Lecture-21

→ Eigen values - Eigen Vectors

→ $\det[A - \lambda I] = 0$

→ Trace $= \lambda_1 + \lambda_2 + \cdots + \lambda_n$

Ax parallel to x

Matrix Vector Eigen-vector

$Ax = \lambda x$

Eigen Value

Eigen Vectors

Eigen Values

If A is Singular, $\lambda = 0$ is eigen value.

What are the x's and λ's for Projection matrix.

⇒ Any x's in the plane will be a eigen vectors

⇒ λ's $= 1$

⇒ $x \perp$ to the plane will be a eigen Vector

⇒ $\lambda = 0$

Example

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\lambda = 1$

$x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\lambda = -1$

$\Rightarrow$ N×N will have N eigen value

$\Rightarrow$ Sum of λ's = $a_{11} + a_{22} + \cdots a_{nn}$

Fact

$\rightarrow$ definition

Trace $\Rightarrow$ Sum of the element on the main diagonal

$$tr(A) = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \cdots a_{nn}$$

How to Solve $Ax = \lambda x$

$$\Rightarrow (A - \lambda I)x = 0$$

$\rightarrow$ It has to be Singular for some non zero x.

$$\Rightarrow \boxed{|A - \lambda I| = 0}$$

$\rightarrow$ Eigen Value Equation

Let Q be matrix which rotate every vector by 90°

$$Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \det(Q - \lambda I)$$

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

In case of upper Δ matrix eigen value are in the diagonal.