ITC 100 Inspiring Years

classmate

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1

Classical Electrostatics & Dynamics

The sparking waters of **Lake McKenzie** *on Frazer Island, Australia, lap gently against the sandy shore*

Notebook

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INDEX

NAME: Aditya STD.: XII SEC.: A ROLL NO.: 01 SUB.: Physics

Introduction to Electric & Magnetic phenomena

1. Electric field & Potential ✓
2. Gauss's Law ✓
3. Capacitor ✓
4. Electric current in conductor ✓
5. Thermal & Electric effect of Electric current ✓
6. Magnetic field ✓
7. Magnetic field due to current ✓
8. Magnetism & matter ✓
9. Electromagnetic Induction ✓
10. Alternating Current ✓
11. Electromagnetic wave ✓

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Electric field and potential

⇌ Elementary particles

→ Matters are made up of certain elementary particles. In pressent hundreds of elementary particles have been discovered.

→ Most common elementary particles are electrons, protons and neutrons.

* Some other elementary particles :-

(i) graviton (ii) Photon

(iii) Higgs boson (iv) quarks

(v) X & Y bosons (vi) W' & Z' bosons

* Electric charge ⇒ Electric charge is a physical property of matter that causes it to experience a force when near other electrically charged matter.

* Electric force ⇒ Force of attraction and repulsion between charged particles.

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⇌ Two kind of Charge

→ It is observed that when two electrons or two Protons are placed near they exert a repulsive force on each other, whereas when one electron and one proton is brought near they exert ~~a repulsive~~ force on ~~each~~ attractive on each other.

→ It is also observed that if one electron and one proton are placed in a little space then, the net charge on the system is zero.

→ As per the above observation and convension an electron has ⊖ charge and a proton has ⊕ charge.

Unit of charge

→ SI unit of charge is coulomb abbreviated as C.

* 1 Coulomb ⇒ 1 Coulomb is the charge flowing through a wire in 1s if the electric current in it is 1A.

→ $\boxed{e = 1.6 \times 10^{-19} C}$

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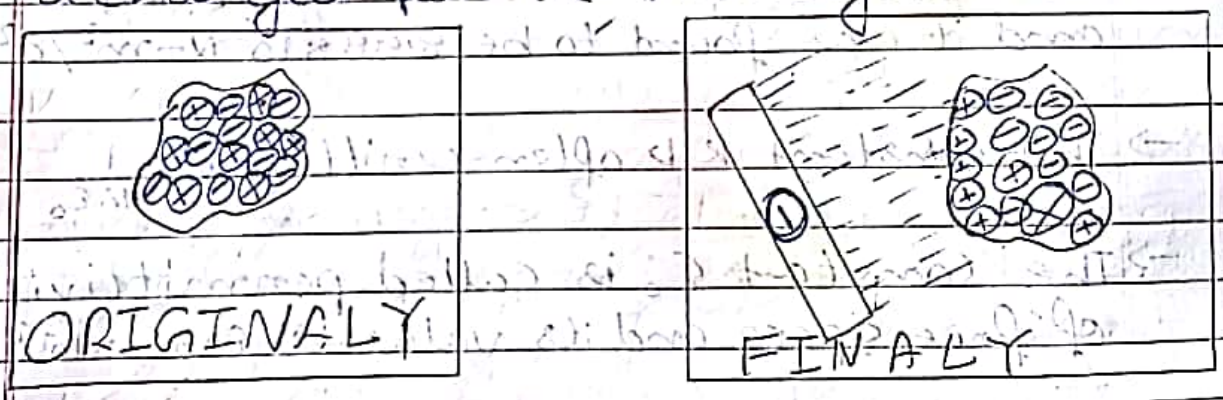
Properties of charge

→ Charge on any thing is quantized and integral multiples of e.

→ Charged particles may be created or distroyed but net charge always remains the same.

* Charge Induction ⇒ The displacement of charge in an isolated system when a charged particle is brought near.



⇒ **Coulomb's Law**

The experiments of Coulomb and others established that the force exerted by a charged on the other is given by :-

$$F = \frac{K q_1 q_2}{r^2}$$ where
- q_1, q_2 are charges
- r is separation
- K = constant

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→ The force is attractive if the charges are of opposite signs and is repulsive if they are of same sign.
We can write Coulomb's Law as :-

$$\vec{F} = \frac{k q_1 q_2 \vec{r}}{r^3}$$

where, $\vec{r}$ is the position vector of the force-experiencing particle with respect to the force-exerting particle.

→ The constant K can be measured experimentally and it was found to be 8.98×10^9 N-m²/C²

→ The constant K is often written as $\frac{1}{4\pi\varepsilon_0}$

→ The constant ε_0 is called permittivity of free space and its value is 8.85×10^{-12} C²/N-m²

⇒ Electric Field $\{\vec{E}\}$.

Electric field is a region around a charged particle or object within which a force would be exerted on other charged particles or objects.

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* Test charge → A test charge is an object {usually a point particle} that has negligible charge; one can ignore the electric field generated by the object itself.

→ One can calculate field at a point of space using expression :-

$\vec{E} = \vec{F}/q$ where, {$\vec{F}$ is the force experienced on test charge q}

→ Field is a vector quantity and can be added according to laws of vector addition.

→ Electric field ($\vec{E}$) by a charge q at a distance r is defined as :-

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

⇒ Lines of Electric force

· Electric field lines are graphical representation of Electric field. They have no physical Existance.

→ Lines of force are drawn in such a way that

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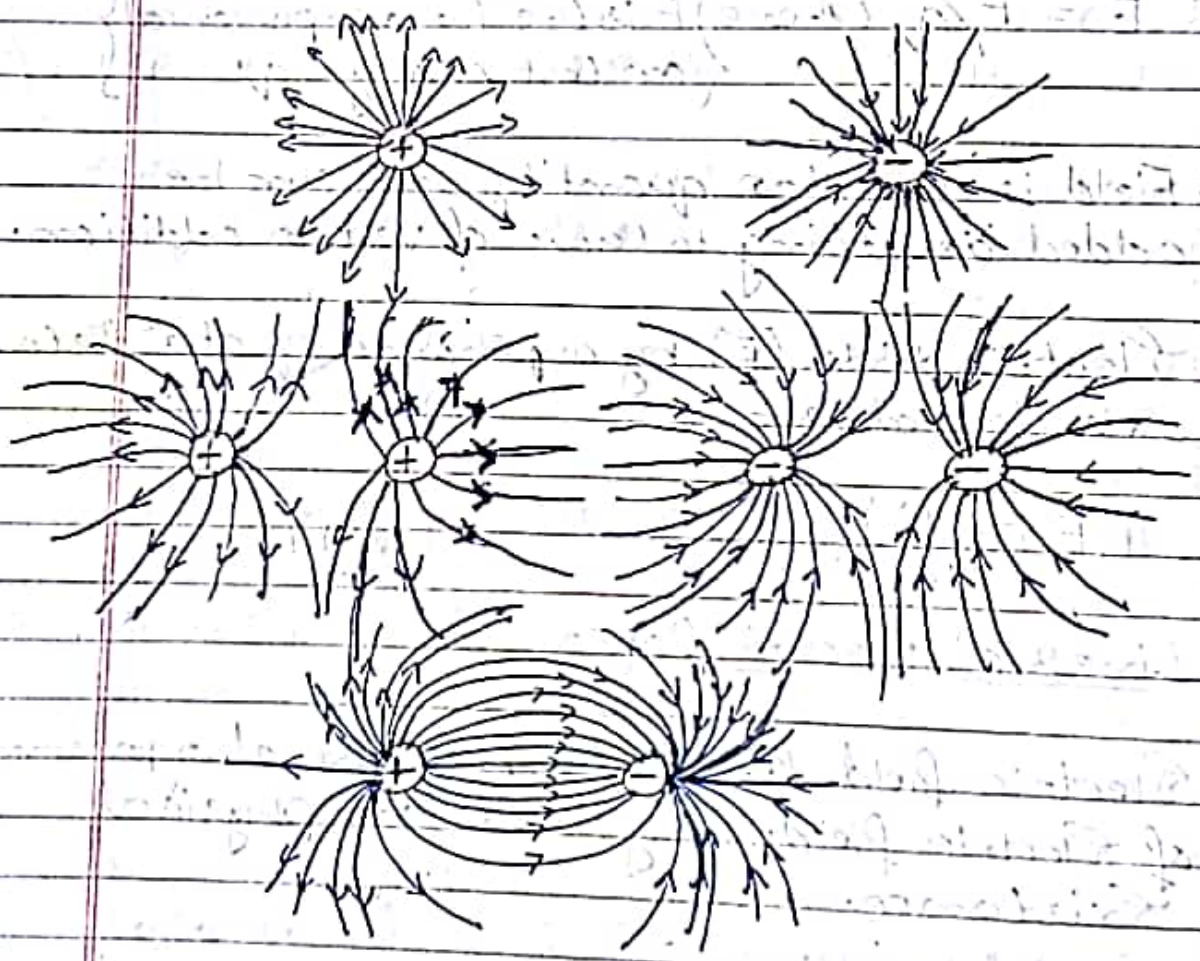
the tangent to the line of force gives the direction of the resultant electric field.

→ As per graphical representation, Intensity of electric field is directly proportional number of lines.

Graphical representation



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⇒ Electric potential Energy

→ Change in electric potential energy of the System is negative of the work done by the electric force.

Let us conciden two charged particle q_1 and q_2 initicly separated by distance r_1 and finally by r_2.

$$\Rightarrow dW = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr$$

$$\Rightarrow W = \int_{r_1}^{r_2} \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\Rightarrow U(r_1) - U(r_2) = \frac{-q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

// as per convention $U(\infty)$ is conciderd to be zero.

$$\Rightarrow U(\infty) - U(r) = \frac{-q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{\infty}\right]$$

$$\Rightarrow U(r) = \frac{q_1 q_2}{4\pi\varepsilon_0}\left[\frac{1}{r}\right] = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

Potential energy of an N particle system can be calculated by adding Potential energy of every possible pairs.

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⇒ **Electric potential**

Potential difference is defined as change in potential energy per unit charge.

$$V_B - V_A = \frac{U_B - U_A}{q}$$

* **Electric potential** ⇒ Electric potential at a point is the amount of electric potential energy that a unitary point charge would have when located at that point.

→ Potential energy and potential are scalar quantities.

* Electric potential by a point charge Q at distance r is :-

$$V_r = \frac{Q}{4\pi\varepsilon_0 r}$$

⇒ **Relation between Electric field and Potential**

* $V(\vec{r}) = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r}$

* **Equipotential surface** ⇒ Equipotential surface are surfaces in which Electric potential is always same.

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⇒ Electric Dipole

A combination of two charges +q and −q separated by a small distance d constitutes an Electric dipole.

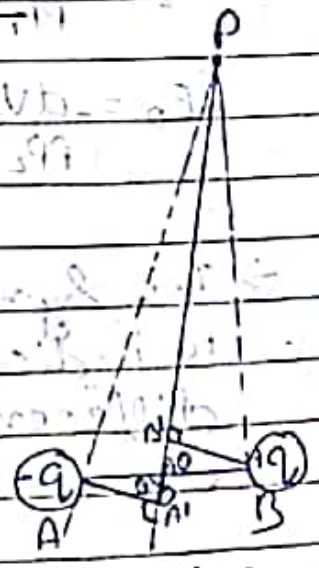
→ The electric dipole moment of this combination is defined as a vector :-

* $\boxed{\vec{p} = q\vec{d}}$

where $\vec{d}$ is the vector joining the negative charge to the positive charge.

→ The line along the direction of the dipole moment is called the axis of the dipole.

// Electric potential due to a Dipole



→ O is the middle point of AB.
→ let potential at point P be V.
→ $\angle POB = \theta$

⇒ $PA \simeq PA' = OP + OA' = r + \frac{d}{2}\cos\theta$

⇒ $PB \simeq P'B = OP - OB' = r - \frac{d}{2}\cos\theta$

$$V_1 = \frac{-q}{4\pi\varepsilon_0}\left[\frac{1}{r + \frac{d}{2}\cos\theta}\right] \quad \text{—(1)}$$

• $AB = d$
• $OP = r$ $\{r \gg d\}$

$$\Rightarrow V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - \frac{d}{2}\cos\theta}\right] \quad \text{--- (1)}$$

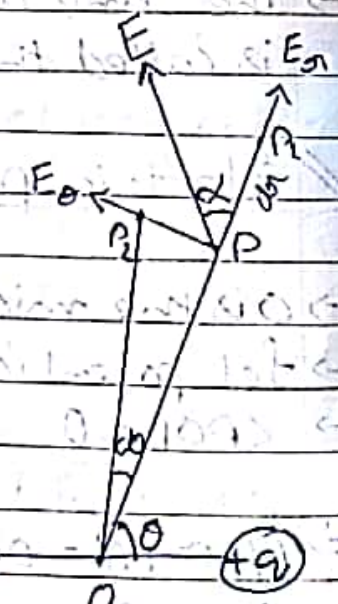
$$\Rightarrow V = V_1 + V_2$$

$$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - \frac{d}{2}\cos\theta} - \frac{1}{r + \frac{d}{2}\cos\theta}\right]$$

$$\Rightarrow V = \frac{q\,d\cos\theta}{4\pi\varepsilon_0\left[r^2 - \frac{d^2}{4}\cos^2\theta\right]}$$

$$\Rightarrow \boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}}$$

// <u>Electric field due to a dipole</u>



$$E_r = -\frac{dV}{PP_1} = \frac{-dV}{dr} = -\frac{\delta V}{\delta r}$$

$$E_\theta = -\frac{dV}{PP_2} = \frac{-dV}{r\,d\theta} = -\frac{\delta V}{r\,\delta\theta}$$

$\Rightarrow$ The symbol δ etc are used to indicate that while differentiating with respect to one coordinate, the others are kept constant.

* $E_r = \frac{-\delta V}{\delta r} = \frac{-P\cos\theta}{4\pi\varepsilon_0}\frac{\delta}{dr}\left(\frac{1}{r^2}\right) = \frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}$

* $E_\theta = \frac{-1}{r}\frac{\delta V}{\delta\theta} = \frac{-1}{r}\times\frac{P}{4\pi\varepsilon_0 r^2}\frac{\delta}{\delta\theta}\cos\theta = \frac{P\sin\theta}{4\pi\varepsilon_0 r^3}$

$\Rightarrow |\vec{E}| = |\vec{E_r} + \vec{E_\theta}| = \sqrt{\left(\frac{2P\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{P\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$

$$\boxed{E = \frac{P}{4\pi\varepsilon_0 r^3}\sqrt{(3\cos^2\theta + 1)}}$$

$\Rightarrow \tan\alpha = E_\theta / E_r = \frac{1}{2}\tan\theta$

$\Rightarrow \boxed{\alpha = \tan^{-1}\left[\frac{1}{2}\tan\theta\right]}$

// Special cases

* If $\theta = 0$

$V = \frac{P}{4\pi\varepsilon_0 r^2}$; $E = \frac{2P}{4\pi\varepsilon_0 r^2}$

* If $\theta = \pi/2$

$V = 0$; $E = \frac{P}{4\pi\varepsilon_0 r^3}$; $\alpha = \pi/2$

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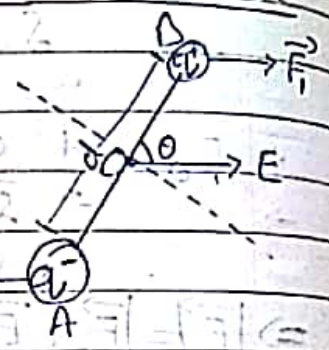
⇌ Torque on an Electric dipole placed in an Electric field



Let the axis of dipole (AB) make angle θ with Electric field at certain instant.

* $\vec{F_1} = q\vec{E}$; $\vec{F_2} = -q\vec{E}$

// $\vec{\Gamma_1} = \vec{OB} \times \vec{F_1} = q(\vec{OB} \times \vec{E})$

// $\vec{\Gamma_2} = \vec{OA} \times \vec{F_2} = -q(\vec{OA} \times \vec{E}) = q(\vec{AO} \times \vec{E})$

$\Rightarrow \vec{\Gamma} = \vec{\Gamma_1} + \vec{\Gamma} = q(\vec{OB} + \vec{AO}) \times \vec{E}$

$\Rightarrow \vec{\Gamma} = q\vec{AB} \times \vec{E} = \vec{p} \times \vec{E}$

⇌ Potential Energy of a dipole placed in uniform Electric field

* Change in potential energy is defined as negative of the work done.

* Let work done by field in rotating dipole by $d\theta$ be W.

$\Rightarrow dW = \tau d\theta$

$\Rightarrow dW = -PE \sin\theta \, d\theta$

$\Rightarrow W = \cancel{+PE\cos\theta}$

$\Rightarrow \Delta U = -dW = \cancel{PE\cos\theta}\ PE \sin\theta \, d\theta$

// Let $U(\pi/2)$ be zero.

$\Rightarrow U(\theta) - U(\pi/2) = \int_{\pi/2}^{\theta} PE \sin\theta \, d\theta = -PE(\cos\theta - \cos 90)$

$= -PE\cos\theta$

$\Rightarrow \boxed{U = -\vec{P} \cdot \vec{E}}$

$\Rightarrow$ Conductors, Insulators & semiconductors

* Conductor $\Rightarrow$ A material that allows free movement of electrons and therefore allows easy flow of electricity.

* Insulator $\Rightarrow$ A material that dose'nt allows free movement of electrons and therefore dosent allows flow of electricity.

* Semiconductor $\Rightarrow$ Any of various solid crystalline substance having electrical conductivity greater than insulators but less than good conductors, is a semiconductor.

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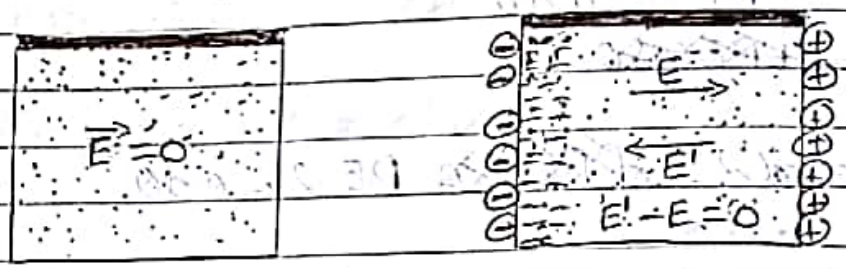
⇌ The Electric field inside a conductor



→ Whenever a conductor is placed in an electric field some of the free electrons redistribute themselves on the surface of the conductor.

→ The redistribution takes place in such a way that Electric field by conductor and original Electric field become equal and opposite. So Net electric field inside the conductor become zero

→ The redistribution takes a time which is in general, less than a millisecond.

→ There can be no electric field inside a conductor in electrostatics.

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Gauss's Law

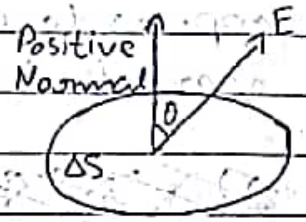
⇒ Flux of an Electric field through a surface

* Let there be a hypothetical plane surface of area ΔS and suppose a uniform electric field $\vec{E}$ acting on that area and making angle θ with positive Normal to the surface.

→ The quantity :-

$\Delta \phi = \vec{E} \cdot \vec{\Delta S} = E \Delta S \cos\theta$

is called the flux of the electric field through the chosen surface.



* Electric flux ⇒ Electric flux is the rate of flow per unit area of the electric field.

→ Flux is a scalar quantity and may be added using the rule of scalar addition.

* open surface ⇒ Surface which dose'nt enclose a volume.
Eg → hemisphere, disc etc...

* closed surface ⇒ Surface which enclose a volume.
Eg → Sphere, cube etc...

→ when flux through a closed surface is required, we use a small circular sign on the integration symbol.

$\phi = \oint \vec{E} \cdot \vec{ds}$

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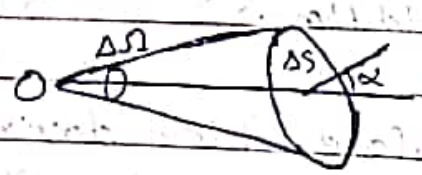
→ It is customary to take the outer normal as positive in this case.

⇒ <u>Solid Angle</u> $\{\Omega\}$

Solid angle is a measure of how large the object appears to an observers looking from that point.

⇒ The solid angle is formed by the line joining the points on the periphery with that point.

→ If a small plane area ΔS subtends a solid angle $\Delta\Omega$ at O then :-



$$\Delta\Omega = \frac{\Delta S \cos\alpha}{r^2}$$

{where, r = distance between O and ΔS
α = angle between r and ΔS}

→ Any closed surface subtends a solid angle 4π at any internal point.

→ A closed surface subtends zero solid angle at an external point.

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Gauss's Law and its derivation from Coulomb's Law

The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by ε_0.

