Math 53 {Multivariable Calculus}

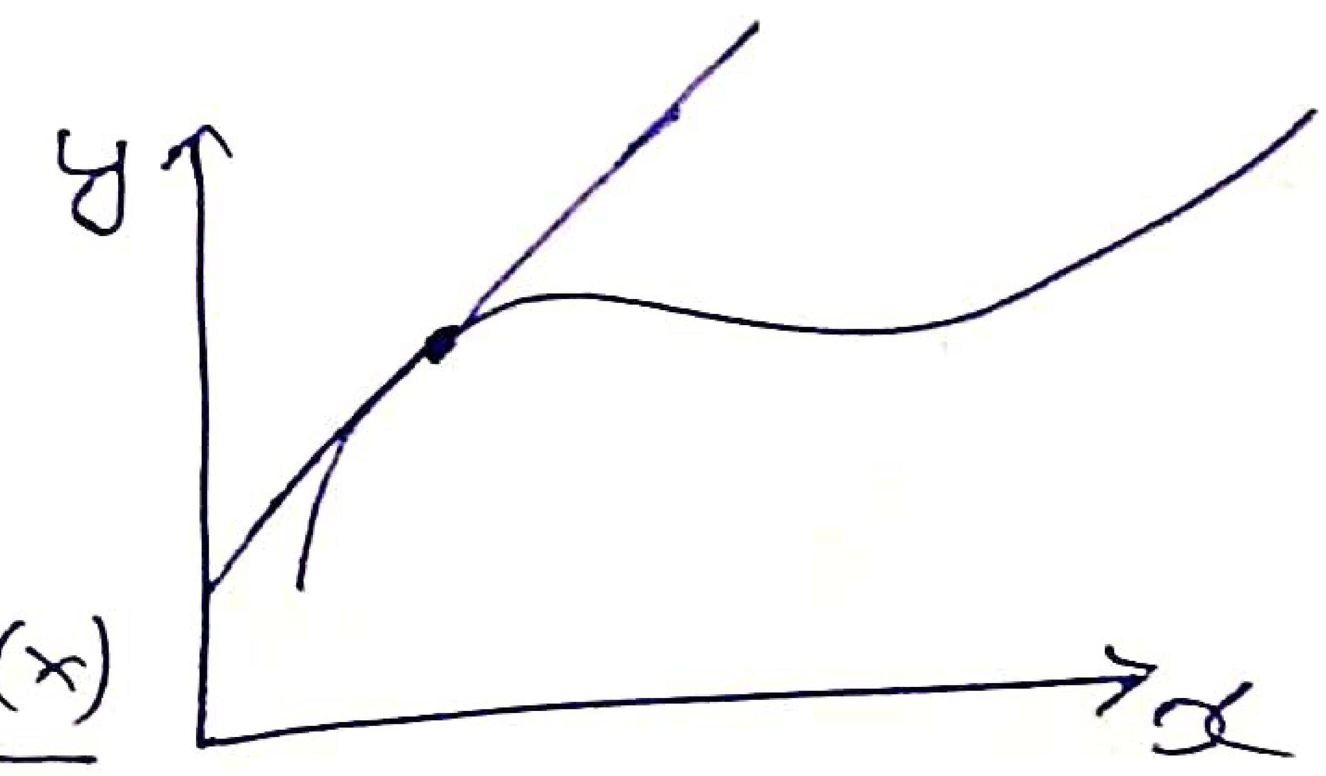
Part 1: Geometric Preliminaries

(1) Introduction to the Course, Parametrized Curves

★ Review & Introduction

Single variable Calculus

$y = f(x)$ function



