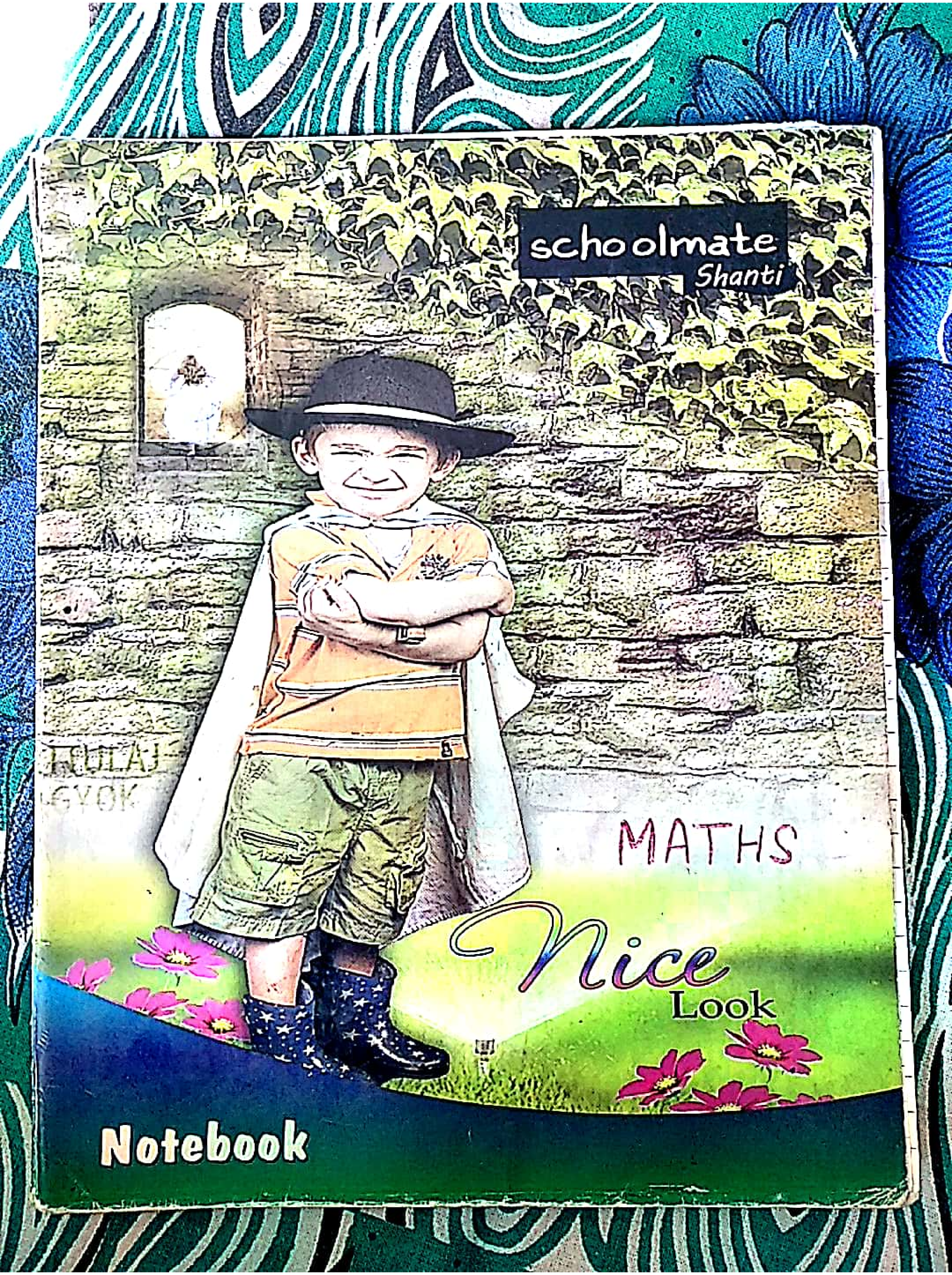
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Quadatic Equation

"The standard form of the quadratic equation is $ax^2+bx+c=0$ where a, b, c are real numbers and $a \neq 0$"

1) Roots of quadratic equation

Let α, β be roots of a quadratic equation $ax^2+bx+c=0$

$\rightarrow (\alpha, \beta) = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\rightarrow \alpha+\beta = -\frac{b}{a}$

$\rightarrow \alpha\beta = c/a$

$\rightarrow$ factorized form $\Rightarrow a(x-\alpha)(x-\beta)$

$\rightarrow$ If S be the sum and P be the product of roots, then quadratic equation is :-

$$x^2-Sx+P=0$$

2) Nature of roots: $\left(D = b^2-4ac\right)$

$\rightarrow D>0$; roots real and un-equal.

$\rightarrow D=0$; roots real and equal.

→ $D<0$, then the roots are non-real {complex of form $p+iq$ & $p-iq$}

→ If a quadratic expression in x has more than two roots, then it is an identity in x.

3) Common roots

ⓐ One root common

Let $ax^2+bx+c=0$ & $px^2+qx+r=0$ have a common root α then:

$\Rightarrow a\alpha^2+b\alpha+c=0 ; p\alpha^2+q\alpha+r=0$

$$\Rightarrow \boxed{\frac{\alpha^2}{(br-cq)} = \frac{-\alpha}{(ar-cp)} = \frac{1}{(aq-pb)}}$$

$$\Rightarrow \boxed{(ar-cp)^2 = (aq-bp)(br-cq)}$$

ⓑ Two common roots

→ both equation should be identical

$$\rightarrow \boxed{\frac{a}{p} = \frac{b}{q} = \frac{c}{r}}$$

4) Basic properties of inequalities

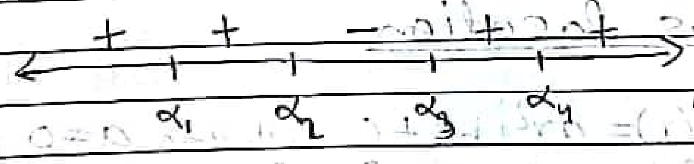
→ $a < b$ & $b < c \Rightarrow a < c$

→ $a < b \Rightarrow a+c < b+c$

→ $a < b \Rightarrow ac < bc \quad \forall\, c > 0$
$\Rightarrow ac > bc \quad \forall\, c < 0$

→ $a < b \Rightarrow a/c < b/c \quad \forall\, c > 0$
$\Rightarrow \frac{a}{c} > b/c \quad \forall\, c < 0$

5) $\dfrac{(x-\alpha_1)^a (x-\alpha_2)^b}{(x-\alpha_3)^c (x-\alpha_4)^d} = y \quad \{\alpha_4 > \alpha_3 > \alpha_2 > \alpha_1\}$ $\forall$ (a & d) are even & (b & c) are odd



Note

⇒ Right most side is always positive

⇒ Sign dosen't change when power is even.

6) Quadratic expression

"The expression ax^2+bx+c where $a \neq 0$ and $a,b,c \in R$ represents quadratic expression"

→ Quadratic expressents parabola with vertex at $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

→ If $a>0$, the parabola will be concave upward and if $a<0$, the parabola will be concave downwards.

→ The parabola cuts the x-axis at two points if $D>0$ or, never intersects the x-axis if $D<0$ or touches the x-axis if $D=0$.

7) Roots location

Let $f(x)=ax^2+bx+c$ where $a \neq 0$ $a,b,c \in R$ and $\delta, \delta_1, \delta_2 \in R$ and $\delta_1 < \delta_2$.

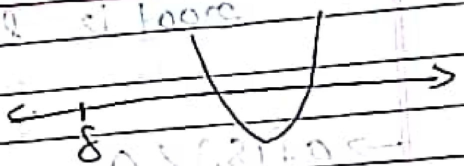
(i) If both roots are less than δ

→ $D \geq 0$

→ $a \cdot f(\delta) > 0$

→ $\frac{-b}{2a} < \delta$

(ii) If both are greater than δ

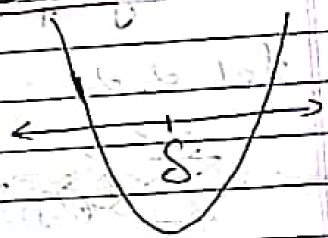


→ $D \geq 0$

→ $af(\delta) > 0$

→ $\delta < -\frac{b}{2a}$

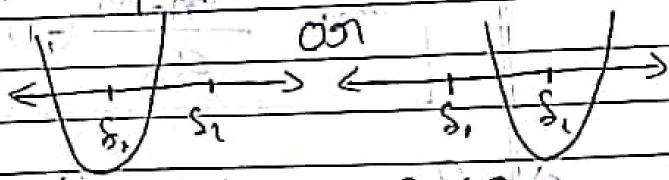
(iii) If δ lies between the roots



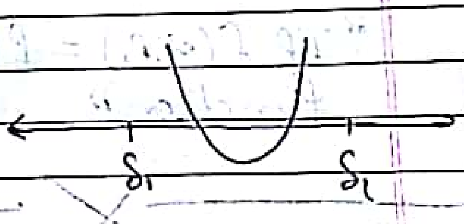
→ $af(\delta) < 0$

(iv) If exactly one roots lies between δ_1 and δ_2

→ $f(\delta_1)f(\delta_2) < 0$.



(v) If both roots lies between δ_1 & δ_2



→ $af(\delta_1) > 0$

→ $af(\delta_2) > 0$

→ $\delta_1 < \frac{-b}{2a} < \delta_2$

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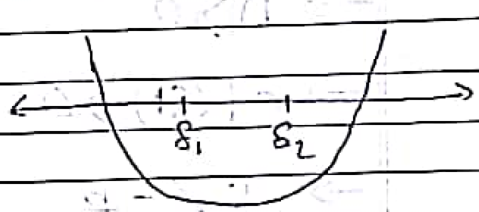
(VI) If one root is greater the δ_1 and the other root is less than δ_1

$\rightarrow af(\delta_1) < 0$



$\rightarrow af(\delta_1) < 0$

8) Theory of polynomial equation

Let $\alpha, \alpha_1 \cdots \alpha_n$ be roots of equation:

$$\sum_{i=0}^{n} a_i x^i = 0$$

$$\boxed{S_m = \frac{a_{n-m}}{a_n}(-1)^m}$$

Where S_m is sum of roots taken m at a time

9) Symmetric function

"If $f(\alpha,\beta) = f(\beta,\alpha)$ then f is symmetric function"

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Trigonometric functions

1) Radian ⇒ 1 radian is the angle subtended at the centre of a circle by an arc whose length is equal to radius.

