Lecture -29

→ A^TA is positive definit!

→ Similar matrices

$$x^T(A^TA)x = (Ax)^T(Ax) = |Ax|^2 > 0$$

Hence positive definit

Similar Matrix

A & B are $n \times n$ matrix

A & B are Similar if for Some M

$$B = M^{-1}AM$$

⇒ Similar matrix have same eigen Value.

$$Ax = \lambda x \qquad \{B = M^{-1}AM\}$$

$$AMM^{-1}x = \lambda x$$

$$\Rightarrow (M^{-1}AM)M^{-1}x = \lambda M^{-1}x$$

$$\Rightarrow B(M^{-1}x) = \lambda(M^{-1}x)$$

So λ is the eigen Value of B.

but eigen Vector is not same

(Eigen Vector of B) = M^{-1} (Eigen Vector of A)

BAD Case $\lambda_1 = \lambda_2 = 4$

Small family $\quad M^{-1}\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}M = \begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$

Big family $\quad \begin{bmatrix}4 & 1\\ 0 & 4\end{bmatrix} \rightarrow$ Jordan form

Jordan Block

$$J_i = \begin{bmatrix}\lambda_i & 1 & 0 & 0 & 0\\ 0 & \lambda_i & 1 & 0 & 0\\ 0 & 0 & \lambda_i & \ddots & 1\end{bmatrix}$$

Every Square matrix A is similar to a Jordan matrix J

$$J = \begin{bmatrix}\boxed{J_1} & & \\ & \boxed{J_1} & \\ & & \boxed{J_3}\end{bmatrix}$$ # block = # Eigen Vector

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