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Probability models and axioms

Probabilistic model

→ It is a quantative description of a Situation or experiment whose outcome is uncertain.

⇒ for building probabilistic model involves two Key steps:

- → We need to describe the possible outcome of the experiment. (i.e. Sample Space)
- → Then we specify a probability law which assigns probability to outcomes or collection of outcomes.

★ Sample Space

⇒ Set of possible outcomes (Ω)

⇒ The Set must be:

- → Mutually exclusive { There can be only one outcome from sample space }
- → Collectively exhaustive
- → At right granularity { All possible outcomes should be in the sample space }

★ Probability axioms

Event

- → A subset of the Sample space.
- → Probability is assigned to Events.

Axioms:

① Nonnegativity: $P(A) \geq 0$

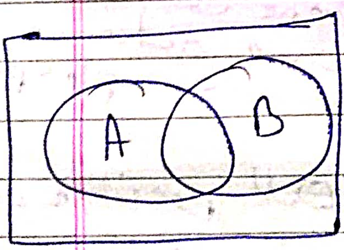
② Normalization: $P(\Omega) = 1$

③ Finite additivity:

If $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$

★ Properties that follows from Axioms

① $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.



$$P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(B \cap A^c) \quad \text{— ①}$$

$$P(A) + P(B) - P(A \cap B) = P(A \cap B^c) + P(A \cap B) + P(B \cap A^c) + \cancel{P(A \cap B)} \quad \text{— ①}$$

$$+ \; - \cancel{P(A \cap B)}$$

Using eq ① & ② we get out expression as

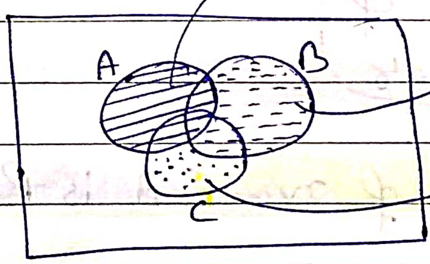
$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$

⇒ as $P(A\cap B) \geq 0$

⇒ $\boxed{P(A\cup B) \leq P(A) + P(B)}$

{Union bound}

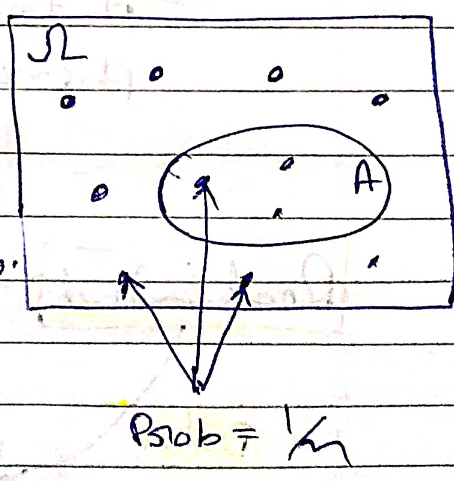
② $P(A\cup B\cup C) = P(A) + P(A^c\cap B) + P(A^c\cap B^c\cap C)$



★ <u>Discrete uniform Law</u>

⇒ Assume Ω (finite) consists of n equally likely elements.

⇒ Assume A consist of K elements.



$$\boxed{P(A) = K \times \frac{1}{n}}$$

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⇒ Strengthens the finite additivity axiom

Countable Additivity Axiom

If $A_1, A_2, A_3 \ldots$ is an infinite sequence of disjoint events, then $P(A_1 \cup A_2 \cup A_3 \cup \cdots)$
$= P(A_1) + P(A_2) + P(A_3) + \cdots$