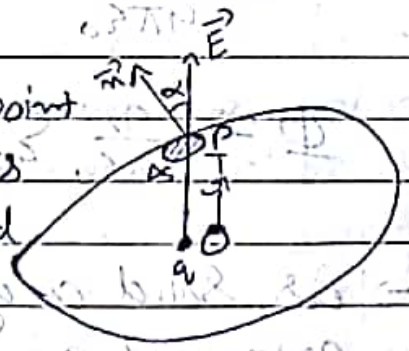
$$\oint \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

{where, q_{in} is the net charge enclosed by the surface through which the flux is calculated}

* Proof of Gauss's Law {Assuming Coulomb's Law}

① flux due to internal charge

Let a charge q is placed at a point O inside a "closed" surface. Let us consider a small area ΔS around point P on the surface.



Electric field at point P due to charge q :-

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

{α = angle between area vector and field.}

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\Delta S\cos\alpha = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S\cos\alpha$$

$$\Delta\Phi = \frac{q\,\Delta\Omega}{4\pi\varepsilon_0}$$

{$\Delta\Omega = \frac{\Delta S\cos\alpha}{r^2}$ = solid angle}

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$$\Rightarrow \Phi = \sum \frac{q\,\Delta\Omega}{4\pi\varepsilon_0} = \frac{q}{4\pi\varepsilon_0}\sum \Delta\Omega = \frac{q}{4\pi\varepsilon_0}\cdot 4\pi$$

$$\Phi = \frac{q}{\varepsilon_0}\left\{\sum \Delta\Omega = 4\pi \text{ as the surface is closed}\right\}$$

② <u>Flux due to an external charge</u>

Let the charge q be outside the closed surface at distance r from ΔS.

$$\Delta\Phi = \frac{q}{4\pi\varepsilon_0}\,\frac{\Delta S\cos\alpha}{r^2} = \frac{q\,\Delta\Omega}{4\pi\varepsilon_0}$$

$$\Phi = \frac{q}{4\pi\varepsilon_0}\sum \Delta\Omega = 0$$

$\Rightarrow$ As, solid angle of closed surface from external point is zero.

* <u>Gaussian surface</u> $\Rightarrow$ A surface on which gauss's law is applied is called a Gaussian surface.

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⇒ Applications of gauss's Law

(A) Charged Conductor

→ In presence or absence of electric field any volume completely inside a conductor is electrically neutral.

→ If a charge is injectet anywhere inside the conductor, then it must come over the surface of the conductor so that the interior is always charge free. Also if the conductor has a cavity, the charge must come over the outer surface.

→ If a charge q is placed in the cavity then opposite charge appears on the cavity surface and alike charge appears on the outer surface.

→ If there is a cavity in the conductor and no charge is placed in the cavity, the electric field all all points of cavity is zero.

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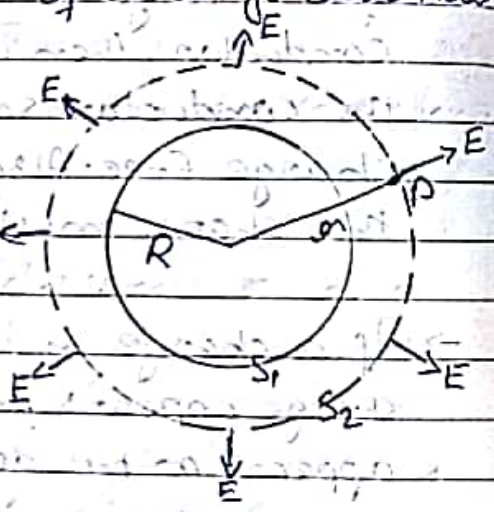
(B) Electric field due to a uniformly charged sphere.

* Let total charge Q in uniformly distributed in a spherical volume of radius R.

(1) Field at an outside point

→ Let point P be outside the charge distribution at distance r from centre of charge distribution ie $r > R$.

→ Let us draw a spherical gaussian surface through point P concentric to sphere.



$\Phi = \int \vec{E} \cdot \vec{ds}$ {flux}

$\Phi = E \int ds$

$\Phi = E \cdot 4\pi r^2 = 4\pi r^2 E$

$\Rightarrow \Phi = \frac{Q_{in}}{\varepsilon_0}$

$\Rightarrow 4\pi r^2 E = \frac{Q}{\varepsilon_0}$

$\Rightarrow \boxed{E = \frac{Q}{4\pi r^2 \varepsilon_0}}$

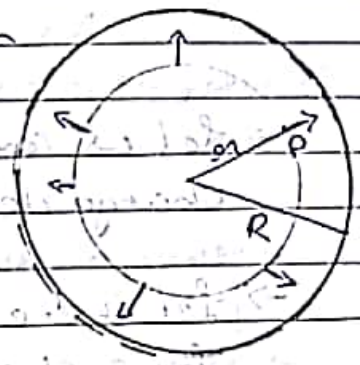
* The electric field due to a uniformly charged sphere at a point outside it, is identical

with the field due to an equal point charge placed at the centre.

(ii) Field at an ~~ext~~ internal point.

→ Let point p be ~~outside~~ inside the charge distribution at distance r from centre of sphere. ie.. $r<R$

→ Let us draw a spherical gaussian surface through point p concentric to sphere.



* $\Phi = \int \vec{E}\cdot d\vec{s} = E\int ds = 4\pi r^2 E$ — (I)

* $\Phi = \frac{Q_{in}}{\varepsilon_0}$ — (II)

* $Q_{in} = \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3 = \frac{Qr^3}{R^3}$ — (III)

// Using Eq (I), (II) and (III) we get :-

⇒ $4\pi r^2 E = \frac{Qr^3}{\varepsilon_0 R^3}$

⇒ $\boxed{E = \frac{Qr}{4\pi\varepsilon_0 R^3}}$

* The electric field due to a uniformly charged sphere at an internal point is $Qr/4\pi\varepsilon_0 R^3$ in radial direction.

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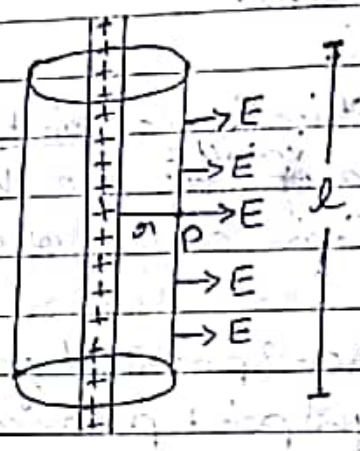
(c) Electric field due to a linear charge distribution



→ Let us consider a long line charge with a linear charge density λ.

→ Let there be point P at r distance from line charge

→ Let us draw a cylinder of length l passing through P and coaxial with the line charge

* $\Phi\{\text{at curved surface}\} = \int \vec{E}\cdot d\vec{s} = E\int ds = 2\pi r l E$

* $\Phi_0\{\text{at lids of cylinder}\} = \int \vec{E}\cdot d\vec{s} = \int E\,ds\cos 90 = 0$

$\Rightarrow \Phi = \frac{Q_{in}}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

$\Rightarrow 2\pi r l E = \frac{\lambda l}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$

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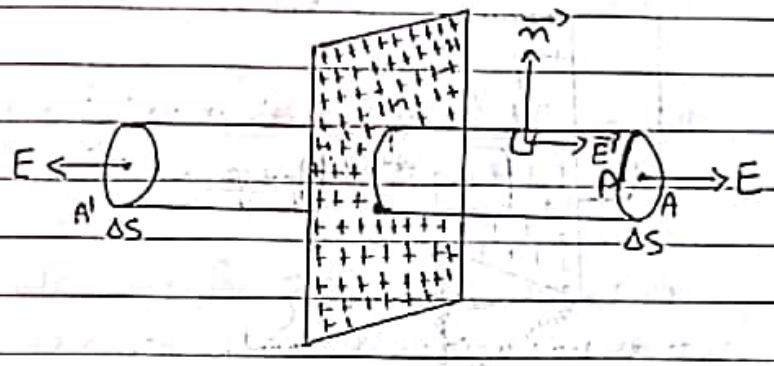
(D) Electric field due to a plane sheet of charge



→ Let us consider a large plane sheet of charge with surface charge density σ.

→ Let there be point P at ⊥ distance from sheet.

→ Draw a plane surface passing through P and ∥ to the charge sheet. Extend the surface to the other side of the charge sheet till equidistant symmetry. Close the cylinder on the other side. This will be our gaussian surface.

* $\Phi_D\{\text{curved surface}\} = \vec{E}\cdot\vec{\Delta S} = E\cdot\Delta S\cos 90^\circ = 0$
* $\Phi_1\{\parallel \text{surface}\} = \vec{E}\cdot\vec{\Delta S} = E\cdot\Delta S\cos 0 = E\Delta S$

* $\Phi = \int \vec{E}\cdot\vec{\Delta S} = E\Delta S + E\Delta S + 0 = 2E\Delta S$

$$\Rightarrow \Phi = \frac{Q_{in}}{\varepsilon_0} = \frac{\sigma\Delta S}{\varepsilon_0} = 2E\Delta S \Rightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

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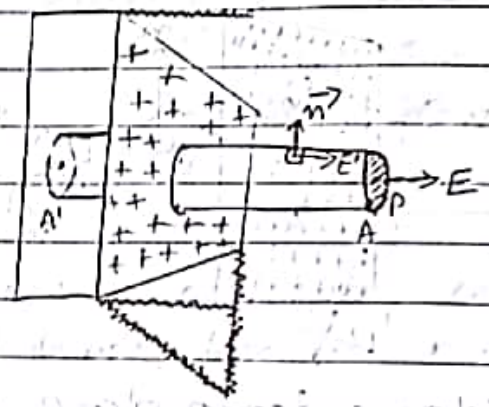
(5) Electric field near a charged conducting surface



→ Let charge density on the surface be σ.
→ Let there be point p near the surface outside the conductor.

* Defining gaussian surface
→ Let us draw a surface ∥ to conductor which includes p.
→ Draw cylinder with the conductor and extend it inside the conductor.

* $\Phi\{\text{at curved surface}\} = \int \vec{E}\cdot\Delta\vec{S} = \int E\Delta S\cos 90 = 0$
* $\Phi\{\text{at ∥ surface}\} = \int \vec{E}\cdot\Delta\vec{S} = E\Delta S$

⇒ $\Phi = \frac{Q_{in}}{\varepsilon_0}$ {using Gauss Law}

⇒ $E\Delta S = \frac{\sigma\Delta S}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$

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* The electric field near a charged, conducting surface is σ/ε_0 and it is normal to the surface.

* <u>Some generalized statement on Spherical Charge distributions:</u>

(a) The electric field due to a uniformly charged spherical shell at ~~a~~ an external point is the same as that due to an equal point charge placed at at the centre of the shell, $E = \frac{Kq}{r^2}$.

(b) The electric field due to a uniformly charged spherical shell at an internal point is zero.

(c) The electric potential due to a uniformly charged thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre.

(d) The electric potential due to a uniformly charged thin spherical shell at an internal point is the same everywhere and is equal to that at the surface. $V = Q/4\pi\varepsilon_0 R$.

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Electric potential energy of a uniformly charged sphere

→ Let there be a uniformly charged sphere of radius R having a total charge Q.

→ The electric potential energy of this sphere is equal to the work done in bringing the charges from infinitive to assemble the sphere.

→ Let at some instant, charge assembled up to a radius x.

→ In the next step, we bring some charge from infinity and put it on this sphere to increase the radius from x to x+dx.

→ The entire sphere is assembled as x varies from 0 to R.

The charge density is :-

$$\rho = \frac{3Q}{4\pi R^3}$$

When the radius of the sphere is x, the charge contained in it is :-

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$$q = \frac{4}{3}\pi x^3 \rho = \frac{Q}{R^3} x^3$$

The potential at the surface is :-

$$V = \frac{q}{4\pi\varepsilon_0 x} = \frac{Q}{4\pi\varepsilon_0 R^3} x^2$$

The charge needed to increase the radius from x to $x+dx$ is

$$dq = 4\pi x^2 dx \rho = \frac{3Q}{R^3} x^2 dx$$

Work done in bringing the charge dq from infinity to the surface of the sphere of radius x is :-

$$W = \int_0^R V\,dq = \int_0^R \frac{3Q^2 x^4}{4\pi\varepsilon_0 R^6} dx = \frac{3Q^2}{20\pi\varepsilon_0 R}$$

⇒ <u>Electric potential energy of a uniformly charged thin spherical shell</u>

→ Let there a a uniformly charged, thin spherical shell of radius R having a total charge Q

→ The electric potential energy is equal to the work done

in bringing charge from infinity and put them on the shell.

→ Let at some instant a charge q is placed on the shell. The potential at the surface is :-

$$V = \frac{q}{4\pi\varepsilon_0 R}$$

→ The work done in bringing a charge dq from infinity to this shell is

$$W = \int_0^Q V \cdot dq = \int_0^Q \frac{q\,dq}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

Earthing a conductor

→ The earth is a good conductor of electricity.

→ The earth surface has a negative charge about $1 nC/m^2$.

→ If a conductor is connected to earth, the potential of the conductor becomes equal to that of the earth i.e. zero.

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→ If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

→ When a conductor is connected to earth, the conductor is said to be earthed or grounded

→ Symbol of earthing.

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31

Capacitors

⇌ Capacitor and capcitance

* Capacitor ⇒ A combination of two conductor placed close to each other is called a capacitor

→ One of the conductor is given a positive charge and the other is given an equal negative charge.

→ The charge on the positive plate is called the charge on the capacitor and the potential difference between the plates is called the potential of the capacitor.

* Symbol used to represent capacitor. ⊣⊢

→ The charge Q on the capacitor is proportional to the potential difference V between the plates.

$$Q \propto V$$
$$Q = CV$$

→ The proportionality constant C is called the capacitance of the capacitor.

→ Capacitance depends on the shape, size and geometrical placing of the conductors

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and the medium between them.

→ The SI unit of capacitance is C/V which is written as farad. The symbol F is used for it.

* Properties of an ideal battery :-

(a) A battery has two terminals.

(b) The potential difference V between the terminals is constant for a given battery. The terminal with higher potential is called the positive terminal and that with lower potential is called the negative terminal.

(c) The value of this fixed potential difference is equal to the electromotive force or emf of the battery.

{ when the two plates of a capacitor are connected to the terminals of a battery the potential difference between the plates of the capacitor becomes equal to the emf of the battery. }

(d) Total charge in a battery always remains zero.

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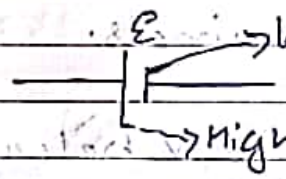
e) When a charge Q passes through a battery of emf ε from the negative terminal to the positive terminal, an amount of $Q\varepsilon$ of work is done by the battery.

{ An ideal battery is represented by the symbol :-



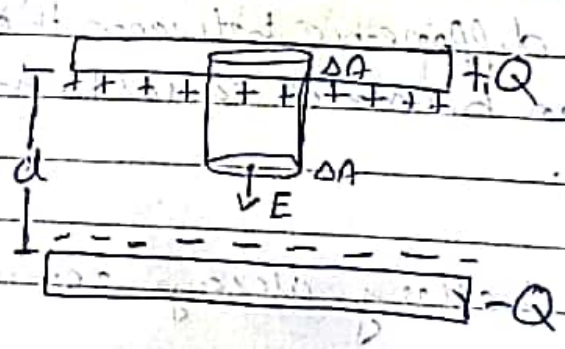
}

⇒ Calculation of capacitance

@ parallel-plate capacitor

→ Let us consider two square plates ∥ to each other of area A and separation d with charges ±Q on the opposit plates.

→ Suppose that the plates are large as compared to the separation between them.



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Defining gaussian surface

→ Let us draw a small area ΔA ∥ to the plates and in between them.

→ Draw a cylinder with ΔA as a cross-section and terminate it by another symmetrically situation area ΔA inside the positive plates.

$$\phi = \vec{E} \cdot \Delta \vec{A} = E \Delta A$$

charge inclosed $= \Delta A \sigma = \frac{Q \Delta A}{A}$

$$\Rightarrow \int \vec{E} \cdot \Delta A = \frac{Q_{in}}{\varepsilon_0} \quad \{\text{Gauss Law}\}$$

$$\Rightarrow E = \frac{Q}{\varepsilon_0 A}$$

$$\Rightarrow V = -\int_A^D \vec{E} \cdot d\vec{r} = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$\Rightarrow \boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}}$$

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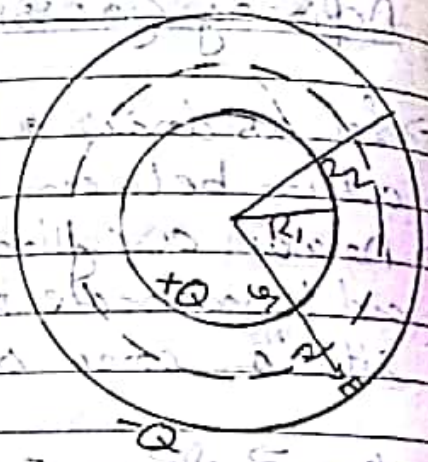
(b) Spherical capacitor



$$\Rightarrow \int \vec{E}\cdot\vec{\Delta S} = \frac{Q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\,4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow V = -\int_A^B E\,dr$$

$$\Rightarrow V = -\int_{R_1}^{R_2} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{-Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r^2}\,dr$$

$$\Rightarrow V = \frac{-Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{R_1}^{R_2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow V = \frac{Q(R_2-R_1)}{4\pi\varepsilon_0 R_1 R_2}$$

$$\Rightarrow \boxed{C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2-R_1}}$$

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* Special cases of Spherical capacitor

(i) Isolated sphere

→ If ~~both R and~~ we assume that the outer sphere is at ~~infinity~~ infinity, we get an isolated single sphere of radius R_1.

→ $R_2 \to \infty$

$$\Rightarrow C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1} \simeq \frac{4\pi\varepsilon_0 R_1 R_2}{R_2} = 4\pi\varepsilon_0 R$$

(ii) Parallel limit

→ If both R_1 and R_2 are made large but $R_2 - R_1 = d$ is kept fixed, we can write:

$$\Rightarrow C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1} = \frac{4\pi\varepsilon_0 R^2}{d} = \frac{\varepsilon_0 A}{d}$$

{where, $R \simeq R_1 \simeq R_2$}

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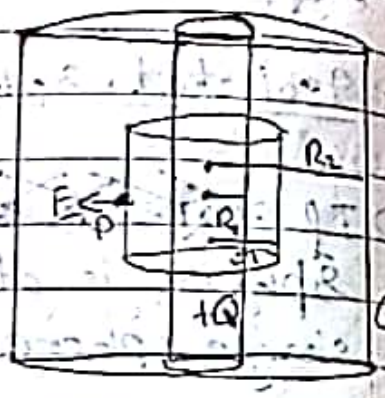
(c) Cylindrical capacitor



$$\Rightarrow \int \vec{E}\cdot\vec{dS} = \frac{Q_{in}}{\varepsilon_0}$$

$$\Rightarrow E\cdot 2\pi r x = \frac{Qx}{l\varepsilon_0}$$

$$\Rightarrow E = \frac{Q}{2\pi r l \varepsilon_0}$$

$$\Rightarrow V = -\int_A^B \vec{E}\cdot\vec{dr} = -\int_{R_1}^{R_2} E\,dr = -\int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 r l}\,dr$$

$$\Rightarrow V = \frac{Q}{2\pi\varepsilon_0 l}\ln\left(\frac{R_2}{R_1}\right)$$

$$\Rightarrow \boxed{C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 l}{\ln(R_2/R_1)}}$$

⇒ Combination of capacitors

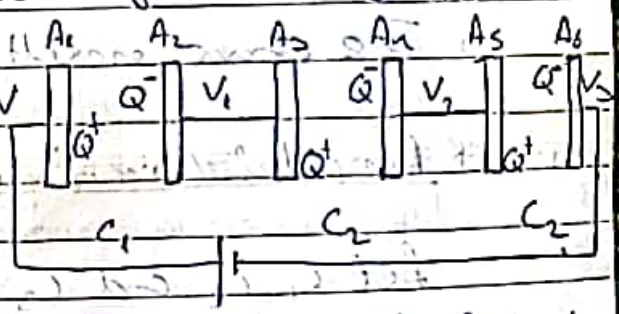
→ If V be the potential difference between the points and Q be the magnitude of the charges supplied by either terminal of the battery.

→ We define equivalent capacitance of the combination between the two points to be :-

$$C = Q/V$$

* Series combination {Voltage divide in inverse ratio}

→ Let C_1, C_2 and C_3 be three capacitors connected in series to each other.



→ The Q^+ is given by the battery to plate A_1 and Q^- to plate A_6.

→ In series combination, each capacitor has equal charge for any value of capacitances.

⇒ $C = \frac{Q}{V}$ {C = capacitance}

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$\Rightarrow C_1 = \frac{Q}{V - V_1}$ * $C_2 = \frac{Q}{V_1 - V_2}$ * $C_3 = \frac{Q}{V_2 - V_3}$

$\Rightarrow$ Let potential at plate A_6 be zero.

