MEC E 539
Lab 4: Unsteady Laminar Flow Around a Circular Cylinder

by: Alex Diep

Date: March 8, 2024

Contents

List of Figures

List of Tables

1 Part I. Problem Setup

1.1 Task 1.1

Calculate the problem Reynolds number ($Re = U_\infty D/\nu$) and confirm that the flow is laminar.

The working fluid is air. The properties of air at 25°C and 1 atm are taken from a textbook [1].

$$\rho = 1.184 \text{ kg/m}^3$$
$$\nu = 1.562 \times 10^{-5} \text{ m}^2/\text{s}$$

The Reynolds number is calculated as

$$\begin{aligned}\text{Re} &= \frac{U_\infty D}{\nu} \\ &= \frac{0.20 \times 0.01}{1.562 \times 10^{-5}} \\ &= \boxed{128.04}\end{aligned}$$

This is less than the critical Reynolds number of 2000, so the flow is laminar.

1.2 Task 1.3

Calculate the minimum timestep for unsteady simulations based on the minimum element size of "Grid 3". You need to use the Courant (CFL) number for this estimation. Set the simulation timestep according to your calculations, while keeping the estimated CFL number below 0.8.

First we desire CFL to be

$$\text{CFL} = \frac{U_\infty \Delta t}{\Delta x} < 0.8$$

For Grid 3, the element size was selected such that $\Delta x = D/15 = 0.667$ mm. The minimum timestep is then

$$\begin{aligned}\Delta t &= \frac{0.8\Delta x}{U_\infty} \\ &= \frac{0.8 \times 0.667 \times 10^{-3}}{0.20} \\ &= 2.67 \times 10^{-3} \text{ s} \\ &\approx \boxed{0.0025} \text{ s}\end{aligned}$$

1.3 Task 1.4

Calculate the Through-Time (T_thr) for this setup, which is defined as the time it takes air to travel from the inlet to the outlet without any disturbance. Knowing the inlet velocity (U_∞)

and length of the computational domain (L_{cv}),

$$T_{thr} = \frac{L_{cv}}{U_\infty}$$

The total time of the simulation should be 10 T_{thr}.

$$\begin{aligned} T_{thr} &= \frac{L_{cv}}{U_\infty} \\ &= \frac{20D}{U_\infty} \\ &= \frac{20 \times 0.01}{0.20} \\ &= \boxed{1} \text{ s} \end{aligned}$$

The total time of the simulation should be 10 T_{thr},

$$\begin{aligned} T_{\text{total}} &= 10T_{thr} \\ &= 10 \times 1 \\ &= \boxed{10} \text{ s} \end{aligned}$$

The number of iterations is then,

$$\begin{aligned} \text{Iterations} &= \frac{T_{\text{total}}}{\Delta t} \\ &= \frac{10}{0.0025} \\ &= \boxed{4000} \end{aligned}$$

1.4 Task 1.9

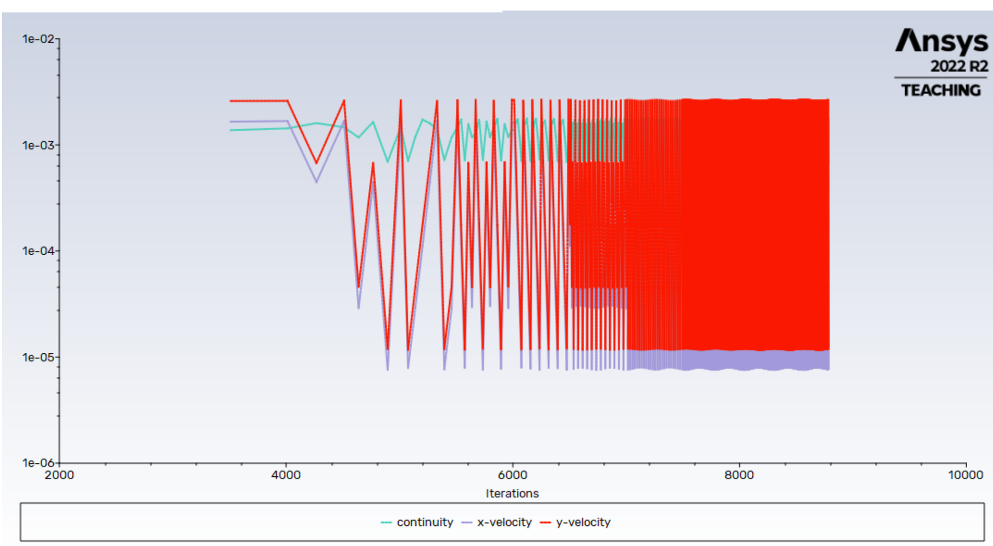

Figure 1: Residuals for Grid 3

2 Part II. Reference Simulation

2.1 (I)

Calculate the Mean Drag and Lift Coefficient for the cylinder. Drag and lift plots are shown in Figures 2 and 3. The mean drag force is

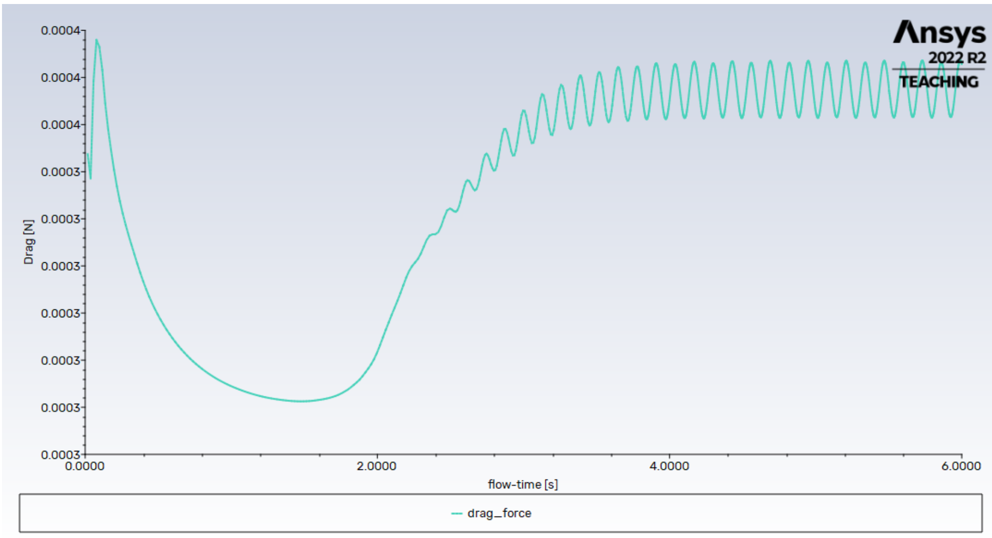

Figure 2: Drag force over time for Grid 3

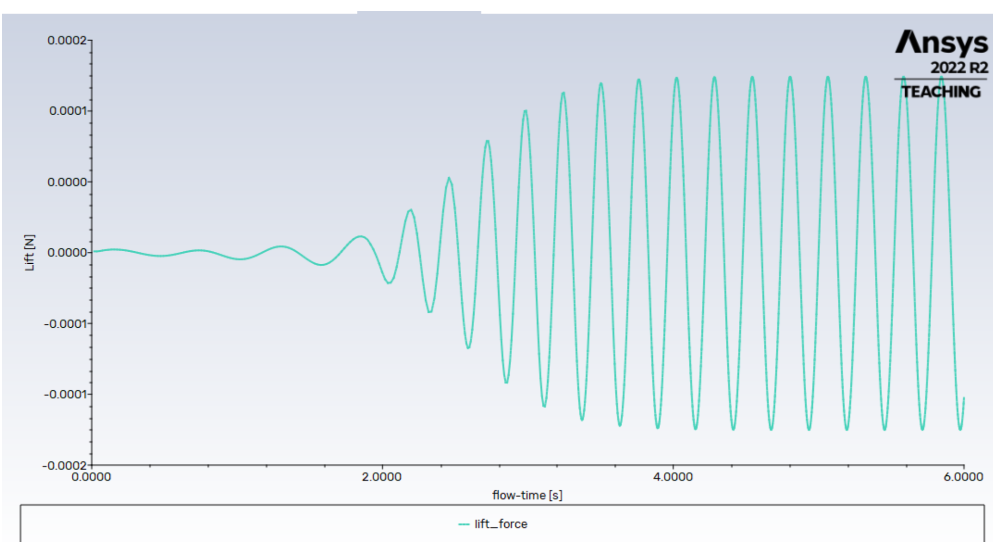

Figure 3: Lift force over time for Grid 3

The mean drag force can be found by averaging the peak-to-trough values of the force over one period of vortex shedding in the quasi steady state region. This value was found to be

$$\begin{aligned}
F_D &= \frac{F_{D_{\max}} + F_{D_{\min}}}{2} \\
&= \frac{3.64 \times 10^{-4} + 3.52 \times 10^{-4}}{2} \\
&= \boxed{3.58 \times 10^{-4}\ \mathrm{N}}
\end{aligned}$$

Similarly, the mean lift force was found to be

$$\begin{aligned}
F_L &= \frac{F_{L_{\max}} + F_{L_{\min}}}{2} \\
&= \frac{1.24 \times 10^{-4} - 1.25 \times 10^{-4}}{2} \\
&= \boxed{-4.40 \times 10^{-7}\ \mathrm{N}}
\end{aligned}$$

The mean drag coefficient is then

$$\begin{aligned}
C_d &= \frac{F_D}{0.5\rho U_\infty^2 D} \\
&= \frac{3.58 \times 10^{-4}}{0.5 \times 1.184 \times 0.20^2 \times 0.01} \\
&= \boxed{1.512}
\end{aligned}$$

The mean lift coefficient is

$$\begin{aligned}
C_l &= \frac{F_L}{0.5\rho U_\infty^2 D} \\
&= \frac{-4.40 \times 10^{-7}}{0.5 \times 1.184 \times 0.20^2 \times 0.01} \\
&= \boxed{-0.00186}
\end{aligned}$$

2.2 (II)

Plot the monitor point data (Pressure and Velocity Components) at P1 as a function of time. What is the period of vortex shedding from these plots? What is the normalized frequency of vortex shedding or the Strouhal number (St)?

