MEC E 451
Lab 2: Forced Damped SDOF Vibration

by: Alex Diep

Date: March 9, 2024

Contents

List of Figures

List of Tables

1 Question 1

Estimate the natural frequency of the system by determining the motor frequency that results in the largest measured displacement amplitude.

The main results are shown in Table 1. The full dataset is shown in Table 2 in Appendix A.

Table 1: Main results for the forced damped SDOF vibration experiment

Dataset #	Frequency, f (Hz)	Displacement Amplitude, $\mathbb{X}$ (m)	ω/p
1	4.141	2.91E-04	0.792
2	2.840	4.74E-04	0.543
3	2.923	5.87E-04	0.559
4	3.976	5.03E-04	0.760
5	4.733	3.82E-04	0.905
6	4.518	7.37E-04	0.864
7	**5.231**	**1.98E-03**	1.000
8	5.847	6.39E-04	1.118
9	6.626	4.51E-04	1.267
10	11.044	1.99E-04	2.111
11	12.424	1.93E-04	2.375
12	11.044	2.85E-04	2.111
13	12.424	2.32E-04	2.375
14	14.199	2.16E-04	2.714

From Table 1, the largest measured displacement amplitude was 0.00198m, which occurs at a motor frequency of $\boxed{5.23\text{Hz}}$.

2 Question 2

Plot $\mathbb{X}$ vs. ω/p to obtain the frequency response curve of the system.

The data from Table 2 was plotted using Matplotlib in Python [1]. The plot is shown in Figure 1.

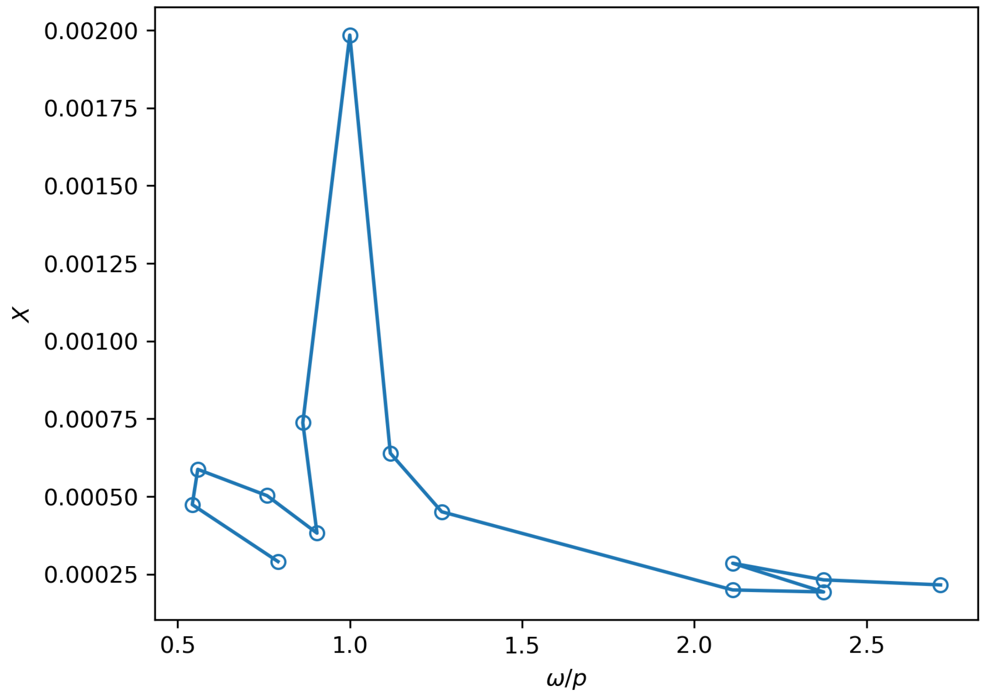

Figure 1: Amplitude vs. ω/p Response Curve

3 Question 3

Using your plot of $\mathbb{X}$ vs. ω/p, estimate the damping ratio using the half-power bandwidth method. ω_1 and ω_2 can be determined using linear interpolation.

