Question 1

Consider the transfer function

$$G(s) = \frac{s}{10s - 1}$$

Sketch the Bode plot of $G(s)$ by hand, using the Bode graph paper provided on eClass, then validate your answer using MATLAB's bode command. Please include both plots with your answer.

Solution

There are two factors, s and $10s - 1$. For $10s - 1$, $1/\tau = 0.1$.

By Matlab,

```
G = tf([1 0], [10 -1]);
bode(G);
```

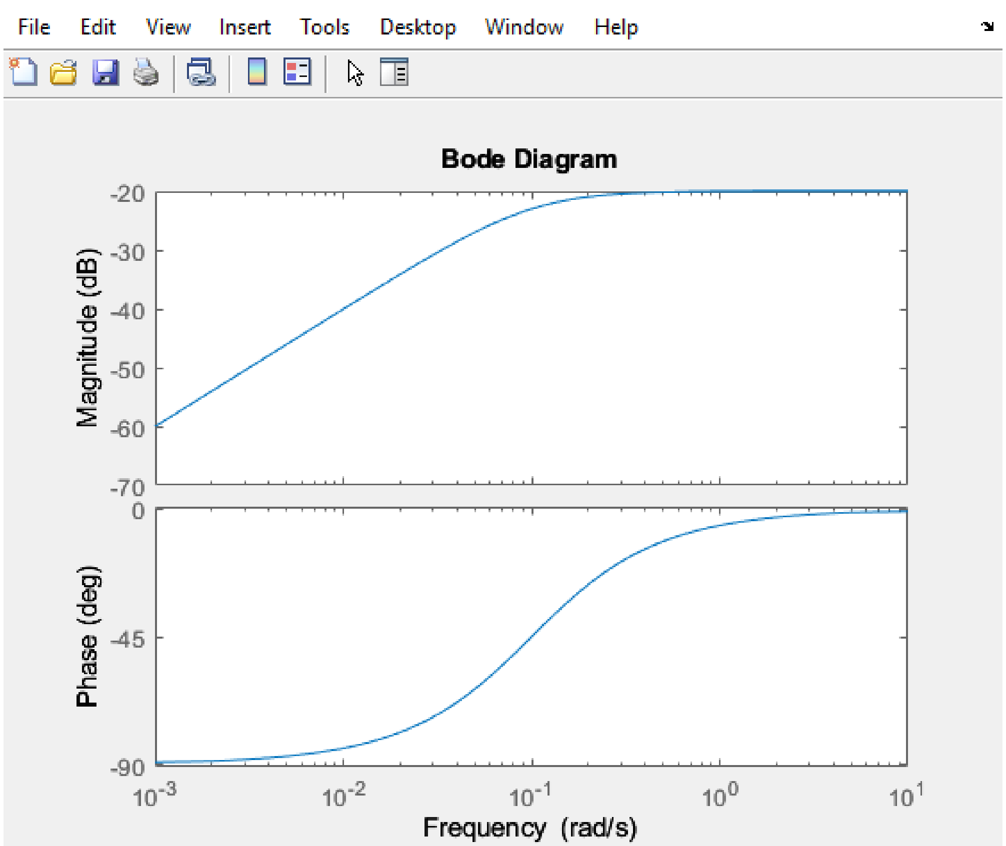
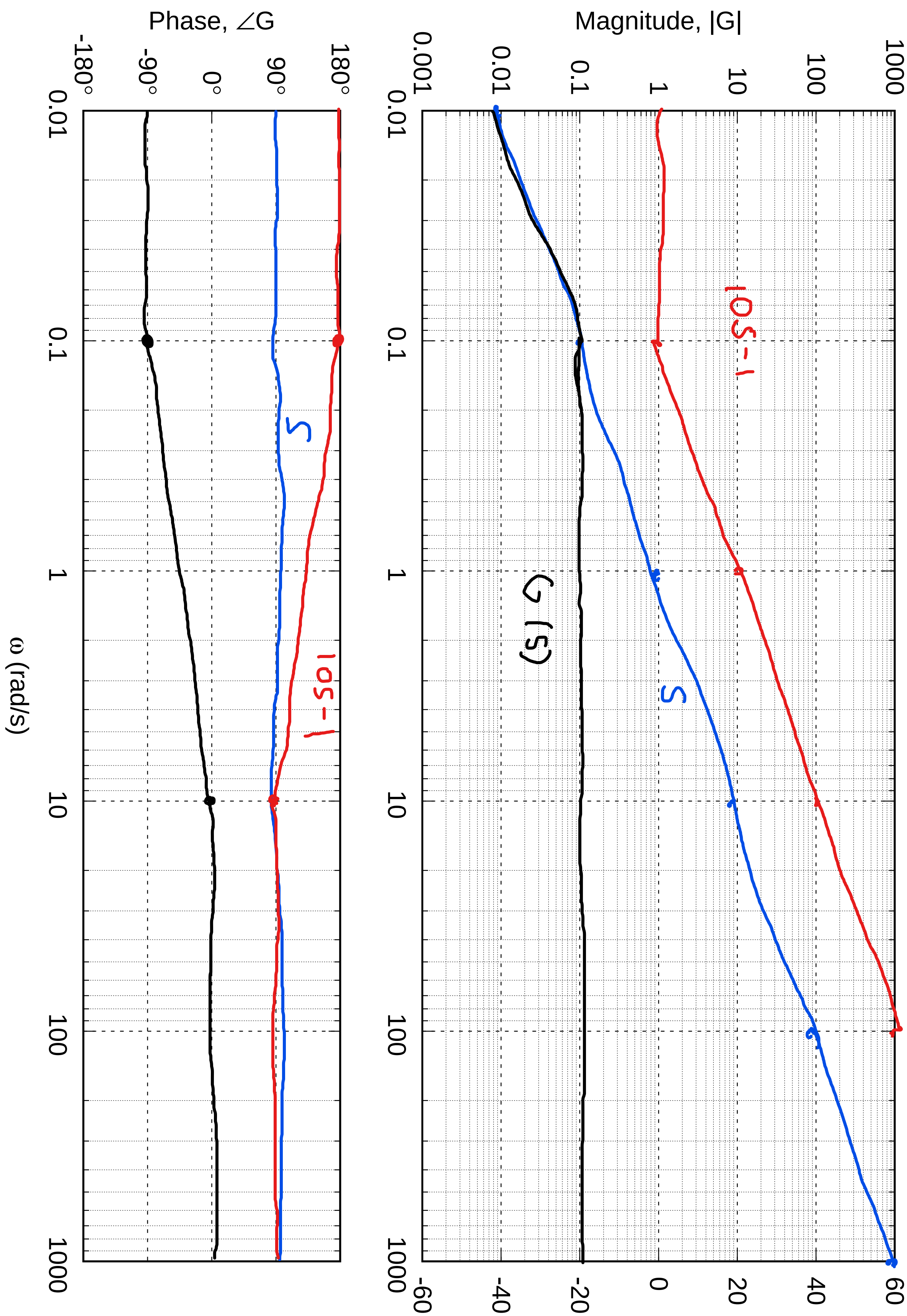


