt) The maximum acceleration felt by the occupants.

(iii) (1 pt) The maximum stress in the cables.

(b) (3 pts) To reduce the maximum tension in the cables and acceleration of the cab, a coil spring (k = 600 kN/m) is inserted between the cable attachment and the cab. How does this change the maximum displacement, acceleration, and stress for both the loaded and unloaded cases?

(c) The results for the vibration analysis of the original elevator system (no coil spring) was done under the assumption of no damping. However, the system components have an inherent damping. a test was done on an UNLOADED cab and it was found that the cab's oscillation amplitude decreased by 50% in two cycles.

(i) (2 pts) Determine the damping ratio for the LOADED case assuming viscous damping.

(ii) (2 pts) For the LOADED case, estimate how much time is needed after reaching the ground floor so that the passengers feel virtually no vibration of the elevator cab. Assume that the vibrations essentially stop when the amplitude decreases to 8% of its maximum value. Hint: recall that the logarithmic decrement is measured between subsequent peaks, so you must account for the time from $t = 0$ to the first peak.

Solution

(a)

The system acts as a simple spring-mass system with initial conditions $x(0) = 0$ and $\dot{x}(0) = 1.5$ m/s. Assume that the equilibrium position is at the basement floor level.

The equation of motion is:

$$\ddot{x} + \frac{k}{m}x = 0$$

with the general solution:

$$x(t) = A\cos(pt) + B\sin(pt)$$

with the solution to the initial conditions:

$$\begin{aligned} x(0) = 0 &\implies A = 0 \\ \dot{x}(0) = 1.5 &\implies B = 1.5/p \end{aligned}$$

So the solution is:

$$x(t) = \frac{1.5}{p}\sin(pt)$$

Next, the spring constant can be found by

$$k = \frac{EA}{L} = \frac{(100 \times 10^9)\pi(0.02)^2}{3.5 \times 10} = 3.59\text{Mn/m}$$

So the natural frequency for both cases is:

$$p_{\text{unloaded}} = \sqrt{\frac{3.59 \times 10^6}{1200}} = 54.7\text{rad/s}$$
$$p_{\text{loaded}} = \sqrt{\frac{3.59 \times 10^6}{1200 + 15 \times 70}} = 39.9\text{rad/s}$$

The overshoot of the cab is simply the amplitude of the solution. So,

$$\boxed{\begin{aligned} x_{\text{unloaded,overshoot}} &= \frac{1.5}{54.7} = 0.0274\text{m} \\ x_{\text{loaded,overshoot}} &= \frac{1.5}{39.9} = 0.0376\text{m} \end{aligned}}$$

To find the maximum acceleration, we can take the second derivative of the solution

$$\ddot{x} = -1.5p\sin(pt)$$

The maximum acceleration is the amplitude of the second derivative. So,

$$\boxed{\begin{aligned} \ddot{x}_{\text{unloaded,max}} &= 1.5 \times 54.7 = 82.0\text{m/s}^2 \\ \ddot{x}_{\text{loaded,max}} &= 1.5 \times 39.9 = 59.9\text{m/s}^2 \end{aligned}}$$

The maximum stress in the cables is at the maximum displacement. So,

$$\sigma_{\text{max}} = \frac{kx_{\text{max}} + mg}{A}$$

$$\boxed{\begin{aligned} \implies \sigma_{\text{unloaded,max}} &= \frac{3.59 \times 10^6 \times 0.0274 + 1200 \times 9.81}{\pi(0.02)^2} = 87.6\text{MPa} \\ \implies \sigma_{\text{loaded,max}} &= \frac{3.59 \times 10^6 \times 0.0376 + (1200 + 15 \times 70) \times 9.81}{\pi(0.02)^2} = 125\text{MPa} \end{aligned}}$$

(b)

An additional spring is added in series with the cable. For a simple oscillator, as seen in the Effective Stiffness Examples.pdf, the effective stiffness is given by:

$$k_{\text{eff}} = \frac{k_1 k_2}{k_1 + k_2}$$

So the effective stiffness is:

$$k_{\text{eff}} = \frac{3.59 \times 10^6 \times 600 \times 10^3}{3.59 \times 10^6 + 600 \times 10^3} = 0.514\text{Mn/m}$$

