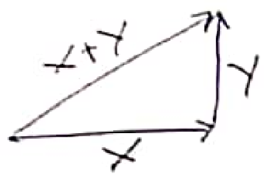
Lecture - 14

→ Orthogonal Vectors & Subspace
→ nullSpace ⊥ row Space
→ $N(A^TA) = N(A)$

Orthogonal Vectors

Condition for orthogonality

$$\boxed{x^Ty = 0}$$



$$|x|^2 + |y|^2 = |x+y|^2$$

$$\cancel{x^Tx} \quad \cancel{y^Ty} \quad (x+y)^T(x+y) \rightarrow \cancel{x^Tx} + \cancel{y^Ty} + 2x^Ty + \cancel{y^Tx}$$

$$x^Ty = 0$$

Subspace S is orthogonal to Subspace T.
Means: Every vector in S is orthogonal to every vector in T.

\# Row space is orthogonal to null space.
\# Column space is orthogonal to null space of A^T.
⇒ Nullspace and row space are orthogonal <u>Complements</u> in R^n.
↳ Nullspace contains all vectors ⊥ row space.

$Ax = b$ when there is no Solution.

$\boxed{A^TA}$ — $n\times m$ $m\times n$ — $n\times n$ → Square

→ Symmetric

$$\boxed{Ax = b}$$

$$\downarrow$$

$$\boxed{A^TA\hat{x} = A^Tb}$$

⇒ A^TA is invertible if exactly A has independent Columns.

$$\text{Rank}(A^TA) = \text{Rank}(A)$$