Figure 1: Bode plot of $G(s)$ using Matlab


Magnitude, |G|
Phase, ∠G
ω (rad/s)
10s-1
s
G(s)
0.001
0.01
0.1
1
10
100
1000
-60
-40
-20
0
20
40
60
-180°
-90°
0°
90°
180°

Question 2

Consider the following closed-loop system, copied from the previous assignment, which was shown to be closed-loop stable:

$$\begin{aligned}
L(s) &= \frac{10(s+1)}{s^2-4} \\
&= \frac{10(s+1)}{(s-2)(s+2)} \\
&= \frac{10(s+1)}{4(\frac{1}{2}s-1)(\frac{1}{2}s+1)} \\
&= \frac{2.5(s+1)}{(0.5s-1)(0.5s+1)}
\end{aligned}$$

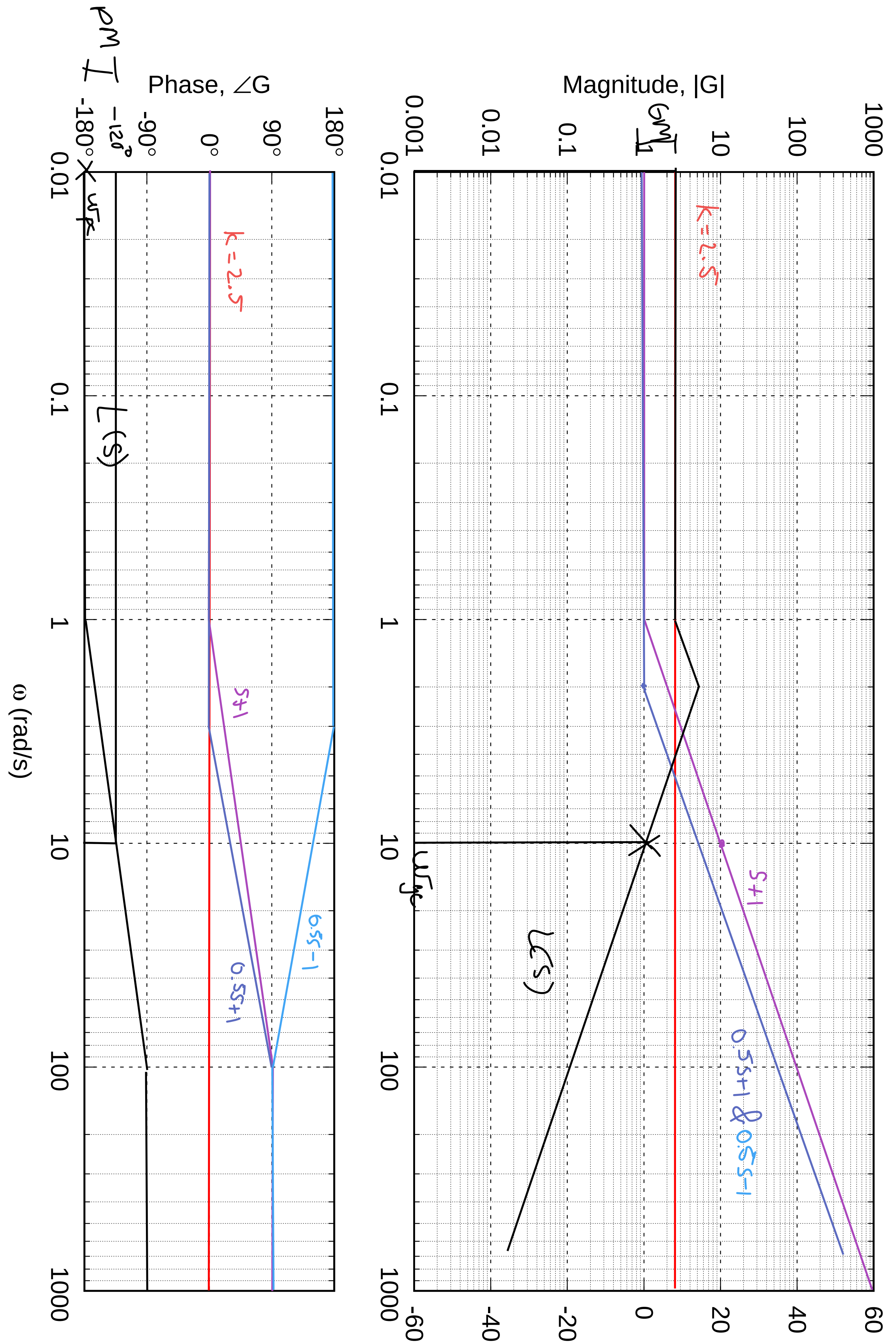
(a)

By hand, sketch the Bode diagram of $L(s)$ using the .pdf template on eClass There are 4 factors

- A gain of 2.5
- $s+1$, $\tau = 1$, $1/\tau = 1$
- $0.5s-1$, $\tau = 0.5$, $1/\tau = 2$
- $0.5s+1$, $\tau = 0.5$, $1/\tau = 2$

(b)

Label the locations of the gain crossover frequency ω_{gc}, the phase crossover frequency ω_{pc}, and use your sketch to read off the (approximate) GM and PM values for this design


Magnitude, |G|
Phase, ∠G
ω (rad/s)
K=2.5
s+1
0.5s+1 & 0.5s-1
0.5s-1
0.5s+1
L(s)
GM
PM
-120°
ω_{gc}
ω_{pc}

- $\omega_{gc} \approx 10$
- $\omega_{pc} = 0$
- $\mathrm{Gm} = 1/2.5 = 0.4$
- $\mathrm{Pm} \approx 60°$