So the new natural frequency for both cases is:

$$p_{\text{unloaded}} = \sqrt{\frac{0.514 \times 10^6}{1200}} = 20.70\text{rad/s}$$
$$p_{\text{loaded}} = \sqrt{\frac{0.514 \times 10^6}{1200 + 15 \times 70}} = 15.1\text{rad/s}$$

The overshoot of the cab is

$$\boxed{\begin{aligned} x_{\text{unloaded,overshoot}} &= \frac{1.5}{20.70} = 0.0725\text{m} \\ x_{\text{loaded,overshoot}} &= \frac{1.5}{15.1} = 0.0993\text{m} \end{aligned}}$$

The maximum acceleration is

$$\boxed{\begin{aligned} \ddot{x}_{\text{unloaded,max}} &= 1.5 \times 20.70 = 31.1\text{m/s}^2 \\ \ddot{x}_{\text{loaded,max}} &= 1.5 \times 15.1 = 22.7\text{m/s}^2 \end{aligned}}$$

The maximum stress in the cables is at the maximum displacement. So,

$$\sigma_{\text{max}} = \frac{kx_{\text{max}} + mg}{A}$$

$$\boxed{\begin{aligned} \implies \sigma_{\text{unloaded,max}} &= \frac{0.514 \times 10^6 \times 0.0725 + 1200 \times 9.81}{\pi(0.02)^2} = 39.0\text{MPa} \\ \implies \sigma_{\text{loaded,max}} &= \frac{0.514 \times 10^6 \times 0.0993 + (1200 + 15 \times 70) \times 9.81}{\pi(0.02)^2} = 58.2\text{MPa} \end{aligned}}$$

(c)

Using the unloaded system, the damping ratio can be found through the logarithmic decrement (eq. 3.19).

$$\delta = \frac{1}{q} \ln\left(\frac{x_p}{x_{p+q}}\right)$$
$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

since $x_0/x_{0+2} = 1/0.5 = 2$. So,

$$\delta = \frac{1}{2}\ln(2) = 0.3466$$
$$\zeta = \frac{0.3466}{\sqrt{4\pi^2 + 0.3466^2}} = 0.055079$$

From the definition of the damping ratio,

$$\zeta = \frac{c}{2mp}$$

So the damping coefficient is:

$$c = 2mp\zeta = 2 \times 1200 \times 54.7 \times 0.055079 = 7230.77\text{N s/m}$$

Calculating the damping ratio for the loaded case,

$$\boxed{\begin{aligned}\zeta &= \frac{c}{2mp} \\ &= \frac{7230.77}{2 \times (1200 + 15 \times 70) \times 39.9} \\ &= 0.0403\end{aligned}}$$

(d)

For the time to reach 8% of the maximum amplitude, (eq. 3.20) can be used to find the time from the zeroth peak to the m-th peak.

$$\frac{\ln(x_p/x_{p+q})\sqrt{1-\zeta^2}}{2\pi\zeta} = q$$

then let $n = 0$, $x_0/x_{0+q} = 1/0.08 = 12.5$. So,

$$\begin{aligned}q &= \frac{\ln(12.5)\sqrt{1-0.0403^2}}{2\pi \times 0.0403} \\ &= 10\end{aligned}$$

since the response is sin, the time from the start to the first peak is

$$t_{\text{zero to peak}} = \frac{\tau}{4}$$

So the total time is

$$\begin{aligned}t_{\text{total}} &= q\tau + \frac{\tau}{4} \\ &= \tau\left(10 + \frac{1}{4}\right) \\ &= \frac{2\pi}{\sqrt{1-0.0403^2} \times 39.9}\left(10 + \frac{1}{4}\right) \\ &= \boxed{1.615\text{s}}\end{aligned}$$

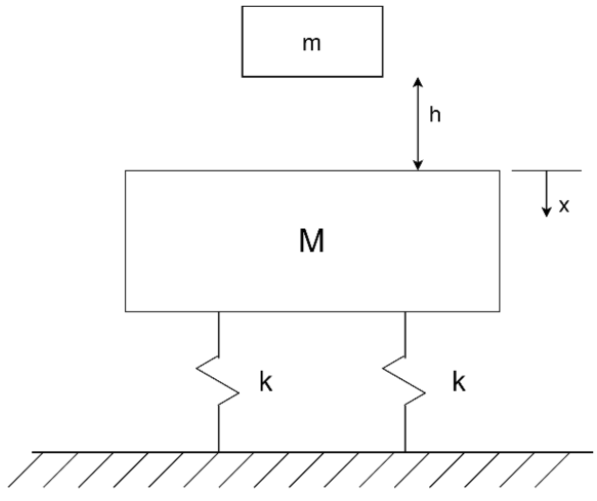

Figure 3: The floor and mass system

Question 5

The system shown, represents a floor of mass M supported by springs of stiffness k. A mass m is dropped on the floor from a height of h.