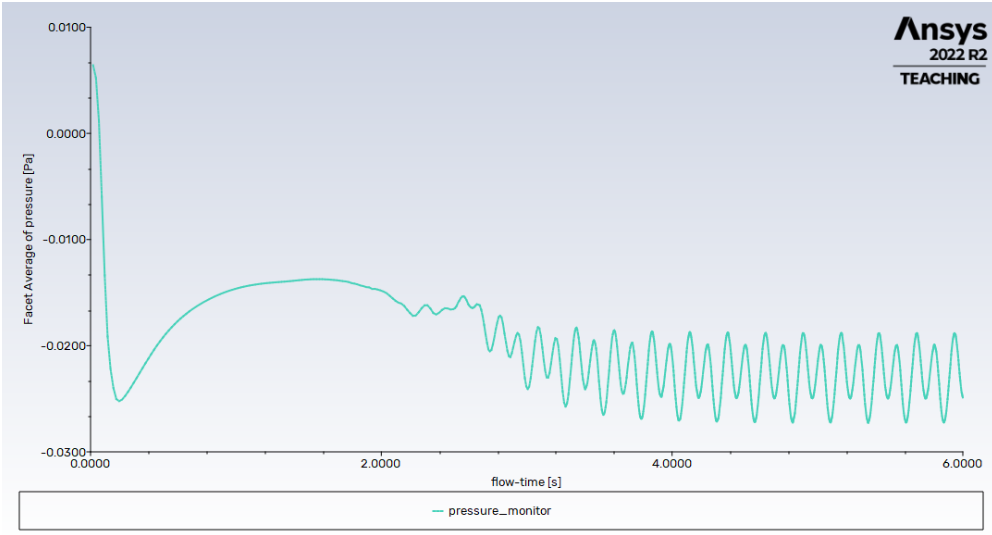

Figure 4: Pressure at P1 over time for Grid 3

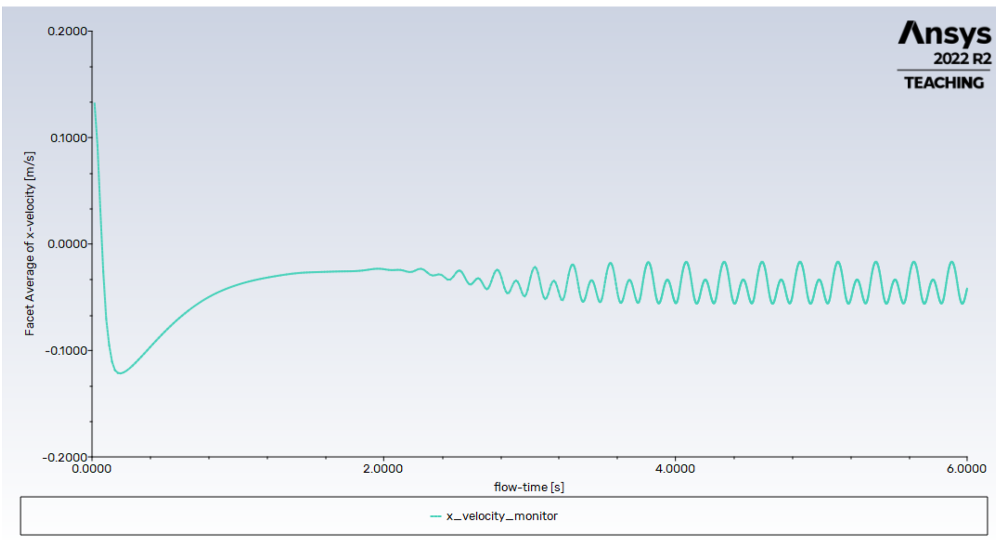

Figure 5: Streamwise velocity at P1 over time for Grid 3

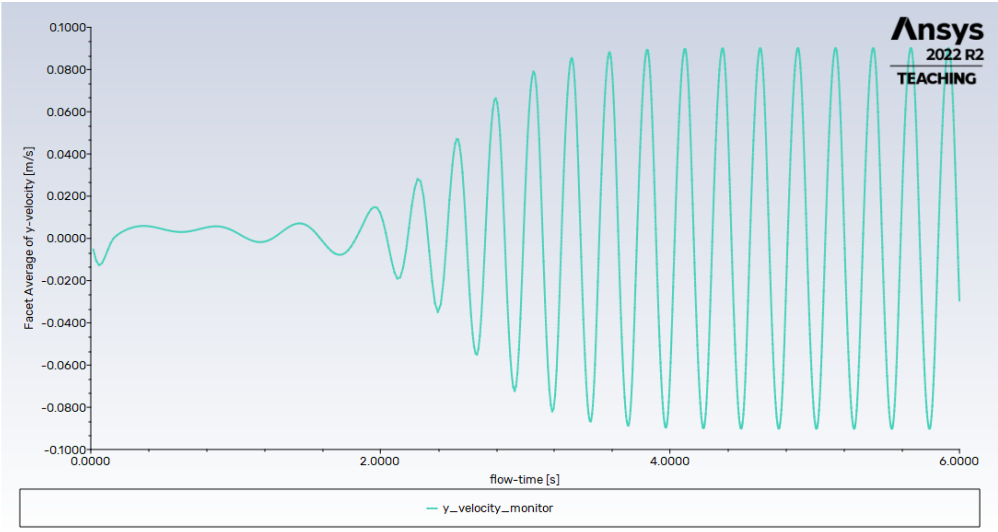

Figure 6: Spanwise velocity at P1 over time for Grid 3

The period of vortex shedding is obtained from analyzing the y-plot in the quasi steady state region $t > 3.5$ s. The period is approximately

$$\tau = 2(t_{\text{peak}} - t_{\text{trough}}) \qquad = 2(5.975 - 5.845)$$
$$= \boxed{0.265}\ \text{s}$$

The Strouhal number is then

$$\begin{aligned}\text{St} &= \frac{fD}{U_\infty} \\ &= \frac{1/0.265 \times 0.01}{0.20} \\ &= \boxed{0.1887}\end{aligned}$$

2.3 (III)

Create a contour plot of pressure (p) on the xy-plane. Adjust the values of maximum and minimum pressure so that the vortex shedding is distinguishable from the flow.

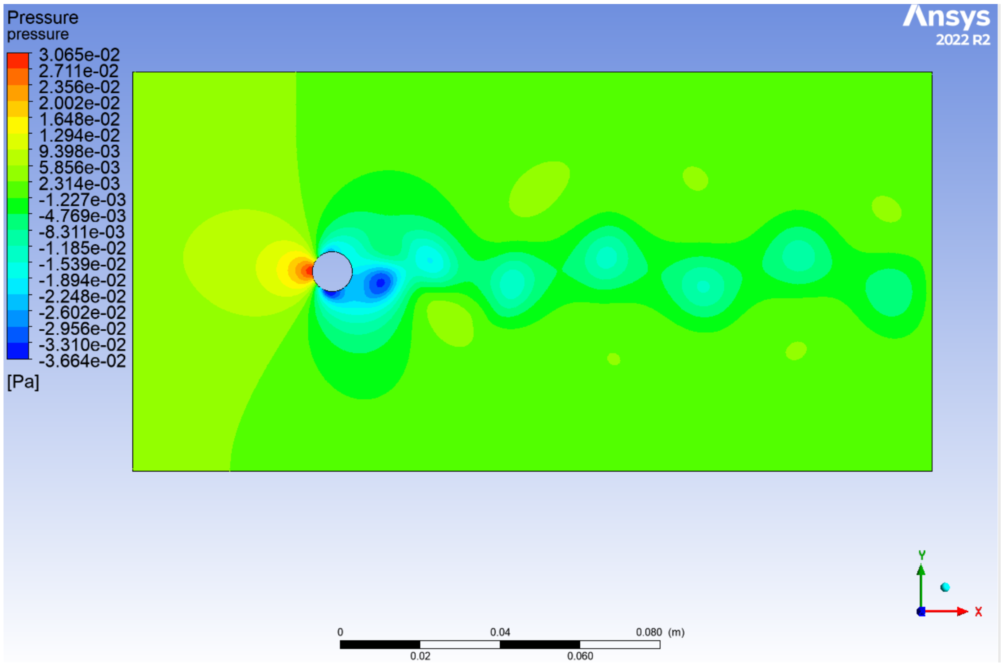

Figure 7: Pressure Contour for Grid 3

2.4 (IV)

Create a contour plot of spanwise vorticity (ω_z) on the xy-plane. Adjust the values of maximum and minimum vorticity so that the vortex shedding is distinguishable from the flow.

