Question 1

Integrate eq. (2) over the internal control volumes (2 - 3) using control volume 2 as a representative volume. Assume constant cross section value $S = 1$.

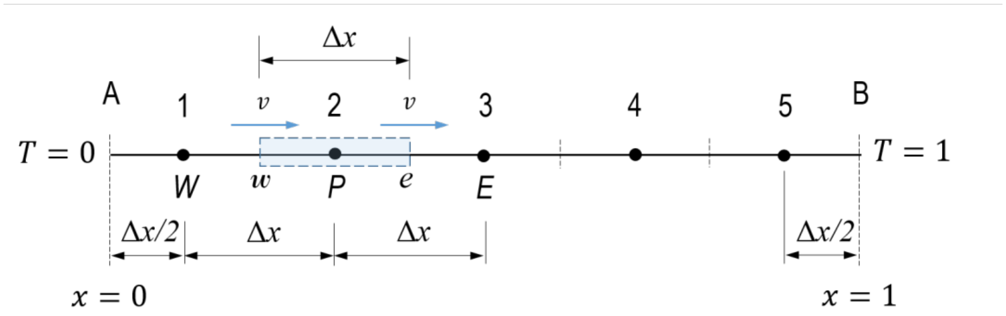

Figure 1: Simulation Domain Mesh

Eq. 2 is given by:

$$\frac{d\hat{T}}{dx} = \frac{d}{dx}\left(\frac{1}{\text{Pe}}\frac{d\hat{T}}{dx}\right)$$

Then integrating over the control volume,

$$\int_{\Delta x} \frac{d\hat{T}}{dx} S dx = \int_{\Delta x} \frac{d}{dx}\left(\frac{1}{\text{Pe}}\frac{d\hat{T}}{dx}\right) S dx$$
$$\hat{T}\Big|_w^e = \left(\frac{1}{\text{Pe}}\frac{d\hat{T}}{dx}\right)\Big|_w^e$$

Then,

$$\boxed{\underbrace{\hat{T}_e - \hat{T}_w}_{\text{Convective}} = \underbrace{\frac{1}{\text{Pe}}\left[\frac{d\hat{T}}{dx}\right]_e - \frac{1}{\text{Pe}}\left[\frac{d\hat{T}}{dx}\right]_w}_{\text{Diffusive}}}$$

Question 2

Approximate the convective term of the obtained equation using the upwind scheme. For upwind scheme with east as positive flow direction,

$$\hat{T}_w = \hat{T}_W$$
$$\hat{T}_e = \hat{T}_P$$

Then, the convective term becomes,

$$\begin{aligned}\text{Convective} &= \hat{T}_e - \hat{T}_w \\ &= \boxed{\hat{T}_P - \hat{T}_W}\end{aligned}$$

Question 3

Approximate the diffusive term of the equation using the central difference scheme.

For central difference scheme,

$$\begin{aligned}\left[\frac{d\hat{T}}{dx}\right]_e &= \frac{\hat{T}_E - \hat{T}_P}{\Delta x} \\ \left[\frac{d\hat{T}}{dx}\right]_w &= \frac{\hat{T}_P - \hat{T}_W}{\Delta x}\end{aligned}$$

Then, the diffusive term becomes,

$$\begin{aligned}\text{Diffusive} &= \frac{1}{\text{Pe}}\left[\frac{d\hat{T}}{dx}\right]_e - \frac{1}{\text{Pe}}\left[\frac{d\hat{T}}{dx}\right]_w \\ &= \frac{1}{\text{Pe}}\left(\frac{\hat{T}_E - \hat{T}_P}{\Delta x}\right) - \frac{1}{\text{Pe}}\left(\frac{\hat{T}_P - \hat{T}_W}{\Delta x}\right) \\ &= \boxed{\frac{1}{\text{Pe}}\left(\frac{\hat{T}_E - 2\hat{T}_P + \hat{T}_W}{\Delta x}\right)}\end{aligned}$$

Question 4

Substitute approximations obtained in Questions 2 and 3 back into equation derived in Question 1. Rearrange the obtained equation into a form that can be used to solve for $\hat{T}_P$.

$$a_P T_P = a_W T_W + a_E T_E$$

and identify coefficients a_P, a_W, a_E, q_T^u, and q_T^P.

Substituting back into the equation derived in Question 1,

$$\hat{T}_P - \hat{T}_W = \frac{1}{\text{Pe}}\left(\frac{\hat{T}_E - 2\hat{T}_P + \hat{T}_W}{\Delta x}\right)$$

Rearranging,

$$\underbrace{\left[\frac{2}{\text{Pe}\Delta x} + 1\right]}_{a_P}\hat{T}_P = \underbrace{\left[\frac{1}{\text{Pe}\Delta x} + 1\right]}_{a_W}\hat{T}_W + \underbrace{\left[\frac{1}{\text{Pe}\Delta x}\right]}_{a_E}\hat{T}_E + [0]q_T^u + [0]q_T^P$$

Question 5

Integrate eq. (2) over boundary control volume 1 and approximate the convective term using upwind scheme. Use central difference for approximation of the diffusive term. Assume constant cross section value $S = 1$.

Integrating Eq. 2, but over control volume 1,

$$\int_{\Delta x} \frac{d\hat{T}}{dx} S dx = \int_{\Delta x} \frac{d}{dx}\left(\frac{1}{\mathrm{Pe}}\frac{d\hat{T}}{dx}\right) S dx$$
$$\hat{T}\Big|_A^e = \left(\frac{1}{\mathrm{Pe}}\frac{d\hat{T}}{dx}\right)\Big|_A^e$$

Then,

$$\boxed{\underbrace{\hat{T}_e - \hat{T}_A}_{\text{Convective}} = \underbrace{\frac{1}{\mathrm{Pe}}\left[\frac{d\hat{T}}{dx}\right]_e - \frac{1}{\mathrm{Pe}}\left[\frac{d\hat{T}}{dx}\right]_A}_{\text{Diffusive}}}$$

For the convective term, upwind scheme is used,

$$\hat{T}_w = \hat{T}_A$$
$$\hat{T}_e = \hat{T}_P$$

Then,

$$\begin{aligned} \text{Convective} &= \hat{T}_e - \hat{T}_A \\ &= \hat{T}_P - \hat{T}_A \end{aligned}$$

For the diffusive term, a central difference scheme is used,

$$\left[\frac{d\hat{T}}{dx}\right]_e = \frac{\hat{T}_E - \hat{T}_P}{\Delta x}$$
$$\left[\frac{d\hat{T}}{dx}\right]_A = \frac{\hat{T}_P - \hat{T}_A}{\Delta x/2}$$

Substituting back into the equation derived,

$$\hat{T}_E - \hat{T}_P = \frac{1}{\mathrm{Pe}}\left(\frac{\hat{T}_E + \hat{T}_P - 2\hat{T}_A}{\Delta x}\right)$$