$1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ} \approx 57^{\circ}17'45''$

2) Basic formulae

① $\sin^2\theta + \cos^2\theta = 1$

② $\sec^2\theta - \tan^2\theta = 1$

③ $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

3) Complementary, Supplementary and negative angles

Angles	sin	cos	tan	cot
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\cot\theta$
$\pi/2+\theta$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$	$-\tan\theta$
$\frac{\pi}{2}-\theta$	$\cos\theta$	$\sin\theta$	$\cot\theta$	$\tan\theta$
$\pi+\theta$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$	$\cot\theta$
$\pi-\theta$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$	$-\cot\theta$

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④ Sum and difference of angle

① $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

② $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

③ $\tan(A \pm B) = \tan A \pm \tan B / 1 \mp \tan A \tan B$

④ $\cot(A \pm B) = \dfrac{\cot A \cot B \mp 1}{\cot B \pm \cot A}$

⑤ 2θ, 3θ & $\theta/2$

① $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

② $\cos 2\theta = \cos^2\theta - \sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

③ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

④ $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

⑤ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

⑥ $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

⑦ $\sin(\theta/2) = \pm\sqrt{\frac{1}{2}(1-\cos\theta)}$

⑧ $\cos(\theta/2) = \pm\sqrt{\frac{1}{2}(1+\cos\theta)}$

⑨ $\tan(\theta/2) = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}} = \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta}$

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⑥ Sum to product transformation

① $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

② $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

③ $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

④ $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

⑦ Product to sum transformation

① $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

② $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

③ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

⑧ Standard Values of Some trigonometric functions

① $\tan 15 = 2-\sqrt{3}$

② $\tan 22.5 = \sqrt{2}-1$

③ $\tan 67.5 = \sqrt{2}+1$

④ $\tan 75 = 2+\sqrt{3}$

⑤ $\sin 18 = \frac{1}{4}(\sqrt{5}-1)$

⑥ $\sin 54 = \frac{1}{4}(\sqrt{5}+1)$

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9 Miscellaneous results

① $\sin(A_1+A_2+A_3\cdots) = \prod C_A (S_1 - S_3 + S_5 - S_7 + \cdots)$

② $\cos(A_1+A_2+A_3+\cdots) = \prod C_A (1 - S_2 + S_4 - S_6 + \cdots)$

③ $\tan(A_1+A_2+A_3+\cdots) = \left(\dfrac{S_1 - S_3 + S_5 \cdots}{1 - S_2 + S_4 - S_6 \cdots}\right)$

$\left\{ S_K = \text{Summation of tangents taken K at a time} \right\}$

④ $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

⑤ $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

$\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

⑥ $\sin A \sin(60+A)\sin(60-A) = \frac{1}{4}\sin 3A$

$\cos A \cos(60+A)\cos(60-A) = \frac{1}{4}\cos 3A$

$\tan A \tan(60+A)\tan(60-A) = \tan 3A$

⑦ $\cot A - \tan A = 2\cot 2A$

⑧ $-\sqrt{a^2+b^2} \leq a\cos x + b\sin x \leq \sqrt{a^2+b^2}$

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(10) Results for sequences

① $\cos A \cos 2A \cos 4A \cdots \cos 2^n A = \frac{\sin 2^{n+1} A}{2^{n+1} \sin A}$

② $\sin A + \sin(A+d) + \cdots + \sin(A+nd) = \frac{\sin\left(\frac{(n+1)d}{2}\right)}{\sin(d/2)} \sin\left(\frac{2A+nd}{2}\right)$

③ $\cos A + \cos(A+d) + \cdots + \cos(A+nd) = \frac{\sin\left(\frac{(n+1)d}{2}\right)}{\sin(d/2)} \cos\left(\frac{2A+nd}{2}\right)$

④ $(1+\tan\theta)\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) = 2$

⑤ $(1+\cot\theta)\left(1-\cot\left(\frac{\pi}{4}-\theta\right)\right) = 2$

⑥ $\operatorname{cosec}\theta = \cot\theta/2 - \cot\theta$

⑦ $\sin\theta \sec 3\theta = \frac{1}{2}(\tan 3\theta - \tan\theta)$

⑧ $\tan\theta = \cot\theta - 2\cot 2\theta$

⑨ ~~$\tan A \tan B \tan C = \tan A + \tan B + \tan C$~~ $\{A+B+C=180^\circ\}$

⑩

Trigonometric Equations

① $\sin x = 0 \Rightarrow x = n\pi$

② $\cos x = 0 \Rightarrow x = (2n+1)\pi/2$

③ $\tan x = 0 \Rightarrow x = n\pi$

④ $\sin x = \sin\alpha \left(-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}\right) \Rightarrow x = n\pi + (-1)^n \alpha$

⑤ $\cos x = \cos\alpha \left(0 \le \alpha \le \pi\right) \Rightarrow x = 2n\pi \pm \alpha$

⑥ $\tan x = \tan\alpha \left(-\frac{\pi}{2} < \alpha < \frac{\pi}{2}\right) \Rightarrow x = n\pi + \alpha$

⑦ $\sin^2 x = \sin^2\alpha \Rightarrow x = n\pi \pm \alpha$

⑧ $\cos^2 x = \cos^2\alpha \Rightarrow x = n\pi \pm \alpha$

⑨ $\tan^2 x = \tan^2\alpha \Rightarrow x = n\pi \pm \alpha$

⑩ $\sin\theta = 1 \Rightarrow \theta = (4n+1)\pi/2$

⑪ $\sin\theta = -1 \Rightarrow \theta = (4n-1)\pi/2$

⑫ $\cos\theta = 1 \Rightarrow \theta = 2n\pi$

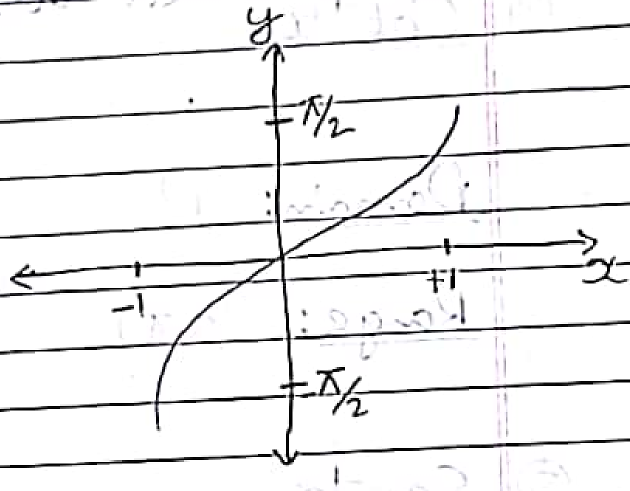
⑬ $\cos\theta = -1 \Rightarrow \theta = (2n+1)\pi$

