Exam - MA284 Discrete Mathematics

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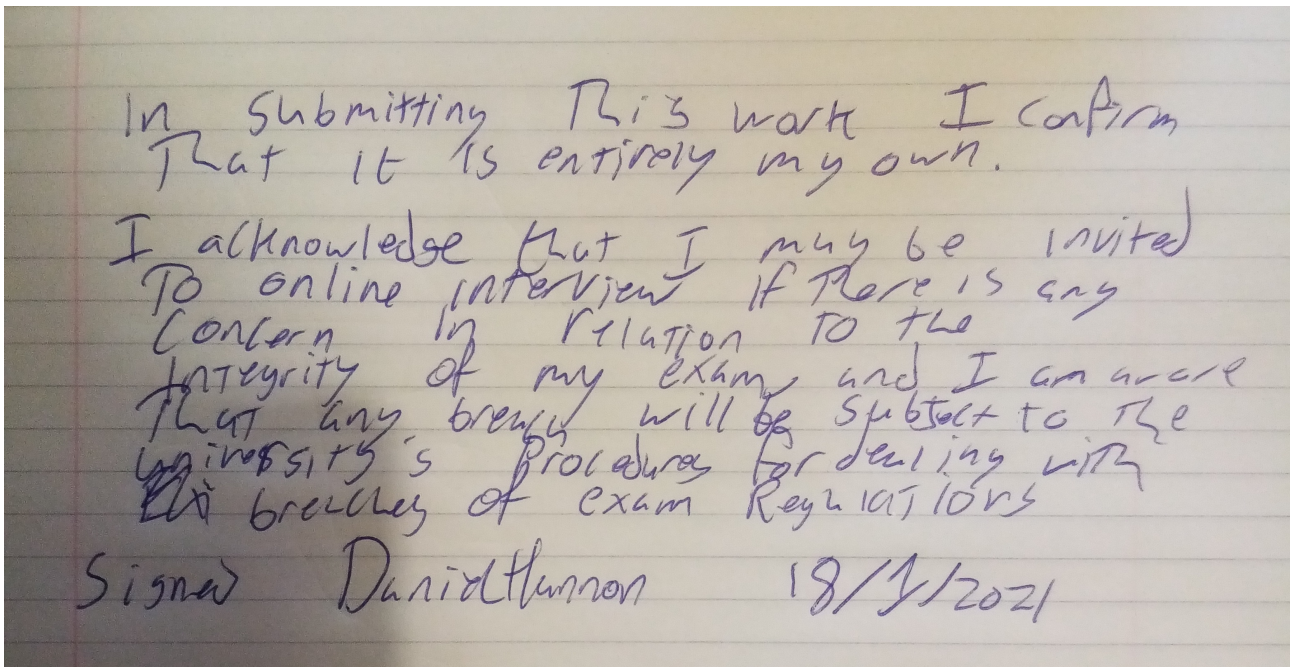
January 2021


In Submitting This work I confirm
That it is entirely my own.
I acknowledge that I may be invited
To online interview if there is any
Concern in relation to the
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Signed Daniel Hannon 18/1/2021

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Question 1

1 a ~~45~~

$|A| = 15 \quad |B| = 10 \quad |C| = 13$

I 15

II 10

III 0

IV 0

✓ {15, 10}

b $S = \{1, 2, 3, 4, 5, 6\}$

I $2^6 - 1 = 63$

II $\binom{6}{2} = \frac{6!}{2!\,4!} = \frac{720}{48} = 15$

III

{1, 2, 3}

{1, 2, 3, 4}

{1, 2, 3, 5}

{1, 2, 3, 6}

4

V

31

63 - Null set

2

IV {4}, {2}, {6}

1, 4, 6

3, 4, 6

5, 4

1, 2

3, 2

5, 2

1, 6

3, 6

5, 6

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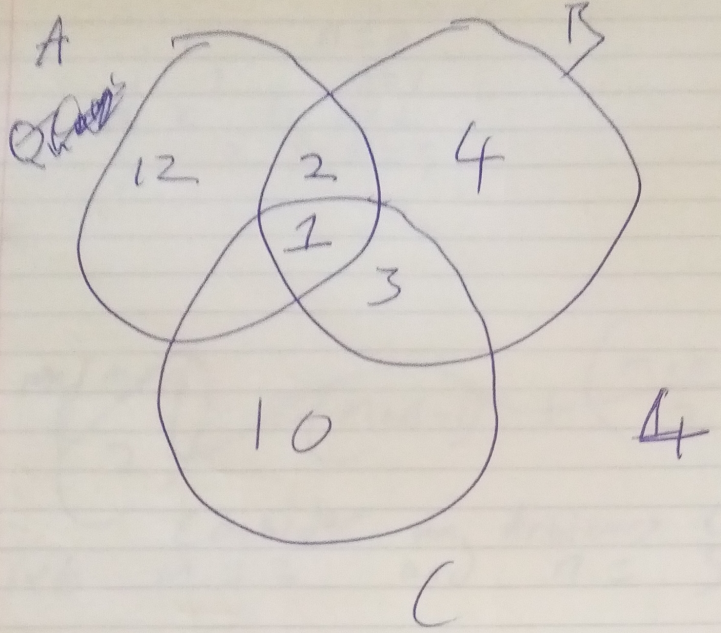
1 (L)