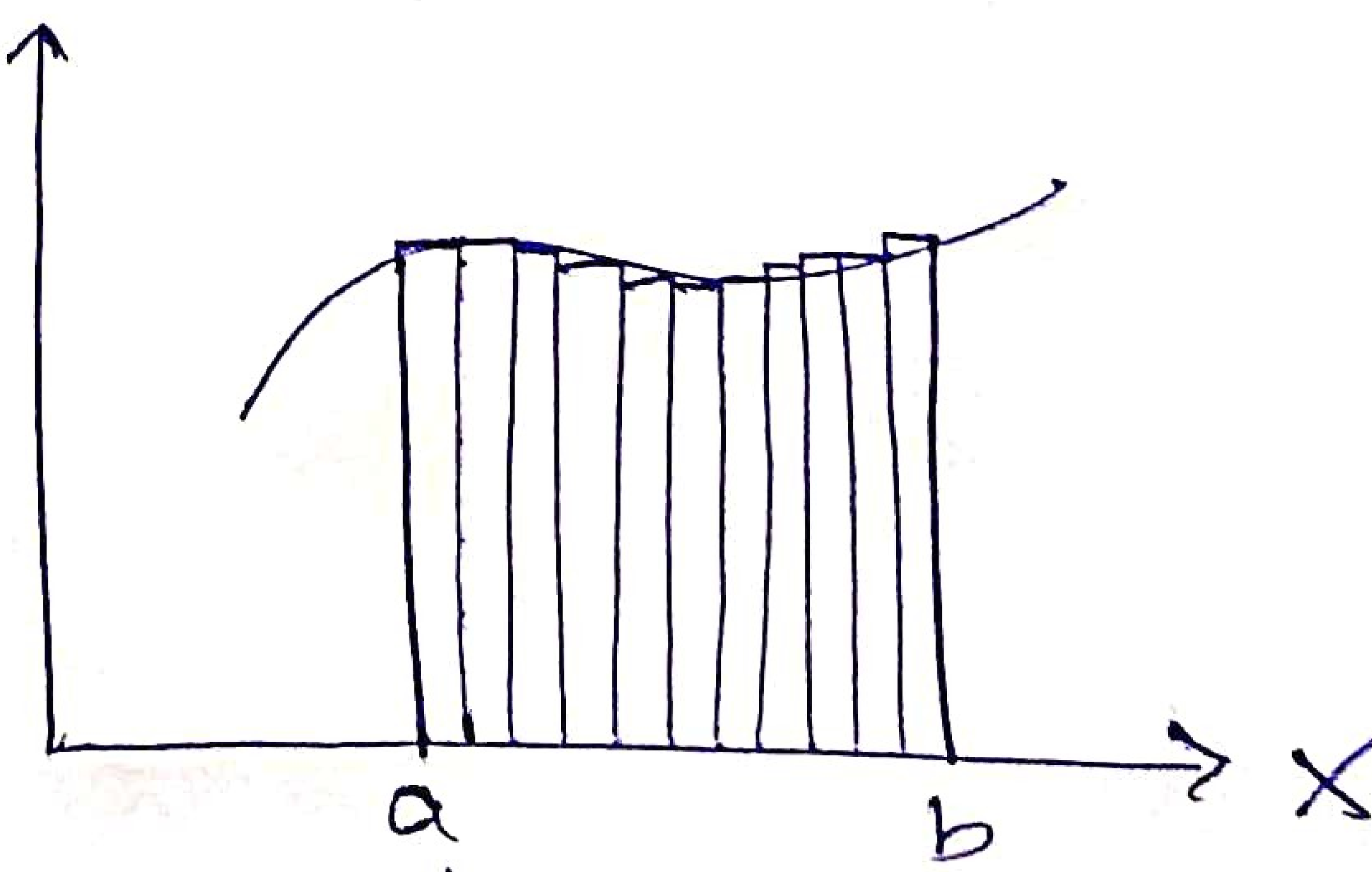
$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

★ (derivative) if this limit exists

→ Useful for optimization problem.
Minimum or maximum of f occurs where $f'=0$ (or on boundary of domain or where f' is not defined)