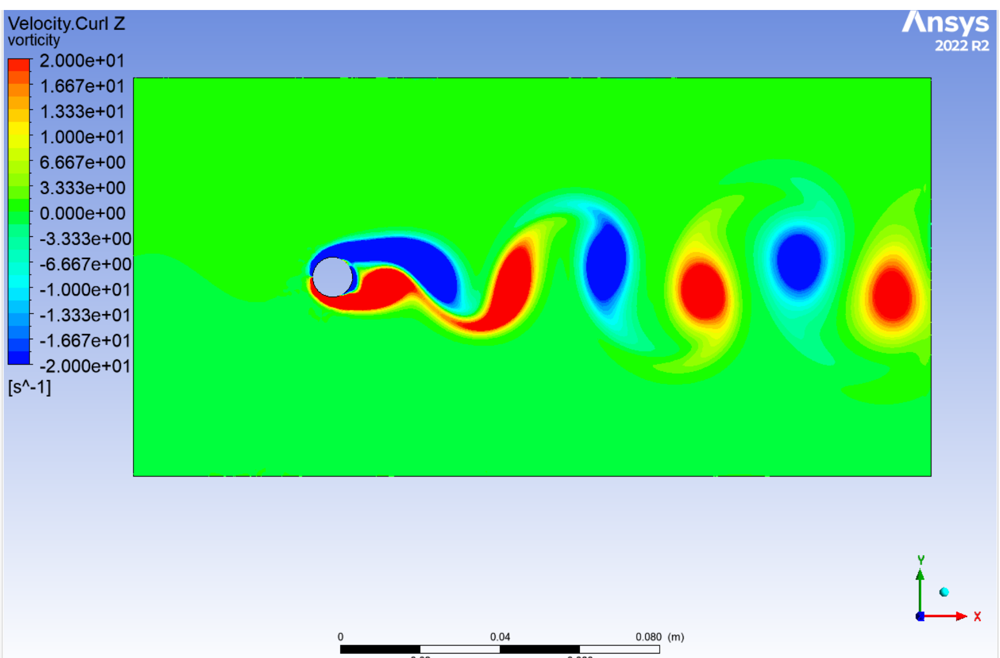

Figure 8: Vorticity Contour for Grid 3

2.5 (V)

Draw the plot of mean streamwise (x-direction) velocity (u) along the wake centerline and calculate the normalized value of L_r, which is the mean recirculation length.

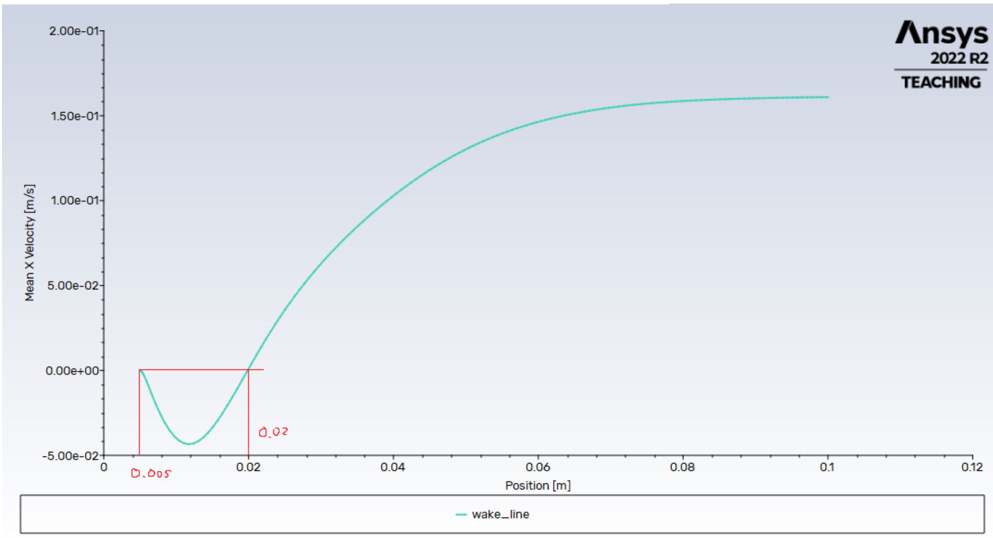

Figure 9: Mean Streamwise Velocity for Grid 3

From the plot, the recirculation length is approximately

$$\begin{aligned}\hat{L}_r &= \frac{\Delta x}{D} \\ &= \frac{0.02 - 0.005}{0.01} \\ &= \boxed{1.5}\end{aligned}$$

2.6 (VI)

Create a contour plot of mean pressure (p) on the xy-plane. Adjust the values of maximum and minimum pressure so that the recirculation vortex is distinguishable from the flow.

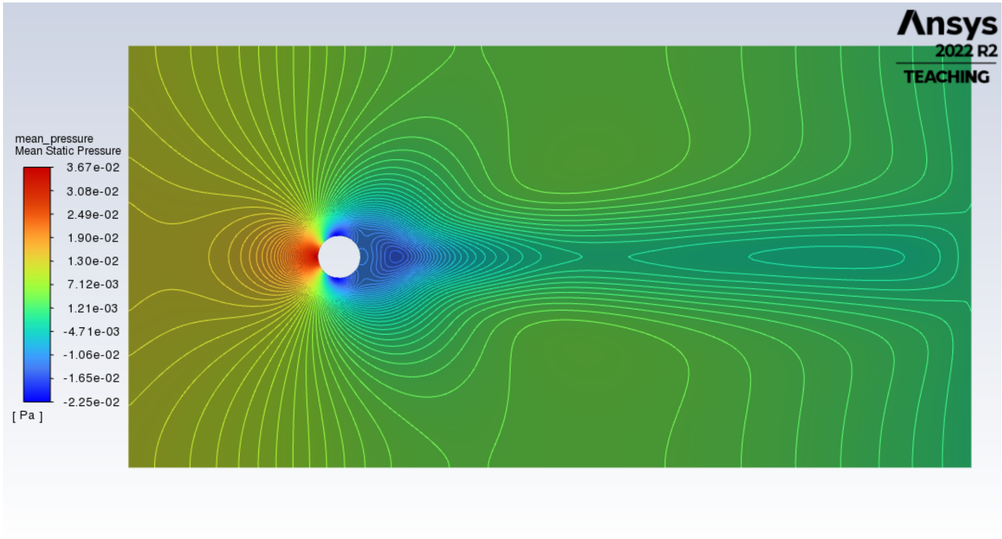

Figure 10: Mean Pressure Contour for Grid 3

3 Part III. Timestep Sensitivity Analysis

Based on the setup from PART I, repeat the simulation after changing the timestep value to $\delta t = 0.025$ *s and* $\delta t = 0.25$ *s, which should be labeled as "Grid 3-2" and "Grid 3-3", respectively.*

3.1 Task 3.1

Once the simulations are complete, proceed to ANSYS CFD-Post for post-processing of the data. Re-generate the plots and calculate the parameters as listed in PART II.

3.1.1 Grid 3-2

The number of iterations has been updated to

$$\begin{aligned}
\text{Iterations} &= \frac{T_{\text{total}}}{\Delta t} \\
&= \frac{10}{0.025} \\
&= \boxed{400}
\end{aligned}$$

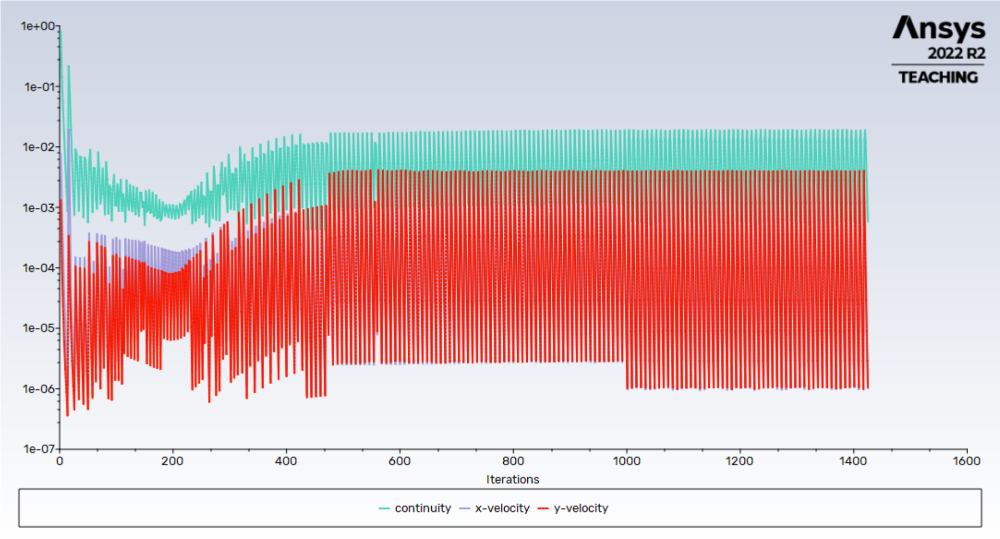

Figure 11: Residuals for Grid 3-2

The mean and drag plots are shown in Figures 12 and 13.

