(Linear Algebra)
Lecture 24a

→ Markov matrices
(Steady State)

→ Fourier Series Projection.

A is Markov matrice if
① All entity ≥ 0
② All Columns add to 1.

eg: $A = \begin{bmatrix} 0.1 & 0.01 & 0.3 \\ 0.2 & 0.99 & 0.3 \\ 0.7 & 0.0 & 0.4 \end{bmatrix}$

→ $\lambda = 1$ is an eigen Value
→ All other $|\lambda_i| < 1$

⇒ Eigen Value of A are Same as Eigen Value of A^T.

⇒ Projection with Orthonormal basis

$V = x_1 q_1 + x_2 q_2 + \dots x_n q_n$

{$x_1 \dots x_n$ are Scalars} {$q_1 \dots q_n$ are othonormal basis}

$V = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ ⇒ $QX = V$

$X = Q^{-1} V$

$\boxed{X = Q^T V}$

$x_1 = q_1^T V$

$$f(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \ldots$$

{fourier Series}

$\Rightarrow$ 1, $\cos x$, $\sin x$, $\cos 2x$ --- are function basis.

for Vectors	for functions
$v^T w = v_1 w_1 + \cdots + v_n w_n$	$f^T g = \int_0^{2\pi} f(x)\, g(x)\, dx$

$$a_1 = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos x \, dx$$

Lecture (24 b)
(Review)

① $Q = [q_1 \; q_2 \; \ldots \; q_n]$

Projection — Least Square

(Gram Schmit)

② det A

→ Properties

→ Big formula

→ $A^{-1} = \text{Cofactor}/|A|$

③ Eigen Values $Ax = \lambda x$

$\det(A - \lambda I) = 0$

Diagonalize $S^{-1}AS = \Lambda$

Powers A^k