★ Integration

$$\int_a^b f(x)\,dx = \text{"area under the graph"}$$



$a = x_0 < x_1 < x_2 \cdots < x_n = b$

$\Delta x = x_i - x_{i-1} = \frac{b-a}{n}$

$x_i^* =$ a sample point with

$x_{i-1} \leq x_i^* \leq x_i$

{n subintervals}

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \underbrace{\sum_{i=1}^{n} f(x_i^*)}_{\{\text{height of the } i^{th} \text{ rectangle}\}} \underbrace{\Delta x}_{\{\text{width}\}}$$

* Fundamental theorem of Calculus

① $\int_a^b f'(x)\,dx = f(b) - f(a)$

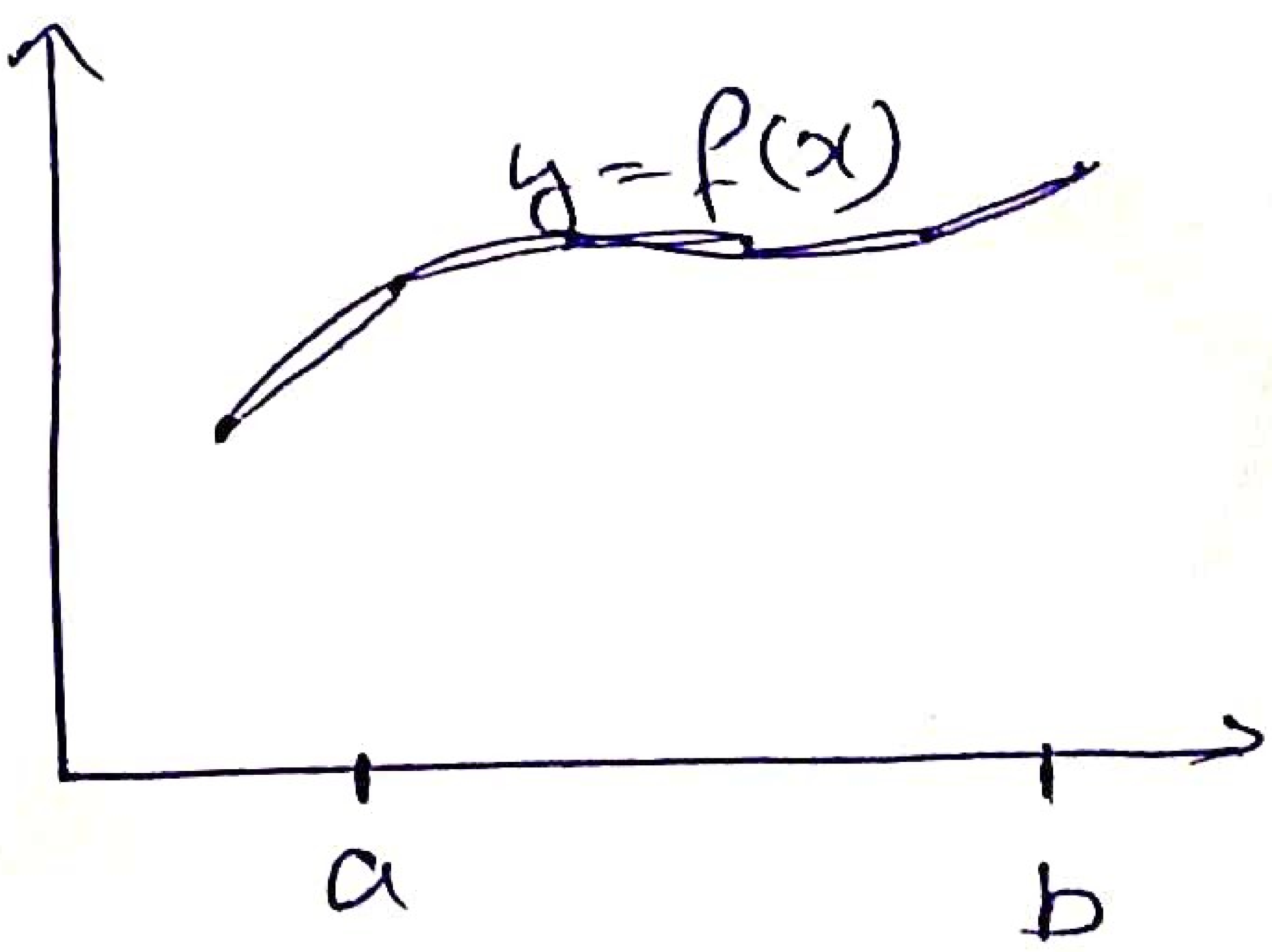
② $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