A = Queens Gambit

B = Normal People

C = Big mouth

$|A \cap B| = 3$

$|B \cap C| = 4$

$|A \cap B \cap C| = 1$

$12 + 2 + 4 + 1 + 3 + 10 - 4 = 30$

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Q2

a

1 n=0
1 1 n=1
1 2 1 n=2
1 3 3 1 n=3
⋮ ⋱

$\binom{m+n}{2} = (m+n-1) + \binom{m+n-2}{2}$

Consider an arbitrary case
let $m = 2$ and $n = 3$

$\binom{5}{2} - \binom{2}{2}\binom{3}{2} = 6$ ✓

$\binom{5}{2} - 4 = 6$

$\binom{4}{1} + \binom{4}{2} - 4 = 6$

$\binom{4}{2} = 6$ True

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2(b) Arbitrary number $n = 5$

$$\binom{5}{3} = \sum_{K=2}^{4} ((K-1)(4-K))$$

$$(1)(2) + (2)(1) + (3)(0)$$

$$\binom{5}{3} = \binom{4}{2} + \binom{4}{3} = 3 + \binom{3}{2} + 3 + 1 = 10$$

$$3$$

False

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- Q3

a

I $\binom{4}{2} = 8 + \binom{8}{2}$ 7 + 6 + 5 + 4 + 3 + 2 + 1

15 + 11 + 7 + 3

26 10

36

- II 0, 2, 8 2, 2, 6 0, 0, 10

0, 4, 6 2, 4, 4

5

III 3 0

- 6

$24 \times \left(\frac{3}{4}\right)^4 = 7.6 = 8$

I $3 \times 3 \times 2 \times 1 = 18$

23 with at least 1 wrong

II $3 \times 2 \times 2 \times 1 = 12$

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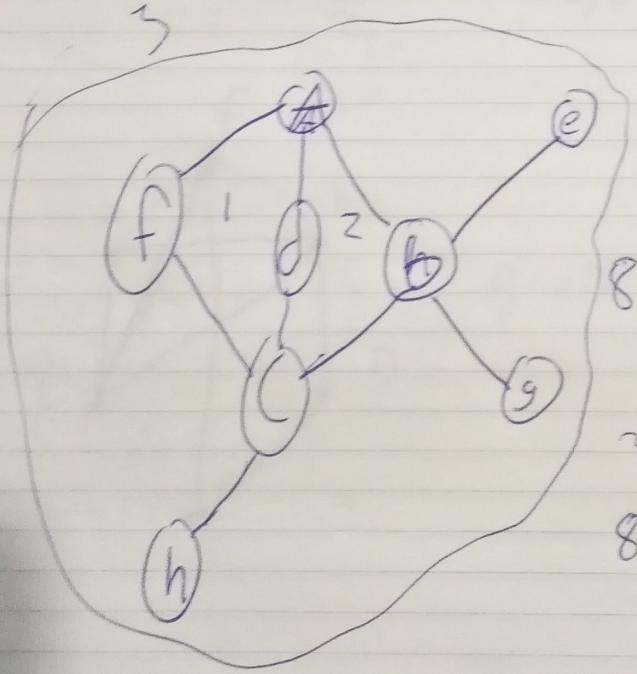
a (d, f, b, h) never touch (a, c, e, g)

