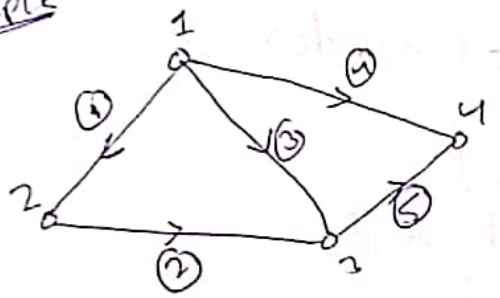
Lecture - 12

→ Graphs & Networks
→ Incidence Matrices
→ Kirchhoff's Laws

Graph : Bunch of nodes and Edges Connecting those nodes.

Example



$n = 4$ nodes

$m = 5$ edges

Incidence Matrix

$$
\begin{array}{c|c}
\text{node} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \quad \text{edge} \\
A = & \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix}
\end{array}
$$

Loop ⇒ Edges 1, 2 & 3 forms a loop

→ Loops correspond to linearly dependent rows.

$$A_x = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \\ x_4 - x_1 \\ x_4 - x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$X = x_1, x_2, x_3\ x_4$
as potential at nodes

$(x_2 - x_1)\ (x_3 - x_2)$ ---
as potential difference across edges

$$X = C \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \dim N(A) = 1$$

Rank = 3

$y_1\ y_2$ --- y_5 be the currents on the edges.

$A^T y = 0$ {Kirchhof's Current Law}

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Basis for null space of A^T

$$\begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

\# Graph with no loop $\Rightarrow$ <u>Tree</u>

$$\dim N(A^T) = m - r$$

(no of loops) (number of Edges) (number of nodes -1)

$$\boxed{\begin{pmatrix}\text{no of}\\ \text{nodes}\end{pmatrix} - \begin{pmatrix}\text{no of}\\ \text{Edges}\end{pmatrix} + \begin{pmatrix}\text{no of}\\ \text{Loops}\end{pmatrix} = 1}$$

Euler's formula

\# $Ax = e$ — Potential diff. / Voltage Source Comes here

\# $y = Ce$

\# $A^T y = 0$ — Current Source Can be put here

$$\boxed{A^T C A x = f}$$

It is always Symmetric.