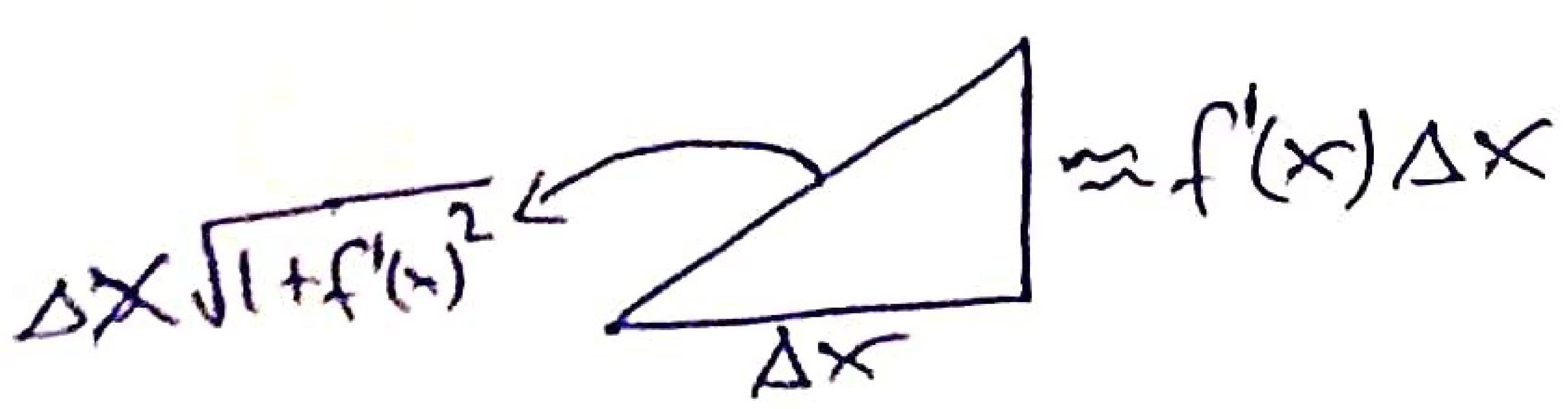
{First one can be deduced from the Second one}

* Other useful formulas

⇒ Length of the graph $y = f(x)$ from $(a, f(a))$ to $(b, f(b))$

$$L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$





⇒ Area of Surface of revolution around X-axis:

$$A = \int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}\,dx$$

⇒ In this Course:

- More general curves & surfaces in 2d and 3d.
- Functions of 2 or 3 variables
- Partial derivatives
  - Minima & Maxima of functions of 2 or 3 variables.
- Integration in 2 or 3 dimension
- Fundamental Theorem of Line Integrals
- Green's Theorem
- Stokes Theorem
- Divergence Theorem

