High School Mathamatics

(Vector, Matrices
Probability)

Vector Algebra

A Euclidean vector represents a line segement with a definit direction.

{graphically as a arrow connecting initial point A and a terminal point B and denoted by $\overrightarrow{AB}$}

→ length of that line segment is called its magnitude.

* Unit Vector ⇒ Vector whose magnitude is unity.

Convention

$\vec{a}$ = Vector of magnitude Unity

$\vec{A}$ = A general vector

A = magnitude of $\vec{A}$

* **Equality of vector**

"By Saying two vectors $\vec{A}$ and $\vec{B}$ are equal to one another {$\vec{A} = \vec{B}$ or $\vec{B} = \vec{A}$} we mean that there magnitudes are equal and they have same direction"

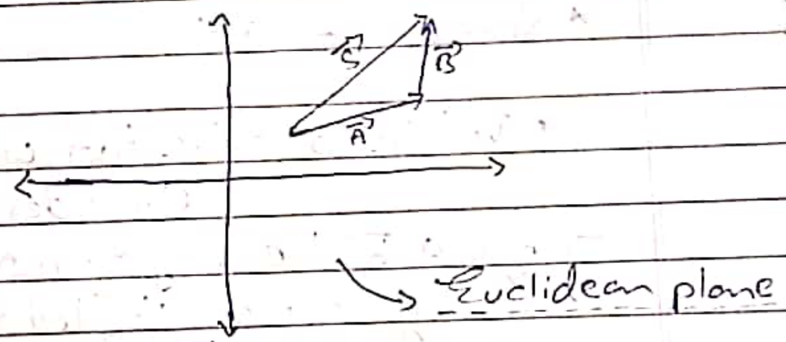
→ A vector can be represented in euclidean spece ~~B~~ by line segement joining ordered pair of ~~th~~ points.

- Rigid translation (or parallel shifting) of a given vector dose not change the vector.

* Addition of Vector

"Let $\vec{A}$ & $\vec{B}$ be any two vectors, drawn anywhere. Shift $\vec{B}$ so as to bring its origin to coincidence with the end point of A. The vector being thus linked up into a chain we call sum of $\vec{A}$ and $\vec{B}$ and denote by a thins vector $\vec{S}$ which runs from the beginning to the end of the chain"

$$\vec{S} = \vec{A} + \vec{B}$$



→ Vector addition is Commutative as well as Associative.

- $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$

→ The magnitude of vector sum are conveniently denoted by:

$$S = |\vec{A} + \vec{B}|$$

* Defination of scalar multiplication

"A vector $\vec{A}$ multiplied by a scalar $a \in R$ is called scalar multiplication and is denoted by $a\vec{A}$. Magnitude of $a\vec{A}$ is $a \times A$ and is directed along $\vec{A}$ if $a > 0$ and directed opposit to that of $\vec{A}$ if $a < 0$"

→ If $\vec{a}$ is unit vector of $\vec{A}$ then:-

$$\vec{A} = A\vec{a}$$

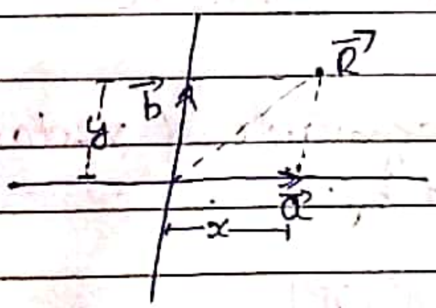
* Some terms :-

• Nul-Vector ⇒ Vector whose initial and terminal coincides.

→ Let $\vec{a}, \vec{b}$ be any two non-collinear unit vector. {Imagine them shifted so as to be coinitial}. Then any vector $\vec{R}$ contained in or parallel to the plane $\vec{a}, \vec{b}$ can obviously be expressed by :-

$$\vec{R} = x\vec{a} + y\vec{b}$$

where x, y are some scalar number.



→ Similarly, if $\vec{a}, \vec{b}, \vec{c}$ be any three non-coplanar vector which we may again take as unit vector, then any vector whatever can be expressed in the form :-

$$\vec{R} = x\vec{a} + y\vec{b} + z\vec{c}$$

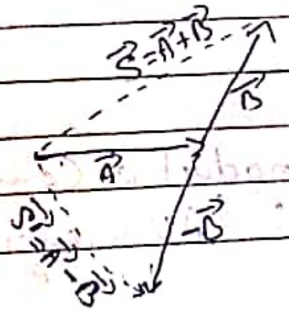
→ The scalars x, y, z are called the component of $\vec{R}$ taken along $\vec{a}, \vec{b}, \vec{c}$ as axis.

