Lecture-8

→ Complete Solution of $Ax=b$.
→ Rank r

Solving $Ax=b$

⇒ Solvability Condition on b

→ $Ax=b$ solvable when b is in $C(A)$

{Column Space of A}

Complete Solution of $Ax=b$

① $X_{particular}$

→ Set all free Variables to Zero.
→ Solve $Ax=b$ for the pivot Variables.

↳ (X_p)

Add

② $X_{nullSpace}$ (X_n)

All vector from null Space

Comple Solution: $\boxed{X_c = X_p + X_n}$

Proof

$$A X_p = b$$

$$A X_n = 0$$

$$A(X_p + X_n) = b$$

$r<m$ $r<n$

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$$

(0 or ∞ Solution to $Ax=b$)

m×n matrix of rank r

{Number of pivots}

Rank of a matrix A is the dimension of the vector space generated by its column.

Tells you everything about number of solution

$\{r \leq m, r \leq n\}$

Full Column rank $r = n$

→ No free variable

→ {Unique solution if it exist}

(zero or one solution)

Full Row rank $r = m$

→ Can solve $Ax = b$ for every b

→ Left with $n - r$ free variables

$\boxed{r = m = n}$ {Full rank}

→ Matrix is invertible

→ $R = I$

→ Solution always exist

$r = m = n$	$r = n < m$	$r = m < n$
$R = I$	$R = \begin{bmatrix} I \\ 0 \end{bmatrix}$	$R = [I \; F]$
1 solution of $Ax = b$	(0 or 1 solution to $Ax = b$)	{∞ solution to $Ax = b$}