→ Generalization of Fundamental Theorem of Calculus

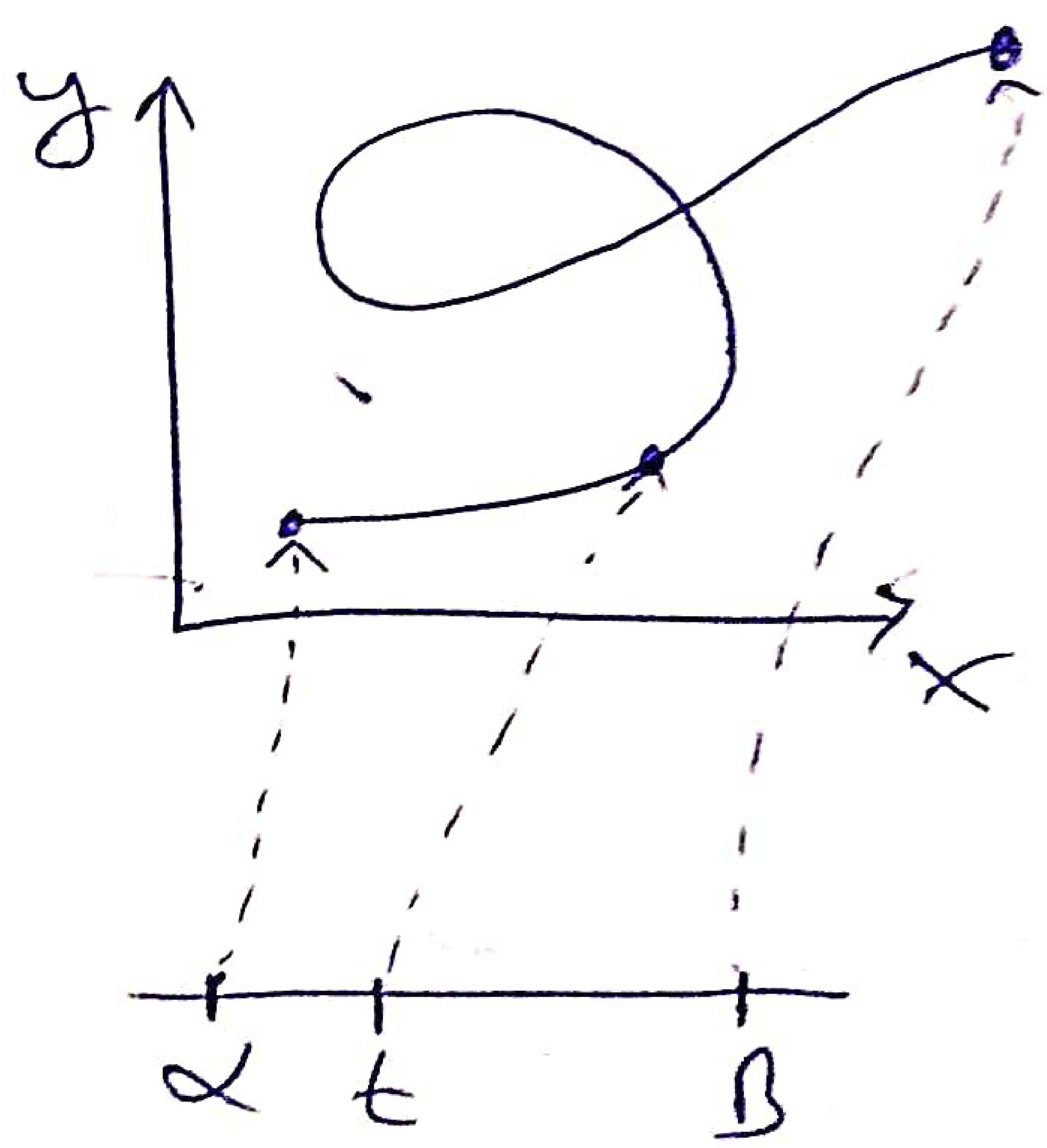
★ <u>Introduction to parametrized curves</u>

