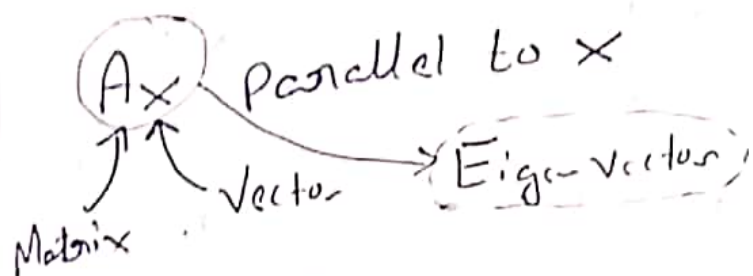


Lecture-21

→ Eigen values - Eigen Vectors

$$\rightarrow \det [A - \lambda I] = 0$$

$$\rightarrow \text{Trace} = \lambda_1 + \lambda_2 + \dots + \lambda_n$$



$$Ax = \lambda x$$

Diagram showing the equation $Ax = \lambda x$. An arrow points from λ to the circled phrase "Eigen Value".

If A is Singular, $\lambda = 0$ is eigen value.

What are the x 's and λ 's for projection matrix.

⇒ Any x 's in the plane will be a eigen Vector.

$$\Rightarrow \lambda = 1$$

⇒ $x \perp$ to the plane will be a eigen Vector

$$\Rightarrow \lambda = 0$$

Example

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Diagram showing the matrix A and its eigen vectors and values:

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \lambda = 1$$

~~$x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \lambda = -1$~~

$$x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \lambda = -1$$

$\Rightarrow N \times N$ will have N eigen values
 \Rightarrow Sum of λ 's = $a_{11} + a_{22} + \dots + a_{nn}$

definition

Fact

Trace \Rightarrow Sum of the element on the main diagonal

$$\text{Tr}(A) = \sum_{i=1}^n a_{ii} = a_{11} + a_{22} + \dots + a_{nn}$$

How to Solve $Ax = \lambda x$

$$\Rightarrow (A - \lambda I)x = 0$$

\rightarrow It has to be singular for some non zero x .

$$\Rightarrow |A - \lambda I| = 0$$

Eigen Value Equation

Let Q be matrix which rotates every vector by 90°

$$Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \det(A - \lambda I)$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

In case of upper Δ matrix eigen values are in the diagonal.