

## Derived Distribution

### ★ Derived Distribution (The discrete case)

$$Y = g(X) \quad \left\{ \begin{array}{l} X \text{ and } Y \text{ are two discrete} \\ \text{random variable} \end{array} \right\}$$

$$P_Y(y) = P(g(X) = y)$$

$$= \sum_{x: g(x)=y} P_X(x)$$

### ⑧ Linear function

$$\text{If } Y = aX + b$$

$$P_Y(y) = P_X\left(\frac{y-b}{a}\right)$$

### ⑧ Linear function of a continuous r.v

$$Y = aX + b \quad \forall a > 0$$

$$F_Y(y) = P(Y \leq y) = P(aX + b \leq y)$$

$$= P\left(X \leq \frac{y-b}{a}\right) = F_X\left(\frac{y-b}{a}\right)$$

$$f_Y(y) = f_X\left(\frac{y-b}{a}\right) \cdot \frac{1}{|a|}$$

★ Linear function of a normal r.v. is normal

$$X \sim N(\mu, \sigma^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

Let  $Y = aX + b$   $a \neq 0$

$$f_Y(y) = \frac{1}{|a|} f_X\left(\frac{y-b}{a}\right)$$

$$= \frac{1}{|a|} \frac{1}{\sqrt{2\pi}\sigma} e^{-\left(\frac{y-b}{a} - \mu\right)^2/2\sigma^2}$$

$$= \frac{1}{\sqrt{2\pi}\sigma|a|} e^{-(y-b-a\mu)^2/2a^2\sigma^2}$$

$$Y \sim N(a\mu + b, a^2\sigma^2)$$

★ A general function  $g(X)$  of a continuous r.v.

⇒ Two step procedure:

① Find the CDF of  $Y$ :  $F_Y(y) = P(Y \leq y)$

② Differentiate:  $f_Y(y) = \frac{dF_Y(y)}{dy}$



⇒ A general formula for the PDF of  $Y=g(X)$  when  $g$  is monotonic.

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dg^{-1}}{dy}(y) \right|$$

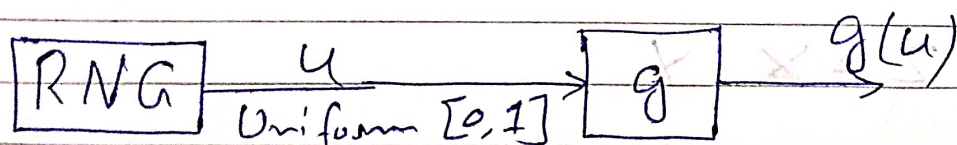
★ A function of multiple r.v's  $Z=g(X,Y)$

① Find the CDF of  $Z$ :  $F_Z(z) = P(Z \leq z)$   
 $= P(g(X,Y) \leq z)$

② Differentiate:  $f_Z(z) = \frac{dF_Z(z)}{dz}$

★ Simulation: Generate sample of  $X$  with given CDF  $F_X(\cdot)$

⇒ Modern computer generally have a random number generator which generates samples from a uniform distribution.



⇒ Find  $g$  so that  $g(u) \sim F_X$

$$g = F_X^{-1}$$