→ These axis may be chosen at our will either perpendicularly or obliquely to one another.

{ Conventionally $\hat{i}$, $\hat{j}$ and $\hat{k}$ are chosen as three mutually perpendicular unit vector as axis. }

★ Subtraction of Vector $\{ -\vec{A} = (-1)\vec{A} \}$

$\vec{S'} = \vec{A} - \vec{B}$, Can be thought as $\vec{A} + (-\vec{B})$, when $-\vec{B}$ means Scalar multiplication of (-1) with $\vec{B}$.



Algebric rules governing Vector addition Substraction and Scalar multiplication

(i) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ {vector addition is Comutative}

(ii) $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$ {Vector addition is Associative}

(iii) $\vec{A} + \vec{O} = \vec{A}$ {Null vector is identity element of vector addition}

(iv) $\vec{A} + (-\vec{A}) = \vec{O}$ {$-\vec{A}$ vector is inverse element of $\vec{A}$}

(i) $\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$ {Vector substraction is not Commutative}

(ii) $(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$ {Vector Substraction is not Associative}

(iii) $\vec{A} - \vec{O} = \vec{A}$ {Null vector is identity element of Vector substraction}

(iv) $\vec{A} - \vec{A} = \vec{O}$ {$\vec{A}$ vector is inverse element of $\vec{A}$}

(i) $a(\vec{A} \pm \vec{B}) = (\vec{A} \pm \vec{B})a = a\vec{A} \pm a\vec{B}$

{Scalar multiplication of vector is distributive over Vector addition and Substraction.}

★ Dot product of two vector

"The dot product of two Euclidean vectors $\vec{A}$ and $\vec{B}$ is defined as :-

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos\phi \quad \{\phi \text{ is angle between } \vec{A} \,\&\, \vec{B}\}$$

Properties

(i) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ {Dot product is Comutative}

(ii) $\vec{A} \cdot (\vec{B} \cdot \vec{C}) \neq (\vec{A} \cdot \vec{B}) \cdot \vec{C}$ {Associative property for dot Product is not defined}

(III) & (IV) Identity element and Inverse element is not defined for dot product

(V) $\vec{A}\cdot(\vec{B}\pm\vec{C}) = (\vec{A}\cdot\vec{B}) \pm (\vec{A}\cdot\vec{C})$ {Dot product is distributive over vector addition and substraction}

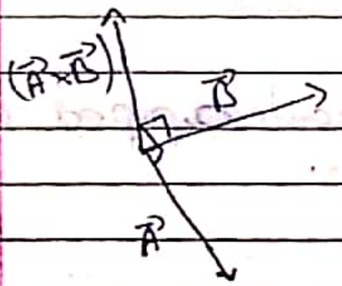
(VI) $(C_1\vec{A})\cdot(C_2\vec{B}) = C_1C_2(\vec{A}\cdot\vec{B})$ {Scalar multiplication is bidistributive over dot product}

(VII) $\vec{a}\cdot\vec{B} = \vec{A}\cdot\vec{C} \not\Rightarrow \vec{B} = \vec{C}$ {elements of dot product do not obey cancellation Law}

★ Cross product of Vectors

"The cross product is defined by the formula :-

- $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$ {$\hat{n}$ is unit vector directed ⊥ to the plane of $\vec{A}$ & $\vec{B}$ accordingly with 'right hand thumb rule"}



Properties

(I) $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ {Cross product is anti commutative}

(II) $\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$ {Cross product is not associative}

(III) (IV) Identity element and inverse element is not defined for Cross product.

