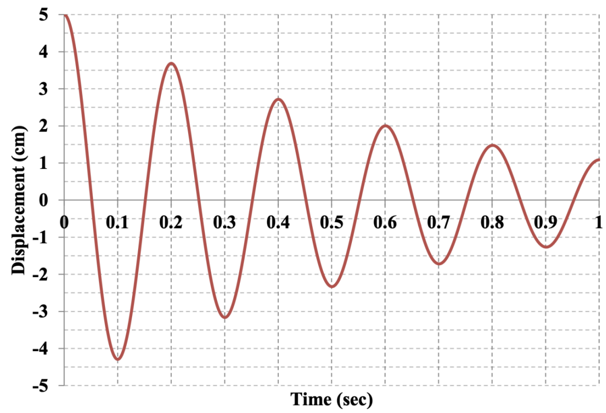
Note to marker:

MATLAB symbolic toolbox was used to numerically solve implicit equations. This could've been done with my CASIO 991EX but I was too lazy to pull out my calculator. Hope this is okay.

Question 1

The free vibration of a viscously damped SDOF system due to a non-zero initial displacement (zero initial velocity) is given in the graph shown below. Determine the following questions using this graph. Clearly indicate the values you obtain from the graph.



(a) (5 pts) Write a differential equation that governs the equation of motion of this system.

(b) (5 pts) If the same system was subjected only to a non-zero initial velocity of 100cm/sec (zero initial displacement), what would be the displacement response at t = 0.25 sec.

Solution

(a)

Notice that the graph exhibits the behaviour of a damped cosine wave. Using $x_0 = 5$ and $x_3 = 2$,

$$
\begin{aligned}
\delta &= \frac{1}{n} \ln\left(\frac{x_0}{x_3}\right) \\
&= \frac{1}{3} \ln\left(\frac{5}{2}\right) \\
&= 0.30543
\end{aligned}
$$

From this, we can find the damping ratio ζ,

$$
\begin{aligned}
\zeta &= \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \\
&= \frac{0.30543}{\sqrt{4\pi^2 + 0.30543^2}} \\
&= 0.048553
\end{aligned}
$$

The period is $\tau = 0.2$ sec, so the natural frequency is

$$\begin{aligned} p &= \frac{2\pi}{\sqrt{1-\zeta^2}\tau} \\ &= \frac{2\pi}{\sqrt{1-0.048553^2}\cdot 0.1} \\ &= 31.45 \text{ rad/sec} \end{aligned}$$

Recall the equation of motion for a damped SDOF system,

$$m\ddot{x} + c\dot{x} + kx = 0$$

dividing by m and letting $p^2 = \frac{k}{m}$ and $2\zeta p = \frac{c}{m}$, we get

$$\ddot{x} + 2\zeta p\dot{x} + p^2 x = 0$$

substituting in the values we found, we get

$$\begin{gathered} \ddot{x} + 2(0.048553)(31.45)\dot{x} + (31.45)^2 x = 0 \\ \boxed{\ddot{x} + 3.054\dot{x} + 989.1x = 0} \end{gathered}$$

(b)

Now, using initial velocity of 100 cm/sec, and zero initial displacement, using (3.13) from the course notes,

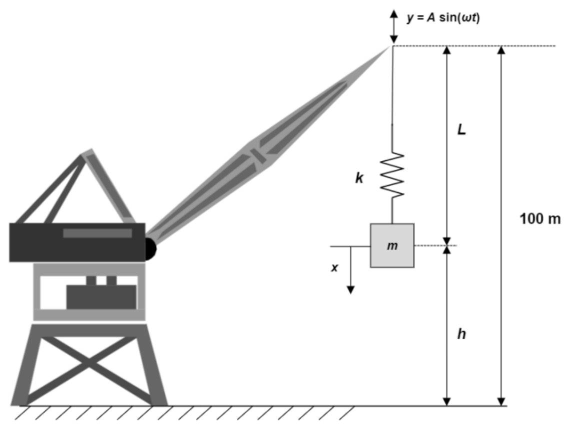
$$x(t) = e^{-\zeta pt}\left(\frac{v_0 + \zeta p x_0}{\sqrt{1-\zeta^2}p}\sin\left(\sqrt{1-\zeta^2}pt\right) + x_0\cos\left(\sqrt{1-\zeta^2}pt\right)\right)$$

so,

$$\begin{aligned} x(0.25) &= e^{-0.048553\cdot 31.45\cdot 0.25}\left(\frac{100+0}{\sqrt{1-0.048553^2}\cdot 31.45}\sin\left(\sqrt{1-0.048553^2}31.45\cdot 0.25\right)\right) \\ &= \boxed{2.173 \text{ cm}} \end{aligned}$$