$\Rightarrow V - V_1 = \frac{Q}{C_1}$ * $V_1 - V_2 = \frac{Q}{C_2}$ * $V_2 - 0 = \frac{Q}{C_3}$

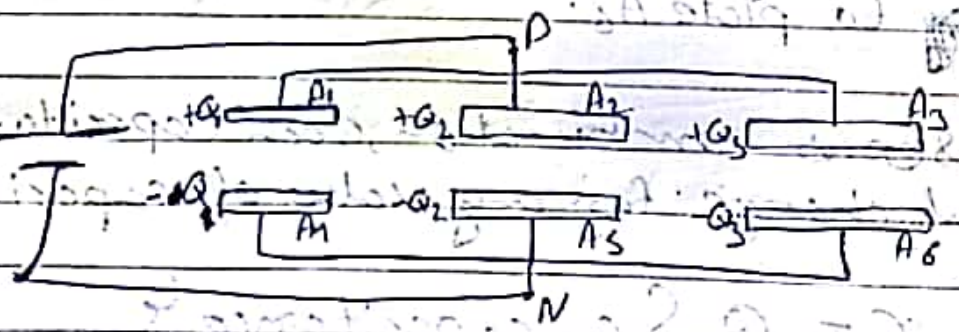
$\Rightarrow V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$

$\Rightarrow$ The above result may be extended to n capacitors.

* Parallel Combination {Charges divides in same ratio}

Let C_1, C_2 and C_3 be the capacitors connected ||l to each other.



→ Charge +Q is given to terminal P and −Q is given to terminal N.

→ In ll combination potential difference for any capacitance is equal.

⇒ $Q = Q_1 + Q_2 + Q_3$

⇒ $C_1 = \frac{Q_1}{V}$, $C_2 = \frac{Q_2}{V}$, $C_3 = \frac{Q_3}{V}$

⇒ $\frac{Q_1 + Q_2 + Q_3}{V} = C_1 + C_2 + C_3$

⇒ $\frac{Q}{V} = C_1 + C_2 + C_3$

⇒ $\boxed{C = C_1 + C_2 + C_3}$

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General rules for combination of capacitors

Step1: Identify the two points between which the equivalent capacitance is to be calculated. Call any one of them as P and the other as N

Step2: Connect (mentally) a battery between P and N with the positive terminal connected to P and the negative terminal to N. Send a charge +Q from the positive terminal of the battery and −Q from the negative terminal of the battery.

Step3: Write the charges appearing on each of the plates of the capacitors. The charge conservation principle may be used. The facing surfaces of a capacitor will always have equal and opposite charge. Assume variables $Q_1, Q_2 \cdots$ etc. for charges wherever needed.

Step4: Take the potential of the negative terminal N to be zero and that of the positive terminal P to be V. Write the potential of each of the plates. If necessary, assume variable $V_1, V_2 \cdots$

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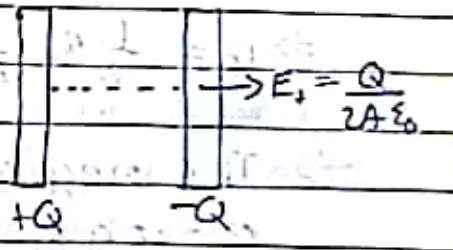
Step5: Write the capacitor equation $Q=CV$ for each capacitor. Eliminate $Q_1, Q_2 \cdots$ and $V_1, V_2 \cdots$ etc to obtain the equivalent capacitance $C = Q/V$.

⇒ Force between plates of a capacitor

⇒ $E = \dfrac{Q}{2A\varepsilon_0}$

⇒ $\boxed{F = \dfrac{Q^2}{2A\varepsilon_0}}$



→ F is the force of attraction between the two charged plates.

⇒ Energy Stored in a capacitor and Energy Density in Electric field

⇒ $W = Fd = \dfrac{Q^2 d}{2A\varepsilon_0} = \dfrac{Q^2}{2C}$

⇒ $\boxed{U = \dfrac{Q^2}{2C} = \dfrac{CV^2}{2} = \dfrac{QV}{2}}$

→ The energy stored in a capacitor is electrostatic potential energy.

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* The energy stored per unit volume in the electric field of capacitor is

$$\Rightarrow u = \frac{\frac{Q^2(d_2-d_1)}{2A\varepsilon_0}}{A(d_2-d_1)} = \frac{Q^2}{2A^2\varepsilon_0}$$

$$\Rightarrow u = \frac{1}{2}\varepsilon_0\left(\frac{Q}{A\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

→ The region containing electric field E has energy $\frac{1}{2}\varepsilon_0 E^2$ per unit volume.

Dielectrics

* Dielectric ⇒ A dielectric is an electrical insulator that can be polarized by an applied electric field.

* Polar material ⇒ Polar material are materials in which ~~centre~~ centre of positive and negative charge of its building blocks (atom, molicule) ~~are~~ dose not coincides.

→ In such material each building blocks have permanent dipole moment.

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→ If such a material is placed in an electric field, the individual dipoles experiences torque due to the field and they try to align along the field.

* As a result, we get a net dipole moment in any volume of the material.

* Non-Polar material ⇒ Non-polar material are materials in which centre of positive and negative charge of its building blocks concides.

→ In such material ~~each~~ building blocks dosent have permanent dipole moment.

→ If such a material is placed in an electric field, the electron charge distribution is slightly shifted opposite to the electric field.

* This induced dipole moment in building blocks ~~per~~ results in dipole moment in any volume of the material.

* Polarization ⇒ Polarization is defined as the dipole moment per unit volume.

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* Dielectric Constant ⇒ Dielectric constant is a factor by which capacitance of capacitor increases when dielectric material is inserted between them.

⇒ Dielectric Constant

* Relative permittivity ⇒ The relative permittivity of a material under given condition reflects the extent to which it concentrates electrostatic lines of flux.

or

⇒ It is the ratio of the amount of electrical energy stored in a material by an applied voltage relative to that stored in a vacuum.

* dielectric breakdown ⇒ If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atom. The dielectric then behaves like a conductor. This phenomenon is known as dielectric breakdown.

* Dielectric Strength ⇒ The minimum field at which the breakdown occures is called the dielectric strength.

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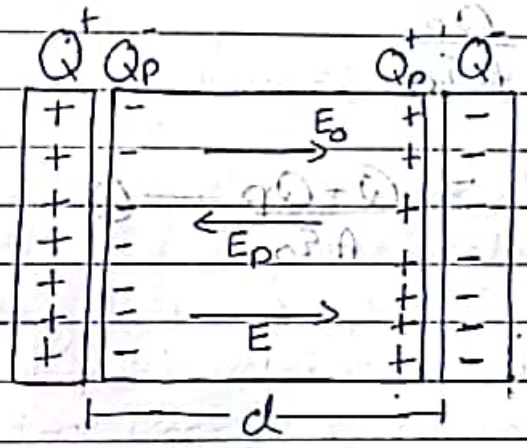
⇒ Parallel-plate capacitor with a dielectric

Let us consider a parallel-plate capacitor with plate area A and separation d between the plates with a dielectric slab of dielectric constant K inserted in the space between the plates.



* $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ (→)

→ From the defination of dielectric constant :-

* $E = \frac{E_0}{K} = \frac{Q}{A\varepsilon_0 K}$

⇒ $V = Ed = \frac{Qd}{A\varepsilon_0 K}$

⇒ $C = \frac{Q}{V} = \frac{KA\varepsilon_0}{d} = KC_0$ $\left\{\text{where, } C_0 = \frac{\varepsilon_0 A}{d}\right\}$

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→ The capacitance of a capacitor is increased by a factor of K when the space between the plates is filled with a dielectric of dielectric constant K.

* Magnitude of the induced charge

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{Q_p}{A\varepsilon_0}$$

$$E = E_0 - E_p = \frac{Q - Q_p}{A\varepsilon_0} \quad \text{--- (I)}$$

$$E = \frac{Q}{\varepsilon_0 A K} \quad \text{--- (II)}$$

// Using (I) and (II) we get :-

$$\Rightarrow \frac{Q - Q_p}{\varepsilon_0 A K} = \frac{Q}{\varepsilon_0 A K}$$

$$\Rightarrow \boxed{Q_p = Q\left(1 - \frac{1}{K}\right)}$$

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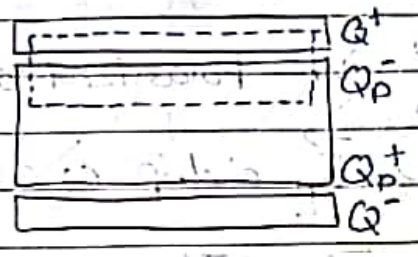
⇒ An alternative form of Gauss's Law



⇒ Charge inclosed $= Q - Q_p$

$$\Rightarrow \oint \vec{E}\cdot d\vec{S} = \frac{Q - Q_p}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left[Q - Q\left(1 - \frac{1}{K}\right)\right] = \frac{Q}{\varepsilon_0 K}$$

$$\Rightarrow \oint K\vec{E}\cdot d\vec{S} = \frac{Q_{free}}{\varepsilon_0} \quad \left\{\text{Alternative form of Gauss's Law}\right\}$$

→ Q_{free} is used in place of Q to emphasise that it is the free charge given to the plates and does not include the bound charge appearing due to polarization.

→ The above equation is true in any situation where the dielectric used is homogeneous and isotropic.

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Displacement vector

$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$ {P = Polarization}

$\vec{E_p} = \frac{-\vec{P}}{\varepsilon_0}$ {as $\vec{E_p}$ and $\vec{P}$ are opp. in direction}

$\vec{E} = \vec{E_0} + \vec{E_p} = \vec{E_0} - \frac{\vec{P}}{\varepsilon_0}$

$\Rightarrow \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E_0}$

$\Rightarrow \oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{s} = \oint \varepsilon_0 \vec{E_0} \cdot d\vec{s}$

→ As $\vec{E_0}$ is the field produced by the free charge Q_{free}

$\Rightarrow \oint \varepsilon_0 \vec{E_0} \cdot d\vec{s} = Q_{free}$ {from gauss's law}

→ The quantity $\varepsilon_0 \vec{E} + \vec{P}$ is known as the electric displacement vector $\vec{D}$

$\Rightarrow \oint \vec{D} \cdot d\vec{s} = Q_{free}$ {Another form of Gauss's Law}

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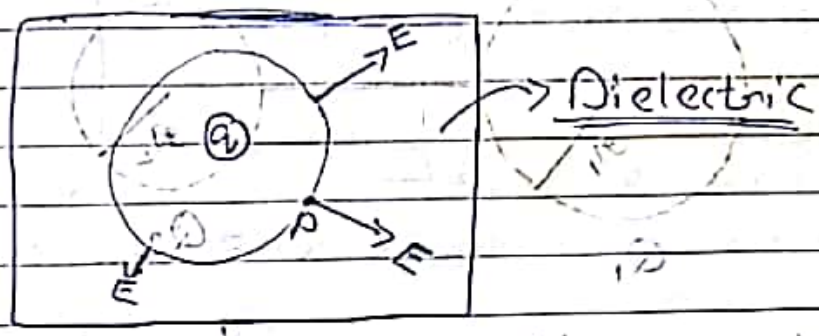
⇒ Electric field due to a point charge q placed in an infinite dielectric



$$\Rightarrow \oint K\vec{E}\cdot d\vec{s} = \frac{q}{\varepsilon_0} \quad \left\{\text{using alternative form of gauss's Law}\right\}$$

$$\Rightarrow K E 4\pi r^2 = q/\varepsilon_0$$

$$\Rightarrow E = \frac{q}{4\pi\varepsilon_0 K r^2} \quad \left\{\text{in radially outward direction}\right\}$$

⇒ Energy in the Electric field in a dielectric

$$u = \frac{U}{Ad} \quad \left\{u = \text{Energy density}\right\}$$

$$u = \frac{\frac{1}{2}\left(\frac{K\varepsilon_0 A}{d}\right)V^2}{Ad} = \frac{1}{2}K\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}K\varepsilon_0 E^2$$

→ where, $E = V/d$ is the electric field between the plates.

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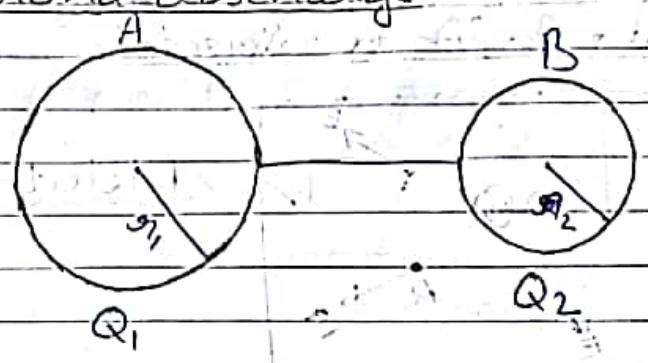
⇒ Corona discharge



// Let us conciden the situation shown in the figure:-

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1} \quad V_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2}$$

→ As the two sphere are connected i.e. potential on both the sphere will be same.

$$\Rightarrow V_1 = V_2 \Rightarrow \frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{Q_2}{4\pi\varepsilon_0 r_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{r_1}{r_2}$$

$$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1} \left\{ \sigma_2 > \sigma_1 \text{ as } r_1 > r_2 \right\}$$

→ Sphere with smaller radius has larger charge density to maintain the same potential.

General result

* In a non uniform single conductor, the charge density will be larger where the radius of curvature is small.

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→ If a conductor has a pointed shape like a needle and a charge is given to it, the charge density at the pointed end will be very high.

→ Correspondingly, the electric field near these pointed ends will be very high which may cause dielectric breakdown in air.

→ The charge may jump from the conductor to the air because of increased conductivity of the air.

→ Often this discharge of air is accompanied by a visible glow surrounding the pointed end.

→ This phenomenon is called corona discharge.

* Corona discharge ⇒ Corona discharge is an electrical discharge brought on by the ionization of a fluid surrounding a conductor that is electrically energized.

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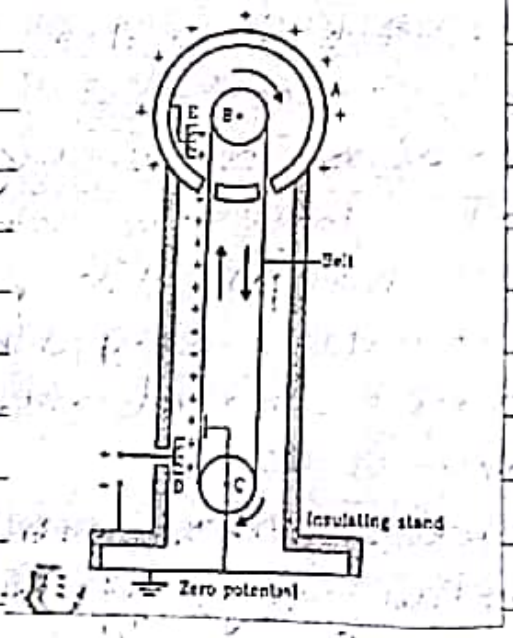
⇌ High-Voltage generation by van de Graaff High-Voltage Generator



Electric Current in Conductor

⇒ Electric current and current density!

* Electric Current ⇒ Rate of flow of Electric Charge.

$$\boxed{i = dQ/dt}$$

→ direction of Current is toward the flow of positive charge.

→ The SI unit of current is Ampere

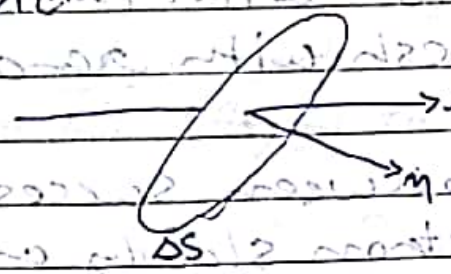
* Current density ⇒ Current per unit area of Crossection

$$\boxed{\vec{J} = di/d\vec{s}} = \frac{di}{ds\cos\theta}$$

θ = angle between current and area vector

$$\boxed{i = \int \vec{J} \cdot d\vec{s}}$$

→ direction of current density is same as current



→ Electric current can be obtained by ⇒

(i) moving Electrons.

(ii) moving ions.

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⇌ Drift Speed

* Drift speed ⇒ Drift Speed is the average speed that a particle such as an electron attains due to an electric field.

* Motion of Electron in Conductor :-

→ As electric field is applied electrons starts moving in direction opp. to electric field and collides with lattice and starts random motion.

→ At each collision, the electrons starts a fresh in a random direction with random speed but gain additional velocity due to the electric field.

→ This velocity increases with time and suddenly become zero as the electron makes a collision with the lattice and starts afresh with random velocity.

→ As the time t between successive collision is small, the electron slowly and stedily drift opposite to the applied field.

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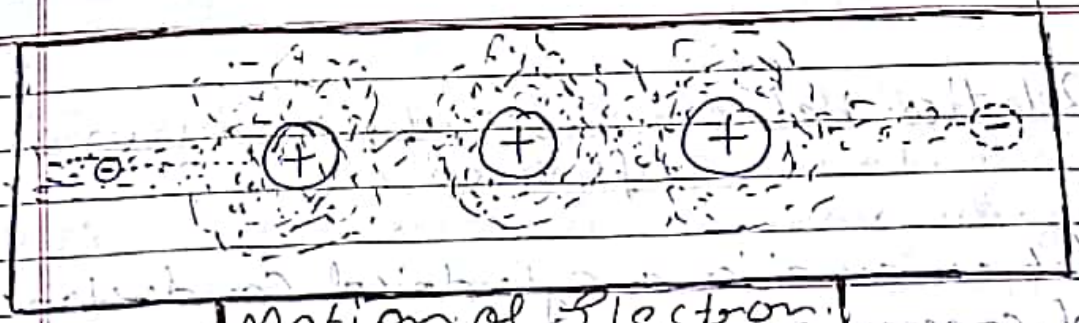

Motion of Electron

* Expression for drift speed

→ If the electron drifts a distance l in a long time t, we define drift speed as

$$V_d = \frac{l}{t}$$

→ If τ (tau) be the average time between successive collisions, the distance drifted during this period is

$$l = \frac{1}{2} a (\tau)^2 = \frac{1}{2} \frac{e\tau^2}{m} E$$

$$\Rightarrow V_d = \frac{l}{\tau} = \frac{e\tau}{2m} E = KE$$

{where, K is constant for a given conductor at a particular temperature}

→ τ (tau) is constant for a given material at a particular temperature.

→ When no electric field is applied electrons are in random motion but have no net drift speed.

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* Relation between current density and drift speed

→ Let us consider a cylindrical conductor of crossectional area A in which an electric field E exists.

→ Concider a length $v_d \Delta t$ of the conductor. The volume of this portion is $A v_d \Delta t$.

→ If there are n free electron per unit volume of wire, the number of free electrons in this portion is $n A v_d \Delta t$

→ All these electrons cross the area A in time Δt. Thus, the charge crossing the area in time Δt is

$$\Delta Q = n A v_d \Delta t e$$

$$i = \frac{\Delta Q}{\Delta t} = n A v_d e$$

$$\boxed{j = \frac{i}{A} = n e v_d}$$

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⇒ OHM'S LAW

Ohm's law states that the conductivity (or resistivity) of a material is independent of the electric field existing in the material.

$$j = \sigma E \quad \left\{ \text{where, } \sigma = \frac{ne^2\tau}{2m} \right\}$$

→ σ depends only on the material of the conductor and its temperature.

→ σ is called Electric Conductivity and the quantity $\left(\rho = \frac{1}{\sigma}\right)$ is called resistivity of material.

→ Let us consider a conductor of length l and uniform crossectional area A. Let us apply a potential difference V between the ends of conductor.

$$\Rightarrow j = \sigma E$$

$$\Rightarrow \frac{i}{A} = \sigma \frac{V}{l}$$

$$\Rightarrow V = \frac{1}{\sigma}\frac{l}{A} i \Rightarrow V = Ri \left\{ R = \rho \frac{l}{A} \right\}$$

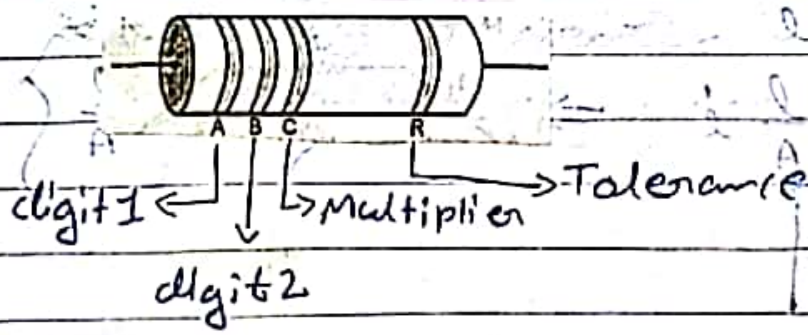
$$\Rightarrow \boxed{R = \rho \frac{l}{A}}$$

→ R is called the resistance of the given conductor. The quantity 1/R is called conductance.

→ We arrived to these results using assumptions which are not valid for semiconductors, insulators, solutions etc.. Ohm's law cannot be applied in such cases.

Colour code for resistors

COLOUR	LETTER AS AN AID TO MEMORY	NUMBER	MULTIPLIER	COLOUR	TOLERANCE
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No colour	20%
Orange	O	3	10^3		
Yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
Violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		
Gold			10^{-1}		
Silver			10^{-2}		



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⇒ Temperature dependence of resistivity

* For small temperature variations we can write for most of the material :-

$$\rho(T) = \rho(T_0)\left[1 + \alpha(T - T_0)\right]$$

where $\rho(T)$ and $\rho(T_0)$ are resistivities at temperatures T and T_0 respectively. α is a constant for the given material.