Inverse circular function

① $\sin^{-1}x$

Domain: $[-1,1]$

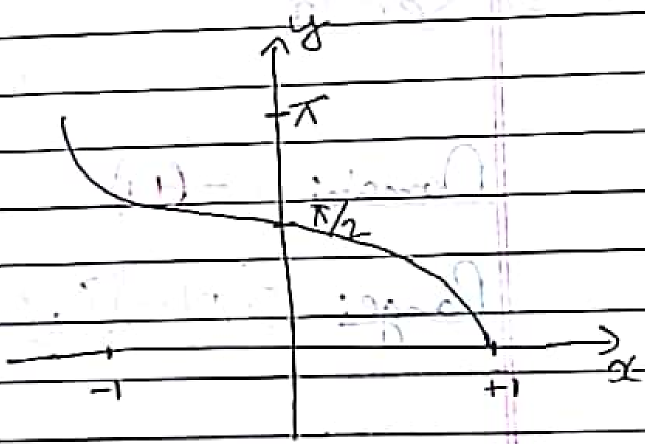
Range: $[-\pi/2, \pi/2]$



② $\cos^{-1}x$

Domain: $[-1,1]$

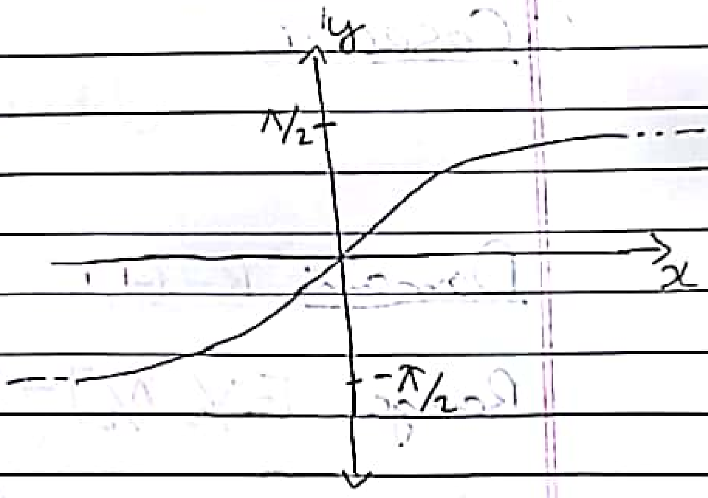
Range: $[0\ \pi]$



③ $\tan^{-1}x$

Domain: R

Range: $(-\pi/2, \pi/2)$

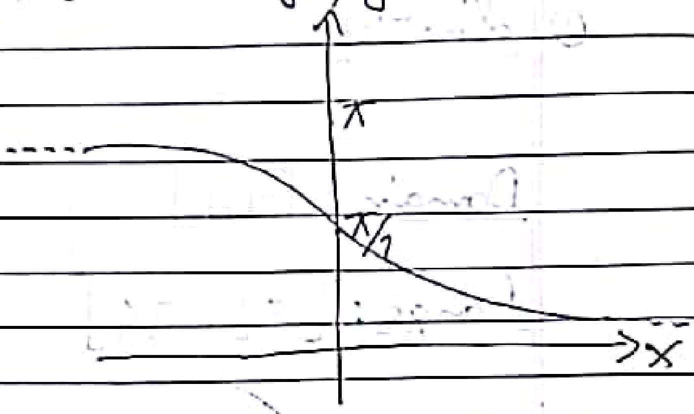


~~{We have exceptional convention for $\cot^{-1}x$ without any good reason}~~

④ $\cot^{-1}x$

Domain: R

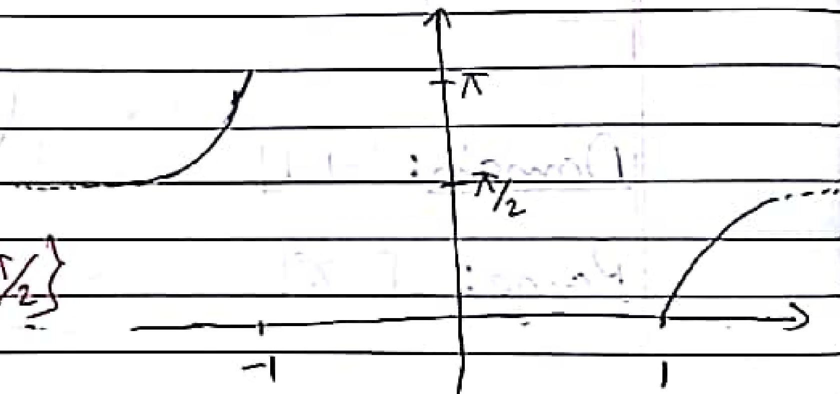
Range: $(0\ \pi)$



⑤ $\sec^{-1}x$

Domain: $R-(-1,1)$

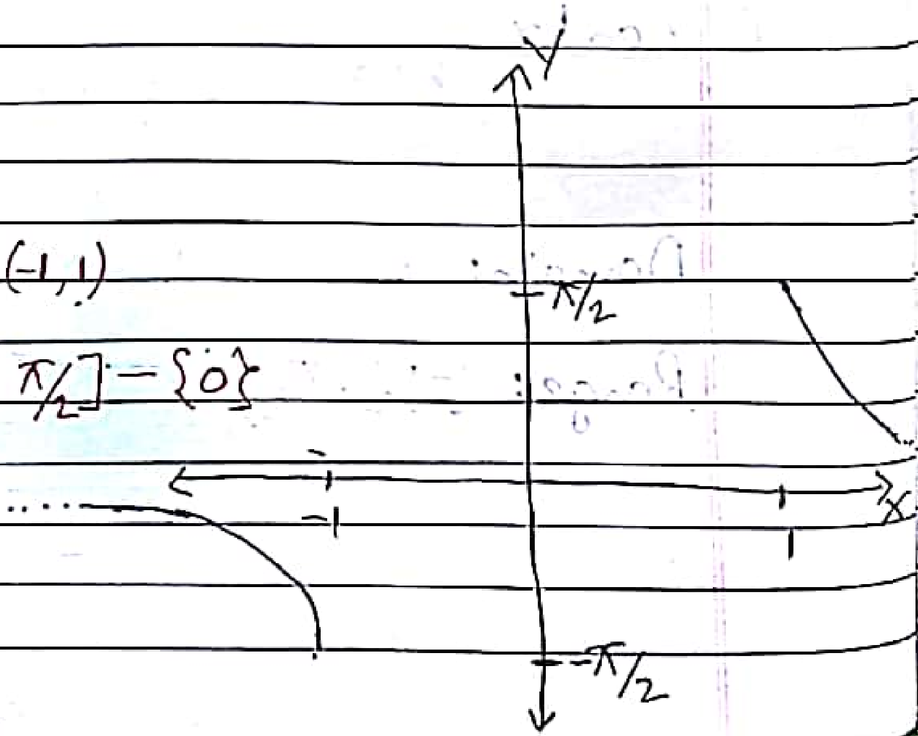
Range: $[0\ \pi]-\{\pi/2\}$



⑥ $\text{Cosec}^{-1}x$

Domain: $R-(-1,1)$

Range: $[-\pi/2\ \ \pi/2]-\{0\}$



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Properties

① $trigo^{-1}(trigo\,x) = x$ { In there respective range only }

$trigo(trigo^{-1}x) = x$ { In there respective domain only }

② $Sin^{-1}(-x) = -Sin^{-1}(x)$

$Cosec^{-1}(-x) = -Cosec^{-1}(x)$

$Cos^{-1}(-x) = \pi - Cos^{-1}x$

$Sec^{-1}(-x) = \pi - Sec^{-1}x$

$tan^{-1}(-x) = -tan^{-1}x$

$cot^{-1}(-x) = \pi - Cot^{-1}x$

③ $trigo^{-1}(x) = a\,trigo^{-1}\left(\frac{1}{x}\right)$

④ $Sin^{-1}x + Cos^{-1}x = \pi/2$

$tan^{-1}x + Cot^{-1}x = \pi/2$

$Cosec^{-1}x + Sec^{-1}x = \pi/2$

⑤ $tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$

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⑥ $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

⑦ $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right\}$

$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left\{xy \mp \sqrt{(1-x^2)(1-y^2)}\right\}$

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Progression

① <u>Sequence</u>

"It is a function whose domain is set of natural number"

② <u>Series</u>

"Sum of terms of Sequence it's series."

③ <u>Arithmetic progression</u>

"It is a sequence in which the difference between two consecutive terms is same"

$$t_n = a+(n-1)d$$
$$S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}[a+l]$$

★ <u>Properties of AP</u>

(i) $a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = \dots$

(ii) $(a_1 + a_2 + \dots + a_n) = \begin{cases} n\cdot(\text{middle term}) & \forall\, n \text{ is odd} \\ \frac{n}{2}\left(\text{Sum of two middle term}\right) & \forall\, n \text{ is even} \end{cases}$

(iii) If in an AP with CD as d, terms are selected in AP with $CD = d_0$, then CD of the AP obtained is $d_1 = dd_0$.

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★ Special form

③ $a-d \quad a \quad a+d$

④ $a-3d \quad a-d \quad a+d \quad a+3d$

⑤ $a-2d \quad a-d \quad a \quad a+d \quad a+2d$

★ Invariance of AP

(i) $a \quad b \quad c$

(ii) $c \quad b \quad a$

(iii) $a\pm\lambda \quad b\pm\lambda \quad c\pm\lambda$

(iv) $a\lambda \quad b\lambda \quad c\lambda$

(v) $a/\lambda \quad b/\lambda \quad c/\lambda$

★ Arithmetic mean

If $a, x_1, x_2 \cdots x_n, b$ are in AP, then $x_1, x_2, \cdots x_n$ are the n arithmetic means between a and b.

• $d = \dfrac{b-a}{n+1}$ • $x_m = a + \left(\dfrac{b-a}{n+1}\right)m$

• $\displaystyle\sum_{i=1}^{n} x_i = n\left(\frac{a+b}{2}\right)$

• $AM = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} x_i$

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(4) Geometric progression

"It is a sequence in which the ratio of any two consecutive terms is same"

$$t_n = ar^{n-1}$$

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

$$S_\infty = a\left(\frac{1}{1-r}\right) \quad \forall \ |r| < 1$$

★ **Properties of GP**

(i) $a_1 a_n = a_2 a_{n-1} = a_3 a_{n-2} = \cdots$

(ii) $a_1 a_2 a_3 \cdots a_n = \begin{cases} (\text{middle term})^n & \forall \ n \text{ odd} \\ (\text{Product of middle term})^{n/2} & \forall \ n \text{ even} \end{cases}$

★ **Special form**

③ $\frac{a}{r} \quad a \quad ar$

④ $\frac{a}{r^3} \quad \frac{a}{r} \quad ar \quad ar^3$

⑤ $\frac{a}{r^2} \quad \frac{a}{r} \quad a \quad ar \quad ar^2$

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★ Invariance of GP

① $a \quad b \quad c$

② $\cancel{c} \quad \cancel{b} \quad \cancel{a}$

③ $\lambda a \quad \lambda b \quad \lambda c$

④ $a/\lambda \quad b/\lambda \quad c/\lambda$

⑤ $a^n \quad b^n \quad c^n$

★ Geometric mean

If $a\, x_1, x_2 \cdots x_n\, b$ are in GP, then $x_1, x_2 \cdots x_n$ are the n "geometric means" between a and b.

- $x_1 = \left(\frac{b}{a}\right)^{\left(\frac{1}{n+1}\right)}$ • $x_m = a\left(\frac{b}{a}\right)^{\left(\frac{m}{n+1}\right)}$

- $\prod_{i=1}^{n} G_i = (ab)^{n/2}$

- $GM = (x_1 x_2 \cdots x_n)^{1/n}$

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5) Harmonic progression

"A sequence is an HP if the reciprocals of its terms form an AP"

- $t_n = \dfrac{1}{a+(n-1)d}$

★ Invariation of H.P.

① $a \quad b \quad c$

② $c \quad b \quad a$

③ $\lambda a \quad \lambda b \quad \lambda c$

④ $\cancel{a/\lambda \quad b/\lambda \quad c/\lambda}$

★ Harmonic mean

If $a, x_1, x_2, \ldots, x_n, b$ are in H.P then $x_1, x_2, \ldots x_n$ are n harmonic means between a and b.

$$x_m = \frac{(n+1)ab}{ma + (n+1-m)b} = \frac{ab}{b + \frac{(b-a)}{(n+1)}m}$$

$$HM = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}$$

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⑥ Inequalities

① $AM \geq GM \geq HM$ $\left\{ \begin{array}{l} \text{Equality holds} \\ \text{when terms are} \\ \text{equal} \end{array} \right\}$

② $a^2+b^2+c^2 \geq ab+bc+ca$

③ $G^2 = AH \Rightarrow A, G, H$ are in GP

⑦ Arithmetic – Geometric progression

- $t_n = (a+(n-1)d)r^{n-1}$

- $S_n = \frac{1}{1-r}\left[a - (a+(n-1)d)r^n + dr\left(\frac{1-r^{n-1}}{1-r}\right)\right]$

- $S_\infty = \frac{a+(d-a)r}{(1-r)^2}$ $\forall$ $|r| < 1$

⑧ Identities of natural number

① $1+2+3+\cdots+n = n(n+1)/2$

② $1^2+2^2+3^2+\cdots+n^2 = n(n+1)(2n+1)/6$

③ $1^3+2^3+3^3+\cdots+n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Logarithm {Properties}

① $a = b^x \Rightarrow x = \log_b a$

② $b^{\log_b x} = x$.

③ $a^{\log_c b} = b^{\log_c a}$

④ $\log_b a = \dfrac{1}{\log_a b}$

⑤ $\log_b a = \dfrac{\log_c a}{\log_c b}$

⑥ $\log(xy) = \log x + \log y$

⑦ $\log(x/y) = \log x - \log y$

⑧ $\log_{b^\beta}(a^\alpha) = \dfrac{\alpha}{\beta}\log_b a$

Properties of Triangle

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⇒ ① Notation {In triangle ΔABC}

① A, B, C are the angles.

② a, b, c are the sides

③ G is the centroid, the point of intersection of the medias.

④ O is the ortho centres, the point of intersection of the altitudes.

{Centre of circumscribed circle}

⑤ S is the circumcentre, the point of intersection of the perpendicular bisectors of the sides.

{Centre of Inscribed Circle}

⑥ I is the incentre, the point of intersection of the internal bisector of angles.

⑦ I_1, I_2, I_3 are the excenteres, the points of intersection of internal bisector of one angle and external bisector of other two angles.

⑧ R is the circumradius, the radius of the circle with centre S which passes through the vertices.

⑨ r is the inradius, the radius of the circle with centre I which touches the sides.

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(10) r_1, r_2, r_3 are the exradii, the radii of circle with centre $I_1 I_2 I_3$ respectively which touches the sides.

(11) S is semiperimeter $= \dfrac{a+b+c}{2}$

(12) Δ is area of triangle.

⇒ (2) Sine rule

(1) $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

⇒ (3) Area of ΔABC

(1) $$\Delta = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ca \sin B$$

(11) $$\Delta = \frac{1}{2} \times (\text{base}) \times (\text{height}) = \sqrt{s(s-a)(s-b)(s-c)}$$

(111) $$\Delta = \frac{b^2 \sin C \sin A}{2 \sin B} = \frac{a^2 \sin B \sin C}{2 \sin A} = \frac{c^2 \sin A \sin B}{2 \sin C}$$

⇒ (4) Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

(1) $$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

⇌ (5) Projection rule

(i) $a = b\cos C + c\cos B$

⇌ (6) Tangent rule

(i) $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$

⇌ (7) Half angle formula

(i) $\sin A/2 = \sqrt{\frac{(s-b)(s-c)}{bc}}$

(ii) $\cos A/2 = \sqrt{\frac{s(s-a)}{bc}}$

(iii) $\tan A/2 = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

⇌ (8) Inscribed circle of a triangle and its radius r

(i) $r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2} = (s-b)\tan B/2 = (s-c)\tan C/2$

(ii) $r = \frac{a\sin B/2\ \sin C/2}{\sin A/2} = \frac{b\sin C/2\ \sin A/2}{\sin B/2}$

(iii) $r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

⇒ (9) Circumscribed circle of a triangle and its radius R

(i) $R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

(ii) $R = \frac{abc}{4\Delta}$

⇒ (10) Excircles and their radii

(i) $r_a = \frac{\Delta}{s-a}$, $r_b = \frac{\Delta}{s-b}$, $r_c = \frac{\Delta}{s-c}$

(ii) $r_a = s\tan\frac{A}{2}$, $r_b = s\tan\frac{B}{2}$, $r_c = s\tan\frac{C}{2}$

(iii) $r_a = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$

$r_a = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iv) $r_b = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

$r_c = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

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⇉ (11) Excentral Triangle

(i) Incentre of ΔABC is ortho centre of $\Delta I_a I_b I_c$.

(ii) ΔABC is the pedal triangle of $\Delta I_a I_b I_c$.