$x = f(t)$

$y = g(t)$

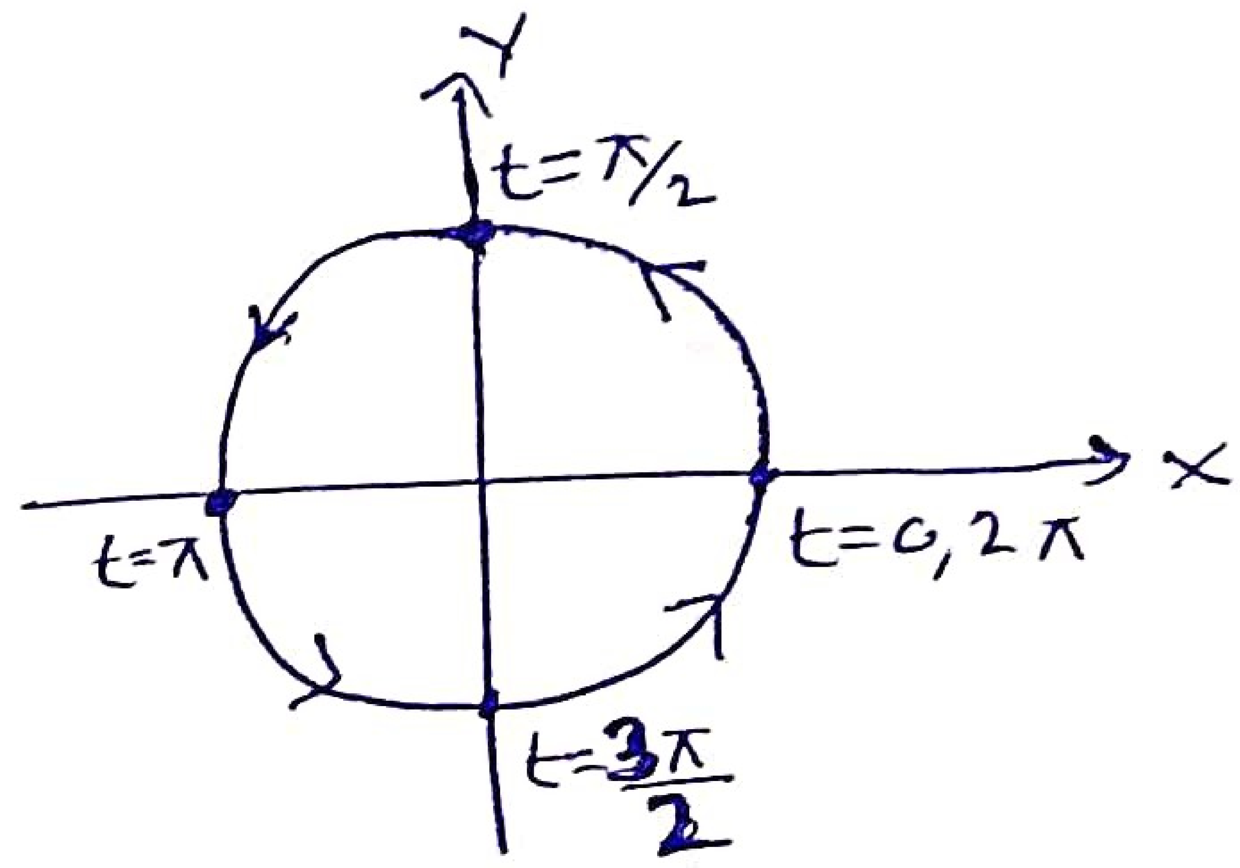
$\alpha \leq t \leq B$

→ Parameter (You can think of it as time)