→ The constant α is called the temperature coefficient of resistivity.

{ In fact α depends to a small extent on the temperature }

→ as $R \propto \rho$ for a given resistor

$$R(T) = R(T_0)\left[1 + \alpha(T - T_0)\right]$$

* Therimistor ⇒ A therimistor is a type of resistor whose resistance varies significantly with temperature, more so than in standard resistors.

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* Semiconductor ⇒ A Semiconductor is a material which has electric conductivity between that of a conductor such as copper and an insulator such as glass.

→ The conductivity of a semiconductor increases with increasing temperature.

→ Eg ⇒ Silicon, germanium etc.

* Superconductivity ⇒ Superconductivity is a phenomenon of exactly zero electrical resistance and ~~exp~~ expulsion of magnetic fields occurring in certain materials when cooled below a characteristic critical temperature.

→ The material in this state is called a Superconductor.

→ If an electric current is set up in a Superconductor, it can persist for long time without any applied emf.

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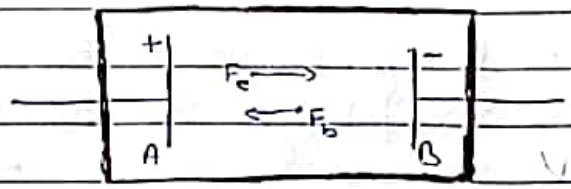
⇒ Battery and Emf



→ Let F_b be the force on positive charge by some internal mechanism of battery

→ Due to potential difference between the terminal an electric field $\vec{E}$ is developed in the battery material from A to B and exert a force $\vec{F_e} = qE$ on charge q

→ In steddy state, the charge accumulated on A and B is such that, $F_b = F_e$ No further accumulation takes place.

* Emf (Electromotive force) ⇒ Emf is defined as work done by battery force per unit charge.

$$\mathcal{E} = \frac{W}{q} = \frac{F_b d}{q} \quad \left\{ d = \text{Separation between terminals} \right\}$$

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→ If nothing is connected externally between A and B :-

⇒ $F_b = F_e = qE$

⇒ $F_b d = qEd = qV$

⇒ $\frac{F_b d}{q} = \mathcal{E} = V$

→ The emf of a battery equals the potential difference between its terminals when the terminals are not connected externally.

⇒ Energy transfered in an Electric Circuit

* Thermal Energy produced in the resistor

→ In time t, a charge ~~q=id~~ q=it goes through the circuit.

$U = qV = (it)(iR) = i^2Rt$

→ This loss in electric potential energy appears as increased thermal energy of the resistor.

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→ The power developed is

$$\boxed{P = \frac{U}{t} = i^2 R}$$

→ Using Ohm's law, this can also be written as :-

$$\boxed{P = \frac{V^2}{R} = Vi}$$

* Internal Resistance of a battery

→ Internal resistance ⇒ A practical electrical power source which is a linear electric circuit may according to Thévenin's theorem be represented as an ideal voltage source in series with an impedance. This resistance is termed as internal resistance of the source.

→ As the charge $q = it$ goes through the battery from the negative terminal B to the positive terminal A, work is done by the nonelectrostatic battery force F_b.

$U_1 = q\mathcal{E} = \mathcal{E}it$

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→ As the potential of A is higher than the potential of B by an amount V, the electric potential energy increases by an amount.

$U_2 = V(it)$

→ The remaining energy $(U_1 - U_2)$ appears as thermal energy of the battery material.

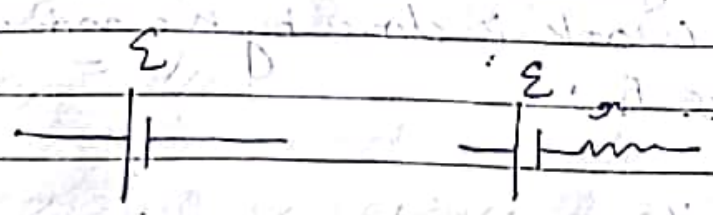
→ The fraction appearing as thermal energy depends on the battery material and the battery mechanism.

→ If no thermal energy is developed as the charge goes through the battery,

⇒ $\mathcal{E} i t = V i t$

⇒ $\mathcal{E} = V$

→ Such a battery is called an <u>ideal battery</u>.



Ideal cell Non-Ideal cell

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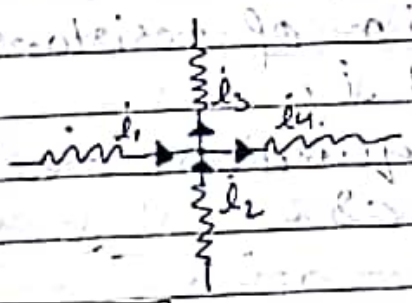
→ A nonideal ~~battery~~ battery develops thermal energy as a current passes throughit and the potential difference between the terminals is smaller than the emf.

$$\boxed{\Delta V = \mathcal{E} - ir}$$ {r is the internal resistance of battery.}

⇒ Kirchhoff's Law

* The junction Law

The sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from the point.



⇒ $\boxed{i_1 + i_2 = i_3 + i_4}$

→ Basis

The junction law follows from the fact that no point in a circuit keeps on accumulating charge or keeps on supplying charge.

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* The loop law

The algebric sum of all the potential differences along a closed loop in a ~~circuits~~ circuit is zero.

→ Basis

The loop law follows directly from the fact that electrostatic force is a conservative force and the work done by it in any closed path is zero.

⇒ Combination of Resistors in series and parallel

If a potential difference V is applied to the combination of resistors and it draws some current i.

We define equivalent resistance of the combination as

$$\boxed{R_{eq} = V/i}$$

→ This single resistance draws the same current as the given combination when the same potential difference is applied across the end points.

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* Series Combination

Resistors are said to be connected in series if the same current passes through all the resistors.

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

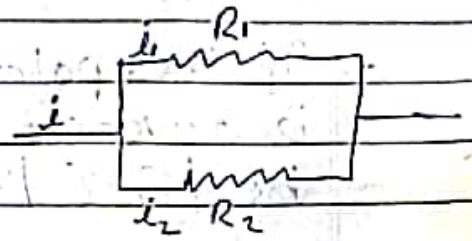
* Parallel combination

Resistors are said to be connected in parallel if the same potential difference exists across all the resistors.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

→ Division of current in Resistors joined in parallel

→ Current is divided in resistors, connected in parallel, in inverse ratio of the resistance



$$\Rightarrow \boxed{i_1 = \frac{R_2}{R_1 + R_2} i}$$

$$\Rightarrow \boxed{i_2 = \frac{R_1}{R_1 + R_2} i}$$

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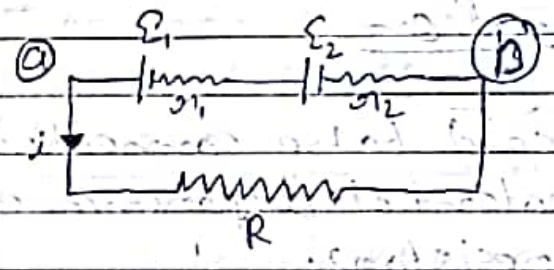
⇒ Grouping of Batteries

* Series connection

→ Let two batteries having Emf ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series as shown.



// Using Kirchhoff's Loop Law we get :-

⇒ $iR + ir_2 - \varepsilon_2 + ir_1 - \varepsilon_1 = 0$

⇒ $i = \dfrac{\varepsilon_1 + \varepsilon_2}{R + (r_1 + r_2)} = \dfrac{\varepsilon_0}{R + r_0}$

→ We see that the combination act as a battery of Emf $\varepsilon_0 = \varepsilon_1 + \varepsilon_2$ and Internal resistance $r_0 = r_1 + r_2$.

→ If the polarity of one of the batteries is reversed, the equivalent Emf will be $|\varepsilon_1 - \varepsilon_2|$.

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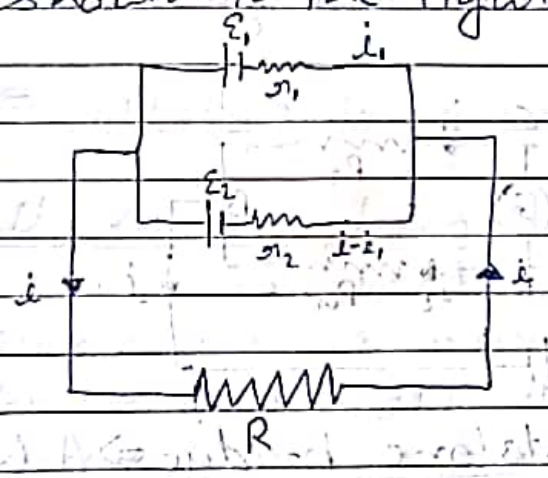
* Parallel Connection

⇒ Let the batteries are connected in parallel as shown in the figure :-



// Applying Kirchhoff's Loop Law we get :-

⇒ $Ri + r_1 i_1 - \mathcal{E}_1 = 0$ —— (I)

⇒ $Ri + r_2(i - i_1) - \mathcal{E}_2 = 0$ —— (II)

⇒ Eleminating i_1 from Eq (I) and (II) we get :-

⇒ $i = \dfrac{\dfrac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}}{R + \dfrac{r_1 r_2}{r_1 + r_2}} = \dfrac{\mathcal{E}_0}{R + r_0}$

→ We see that combination acts as a battery of emf, $\mathcal{E}_0 = \dfrac{\mathcal{E}_1 r_2 + \mathcal{E}_2 r_1}{r_1 + r_2}$ and internal resistance, $r_0 = \dfrac{r_1 r_2}{r_1 + r_2}$,

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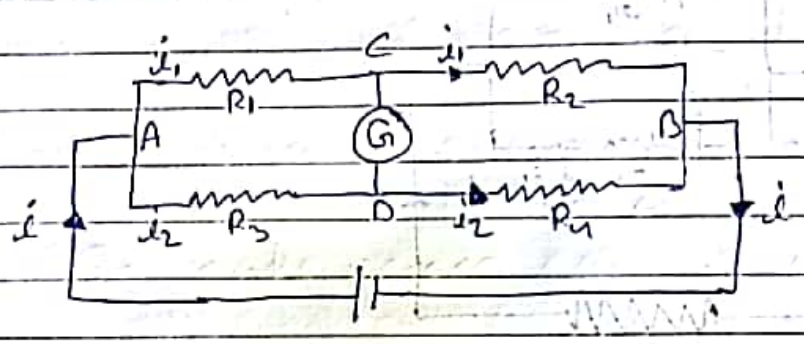
⇒ Wheatstone Bridge

→ Wheatstone bridge is an arrangement of ~~four~~ four resistance and a galvanometer as shown :-



* Balanced Wheatstone bridge ⇒ A Wheatstone bridge is said to be balanced if current through the galvanometer is zero.

→ Let us assume a balaced wheatstone bridge:

⇒ $V_C = V_D$ {as current through ~~gadgp~~ galvanometer is zero}

⇒ $V_A - V_C = i_1 R_1$ — (I)
⇒ $V_C - V_B = i_1 R_2$ — (II) {Ohm's Law}

⇒ $\frac{V_A - V_C}{V_C - V_B} = \frac{R_1}{R_2}$ — (b)

⇒ $\frac{V_A - V_D}{V_D - V_B} = \frac{R_3}{R_4}$ {Ohm's Law} — (c)

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// Using Equation ⓐ, ⓑ and ⓒ we get :-

$$\Rightarrow \boxed{\frac{R_1}{R_2} = \frac{R_3}{R_4}}$$

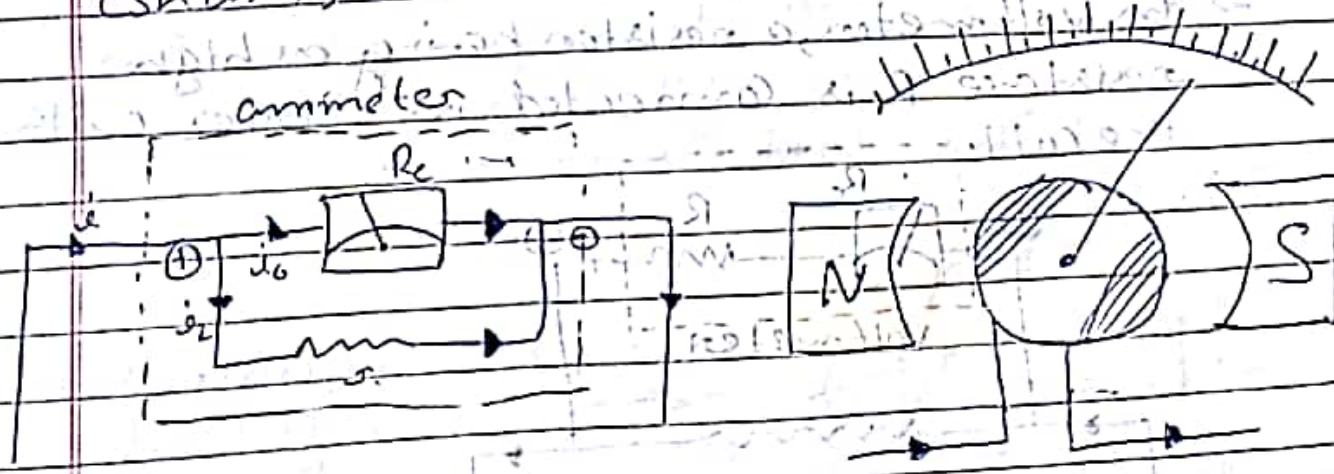
⇒ Ammeter and voltmeter:

* Ammeter

Ammeter is a device to measure electric current.

→ In an ammeter a resistor having a small resistance is connected in parallel with the coil. This resistor is called the shunt.

→ Suppose the coil has a resistance R_c and the small resistance connected in parallel (shunt) has a value r.



* Current through Coil, $i_0 = \frac{S}{R_c + S} i$

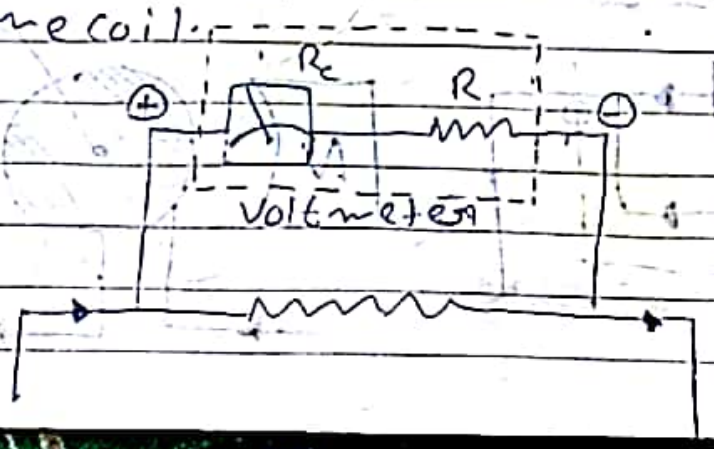
→ The deflection is proportional to i_0 and hence to i and the scale is graduated to read the value of i directly.

→ If resistance of ammeter is high then main current will increase, so to ~~reduce~~ make main current unaltered a small resistance (shunt) is connected parallel to coil and hence reduce resistance of ammeter.

* Voltmeter

Voltmeter is a device to measure potential difference.

→ In voltmeter, a resistor having a high resistance R is connected in series with the coil.



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* Current through voltmeter, $i = \frac{V}{R_c + R}$

→ The deflection is proportional to the current i and hence V. and the scale is graduated to read the potential difference directly.

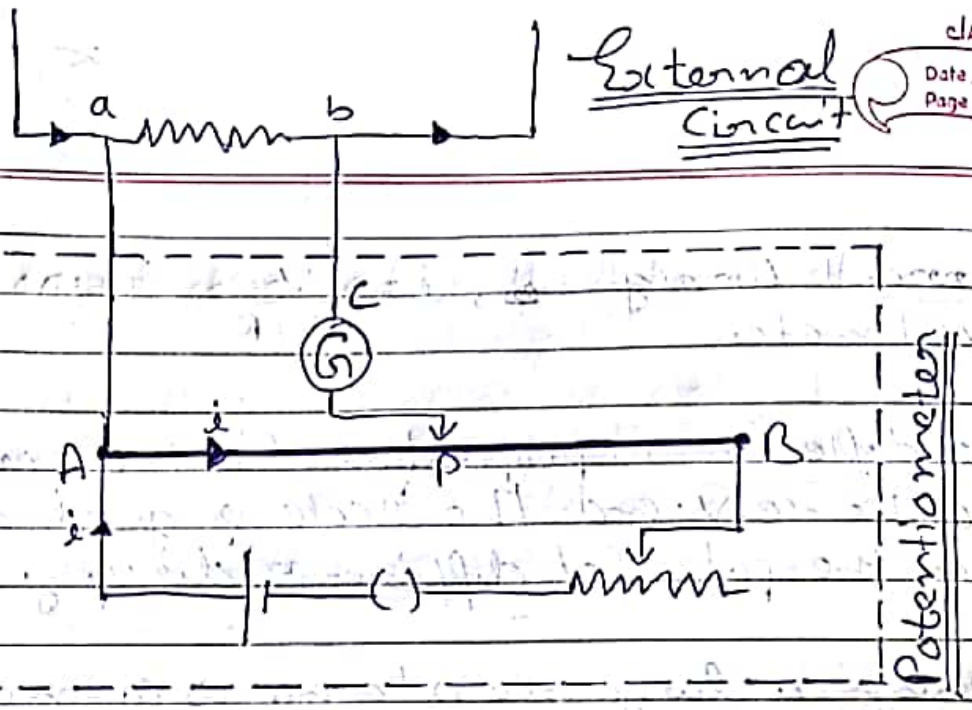
→ When a large resistance is connected in parallel to a small resistance, the equivalent resistance is only slightly less than the small one. That is why a large resistance R is added in series with the coil of a voltmeter.

⇒ Stretched Wire Potentiometer

→ Potentiometer is a device which dose not draw any current from the given circuit and still measures the potential difference.

→ It is equivalent to an ideal voltmeter.

External Circuit



→ We choose wire AB such that potential drop between A and B is greater than a and b.

→ Then there will be a point p on AB which will have the same potential as b.

→ Let the driving current sets up a potential difference V_0 between the ends A and B of the potentiometer wire.

Let $AB = L$ and $AP = l$

$$\Rightarrow \frac{V}{V_0} = \frac{l}{L}$$

$$\Rightarrow \boxed{V = V_0 \frac{l}{L}}$$

{ V = Potential difference across a and b }

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* Measurement of V_0

→ V_0 can be measured by putting battery of known Emf $\{\mathcal{E}\}$ between a and b.