(v) $\vec{A}\times(\vec{B}\pm\vec{C}) = (\vec{A}\times\vec{B}) \pm (\vec{A}\times\vec{C})$ {Cross product is distributive over vector addition & substraction}

(vi) $C_1\vec{A}\times C_2\vec{B} = C_1C_2(\vec{A}\times\vec{B})$ {Scalar multiplication is bi distributive over Cross product}

(vii) $\vec{A}\times\vec{B} = \vec{A}\times\vec{C} \not\Rightarrow \vec{B}=\vec{C}$ {Cross product do not obeay Cancellation Law}

(viii) $\vec{a}\times(\vec{B}\times\vec{C}) + \vec{b}\times(\vec{C}\times\vec{a}) + \vec{C}\times(\vec{a}\times\vec{b}) = 0$ {Jacobi identity}

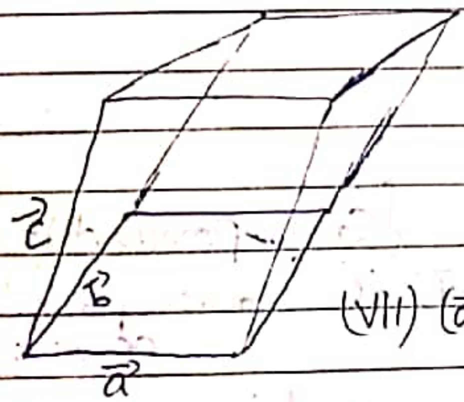
(ix) $|\vec{a}\times\vec{b}|^2 = a^2b^2 - (a\cdot b)^2$ {Lagranges identity}

★ <u>Scalar triple product</u>

Geometrically, the scalar triple product :-

$$\vec{a}\cdot(\vec{b}\times\vec{c})$$

is the volume of the parallelopiped defined by the three vector given.



(VIII) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$

{Lagranges Identy}

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Properties

(i) The scalar triple product is invariant under a circular shift of its three operands $(\vec{a}, \vec{b}, \vec{c})$

$$\vec{a}\cdot(\vec{b}\times\vec{c}) = \vec{b}\cdot(\vec{c}\times\vec{a}) = \vec{c}\cdot(\vec{a}\times\vec{b})$$

(ii) Circular shift property is anti commutative.

$$(a,b,c) = -(b,a,c)$$

(iii) $$\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \\ \vec{b}_1 & \vec{b}_2 & \vec{b}_3 \\ \vec{c}_1 & \vec{c}_2 & \vec{c}_3 \end{vmatrix}$$

(iv) If scalar product is zero, then the three vectors must be co-planer.

(v) Dot product is not distributive over Cross product.

(vi) $$[\vec{a}\vec{b}\vec{c}]^2 = \begin{vmatrix} \vec{a}\cdot\vec{a} & \vec{b}\cdot\vec{a} & \vec{c}\cdot\vec{a} \\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{c}\cdot\vec{b} \\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} = [\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]$$

$$\vec{x}\times(\vec{y}\times\vec{z}) = (\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z}$$ {Vector tripple product}

Matrix {Algebra}

Syllabi

-> Matrices a rectangular array of number
-> Equality of Matrices.
-> Operation of Matrices
 * Unary operation
  -> Transpose of matrix
  -> determinant of square matrices up to three.
  -> Inverse
 * Binary
  -> addition and Substraction
  -> Scalar multiplication
  -> Product of matrices

with properties of these operation

-> Diagonal, Symmetric and Skew-symmetric with there properties.
-> Solutions to System of linear equation in two or three variable.

① Defination

"A matrix is a rectangular array of numbers or any mathematical object for which operations such as addition and multiplication are defined"

$$\begin{array}{c} \text{Rows} \downarrow \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ \vdots & \vdots & \vdots & \end{bmatrix}$$

$\xrightarrow{\text{Columns}}$

$a_{ij} \Rightarrow$ General Element

"Horizontal rows & Vertical Columns"

② ★ Size $\Rightarrow$ The size of a matrix is defined by the number of rows and Column that it Contain.

Eg $\Rightarrow$ A matrix with n rows and m Column is of $n \times m$ Size.

② Equality of Matrices

"Two matrices A & B of same Size $(n \times m)$ are equal if $a_{ij} = b_{ij}$ $\forall$ $i \leq n$ $j \leq m$"

$\rightarrow$ Equality of matrices is denoted as $A = B$.

★ Convention

→ Matrices are denoted by Capetal leters and its elements are denoted by Small leters.

③ Operations on Matrices

★ Binary Operations.