Question 2

A crane 100 m tall is loading a container full of feathers and delicate glass figurines weighing 5 metric tons onto a cargo ship. While the container is in the air, there is an emergency shutdown of the crane and the case is left hanging for a short time. During this time, winds cause the arm of the crane to vibrate vertically at a frequency of 4 Hz with an amplitude of 5 cm. The supporting cable has an effective stiffness of $k = \frac{100}{L}$ MN/m, where L is the length of exposed cable in metres (neglect changes in L due to vibration).



(a) (2 pts) Determine the cable length L at which the transmissibility would be exactly 1.

(b) (5 pts) Assuming steady state vibration, determine all possible heights h at which the crate would hit the ground.

(c) (3 pts) After the emergency is dealt with, operations resume as the wind continues to excite the crane arm. The crane needs to lift shipments 80 m high to place it onto the cargo ships. What is the smallest mass of cargo that can be lifted in these conditions without shaking with an amplitude greater than 4 cm? Assume $\omega > p$.

Solution

(a)

First, we must choose the proper model for the system. A simple undamped spring-mass with base excitation model is appropriate. Assume the weight of the cable is negligible.

The transmissibility is then

$$\mathrm{TR} = \frac{\left(\frac{\omega}{p}\right)^2}{1 - \left(\frac{\omega}{p}\right)^2}$$

converting $f = 4$ Hz to $\omega = 2\pi f = 8\pi$ rad/sec, we can solve for p when $\text{TR} = 1$

$$
\begin{aligned}
1 &= \frac{\left(\frac{8\pi}{p}\right)^2}{1 - \left(\frac{8\pi}{p}\right)^2} \\
1 - \left(\frac{8\pi}{p}\right)^2 &= \left(\frac{8\pi}{p}\right)^2 \\
1 &= 2\left(\frac{8\pi}{p}\right)^2 \\
\implies p &= 8\sqrt{2}\pi
\end{aligned}
$$

Since p is defined as

$$
\begin{aligned}
p &= \sqrt{\frac{k}{m}} \\
\implies k &= p^2 m \\
&= (8\sqrt{2}\pi)^2(5 \times 10^3) \\
&= 0.64\pi^2 \times 10^6 \text{ N/m} \\
&= 0.64\pi^2 \text{ MN/m}
\end{aligned}
$$

and since $k = \frac{100}{L}$, we can solve for L,

$$\boxed{L = \frac{100}{k} = \frac{100}{0.64\pi^2} = 15.8 \text{ m}}$$

(b)

The DMF is

$$\frac{\mathbb{X}}{\mathbb{X}_0} = \frac{1}{\left|1 - \left(\frac{\omega}{p}\right)^2\right|}$$

Since we want the amplitude to be equal to or greater than the height of the crate, we're interested in the region between resonance and the value where the amplitude is equal to the height of the crate. First, let us find the point past resonance where the DMF is

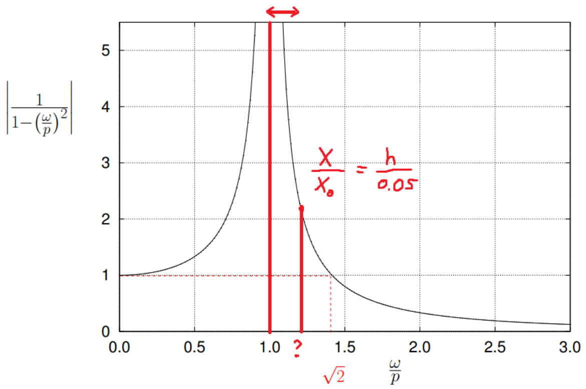


Figure 1: Operating range for b)