→ Let in this case A_p be equal to l_0

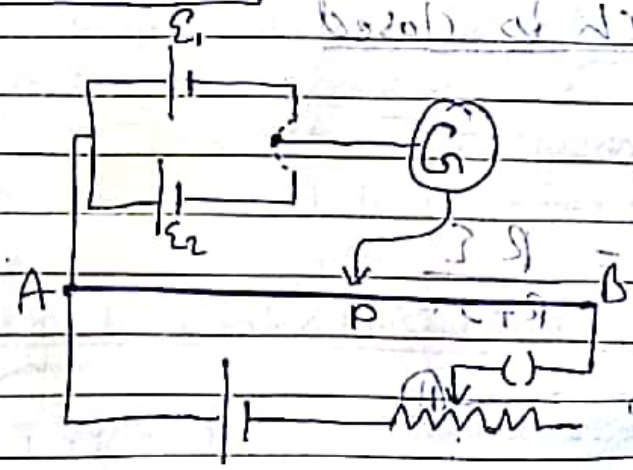
$$\Rightarrow \mathcal{E} = \frac{l_0}{L} V_0 \Rightarrow \boxed{V_0 = \frac{\mathcal{E} L}{l_0}}$$

{This process of finding V_0 is called Calibration of Potentiometer}

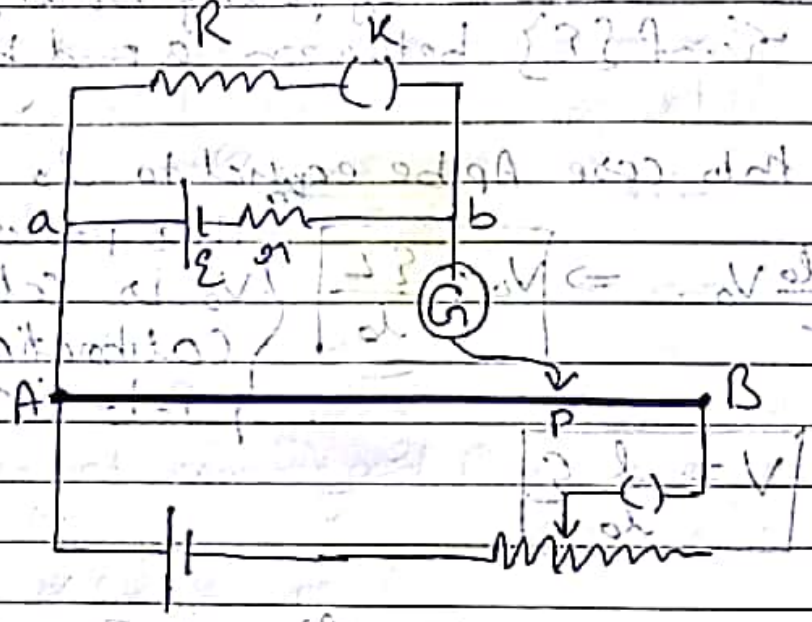
Hence, $\boxed{V = \frac{l}{l_0} \mathcal{E}}$

* Comparision of Emf's of Two batteries

$$\boxed{\frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{l_1}{l_2}}$$



* Measurement of Internal resistance of a battery



// → When switch is ~~[illegible]~~ open

$$\varepsilon = \frac{l}{L} V_0 \quad \text{—— (I)}$$

// When switch is closed

$$i = \frac{\varepsilon}{R+r}$$

$$V_a - V_b = iR = \frac{R\varepsilon}{R+r}$$

$$\frac{R\varepsilon}{R+r} = \frac{l'}{L} V_0 \quad \text{—— (II)}$$

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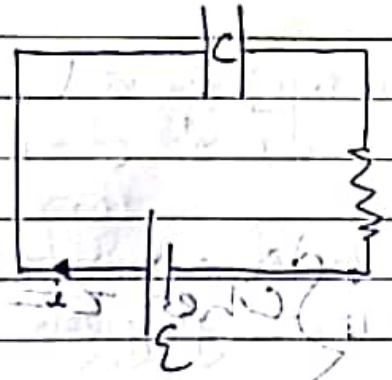
→ Using Eq (I) and (II) we get :-

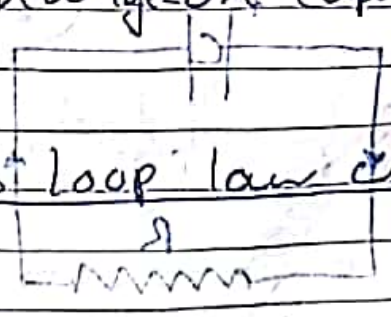
$$\boxed{r = \frac{R(l-l')}{l'}}$$

⇒ Charging and discharging of Capacitors

(1) Charging



Let us consider the circuit shown above.
Let battery is connected to the circuit at $t=0$
Let at time $=t$ charge on capacitor be q.



Using Kirchhoff's loop law we get :-

$$\frac{q}{C} + iR = \varepsilon$$

$$\Rightarrow iRC = C\mathcal{E} - q$$

$$\Rightarrow \frac{dq}{dt} RC = C\mathcal{E} - q$$

$$\Rightarrow \int_0^q \frac{dq}{C\mathcal{E} - q} = \int_0^t \frac{dt}{RC}$$

$$\Rightarrow \Big[\ln|C\mathcal{E} - q|\Big]_0^q = -\frac{t}{RC}$$

$$\Rightarrow \ln\left|\frac{C\mathcal{E} - q}{C\mathcal{E}}\right| = -\frac{t}{RC}$$

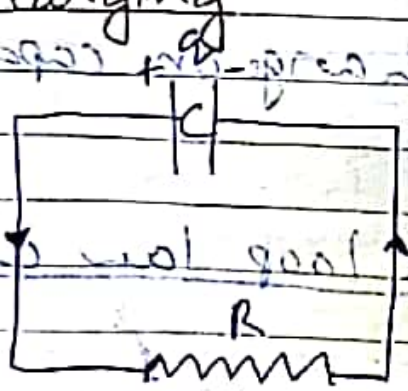
$$\Rightarrow C\mathcal{E} - q = C\mathcal{E}\, e^{-\frac{t}{RC}}$$

$$\Rightarrow q = C\mathcal{E}\left\{1 - e^{-\frac{t}{RC}}\right\}$$

$$\boxed{q = C\mathcal{E}\left\{1 - e^{-\frac{t}{\tau}}\right\}} \quad \left\{\text{where } \tau = RC\right\}$$

$\Rightarrow$ τ {tau} is called time constant

(ii) Discharging



Let us consider the circuit shown above.
Let initial charge on capacitor be Q.

⇒ Let at time = t, charge on capacitor be q and current in circuit be i.

⇒ Using Kirchhoff's loop law we get

$$\Rightarrow \frac{q}{C} - Ri = 0$$

$$\Rightarrow -\frac{dq}{dt}R = \frac{q}{C}$$

$$\Rightarrow \int_Q^q \frac{dq}{q} = \int_0^t \frac{-dt}{RC}$$

$$\Rightarrow \left[\ln|q|\right]_Q^q = \frac{-t}{RC}$$

$$\Rightarrow \ln\left|\frac{q}{Q}\right| = \frac{-t}{RC}$$

$$\Rightarrow q = Qe^{-\frac{t}{RC}}$$

$$\boxed{q = Qe^{-\frac{t}{\tau}}} \quad \{\tau = RC\}$$

⇒ Atmospheric Electricity

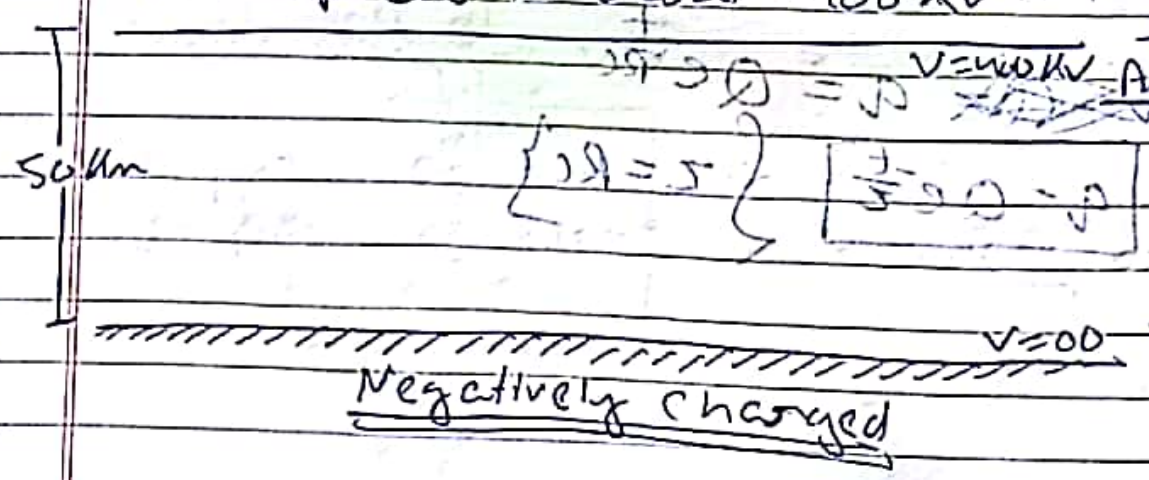
* Cosmic ray ⇒ Cosmic ray is the term given to high energy radiation which strikes the Earth from outer space.

→ • Earth has negative charge is spread with approximately uniform density over its surface

• This result in Electric field directed in vertically downward direction.

• At about 50 km from the earth's surface, the field is negligible.

• Total potential difference between the earth's surface and the top of the atmosphere is about 400 KV.



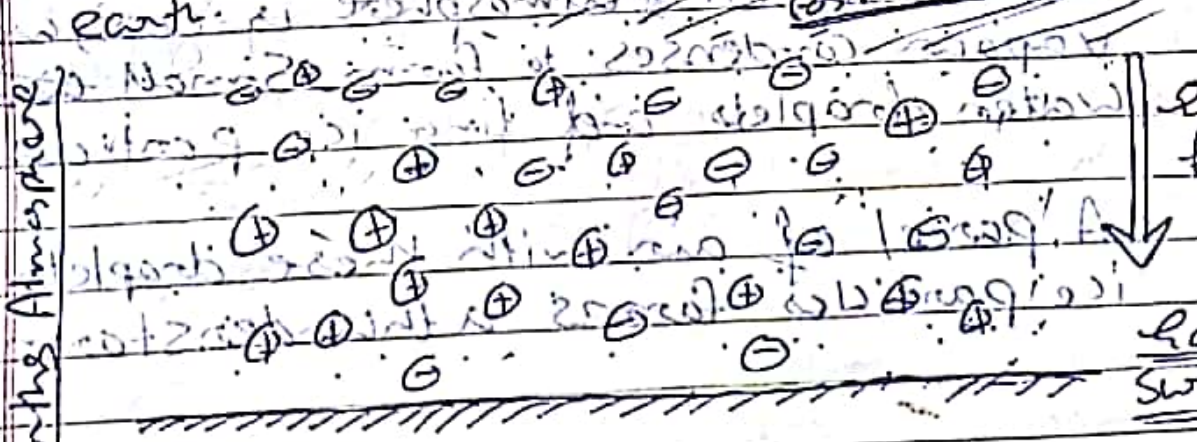
→ • The atmosphere contain large number of ions, both positively charged and negatively charged.

• The main source of these ions are :-
(i) Cosmic rays
(ii) dust particles charged by friction.

• Because of Electric field in the atmosphere negative charge goes up and positive charge comes down.

• Thus there is an electric current in the atmosphere.

• When total surface area of the earth is considered, 1800 A of current reaches the earth.



→ If 1800 A of current flow towards the earth, the entire negative charge of the earth should get neutralised in about half an hour and the electric field in the atmosphere shold reduce to zero.

- But this dosent happne

- So mechanism which brings negative charge back to the earth is thunderstorm and lightning.

→ Because of difference in temperature and pressure between different parts of the atmosphere, air packets keeps on moving in a rather systematic fashion.

- As the upper atmosphere is cool, water vapour condenses to form small water droplets and tiny ice particles.

- A parcel of air with these droplets and ice particles forms a thunderstorm.

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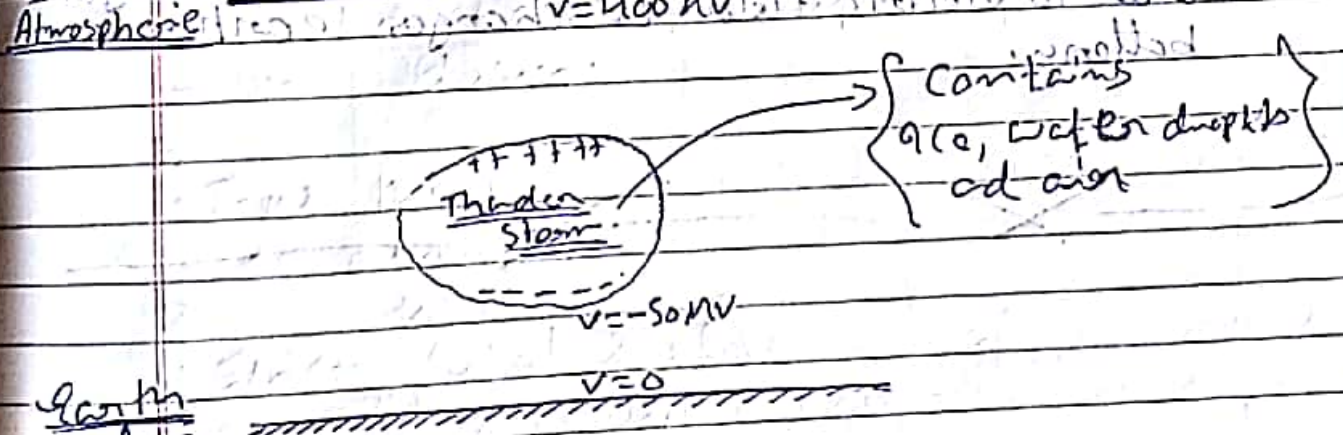
→ • The upper part of a thunderstorm contains excess positive charge and the lower part contains excess negative charge.

• This negative charge creates a potential difference of 20 to 100 MV between these clouds and the earth.



→ • The high electric field between the lower part of the storm and the earth is often sufficient for the dielectric breakdown of air and the air becomes conducting.

• Negative charge jumps from the cloud to the earth's surface. This phenomenon is called lightning.

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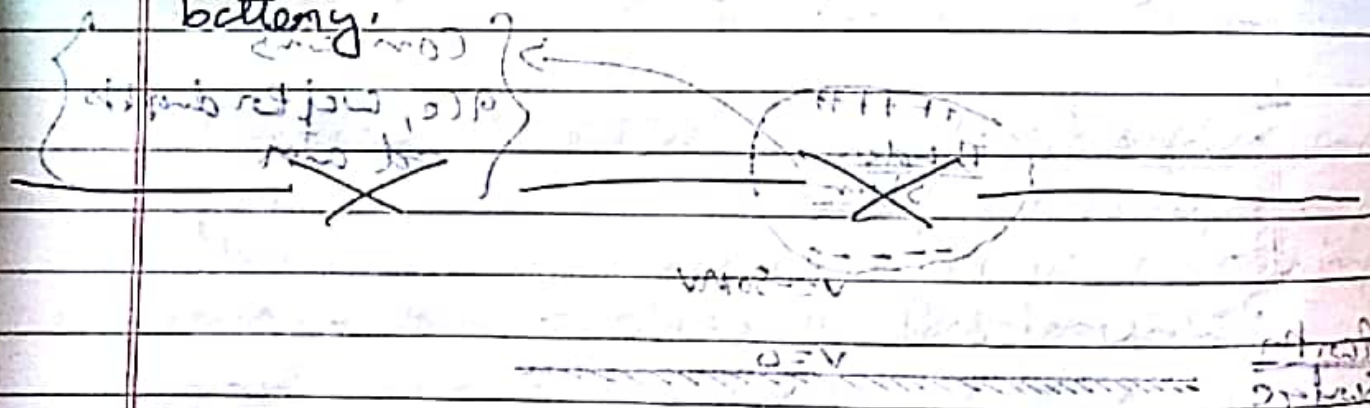
- The positive charge jumps up to high Altitude layer of high Conductivity.

→ Thunderstorms charges the earths battery by supplying negative charge to the earth and positive charge to the upper atmosphere.

- Movement of positive ion toward earth and negative ion away from it due to electric field discharges earths battery.



Magnetic field

Magnetic field $\{\vec{B}\}$

If a charged particle is projected in a magnetic field, in general, we observe following facts :-

(a) There is one line through the point P, such that, if the velocity of the particle is along this line, there is no magnetic force. We define the direction of the magnetic field to be along this line.

{the direction is not uniquely defined yet, because there are two opposite directions along any line}

(b) If the speed of the particle is v and it makes an angle θ with the line identified in (a), the magnitude of the magnetic force is proportional to $|v \sin\theta|$.

(c) The direction of the magnetic force is perpendicular to the directions of the magnetic field as well as to the direction of the velocity.

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(d) The force is proportional to the magnitude of the charge q and its direction is opposite for positive and negative charge.

All the above facts may be explained if we define the magnetic field by the equation :-

$$\vec{F} = q\vec{v} \times \vec{B}$$

$\vec{F}$ = magnetic force acting on charge q.

$\vec{B}$ = magnetic field

UNIT

→ The SI unit of magnetic field is newton/ampere-metre.

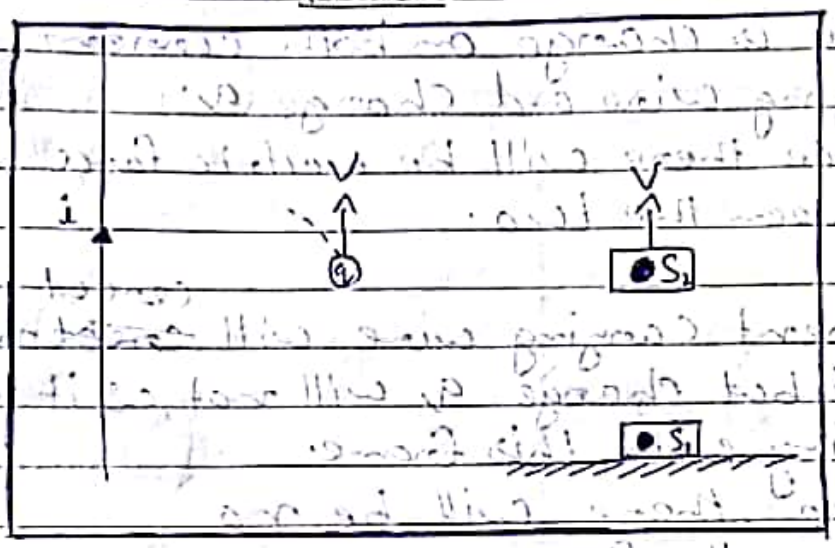
→ This is written as tesla and abbreviated at T.

→ Another unit in common use is gauss.

$$\boxed{1T = 10^4 \text{ gauss}}$$

→ The unit weber/m² is also used for magnetic field and is the same as tesla.

Relation between Electric and magnetic fields



* Let us consider the above situation. It is observed that charge q is attracted toward current carrying wire.

Case 1: With respect to frame S_1

→ • there is no charge on current carrying wire.
• hence there will be no electric force between wire and charge q.

→ • Current carrying wire and charge q will both create magnetic field.
• hence there will be magnetic force between the two.

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Case 2: With respect to frame S_2

→ • There is charge on both current carrying wire and charge q.
• Hence there will be electric force between the two.

→ • Current carrying wire will ~~create~~ create magnetic field but charge q will not, as it is not moving with this frame.
• Hence there will be no magnetic force.

* With the above observations we conclude that the electric field and the magnetic field are not basically independent.
• They are two aspects of the same entity which we call electromagnetic field.

* Whether the electromagnetic field will show up as an electric field or a magnetic field or a combination, depends on the frame from which we are looking at the field.

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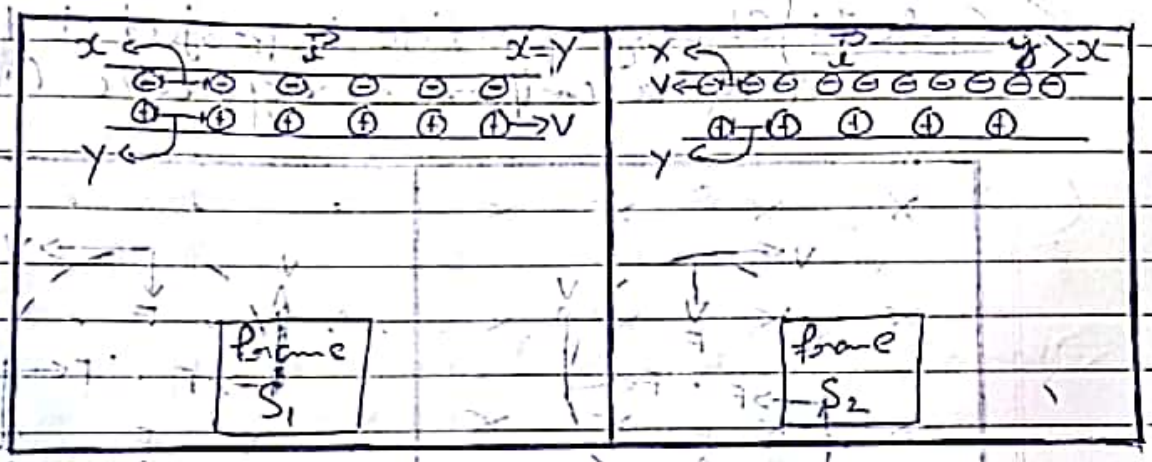
* Appearing of charge on wire with respect to frame S_2



→ In case of frame S_2 Y becomes greater than X due to Lorentz contraction by a factor

$$\therefore \quad Y = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}$$

→ Hence wrt S_2 negative charge appears on the wire with current i.

* Direction notation

→ ⊗ ⇒ This represents the direction going into the paper.

→ ⊙ ⇒ This represents the direction going out of the paper.

