PHIL 120 Assignment 1 – Symbolic Logic I

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I: *Symbolize the following English sentences into TFL, using the following symbolization key. (0.5 point each)*

A: Adam goes/will go to the hockey game.

B: Betty goes/will go to the hockey game.

C: Carol goes/will go to the hockey game.

D: Daniel goes/will go to the hockey game.

1. *Adam and Carol will go to the hockey game.*
$\boxed{(A \wedge C)}$

2. *Not both Adam and Betty are going to the hockey game if Carol is going to the hockey game.*
$\boxed{[C \rightarrow \neg(A \wedge B)]}$

3. *Adam goes to the hockey game if and only if Carol goes to the hockey game.*
$\boxed{(A \leftrightarrow C)}$

4. *Either Carol goes to the hockey game or Daniel goes to the hockey game, but both Adam and Betty go to the hockey game only if Carol goes to the hockey game.*
$\boxed{[(C \vee D) \wedge ((A \wedge B) \rightarrow C)]}$

5. *Someone between Adam, Betty, Carol, and Daniel will go to the hockey game, but not all of them.*
$\boxed{[(A \vee B \vee C \vee D) \wedge \neg(A \wedge B \wedge C \wedge D)]}$

II: *In each of the following three expressions, determine whether it is a sentence of TFL or not. (0.25 point each) Explain why it is or it is not. (0.25 point each)*

6. $[G \rightarrow (\neg\neg L \vee (D \vee \neg C))] \leftrightarrow [(C \leftrightarrow (H \wedge \neg D)) \leftrightarrow R]$
Let us examine the LHS of the biconditional. The LHS is a sentence of TFL since

- G, L, D, C are atomic sentences.
- $\neg\neg L = L$ is a double negation sentence.
- $\mathbb{X} = (D \vee \neg C)$ is an inclusive disjunction sentence.
- $\mathbb{Y} = (\neg\neg L \vee \mathbb{X})$ is an inclusive disjunction sentence.
- $(G \rightarrow \mathbb{Y})$ is a conditional sentence.

Let us examine the RHS of the biconditional. The RHS is a sentence of TFL since

- C, H, D, R are atomic sentences.
- $\neg D$ is a negation sentence.
- $\mathbb{Z} = (H \wedge \neg D)$ is a conjunction sentence.
- $\mathbb{W} = (C \leftrightarrow \mathbb{Z})$ is a biconditional sentence.
- $(\mathbb{W} \leftrightarrow R)$ is a biconditional sentence.

Since both the LHS and RHS of the biconditional are sentences of TFL, the entire expression is a sentence of TFL.

7. $[(G \rightarrow (\neg\neg L \vee (D \vee \neg C))) \leftrightarrow C] \leftrightarrow [(C \wedge \neg D) \leftrightarrow R]$
Let us examine the LHS of the biconditional. The LHS is a sentence of TFL since

- G, L, D, C are atomic sentences.
- $\mathbb{X} = (G \rightarrow (\neg\neg L \vee (D \vee \neg C)))$ was established as a sentence of TFL in question 6.
- $(\mathbb{X} \leftrightarrow C)$ is a biconditional sentence.

Let us examine the RHS of the biconditional. The RHS is a sentence of TFL since

- C, D, R are atomic sentences.
- $\neg D$ is a negation sentence.
- $\mathbb{Z} = (C \wedge \neg D)$ is a conjunction sentence.
- $(\mathbb{Z} \leftrightarrow R)$ is a biconditional sentence.

Since both the LHS and RHS of the biconditional are sentences of TFL, the entire expression is a sentence of TFL.

8. $(G \rightarrow \neg(\neg L \vee (D \vee \neg C))) \leftrightarrow ((C \leftrightarrow (H \wedge \neg D)) \leftrightarrow R)$
The RHS is a sentence since it is equivalent to the RHS of the biconditional in question 6.

Let us examine the LHS of the biconditional. The LHS is a sentence of TFL since

- G, L, D, C are atomic sentences.
- $\neg L$ is a negation sentence.
- $\neg C$ is a negation sentence.
- $\mathbb{X} = (D \vee \neg C)$ is an inclusive disjunction sentence.

- $\mathbb{Y} = (\neg L \vee \mathbb{X})$ is an inclusive disjunction sentence.
- $\neg\mathbb{Y}$ is a negation sentence.
- $(G \to \neg\mathbb{Y})$ is a conditional sentence.

Since both the LHS and RHS of the biconditional are sentences of TFL, the entire expression is a sentence of TFL.

III: *In each of the following two sentences, find the main connective. (0.5 point each)*

9. $[((E \leftrightarrow (\neg B \vee (D \to \neg C))) \vee C) \to ((C \wedge \neg D) \leftrightarrow A)]$
Using the counting method,

$$\underbrace{[((}_{+3} E \leftrightarrow \underbrace{(}_{+1} \neg B \vee \underbrace{(}_{+1} D \to \neg C \underbrace{)))}_{-3} \vee C \underbrace{)}_{-1} \underbrace{\to}_{=1} ((C \wedge \neg D) \leftrightarrow A)]$$

Therefore, the main connective is the conditional connective $\to$.

10. $\neg((E \leftrightarrow \neg(\neg\neg B \vee D)) \wedge (\neg(\neg C \to C) \to ((C \wedge \neg D) \leftrightarrow A)))$

Since the sentence is in the form $\neg(\mathbb{X})$, the main connective is the negation connective $\neg$.