$$
\begin{aligned}
\text{DMF} := \frac{\mathbb{X}}{\mathbb{X}_0} &= \frac{h}{0.05} \\
&= \frac{1}{\left|1 - \left(\frac{\omega}{p}\right)^2\right|} \\
&= \frac{1}{\left|1 - \frac{m\omega^2}{k}\right|} \\
&= \frac{1}{\left|1 - \frac{mL\omega^2}{100\times10^6}\right|} \\
&= \frac{1}{\left|1 - \frac{m(100-h)\omega^2}{10^8}\right|} \\
&= \frac{1}{\left|1 - \frac{5\times10^3(100-h)(8\pi)^2}{10^8}\right|}
\end{aligned}
$$

```
syms h
eqn = 1/abs(1 - 5*10^3*(100-h)*(8*pi)^2/10^8) == h/0.05;
h = solve(eqn, h);
h = double(h)

>> h =
        0.0232
        68.3140
        68.3603
```

The natural frequency at each of these points is

$$p = \sqrt{\frac{100}{(100-h)m} \times 10^6}$$
$$p_1 = \sqrt{\frac{100}{(100-0.0232)\times 5\times 10^3} \times 10^6} = 14.144$$
$$p_2 = \sqrt{\frac{100}{(100-68.314)\times 5\times 10^3} \times 10^6} = 25.124$$
$$p_3 = \sqrt{\frac{100}{(100-68.3603)\times 5\times 10^3} \times 10^6} = 25.142$$

Since the driving frequency is $8\pi = 25.133$, and $\omega > p$, the only solution is 68.3403 m.

Now let's solve for resonance. This happens when the DMF is infinite, or when the denominator is zero. Then,

$$1 - \frac{m(100-h)\omega^2}{10^8} = 0$$

```
syms h
eqn = 1 - 5*10^3*(100-h)*(8*pi)^2/10^8 == 0;
h = solve(eqn, h);
h = double(h)

h =
    68.3371
```

So the possible heights are $\boxed{h = (68.337, 68.360] \text{ m}}$.

(c)

Let $h = 80$, and the operating range is $\omega > p$. Then, the DMF is

$$\text{DMF} = \frac{1}{\left(\frac{\omega}{p}\right)^2 - 1}$$

Since the shaking specification is 4 cm, then

$$\frac{4}{5} = \frac{1}{\left(\frac{\omega}{p}\right)^2 - 1}$$
$$\frac{4}{5} = \frac{1}{\left(\frac{8\pi}{p}\right)^2 - 1}$$

Solving for p,

```
syms p
eqn = 4/5 == 1/((8*pi/p)^2 - 1);
p = solve(eqn, p);
p = double(p)

>> p = -15.7003
        15.7003
```

The only physical solution is $p = 15.7$ rad/sec. By the definition of p,

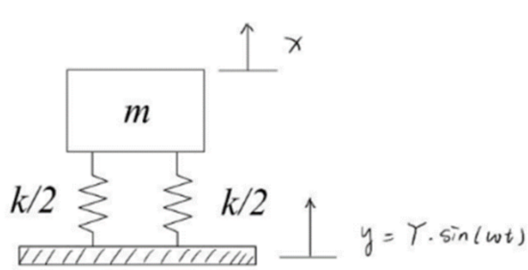
$$
\begin{aligned}
p &= \sqrt{\frac{k}{m}} \\
&= \sqrt{\frac{100/L \times 10^6}{m}}
\end{aligned}
$$

Solving for m,

$$
\begin{aligned}
m &= \frac{100/L \times 10^6}{p^2} \\
&= \frac{100/20 \times 10^6}{(15.7)^2} \\
&= \boxed{20.28 \times 10^3 \text{ kg}}
\end{aligned}
$$

Question 3

National Institute of Nanotechnology (NINT) is a first generation Nano-research facility and the first of its kind in Canada. The facility is a six-storey building located on the University of Alberta Campus. Along with research offices, wet laboratories and clean nano-fab space, the facility features several ultra sensitive electron microscopes. In order for these microscopes to operate in the nanoscale, they must be provided with an extremely stable environment that is free from movement and vibration. One model of isolation setup for the electron microscopes is shown as (1) where an excitation displacement is applied to the base in response of floor vibration. The floor vibration is assumed to have a frequency of 50 Hz. The natural frequency of these whole setup is measured as 10 Hz.