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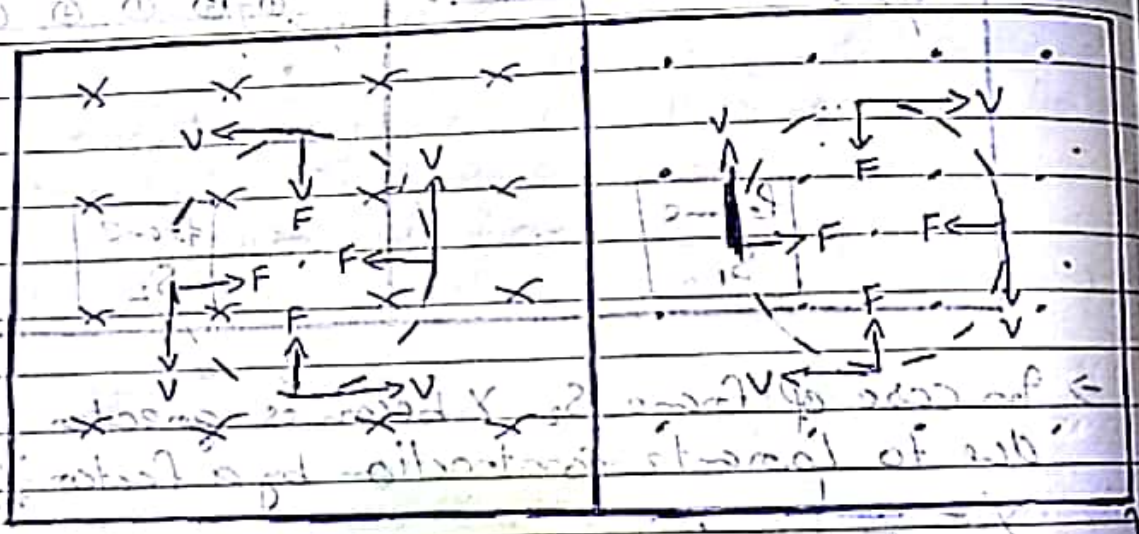
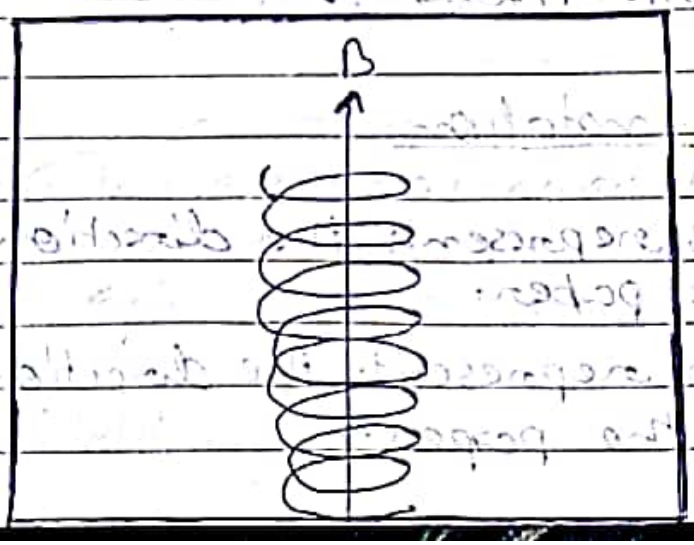
⇒ Motion of a charged particle in a uniform magnetic field

Case 1: {Charge particle is project ⊥ to a uniform magnetic field.}



Case 2: {Charge particle is projected at some angle (other than ⊥) to a uniform magnetic field.}



{helix path}

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⇒ Magnetic force on a Current-Carrying Wire

→ In a wire each electron experience on average ~~elect~~ magnetic force of :-

$$\boxed{f = -e\vec{V_d} \times \vec{B}}$$

V_d = drift speed of electron
f = force on electron

→ The number of free electrons in the small element considered is $nAdl$.

n = number of free electron per unit volume
A = Crossectional area of wire
dl = Small length of wire

• Thus the magnetic force on the wire of length dl is :-

$$d\vec{F} = (nAdl)(-e\vec{V_d} \times \vec{B})$$

If we denote the length dl along the direction of the current by $d\vec{l}$, the above equation becomes

$$d\vec{F} = nAeV_d\, d\vec{l} \times \vec{B}$$

$$d\vec{F} = i\, d\vec{l} \times \vec{B} \quad \{i = neV_dA\}$$

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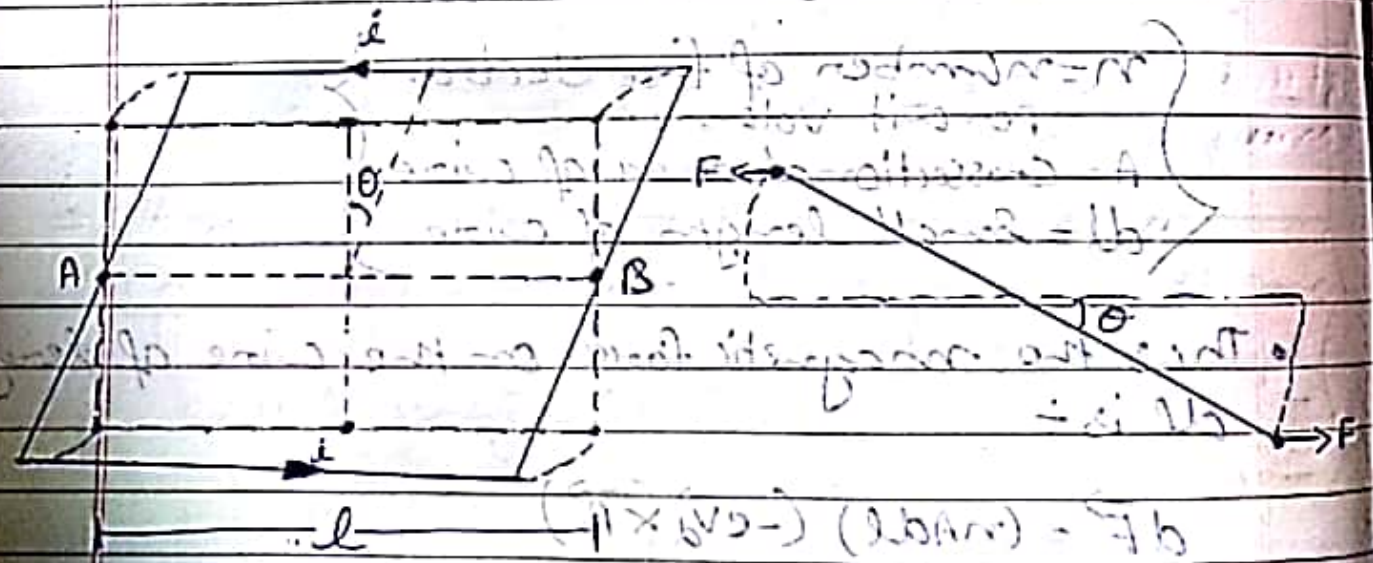
→ The quantity $i\vec{dl}$ is called a current element.

→ If a straight wire of length l carrying a current i is placed in a uniform magnetic field $\vec{B}$, the force on it is :-

$$F = i\vec{l} \times \vec{B}$$

⇒ <u>Torque on a Current loop</u>



Let us consider a rectangular (b×l) current loop at angle θ with the plane of paper about AB.

$\Rightarrow \Gamma\{\text{about } AB\} = 2\times\left\{\frac{b}{2} F\sin\theta\right\} = bF\sin\theta$

$\Rightarrow b(ilB)\sin\theta$

$\Rightarrow ilbB\sin\theta$

$\Rightarrow iAB\sin\theta$ {A = Area inclosed under loop}

// Let us define area vector $\vec{A}$ such that "its magnitude is equal to area inclosed by the loop and directed ⊥ to the plane of loop towards the side from which current looks anticlockwise"

$\Rightarrow \vec{\Gamma} = i\vec{A}\times\vec{B} = \vec{\mu}\times\vec{B}$ $\{\vec{\mu} = i\vec{A}\}$

{where, $\vec{\mu}$ is called the magnetic dipole moment or simply magnetic moment of the current loop.}

$\Rightarrow$ If there are n turns in the loop, each turn experiences a torque. The net torque is :-

$$\vec{\Gamma} = ni\vec{A}\times\vec{B} = \vec{\mu}\times\vec{B} \quad \{\vec{\mu} = ni\vec{A}\}$$

$\Rightarrow$ Above equations are general and are valid for plane loops of any shape.

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Magnetic field due to a Current

⇒ **Introduction**

A magnetic field can be produced by moving charges or electric currents.

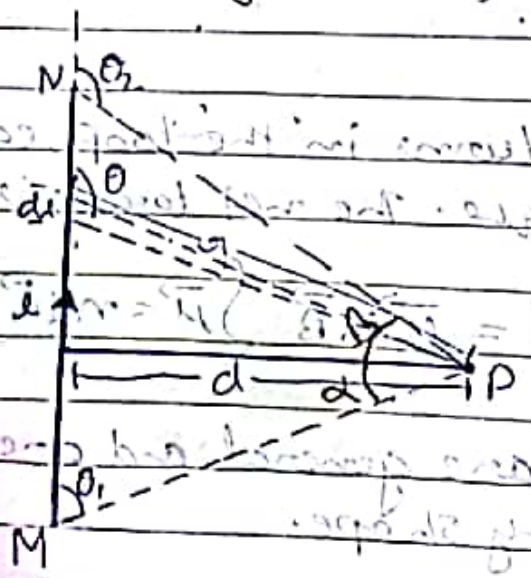
⇒ **BIOT-SAVART LAW**

BIOT-SAVART Law states that magnetic field by a current element at a point is given by

$$\boxed{d\vec{B} = \frac{1}{4\pi\varepsilon_0 c^2} i \frac{d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} i \frac{d\vec{l} \times \vec{r}}{r^3}}$$

$\left\{ \mu = \frac{1}{\varepsilon_0 c^2} = \text{Permeability of vacuum} \right\}$

⇒ **Magnetic field due to Current in a Straight wire**



$$dB = \frac{\mu_0}{4\pi} i \frac{dl \sin\theta}{r^2}$$

$l = -d\cot\theta$

$dl = d\,\mathrm{cosec}^2\theta\, d\theta$ $\left\{ d = \perp \text{ distance of wire from point } P \right\}$

$r = d\,\mathrm{cosec}\,\theta$

$$\Rightarrow \int_0^B dB = \frac{\mu_0 i}{4\pi d} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi d} (\cos\theta_1 - \cos\theta_2) \Rightarrow \boxed{|\vec{B}| = \frac{\mu_0 i}{4\pi d} |\sin\alpha + \sin\beta|}$$

→ Special cases

* Case 1: {Field on ⊥ bisector}

$\theta_2 = \pi - \theta_1$

$\Rightarrow \cos\theta_2 = -\cos\theta_1$

$$\Rightarrow B = \frac{\mu_0 i}{4\pi d} 2\cos\theta_1$$

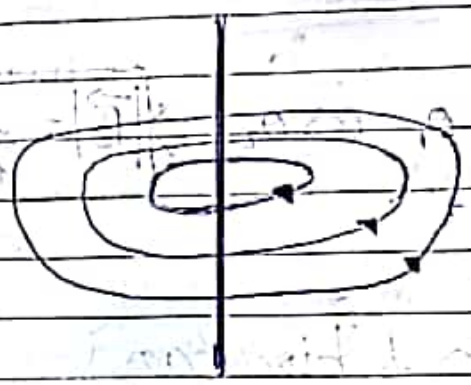
$$\Rightarrow B = \frac{\mu_0 i a}{2\pi d \sqrt{a^2 + 4d^2}}$$ $\left\{ a = \text{length of current wire} \right\}$

* Case 2: {Field due to a long straight wire}

$\theta_1 = 0$ & $\theta_2 = \pi$

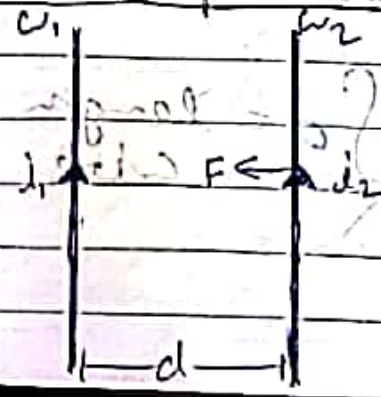
$B = \frac{\mu_0 i}{2\pi d}$

→ Magnetic field lines due to a straight current carrying wire



A tangent to a magnetic field line gives the direction of the magnetic field existing at that point.

⇒ Force between parallel conductors



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Let us consider field due to i_1 at i_2.

$$B = \frac{\mu_0 i_1}{2\pi d}$$

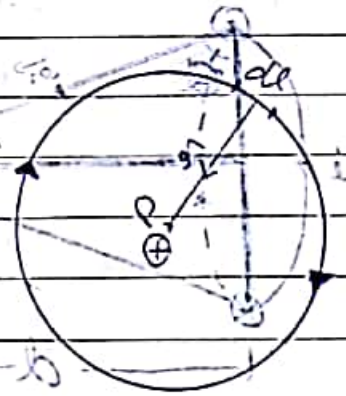
$$d\vec{F} = i_2 d\vec{l} \times B = i_2 dl \frac{\mu_0 i_1}{2\pi d}$$

$$\frac{d\vec{F}}{dl} = \frac{\mu_0 i_1 i_2}{2\pi d}$$ {Force per unit length of wire}

⇒ Field due to a Circular Current

* Field at centre:

Let us consider a circular loop of radius a carrying a current i.



$$dB = \frac{\mu_0}{4\pi} \frac{i\, d\vec{l} \times \vec{a}}{a^3} = \frac{\mu_0}{4\pi} \frac{i\, dl}{a^2}$$

$$B = \frac{\mu_0 i}{4\pi a^2} \int dl = \frac{\mu_0 i}{4\pi a^2} \times 2\pi a = \frac{\mu_0 i}{2a}$$

→ If the current is in anticlockwise direction, the magnetic field is toward the viewer. If the current is in clockwise direction, the field is away from the viewer.

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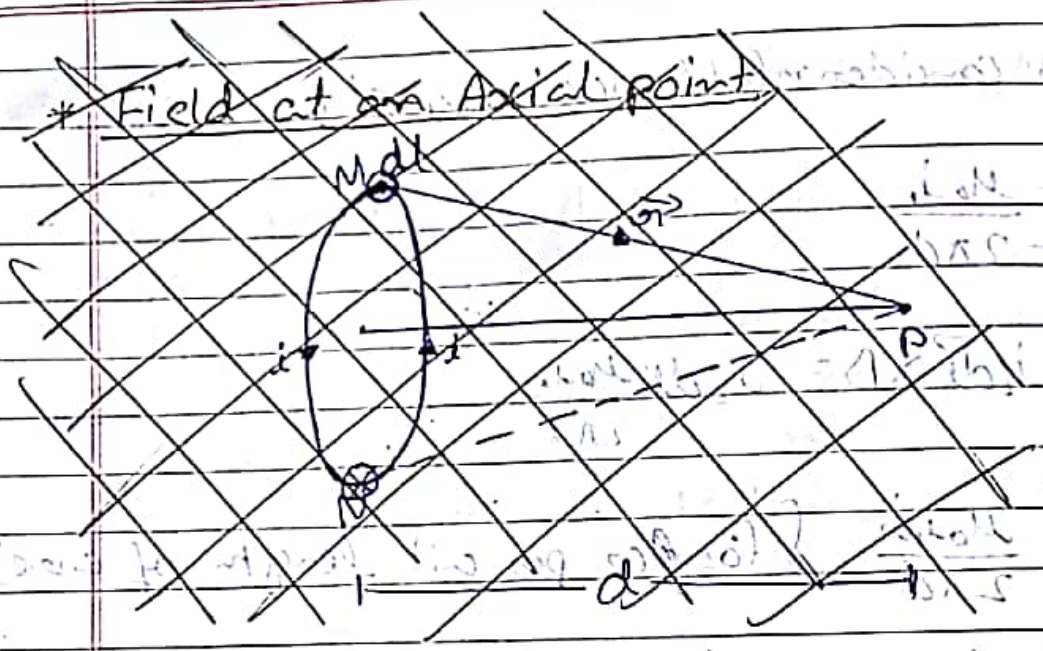
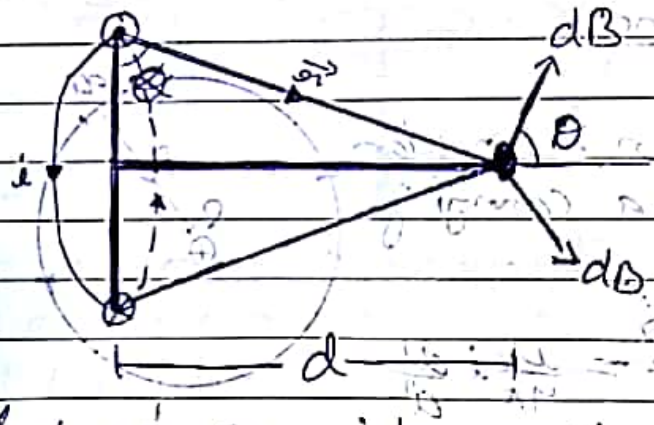
* Field at an Axial point



* Field at an Axial point



Let us concider a circular loop of radius a carrying a current i. Let the seperation of point p from loop be d.

$$dB = \frac{\mu_0}{4\pi} i \frac{d\vec{l} \times \vec{r}}{r^3} = \frac{\mu_0 i}{4\pi} \times \frac{dl}{a^2+d^2}$$

Vertical component of field is zero

$\Rightarrow dB\cos\theta = \frac{\mu_0 i a\, dl}{4\pi(a^2+d^2)^{3/2}}$

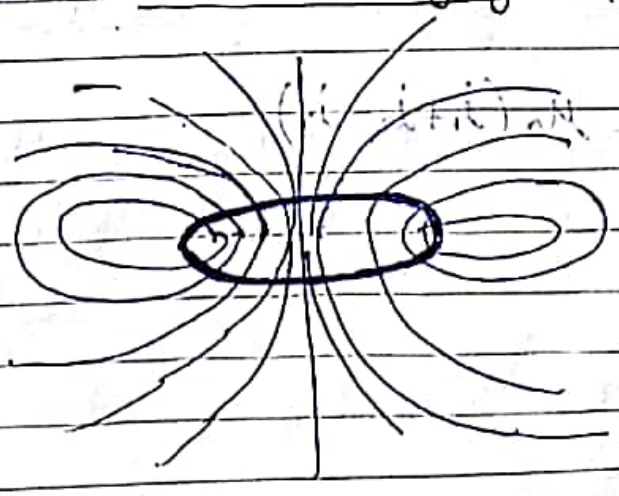
$\Rightarrow B = \int \frac{\mu_0 i a}{4\pi(a^2+d^2)^{3/2}} dl = \frac{\mu_0 i a}{4\pi(a^2+d^2)^{3/2}} \times 2\pi a$

$B = \frac{\mu_0 i a^2}{2(a^2+d^2)^{3/2}}$

→ Field at point far away from the centre

$$\boxed{B = \frac{\mu_0 i a^2}{2d^3} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{d^3}}$$

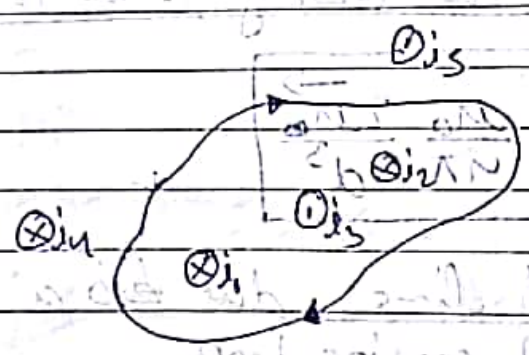
→ Magnetic field lines due to a circular current carrying loop



Ampere's law

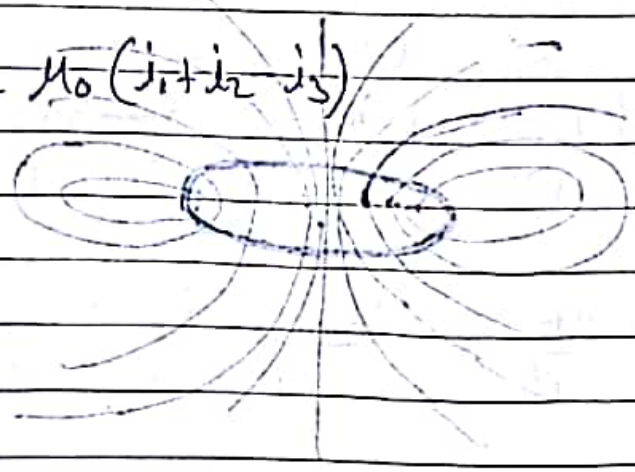
The circulation $\oint \vec{B} \cdot d\vec{l}$ of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

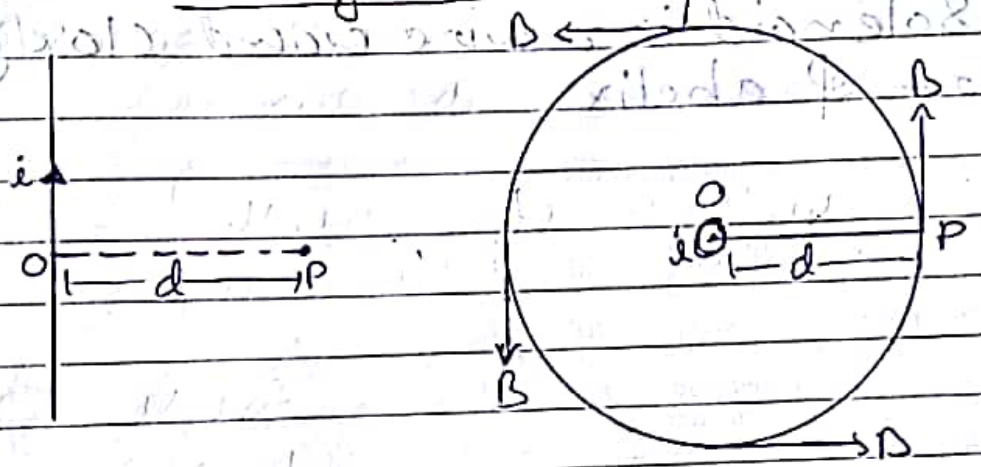


Example

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_1 + i_2 - i_3)$$



⇒ Magnetic field at a point due to a long Straight current



$$\Rightarrow \oint \vec{B} \cdot dl = \mu_0 i$$

$$\Rightarrow B \int dl = \mu_0 i$$

$$\Rightarrow B \times 2\pi d = \mu_0 i$$

$$\boxed{B = \frac{\mu_0 i}{2\pi d}}$$

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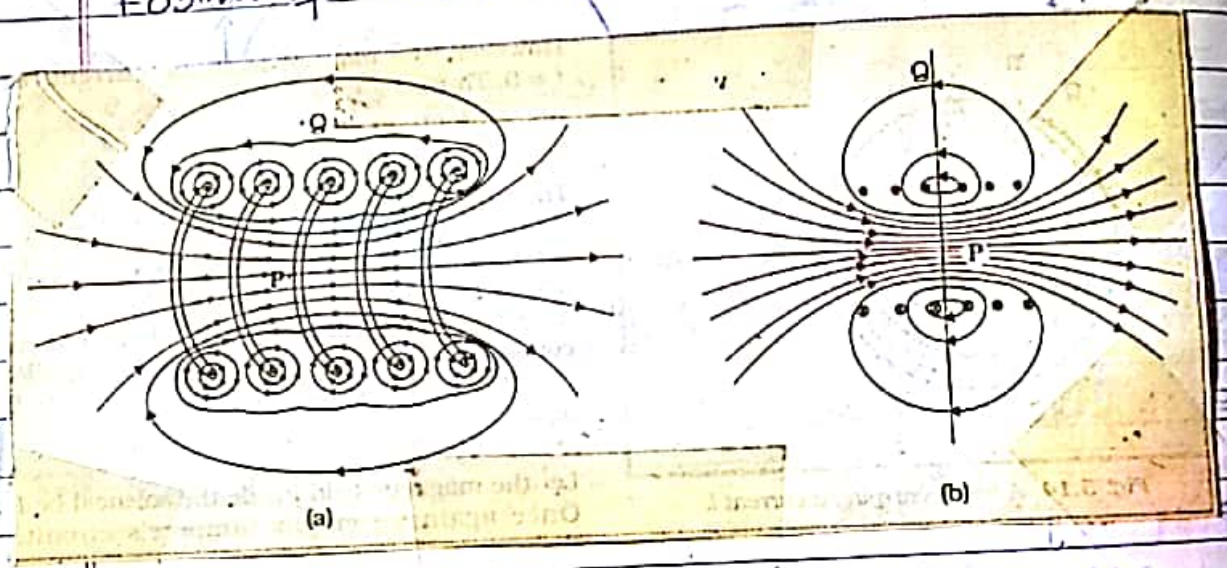
Solenoid

A Solenoid is a wire wound closely in the form of a helix.



