PHIL 120 - Winter 2025

Course Notes

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These are distilled notes taken from *forallx CALGARY An Introduction to Formal Logic* taught by Paolo Verdini.

Contents

Key Notions of Logic

Arguments

Definition 1. *A **sentence** is a statement that can be true or false.*

Example 1. The following are sentences:

1. The cat is on the mat.
2. Rhubarb is tasty.

The following are not sentences:

1. Are you sleepy yet?
2. Go to your room!
3. Ouch!

Definition 2. *An **argument** is a sequence of statements. One of these statements is the **conclusion** and the remaining statements are the **premises**.*

Example 2. Consider the following argument:

Premise 1: Either the butler or the cook did it.

Premise 2: The butler did not do it.

Conclusion: $\therefore$ The cook did it.

Definition 3. *A **premise indicator word** is a word that indicates that what follows is a premise of an argument. Some common premise indicator words include since, because, given that, as, for, seeing that, in as much as*

Definition 4. *A **conclusion indicator word** is a word that indicates that what follows is the conclusion of an argument. Some common conclusion indicator words include therefore, thus, so, hence, consequently, accordingnly*

The Scope of Logic

Definition 5. *A sentence $\mathcal{A}$ is a **consequence** of the set of sentences $\mathcal{B} = \{B_1, B_2, \ldots, B_n\}$ if and only if it is impossible for all of the sentences in $\mathcal{B}$ to be true while $\mathcal{A}$ is false.*

Definition 6. *An argument is **valid** or **deductively valid** if and only if the conclusion is a consequence of the premises.*
*An argument is **invalid** if and only if the conclusion is not a consequence of the premises.*

Example 3. Can a valid argument be made invalid by the addition of a new premise?

No. If an argument is valid, then by definition, it's impossible for the premises to be true and the conclusion false. This implies there is at least one presmise that is false when the conclusion is false. Thus, adding a new premise will not change the fact that not all premises can be true while the conclusion is false.

Definition 7. *A **counterexample** is an example that shows that an argument is invalid.*

Definition 8. *An argument is **nomologically valid** if the conclusion is a consequence of the premises in virtue of the laws of nature.*

Example 4. Consider the following argument:

Premise 1: The spaceship took six hours to reach Jupiter from Tycho space station

Conclusion: $\therefore$ The distance between Jupiter and Tycho space station is ≤ 6 light hours

This argument is nomologically valid.

Definition 9. *An argument is **conceptually valid** if there are no counterexamples that don't violate conceuptual connections between the premises words.*

Example 5. Consider the following argument:

Premise 1: Priya is an opthalmologist

Conclusion: $\therefore$ Priya is an eye doctor

This argument is conceptually valid.

An argument is **formally valid** if the conclusion is a consequence of the premises in virtue of the logical form of the argument.

Example 6. Consider the following argument:

Premise 1: $(A = X) \vee (A = Y)$

Premise 2: $A \neq Y$

Conclusion: $\therefore A = X$

Definition 10. *An argument is **sound** if and only if it is valid and all of its premises are true.*

Example 7. Consider the following argument:

Premise 1: Oranges are either fruit or musical instruments

Premise 2: Oranges are not fruit

Conclusion: $\therefore$ Oranges are musical instruments

This argument is valid but not sound since one of the premises is false.

Definition 11. *An argument is **inductive** if the premises are intended to provide probable support for the conclusion. They are not valid arguments.*

Example 8. Consider the following argument:

Premise 1: 90% of the students at the University of Calgary are from Alberta

Conclusion: $\therefore$ The next student I meet will be from Alberta

Other Logical Notions

Definition 12. *Sentences are **jointly possible** if and only if it is possible for all of them to be true at the same time.*

Example 9. Consider the following sentences:

1. The cat is on the mat.
2. The cat is not on the mat.

These sentences jointly impossible.

Remark 1. If a set of sentences is jointly impossible, then adding any sentence to the set will not make the set jointly possible.