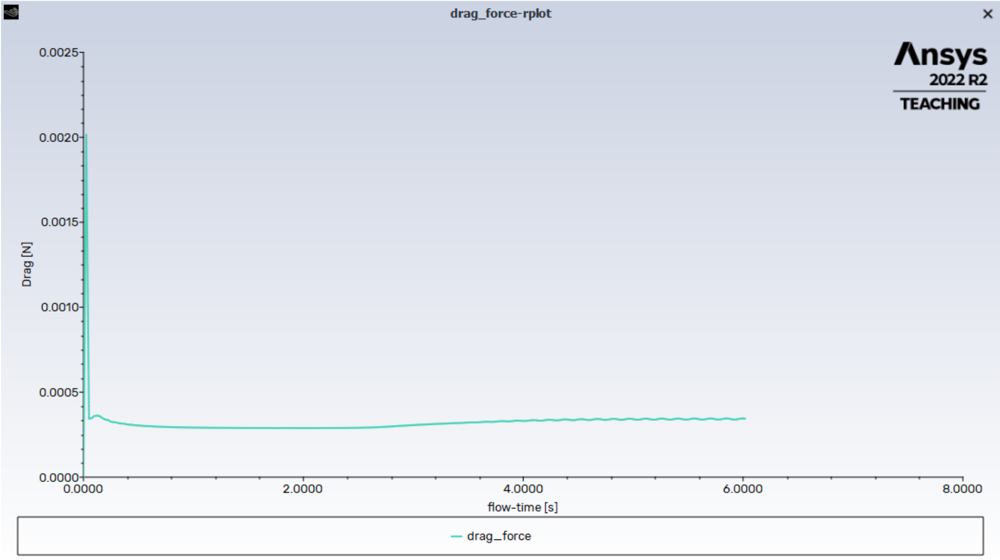

Figure 12: Drag force over time for Grid 3-2

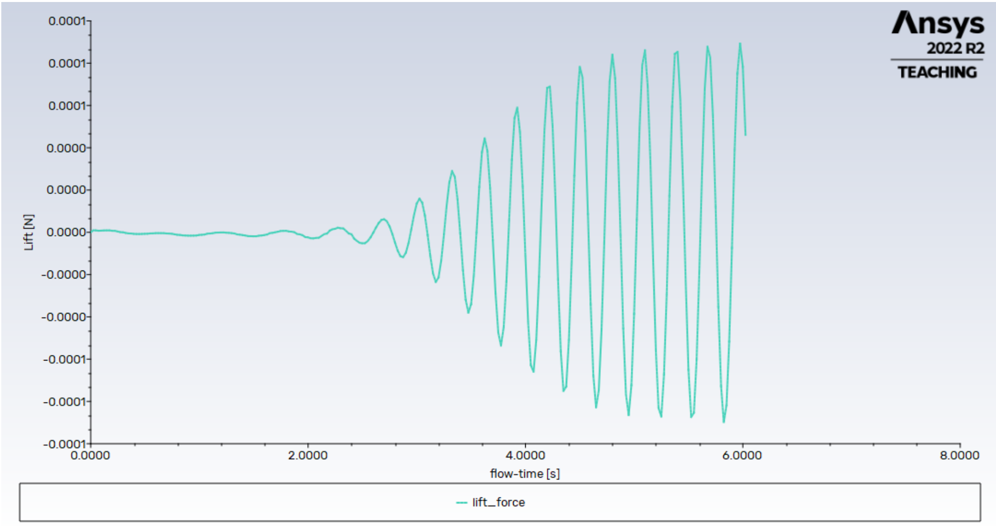

Figure 13: Lift force over time for Grid 3-2

The mean drag force is

$$
\begin{aligned}
F_D &= \frac{F_{D_{\max}} + F_{D_{\min}}}{2} \\
&= \frac{3.46 \times 10^{-4} + 3.38 \times 10^{-4}}{2} \\
&= \boxed{3.42 \times 10^{-4} \text{ N}}
\end{aligned}
$$

The mean lift force is

$$
\begin{aligned}
F_L &= \frac{F_{L_{\max}} + F_{L_{\min}}}{2} \\
&= \frac{8.92 \times 10^{-5} - 8.98 \times 10^{-5}}{2} \\
&= \boxed{-2.84 \times 10^{-7} \text{ N}}
\end{aligned}
$$

Then, the mean drag coefficient is

$$
\begin{aligned}
C_d &= \frac{F_D}{0.5 \rho U_\infty^2 D} \\
&= \frac{3.42 \times 10^{-4}}{0.5 \times 1.184 \times 0.20^2 \times 0.01} \\
&= \boxed{1.444}
\end{aligned}
$$

The mean lift coefficient is

$$
\begin{aligned}
C_l &= \frac{F_L}{0.5 \rho U_\infty^2 D} \\
&= \frac{-2.84 \times 10^{-7}}{0.5 \times 1.184 \times 0.20^2 \times 0.01} \\
&= \boxed{-0.00112}
\end{aligned}
$$

The pressure and velocity plots are shown in Figures 14, 15, and 16.