(iii) The sides of the excentral triangle are $4R\cos\frac{A}{2}$, $4R\cos\frac{B}{2}$, $4R\cos\frac{C}{2}$ and its angles are $\left(\frac{\pi}{2}-\frac{A}{2}\right)$, $\left(\frac{\pi}{2}-\frac{B}{2}\right)$ & $\left(\frac{\pi}{2}-\frac{C}{2}\right)$.

$$II_a = 4R\sin\frac{A}{2}$$

(iv) $$II_b = 4R\sin B/2$$

$$II_c = 4R\sin C/2$$

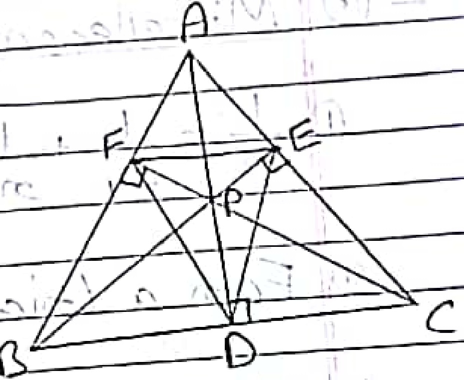
⇉ (12) Cyclic Quadrilateral

$$\text{area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

$$R = \frac{1}{4\Delta}\sqrt{(ac+bd)(ad+bc)(ab+cd)}$$

$$\angle C + \angle A = \pi$$

$\Rightarrow$ (13) Pedal triangle



$\Rightarrow \Delta DEF$ is called pedal triangle of triangle ΔABC.

(I) $AP = 2R\cos A$

$BP = 2R\cos B$

$CP = 2R\cos C$

(II) $PD = 2R\cos B\cos C$

$PE = 2R\cos A\cos C$

$PF = 2R\cos A\cos B$

$\Rightarrow$ (14) Regular polygon of n side

(I) $\underset{\{\text{inscribed}\}}{\text{Area}} = \frac{1}{2} nR^2 \sin\frac{2\pi}{n}$

(II) $\underset{\{\text{circumscribed}\}}{\text{Area}} = \frac{1}{2} nr^2 \tan\pi/n$

(III) $\underset{\{\text{inscribed}\}}{\text{Side}} = 2R\sin\pi/n$

(IV) $\underset{\{\text{circumscribed}\}}{\text{Side}} = 2r\tan\pi/n$

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⇒ (13) Miscellaneous

(i) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

(ii) For a triangle O, S, G are collinear

(iii) $r_1 + r_2 + r_3 - r = 4R$

(iv) $r_1 r_2 + r_2 r_3 + r_1 r_3 = s^2$

(v) $r r_1 r_2 r_3 = \Delta^2$

Permutation and Combination

① Counting Principle

If a certain work A is done in m ways and another work B in n ways then:

(i) the number of ways of doing work A or B = m+n.

(ii) the number of ways of doing both the work is = m×n.

② Permutation {Arrangement}

ⓐ The number of permutation of n different things taken r at a time is

$$^nP_r = \frac{n!}{(n-r)!} \quad \left\{ n! = \prod_{i=1}^{n} i \right\}$$

ⓑ The number of permutation of n different things taken r at a time allowing repetitions is n^r.

ⓒ The permutations of n things of which P are identical of one type, q are identical of a second type, r are identical of a third type where $p+q+r=n$ is

$$\frac{n!}{p!\,q!\,r!}$$

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(d) The number of arrangements of n different objects around a closed curve.

$= (n-1)!$ if Clockwise and anticlockwise arrangements a considered different.

$= \frac{(n-1)!}{2}$ If clockwise and anticlockwise arrangements are considered identical $\Rightarrow$ {fliping symmetry}

③ Combination {Selection}

(a) The number of combinations of n different things taken r at a time.

$${}^{n}C_{r} = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

(b) The number of combinations of n different things taken r at a time allowing repetitions is

$$\binom{n+r-1}{r}$$

(c) Total number of selection of p like object, q like object of another type and r distinct object is :-

$$(p+1)(q+1)2^{r} - 1$$

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(4) Division of different things

(a) The number of ways of dividing n different things into 3 groups of p, q, r things (p+q+r=n) is :-

$$= \frac{n!}{p!q!r!} \quad \{p, q, r \text{ are unequal}\}$$

$$= \frac{n!}{2!\,p!(q!)^2} \quad \{q = r\}$$

$$= \frac{n!}{3!\,(p!)^3} \quad \{p = q = r\}$$

(b) Number of way of dividing n different objects in r groups.

$$= \sum_{i=0}^{r} (r-i)^n \binom{r}{i} (-1)^i = r^n \binom{n}{0} - (r-1)^n \binom{n}{1} + (r-2)^n \binom{n}{2} + \dots$$

(5) Division of identical things

(a) The number of ways of dividing n identical things among r persons such that each one may get atmost n things is

$$\binom{n+r-1}{r-1}$$

Which is also the number of non negative integer solutions of the equation $x_1+x_2+x_3+\cdots+x_r=n$.

(b) The number of ways of dividing n identical things among r ~~people~~ persons such that each one gets atleast one is

$$\binom{n-1}{r-1}$$

Which is also the number of positive integer solutions of the equation $x_1+x_2+x_3+\cdots+x_r=n$.

⑥ Prime factor

$$N = P_1^{\alpha_1} P_2^{\alpha_2} \cdots P_k^{\alpha_k} \quad \left\{ \begin{array}{l} P_1, P_2 \cdots P_k \text{ are Prime} \\ \alpha_1, \alpha_2 \cdots \alpha_k \text{ are natural numbers} \end{array} \right\}$$

- Sum of factors $= (P_1^0+P_1^1+\cdots+P_1^{\alpha_1})(P_2^0+P_2^1+\cdots P_2^{\alpha_2})\cdots(P_k^0+P_k^1+\cdots+P_k^{\alpha_k})$

- Number of factors of N $= (\alpha_1+1)(\alpha_2+2)\cdots(\alpha_k+k)$

⑦ Derangements $\Rightarrow$ If n things form an arrangement in a row, the number of ways in which they can be arranged so that

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no one of them occupies its original place is :-

$$n!\left\{\frac{1}{0!}-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^n\frac{1}{n!}\right\} = n!\sum_{i=0}^{n}\frac{1}{(i!)}(-1)^i$$

(8) Properties of $\binom{n}{r}$

(a) $\binom{n}{r}+\binom{n}{r-1}=\binom{n+1}{r}$

(b) $\binom{n}{r}=\binom{n}{n-r}$

(c) $\sum_{i=0}^{n}\binom{n}{i}=2^n$

Binomial Theorem

① Let $n \in N$ and $a, b \in R$

$$\sum_{r=0}^{n} \binom{n}{r} a^{n} b^{n-r} = (a+b)^{n}$$

→ There are n+1 terms

② Pascal rule

ⓐ $\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$ ⓒ $\binom{n}{r} = \binom{n}{n-r}$

ⓑ $\binom{n}{r} \Big/ \binom{n}{r-1} = \left(\frac{n-r+1}{r}\right)$

③ ⓐ Greatest term in the expansion of $(1+x)^n$ in $\binom{n}{r}x^r$ where,

$$r = \left[\frac{(n+1)x}{1+x}\right]$$

ⓑ $$\frac{n!}{x(x+1)(x+2)\cdots(x+n)} = \sum_{r=0}^{n} \frac{{}^{n}C_{r}}{x+r}(-1)^{r}$$

③ Properties of binomial Coefficient

ⓐ $C_0 + C_1 + C_2 + \cdots + C_n = 2^n$

ⓑ $C_0 - C_1 + C_2 - C_3 + \cdots \pm C_n = 0$

(c) $C_0 + C_2 + C_4 + \cdots = 2^{n-1}$

(d) $C_1 + C_3 + C_5 + \cdots = 2^{n-1}$

(4) Multinomial theorem

$$(x_1 + x_2 + \cdots + x_n)^n = \sum \binom{n}{\alpha_1, \alpha_2 \cdots \alpha_n} x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$$

$$\left\{ \binom{n}{\alpha_1, \alpha_2 \cdots \alpha_n} = \frac{n!}{\alpha_1!\,\alpha_2! \cdots \alpha_n!} \right\}$$

(5) $$(a+b)^{-n} = \sum_{r=0}^{\infty} \binom{n+r-1}{r} a^r\, b^{-(n+r)} (-1)^r$$

★ $$(1-x)^{-n} = \sum_{r=0}^{\infty} \binom{n+r-1}{r} x^r$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots + x^r$$

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots + (r+1)x^r$$

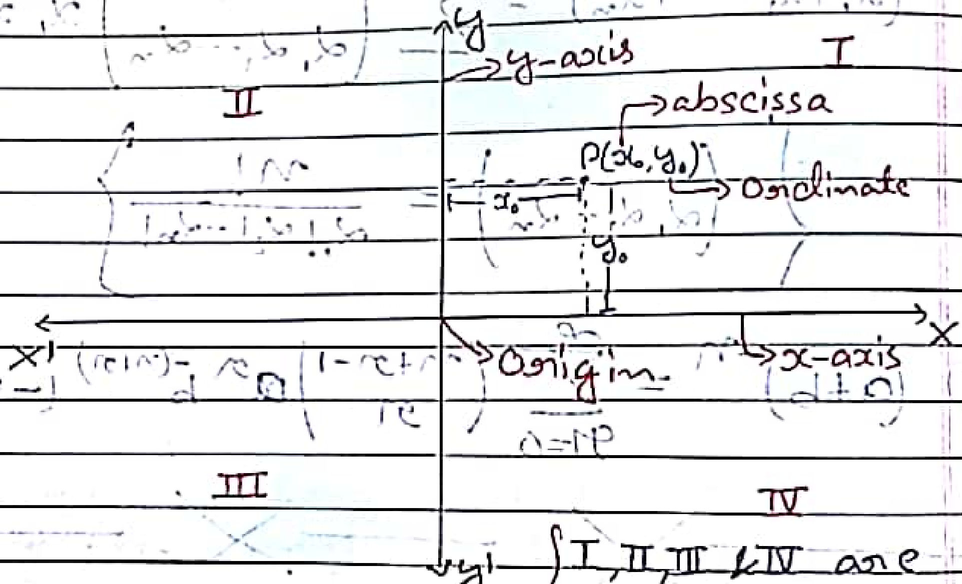
$$(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \cdots + \frac{(r+1)(r+2)}{1 \cdot 2} x^r$$

Straight line {20}

★ Fundamental

① Cartesian coordinate system

Let two perpendicular lines X'OX & Y'OY divide the plane into four quadrants.



{I, II, III & IV are respective quadrants}

② Distance between two points

The distance between $A(x_1, y_1)$ & $B(x_2, y_2)$ is :-

$$S = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

③ Section formulae

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If a line segment joining $A(x_1, y_1)$ & $B(x_2, y_2)$ is divided by C internally and D externally in the ratio $m_1 : m_2$ then:-

$$C = \left(\frac{x_1 m_2 + x_2 m_1}{m_1 + m_2}, \frac{y_1 m_2 + y_2 m_1}{m_1 + m_2} \right).$$

$$D = \left(\frac{x_1 m_2 - x_2 m_1}{m_2 - m_1}, \frac{y_1 m_2 - y_2 m_1}{m_2 - m_1} \right).$$

④ Shifting of the Origin

Let position of point P with respect to Origin O_1 be $P \equiv (x, y)$ and let position of another origin O_2 with respect to O_1 be $O_2 \equiv (h, k)$.