① Addition and Substraction of Matrices

"for two matrix A & B of Size $(n \times m)$

$A \pm B = [a_{ij} \pm b_{ij}]$ $\forall$ $1 \leq i \leq n$ & $1 \leq j \leq m$"

Properties {Exactly Same as that of Number}

(I) $A+B = B+A$ && $A-B \neq B-A$

{Matrix addition is commutative but Matrix Substraction is not Commutative}

(II) $A+(B+C) = (A+B)+C$ && $A-(B-C) \neq (A-B)-C$

{Matrix addition is associative but Matrix substraction is not associative}

(III) $A \pm O = A$ {O is identity element of Matrix addition and substraction}

$$O \Rightarrow [O_{ij} = 0 \{zero\}] \Rightarrow \begin{bmatrix} 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

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(iv) $A + (-A) = 0$ & $A - A = 0$

{ ~~$\mp A$ is Inverse element of addition of A and substraction respectively of A~~ }

② Scalar multiplication :

'If $B = KA$ $\forall$ $A, B \in$ Matrix & $K \in R$ then $b_{ij} = Ka_{ij}$'

Properties

(i) $K(A \pm B) = KA \pm KB$ { Scalar multiplication is distributive over Matrix Addition and Substraction. }

③ Matrix Multiplication.

"Multiplication of two matrices is defines if and only if the number of Columns of the left matrix is the same as the number of rows of the right matrix"

→ If A is a $n \times P$ matrix and B is an $P \times m$ matrix, then their matrix product AB is the $n \times m$ matrix defined as :

$$C = AB \Rightarrow C_{ij} = \sum_{K=1}^{P} a_{iK} b_{Kj} \quad \forall 1 \leq i \leq n, \ 1 \leq j \leq m$$

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Properties

⟨I⟩ Matrix multiplication is not Commutative.

$$AB \Rightarrow [(AB)_{ij}] = \left[\sum_{K=1}^{p} a_{iK} b_{Kj}\right]$$

$$BA \Rightarrow [(BA)_{ij}] = \left[\sum_{K=1}^{p} b_{iK} a_{Kj}\right]$$

So $AB \neq BA$ {In general}

⟨II⟩ Matrix multiplication is Associative

$\cancel{[(AB)C]_{ij} = [\sum a}$

$\begin{cases} A = n \times p \\ B = p \times q \\ C = q \times m \end{cases}$

$(AB)C$

Let general element of AB be $(AB)_{ij}$ and general element of C be C_{ij}

$$(AB)_{ij} = \sum_{K=1}^{p} a_{iK} b_{Kj}$$

$\Rightarrow$ $\cancel{(AB)_{ij} C_{ij}} \Rightarrow$ Let general term of $(AB)C$ be $[(AB)C]_{ij}$

$$[(AB)C]_{ij} = \sum_{l=1}^{q} (AB)_{il} C_{lj}$$

$$\Rightarrow \sum_{l=1}^{q} C_{lj} \sum_{K=1}^{p} a_{iK} b_{Kl}$$

$$\Rightarrow C_{ij} [a_{i1} b_{11} + a_{i2} b_{21} + \cdots + a_{ip} b_{p1}]$$

$$+ C_{2j}\left[a_{i1} b_{12} + a_{i2} b_{22} + \cdots + a_{ip} b_{p2}\right]$$

$$+ \cdots$$

$$+ \vdots$$

$$+ C_{qj}\left[a_{i1} b_{1q} + a_{i2} b_{2q} + \cdots + a_{ip} b_{pq}\right]$$

$$\Rightarrow a_{i1}\left[b_{11} C_{1j} + b_{12} C_{2j} + \cdots\right]$$

$$\vdots$$

$$a_{i2}\left[b_{21} C_{1j} + b_{22} C_{2j} + \cdots\right]$$

$$\Rightarrow \sum_{l=1}^{p} a_{il} \sum_{k=1}^{q} b_{lk} C_{kj} \quad \text{--- (I)}$$

<u>$A(BC)$</u>

$$(BC)_{ij} = \sum_{k=1}^{q} b_{ik} C_{kj}$$

$$[A(BC)]_{ij} = \sum_{l=1}^{p} a_{il} (BC)_{lj}$$

$$\Rightarrow \sum_{l=1}^{p} a_{il} \sum_{k=1}^{q} b_{lk} C_{kj} \quad \text{--- (II)}$$

So from (I) & (II) we get :-

$$[(AB)C]_{ij} = [A(BC)]_{ij}$$

$$\Rightarrow (AB)C = A(BC)$$ // <u>Proved</u>

(III) Identity element

$AI_R = A$ {where I_R is Right Identity element of A}

→ If A is of Size $p\times q$ than I_R must be of size $q\times q$ as $[p\times q]\times[q\times q] = [p\times q]$

$$[AI]_{ij} = \sum_{k=1}^{q} a_{ik} I_{kj} = a_{ij}$$

$$\text{So, } I_{kj} = \begin{cases} 0 & \forall\, k \neq j \\ 1 & \forall\, k = 1 \end{cases}$$

$$\Rightarrow I_{jj} = 1 \quad \forall\, 1 \leq J \leq q$$

⇒ Right Identity element of a $p\times q$ matrix is a $q\times q$ square diagonal matrix with diagonal element 1.

