PHIL 120 Assignment 2 – Symbolic Logic I

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Question 1: *Suppose $\neg A$ is True. What can you say about the following sentence? (0.5 points)*

$$(A \to (C \to B)) \to ((A \to B) \to (\neg A \to (C \to A)))$$

  A: It is True

  B: The truth value of the sentence depends on the assignment values for B and C

  C: It is False

  D: It can't be determined

Since $\neg A$ is True, A is False. Therefore any statement of the form $A \to \mathbb{X}$ is True. Writing a partial truth table:

A	B	C	$(A \to (C \to B))$	$\to$	$((A \to B)$	$\to$	$(\neg A$	$\to$	$(C$	$\to$	A	$)))$
F	T	T	T	**F**	T	F	T	F	T	F	F	
F	T	F	T	**T**	T	T	T	T	F	T	F	

> As we can see, the truth value of the sentence depends on the assignment value of C. Therefore, the correct answer is **B**.

Question 2: *Suppose A is a Tautology and B is a Contradiction. Note that A, B, C are metavariables (an atomic sentence can't be a Tautology or a Contradiction). What can you say about the following sentence? (0.5 points)*

$$(\neg A \wedge C) \to \neg(\neg B \vee C)$$

  A: It can't be determined

  B: It is a Tautology

  C: It is a Contradiction

  D: It is a contingent sentence (can be either True or False)

First observe that $(\neg A \wedge C)$ is a contradiction and $(\neg B \vee C)$ is a tautology. Let us construct a truth table to determine the truth value of the sentence:

A	B	C	$(\neg A \wedge C)$	$\rightarrow$	$\neg$	$(\neg B \vee C)$
T	F	T	F	**T**	F	T
T	F	F	F	**T**	F	T

Under the given conditions that A is a Tautology and B is a Contradiction, the sentence is a Tautology. Therefore, the correct answer is **B**.

Question 3: *Suppose C is True, and H is False. What is the truth value of the following sentence? (0.5 points)*

$$\neg(C \vee \neg E) \rightarrow (G \wedge \neg H)$$

A: *It can't be determined*

B: *It is True*

C: *It depends on the truth value of E*

D: *It is False*

Given that C is True, then $\neg(C \vee \neg E)$ is False. Seeing the structure of the sentence is $\mathbb{A} \rightarrow \mathbb{B}$, where $\mathbb{A}$ is a Contradiction, the sentence is a Tautology. Therefore, the correct answer is **B**.

Question 4: *The following argument is invalid:*

$$P \rightarrow Q, \neg R \rightarrow \neg Q, R \rightarrow S \therefore \neg P$$

We want to add another premise to the existing premises of the argument to make it valid. Which one of the following choices works? That is, which one of the following choices, if added to the argument as another premise, would make the argument valid? Help yourself with the use of the Truth Tables (it is not required that you write down the Truth Table in your answer!). (0.5 points)

A: Q

B: R

C: $\neg S$

D: $Q \vee S$

All answers but $\neg S$ lead to a dead end. By $\neg S$ to the premises,

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1 │ P → Q
2 │ ¬R → ¬Q
3 │ R → S
4 │ ¬S
  ├─────
5 │ ¬R          MT, 3, 4
6 │ ¬Q          →E, 2, 5
7 │ ¬P          MT, 1, 6
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Therefore, the correct answer is **C**.

Proof 1: *Fill in the missing citations for the proof. (0.5 point each)*

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1  │ C ∧ D
2  │ (D ∨ E) → G
3  │ (G ∧ C) → ¬L
4  │ ¬N → L
   ├─────────
5  │ D                ∧E, 1
6  │ D ∨ E            Question 5
7  │ G                →E, 2, 6
8  │ C                ∧E, 1
9  │ G ∧ C            ∧I, 7, 8
10 │ ¬L               →E, 3, 9
11 │   │ ¬N
   │   ├────
12 │   │ L            →E, 4, 11
13 │   │ ⊥            ¬E, 10, 12
14 │ N                Question 6
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1	$C \wedge D$	
2	$(D \vee E) \to G$	
3	$(G \wedge C) \to \neg L$	
4	$\neg N \to L$	
5	D	$\wedge$E, 1
6	$D \vee E$	$\vee$**I, 5**
7	G	$\to$E, 2, 6
8	C	$\wedge$E, 1
9	$G \wedge C$	$\wedge$I, 7, 8
10	$\neg L$	$\to$E, 3, 9
11	$\neg N$	
12	L	$\to$E, 4, 11
13	$\bot$	$\neg$E, 10, 12
14	N	**IP, 11-13**

Proof 2: *Fill in the missing citations for the proof. (0.5 point each)*

1	$P \lor Q$	
2	$\neg C \land R$	
3	$\neg S \to \neg P$	
4	$Q \to (S \land R)$	
5	P	
6	$\neg S$	
7	$\neg P$	**Question 7**
8	$\bot$	$\neg$E, 7, 5
9	S	IP, 6–8
10	Q	
11	$S \land R$	$\to$E, 4, 10
12	S	$\land$E, 11
13	S	$\lor$E, 1, 5–9, 10–12
14	R	**Question 8**
15	$S \land R$	$\land$I, 13, 14

1	$P \lor Q$	
2	$\neg C \land R$	
3	$\neg S \to \neg P$	
4	$Q \to (S \land R)$	
5	P	
6	$\neg S$	
7	$\neg P$	**$\to$E, 3, 6**
8	$\bot$	$\neg$E, 7, 5
9	S	IP, 6–8
10	Q	
11	$S \land R$	$\to$E, 4, 10
12	S	$\land$E, 11
13	S	$\lor$E, 1, 5–9, 10–12
14	R	**$\land$E, 11**
15	$S \land R$	$\land$I, 13, 14

Proof 3: *Fill in the missing citations for the proof. (0.5 point each)*

1	$M \to \neg\neg R$	
2	$G \to D$	
3	$\neg M \to \neg\neg D$	
4	$\neg D$	
5	G	
6	D	$\to$E, 2, 5
7	$\bot$	$\neg$E, 4, 6
8	$\neg G$	$\neg$I, 5–7
9	$\neg R$	
10	$\neg M$	
11	$\neg\neg D$	$\to$E, 3, 10
12	$\bot$	$\neg$E, 11, 4
13	M	IP, 10–12
14	$\neg\neg R$	$\to$E, 1, 13
15	$\bot$	**Question 9**
16	R	IP, 9–15
17	$\neg G \wedge R$	$\wedge$I, 8, 16
18	$\neg D \to (\neg G \wedge R)$	**Question 10**

1	$M \rightarrow \neg\neg R$		
2	$G \rightarrow D$		
3	$\neg M \rightarrow \neg\neg D$		
4	$\neg D$		
5	G		
6	D	$\rightarrow$E, 2, 5	
7	$\bot$	$\neg$E, 4, 6	
8	$\neg G$	$\neg$I, 5–7	
9	$\neg R$		
10	$\neg M$		
11	$\neg\neg D$	$\rightarrow$E, 3, 10	
12	$\bot$	$\neg$E, 11, 4	
13	M	IP, 10–12	
14	$\neg\neg R$	$\rightarrow$E, 1, 13	
15	$\bot$	**$\neg$E, 14, 9**	
16	R	IP, 9–15	
17	$\neg G \wedge R$	$\wedge$I, 8, 16	
18	$\neg D \rightarrow (\neg G \wedge R)$	**$\rightarrow$I, 4-17**	