Observability and State estimation

★ State estimation

⇒ Lets consider a discrete-time system

$$x(t+1) = Ax(t) + Bu(t) + w(t)$$
$$y(t) = Cx(t) + Du(t) + v(t)$$

→ w is state disturbance or noise
→ v is sensor noise or error.

⇒ State estimation problem: estimate $x(s)$ from
$$u(0) \cdots u(t-1),\ y(0), \cdots y(t-1)$$

⇒ An algorithm or system that yields an estimate $x(s)$ is called an observer or state estimator.

⇒ Let's look at finding $x(0)$, with no state or measurement noise

$$\begin{bmatrix} y(0) \\ \vdots \\ y(t-1) \end{bmatrix} = O_t\, x(0) + T_t \begin{bmatrix} u(0) \\ \vdots \\ u(t-1) \end{bmatrix}$$

Where,

$$O_t = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{t-1} \end{bmatrix} \quad T_t = \begin{bmatrix} D & 0 & \cdots & \cdots \\ CB & D & 0 & \cdots \\ \vdots & \vdots & & \\ CA^{t-2}B & CA^{t-3}B & \cdots\ CB & D \end{bmatrix}$$

⇒ hence we have

$$O_t x(0) = \begin{bmatrix} y(0) \\ \vdots \\ y(t-1) \end{bmatrix} - T_t \begin{bmatrix} u(0) \\ \vdots \\ u(t-1) \end{bmatrix}$$

⇒ RHS is Known, $x(0)$ is to be determined.

★ <u>Observability matrix</u>

⇒ By CH theorem, each A^K is linear combination of $A^0, \dots A^{n-1}$, hence for $t \geq n$, $N(O_t) = N(O)$ where

$$O = O_n = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

is called the **observability matrix.**