Lecture-6

→ Vector Space and Subspace
→ Column Space of A : Solving $Ax = b$
→ Nullspace of A.

If S and T are two Subspaces :-

⇒ $S \cup T$ is not a Subspace
⇒ $S \cap T$ is a Subspace.

→ Let us consider U & V vectors in the intersection of S & T.
⇒ Linear combination of U & V is in S because U & V is in S.
⇒ linear combination of " " T " " " T.
⇒ So linear combination of U & V is in the intersection of S & T
⇒ So $S \cap T$ is a Subspace

Column Space of Matrix A.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}_{4\times 3} \rightarrow R^4 \text{ Subspace} \rightarrow C(A)$$

C(A) → all linear combination of the column.

Dose $Ax=b$ have Solution for every b?

4 equation, 3 unknown

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 3 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

{Not independent (i.e. sum of first two)} ⇒ So for every b there is no x

I Can Solve $Ax=b$ exactly when b is in Column Space of A.

Null Space of A $N(A)$

→ all Solutions $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ to $Ax=0$.

always R^3

⇒ Solutions of $Ax=0$ gives a Subspace

$$\left\{ \begin{array}{l} Av=0 \ \ \& \ \ Aw=0 \\ \text{then } A(v+w)=0 \end{array} \right\}$$