Then position of point P with respect to origin O_2 will be $P \equiv (x-h, y-k)$

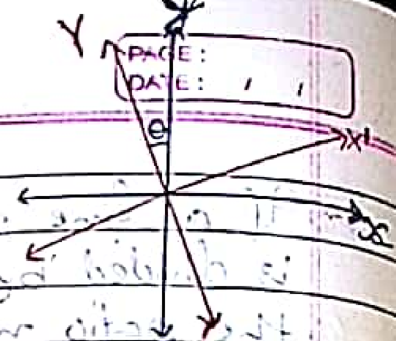
$$\left\{ \begin{array}{l} x_2 = x_1 - h \\ y_2 = y_1 - k \end{array} \right\}$$

Note: O_1 & O_2 has same orientation

⑤ Rotation of Axes

Let the axis be rotated through angle θ in counter clockwise direction about the origin. The new coordinat X, Y and Old Coordinate x, y are related as:-

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



⑥ Straight line {Slope and Intercept}

- Slope $(m) = \tan\phi$ {where ϕ is angle made by line with positive x-axis}
- y-Intercept = Value of y when $x=0$
- x-Intercept = Value of x when $y=0$

⑦ Equation of Straight line

ⓐ Intercept form

$$\boxed{\frac{x}{a}+\frac{y}{b}=1}$$ {a & b are x-and y-Intercept respectively}

ⓑ Slope-Intercept form

$$\boxed{y = mx + c}$$ {m = Slope, c = y-Intercept}

ⓒ Point slope form

$$\boxed{\frac{y-y_1}{x-x_1} = m}$$ {m = slope, $(x_1, y_1) \Rightarrow$ Point}

ⓓ two point form

$$\boxed{\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}}$$ {(x_1,y_1) & (x_2,y_2) are the two point}

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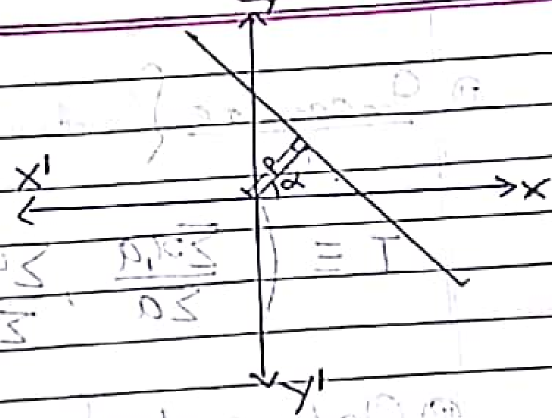
(e) Normal form

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$



★ Analysis

(1) Three points

(a) Area of Δ formed by three points

$$ar.(\Delta) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$ {where (x_1, y_1), (x_2, y_2) & (x_3, y_3) are the vertices of triangle}

(b) Collinearity of three points

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$; {where (x_1, y_1) (x_2, y_2) & (x_3, y_3) are three points}

(c) Centers of Δ formed by three points

(i) Centroid {Centre of gravity}

$$G \equiv \left(\frac{1}{3}\Sigma x_1, \frac{1}{3}\Sigma y_1\right)$$

(ii) Incentre {Centre of Incircle} {Intersection of internal angle bisector}

$$I \equiv \left(\frac{\sum x_1 a}{\sum a}, \frac{\sum y_1 a}{\sum a} \right)$$

(iii) Orthocentre {Intersection of Altitude}

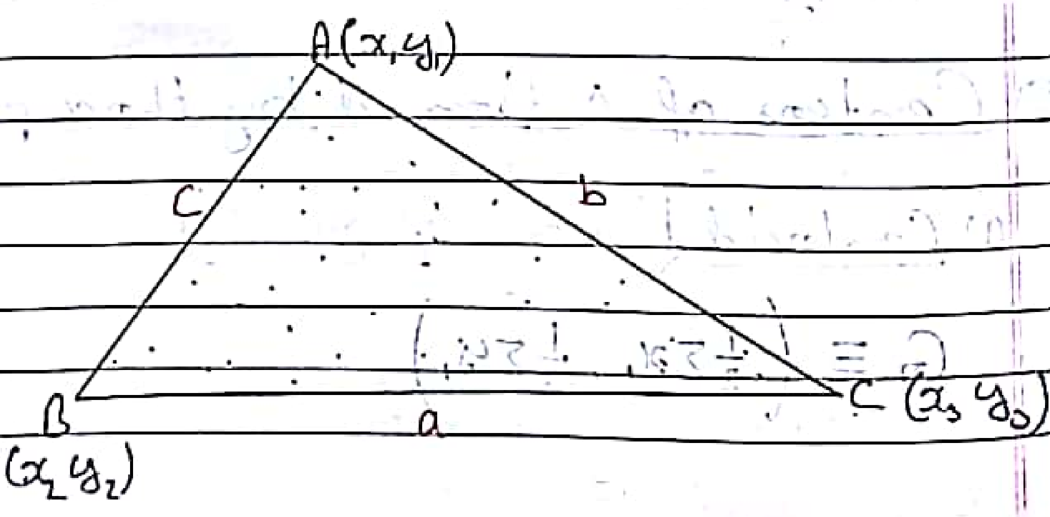
$$O \equiv \left(\frac{\sum x_1 \tan A}{\sum \tan A}, \frac{\sum y_1 \tan A}{\sum \tan A} \right)$$

O is (x_1, y_1) if $A = \pi/2$

(iv) Circumcentre {Centre of Circumcircle} {⊥ bisector of sides}

$$S \equiv \left(\frac{\sum x_1 \sin 2A}{\sum \sin 2A}, \frac{\sum y_1 \sin 2A}{\sum \sin 2A} \right)$$

$$S = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2} \right) \text{ if } A = \pi/2$$



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② A line and a point $\{(x_1, y_1)$ & $ax + by + c = 0\}$

ⓐ ⊥ distance of point from line

$$S = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

ⓑ foot of ⊥ from a point

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

ⓒ Image of a point with line as mirror

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

③ A line and two points $\{(x_1, y_1), (x_2, y_2)$ & $ax + by + c = 0\}$

ⓐ Points are on same side of line L if :-

$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$$

ⓑ Points are on opposit side of line L if :-

$$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$$

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(4) Two lines $\{a_1x+b_1y+c_1=0$ & $a_2x+b_2y+c_2=0\}$

(a) Parallel lines

(i) lines are ∥ if $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

(ii) $d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$ {distance between ∥ lines}

(iii) The point $P(x_1,y_1)$ lies between ∥ lines if :-

$$\frac{ax_1+by_1+c_1}{ax_1+by_1+c_2}<0$$

(b) Concident lines

Two lines are coinciding if :-

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

(c) Intersecting lines

(i) The angle between the line with slopes m_1 and m_2 is :-

$$\phi=\tan^{-1}\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

→ line are ⊥ if $m_1m_2=-1$

ⓓ Image of line L_1 wrt L_2

$$2(a_1a_2+b_1b_2)(a_2x+b_2y+c_2) = (a_2^2+b_2^2)(a_1x+b_1y+c_1)$$

ⓔ Bisector of angle between two lines

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$$

(i) Equation of bisector in which origin lies

→ First make the constant c_1 & c_2 either both (+)ve or both (−)ve in the two equation and then take only (+)ve sign.

(ii) Bisector of acute or obtuse angle

→ Firstly make the constants c_1 & c_2 of the same sign.

→ Now evaluate $a_1a_2+b_1b_2$.

• If $a_1a_2+b_1b_2$ is ⊕, origin lies in obtuse angle.

• If $a_1a_2+b_1b_2$ is ⊖, origin lies in acute angle.

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(f) <u>Points lying between Intersecting lines</u>

The point (x_1, y_1) lies in obtuse angle if:

$$\frac{a_1a_2 + b_1b_2}{(a_1x_1 + b_1y_1 + c_1)(a_2x_2 + b_2y_2 + c_2)} > 0$$

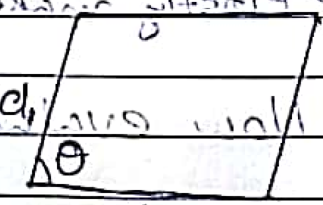
{Similarly for acute angle}

(5) <u>Three lines</u>
$$\left\{\begin{aligned} L_1 &\equiv a_1x + b_1y + c_1 \\ L_2 &\equiv a_2x + b_2y + c_2 \\ L_3 &\equiv a_3x + b_3y + c_3 \end{aligned}\right\}$$

(a) <u>Condition for concurrent lines</u>

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

• <u>Area of ||gm</u> $= \dfrac{d_1d_2}{\sin\theta}$



Circle

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① Definition

"A Circle is the locus of a point which moves in a plane such that its distance from a fixed point is constant"

→ The fixed point is called its centre and the constant distance is called ~~its~~ radius of the circle.

② Equation of the circle in various form

I) The equation $(x-a)^2+(y-b)^2=r^2$ represents a circle with centre (a,b) and radius r.

II) The equation $x^2+y^2+2gx+2fy+c=0$ is the general equation of a circle with centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$

III) Equation of the circle with points $P(x_1,y_1)$ and $Q(x_2,y_2)$ as extremities of a diameter is:-

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

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③ Parametric form

The parametric coordinates of any point on the Circle $(x-h)^2+(y-k)^2=a^2$ are given by :-

$$P = (h + a\cos\theta,\ k + a\sin\theta)$$

Note

→ Since there are three independent Constants g, f, c in general equation of circle $x^2+y^2+2gx+2fy+c=0$, So a Circle can be found to satisfy three independent geometrical conditions and no more.

④ Tangents

"A tangent to a curve at a point is defined as the limiting positions of a Secant obtained by joining the given point to another point in the vicinity on the curve as the second point tends to the first point along the curve"

→ $y = mx \pm a\sqrt{1+m^2}$ is always a tangent to the circle $x^2+y^2=a^2$ whatever be the value of m.

⇒ Equation of the tangent to $x^2+y^2+2gx+2fy+c=0$ at $A(x_1,y_1)$ on the curve is

$$xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$$

or

$$\boxed{S_1 = 0}$$

⑤ Normal

"The normal to a curve at a point is defined as the straight line passing through the point and ⊥ to the tangent at that point"

⇒ In case of a circle, every normal passes through the centre of the circle.

⇒ The equation of the normal to the circle $x^2+y^2+2gx+2fy+c=0$ at any point (x_1,y_1) lying on the circle is
$$\boxed{\frac{x-x_1}{x_1+g} = \frac{y-y_1}{y_1+f}}$$

★ Analysis

① Circle and a point

$S = x^2 + y^2 + 2gx + 2fy + c$

$S_1 = x_1 x + y_1 y + g(x + x_1) + f(y_1 + y_1) + c$

$S_{11} = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

S	S_1	S_{11}
x^2	$x_1 x$	x_1^2
y^2	$y_1 y$	y_1^2
$2x$	$x_1 + x$	$2x_1$
$2y$	$y_1 + y$	$2y_1$

ⓐ Location of $P(x_1, y_1)$ with circle $S = 0$

Point P is In, Out, on the circle for $S_{11} < 0$, $S_{11} > 0$, $S_{11} = 0$ respectively.