Example 1

$$X = \cos t$$
$$Y = \sin t$$
$$0 \leq t \leq 2\pi$$

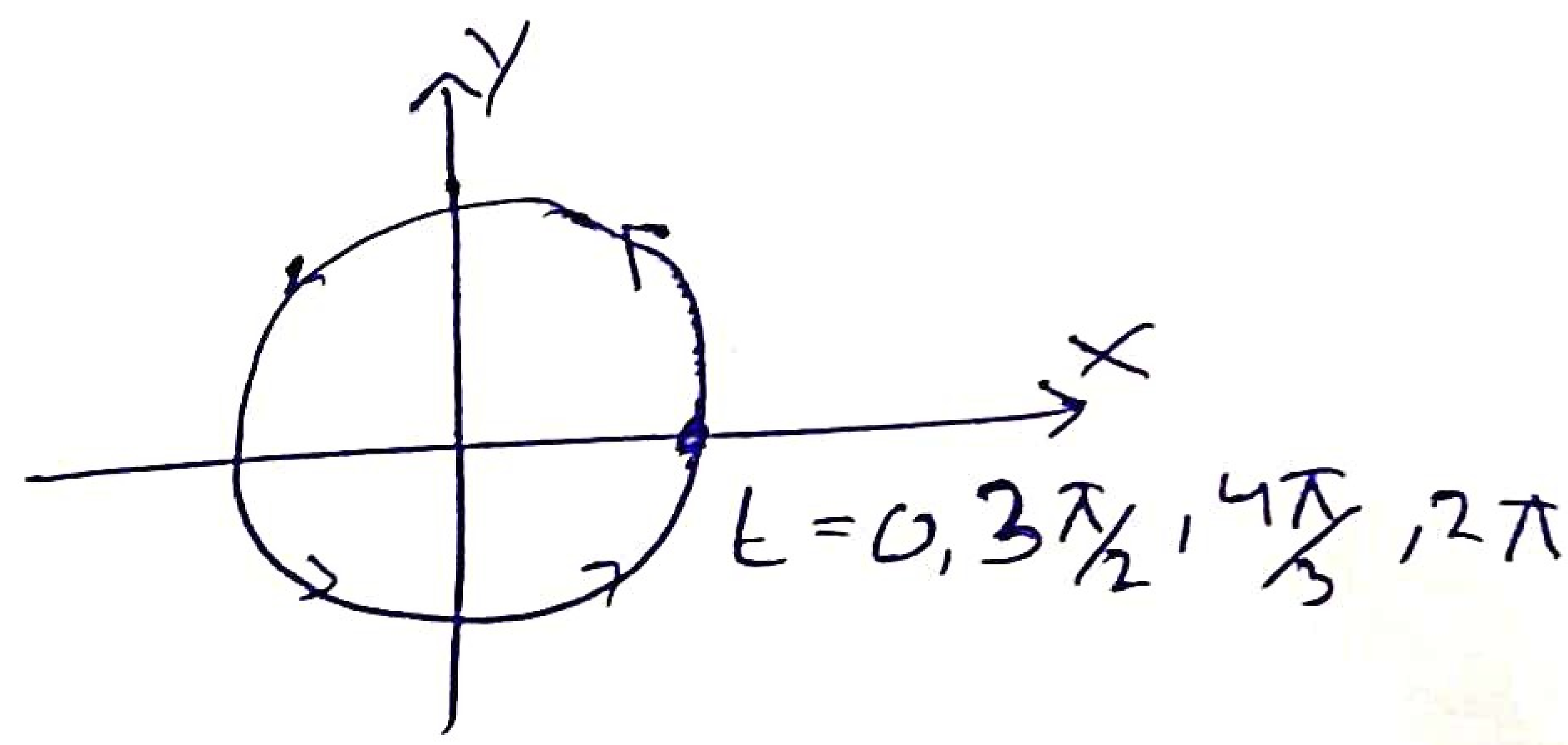


It is customary to draw arrows which indicate the direction in which t is increasing

Unit circle, going around counterclockwise at unit speed.