(c)

Use MATLAB's margin command to validate your results from (a) and (b). Include a print-out of the resulting plot By Matlab,

```
syms s
L = 2.5*(s+1)/((0.5*s-1)*(0.5*s+1));
[n, d] = numden(L);
n = sym2poly(n);
d = sym2poly(d);
margin(tf(n, d));
```

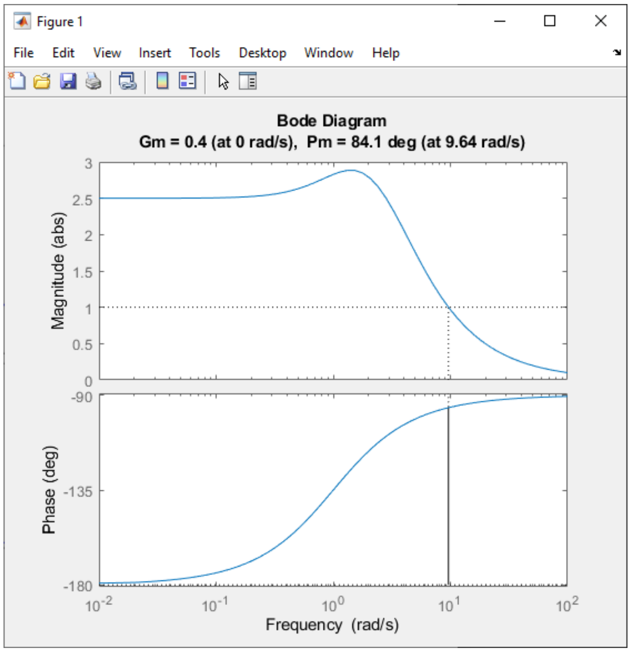


Figure 2: Margin plot of $L(s)$ using Matlab

From the plot,

- $\omega_{gc} = 9.64$
- $\omega_{pc} = 0$
- $\mathrm{Gm} = 0.4$
- $\mathrm{Pm} = 84.1°$

(d)

Using the values identified in (c), calculate the delay margin t_d^{max} for this closed-loop system

$$t_d^{\max} = \frac{\text{PM}}{\omega_{\text{gc}}} = \frac{84.1 \times \frac{\pi}{180}}{9.64} = 0.152 \text{ s}$$

(e)

Calculate the Nyquist Margin NM of this design (give the MATLAB commands you used)
By Matlab,

```
syms s
L = 2.5*(s+1)/((0.5*s-1)*(0.5*s+1));
[n, d] = numden(L);
n = sym2poly(n);
d = sym2poly(d);

bodemag(tf(n, d));
```

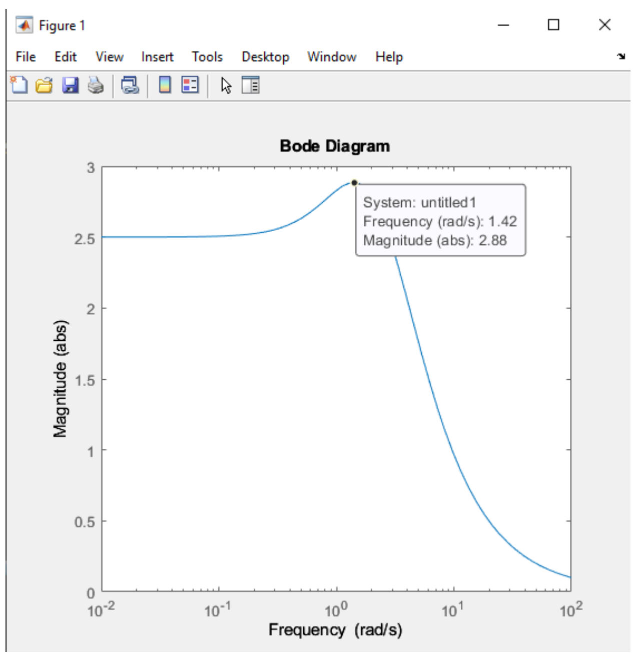


Figure 3: Bode magnitude plot of $L(s)$ using Matlab

On the plot, the peak is 2.88. Therefore, $\boxed{\text{NM} = 1/2.88 = 0.347}$.