ⓑ Equation of pair of tangent from (x_1, y_1) on $S = 0$

$\boxed{S_1^2 = S_{11} S}$

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ⓒ length of tangent from (x_1, y_1) on $S=0$ –

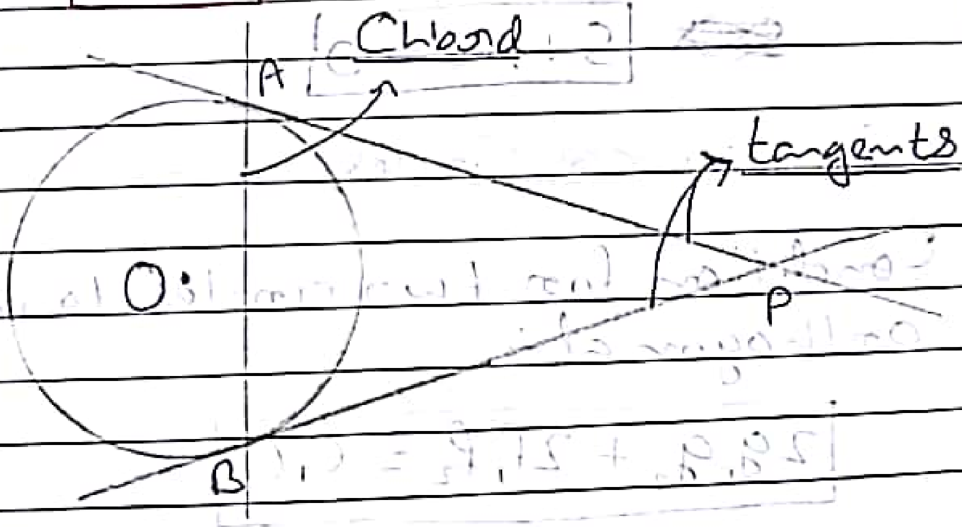
$$\boxed{L = \sqrt{S_{11}}}$$

ⓓ Equation of Chords from tangent from (x_1,y_1) on $S=0$

$$\boxed{S_1 = 0}$$

ⓔ Equation of chord with midpoint (x_1, y_1) on $S=0$

$$\boxed{S_1 = S_{11}}$$



⑦ Two Circles

ⓐ Radical axis

"The radical axis of two circles is the locus of a point which moves so that the length of the tangents drawn from it to the two circles are equal"

→ For two circles $S=0$ & $S'=0$ Equation of radical axis is ÷

$$\boxed{S-S'=0}$$

★ Properties ÷

(i) It is ⊥ to line joining centers of the circles.

(ii) It passes through the points of intersection of the two circles.

(iii) Circles through the points of intersection of $S=0$ & $S'=0$ is ÷

$$\boxed{S+\lambda S'=0}$$

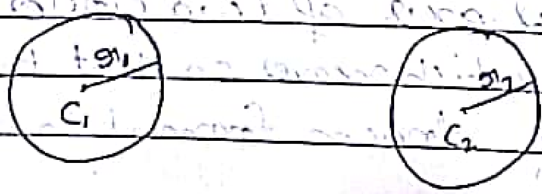
ⓑ Orthogonal Circles

Condition for two circles to intersect orthogonal ÷

$$\boxed{2g_1g_2+2f_1f_2=C_1C_2}$$

ⓒ Common tangents to two Circles

C_1 & C_2 are Centres, r_1 and r_2 are the radius $(r_1>r_2)$ Of two given circles.



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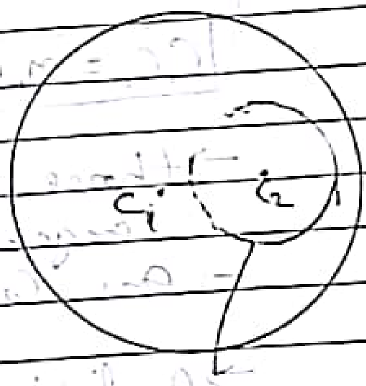
(I) One circle contain other

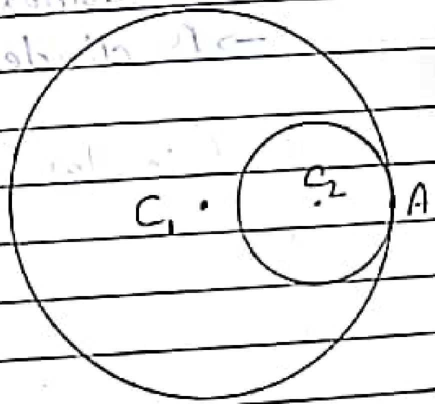
$$\boxed{C_1C_2 < |r_1 - r_2|}$$



→ No Common tangent
→ One Common normal.

(II) Circle touches internally

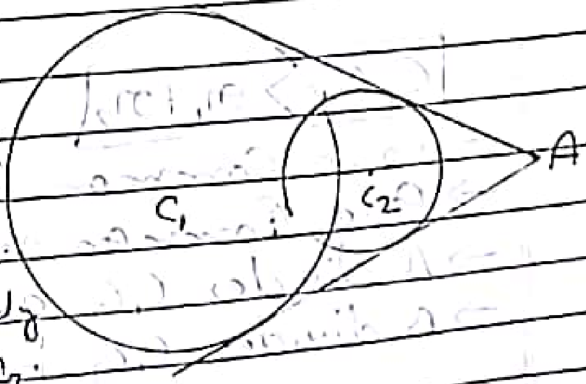
$$\boxed{C_1C_2 = r_1 - r_2}$$



→ One Common tangent
→ One Common normal
→ A divides C_1C_2 externally in the ratio $r_1 : r_2$.

(III) Intersecting circles

$$\boxed{r_1 - r_2 < C_1C_2 < r_1 + r_2}$$



→ Two Common tangent.
→ One Common normal.
→ A divides C_1C_2 externally in the ratio $r_1 : r_2$.

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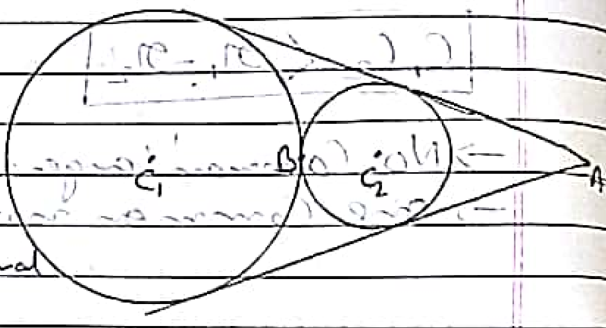
(iv) Circles touch externally

$\boxed{C_1C_2 = r_1 + r_2}$



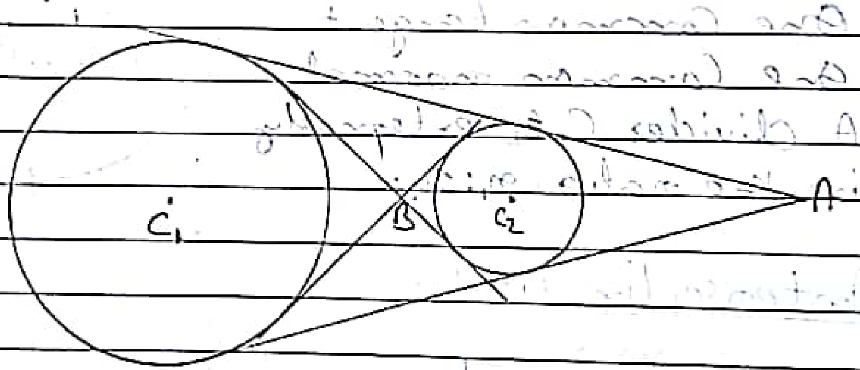
→ three Common tangents

→ One Common normal

→ A divides C_1C_2 externally in ratio $r_1 : r_2$.

→ B divides C_1C_2 internally in ratio $r_1 : r_2$.

(v) Circles are Separated



$\boxed{C_1C_2 > r_1 + r_2}$

→ four Common tangents

→ One Common normal

→ A divides C_1C_2 externally in ratio $r_1 : r_2$

→ B divides C_1C_2 internally in ratio $r_1 : r_2$

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③ Circum circle

① If the sides of a triangle are

$L_i \equiv a_i x + b_i y + C_i = 0 \;\; \forall\, i = 1, 2, 3$

then its circum circle is:

$\alpha L_1 L_2 + \beta L_2 L_3 + \gamma L_1 L_3 = 0$

where, α, β, γ are found using conditions:

→ Coeff $xy = 0$

→ Coeff x^2 = Coeff y^2

⑪ If the sides of a cyclic quadrilateral are $L_i \equiv a_i x + b_i y + C_i = 0 \;\forall\, i = 1, 2, 3$ in that order then the equation of the circle through the vertices is $L_1 L_3 + \lambda L_2 L_4 = 0$

⇒ λ is determined by the condition:

Coeff $xy = 0$.

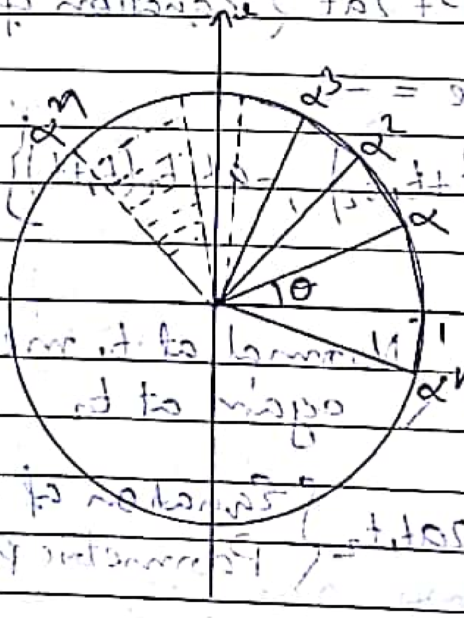
Complex Number Cont...

Basics
→ Defination
→ Algebra
→ Conjugate and its properties
→ Modulus and its properties
→ Argument and its properties
→ Geometrical representation
→ Polar representation and De, Moivre's Theorem
→ Cube root of Unity and its properties.

① n^{th} root of Unity

$z^k - 1 = 0$

The K roots of above equation will be vertices of a regular polygon of K sides inscribe in circle $|z| = 1$ in Complex plane.



$\theta = \frac{2\pi}{K}$

$\alpha = e^{\frac{2\pi}{k}i}$

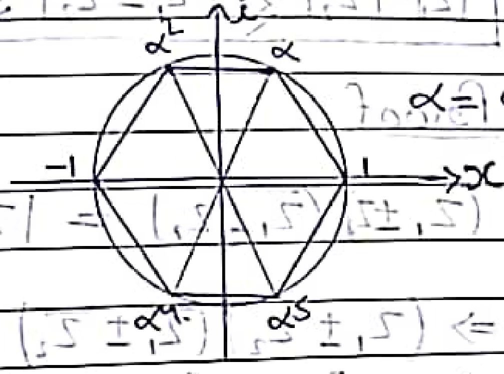
$\alpha^n = e^{\frac{2\pi n}{K}i}$

⇒ The roots will be :

$\{1, \alpha, \alpha^2, \ldots, \alpha^{k-1}\}$ where, $\alpha = e^{\frac{2\pi}{k} i}$

★ Example

$z^6 - 1 = 0$ ⇒



$\alpha = e^{\frac{\pi}{3} i}$

★ Properties

Let $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_k$ be all the roots of $z^k - 1 = 0$

(I) $\sum_{i=1}^{k} \alpha_i = 0$

(II) $\sum_{i=1}^{k} \alpha_i^m = n$ or 0, depending on whether m is divisible by K or not.

(III) $\overline{\alpha^r} = \frac{1}{\alpha^r} = \alpha^{n-r}$

★ $z^K - b^K = 0$ $\{b \in R\}$

The K roots of above equation will be vertices of a regular polygon of K sides inscribed in a circle $|z| = b$ in complex plane.