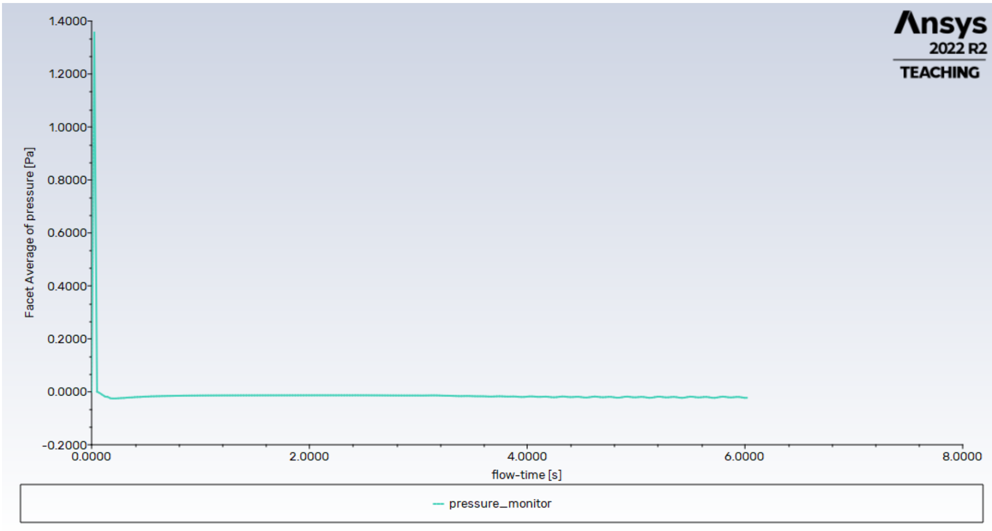

Figure 14: Pressure at P1 over time for Grid 3-2

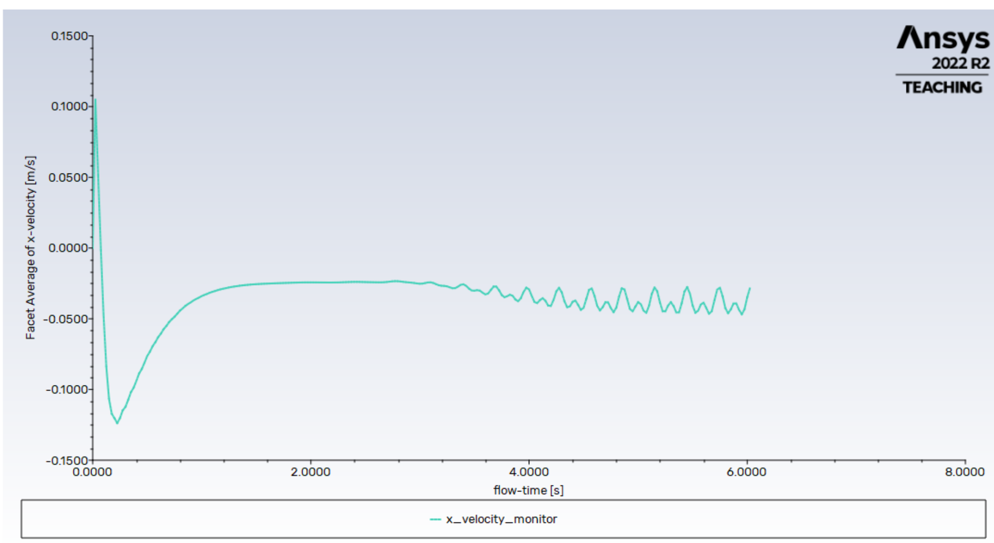

Figure 15: Streamwise velocity at P1 over time for Grid 3-2

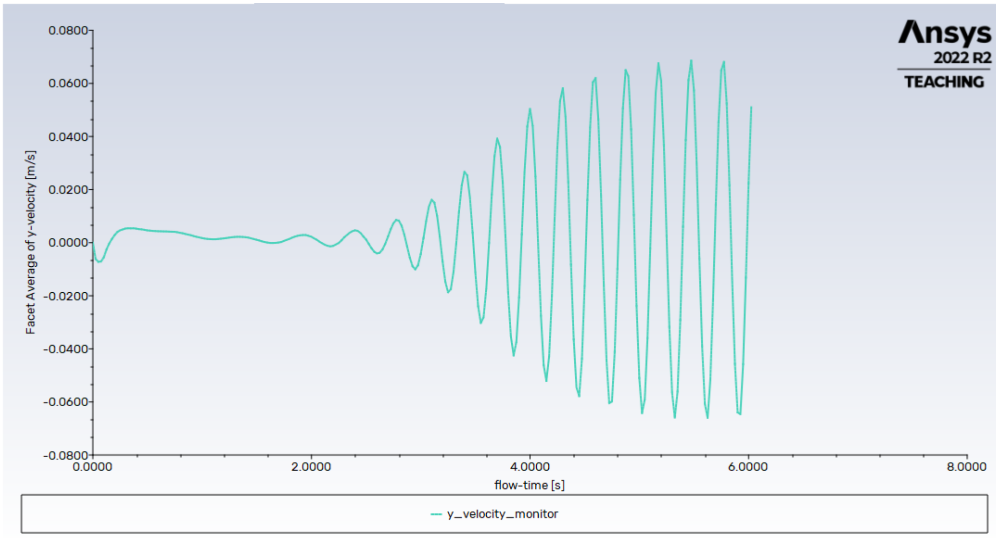

Figure 16: Spanwise velocity at P1 over time for Grid 3-2

The period of vortex shedding is approximated from the y-velocity plot to be

$$\tau = 2(t_{\text{peak}} - t_{\text{trough}}) = 2(5.85 - 5.625) = 0.45 \text{ s}$$

The Strouhal number is then

$$\begin{aligned} \text{St} &= \frac{fD}{U_\infty} \\ &= \frac{1/0.45 \times 0.01}{0.20} \\ &= \boxed{0.1111} \end{aligned}$$

The pressure and vorticity contours are shown in Figures 17 and 18.

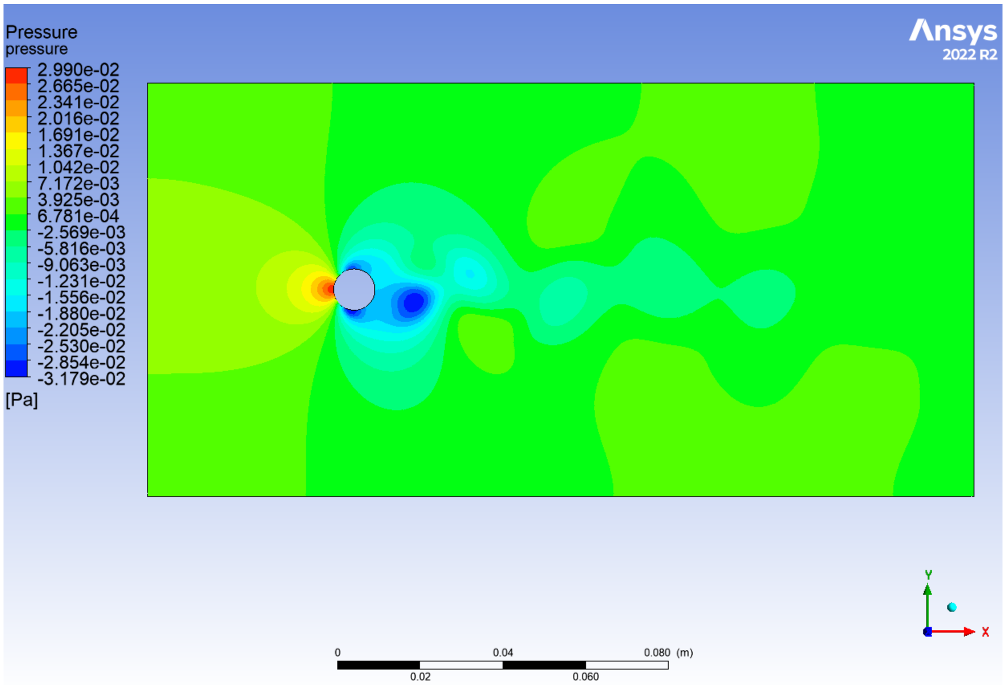

Figure 17: Pressure Contour for Grid 3-2

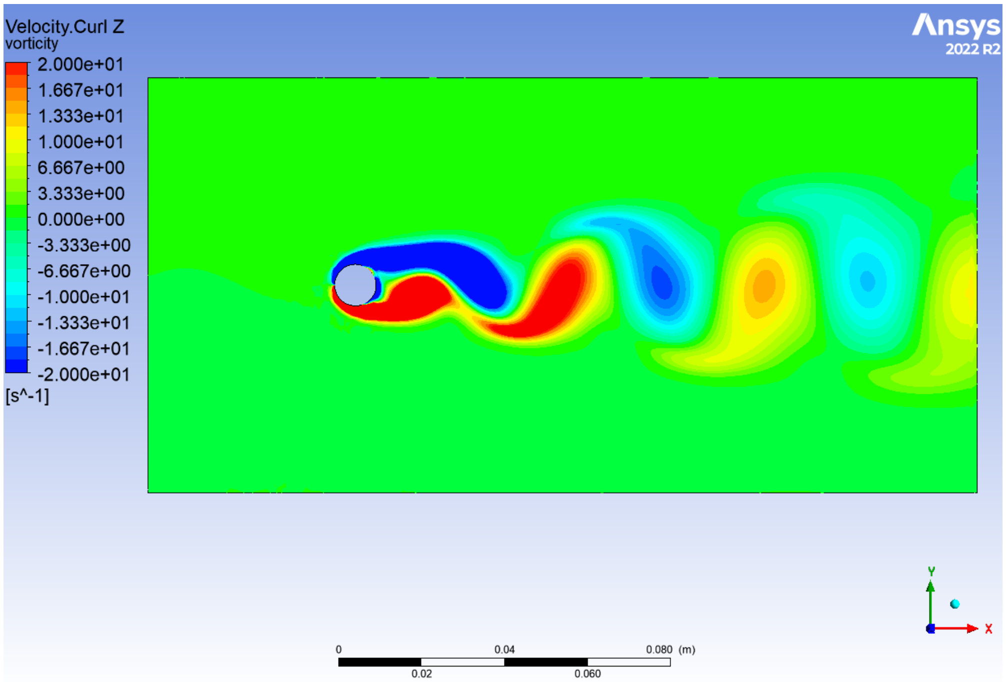

Figure 18: Vorticity Contour for Grid 3-2

The mean streamwise velocity and mean pressure contours are shown in Figures 19 and 20.