From the annotated frequency response curve in Figure 2, the half-power bandwidth method was used to estimate the damping ratio. From linear interpolation of the LHS of the peak, denoting $(x, y) = (\omega/p, \mathbb{X})$,

$$\begin{aligned} x_{\text{LHS}} &= \frac{x_1 - x_2}{y_1 - y_2}\left(\frac{y_1}{\sqrt{2}} - y_1\right) + x_1 \\ &= \frac{1 - 0.864}{0.00198 - 0.000737}\left(\frac{0.00198}{\sqrt{2}} - 0.00198\right) + 1 \\ &= 0.9364 \end{aligned}$$

For the RHS of the peak,

$$\begin{aligned} x_{\text{RHS}} &= \frac{x_1 - x_2}{y_1 - y_2}\left(\frac{y_1}{\sqrt{2}} - y_1\right) + x_1 \\ &= \frac{1 - 1.118}{0.00198 - 0.000639}\left(\frac{0.00198}{\sqrt{2}} - 0.00198\right) + 1 \\ &= 1.0508 \end{aligned}$$

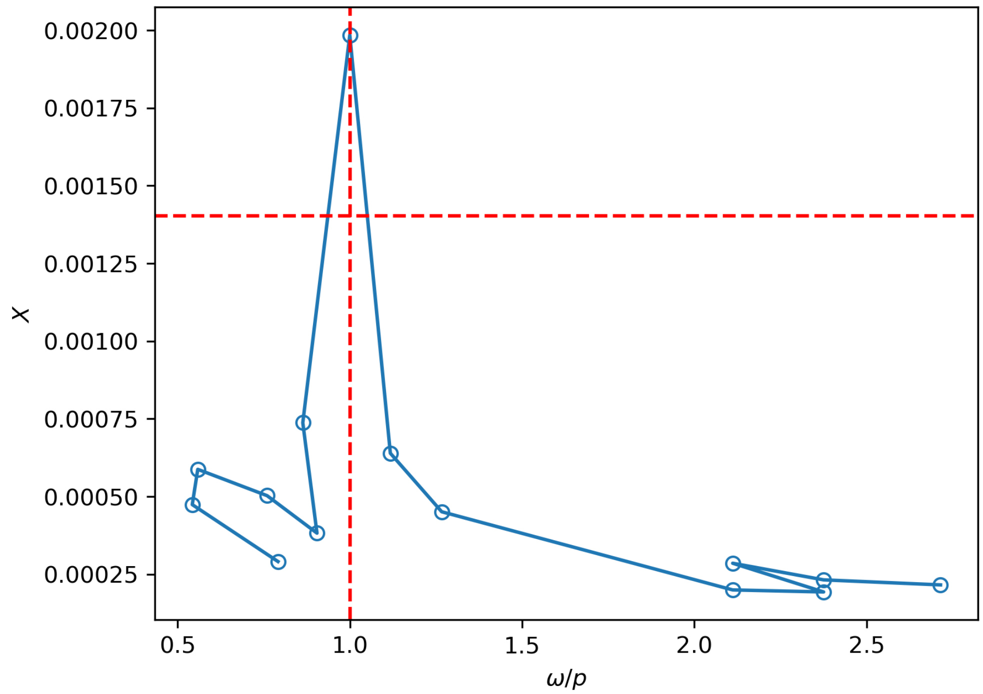

Figure 2: Annotated Amplitude vs. ω/p Response Curve

Then by the half-power bandwidth method, the damping ratio, ζ, was calculated by

$$\begin{aligned}\zeta = \frac{\omega_2 - \omega_1}{2p} &= \frac{\omega_{\text{RHS}}/p - \omega_{\text{LHS}}/p}{2} \\ &= \frac{1.0508 - 0.9364}{2} \\ &= \boxed{0.0572}\end{aligned}$$

4 Question 4

Using your calculated damping ratio, estimate the mass of a single imbalance (remember that there are two imbalances, not one), if they each have an eccentricity of 25 mm. Assume the total mass of the system is 13 kg.

From Eq. (5.22),

$$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{p}\right)^2}}$$

Rearranging,

$$\tilde{m} = \frac{M\mathbb{X}}{e}\frac{\sqrt{\left[1-\left(\frac{\omega}{p}\right)^2\right]^2+\left(2\zeta\frac{\omega}{p}\right)^2}}{\left(\frac{\omega}{p}\right)^2}$$

Using Dataset #7 from Table 2,

$$\begin{aligned}\tilde{m} &= \frac{13\times 1.98\times 10^{-3}}{0.025}\frac{\sqrt{\left[1-(1)^2\right]^2+(2\times 0.0572\times 1)^2}}{(1)^2}\\ &= 0.11778624 \text{ kg}\end{aligned}$$

Since there are two imbalances, the mass of a single imbalance is

$$\boxed{\tilde{m}_{\text{single}} = 0.0589 \text{ kg}}$$

5 Question 5

Estimate the natural frequency of the system using the vertical acceleration data recorded during beating of the platform and the motor frequency measured with the stroboscope.