Example

If $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times 3}$ $I_{AR} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$I_L A = A$ {Where I_L is left Identity element of A}

→ If A is of size $p \times q$ the I_L must be of size $p \times p$ as $[p \times p] \times [p \times q] = [p \times q]$.

$$[I_L A]_{ij} = \sum_{k=1}^{p} I_{ik} a_{kj} = a_{ij}$$

$$\text{So } I_{ik} = \begin{cases} 0 & \forall k \neq i \\ 1 & \forall k = i \end{cases}$$

$$\Rightarrow I_{ii} = 1 \quad \forall 1 \leq i \leq p$$

⇒ Left identity element of a $p \times q$ matrix is a $p \times p$ square diagonal matrix with diagonal element 1.

Example

If $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}_{2\times3}$ $\quad I_{AL} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

→ $I_{AL} = I_{AR}$ if and only if A is a square matrix.

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(IV) Distributive over matrix Addition and Substraction

$A(B+C) = AB + AC$ {Left distributive}
$(B+C)A = BA + CA$ {Right distributive}

(V) Inverse Element

$A_{p\times q} A_R^{-1} = I_{q\times q}$ {Right inverse}
$A_L^{-1} A_{p\times q} = I_{p\times p}$ {Left inverse}

From above expression Right inverse or left Inverse exists only if $p=q$.
Hence Inverse of a matrix exists if and only if it is a Square matrix.

★ Singular matrix ⇒ Square matrix that is non invertible is called Singular.

→ Determinant of every Singular matrix is zero.

Note
→ Inverse of a matrix will be discussed in detail afterward.

* Unary Operation

① Transpose

"If A be an matrix, then the matrix obtained by interchanging the rows and Column of A is called the transpose of A"

$A = [a_{ij}]_{m\times n}$ then $A^T = [a_{ji}]_{n\times m}$

Properties

① $(A^T)^T = A$

If $A = [a_{ij}]_{m\times n}$

$A^T = [a_{ji}]_{n\times m}$

$(A^T)^T = [a_{ij}]_{m\times n}$

$\Rightarrow (A^T)^T = A$

② $(KA)^T = KA^T$

$KA = [ka_{ij}]$

$(KA)^T = [Ka_{ji}] = K[a_{ji}]$

$\Rightarrow (KA)^T = K A^T$

③ $(A \pm B)^T = A^T \pm B^T$

$[A \pm B]_{ij} = a_{ij} \pm b_{ij}$

$[A \pm B]^T_{ij} = a_{ji} \pm b_{ji} = [A]^T_{ij} \pm [B]^T_{ij}$

$\Rightarrow (A \pm B)^T = A^T \pm B^T$

④ $(AB)^T = B^T A^T$

$[AB]_{ij} = \sum_{k=1}^{p} a_{ik} b_{kj}$

$[AB]^T_{ij} = \sum_{k=1}^{p} \cancel{a_{jk} b_{ki}} \; a_{ki} b_{jk}$ — (I)

$[B]^T_{ij} = b_{ji}$ $\quad [A]^T_{ij} = a_{ji}$

$\cancel{[BA]}\; [B^T A^T]_{ij} = \sum b_{jk} a_{ki}$ — (II)

// using (I) and (II) we get :

$[AB]^T_{ij} = [B^T A^T]_{ij}$

$(AB)^T = B^T A^T$

② Determinant

Determinat is a uniary operator on square matrix defined as :-

① $\forall$ 2×2 matrix {Area of parallelogram}

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

(ii) $\forall$ 3×3 matrix {Volume of parallelopiped}

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{21}\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}$$

