Lecture - 4

→ Inverse of $AB\ A^T$

→ Product of Elimination matrix

$A = LU$ (no row exchange)

→ Upper triangular

→ Lower triangle

$(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^T = B^T A^T$

$AA^{-1} = I$

$(AA^{-1})^T = I^T \Rightarrow (A^{-1})^T (A)^T = I$

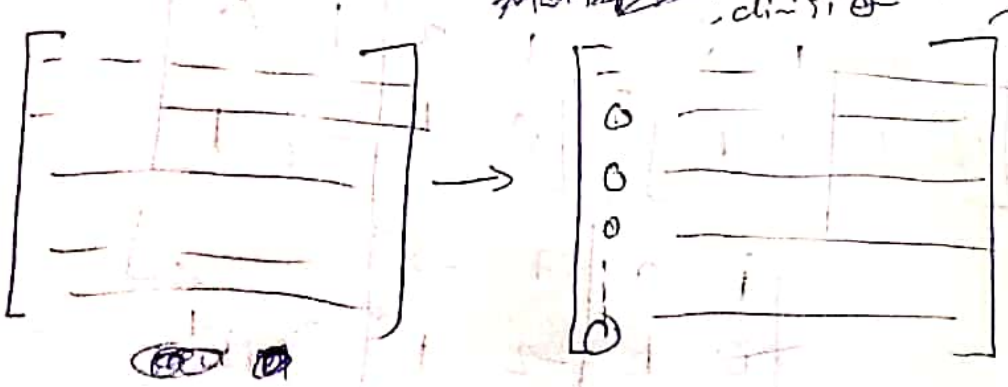
$(A^T)^{-1} = (A^{-1})^T$

$E_{32} E_{31} E_{21} A = U$ (No row exchange)

$A = \underbrace{E_{21}^{-1} E_{31}^{-1} E_{32}^{-1}}_{L} U$

#How many Operations on $n \times n$ matrix A?

(Addition, Subtraction, ~~Multiplication~~ division) → about n^2



Count = $n^2 + (n-1)^2 + \cdots + 2^2 + 1^2$ {Roughly}

$\approx \frac{n^3}{3}$

Permutation matrix (3×3)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{P^{-1} = P^T}$$