Example 2

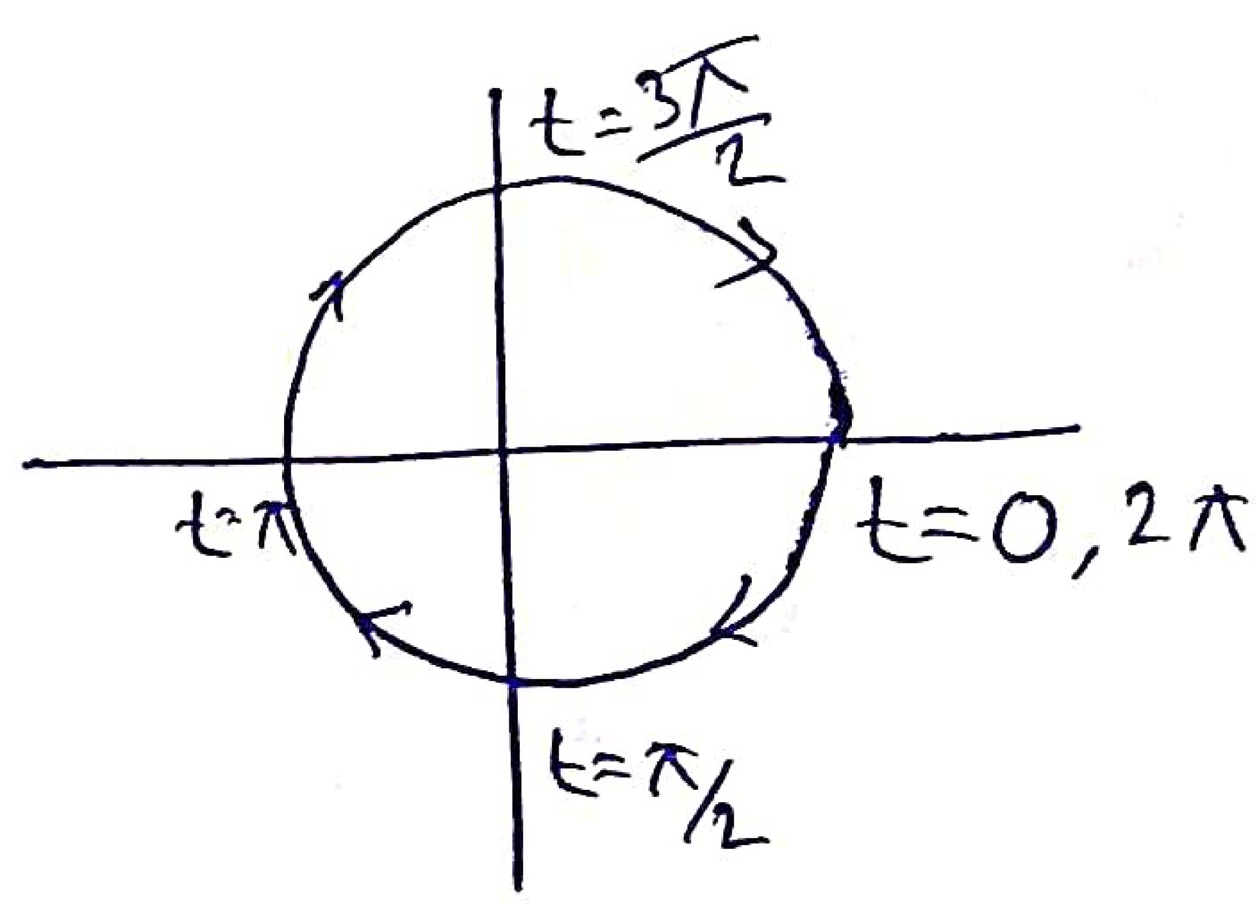
$$X = \cos(3t)$$
$$Y = \sin(3t)$$
$$0 \leq t \leq 2\pi$$



Unit circle going around counterclockwise three times.

Example 3

$$X = \cos(-t) = \cos t$$
$$Y = \sin(-t) = -\sin t$$



Unit circle going around clockwise.

$\Rightarrow$ Parametrize Curve

↳ Curve + Parametrization
(time-table)