Lecture - 26

Complex — inner product — Fourier matrix F_n

Vector — Discrete Fourier

Matrices — Fast Fourier Transform = FFT

Hermitian

$$Z^H \longrightarrow \bar{Z}^T$$

Inner Product of Y & X ⇒ $Y^H X$ {∀ Complex numbers}

⇒ Symmetric {for complex} ⇒ $Y^H = Y$

eg ⇒ $\begin{bmatrix} 2 & 3+i \\ 3-i & 5 \end{bmatrix}$

Called Hermitian matrix.

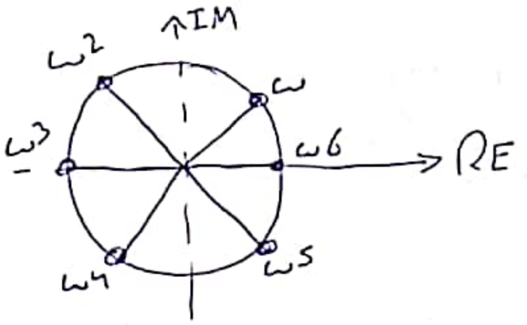
Fourier Matrix → Columns are orthogonal & normal

$$F_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & w & w^2 & \cdots & w^{n-1} \\ 1 & w^2 & w^4 & \cdots & w^{2(n-1)} \\ 1 & w^3 & w^6 & \cdots & w^{3(n-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & w^{n-1} & w^{2(n-1)} & \cdots & w^{(n-1)^2} \end{bmatrix}$$

w^{ij} {i, j begins from 0 to n-1}

$w^n = 1$ $\quad W = e^{\frac{i2\pi}{n}}$

for $n=0$ $\qquad \omega = e^{i\frac{2\pi}{6}}$



$F_n^H F_n = I$

$F_n^H \rightarrow$ Inverse of F_n

$$\boxed{\omega_{(n)}^2 = \omega_{(n/2)}}$$

Permutation matrix

$$[F_{64}] = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} 1 & 1 & & \\ & & 1 & 1 \\ 1 & & 1 & 1 \end{bmatrix} \quad \{FFT\}$$

64^2 Calculation

$2(32)^2 + \text{fix} \rightarrow 32$ Calculation

$D = \begin{bmatrix} 1 & \omega^1 & \cdots & \omega^{31} \end{bmatrix}$