Yes, graph is Bipartite as it has a Chromatic number of Two and as a result contains no odd cycles

b The Graph is not a Tree as it contains cycles

c 3



Eulers formula

Vertices − edges + faces = 2

8 ~~edges~~ Vertices

9 edges

3 faces

8 − 9 + 3 = 2 True

D Subgraph = {A, b, C, f, h}

has 5 vertices and edges

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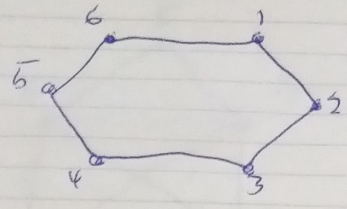
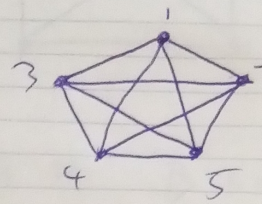
Q5

a



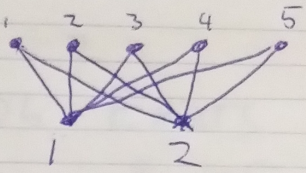
C_6

b



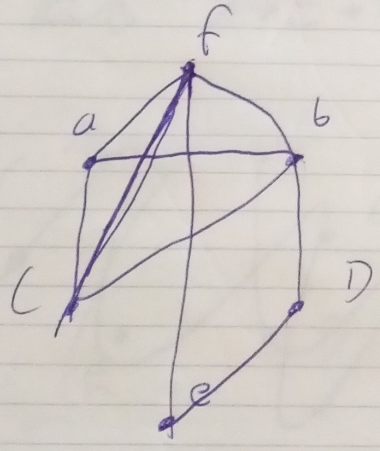
K_5

c



$K_{2,5}$

d



graph G

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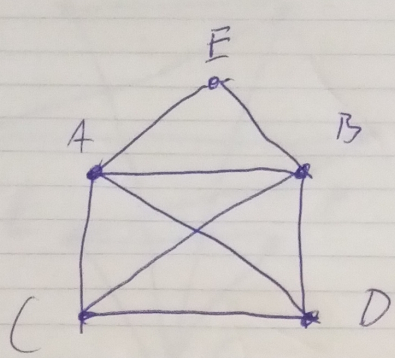
Q6

a



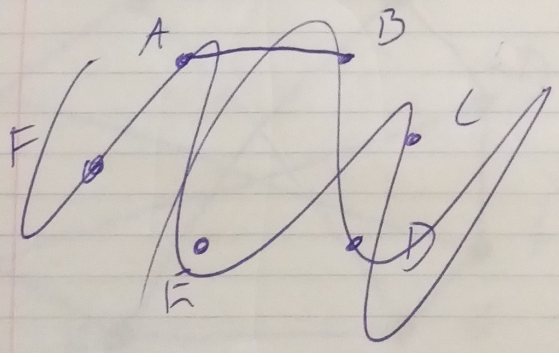
A has 4 B has 4 C has 3 D has 3

E has 2

Graph Exists

Euler Path within The Graph

B A→E→B→C→D



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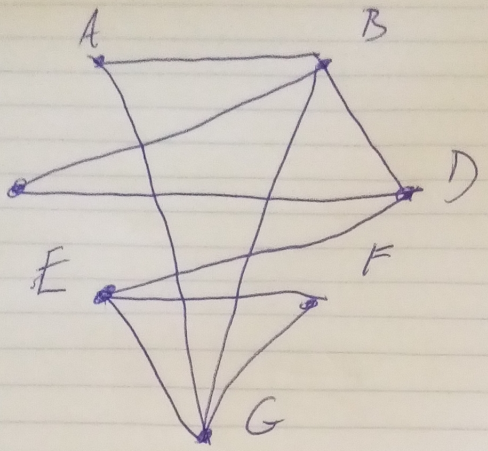
6(B)

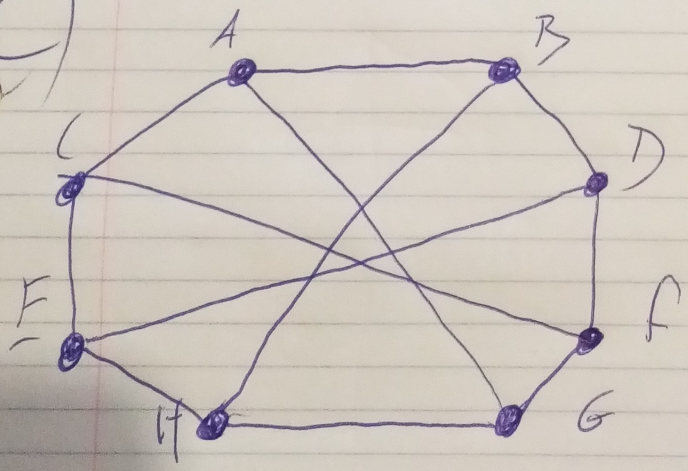


F → G → A → C → D → E

F → G → A → B → C → D → E

would be an Euler Circuit within the Graph

6(C)



A → B → D → f → G → h → e → C

Hamiltonian Cycle