Definition 13. *A sentence is **contingent** if and only if it is possible for it to be true and possible for it to be false.*

Definition 14. *A sentence is a **necessary truth** if and only if it is not possible for it to be false.*

Definition 15. *A sentence is a **necessary falsehood** if and only if it is not possible for it to be true.*

Remark 2. A sentence can *always* be true and still be contingent.

Example 10. Consider the following sentences:

1. The cat is on the mat.
2. Either the cat is on the mat or the cat is not on the mat.

The first sentence is contingent while the second sentence is a necessary truth.

Definition 16. *Two sentences are **necessarily equivalent** if and only if they have the same truth value in every possible case.*

Truth Functional Logic

First Steps to Symbolization

Definition 17. ***Validity in virtue of form*** *is when the form of the argument guarantees the truth of the conclusion.*

Example 11. Consider the following argument:

Premise 1: A

Premise 2: $A \to C$

Conclusion: $\therefore C$

This argument is valid in virtue of form.

Example 12. Consider the following argument:

Premise 1: Alice is a vixen

Conclusion: $\therefore$ Alice is a fox

This argument is *conceptually valid*, but not *valid in virtue of form.*

Connectives

Definition 18. *A sentence $\mathcal{A}$ can be* ***negated****, symbolized by $\neg\mathcal{A}$, if it can be paraphrased as "It is not the case that $\mathcal{A}$".*

Example 13. Consider the following sentences:

1. The widget can be replaced.
2. The widget is irreplacable.
3. The widget is not irreplacable.

Can be rewritten as:

1. R
2. $\neg R$
3. $\neg\neg R$

Definition 19. *A sentence $\mathcal{A}$ can form a **conjunction** with another sentence $\mathcal{B}$, symbolized by $\mathcal{A} \wedge \mathcal{B}$, if it can be paraphrased as "Both $\mathcal{A}$ and $\mathcal{B}$", where $\mathcal{A}$ and $\mathcal{B}$ are called **conjuncts**.*

*Conjunctions are **symmetric** (**commutative**), meaning the order of the conjuncts does not matter.*

Example 14. Consider the following sentences:

1. Alice is atheletic and energetic.
2. Although Alice is energetic, she is not atheletic.
3. Bob is athletic, but Alice is more athletic than him.

Can be rewritten as:

1. $A \wedge E$
2. $E \wedge \neg A$
3. $B \wedge R$

Remark 3. Conjunctions can not represent temporal order or asymmetrical relationships.

Definition 20. *A sentence $\mathcal{A}$ can form a **disjunction** with another sentence $\mathcal{B}$, if it can be paraphrased as "Either $\mathcal{A}$ or $\mathcal{B}$", where $\mathcal{A}$ and $\mathcal{B}$ are called **disjuncts**.*

*A disjunction where both disjuncts can be true is called an **inclusive disjunction**, symbolized by $\mathcal{A} \vee \mathcal{B}$.*

*A disjunction where both disjuncts cannot be true is called an **exclusive disjunction**, symbolized by $(\mathcal{A} \vee \mathcal{B}) \wedge \neg(\mathcal{A} \wedge \mathcal{B}) = \mathcal{A} \oplus \mathcal{B}$.*

*Disjunctions are **symmetric** (**commutative**), meaning the order of the disjuncts does not matter.*

Example 15. Consider the following sentences:

1. Either you will not have soup, or you will not have salad.
2. You will have neither soup nor salad.
3. You get either soup or salad, but not both.

Can be rewritten as:

1. $\neg S_1 \vee \neg S_2$

2. $\neg(S_1 \vee S_2)$
3. $(S_1 \vee S_2) \wedge \neg(S_1 \wedge S_2) = S_1 \oplus S_2$

Definition 21. *A sentence $\mathcal{A}$ can form a **conditional** with another sentence $\mathcal{B}$, if it can be paraphrased as "If $\mathcal{A}$, then $\mathcal{B}$" or "$\mathcal{A}$ only if $\mathcal{B}$", where $\mathcal{A}$ is the **antecedent** and $\mathcal{B}$ is the **consequent**. Symbolically, this is represented as $\mathcal{A} \to \mathcal{B}$.*

*A conditional is **asymmetric** (**non-commutative**), meaning the order of the antecedent and consequent matters.*

Example 16. Consider the following sentences:

1. For Jean to be in Paris, it is necessary that Jean be in France.
2. It is a necessary condition on Jean's being in Paris that she be in France.
3. For Jean to be in France, it is sufficient that Jean be in Paris.
4. It is a sufficient condition on Jean's being in France that she be in Paris.

