Lecture-2

1 → Elimination
  → Back-Substitution
2 → Elimination matrix
  → Matrix multiplication

Elimination

$$x + 2y + z = 2$$
$$3x + 8y + z = 12$$
$$4y + z = 2$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \\ 2 \end{bmatrix}$$

$$Ax = b$$

Pivot

$$\left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 2 \\ 3 & 8 & 1 & 12 \\ 0 & 4 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 4 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{2} & -2 & 6 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

A b → U c

back Substitution

$$x + 2y + z = 2 \qquad x = 2$$
$$2y - 2z = 6 \qquad y = 1$$
$$5z = -10 \qquad z = -2$$

Elimination matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Row 1, Row 2, Row 3

Column 1, Column 2, Column 3

a_{ij} → Row no, Column no

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}$$

$$= b_{11}\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} + b_{21}\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} + b_{31}\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$ {Column method}

$$= \begin{bmatrix} (a_{11}\ a_{12}\ a_{13}) \cdot (b_{11}\ b_{21}\ b_{31}) \\ (a_{21}\ a_{22}\ a_{23}) \cdot (b_{11}\ b_{21}\ b_{31}) \\ (a_{31}\ a_{32}\ a_{33}) \cdot (b_{11}\ b_{21}\ b_{31}) \end{bmatrix}$$ {Row method}

Dot Product

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} =$$

$$\begin{bmatrix} (a_{11}\ a_{12}\ a_{13}) \cdot (b_{11}\ b_{21}\ b_{31}) & (a_{11}\ a_{12}\ a_{13}) \cdot (b_{12}\ b_{22}\ b_{32}) & (a_{11}\ a_{12}\ a_{13}) \cdot (b_{13}\ b_{23}\ b_{33}) \\ (a_{21}\ a_{22}\ a_{23}) \cdot (b_{11}\ b_{21}\ b_{31}) & (a_{21}\ a_{22}\ a_{23}) \cdot (b_{12}\ b_{22}\ b_{32}) & (a_{21}\ a_{22}\ a_{23}) \cdot (b_{13}\ b_{23}\ b_{33}) \\ (a_{31}\ a_{32}\ a_{33}) \cdot (b_{11}\ b_{21}\ b_{31}) & (a_{31}\ a_{32}\ a_{33}) \cdot (b_{11}\ b_{21}\ b_{31}) & (a_{31}\ a_{32}\ a_{33}) \cdot (b_{11}\ b_{21}\ b_{31}) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

→ Row Operation

$$= \begin{bmatrix} a_{11}(b_{11}\ b_{12}\ b_{13}) + a_{12}(b_{21}\ b_{22}\ b_{23}) + a_{13}(b_{31}\ b_{32}\ b_{33}) \\ a_{21}(b_{11}\ b_{12}\ b_{13}) + a_{22}(b_{11}\ b_{12}\ b_{23}) + a_{23}(b_{31}\ b_{32}\ b_{33}) \\ a_{31}(b_{11}\ b_{12}\ b_{13}) + a_{32}(b_{21}\ b_{22}\ b_{23}) + a_{33}(b_{31}\ b_{32}\ b_{33}) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

E_{32} E_{21}

(Elimination matrix)

$$E_{32}(E_{21}A) = U$$

$$\Rightarrow (E_{32}E_{21})A = U$$

{Associative law}

~~{Commutative Law}~~

$$= \left[b_{11}\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + b_{21}\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + b_{31}\begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} \right.$$

$$, b_{12}\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + b_{22}\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + b_{32}\begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix}$$

$$\left. , b_{13}\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} \quad b_{23}\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} \quad b_{33}\begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} \right]$$

→ Column Operation

If A^{-1} is inverse of matrix A then

$$A^{-1}A = I$$