(a) (4 pts) Estimate the amplitude of vibration of electron microscope by comparing it to the amplitude of the floor for the setup as (1).

(b) (5 pts) The quality of the image from electron microscope is not satisfying since the amplitude is still too large. A dampler was added between the electron microscope and floor. What damping ratio should be chosen to make the amplitude of the vibration of electron microscope two times the amplitude in part a)?

(c) (1 pts) If the damping ratio of the damper needs to be adjusted to increase the amplitude of vibration of electron microscope, should the laboratory staff adjust the damping factor up or down?

Solution

(a)

First, we decide the proper model for the problem. An appropriate model is an undamped SDOF system with base excitation. Since the springs are in parallel, the stiffness is $k_{\text{eff}} = k$. The DMF for this system is

$$\text{DMF} = \frac{1}{1 - \left(\frac{\omega}{p}\right)^2}$$

substituting,

$$\begin{aligned}\text{DMF} &= \frac{1}{1 - \left(\frac{50}{10}\right)^2} \\ &= \frac{1}{1 - 25} \\ &= -0.04167\end{aligned}$$

This means that the amplitude of the electron microscope is $\boxed{4.167\%}$ of the amplitude of the floor. The negative sign indicated that the response of the electron microscope is 180 degrees out of phase with the floor.

(b)

For a damped SDOF system with base excitation, the DMF is

$$\text{DMF} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{p}\right]^2}}$$

The DMF in a) was 1/24, therefore we want the DMF to be 1/12.

$$\frac{1}{12} = \frac{\sqrt{1 + \left(2\zeta\frac{50}{10}\right)^2}}{\sqrt{\left[1 - \left(\frac{50}{10}\right)^2\right]^2 + \left[2\zeta\frac{50}{10}\right]^2}}$$

Solving for ζ,

```
syms zeta
eqn = 1/12 == sqrt(1+(2*zeta*5)^2)/sqrt((1-(5)^2)^2+(2*zeta*5)^2);
zeta = solve(eqn, zeta);
zeta = double(zeta)

>> zeta =
        -0.1738
         0.1738
```

Therefore, the damping ratio should be $\boxed{0.1738}$.

(c)

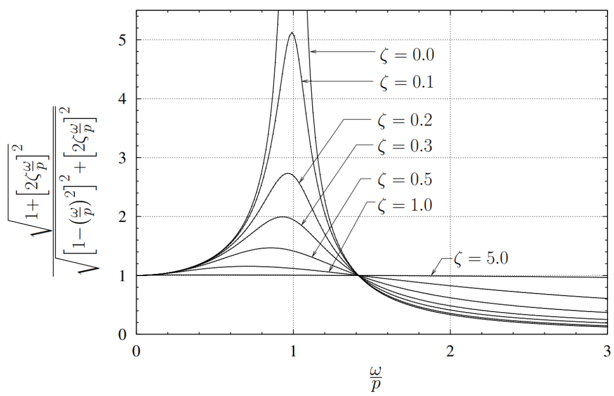


Figure 2: Response amplitude of damped spring–mass system subjected to harmonic base excitation

Since we are operating on the RHS of the resonance peak, we should adjust the damping factor **up** to increase the amplitude of vibration of the electron microscope.

Question 4

A washing machine produces disruptive noise during its spin cycle due to an uneven distribution of clothes around its circumference. To reduce the noise, an engineering consulting firm has proposed that the machine be mounted on spring isolators at each corner (four in total). The machine has a capacity of 10 kg of laundry, an unloaded mass of 450 kg, a drum diameter of 0.50 m, and spin cycle speed of 955 rpm. To address the noise levels, the transmitted force should be reduced by 75%. Assume vertical vibrations only and that c_{eff} is inherent to the system (damping from the spring isolators are negligible).

(a) (5 pts) Determine the isolator spring constant needed to resolve the noise issue. Assume that the system is 25% damped at full capacity ($\zeta = 0.25$).

(b) After installing an appropriate set of spring isolators, it was found that a 0.3 kg wet hoodie was spinning out of balance.

(i) (2.5 pts) Using the stiffness from part a), find the response amplitude due to this imbalance during the spin cycle for the washing machine when it is fully loaded.

(ii) (2.5 pts) After removing the hoodie and resuming the same wash, a wet towel with a mass of 0.5 kg began to spin out of balance. Determine the spring constants needed to maintain the same amplitude as part i).