Can all be rewritten as the conditional $P \to F$.

Remark 4. A necessary condition is a condition that must be met in order for something to be true. "X is a necessary condition for Y" means that if Y is true, then X must be true, or $Y \to X$.

A sufficient condition is a condition that, if met, guarantees that something is true. "X is a sufficient condition for Y" means that if X is true, then Y must be true, or $X \to Y$.

Definition 22. *A sentence $\mathcal{A}$ can form a **biconditional** with another sentence $\mathcal{B}$, if it can be paraphrased as "$\mathcal{A}$ if and only if $\mathcal{B}$" or "$\mathcal{A}$ iff $\mathcal{B}$", where $\mathcal{A}$ and $\mathcal{B}$ are called **biconditionals**. Symbolically, this is represented as $\mathcal{A} \leftrightarrow \mathcal{B} = (\mathcal{A} \to \mathcal{B}) \wedge (\mathcal{B} \to \mathcal{A})$.*

*A biconditional is **symmetric** (**commutative**), meaning the order of the biconditionals does not matter.*

Example 17. Consider the following sentences:

1. Alice is a dog only if she is a mammal.
2. Alice is a dog if she is a mammal.
3. Alice is a dog if and only if she is a mammal.

Each can be rewritten as:

1. $D \to M$
2. $M \to D$
3. $(D \to M) \wedge (M \to D) = D \leftrightarrow M$

Definition 23. *If a sentence can be paraphrased as Unless $\mathcal{A}$, $\mathcal{B}$, then it can be symbolized as $(\mathcal{A} \vee \mathcal{B})$.*

Example 18. Consider the following sentences:

1. Unless you wear a jacket, you will catch a cold.
2. You will catch a cold unless you wear a jacket.

There are three interpretations:

1. $\neg J \rightarrow C$
2. $\neg C \rightarrow J$
3. $(J \vee C)$

All three interpretations are valid and equivalent.

Sentences of Truth Functional Logic

Definition 24. *An **expression** is a sequence of TFL symbols in any order.*

Example 19. The following are expressions:

1. $A \wedge B$
2. $\neg)(\vee() \wedge (\neg\neg())((B$

Definition 25. *Axioms of TFL:*

1. *Every sentence letter is an (atomic) sentence.*
2. *If $\mathcal{A}$ is a sentence, then $\neg\mathcal{A}$ is a sentence.*
3. *If $\mathcal{A}$ and $\mathcal{B}$ are sentences, then*
   - *i.* $(\mathcal{A} \wedge \mathcal{B})$
   - *ii.* $(\mathcal{A} \vee \mathcal{B})$
   - *iii.* $(\mathcal{A} \rightarrow \mathcal{B})$
   - *iv.* $(\mathcal{A} \leftrightarrow \mathcal{B})$

   are sentences.
4. *Nothing else is a sentence.*

Ambiguity

Definition 26. *Define some types of ambiguity:*

1. ***Lexical ambiguity*** *is when a sentence has more than one meaning due to the meaning of the words.*
2. ***Structural ambiguity*** *is when a sentence has more than one meaning due to the structure of the sentence.*

Example 20. Consider the following sentences:

1. "I went to the bank" could mean a financial institution or the side of a river.
2. "Flying planes can be dangerous" could be referring to the act of flying planes or planes that are flying.

Definition 27. ***Vagueness*** *is when a sentence lacks a precise meaning.*

Example 21. Consider the following sentences:

1. "I am tall" depends on the context of what is considered tall.

Example 22. Case Study: Roger Casement and the Treason Act (1351)
The law stated that treason occurs if someone:

> is adherent to the King's enemies in his realm, giving to them aid and comfort in the realm, or elsewhere.

