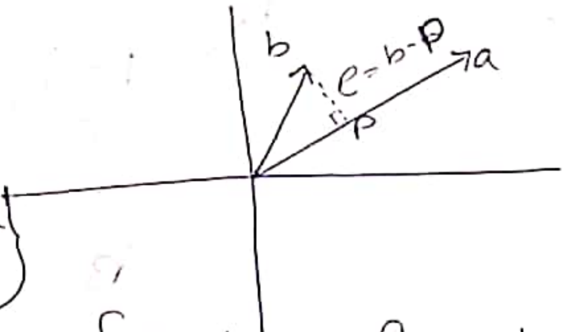
Lecture-15

→ Projection

→ Least Square projection Matrix

$$\boxed{P = xa}$$

{P is along a with some magnitude}



$$a^T(b - xa) = 0$$ {Condition for e to be ⊥ to a}

$$x a^T a = a^T b$$

$$\boxed{x = \frac{a^T b}{a^T a}} \rightarrow \boxed{P = a\frac{a^T b}{a^T a}}$$

Projection Matrix $= Pb = \frac{aa^T}{a^Ta} b$

→ Symmetric

→ $P^2 = P$

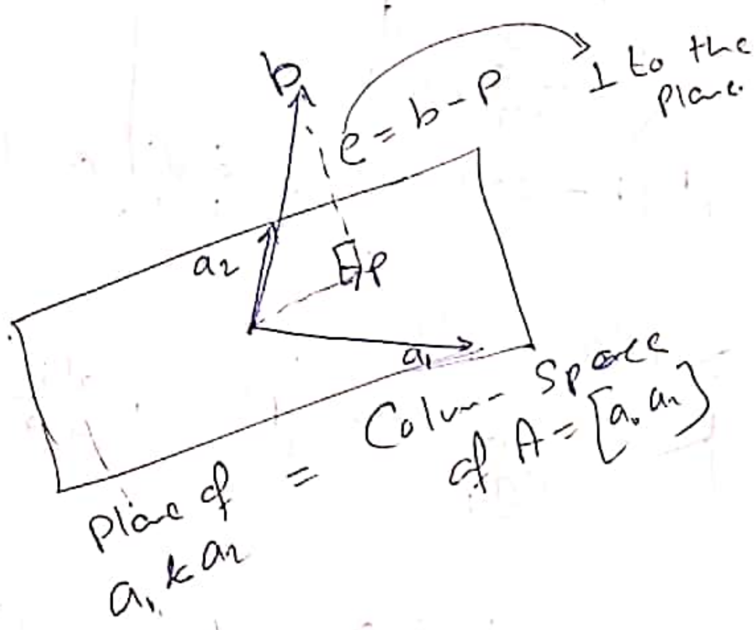
Column Space of P is a line thru a

Rank $= \underline{\underline{1}}$

Why Projection

Because $Ax = b$ may have no Solution

Solve $A\hat{x} = P$ → Projection of b onto Column Space



$$P = \hat{x}_1 a_1 + \hat{x}_2 a_2 = A\hat{x}$$

$P = A\hat{x}$ Find $\hat{x}$

Key: $b - A\hat{x}$ is $\perp$ to Columnn Space of A.

$a_1^T(b - A\hat{x}) = 0$

$a_2^T(b - A\hat{x}) = 0$

$$\begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix}(b - A\hat{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$A^T(b - A\hat{x}) = 0$$

e → It is in null space of A^T

$$A^TA\hat{x} = A^Tb$$

$$\hat{x} = (A^TA)^{-1}A^Tb$$

$$P = A\hat{x} = A(A^TA)^{-1}A^Tb$$

→ Projection Matrix

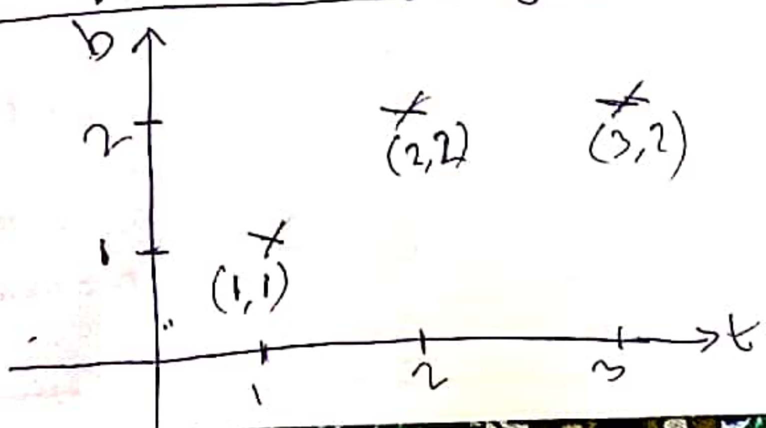
$$(A^TA)^{-1} \neq A^{-1}A^{T^{-1}}$$

→ because A is not invertible. Not a Square Matrix.

→ $P^T = P$

→ $P^2 = P$

Least Square fitting by a line



$b = C + Dt$ {Equation of line with C & D unknown}

$$\boxed{\begin{aligned} C + D &= 1 \\ C + 2D &= 2 \\ C + 3D &= 2 \end{aligned}} \rightarrow \underset{A}{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}} \underset{x}{\begin{bmatrix} C \\ D \end{bmatrix}} = \underset{b}{\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}$$

{Conditions the line Passing through all the point will satisfy}

$$A\hat{x} = P$$

best C & D

{Projection of b onto Column Space of A}

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