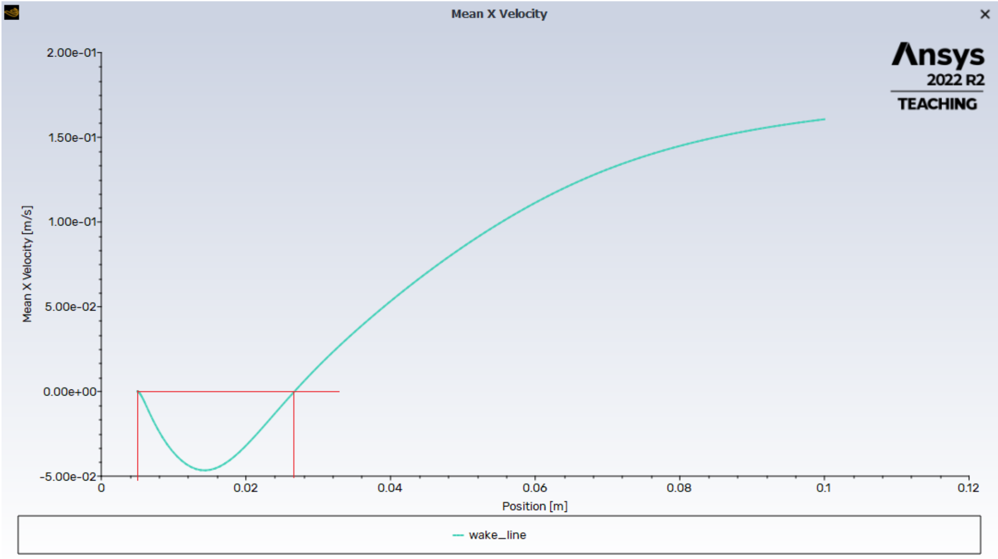

Figure 19: Mean Streamwise Velocity for Grid 3-2

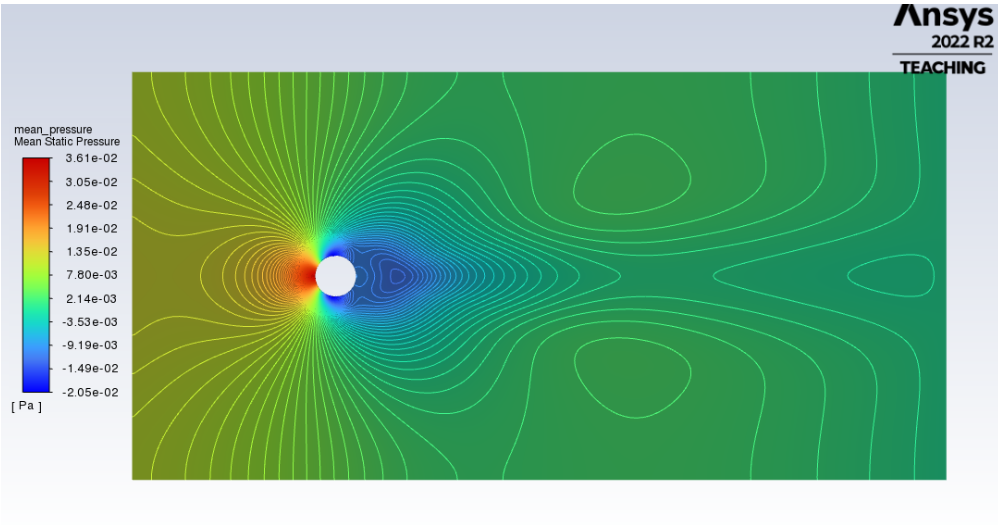

Figure 20: Mean Pressure Contour for Grid 3-2

The recirculation length is approximately

$$\begin{aligned}\hat{L}_r &= \frac{\Delta x}{D} \\ &= \frac{0.026 - 0.005}{0.01} \\ &= \boxed{2.1}\end{aligned}$$

3.1.2 Grid 3-3

The number of iterations has been updated to

$$\begin{aligned}\text{Iterations} &= \frac{T_{\text{total}}}{\Delta t} \\ &= \frac{10}{0.25} \\ &= \boxed{40}\end{aligned}$$

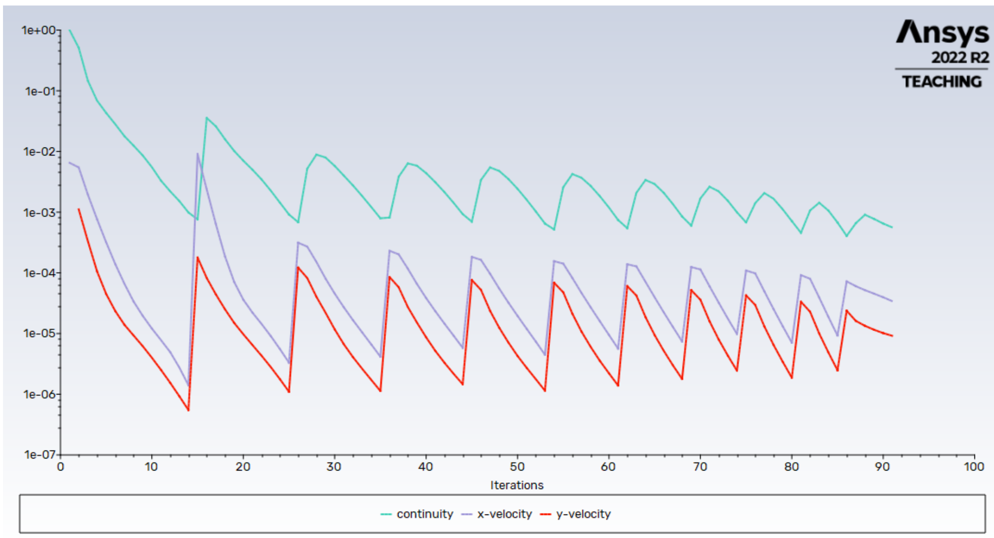

Figure 21: Residuals for Grid 3-3

The mean and drag plots are shown in Figures 22 and 23.

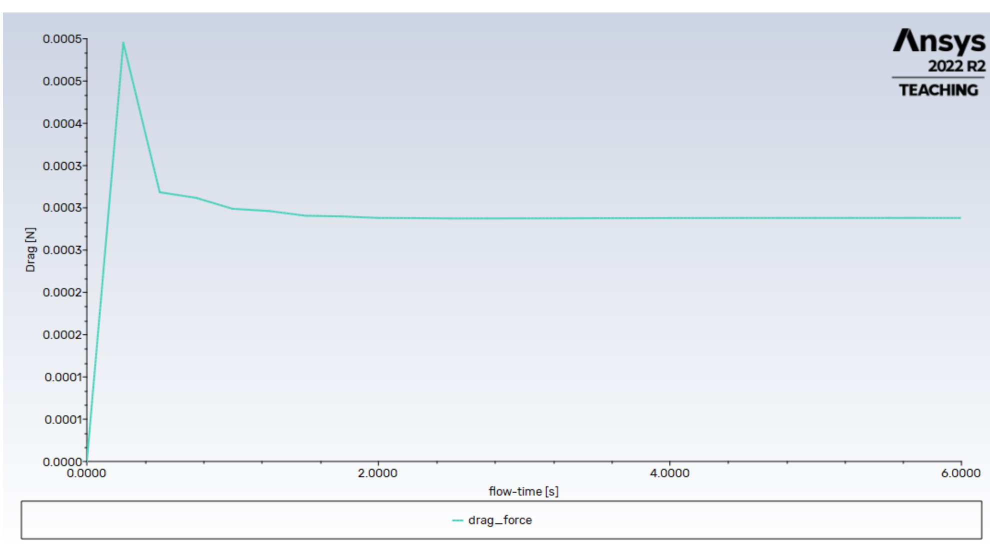

Figure 22: Drag force over time for Grid 3-3

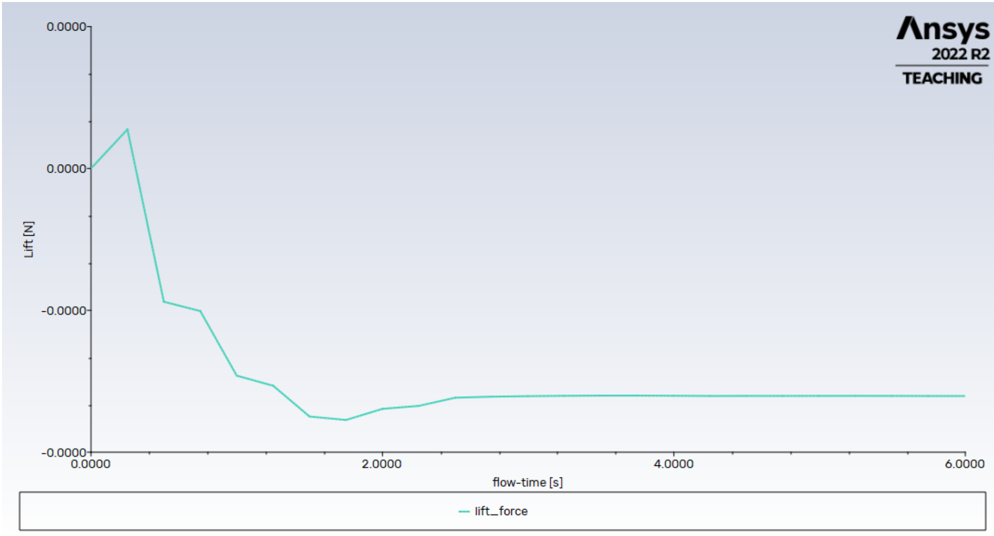

Figure 23: Lift force over time for Grid 3-3

The mean drag force is

$$\begin{aligned} F_D &= \frac{F_{D_{\max}} + F_{D_{\min}}}{2} \\ &= \frac{2.88 \times 10^{-4} + 2.88 \times 10^{-4}}{2} \\ &= \boxed{2.88 \times 10^{-4} \text{ N}} \end{aligned}$$

The mean lift force is

$$\begin{aligned} F_L &= \frac{F_{L_{\max}} + F_{L_{\min}}}{2} \\ &= \frac{-8.02 \times 10^{-7} - 8.02 \times 10^{-7}}{2} \\ &= \boxed{-8.02 \times 10^{-7} \text{ N}} \end{aligned}$$

Then, the mean drag coefficient is

$$\begin{aligned} C_d &= \frac{F_D}{0.5 \rho U_\infty^2 D} \\ &= \frac{2.88 \times 10^{-4}}{0.5 \times 1.184 \times 0.20^2 \times 0.01} \\ &= \boxed{1.216} \end{aligned}$$

The mean lift coefficient is

$$\begin{aligned} C_l &= \frac{F_L}{0.5 \rho U_\infty^2 D} \\ &= \frac{-8.02 \times 10^{-7}}{0.5 \times 1.184 \times 0.20^2 \times 0.01} \\ &= \boxed{-3.39 \times 10^{-3}} \end{aligned}$$

The pressure and velocity plots are shown in Figures 24, 25, and 26.