{All the foregiven properties then follows similarly}

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② <u>Triangle inequality</u>

$$\boxed{|z_1|-|z_2| \leq |z_1 \pm z_2| \leq |z_1|+|z_2|}$$

<u>Proof</u>

$$(z_1 \pm z_2)\overline{(z_1 \pm z_2)} = |z_1 \pm z_2|$$

$$\Rightarrow (z_1 \pm z_2)(\bar{z}_1 \pm \bar{z}_2)$$

$$\Rightarrow |z_1|^2+|z_2|^2 \pm z_1\bar{z}_2 \pm z_2\bar{z}_1$$

$$\Rightarrow |z_1|^2+|z_2|^2 \pm (z_1\bar{z}_2 + \overline{z_1\bar{z}_2})$$

$$\Rightarrow |z_1|^2+|z_2|^2 \pm 2\,Re(z_1\bar{z}_2)$$

So $|z_1+z_2|^2 = |z_1|^2+|z_2|^2+2\,Re(z_1\bar{z}_2)$ — ①

$|z_1-z_2|^2 = |z_1|^2+|z_2|^2-2\,Re(z_1\bar{z}_2)$ — ②

<u>from ①</u>

$$-2|z_1\bar{z}_2| \leq 2\,Re(z_1\bar{z}_2) \leq 2|z_1\bar{z}_2|$$

$$\Rightarrow |z_1|^2+|z_2|^2-2|z_1||z_2| \leq |z_1+z_2|^2 \leq |z_1|^2+|z_2|^2+2|z_1||z_2|$$

$$\Rightarrow \big||z_1|-|z_2|\big| \leq |z_1+z_2| \leq |z_1|+|z_2|$$

from ②

$$-2|z_1\bar{z}_2| \leq -2Re(z_1\bar{z}_2) \leq 2|z_1\bar{z}_2|$$

$$\Rightarrow |z_1|^2+|z_2|^2+2|z_1\bar{z}_2| \leq |z_1-z_2|^2 \leq |z_1|^2+|z_2|^2+2|z_1z_2|$$

$$\Rightarrow |z_1|-|z_2| \leq |z_1-z_2| \leq |z_1|+|z_2|$$

$\Rightarrow$ On combining above results we finally obtain:

$$|z_1|-|z_2| \leq |z_1 \pm z_2| \leq |z_1|+|z_2|$$

③ Section formula

Let z_1 and z_2 represent the point A and B.
Let z_3 and z_4 represent the point C and D which divide AB internally and externally respectively in the ratio m:1. Then:

$$z_3 = \frac{z_1+mz_2}{1+m} \quad ; \quad z_4 = \frac{z_1-mz_2}{1-m}$$

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④ Geometrical meaning of Algebraic Operation

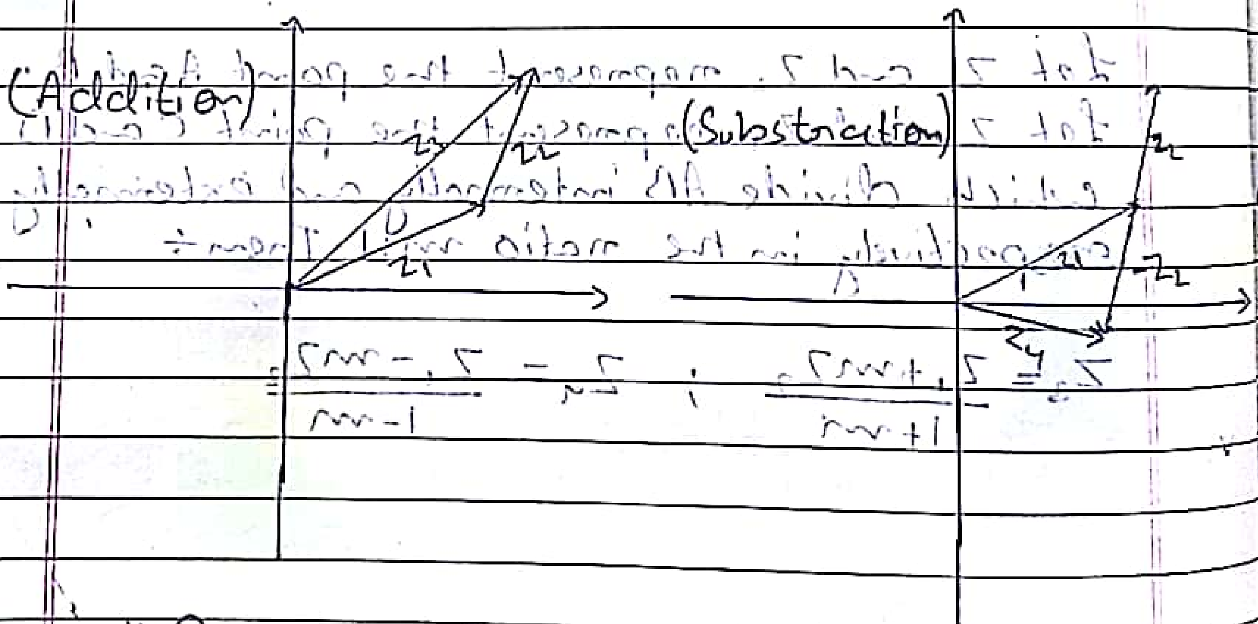
Addition and Substraction

A complex number contains two information length and direction, translating it in complex plane doesn't make any difference, same as 2D vector.

Let us concider two complex numbers z_1 & z_2.

Let, $z_3 = z_1 + z_2$

$z_4 = z_1 - z_2$

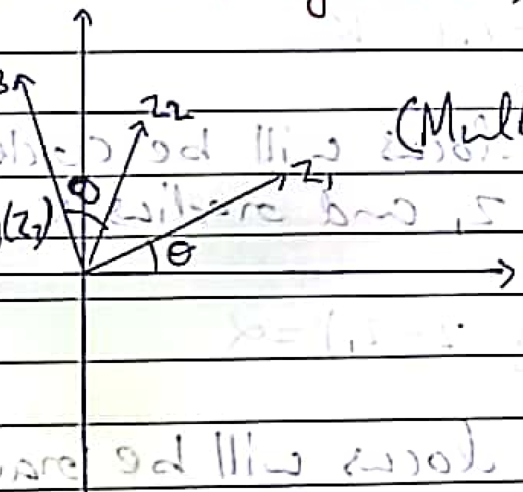


"Polygon law of Vector addition"

Multiplication and Division

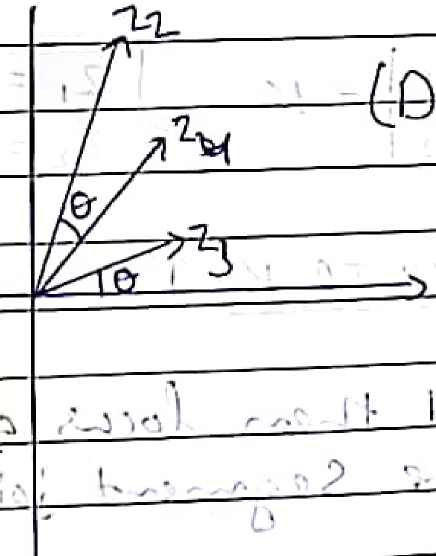
"To multiply (Divide) Complex number z_1 by a complex number z_2, it is necessary to multiply (Divide) by $|C_2|$ to the length of the vector representing C_1 and then rotating the altered vector about point O through an angle equal to the (negative of) argument of C_2"

- $Z_3 = Z_2 Z_1$
- $\arg(z_3) = \arg(z_1) + \arg(z_2)$
- $|Z_3| = |Z_2||Z_1|$



(Multiplication)

- $Z_4 = \dfrac{Z_2}{Z_3}$
- $\arg(z_4) = \arg(z_2) - \arg(z_3)$
- $|Z_4| = \dfrac{|Z_2|}{|Z_3|}$



(Division)

Cor: To rotate complex number z_0 by angle ϕ clockwise. Let z be the new complex number

$$z = z_0 e^{-i\phi}$$

5. Loci in Argand diagram

(a) $|z - z_1| = K$

The locus will be circle with centre at the z_1 and radius K.

(b) $\arg(z - z_1) = \alpha$

The locus will be ray starting from point z_1 and making angle α with real axis.

(c) $\left|\dfrac{z - z_1}{z - z_2}\right| = K$ $\quad \begin{Bmatrix} z_1 = a + ib \\ z_2 = c + id \end{Bmatrix}$

Case 1: If $K = 1$

If $K = 1$ then locus of z will be $\perp$ bisector of line segment joining z_1 and z_2.

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Case 2: If $K \neq 1$

In general locus is a circle and is reffered as Applonius Circle.

$z(x+iy)$

z_1 $(a+ib)$ z_2 $(c+id)$

$$\Rightarrow \left|\frac{z-z_1}{z-z_2}\right| = K$$

$$\Rightarrow \sqrt{\frac{(x-a)^2+(y-b)^2}{(x-c)^2+(y-d)^2}} = K$$

$$\Rightarrow x^2 + y^2 + 2\left(\frac{a-K^2c}{K^2-1}\right)x + 2\left(\frac{b-K^2d}{K^2-1}\right)y + K' = 0$$

$\hookrightarrow$ Some fixed Constant

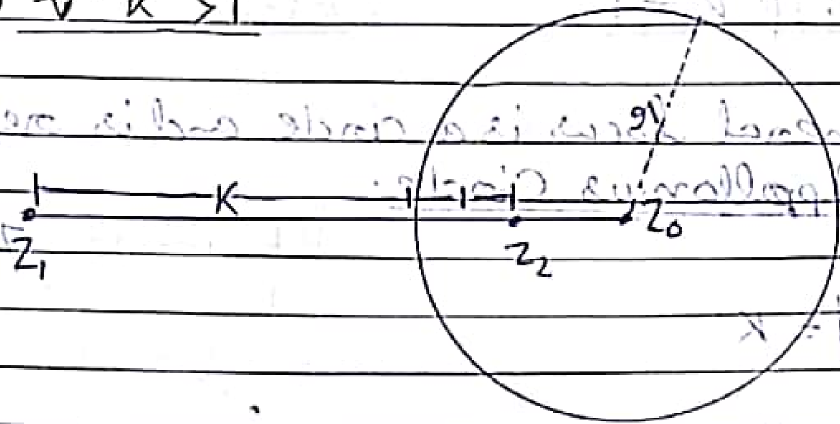
Hence locus is a circle.
Let z_0 be circle's centre and r as radius.

$$z_0 = \frac{z_1 - K^2 z_2}{1-K^2} \qquad r = K\left|\frac{z_1-z_2}{1-K^2}\right|$$

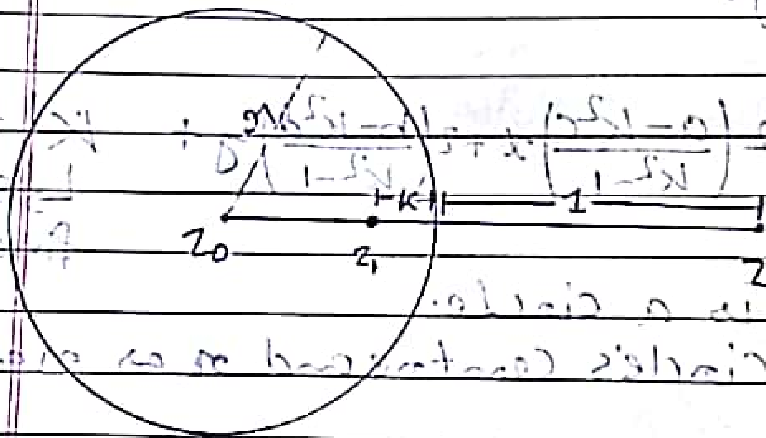
$\Rightarrow z_0$ divide $z_1 z_2$ externally in ratio $K^2 : 1$