Properties

(i) $|AB| = |A||B|$

(ii) $|A^{-1}| = 1/|A|$

(iii) $|A^T| = |A|$

(iv) $|A \pm B| \neq |A| \pm |B|$

(v) $|KA| = K^n|A|$ $K \in R$ and n = Order or Size of A

(vi) If any 2 rows (or Columns) are Interchanged, then Δ become $-\Delta$

(vii) $k\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & I \end{vmatrix} = \begin{vmatrix} Ka & Kb & KC \\ d & e & f \\ g & h & I \end{vmatrix}$

(viii) $\begin{vmatrix} a+x & b+y & c+z \\ d & e & f \\ g & h & I \end{vmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & I \end{vmatrix} + \begin{vmatrix} x & y & z \\ d & e & f \\ g & h & I \end{vmatrix}$

(ix) $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & I \end{vmatrix} = \begin{vmatrix} a+Kd & b+Ke & c+Kf \\ d & e & f \\ g & h & I \end{vmatrix}$

* Area of triangle with vertex as (x_1, y_1), (x_2, y_2) & (x_3, y_3)

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_1 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

of a

* Minors ⇒ The minor M_{ij} of the element a_{ij} of a square matrix A is determinant of matrix obtained by deleting the i^{th} row and j^{th} column.

* Cofactor ⇒ The Cofactor C_{ij} of element a_{ij} of a square matrix A is $(-1)^{i+j}M_{ij}$

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* Adjoint ⇒ The adjoint of a square matrix $A = [a_{ij}]$ is defined as transpose of the matrix $[C_{ij}]$ or $[C_{ij}]^T$.

→ Adjoint of the matrix A is denoted as Adj A.

③ Inverse

$$A^{-1} = \frac{\text{adj} A}{|A|}$$ ~~{Cramer's rule}~~

Properties

(i) $(A^{-1})^{-1} = A$
(ii) $(KA)^{-1} = K^{-1} A^{-1}$
(iii) $(A^T)^{-1} = (A^{-1})^T$
(iv) $(AB)^{-1} = B^{-1} A^{-1}$

④ Diagonal, Symmetric and Skew-Symmetric matrix with their properties

Ⓐ Diagonal matrix

"A diagonal matrix is a matrix in which the entity outside the main diagonal are all zero"

$$d_{ij} = 0 \text{ if } i \neq j \ \forall \ i, j \in \{1, 2, \ldots n\}$$

(B) Symmetric matrix ⇒ Square matrix that is equal to its transpose.

$$A = A^T$$

(C) Skew Symmetric matrix ⇒ Square matrix that is equal to negative of its transpose.

$$\boxed{A = -A^T}$$

(5) Solution of system of linear equation upto three variable

$$a_1x + b_1y + c_1z + K_1 = 0$$
$$a_2x + b_2y + c_2z + K_2 = 0$$
$$a_3x + b_3y + c_3z + K_3 = 0$$

$$\Downarrow$$

$$\frac{x}{\begin{vmatrix} b_1 & c_1 & K_1 \\ b_2 & c_2 & K_2 \\ b_3 & c_3 & K_3 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a_1 & c_1 & K_1 \\ a_2 & c_2 & K_2 \\ a_3 & c_3 & K_3 \end{vmatrix}} = \frac{z}{\begin{vmatrix} a_1 & b_1 & K_1 \\ a_2 & b_2 & K_2 \\ a_3 & b_3 & K_3 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

Probability Revision

Content

-> Addition and Multiplication rules of probability.
-> Bayes theorem
-> Independence of event.
-> Computation of probability using permutation and Combination.

★ Defination => If an event can happen in a ways and fail in b ways and each of these ways is equally likely, the probability or chance, of its happening is a/(a+b) and that of its failing b/(a+b).

=> If P is the probability of the happening of an event, the probability of not happening is 1-P.

★ Compound event => Events for which completion requires completion of two or more events.

=> Events are said to be dependent or independent according as the occurrence of one does or does not affect the occurrence of the other.

=> Events are said to be mutually exclusive if they cannot occure at the same time.

* Sample space ⇒ Sample space of an experiment is a set containing all possible outcome.

* Event ⇒ Event is a set of outcomes of an experiment to which probability is assigined.

→ Elementary event is event which contain single outcome.

→ Event is Subset of Sample Space.