The period was determined from the beating results to be

$$\begin{aligned}\tau_b &= t_{\text{node1}} - t_{\text{node2}}\\ &= 3.87756 - 2.77582\\ &= 1.10174 \text{ s}\end{aligned}$$

From the TA announcement, the formula for the natural frequency is given by

$$\tau_b = \frac{2\pi}{\omega - p}$$

From using the stroboscope, the motor frequency was determined to be $f = 5$ Hz. Then,

$$\begin{aligned}\omega &= 2\pi f\\ &= 2\pi\times 5\\ &= 31.416 \text{ rad/s}\end{aligned}$$

Then,

$$\begin{aligned}p &= \omega - \frac{2\pi}{\tau}\\ &= 31.416 - \frac{2\pi}{1.10174}\\ &= 31.416 - 5.694\\ &= 25.713 \text{ rad/s}\end{aligned}$$

Which is

$$\begin{aligned} f &= \frac{p}{2\pi} \\ &= \frac{25.713}{2\pi} \\ &= \boxed{4.09 \text{ Hz}} \end{aligned}$$

6 References

[1] J. D. Hunter, "Matplotlib: A 2d graphics environment," *Computing in Science & Engineering*, vol. 9, no. 3, pp. 90–95, 2007.

A Appendix: Sample Calculations

Use your accelerometer data (in the z direction) to estimate the frequency of the motor ω and the corresponding displacement amplitude $\mathbb{X}$ of the platform for each motor speed

Table 2: Frequency and Displacement Results

Dataset #	Max Acceleration, a_{max} (m/s^2)	Min Acceleration, a_{min} (m/s^2)	Time of Max, t_{max} (s)	Time of Min, t_{min} (s)	Acceleration Amplitude, $\mathbb{A}$ (m/s^2)	Frequency, f (Hz)	Angular Frequency, ω (rad/s)	Displacement Amplitude, $\mathbb{X}$ (m)	ω/p
1	0.210	-0.184	5.960	6.081	0.197	4.141	26.021	2.91E-04	0.792
2	0.147	-0.155	7.130	7.306	0.151	2.840	17.843	4.74E-04	0.543
3	0.176	-0.220	12.155	12.326	0.198	2.923	18.368	5.87E-04	0.559
4	0.306	-0.321	16.422	16.548	0.314	3.976	24.980	5.03E-04	0.760
5	0.284	-0.392	20.974	21.080	0.338	4.733	29.738	3.82E-04	0.905
6	0.578	-0.610	28.302	28.191	0.594	4.518	28.387	7.37E-04	0.864
7	2.044	-2.244	35.249	35.345	2.144	**5.231**	32.869	1.98E-03	1.000
8	0.897	-0.828	43.280	43.194	0.863	5.847	36.736	6.39E-04	1.118
9	0.854	-0.710	52.083	52.008	0.782	6.626	41.634	4.51E-04	1.267
10	1.019	-0.901	58.312	58.267	0.960	11.044	69.389	1.99E-04	2.111
11	1.282	-1.068	64.397	64.357	1.175	12.424	78.063	1.93E-04	2.375
12	1.385	-1.362	69.977	69.932	1.373	11.044	69.389	2.85E-04	2.111
13	1.629	-1.195	74.560	74.520	1.412	12.424	78.063	2.32E-04	2.375
14	1.896	-1.536	86.093	86.057	1.716	14.199	89.215	2.16E-04	2.714

Sample calculations will be shown for dataset #1. From the data, a_{max}, a_{min}, t_{max}, and t_{min} were determined by inspection. Then, the amplitude of the acceleration, $\mathbb{A}$, was calculated by

$$\mathbb{A} = \frac{a_{max} - a_{min}}{2} = \frac{0.210 - (-0.184)}{2} = 0.197 \text{ m/s}^2$$

The frequency, f, was calculated by

$$f = \frac{1}{2(t_{min} - t_{max})} = \frac{1}{2(6.081 - 5.960)} = 4.141 \text{ Hz}$$

The angular frequency, ω, was calculated by

$$\omega = 2\pi f = 2\pi(4.141) = 26.021 \text{ rad/s}$$

The displacement amplitude, $\mathbb{X}$, was calculated by

$$\mathbb{X} = \frac{\mathbb{A}}{\omega^2} = \frac{0.197}{(26.021)^2} = 2.91 \times 10^{-4} \text{ m}$$

The ratio of the angular frequency to the natural frequency, ω/p, was calculated by

$$\omega/p = \frac{\omega}{p} = \frac{4.141}{5.231} = 0.792$$