Lecture - 17

→ Orthogonal basis $q_1, \cdots q_n$
→ Orthogonal matrix Q
→ Gram - Schmidt $A \rightarrow Q$

Orthonormal Vector

$$q_i^T q_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$$Q = \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}$$

$$Q^T Q = \begin{bmatrix} q_1^T \\ \vdots \\ q_n^T \end{bmatrix} \begin{bmatrix} q_1 \cdots q_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

I

Orthonormal matrix

If Q is square then $Q^T Q = I$
$\Rightarrow Q^T = Q^{-1}$

Q has Orthonormal Columns

Project onto its column space

$$P = Q(\underbrace{Q^T Q}_{I})^{-1} Q^T = \underbrace{Q Q^T}_{I \text{ if } Q \text{ is square}}$$

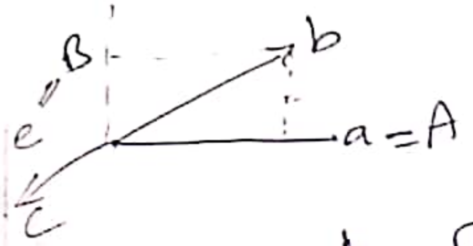
Gram - Schmidt

↓ Vectors $a, b \to A \;\; B$ ⟶ Orthonormal

Independent $\quad c \quad$ Orthogonal $\; C$

$$q_1 = \frac{A}{\|A\|} \quad q_2 = \frac{B}{\|B\|}$$

$$q_3 = \frac{C}{\|C\|}$$



$$B = b - Pb = b - A\frac{A^Tb}{A^TA}$$

$$A^TB = A^T\left(b - A\frac{A^Tb}{A^TA}\right) = A^Tb - A^TA\frac{A^Tb}{A^TA} = 0$$

$$C = c - A\frac{A^T}{A^TA}c - B\frac{B^T}{B^TB}c$$

$A \longrightarrow Q$

A: Normal Independent Column Matrix

Q: Orthonormal Matrix

$$A = QR$$

R: Upper Δ