① Addition theorem of Probability

Let us concider two events A & B associated with Sample space S; $A, B \subset S$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

If A & B are mutually exclusive then $P(A \cap B) = 0$

$$\Rightarrow P(A \cup B) = P(A) + P(B)$$

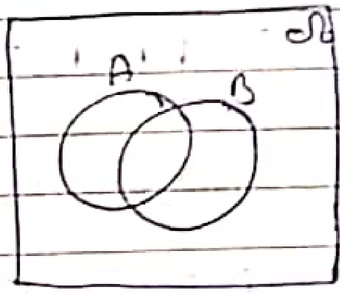
$P((A \cap B) \cup (A \cap C))$

{Corolary} # $P(A \cup B \cup C) = P(A \cup (B \cup C)) = P(A) + P(B \cup C) - P(A \cap (B \cup C))$

$$\Rightarrow P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$$

Venn diagram proof

$$P(A\cup B) = \frac{A+B-(A\cap B)}{S}$$



$$= \frac{A}{S} + \frac{B}{S} - \frac{(A\cap B)}{S}$$

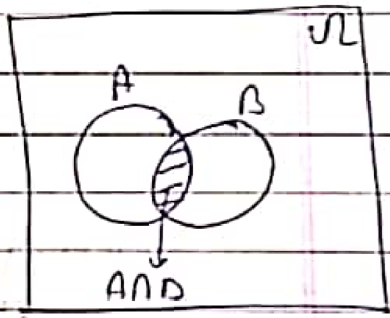
$$\Rightarrow P(A)+P(B)-P(A\cap B)$$

② Multiplication Rule and Conditional Probability

★ Conditional probability ⇒ Conditional probability measures the probability of an event given that another event has occured.

→ It is written as $P(A|B)$; {Probability of A in which B has occured}

$$P(A|B) = \frac{A\cap B}{B} = \frac{P(A\cap B)}{P(B)}$$



$\Rightarrow$ ~~B(A|B)~~ $\boxed{P(A|B) = \frac{P(A\cap B)}{P(B)}}$

{Corollary}★ $P(A\cap B) = P(B)\cdot P(A|B)$

$\Rightarrow$ $\boxed{P(A\cap B\cap C\cap \cdots) = P(A)\cdot P(B/A)\cdot P(C/AB)}$

{Multiplication Rule}

If A & B are two Indipendent event then $P(A/B) = P(A)$

$\Rightarrow \boxed{P(A \cap B) = P(A)\,P(B)}$

③ Law of Total Probability and Bayers theorem

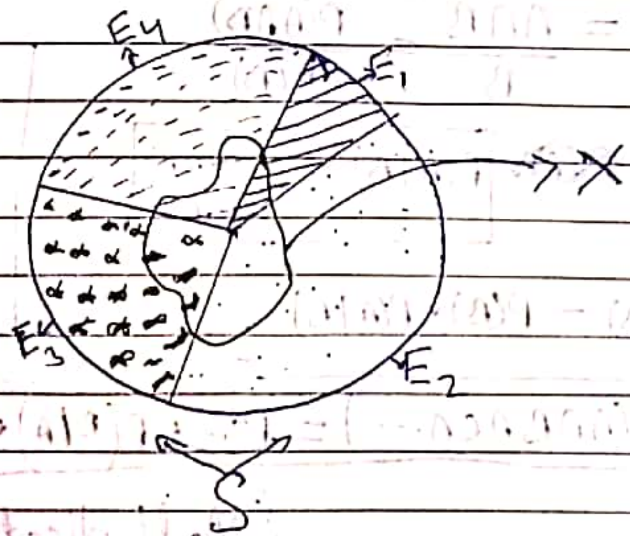
Let $E_1, E_2 \cdots E_n$ be the paintition of Sample spah such that :-

$\Rightarrow E_1 E_2 \cdots E_n \subset S$

$\Rightarrow E_1 \cup E_2 \cup \cdots E_n \equiv S$

Let us concider any event X in Sample Space S; $X \subset S$.

$$\boxed{P(X) = \sum_{i=1}^{n} P(E_i)\,P(X/E_i)}$$



$$P(E_i/x) = \frac{P(\cancel{X} \cap E_i)}{P(\cancel{X})} = \frac{P(E_i)\,P(\cancel{X}/E_i)}{P(x)}$$

$$\Rightarrow \boxed{P(E_i/x) = \frac{P(E_i)\,P(x/E_i)}{\sum_{j=1}^{n} P(E_j)\,P(x/E_j)}} \quad \left\{ \begin{array}{l} \text{Bayes} \\ \text{Theorem} \end{array} \right\}$$

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Random Variable and Probability distribution

★ Random Variable ⇒ A random variable is a real valued function whose domain includes Partitions of sample space of a random experiment

→ Partitions must be mutualy exclusive.

