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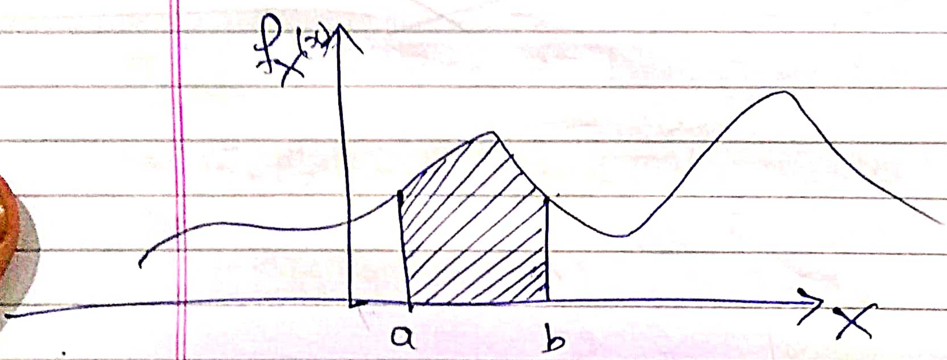
Continuous Random Variable I

★ Probability density functions (PDFs)

$$X=x \longrightarrow \boxed{PDF} \longrightarrow \frac{dP(X=x)}{dx}$$

So for event $a \le X \le b$

$$\boxed{P(a \le X \le b) = \int_a^b f_X(x)\,dx}$$



$$\boxed{\int_{-\infty}^{\infty} f_X(x)\,dx = 1}$$

Continuous random variable

↳ A random variable is continuous if it can be described by a PDF.

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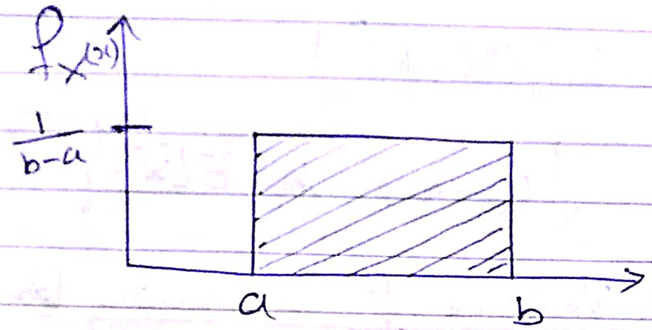
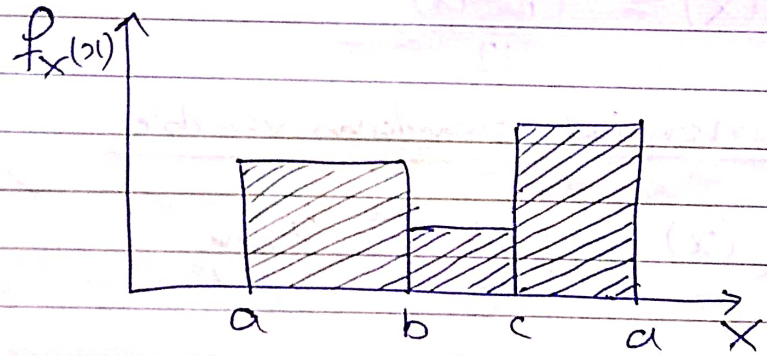
★ **Continuous uniform PDF**



⇒ It models a situation where we know X lies between a and b and nothing else.

⇒ Generalization: piecewise constant PDF



★ **Expectation of a continuous random variable**

$$\boxed{E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx}$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$$

{Expected value rule}

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★ **Variance**

$$Var(X) = E[(X-\mu)^2]$$

$$\{ \text{where, } \mu = E[X] \}$$

★ **Mean and Variance of Continuous Uniform random Variable**

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \frac{a+b}{2}$$

$$Var(X) = \frac{(b-a)^2}{12}$$

★ **Exponential random variable**

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$E[X] = 1/\lambda \qquad Var(X) = 1/\lambda^2$$

★ **Cumulative distribution function (CDF)**

$$\boxed{F_X(x) = P(X \leq x)}$$

⇒ Valid for both continuous & discrete random variable.

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★ Normal (Gaussian) random variable

⇒ Most important probability density function is probability theory.

⇒ Most common model for random noise

⇒ Standard normal $N(0,1)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

⇒ General normal $N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)}$$

★ Linear function of a normal random variable

⇒ Let $X \sim N(\mu, \sigma^2)$ {X is distributed normally with mean μ, & ~~variance~~ variance σ^2}

⇒ If $Y = aX + b$

$$Y \sim N(a\mu + b, a^2\sigma^2)$$