~~Set is coincide~~

→ Magnetic field inside Solenoid

$$\int \vec{B} \cdot dl = \mu_0 i$$

Let n be the number of turn per unit length along the length of the solenoid.

$$Bl = \mu_0 n i l$$

$$\Rightarrow \boxed{B = \mu_0 n i}$$

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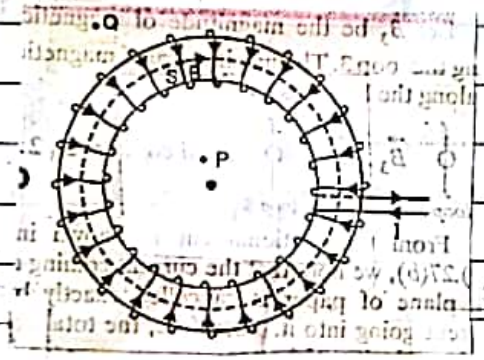
Toroid

If a solenoid is bent in a circular shape and the ends are joined, we get a toroid.



→ Magnetic field inside de Toroid

Let Total number of turns be N.

⇒ Total current = Ni

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 N i$$

$$\Rightarrow 2\pi r B = \mu_0 N i$$

$$\Rightarrow \boxed{B = \frac{\mu_0 N i}{2\pi r}}$$

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Thermal and Chemical Effects of Electric Current

⇒ JOULE'S HEATIND

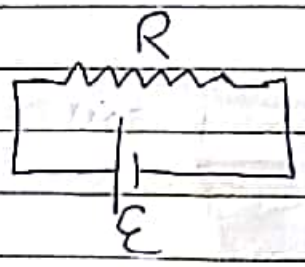
When there is electric current in a Resistor

Electron Starts moving ⟹ It collides with latice

⇓

Kinetic Energy of latice Increases ⟹ consecutively Kinetic Energy of molicule Increases

⟹ And that is heat



⇒ Let potential difference ε is applied across R one resistor.

$\varepsilon = iR$

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⇒ Work done on a resistor ⇒ (Potential difference) × (Charge flow)

⇒ $(iR) \times (iT)$

$$\boxed{H = i^2RT}$$

⇒ Seebeck Effect

⇒ When two different metal strips are joint and the junctions are kept at different temperature, electric current flow in the loop.

• This effect is known as seebeck effect and the emf developed is called thermo-emf.

⇒ The magnitude and the direction of the emf depends on the metals and the temperature of the hot and cold junction.

⇒ Such a combination of two metals is called thermocouple.

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* **Thermoelectric Series** ⇒ Metals are arranged in a particular sequence which may be used to predict the direction of the current in the temperature range 0°C to 100°C. This sequence is known as the thermoelectric series.

Antimony ⇒ nichrome ⇒ iron ⇒ zinc ⇒ copper ⇓

mercury ⇐ aluminium ⇐ lead ⇐ silver ⇐ gold

⇓

Platinum-rhodium ⇒ Platinum ⇒ nickel ⇓

bismuth ⇐ costantan ⇐

→ At cold junction, the current is from the metal coming earlier in the series to the metal coming latter in the series.

* **Neutral temperature** ⇒ The temperature of the hot junction at which the thermo-emf is maximum is called the neutral temperature.

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* Inversion temperature ⇒ The temperature of hot junction at which the thermo-emf changes its sign is called the inversion temperature.

$$\theta_n - \theta_c = \theta_i - \theta_n$$

⇒ If the cold junction is at 0°C and the hot junction at θ, the thermo emf depends on the temperature as

$$\boxed{\mathcal{E}_{AB} = a_{AB}\theta + \frac{1}{2} b_{AB}\theta^2}$$

* Law of Intermediate metal

$$\boxed{\mathcal{E}_{AB} = \mathcal{E}_{AC} - \mathcal{E}_{BC}}$$

$$\boxed{\{a_{AB} = a_{AC} - a_{BC}\} \quad \{b_{AB} = b_{AC} - b_{BC}\}}$$

* Law of Intermediate Temperature

$$\boxed{\mathcal{E}_{\theta_1\theta_2} = \mathcal{E}_{\theta_1\theta_3} + \mathcal{E}_{\theta_3\theta_2}}$$

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⇒ PELTIER Effect

⇒ Suppose two junctions of a thermocouple are initially at the same temperature and an electric current is passed through the circuit by using an external battery.

- It is observed that heat is ~~absorbed~~ produced at ~~the other~~ one junction and is absorbed at the other.

- It is reverse of the Seebeck effect and is called the peltier effect.

⇒ If an amount ΔH of heat is produced or absorbed when a charge ΔQ is passed through the junction, we define peltier emf as

$$\pi_{AB} = \frac{\Delta H}{\Delta Q} = \frac{\text{Peltier heat}}{\text{Charge transferred}}$$

* Difference Peltier heating & Joule heating

→ Joule heat always warms up the resistor whatever be the direction of the current. But one has both Peltier heating & Peltier cooling of the junction depending on the direction of the current

→ Peltier heat in a given time is proportional to the current through the junction, whereas the Joule heating is proportional to the square of the current.

→ Peltier heating or cooling is observed only at the junction, where as the joule heating is throughout the resistor.

⇒ Thomson Effect

⇒ If a metallic wire has a non uniform temperature and a current is passed through it, heat may be absorbed or ~~then~~ produced in different sections of the wire.

• This heat is over and above the Joule heat i^2Rt and is called the Thomson heat. The effect itself is called the Thomson effect.

⇒ Thomson heat, produced or absorbed in a small section of a given wire is proportional to the charge passed through the section and ~~the differen~~ the temperature difference between the ends of the section.

$$\boxed{\Delta H = \sigma (\Delta Q)(\Delta T)}$$

{ where, σ is Thomson coefficient which is constant for a given metal }

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The quantity, $\sigma(\Delta T) = \frac{\Delta H}{\Delta Q} = \frac{\text{Thomson heat}}{\text{Charge transferred}}$ is called Thomson emf.

⇒ Electrolysis

⊛ Terminology

→ Electrolyte ⇒ Liquids that are good conductor of electricity are called electrolyte.

⇒ Liquids that are good conductor of electricity

- Solution of inorganic Salt in water.
- Dilute Acids & Base

→ Electrolytic Cell ⇒ Vessel with electrolyte is called electrolytic cell.

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→ **Electrode** ⇒ An electrode is an electrical conductor used to make contact with a nonmetallic part of a circuit.

- **Cathode** ⇒ Electrode connected to ⊕ terminal of battery.

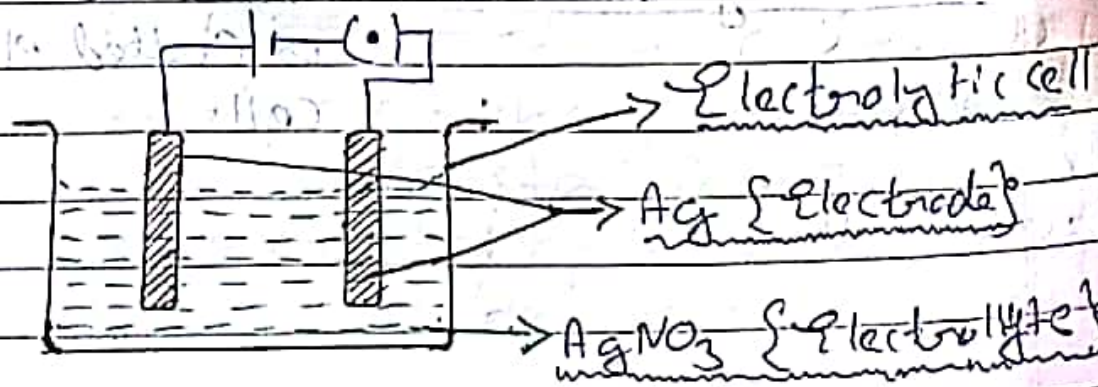
- **Anode** ⇒ Electrod connected to ⊖ terminal of battery.

→ **Ion** ⇒ An ion is an atom or molecule in which the total number of electrons is not equal to the total number of protons.

- **Cation** ⇒ Positively charged ion.

- **Anion** ⇒ Negatively charged ion.

⊛ Mechanism



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Electrolysis ⇒ Electrolysis is a method of using a direct electric current (DC) to drive non-spontaneous chemical reaction.

⇒ Faraday's laws of Electrolysis

* First law ⇒ The mass of a substance liberated at an electrode is proportional to the charge passing through the electrolyte.

⇒ Let m be the mass liberated at Electrode.

$m \propto Q$ {Faraday 1st law}

$m \propto it$ {as $Q = it$}

⇒ $\boxed{m = Zit}$ {Z is constant for substance being liberated}

⇒ Z is called Electrochemical equivalent {ECE}

* Second law ⇒ The mass of a substance liberated at an electrode by a given amount of charge is proportional to the chemical equivalent of the substance.

→ Chemical Equivalent ⇒ The chemical equivalent of a substance is equal to its relative atomic mass divided by its valency.

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If E denotes the chemical equivalent and m be mass liberated at an electrode, from the second law.

$m \propto E$ {Faraday's 2nd law}

$m \propto EQ$ {Combining 1st & 2nd law}

$$\boxed{m = \frac{1}{K} EQ}$$

where K is universal constant

$K = 9.64 \times 10^{7}$ C/Kg

* Faraday's Law and Ionic Theory

⇒ Let the substance of valency V is liberated at an electrode.

⇒ Hence it ions carry charge Ve

⇒ To neutralise this ion, the battery has to supply an take up a charge.

⇒ If total charge passed by the battery is Q, the number of ions neutralised is

$$N = \frac{Q}{Ve} \quad \text{—} (1)$$

Let m be the mass of substance deposited on electrode.

$$\Rightarrow m = NA\, m_u \left\{ \begin{array}{l} N = \text{number of ion} \\ A = \text{relative atomic mass of ion} \\ m_u = \frac{1}{12} C_{12} \text{ mass} \end{array} \right\}$$

$$\Rightarrow m = \frac{QAm_u}{Ve} \quad \{\text{using } (1)\}$$

$$\Rightarrow m = \frac{1}{e/m_u} EQ \quad \left\{\text{as } \frac{A}{V} = E\right\}$$

Comparing it with Faraday equation we get:-

$$\Rightarrow K = \frac{e}{m_u} = \frac{1.60 \times 10^{-19}}{1.66 \times 10^{-27}} = 9.64 \times 10^{7}\ C/kg$$

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* The Unit "faraday".

→ 1 faraday ⇒ The charge of one mole of electrons is called 1 faraday.

• It is denoted by F.

⇒ 1F = 96485 C

→ Set 1F charge is passed through an electrolyte.

→ 1 mole of electron is supplied at electrode.

→ $\frac{1}{V}$ moles of ~~ele~~ ions are supplied at electrode.

→ $\frac{A}{V}$ = E gram of ions are supplied at electrode.

Hence, 1F of charge liberate one grame equivalent of any substance in electrolysis.

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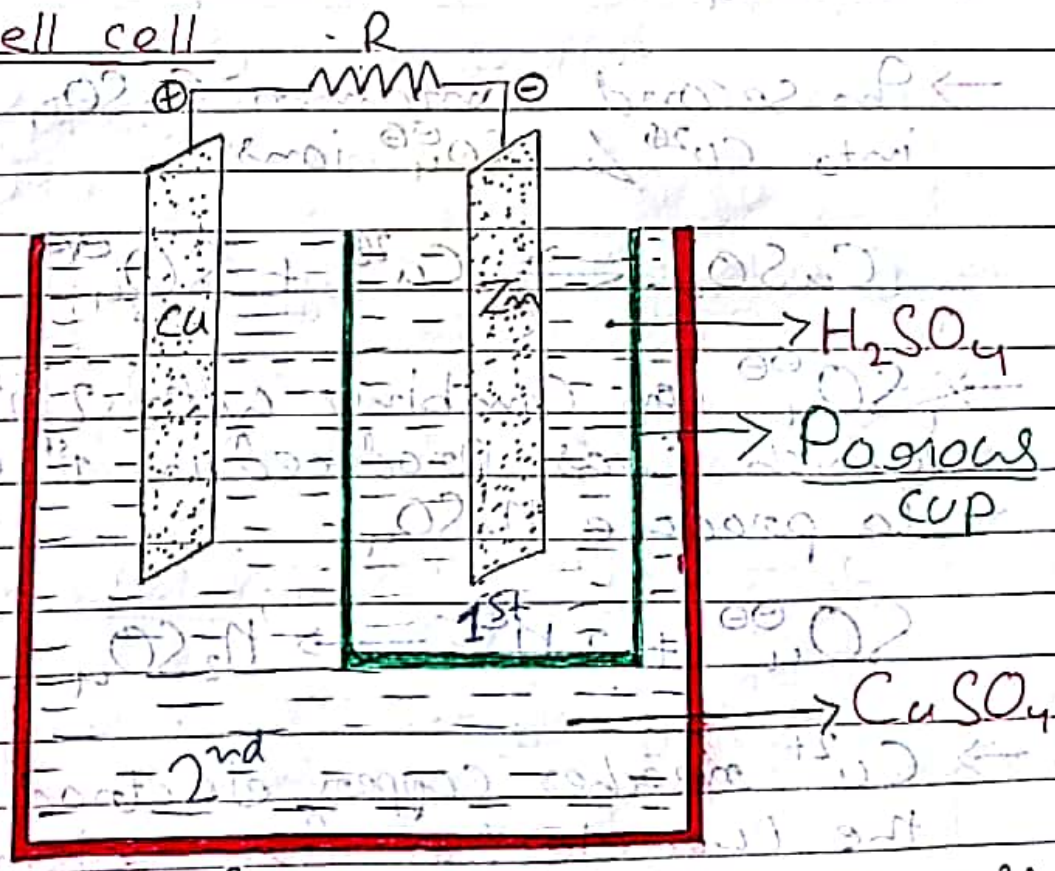
Primary & Secondary Cell

* **Primary Cell** ⇒ A cell is called primary if it is used only for discharge.

* **Secondary cell** ⇒ A cell is called Secondary if it can be used for both charging and discharging.

* Primary cell

→ **Daniell cell**



* **Porous Cup** ⇒ It allows only gas to pass through it & prevents liquids to mix.

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* Mechanism

→ In first container H_2SO_4 dissociats into $2H^{\oplus}$ & $SO_4^{\ominus\ominus}$ ion.

$$H_2SO_4 \rightleftharpoons 2H^{\oplus} + SO_4^{\ominus\ominus}$$

→ $SO_4^{\ominus\ominus}$ Combines with zinc electrod & release $2e^-$ in external circuit.

$$SO_4^{\ominus\ominus} + Zn \longrightarrow ZnSO_4 + 2e^-$$

→ In second container $CuSO_4$ dissociats into $Cu^{2\oplus}$ & $SO_4^{\ominus\ominus}$ ions.

$$CuSO_4 \rightleftharpoons Cu^{2\oplus} + SO_4^{\ominus\ominus}$$

→ $SO_4^{\ominus\ominus}$ ion combines with $2H^+$ ions {which was produced in 1st container} to produce H_2SO_4.

$$SO_4^{\ominus\ominus} + 2H^{\oplus} \longrightarrow H_2SO_4$$

→ Cu^{2+} reaches copper electrod & absorbs the two electron from external circuit.

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→ The above Process goes on continuously and steady current is maintained.

"Cupper electrod in second container dosent reacts with H_2SO_4 to produce current in opposit direction as Cu lies bellow H in reactivity Series"

* Variation in daniell cell as time passes

→ Conc. of H_2SO_4 in 1st container decreases & conc. of $ZnSO_4$ increases.

→ Conc. of H_2SO_4 in 2nd Container increases & conc of $CuSO_4$ decreases.

→ Cu electrod gets thicker & Zn gets thiner.

* When daniel cell stops working

2nd Container gets Compleatly filled with H_2SO_4 OR 1st container gets compleatly filled with $ZnSO_4$.

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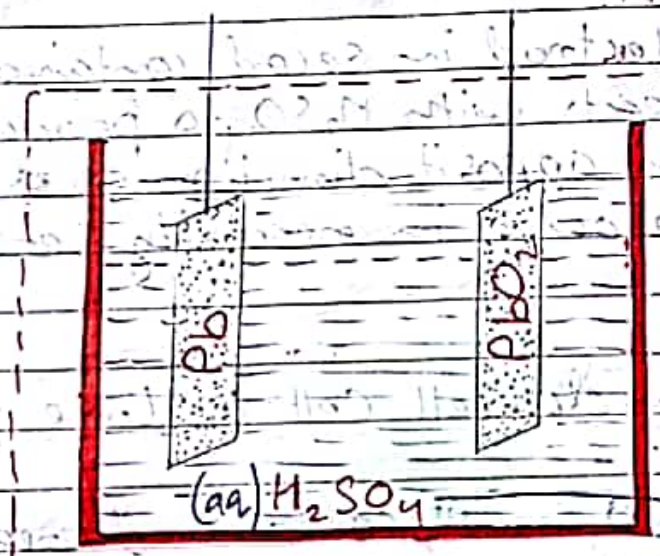
* Secondary Cell

→ Lead Accumulator



→ Lead Accumulator

→ Specific gravity (1.2 – 1.28)

* Discharging

$$\Rightarrow H_2SO_4 \rightleftharpoons 2H^+ + SO_4^{--}$$ } Self Ionization

$$2e + 2H^+ + PbO_2 \rightarrow PbO + H_2O$$

$$PbO + H_2SO_4 \rightarrow PbSO_4 + H_2O$$

$$Pb + SO_4^{--} \rightarrow PbSO_4 + 2e^-$$

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The electrods gets charged up to some extint. And reaction stops as electrods starts repaling ions.

⇒ When electrods are connected to a resistor, the electrods starts neutrilizing. The above reaction starts again and a steady current is observed in external circuit.

⇒ As the sulphuric acid is used up in discharging, the specific gravity of the acid decreases. When the specific gravity falls to 1.15, the cell is considered to be fully discharged.