Argand diagram representation:

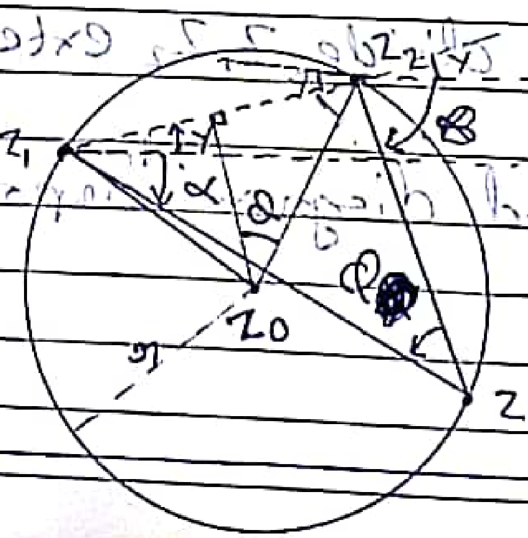
(i) $\forall\ K > 1$



(ii) $\forall\ K < 1$



(d) $\arg\left(\dfrac{z-z_1}{z-z_2}\right) = \varphi$



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Let $\gamma = |\arg(z_2 - z_1)|$

$\alpha = |\arg(z - z_1)|$ $\qquad \phi = \alpha - \beta$

$\beta = |\arg(z - z_2)|$

$\angle z_2 z z_1 + (-\alpha) = (-\beta)$

$\angle z_2 z z_1 = \alpha - \beta = \phi$

$\Rightarrow 2r \sin\phi = |z_1 - z_2|$

$$\boxed{r = \frac{|z_1 - z_2|}{2\sin\phi}} \quad \text{—— (I)}$$

$$(z_0 - z_1) = |z_0 - z_1| e^{i(\phi + \gamma - \pi/2)}$$

$$= r e^{i(\phi + \gamma - \pi/2)}$$

$$z_0 = (z_0 - z_1) + z_1$$

$$\boxed{z_0 = z_1 + r e^{i(\phi + \gamma - \pi/2)}} \quad \text{—— (II)}$$

⑥ <u>Miscellaneous</u>

$z_1 z_2 z_3$ and $\omega_1 \omega_2 \omega_3$ are similar iff :

$$\begin{vmatrix} z_1 & \omega_1 & 1 \\ z_2 & \omega_2 & 1 \\ z_3 & \omega_3 & 1 \end{vmatrix} = 0$$

Permutation and Combination Cont...

① To find the number of ways in which different $m+n+p$ things can be divided into three different groups containing m, n, p things severally.

→ First divide $m+n+p$ things into two groups containing m and $n+p$ things respectively.

→ Selecting m things from $m+n+p$ is equivalent to dividing $m+n+p$ things into two groups containing m and $n+p$ things respectively.

$$X_1 = \frac{(m+n+p)!}{m!\,(n+p)!}$$

→ Now let similarly divide $(n+p)$ things in two group containing n and p things respectively.

$$X_2 = \frac{(n+p)!}{n!\,p!}$$

$$\text{So } X = X_1 X_2 = \frac{(m+n+p)!}{m!\,(n+p)!} \times \frac{(n+p)!}{n!\,p!}$$

$$\Rightarrow X = \frac{(m+n+p)!}{m!\,n!\,p!}$$

Note: This can even be extended.

② To find the total number of ways in which it is possible to make a selection by taking some or all out of $p+q+r$ things, where p are alike of one kind, q are alike of second kind, and r are distinct.

⇒ The p things may be disposed of in $p+1$ ways; for we may take $0, 1, 2, \ldots p$ of them. Similarly the q things may be disposed of in $q+1$ ways.

$$X_1 = (q+1)(p+1)$$

→ Selection from r things may be thought off as either accepting or disposing things.

$$X_2 = 2 \times 2 \times \ldots r \text{ factors} = 2^r$$

So $X = X_1 X_2 - 1$

$$X = (p+1)(q+1)2^r - 1$$

Note
→ -1 is for rejecting case in which none of the things are selected.

~~③ Combination allowing repetition~~

③ $x_1 x_2 \cdots x_n = a_1^{K_1} a_2^{K_2} \cdots a_m^{K_m}$. Find the number of positive integral solution of $x_1, x_2, \cdots x_n$. $a_1, a_2, \cdots a_m \in$ Prime number.

$\Rightarrow$ Imagin first distribut K_1 a_1's in n boxes namely $x_1, x_2, \cdots x_n$.

→ So for this Imagin K_1 a_1's and $(n-1)$ Slas (/) in a row.

$$(a_1\ a_1\ a_1 \cdots K_1\ a\text{'s})(/\ /\ / \cdots (n-1)\ /\text{'s})$$

→ So number of way in which K_1 a_1's can be distributed in n boxes is:

$$\binom{K_1+n-1}{K_1} = \frac{(K_1+n-1)!}{K_1!\,(n-1)!}$$

Similarly K_2 a_2's can be distributed in $\binom{K_2+n-1}{K_2}$

Hence Total number of Solution is

$$\cancel{X = \prod_{i=1}^{m} \binom{K_i+n-1}{n-1}}$$

$$\boxed{X = \prod_{i=1}^{m} \binom{K_i+n-1}{n-1}}$$

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Conic Section in parametric form

Curves	Common Variable used	Parametric Point	Slope of Normal
Parabola $y^2=4ax$	t	$(at^2, 2at)$	$-t$
Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	φ	$(a\cos\varphi, b\sin\varphi)$	$\frac{a}{b}\tan\varphi$
Hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	ψ	$(a\sec\psi, b\tan\psi)$	$-\frac{a}{b}\sin\psi$

Solution to some general problems

① In a GP, if the common ratio is positive and less than 1/2 and the first term is positive, then each term of the GP is greater than sum of infinity of all the terms of the GP that follows it.

If: $0 < r < \frac{1}{2}$ & $a > 0$

$$\Rightarrow a \quad ar^2 \cdots ar^{\infty} \quad \{\text{Infinite GP}\}$$

$$\Rightarrow ar^{n-1} > ar^n + ar^{n+1} + \cdots + ar^{\infty}$$

② If $a_1, a_2 \cdots a_n$ are in AP

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \cdots + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}} = ?$$

$\Rightarrow$ let d be the common ratio.

$$\frac{\cancel{\sqrt{a_2}}-\sqrt{a_1}}{d} + \frac{\sqrt{a_3}-\cancel{\sqrt{a_2}}}{d} + \cdots + \frac{\sqrt{a_n}-\cancel{\sqrt{a_{n-1}}}}{d} \quad \{\text{Rationalisation}$$

$$\Rightarrow \frac{\sqrt{a_n}-\sqrt{a_1}}{d} = \frac{a_n - a_1}{d(\sqrt{a_n}+\sqrt{a_1})} = \frac{d(n-1)}{d(\sqrt{a_n}+\sqrt{a_1})}$$

$$\Rightarrow \frac{(n-1)}{\sqrt{a_n}+\sqrt{a_1}}$$

③ find remainder when 2014^{2015} is divided by 7

$\Rightarrow 2014^{2015} = (2009+5)^{2015}$

$\Rightarrow 5^{2015} = 5^{2013+2} = 25 \times 5^{2013}$

$= 25 \times (125)^{671} \Rightarrow 25 \times (126-1)^{671}$

$\Rightarrow 25 \times (-1)^{676} = -25 \Rightarrow 28 - 25 = 3$

④ find last two digits of 2014^{2015}

$\Rightarrow$ It could be solved by above method by finding remainder when 2014^{2015} is divided by 100.

⑧ Solving AGP

Let sum of AGP be S.

(i) Substract $\left(S - \frac{S}{r}\right)$

(ii) Then the series obtaind will be Simple AP or GP.

⑥ If $x = (\sqrt{2}+1)^6$ then find $[x]$

$\Rightarrow I + f = (\sqrt{2}+1)^6$

$f' = (\sqrt{2}-1)^6$

$I + f + f' = 2\left\{\binom{6}{0}(\sqrt{2})^0 + \binom{6}{2}2^1 + \binom{6}{4}2^2 + \binom{6}{6}2^3\right\}$

$0 < f < 1$ $\Rightarrow 0 < f + f' < 2$

$0 < f' < 1$

$\Rightarrow f + f' = 1$

$I + 1 = 2(1 + 30 + 60 + 8)$

$I = 197$ $\Rightarrow [x] = 197$

Some basic formulae

① $a^n - b^n = (a-b)\left(a^0 b^{n-1} + a b^{(n-2)} + a^2 b^{(n-3)} + \cdots\right)$

② $a^n + b^n = (a+b)\left(a^0 b^{n-1} - a b^{(n-2)} + a^2 b^{(n-3)} + \cdots\right)$

for n is odd

③ $\frac{x}{y} = \frac{a}{b} \Rightarrow \frac{x+y}{x-y} = \frac{a+b}{a-b}$ {Componendo & Dividendo}

④ $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{(n-i)}$ {binomial theorem}

⑤ $a^3+b^3+c^3 = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)+3abc$

⑥ $x^2+ax+c = \left(x \pm \frac{a}{2}\right)^2 + \left\{c - \left(\frac{a}{2}\right)^2\right\}$ {Compleating Square}

⑦ $\boxed{(a+b+c+d)^n = \sum \left(\frac{n!}{x!\,y!\,z!\,p!}\right) a^x b^y c^z d^p}$

{Where, $x+y+z+p=n$}

{Multinomial theorem}

⑧ $(1-x)^{-n} = \sum_{i=0}^{\infty} \binom{n+i-1}{i} x^i$

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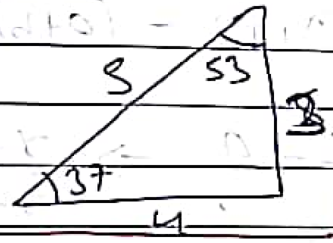
Miscellaneous

(i) $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

(ii) $\cos 37° = 4/5$



(iii) $\cancel{\frac{\cos A - \cos B}{\cos A \cos B}}$ $\boxed{\frac{1}{\cos A \cos B} = \frac{\tan A - \tan B}{\sin(A-B)}}$

(iv) $\boxed{S = 180(n-2)}$ {Sum of ~~i~~nternal angle of an $n°$ Polygon}

(v) 1, 2, 3 → AP
1, 2, 4 → GP
2, 3, 6 → HP

(vi) $\frac{r}{R} = \cos A + \cos B + \cos C - 1$

(vii) Number of ways of dividing n identical objects among r groups such that each gets atmost K is Coefficient of x^n in the expantion of $(1 + x + x^2 + \cdots + x^K)^r$

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(VIII) No of ways of Selecting K objects out of a like objects of one type, b like objects of 2nd type, c like object of 3rd type and n different object is:-

$\Rightarrow$ Cofficient of x^K in the expantion of:-

$$(1+x+\cdots+x^a)(1+x+x^2+\cdots x^b)(1+x+\cdots+x^c)(1+x)^n$$