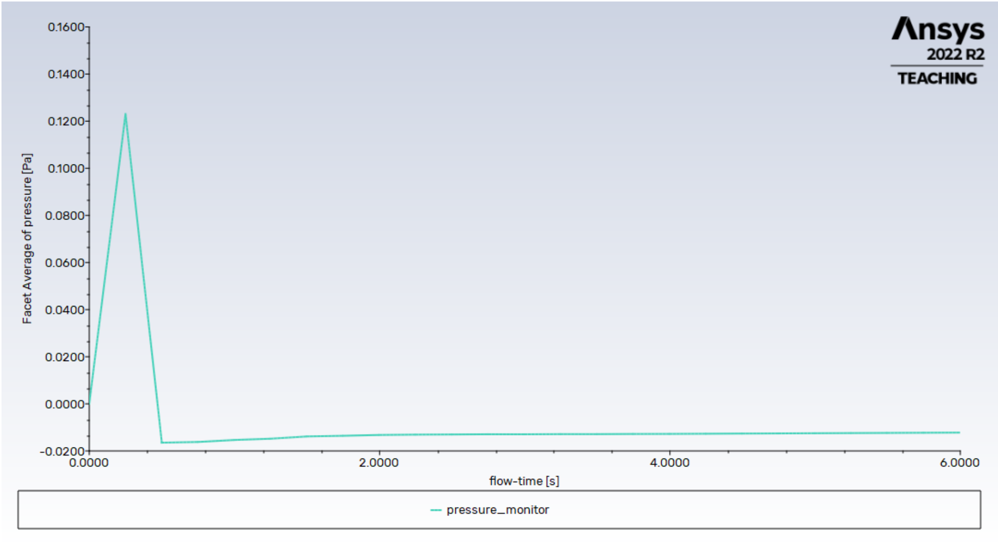

Figure 24: Pressure at P1 over time for Grid 3-3

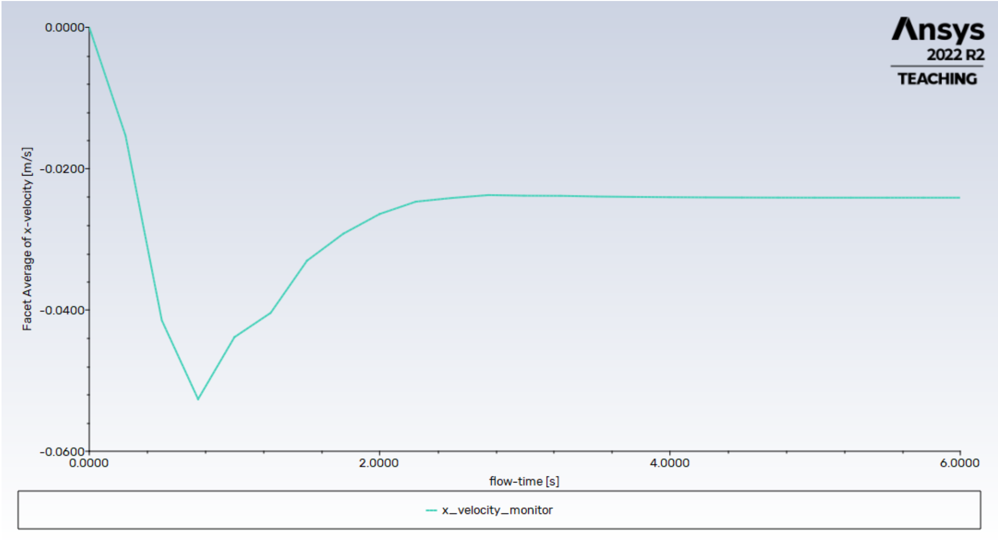

Figure 25: Streamwise velocity at P1 over time for Grid 3-3

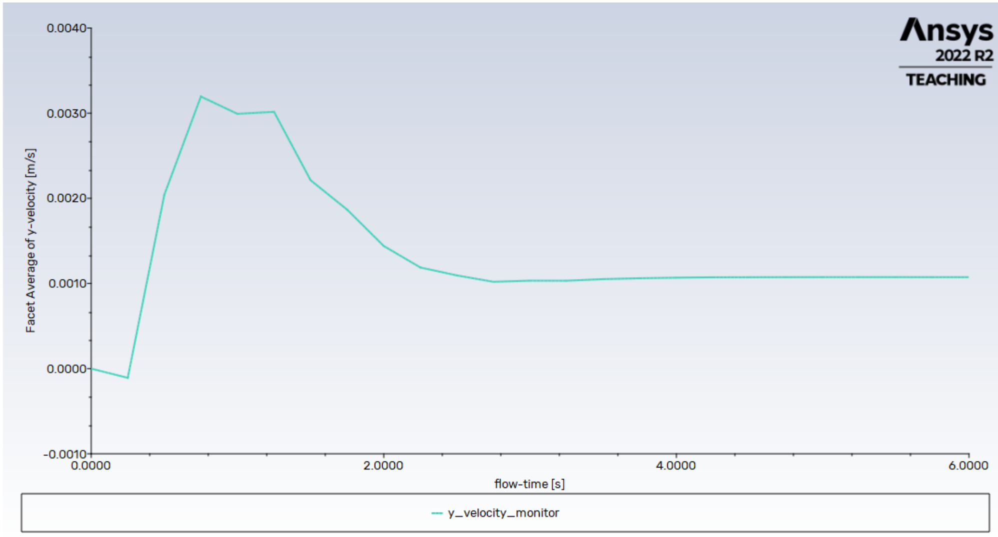

Figure 26: Spanwise velocity at P1 over time for Grid 3-3

The period of vortex shedding in the y-velocity plot could not be determined due to the lack of a clear quasi-steady state region. Defining period as 2, the duration of the steady region, the Strouhal number is then

$$\begin{aligned} \mathrm{St} &= \frac{fD}{U_\infty} \\ &= \frac{1/2 \times 0.01}{0.20} \\ &= \boxed{0.025} \end{aligned}$$

The pressure and vorticity contours are shown in Figures 27 and 28.