The defense argued that "or elsewhere" only applied to giving aid (not adherence). The prosecution argued it applied to both adherence and giving aid. The court ruled in favor of the latter interpretation, and Casement was executed —he claimed he was "anged by a comma."

Definition 28. ***Scope ambiguity*** *is when a sentence has more than one meaning due to interactivity of the logical connectives.*

Example 23. "I like movies that are not long and boring" could mean:

1. I like movies that are not long and I like movies that are not boring.
2. I like movies that are not both long and boring.

Use and Mention in Logic

Notation 1. When we use a **name**, we refer to the object it represents:

- Example: *Justin Trudeau is the Prime Minister.*

When we mention a **name**, we refer to the name itself:

- Example: *"Justin Trudeau" has three syllables.*

Definition 29. • *The **object language** is the language being studied—here, TFL.*

- *The **metalanguage** is the language we use to talk about the object language—here, English.*

Example 24. Consider this example:

- **Incorrect**: D is a sentence letter of TFL.
- **Correct**: $'D'$ is a sentence letter of TFL.

Figures and Tables

Images and graphics play an integral role in Tufte's work. In addition to the standard figure and tabular environments, this style provides special figure and table environments for full-width floats.

Full page–width figures and tables may be placed in figure* or table* environments. To place figures or tables in the margin, use the marginfigure or margintable environments as follows (see figure 1):

```
\begin{marginfigure}
  \includegraphics{helix}
  \caption{This is a margin figure.}
\end{marginfigure}
```

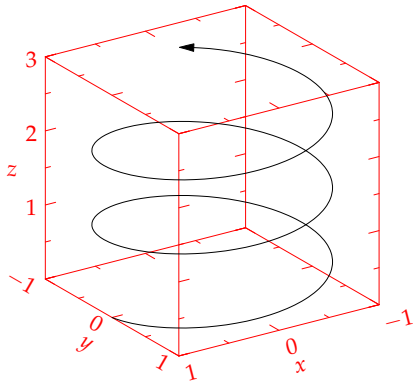

Figure 1: This is a margin figure. The helix is defined by $x = \cos(2\pi z)$, $y = \sin(2\pi z)$, and $z = [0, 2.7]$. The figure was drawn using Asymptote (`http://asymptote.sf.net/`).

The marginfigure and margintable environments accept an optional parameter ⟨*offset*⟩ that adjusts the vertical position of the figure or table. See the "**??**" section above for examples. The specifications are:

```
\begin{marginfigure}[⟨offset⟩]
   ...
\end{marginfigure}

\begin{margintable}[⟨offset⟩]
   ...
\end{margintable}
```

Figure 2 is an example of the `figure*` environment and figure 3 is an example of the normal `figure` environment.

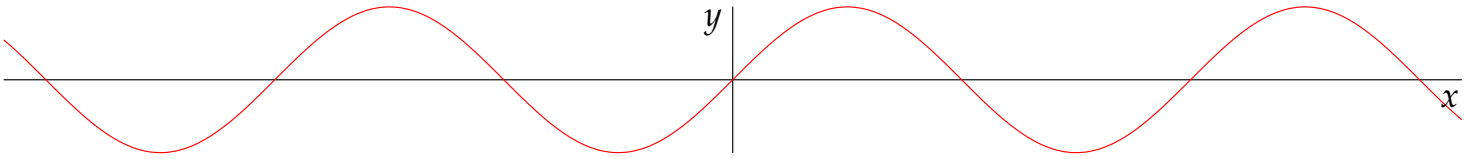

Figure 2: This graph shows $y = \sin x$ from about $x = [-10, 10]$. *Notice that this figure takes up the full page width.*

Table 1 shows table created with the `booktabs` package. Notice the lack of vertical rules—they serve only to clutter the table's data.

