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Sums of independent random variables
Covariance and correlation

★ **The distribution of X+Y: The discrete case**

$Z = X + Y$; ⇒ X, Y independent, discrete
⇒ X, Y have known PMFs

$$P_Z(z) = \sum_x P(X=x, Y=z-x)$$

$$\boxed{P_Z(z) = \sum_x P_X(x)\, P_Y(z-x)}$$

{This is known as Convolution}

★ **The distribution of X+Y: The Continuous case**

$$\boxed{f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\, dx}$$

★ **The sum of independent normal r.v**

$X \sim N(\mu_x, \sigma_x^2) \quad Y \sim N(\mu_y, \sigma_y^2)$

$Z = X + Y$

X and Y are independent.

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} e^{-(x-\mu_x)^2/2\sigma_x^2}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\,\sigma_y} e^{-(y-\mu_y)^2/2\sigma_y^2}$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x)\, f_Y(z-x)\, dx$$

$$= \frac{1}{\sqrt{2\pi(\sigma_x^2+\sigma_y^2)^2}}\, e^{\left(\frac{-1}{2}\left(\frac{(z-\mu_x-\mu_y)^2}{\sigma_x^2+\sigma_y^2}\right)\right)}$$

So $Z \sim N(\mu_x+\mu_y,\ \sigma_x^2+\sigma_y^2)$

★ **Covariance**

$$Cov(X,Y) = E\left[(X-E[X])\cdot(Y-E[Y])\right]$$

★ **Covariance properties**

1. $Cov(X,X) = Var(X)$
2. $Cov(X,Y) = E[XY] - E[X]E[Y]$
3. $Cov(aX+b, Y) = a\,Cov(X,Y)$
4. $Cov(X, Y+Z) = Cov(X,Y) + Cov(X,Z)$

★ **Variance of a sum of random variables**

$$Var(X_1+X_2) = Var(X_1) + Var(X_2) + 2Cov(X_1,X_2)$$

★ The Correlation Coefficient

⇒ It is a dimensionless version of covariance

$$\rho = E\left[\frac{(X-E[X])}{\sigma_X} \cdot \frac{(Y-E[Y])}{\sigma_Y}\right]$$

$$\boxed{\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}}$$

$$\boxed{-1 \leq \rho \leq 1}$$