PHIL 120 **Assignment 3** **Solutions**

Q1)

Yes, all the symbols are accepted in the vocabulary of FOL. Expressions $\supset$ Formulas $\supset$ Sentences.

Q2)

$\forall x$: $Q(p) \leftrightarrow (A(x,c) \wedge \exists y(R(y) \wedge Q(x)))$

$\exists x$: $R(y) \wedge Q(x)$

Q3)

y is free in G(b, y)

Q4)

Yes, Expressions $\supset$ Formulas $\supset$ Sentences.

Q5)

Yes, it is a formula with no free variables.

Q6)-Q10)

Domain: People and Movies

1-place predicates:
A(x): x is artsy
B(x): x is boring
M(x): x is a movie
P(x): x is a person
T(x): x is a teacher

2-places predicates:
L(x,y): x likes y

Names:
a: Amy

Q6)

$\neg \forall x(M(x) \rightarrow A(x))$

Or, equivalently:

$\neg\neg \exists x \neg(M(x) \rightarrow A(x))$, *i.e.* (Quantifier equivalence)

$\exists x \neg(M(x) \rightarrow A(x))$, *i.e.* (Double Negation elimination)

$\exists x \neg(\neg M(x) \lor A(x))$, *i.e.* (Disjunction to Implication equivalence)

$\exists x(\neg\neg M(x) \land \neg A(x))$, *i.e.* (DeMorgan's Law)

$\exists x(M(x) \land \neg A(x))$ (Double Negation elimination)

Q7)

We assume that being a teacher is a subset of being a person, so no other qualification is required:

$\forall x(T(x) \rightarrow \exists y(M(y) \land L(x,y)))$

Q8)

$\exists x((M(x) \land A(x)) \land B(x)) \land \exists x((T(x) \land B(x)) \land A(x))$

Note: the choice of the variable (x, y, z, etc.) is irrelevant. The MLO here is the conjunction, therefore the two scopes of the existential quantifiers do not overlap. The two variables 'x' are different in each case (under each quantifier). For instance, we could have written the following symbolization equivalently:

$\exists x((M(x) \land A(x)) \land B(x)) \land \exists y((T(y) \land B(y)) \land A(y))$

Q9)

We can break down this sentence into two sentences that will be *conjoined* (for reference, check page 230 of the textbook):

1. Amy likes something that is a movie and it is artsy

   AND

2. If anything that is a person likes anything that is a movie and artsy, then this person is Amy.

Sentence 1: $\forall x((M(x) \land A(x)) \rightarrow L(a,x))$

$\land$

Sentence 2: $\forall y[P(y) \rightarrow \forall x[((M(x) \land A(x)) \rightarrow L(y,x)) \rightarrow y=a]]$

Full-sentence(A): $\forall x[(M(x) \land A(x)) \rightarrow L(a,x)] \land \forall y[P(y) \rightarrow \forall x(((M(x) \land A(x)) \rightarrow L(y,x)) \rightarrow y=a)]$

Now, Sentence 1 can be rewritten like this (check the lecture on "Only" symbolization):

$\forall y[P(y) \rightarrow \forall x(y=a \rightarrow ((M(x) \land A(x)) \rightarrow L(a,x))]$, *i.e.*
$\forall x \forall y[P(y) \rightarrow (y=a \rightarrow ((M(x) \land A(x)) \rightarrow L(a,x))]$

So, now we have:

Sentence 1: $\forall x \forall y[P(y) \rightarrow (y=a \rightarrow ((M(x) \land A(x)) \rightarrow L(a,x))]$

$\land$

Sentence 2: $\forall x \forall y[P(y) \rightarrow (((M(x) \land A(x)) \rightarrow L(y,x)) \rightarrow y=a)]$

Full-sentence(B): $\forall y[P(y) \rightarrow \forall x[((M(x) \land A(x)) \rightarrow L(y,x)) \leftrightarrow y=a]]$

A few *remarks* are in order here.

This one was a **very difficult** sentence to symbolize. It takes inspiration from the example the textbook makes about Pavel and Hikaru on page 230 and adds a few more layers by imposing a composite domain and a relation between a variable and a name instead of two names. You had a few days to think this over, and my idea was to challenge your *survival symbolization abilities* in such a hostile environment. My goal was to make you reason through it, and the marking reflects your reasoning and attempt, rather than the "correct" symbolization. Hopefully, you have picked up the challenge with a good spirit.

Now, another remark concerns the symbolization itself. It can be argued that instead of writing: $\forall x \forall y[P(y) \rightarrow (((M(x) \wedge A(x)) \rightarrow L(y,x)) \rightarrow y=a)]$, one could have written $\forall x \forall y[P(y) \rightarrow (((M(x) \wedge A(x)) \wedge L(y,x)) \rightarrow y=a)]$, that is, replacing the first implication with a conjunction. It really is a *matter of interpretation*.

The **first sentence** is ***true*** when:

- y is not a person (but we'd like to avoid this)
- $((M(x) \wedge A(x)) \rightarrow L(y,x))$ is ***false***, that is when *nobody* actually likes artsy movies
- $((M(x) \wedge A(x)) \rightarrow L(y,x))$ is ***true***, provided the person who actually likes artsy movies is Amy
  - ***True*** when either there are no movies at all, or none of them are artsy
  - ***True*** when all the claims are true

The **second sentence** is ***true*** when

- y is not a person (same as above)
- $(M(x) \wedge A(x)) \wedge L(y,x))$ is ***false***, that is when either there are no movies or none of them are artsy, or *nobody* actually likes artsy movies
- $(M(x) \wedge A(x)) \wedge L(y,x))$ is ***true***, provided the person who actually likes artsy movies is Amy
  - ***True*** when all the claims are true

As you can see, the statement with $\wedge$ is *stronger* than the statement with $\rightarrow$

Q10)

$\exists x((M(x) \wedge A(x)) \wedge \neg B(x))$

Or, equivalently:

$\neg \forall x \neg((M(x) \wedge A(x)) \wedge \neg B(x))$, *i.e.* (Quantifier equivalence)
$\neg \forall x(\neg(M(x) \wedge A(x)) \vee \neg\neg B(x))$, *i.e.* (DeMorgan's Law)
$\neg \forall x(\neg(M(x) \wedge A(x)) \vee B(x))$, *i.e.* (Double Negation elimination)
$\neg \forall x((M(x) \wedge A(x)) \rightarrow B(x))$ (Disjunction to Implication equivalence)