

Question 1

In this lab, the apparatus is a simple platform suspended by springs as shown above. When modelling a stiffness/elastic element as a spring, it is typically assumed that the spring provides a stiffness in only the axial direction (ie. in the z – direction).

- (a) How much would the natural frequency of vibrations in the vertical direction increase if the stiffness of each spring is doubled?
- (b) The springs are manufactured such that they are also able to resist lateral forces (ie. in the x and y directions). If the springs have both an axial and lateral stiffness, determine how many degrees of freedom the system has and state each degree of freedom.

(a)

First let the springs be k . Since they are in parallel,

$$k_{\text{eff}} = 4(k) = 4k$$

$$p_1 = \sqrt{\frac{4k}{m}} = 2\sqrt{\frac{k}{m}}$$

Now, let all the springs be $2k$. Then,

$$k_{\text{eff}} = 4(2k) = 8k$$

$$p_2 = \sqrt{\frac{8k}{m}} = 2\sqrt{2}\sqrt{\frac{k}{m}}$$

So the natural frequency would increase by a factor of

$$\boxed{\sqrt{2}}$$

(b)

If the springs can move in the x , y , and z directions, then the plate has 6 degrees of freedom:

1. Displacement in the x direction
2. Displacement in the y direction
3. Displacement in the z direction
4. Rotation about the x axis
5. Rotation about the y axis
6. Rotation about the z axis

Question 2 (4pts)

Plot the vertical acceleration versus time. Using your plot: Calculate the damping ratio using the logarithmic decrement. Use a set of peaks away from the beginning of the measured response due to the initial lateral motion of the platform when it is released.

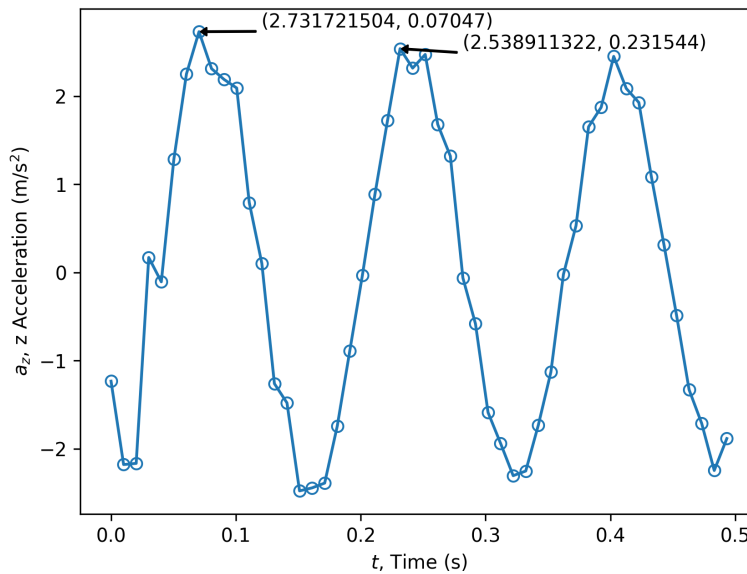


Figure 1: Vertical (z) acceleration of the platform after initial excitation.

Using the logarithmic decrement method,

$$\begin{aligned}
 \delta &= \frac{1}{n} \ln \left(\frac{z_0}{z_n} \right) \\
 &= \frac{1}{1} \ln \left(\frac{2.731721504}{2.538911322} \right) \\
 &= 0.07320
 \end{aligned}$$

then,

$$\begin{aligned}
 \zeta &= \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \\
 &= \frac{0.07320}{\sqrt{4\pi^2 + 0.07320^2}} \\
 &= \boxed{0.01165}
 \end{aligned}$$

Question 3 (3pt)

If each spring has a stiffness of 2.8 kN/m, calculate the mass of the platform.

Experimentally, the period was determined to be

$$\tau = t_2 - t_1 = 0.231544 - 0.07047 = 0.161074$$

We can determine natural frequency from from Eq. 3.15,

$$\begin{aligned}\tau &= \frac{2\pi}{\sqrt{1 - \zeta^2}p} \\ \Rightarrow p &= \frac{2\pi}{\tau\sqrt{1 - \zeta^2}} \\ &= \frac{2\pi}{0.161074\sqrt{1 - 0.01165^2}} \\ &= 39.012 \text{ rad/s}\end{aligned}$$

Then, by definition of the natural frequency,

$$\begin{aligned}p &= \sqrt{\frac{k_{\text{eff}}}{m_{\text{eff}}}} \\ \Rightarrow m_{\text{eff}} &= \frac{k_{\text{eff}}}{p^2} \\ \Rightarrow m_{\text{eff}} &= \frac{4k}{p^2} \\ &= \frac{4 \times 2.8 \times 10^3}{39.012^2} \\ &= \boxed{7.359 \text{ kg}}\end{aligned}$$

Since the typical mass of a smartphone is 0.2 kg, the mass of the platform is

$$\begin{aligned}m_{\text{platform}} &= m_{\text{eff}} - m_{\text{smartphone}} \\ &= 7.359 - 0.2 \\ &= \boxed{7.159 \text{ kg}}\end{aligned}$$

Question 4 (2 pts)

Determine the effective damping of the system.

From equation 3.6,

$$\begin{aligned}\zeta &= \frac{c}{C_c} = \frac{c}{2mp} \\ \Rightarrow c &= 2mp\zeta \\ &= 2(7.3590)(39.012)(0.01165) \\ &= \boxed{6.689 \text{ Ns/m}}\end{aligned}$$

Question 5 (2pts)

If the metal springs were replaced with rubber springs of the same stiffness, would you expect the oscillation period to increase or decrease? Why?

The equation of motion for a damped spring-mass system is given by

$$m\ddot{x} + c\dot{x} + kx = 0$$

If k does not change, then the period does not change. However, if the rubber springs have a weaker damping effect, then damping ratio ζ will decrease, and hence, period would increase.