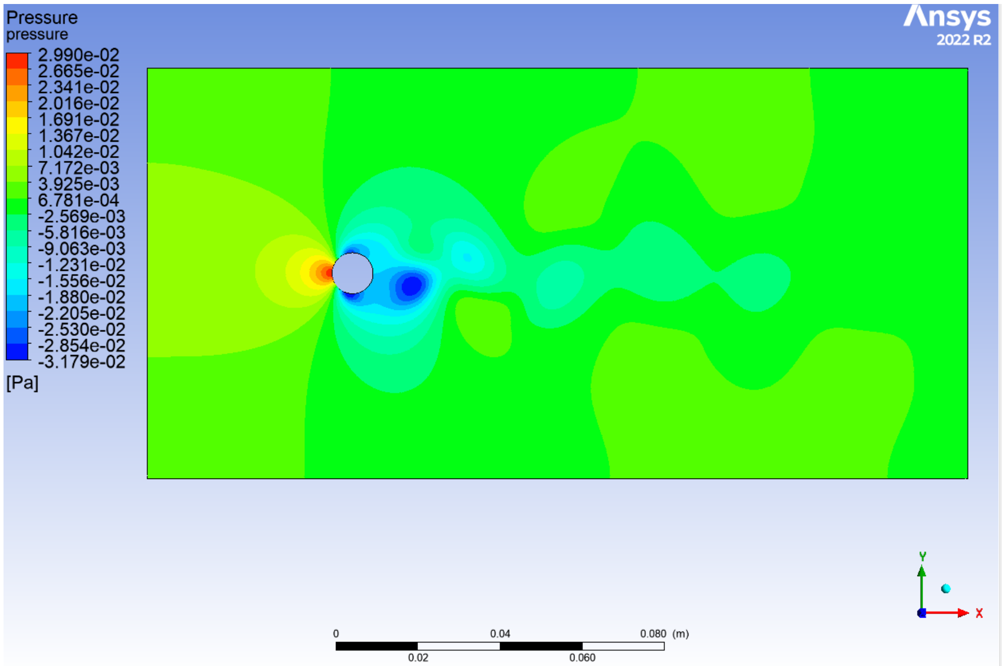

Figure 27: Pressure Contour for Grid 3-3

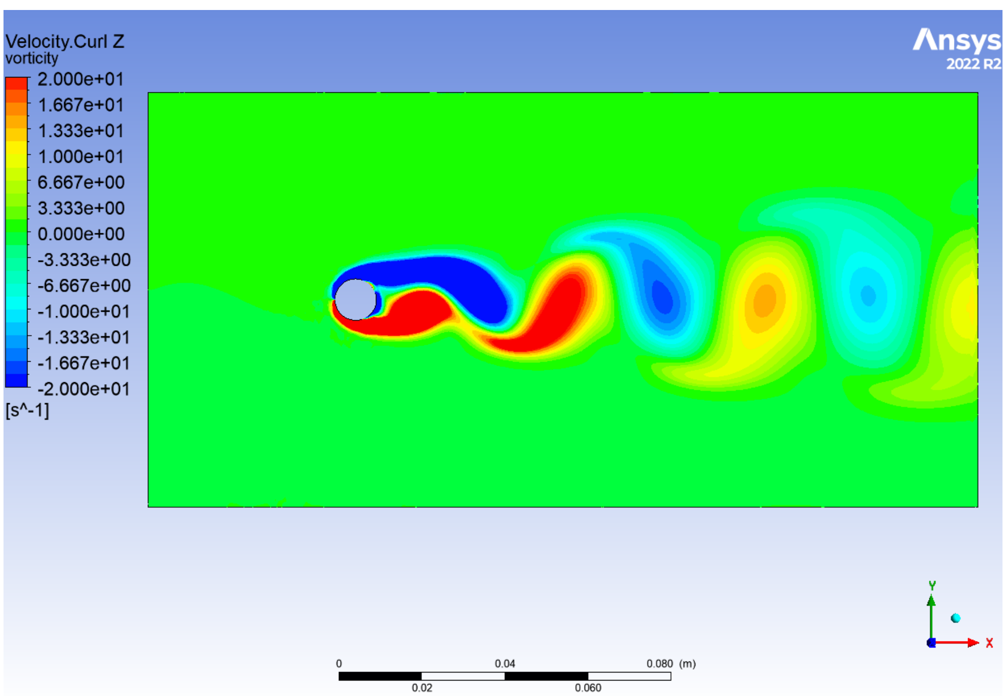

Figure 28: Vorticity Contour for Grid 3-3

The mean streamwise velocity and mean pressure contours are shown in Figures 29 and 30.

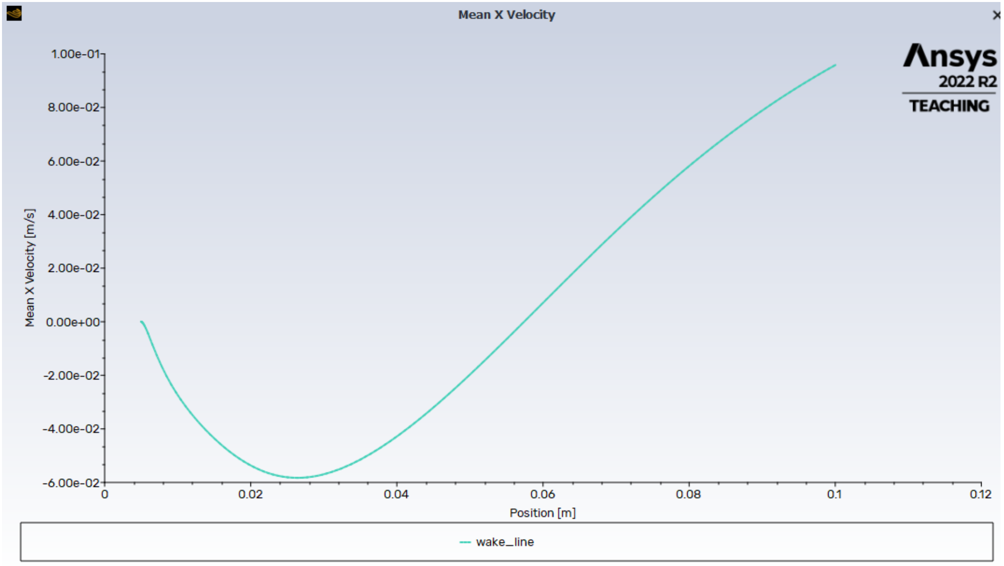

Figure 29: Mean Streamwise Velocity for Grid 3-3

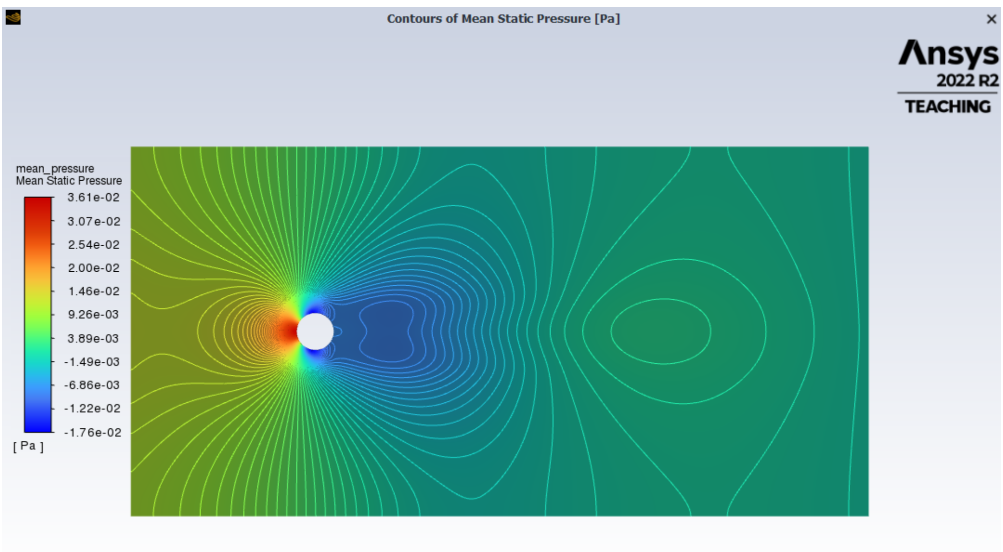

Figure 30: Mean Pressure Contour for Grid 3-3

The recirculation length is approximately

$$\begin{aligned}\hat{L}_r &= \frac{\Delta x}{D} \\ &= \frac{0.057 - 0.005}{0.01} \\ &= \boxed{5.2}\end{aligned}$$

3.2 Task 3.2

Create a table that shows the main wake parameters (Mean Recirculation Length, Mean Drag Coefficient, Mean Lift Coefficient, and St) you calculated for "Grid 3", "Grid 3-2" and "Grid 3-3". Include their percentage change with respect to those of "Grid 3". Also, compare the contour plots of pressure and mean pressure for the three cases.

The results are summarized in Table 1.

Table 1: Summary of Results for Grids 3, 3-2, and 3-3

Grid	C_d	C_l	$\hat{L}_r$	St	$\Delta C_d\%$	$\Delta C_l\%$	$\Delta \hat{L}_r\%$	ΔSt%
3	1.512	-0.00186	1.5	0.1887	-	-	-	-
3-2	1.444	-0.00112	2.1	0.1111	-4.497	-39.785	40	-40
3-3	1.216	-0.00339	5.2	0.025	-19.577	82.258	246.667	-86.957

Refering to Figures 7, 17 and 27, the vortex shedding is most distinguishable in Grid 3. Some detail is lost in Grid 3-2, and the vortex shedding is not visible in Grid 3-3. In fact, the pressure contour for Grid 3-3 almost looks like a mean pressure contour.

Refering to Figures 10, 20 and 30, the grids look very similar. The detail of the pressure contours have been smoothed out by taking the mean.

3.3 Task 3.3

Comment on the differences between the three temporal grids. Which case should be appropriate for wake analysis? Why?

For wake analysis, Grid 3 is deemed the most suitable option. Its smaller timestep effectively captures a greater level of detail in the unsteady flow characteristics. This is evident in the pressure and vorticity contours, which exhibit more detailed representations, particularly in displaying vortex shedding, compared to larger timesteps. Larger timesteps tend to emphasize the mean flow rather than the specific flow characteristics at a given location.

Of course, smaller timesteps will take longer to simulate. If time is a constraint, Grid 3-2 is a suitable compromise. It still captures the vortex shedding, but with less detail. Grid 3-3 is not suitable for wake analysis, as it does not capture the vortex shedding detail appropriately.

References

[1] Y. A. Cengel and J. M. Cimbala, *Fluid mechanics: Fundamentals and applications*. Columbus, OH: McGraw-Hill Education, 4 ed., Feb. 2017.