Lecture-11

→ Bases of new Vector Space
→ Rank one matrices
→ Small world graphs

Basis for M = all 3×3 matrix

$$\begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}, \cdots, \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

→ dimension of Space 9

Basis for M = all 3×3 Symmetric Matrix.

$$\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix} \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix} \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix} \begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix} \begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

→ dimension of Space 6

Sum of element of $S \in U$.

$S+U$ = all 3×3 matrix

→ dimension of Space 9

$$\dim(S+U) = \dim(S) + \dim(U) - \dim(S \cap U)$$

$\frac{d^2y}{dx^2} + y = 0$

$y = \underbrace{\cos x \ , \ \sin x}_{\text{Basis}}$

Complete Solution: $y = C_1 \cos x + C_2 \sin x$ {Null Space}

dimension = 2 = order of differential equation.

Rank 1 matrix

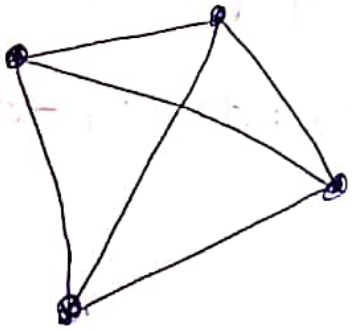
Example $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 8 & 10 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 & 5 \end{bmatrix}$

Any Rank 1 matrix $A = uv^T$

Column Vectors

Any matrix of Rank r can be broken into r matrix of rank 1.

Graph ⇒ It a bunch of nodes and edges connecting n nodes



→ 4 node
→ 6 edges

(2) Minimum Separation of two nodes.