Lecture-7

- → Computing the null Space ($Ax=0$)
- → Pivot Variables – Free Variables
- → Special Solution – $\text{rref}(A) = R$

Consider

$$A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix}$$

we are interested in finding null Space of A.

$\Rightarrow \quad Ax = 0$

{we need to find all x for which above equation is true}

$$A \rightarrow \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$U \leftarrow \begin{bmatrix} \boxed{1} & 2 & 2 & 2 \\ 0 & 0 & \boxed{2} & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Pivots

Pivot Columns

Free Columns

{Rank of A = no of Pivots = 2}

$Ux = 0$

$$x_1 + 2x_2 + 2x_3 + 2x_4 = 0$$
$$2x_3 + 4x_4 = 0$$

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

{echelon form}

{We can take any value of x_2 & x_4 as they combine with free columns}

Let us take $x_2 = 1$ & $x_4 = 0$

$\cancel{1 \quad 2 \quad 2}$ So $x_3 = 0$

$x_1 = -2$

$$X = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Let us take $x_2 = 0$ & $x_4 = 1$

So $x_3 = -2$

$x_1 = 2$

$$X = \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

So, Null Space A is

$$N(A) = c\begin{bmatrix}-2\\1\\0\\0\end{bmatrix} + d\begin{bmatrix}2\\0\\-2\\1\end{bmatrix}$$

R = reduced row echelon form

→ Zero above & below Pivots

→ & Pivots = 1

$$A \to \begin{bmatrix}1 & 2 & 2 & 2\\2 & 4 & 6 & 8\\3 & 6 & 8 & 10\end{bmatrix} \to \begin{bmatrix}1 & 2 & 2 & 2\\0 & 0 & 2 & 4\\0 & 0 & 0 & 0\end{bmatrix}$$

Echelon form

{ Matlab command to get R ⇒ rref(A) }

$$\begin{bmatrix}\boxed{1} & 2 & 0 & -2\\0 & 0 & \boxed{1} & 2\\0 & 0 & 0 & 0\end{bmatrix}$$

Reduced row echelon form

$$Rx = 0$$

$$\begin{bmatrix}I & F\\0 & 0\end{bmatrix}\begin{bmatrix}x_{pivot}\\x_{free}\end{bmatrix} = 0 \Rightarrow x_{pivot} I + x_{free} F = 0$$

$$x_{pivot} = -x_{free} F$$

$$\begin{bmatrix}-F\\I\end{bmatrix}$$