PHIL 120 **Assignment 4** **Winter 2025**

Guidelines:

There are **10 questions** in this assignment. They are all worth **0.5 points each**. Together they amount to 5 points. This assignment is worth **5% of your final grade**.

Please, keep in mind:

1. You should *type* your answers in the designated place in Canvas.
2. You should type down your answers *directly* in Canvas. If you type them down in another software and then copy-paste them into Canvas, the symbols and some other items might change. It is your responsibility to make sure that your answers are readable in Canvas.
3. It is your responsibility to ascertain that the file you submit is **clear** and **readable**.
4. Do not forget to submit your assignment after you are done typing down your answers or uploading your file. If you do not submit your assignment, it remains as a draft. Drafts are not marked.
5. Please number your answers so I can recognize which answer belongs to which question.

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Questions 1-4 are multiple choice answers, given a *fixed* **Extensional Interpretation**, and they are about the semantics of FOL.

Questions 5 and 6 require you to **set up an Interpretation in FOL**. These questions are about the Semantics of FOL.

Question 7 presents an argument, which you are required to **Symbolize in FOL** by *first setting up a proper key and a domain*. This question is about the Syntax of FOL.

Questions 8, 9, and 10 are about a *given* **derivation in Natural Deduction for FOL**. In *Question 8* you are asked to **write the sentence** that corresponds to the rule for the same line; in *Questions 9 and 10* you are given the derived sentences, and you are asked to **cite the proper rule**. These questions are about the Proof Theory of FOL.

Questions:

For Questions 1-4, use the following interpretation (note that both the domain and predicates are defined *extensionally*)

Domain: {Amy, Ben, Cate, Dave} = {a, b, c, d}

a: Amy

b: Ben

c: Cate

d: Dave

The extension of S(x): {a, b, c}

The extension of T(x): {a, d}

The extension of P(x): {}

The extension of H(x): {a, b, c, d}

The extension of L(x, y): {<a, c>, <a, d>, <b, a>, <b, b>, <b, c>, <b, d>, <c, a>, <c, c>}

Question 1 (0.5 points)

Which of the following is True in the given Interpretation?

(I) $\exists x \neg(L(a,x) \wedge H(x))$

(II) $\neg \forall x H(x)$

a. I is true; II is true

b. I is true; II is false

c. I is false; II is true

d. I is false; II is false

Question 2 (0.5 points)

Which of the following is True in the given Interpretation?

(I) $\exists x \neg P(x) \leftrightarrow \forall y \neg L(d,y)$

(II) $P(b) \leftrightarrow \forall x (P(x) \rightarrow \neg L(x,x))$

a. I is true; II is true

b. I is true; II is false

c. I is false; II is true

d. I is false; II is false

Question 3 (0.5 points)

Which of the following is True in the given Interpretation?

(I) $\forall x(S(x) \rightarrow \forall y (T(y) \rightarrow L(x,y)))$

(II) $\exists x((S(x) \wedge H(x)) \wedge (L(x,c) \wedge L(b,x)))$

a. I is true; II is true

b. I is true; II is false

c. I is false; II is true

d. I is false; II is false

Question 4 (0.5 points)

Which of the following is True in the given Interpretation?

(I) $\forall x(\neg L(b,x) \vee \neg P(x))$

(II) $\forall y (\neg H(y) \rightarrow L(a,y))$

a. I is true; II is true

b. I is true; II is false

c. I is false; II is true

d. I is false; II is false

Question 5 (0.5 points)

Construct an Interpretation and show that the following two sentences are Not Equivalent:

$\forall x(A(x) \vee (P(x) \wedge Q(x)))$

$\forall x((A(x) \vee P(x)) \wedge Q(x))$

Question 6 (0.5 points)

Construct an Interpretation and show that the following Entailment *does not hold* (i.e., the argument is *invalid*):

$\forall x (P(x) \rightarrow G(x)), \exists x P(x) \nvDash \forall x G(x)$

Question 7 (0.5 points)

Set up a Key and Symbolize the following argument in FOL:

"All geniuses are misunderstood. All misunderstood people suffer from loneliness. (For instance,) Tesla was a genius. (We conclude that) Tesla suffered from loneliness".

Note: the parts in parentheses are expressions in the English language that do not fit into a formal symbolization of FOL. Just to avoid confusion, I wanted to signal that they are not to be formalized!

For Questions 8-10, consider the following (valid!) proof:

1	$\forall x\, \forall y(H(x,y) \rightarrow \neg H(y,x))$	Premise
2	$\forall x\, \exists y\, H(x,y)$	Premise
3	$\exists y\, H(a,y)$	$\forall$E 2
4	$\quad H(a,b)$	Assumption
5	$\quad \forall y(H(a,y) \rightarrow \neg H(y,a))$	$\forall$E 1
6	$\quad$? [Question 8]	$\forall$E 5
7	$\quad \neg H(b,a)$	$\rightarrow$E 6,4
8	$\quad \exists y\, \neg H(y,a)$	$\exists$I 7
9	$\exists y\, \neg H(y,a)$	? [Question 9]
10	$\forall x\, \exists y\, \neg H(y,x)$	? [Question 10]

Question 8 (0.5 points)

What Sentence should we write on line 6 of the proof?

Question 9 (0.5 points)

Which Natural Deduction Rule should we cite on line 9 of the proof?

Question 10 (0.5 points)

Which Natural Deduction Rule should we cite on line 10 of the proof?