(a) (5 pts) Determine the motion of the floor using x, the displacement from the static equilibrium configuration before impact.

(b) (5 pts) Determine the maximum displacement for the case when $h = 0$. NOTE: Do not neglect the weight of the floor and the weight of mass m

Solution

(a)

First define

- State 1 is the initial state of m at rest at height h.
- State 2 is the state of m just before impact at height $h = 0$.
- State 3 is the state of m just after impact with M.

and assume

- The collision is perfectly inelastic.
- m is dropped with no initial velocity.
- M is initially at rest.
- The springs are initially at their equilibrium length.

We begin the analysis by considering Work-Energy Theorem on mass m initially and just before impact:

$$\begin{aligned} T_1 + U_{1\to 2} &= T_2 \\ \implies 0 + mgh &= \frac{1}{2}mv_2^2 \\ \implies v_2 &= \sqrt{2gh} \end{aligned}$$

Next, we employ Conservation of Momentum on the system of m and M before and after impact.

$$\begin{aligned} mv_2 &= (m+M)v_3 \\ \implies v_3 &= \frac{m}{m+M}v_2 \\ \implies v_3 &= \frac{m}{m+M}\sqrt{2gh} \end{aligned}$$

This gives us the initial velocity of the oscillator with $m_{\text{eff}} = m+M$. Next, we must determine the effect of m on the equilibrium position. The equilibrium position of the oscillator can be

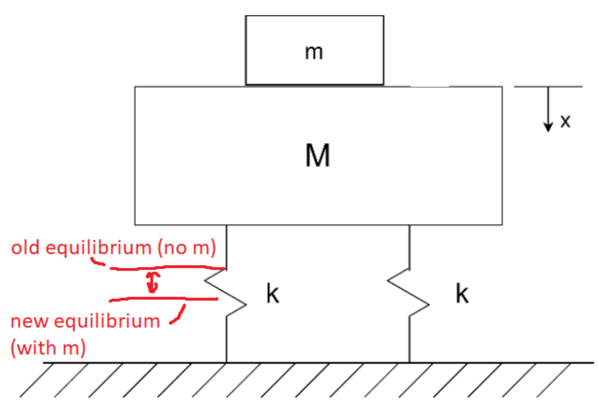

Figure 4: The equilibrium position of the oscillator

found using Hooke's Law. Since the springs are in parallel, the effective stiffness is the sum of the individual stiffnesses.

$$\begin{aligned} F &= k_{\text{eff}}\delta_{st} \\ \implies \delta_{st} &= \frac{mg}{k_{\text{eff}}} \\ &= \frac{mg}{2k} \end{aligned}$$

Since x is defined as positive sense downwards, the initial conditions are:

$$\begin{aligned} x(0) &= -\frac{mg}{2k} \\ \dot{x}(0) &= \frac{m}{m+M}\sqrt{2gh} \end{aligned}$$

Note that the negative sign on $x(0)$ is because the new equilibrium position is below the original equilibrium position. Now we define the natural frequency of this system:

$$\boxed{p = \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} = \sqrt{\frac{2k}{m+M}}}$$

From (eq. 2.7),

$$x(t) = x_0 \cos(pt) + \frac{\dot{x}_0}{p}\sin(pt)$$
$$\boxed{x(t) = -\frac{mg}{2k}\cos(pt) + \frac{m\sqrt{2gh}}{p(m+M)}\sin(pt)}$$

(b)

Using the single-term form of the response,

$$x(t) = X\sin(pt+\phi)$$
$$X = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{p}\right)^2}$$
$$\phi = \tan^{-1}\left(\frac{x_0}{\dot{x}_0/p}\right)$$

So the maximum displacement is the amplitude, X, which is

$$\boxed{x_{\max} = \sqrt{\left(-\frac{mg}{2k}\right)^2 + \left(\frac{m\sqrt{2gh}}{p(m+M)}\right)^2}}$$