Solution

(a)

This is an undamped SDOF with rotating imbalance. The transmissibility is (eq. 5.19)

$$\mathrm{TR} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{p}\right]^2}}$$

Substituting $\mathrm{TR} = 0.25$, $\zeta = 0.25$, $\omega = 955 \times \frac{2\pi}{60}$,

$$0.25 = \frac{\sqrt{1 + \left(2 \times 0.25 \times \frac{955\times 2\pi/60}{p}\right)^2}}{\sqrt{\left[1 - \left(\frac{955\times 2\pi/60}{p}\right)^2\right]^2 + \left[2 \times 0.25 \times \frac{955\times 2\pi/60}{p}\right]^2}}$$

Solving for p,

```
syms p real
eqn = 0.25 == sqrt(1+(2*0.25*955*2*pi/60/p)^2) / ...
sqrt((1-(955*2*pi/60/p)^2)^2+(2*0.25*955*2*pi/60/p)^2);
p = solve(eqn, p);
p = double(p)

>>p =
    -36.0438
     36.0438
```

Therefore, the natural frequency is 36.0438 rad/sec. The spring constant is then

$$\begin{aligned} k &= p^2(M + \tilde{m}) \\ &= (36.0438)^2(450 + 10) \\ &= \boxed{0.5976 \times 10^6 \text{ N/m}} \end{aligned}$$

(b)

(i)

Since ζ was only given for the full load, we must find the ζ for the hoodie. By definition of ζ,

$$\begin{aligned}
\zeta &= \frac{c_{\text{eff}}}{2m_{\text{eff}}p} \\
\implies c_{\text{eff}} &= 2\zeta m_{\text{eff}}p \\
&= 2 \times 0.25 \times (450 + 10) \times 36.0438 \\
&= 8290.074 \text{ Ns/m}
\end{aligned}$$

Therefore with a new $m_{\text{eff}} = 450 + 0.3$,

$$\begin{aligned}
\zeta &= \frac{8290.074}{2(450 + 0.3)36.0438} \\
&= 0.25539
\end{aligned}$$

From eq. 5.22, the ampltitude response is

$$\frac{M\mathbb{X}}{\tilde{m}e} = \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{p}\right]^2}}$$

Substituting $\omega = 955 \times \frac{2\pi}{60}$, $p = 36.0438$, $\zeta = 0.25539$,

$$\begin{aligned}
\frac{M\mathbb{X}}{\tilde{m}e} &= \frac{\left(\frac{955\times 2\pi/60}{36.0438}\right)^2}{\sqrt{\left[1 - \left(\frac{955\times 2\pi/60}{36.0438}\right)^2\right]^2 + \left[2 \times 0.25539 \times \frac{955\times 2\pi/60}{36.0438}\right]^2}} \\
&= \boxed{1.1244}
\end{aligned}$$

(ii)

Again consider the amplitude response

$$\begin{aligned}
\frac{M\mathbb{X}}{\tilde{m}e} &= \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{p}\right]^2}} \\
&= \frac{\left(\frac{\omega}{p}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{p}\right)^2\right]^2 + \left[2\frac{c_{\text{eff}}}{2m_{\text{eff}}p}\frac{\omega}{p}\right]^2}}
\end{aligned}$$

This time, let $M\mathbb{X}/\tilde{m}e = 1.1244$, $\omega = 955 \times \frac{2\pi}{60}$, $m_{\text{eff}} = 450 + 0.5$, $c_{\text{eff}} = 8290.074$,

$$1.1244 = \frac{\left(\frac{955\times 2\pi/60}{p}\right)^2}{\sqrt{\left[1-\left(\frac{955\times 2\pi/60}{p}\right)^2\right]^2 + \left[2\frac{8290.074}{2(450+0.5)p}\frac{955\times 2\pi/60}{p}\right]^2}}$$

