The Central limit Theorem

★ Different scalings of the sum of iid random variables

⇒ $X_1, X_2 \cdots X_n$ iid, finite mean μ & Variance σ^2

⇒ $S_n = X_1 + \cdots + X_n$

↳ Variance : $n\sigma^2$

⇒ $M_n = \frac{S_n}{n} = \frac{X_1 + \cdots + X_n}{n}$

↳ Variance : $\frac{\sigma^2}{n}$

⇒ $\frac{S_n}{\sqrt{n}} = \frac{X_1 + \cdots + X_n}{\sqrt{n}}$

↳ Variance = $\frac{n\sigma^2}{(\sqrt{n})^2} = \sigma^2$

★ The Central limit Theorem (CLT)

⇒ $X_1 \cdots X_n$ iid, finite mean μ & Variance σ^2

⇒ Let $S_n = X_1 + \cdots + X_n$

⇒ Let $Z_n = \frac{S_n - n\mu}{\sqrt{n}\sigma}$

⇒ $E[Z_n] = 0$

⇒ $Var(Z_n) = 1$

⇒ Let Z be a standard normal rv. $\sim N(0,1)$

⇒ Central limit theorem

↳ For every z: $\lim_{n \to \infty} P(Z_n < z) = P(Z \leq z)$