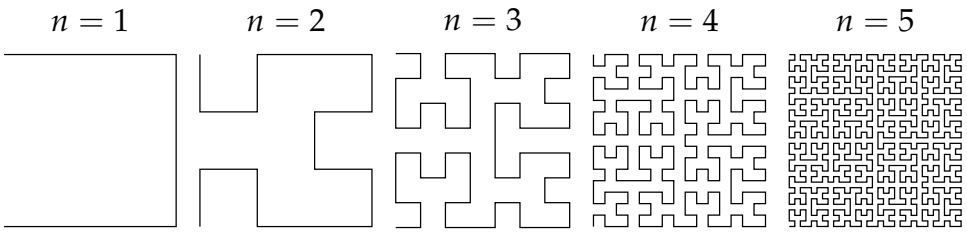

Figure 3: Hilbert curves of various degrees n. *Notice that this figure only takes up the main textblock width.*

Margin	Length
Paper width	8 1/2 inches
Paper height	11 inches
Textblock width	6 1/2 inches
Textblock/sidenote gutter	3/8 inches
Sidenote width	2 inches

Table 1: Here are the dimensions of the various margins used in the Tufte-handout class.

References

References are placed alongside their citations as sidenotes, as well. This can be accomplished using the normal `\cite` command.[1]

The complete list of references may also be printed automatically by using the `\bibliography` command. (See the end of this document for an example.) If you do not want to print a bibliography at the end of your document, use the `\nobibliography` command in its place.

To enter multiple citations at one location,[2] you can provide a list of keys separated by commas and the same optional vertical offset argument: `\cite{Tufte2006,Tufte1990}`.

`\cite[`⟨*offset*⟩`]{`*bibkey1,bibkey2,...*`}`

[1] The first paragraph of this document includes a citation.

[2] Edward R. Tufte. *Beautiful Evidence*. Graphics Press, LLC, first edition, May 2006. ISBN 0-9613921-7-7; and Edward R. Tufte. *Envisioning Information*. Graphics Press, Cheshire, Connecticut, 1990. ISBN 0-9613921-1-8

Full-width text blocks

In addition to the new float types, there is a fullwidth environment that stretches across the main text block and the sidenotes area.

```
\begin{fullwidth}
  Lorem ipsum dolor sit amet...
\end{fullwidth}
```

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Typography

Typefaces

If the Palatino, Helvetica, and `Bera Mono` typefaces are installed, this style will use them automatically. Otherwise, we'll fall back on the Computer Modern typefaces.

Letterspacing

This document class includes two new commands and some improvements on existing commands for letterspacing.

When setting strings of A L L C A P S or SMALL CAPS, the letterspacing—that is, the spacing between the letters—should be increased slightly.[3] The `\allcaps` command has proper letterspacing for strings of F U L L C A P I T A L L E T T E R S, and the `\smallcaps` command has letterspacing for SMALL CAPITAL LETTERS. These commands will also automatically convert the case of the text to upper- or lowercase, respectively.

The `\textsc` command has also been redefined to include letterspacing. The case of the `\textsc` argument is left as is, however. This allows one to use both uppercase and lowercase letters: The Initial Letters Of The Words In This Sentence Are Capitalized.

[3] Robert Bringhurst. *The Elements of Typography*. Hartley & Marks, 3.1 edition, 2005. ISBN 0-88179-205-5

Installation

To install the Tufte-LaTeX classes, simply drop the following files into the same directory as your `.tex` file:

```
tufte-common.def
tufte-handout.cls
tufte-book.cls
```

More Documentation

For more documentation on the Tufte-LaTeX document classes (including commands not mentioned in this handout), please see the sample book.

Support

The website for the Tufte-LaTeX packages is located at `http://code.google.com/p/tufte-latex/`. On our website, you'll find links to our SVN repository, mailing lists, bug tracker, and documentation.

References

Robert Bringhurst. *The Elements of Typography*. Hartley & Marks, 3.1 edition, 2005. ISBN 0-88179-205-5.

Edward R. Tufte. *Envisioning Information*. Graphics Press, Cheshire, Connecticut, 1990. ISBN 0-9613921-1-8.

Edward R. Tufte. *Beautiful Evidence*. Graphics Press, LLC, first edition, May 2006. ISBN 0-9613921-7-7.