→ If $E_1, E_2 \cdots E_n$ denoted the partitions then:-

$$\sum_{i=1}^{n} P(E_i) = 1$$

★ example 1: Let us consider a random experiment of throwing n dies.

→ Let us divide Sample space as Events:-

E_1 = ~~getting~~ scoring a total of 1

E_2 = Scoring a total of 2.

| | | |

and so on.

⇒ So random variable for above partition to the given random experiment will take events $E_1, E_2 \cdots E_n$ and return probability of occuring of $E_1, E_2 \cdots E_n$ respectively.

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★ Example 2: Let a consider a random experiment of throwing n coins:

→ Let us divide the Sample Space as Events ÷
E_1 = getting exactly one head
E_2 = getting exactly two head.
| | | | | | | | |
| | | | | | | | |

and so on.

⇒ So random variable for above partition to the given random experiment will take events $E_1, E_2 \cdots E_n$ and return probability of occuring of $E_1, E_2 \cdots E_n$ respectively.

★ Probability distribution ⇒ Plote of Random variable in its domain is called probability distribution of that particular partition for that particular distribution.
{Plot of Probability Vs. Events}

★ Trial ⇒ Trial is any procedure that can be infinitely repeated and has a well-defined set of possible outcome.

→ A Random experement might inglude many trial.
Example ⇒ Throughting n dies. {Trial ⇒ Throughting die}

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* Binomial distribution ⇒ Binomial distribution is a probability distribution in which ~~each~~ trials has only two possibility success or failure.

→ Trials of Binomial distribution are called Bernoulli trials.

Let us consider a random experiment in which n bernoulli trials occurs. Let probability of success of a trial be P. Then probability of x $(x \leq n)$ succes is :-

$$\boxed{P(x,n,P) = \binom{n}{x} p^x (1-p)^{n-x}}$$

Proof

Let there be x success in n trials.

⇒ So probability of x sussess and $(n-x)$ failure in a particular order will be $\left(p^x (1-p)^{n-x}\right)$ where p is probability of Sucess in one trial.

⇒ x Sucess or $(n-x)$ failure can occure in $\binom{n}{x}$ or $\binom{n}{n-x}$ ways.

$$\Rightarrow P(x,n,P) = p^x (1-p)^{n-x} + p^x (1-p)^{n-x} + \cdots \binom{n}{x} \text{ times}$$

So, $\boxed{P(x,n,p) = \binom{n}{x} p^x (1-p)^{n-x}}$

Integral Calculus

Indefinite (formulae)

1) $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$

2) $\int \cos x \, dx = \sin x + c$

3) $\int \sin x \, dx = -\cos x + c$

4) $\int \sec^2 x \, dx = \tan x + c$

5) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

6) $\int \sec x \tan x \, dx = \sec x + c$

7) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

8) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$

9) $\int \frac{dx}{\sqrt{1+x^2}} = \tan^{-1} x + c = -\cot^{-1} x + c$

10) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\operatorname{cosec}^{-1} x + c$

11) $\int e^x \, dx = e^x + c$

12) $\int \frac{dx}{x} = \ln|x| + c$

13) $\int a^x \, dx = \frac{a^x}{\ln|a|} + C$

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14) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$

15) $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c$

16) $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + c$

17) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

18) $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left|x + \sqrt{x^2+a^2}\right| + c$

19) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + c$

20) $\int \sqrt{x^2+a^2}\, dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2+a^2}\right| + c$

21) $\int \sqrt{a^2-x^2}\, dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

Definite Integral (Properties)

1) $\int_a^b f(x)\, dx = \int_a^b f(t)\, dt$

2) $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

3) $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

4) $\int_a^b f(x)\, dx = \int_a^b f(a+b-x)\, dx$

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5) $\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$

6) $\int_{-a}^{a} f(x)\,dx \Rightarrow 2\int_0^{a} f(x)\,dx$ if f is even function $f(-x) = f(x)$

$\Rightarrow 0$ if f is odd function $f(-x) = -f(x)$