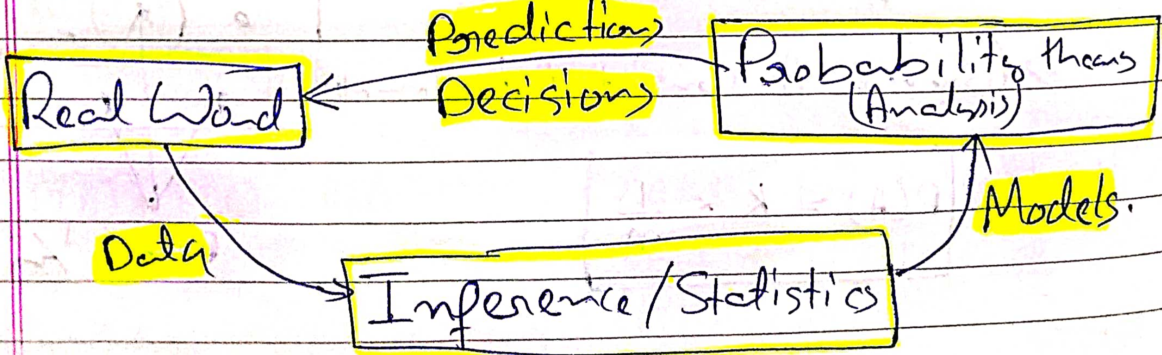
function whose domain is an integer

Interpretations of probability theory

→ "Frequency of event" A is P(A)

→ Description of beliefs.

→ A framework for analyzing phenomena with uncertain outcomes.



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1 Supplement
Mathematical Background

★ Set

"A collection of distinct elements"

Example :→ {'a', 'b', 'c', 'd'} {finite}:

→ R: real number {Infinite}

Notations

$x \in S$ ⇒ x belongs to set S

$x \notin S$ ⇒ x does not belongs to set S

$\{x \in A \mid \text{Constraints}\}$ ⇒ {A way of defining set is x belongs to a bigger Set A & satisfies the following Constraints.}

Ω : Universal set {Collection of all possibe object we may ever want to consider}

S^c ⇒ Complement of Set S

$S \subset T$ ⇒ S is subset of set T {equivalent to $\subseteq$}

$S \cup T$ ⇒ Union of Set S & T

$S \cap T$ ⇒ Intersection of Set S & T

$\bigcup_n S_n$ ⇒ Union of n Set $S_1 \ldots S_n$

$\bigcap_n S_n \Rightarrow$ Intersection of n sets $S_1 \cdots S_n$

★ Set properties

① $S \cup T = T \cup S$

② $S \cup (T \cup U) = (S \cup T) \cup U$

③ $(S^c)^c = S$

④ $(S \cap S^c) = \phi$

⑤ $S \cup \Omega = \Omega$

⑥ $S \cap \Omega = S$

⑦ $S \cap (T \cup U) = (S \cap T) \cup (S \cap U)$

⑧ $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$

★ De Morgan's Laws

① $(S \cap T)^c = S^c \cup T^c$

② $(S \cup T)^c = S^c \cap T^c$

③ $\left(\bigcap_n S_n\right)^c = \bigcup_n S_n^c$

④ $\left(\bigcup_n S_n\right)^c = \bigcap_n S_n^c$

★ Sequences

{Collection of elements, indexed by natural numbers.}

Notation: a_i, $\{a_i\}$

$i \in \mathbb{N} - \{1, 2 \ldots\}$

$a_i \in S$

Formally: $f: \mathbb{N} \to S$

$f(i) = a_i$

⇒ A sequence is said to converge if:

$$\boxed{\lim_{i \to \infty} a_i = a}$$

Properties

① if $a_i \to a$, $b_i \to b$ $a_i \pm b_i \to a \pm b$ *

② if $a_i \to a$ $g(a_i) \to g(a)$ $\forall$ continuous functions

★ Infinite Series

"Operation of adding infinitely many quantities, one after the other"

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$$\lim_{n \to \infty} \sum_{i=1}^{n} a_i \quad \{\text{Provided limit exists}\}$$

⇒ If $a_i \geq 0$: limit exists.

⇒ If terms a_i do not all have the same sign:

→ Limit need not exist
→ Limit may exist but be different if we sum in a different order.

→ limit exist & independent of order of Summation if

$$\sum_{i=1}^{\infty} |a_i| < \infty$$

★ Countable versus un-countable infinite set

- Countable : Can be put in 1-1 correspondence with positive integers

- Uncountable: not Countable.