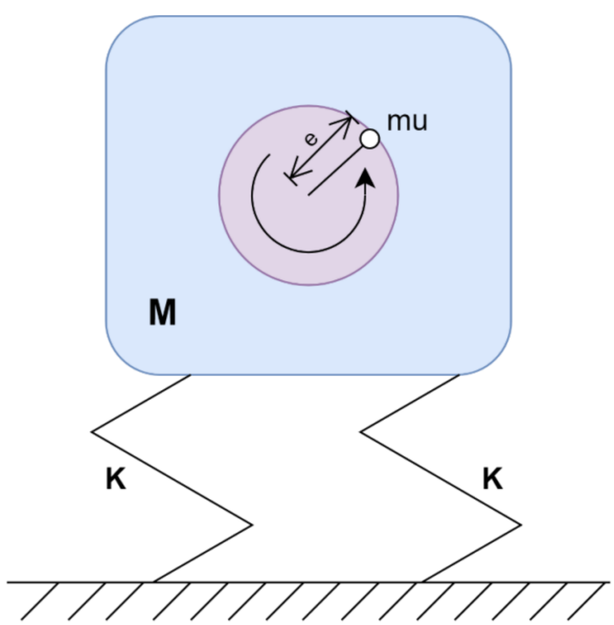
Solving for p,

```
syms p
eqn = 1.1244 == (955*2*pi/60/p)^2/sqrt((1-(955*2*pi/60/p)^2)^2 ...
+(2*8290.074/2/(450+0.5)*955*2*pi/60/p)^2);
p = solve(eqn, p);
p = double(p)

>>p =

    0x1 empty double column vector
```

not good.

Question 5

A motor with the mass 40 kg is supported with 4 springs, each of stiffness 250 N/m as it is illustrated in following figure. The rotor is unbalanced such that the unbalanced effect is equivalent mass of 5 kg located 50 mm from the axis of rotation. (Note: There is no damping in the system.)



(a) (5 pts) Find the amplitude of vibration and the force transmitted to the foundation when the speed of motor is 1000 rpm.

(b) (5 pts.) Compare the amplitude of vibration and the transmitted force calculated in part (a) with the case that the motor is running at speed of 60 rpm.

Solution

(a)

This is an undamped SDOF with rotating imbalance. The amplitude of vibration is given from (4.11),

$$\mathbb{X} = \frac{\frac{\tilde{m}e\omega^2}{k}}{\left|1 - \left(\frac{\omega}{p}\right)^2\right|}$$

The natural frequency is

$$\begin{aligned} p &= \sqrt{\frac{k}{m}} \\ &= \sqrt{\frac{250 \times 4}{40}} \\ &= 5 \text{ rad/sec} \end{aligned}$$

then,

$$\begin{aligned} \mathbb{X} &= \frac{\frac{5\times 50\times 10^{-3}\times(1000\times 2\pi/60)^2}{250\times 4}}{\left|1 - \left(\frac{1000\times 2\pi/60}{5}\right)^2\right|} \\ &= \boxed{6.264 \text{ mm}} \end{aligned}$$

Transmissibility is defined as

$$\begin{aligned} \text{TR} :&= \frac{F_{T_{\max}}}{F_D} \\ &= \frac{1}{\left|1 - \left(\frac{\omega}{p}\right)^2\right|} \end{aligned}$$

where the disturbing force, $F_D = \tilde{m}e\omega^2$. Then the transmitted force $F_{T_{\max}}$ is

$$\begin{aligned} F_{T_{\max}} &= \frac{\tilde{m}e\omega^2}{\left|1 - \left(\frac{\omega}{p}\right)^2\right|} \\ &= \frac{5 \times 50 \times 10^{-3} \times (1000 \times 2\pi/60)^2}{\left|1 - \left(\frac{1000\times 2\pi/60}{5}\right)^2\right|} \\ &= \boxed{6.264 \text{ N}} \end{aligned}$$

(b)

The amplitude of vibration is then

$$\begin{aligned}
\mathbb{X} &= \frac{\frac{5\times 50\times 10^{-3}\times(60\times 2\pi/60)^2}{250\times 4}}{\left|1-\left(\frac{60\times 2\pi/60}{5}\right)^2\right|} \\
&= \boxed{17.042\text{ mm}}
\end{aligned}$$

The transmitted force is

$$\begin{aligned}
F_{T_{\max}} &= \frac{5\times 50\times 10^{-3}\times(60\times 2\pi/60)^2}{\left|1-\left(\frac{60\times 2\pi/60}{5}\right)^2\right|} \\
&= \boxed{17.042\text{ N}}
\end{aligned}$$