* Charging

⇒ A current is forced from the positive to the negative electrod inside the cell

$PbSO_4 + 2H = Pb + H_2SO_4$ {Negative electrod}

$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$ {Positive electrod}

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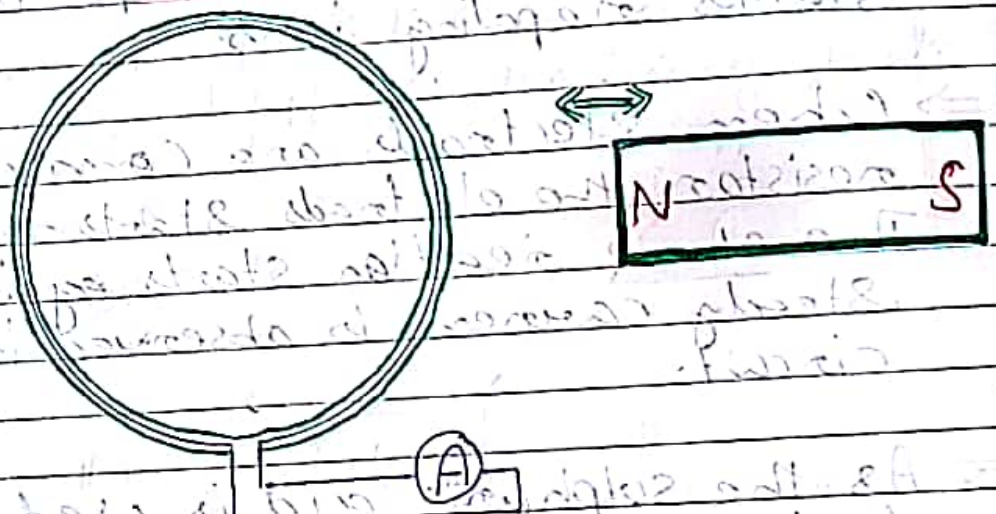
38

Electromagnetic Induction

⇒ Faraday's Law of Electromagnetic Induction



Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$$

Where, $\phi = \int \vec{B} \cdot d\vec{s}$ is the flux of the magnetic field through the area

→ We shall call the quantity ϕ as magnetic flux.

→ The emf developed by a changing flux is called induced emf and the current produced by this emf is called induced current.

* Direction of induced current

(i) Put an arrow on the loop to choose the positive sense of current. {This choice is arbitrary}
Use right hand thumb rule to find the positive direction of the normal to the area bounded by the loop.

(ii) Calculate the flux through the area bounded by the loop.

(iii) Use Faraday law to find Emf.

(iv) If Emf is negative the current induced is opposite to our consideration and vica via.

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⇒ Lenz's Law

The direction of the induced current is such that it opposes the change that has induced it.

⇒ The origin of Induced Emf

{Flow of Electric Current in Conductor}

⇒ * An Electric current is established in a Conducting wire when an electric field exists in it.

* The flow of charge tends to distroy the field.

* Some external mechanism is needed to maintain the electric field in the wire.

* It is the work done per unit charge by this external mechanism that we call emf.

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Mechanism to Produce Emf

⇒ * When the magnetic flux through a closed loop changes, an electric current results.

* The flux $\int \vec{B} \cdot d\vec{S}$ can be changed by :-

(I) Keeping the magnetic field constant with time and moving whole or part of the loop.

(II) Keeping the loop at rest and changing the magnetic field.

(III) Combination of (I) & (II)

→ Will be Studied under heading "Induced electric field"

→ Will be Studied under heading "Motional Emf"

$\boxed{\int dV = 0}$ ⇐ $\int dV$

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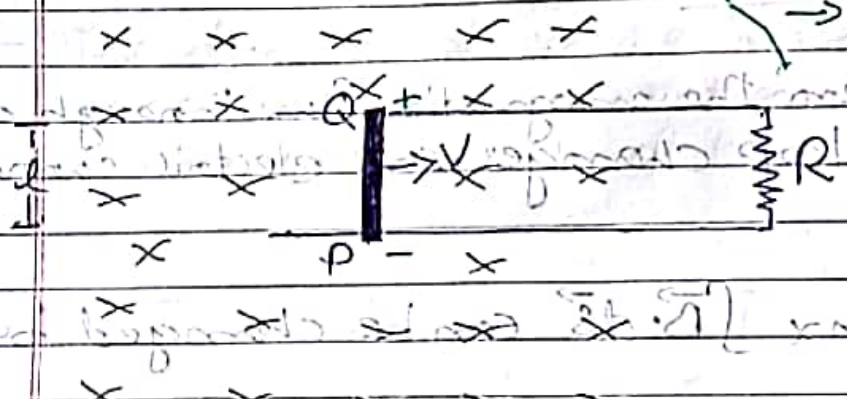
Work is done by force that maintain velocity

* Motional Emf

Mechanism
→ Magnetic field is Polarizing rod.
→ Electric field is Neutralising rod.



⇒ As the rod is moving magnetic force will be exerted on electrons and the rod will get polarised

$F_m = q\vec{v} \times \vec{B}$

⇒ Rod will polarise to some extent and then stop when electrostatic force will become equal to magnetic force.

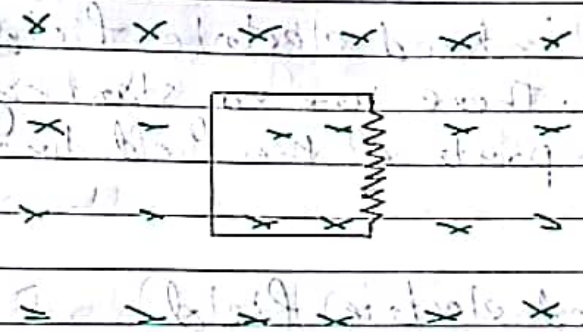
$F_e = \frac{Kq^2}{l^2}$

⇒ $F_e = F_m$

⇒ $\frac{Kq^2}{l^2} = qvB$

⇒ $\frac{Kq}{l} = vBl$ ⇒ $\boxed{\mathcal{E} = vBl}$

* Induced Electric field



→ Till $t=0$, B is constant and afterward it changes with time.

→ The free electrons where at rest till $t=0$.

{We are not intreasted in the random motion of Electron}

→ The magnetic field cannot exert force on electrons at rest.

→ The electrons may be forced to move only by an electric field and hence we conclude that an electric field appears at $t=0$

{The electric field is produced by Changing magnetic field}

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* We call it induced electric field.

→ The lines of induced electric field are closed curves. There are no starting & terminating points of the field lines.

If $\vec{E}$ is induced electric field.

$\vec{F} = q\vec{E}$

$\mathcal{E}$ = Work done per unit charge $= \frac{1}{q}\oint q\vec{E}\cdot d\vec{l} = \oint \vec{E}\cdot d\vec{l}$

<u>Using faraday law of induction</u>

$$\mathcal{E} = -\frac{\delta\phi}{\delta t}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\delta\phi}{\delta t}$$

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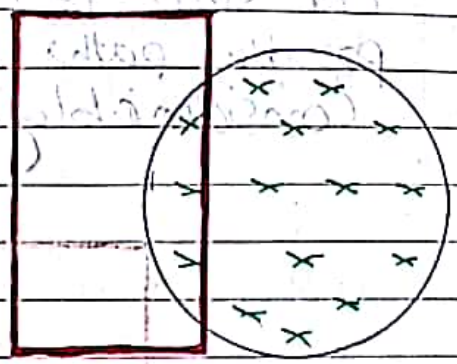
Eddy Current



→ Consider a solid plate of metal which enters a region having a magnetic field.

→ As the plate moves, the magnetic flux through ~~the area bounded by~~ it changes and hence current is induced in loops.

→ There may be a number of such loops on the plate and hence currents are ~~induced~~ induced on the surface along a veriety of paths.

→ Such current is called Eddy Current.

→ Because of the eddy current in the metal plate, thermal energy is produced in it. This energy comes at the cost of the kinetic energy of the plate and the plate slows down.

* This is known as Electromagnetic damping.

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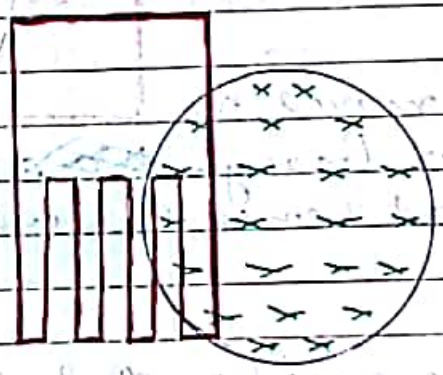
⇒ To reduce electromagnetic damping, one can cut slots in the plate. This reduces the possible paths of the eddy current considerably.



⇒ Self-Induction

⇒ When a current is established in a closed conducting loop, it produces a magnetic field. This magnetic field has its flux through the area bounded by the loop. If the current changes with time, the flux through the loop changes and hence an emf is induced in the loop. This process is called Self induction

$\Rightarrow B \propto i \Rightarrow \phi \propto i$

$\Rightarrow \boxed{\phi = Li}$ {where, L is a constant depending on the geometrical constration of the loop.}

* L is called Self-inductance of the loop.

$\mathcal{E} = -\frac{d\phi}{dt}$ {faraday law}

$\Rightarrow \boxed{\mathcal{E} = -L\frac{di}{dt}}$ (1)

* The SI unit of self-inductance L is weber/ampere. It is given a special name henry and is abbreviated as H.

* <u>Self-Inductance of a long Solenoid</u>

Consider a long solenoid of radius r having n turns per unit length. Suppose a current i is passed through the solenoid.

$B = \mu_0 n i$

// <u>flux through each turn</u>

$\phi = \int B \cdot dS = (\mu_0 n i)(\pi r^2)$

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// The emf induced in each turn

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\mu_0 n \pi r^2 \frac{di}{dt}$$

// As there are nl turns in a length l of the solenoid, the net emf across a length l is :-

$$\mathcal{E} = -(nl)(\mu_0 n \pi r^2)\frac{di}{dt} \quad \text{--- (I)}$$

$$\mathcal{E} = -L\frac{di}{dt} \quad \text{--- (II)}$$

// Comparing eq (I) & (II) we get :-

$$\boxed{L = \mu_0 n^2 \pi r^2 l}$$

* Ideal Inductor

⇒ negligible resistance but considerable self inductance.

Symbol :-

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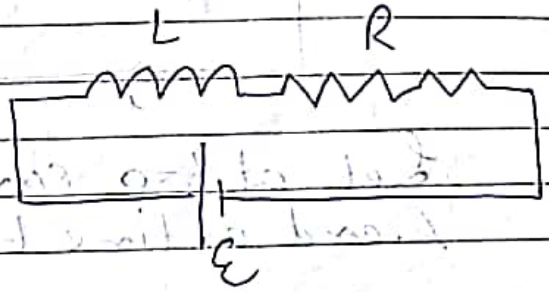
⇒ Growth & decay of current in an LR-Circuit

* Growth of current



Let at time t current in the circuit be i

$$\Rightarrow \quad -\varepsilon + L\frac{di}{dt} + iR = 0$$

$$\Rightarrow \quad L\frac{di}{dt} = \varepsilon - iR$$

$$\int_0^i \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$$

$$\frac{1}{-R}\left[\ln|\varepsilon - iR|\right]_0^i = \frac{t}{L}$$

$$\Rightarrow \quad \ln\left|\frac{\varepsilon - iR}{\varepsilon}\right| = \frac{-Rt}{L}$$

$$\varepsilon - iR = \varepsilon e^{-\frac{Rt}{L}}$$

$$\boxed{i = \frac{\varepsilon}{R}\left\{1 - e^{-\frac{t}{\tau}}\right\}} \quad \left\{\text{where, } \tau = L/R \text{ (Time constant)}\right\}$$

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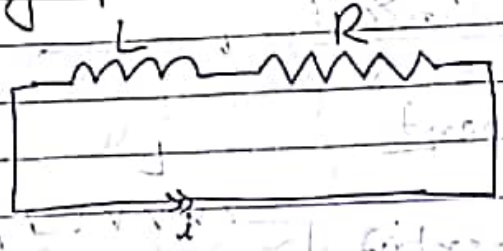
* Decay of current



Let at $t=0$ current in the circuit be i_0 & and at time t current be i.

$$\Rightarrow -L\frac{di}{dt} = iR$$

$$\Rightarrow \int_{i_0}^{i}\frac{di}{i} = -\int_{0}^{t}\frac{R}{L}\,dt$$

$$\Rightarrow \ln\left|\frac{i}{i_0}\right| = -\frac{Rt}{L}$$

$$\Rightarrow i = i_0 e^{-\frac{Rt}{L}}$$

$$\Rightarrow \boxed{i = i_0 e^{-\frac{t}{\tau}}} \quad \left\{\tau = L/R \text{ (Time constant)}\right\}$$

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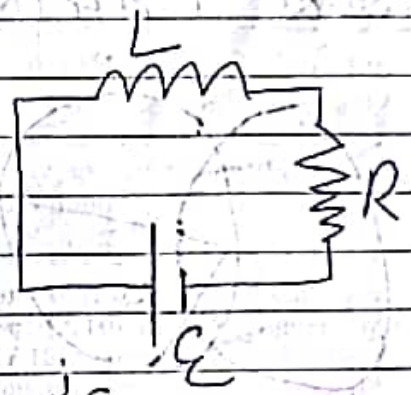
⇒ Energy Stored in an Inductor

⇒ When inductor carries a current, a magnetic field is built in it & magnetic energy is stored in it.

⇒ If i be the current in the circuit at time t we have :-



⇒ $\mathcal{E} - L\frac{di}{dt} = iR$

⇒ $\int_0^t \mathcal{E} i\,dt = \int_0^t i^2 R\,dt + \int_0^i Li\,di$

⇒ $\int_0^t \mathcal{E} i\,dt = \int_0^t i^2 R\,dt + \frac{1}{2}Li^2$

{Work done by the battery} → {Thermal energy liberated at resistor}

{Magnetic energy stored in inductor}

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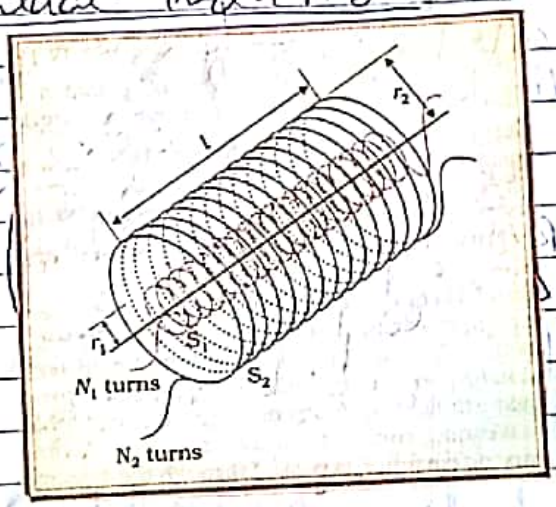
* Energy density in magnetic field

$$\boxed{u = \frac{1}{2\mu_0} B^2}$$ {Can be varified from Special case}

⇒ Mutual Induction



⇒ Suppose two coaxial solenoide are arranged as shown in the figure.

⇒ Let inner solenoid carry current i at time t.

⇒ Flux through outer solenoid is proportional to current in inner solenoid.

$$\phi \propto i$$

$$\phi = Mi$$ {where M is constant depending on geometrical arrangement}

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→ M is called mutual inductance.

$\Rightarrow \mathcal{E} = -\frac{d\phi}{dt}$ {faraday law}

$\Rightarrow \boxed{\mathcal{E} = -M\frac{di}{dt}}$

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Alternating Current

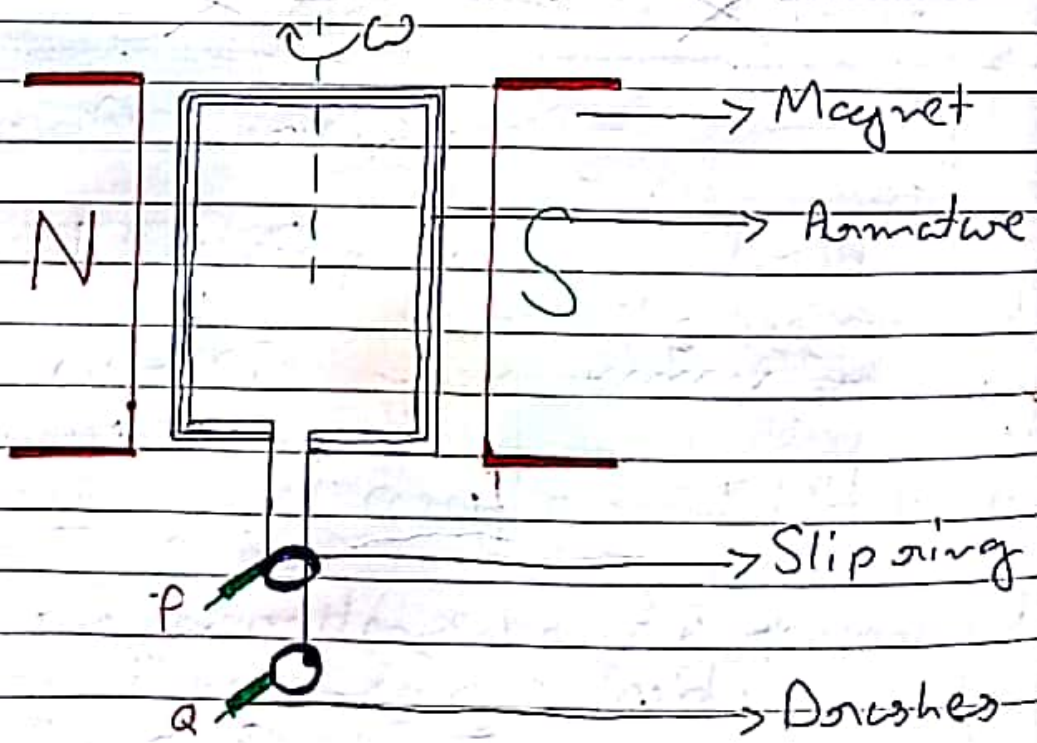
⇒ Alternating Current

Current in which flow of electric charge periodically reverses its direction.

⇒ ~~Sinusoid~~ Sinusoidal alternating current is given by :-

$$\boxed{i = i_0 \sin(\omega t + \phi)}$$

⇒ AC Generator and AC Dynamo



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⇒ Instantaneous and Rms Current

* mean Current; $\bar{i} = \frac{\int_0^t i\,dt}{\int_0^t dt} = \frac{1}{t}\int_0^t i\,dt$ {defination}

$$\bar{i} = \frac{i_0}{t}\int_0^t \sin(\omega t+\phi)\,dt = \frac{-i_0}{t\omega}\{\cos(\omega t+\phi) - \cos\phi\}$$

// For a Time period

$$\bar{i} = \frac{-i_0}{T\omega}\{\cos(2\pi+\phi) - \cos\phi\} = 0$$

⇒ The instantaneous current i could be positive or negative at a given instant but the quantity i^2 always remains positive and hence its average is also positive.

// The average of i^2 over a time period is :-

$$\overline{i^2} = \frac{\int_0^T i^2\,dt}{\int_0^T dt} = \frac{1}{T}\int_0^T i_0^2 \sin^2(\omega t+\phi)\,dt = \frac{i_0^2}{2}$$

⇒ This is known as mean Square current.

* Rms Current ⇒ Square root of mean Square current.

$$\boxed{i_{rms} = \sqrt{\overline{i^2}} = \frac{i_0}{\sqrt{2}}}$$

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Alternating voltage

⇒ An alternating voltage may be written as.

$$V = V_0 \sin(\omega t + \phi)$$

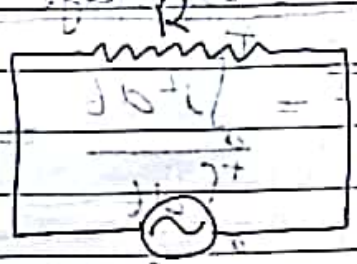
$$\boxed{V_{rms} = \frac{V_0}{\sqrt{2}}} \quad \{\text{Similarly}\}$$

⇒ V_{rms} & i_{rms} acts as an equivalent alternet for alternating voltage & current.

⇒ Simple AC Circuits

* AC Circuit Containing only a Resistor

$$\mathcal{E}_0 \sin\omega t = Ri$$



$$\Rightarrow \boxed{i = \frac{\mathcal{E}_0}{R} \sin\omega t}$$

$$= i_0 \sin\omega t$$

$$\left\{\text{where } i_0 = \frac{\mathcal{E}_0}{R}\right\}$$

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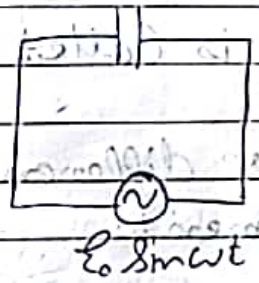
* AC Circuit Containing only a Capacitor

⇒ Let the charge on the capacitor is q and the current is i at a general time t.



⇒ $\mathcal{E}_0 \sin\omega t = \frac{q}{C}$ {Using Kirchhoff's Law}

⇒ $q = C\mathcal{E}_0 \sin\omega t$

⇒ $\frac{dq}{dt} = \omega C \mathcal{E}_0 \cos\omega t$

⇒ $\boxed{i = \omega C \mathcal{E}_0 \cos\omega t} = i_0 \cos\omega t$

$\left\{ \text{where, } i_0 = C\mathcal{E}_0\omega = \frac{\mathcal{E}_0}{1/\omega C} \right\}$

⇒ **Note**

(i) In case of AC Source, the current exists as long as the Source is Connected.

(ii) Relation between Peak Emf and Peak Current.

$i_0 = \frac{\mathcal{E}_0}{1/\omega C} = \frac{\mathcal{E}_0}{X_C}$ $\left\{ \text{where, } X_C = \frac{1}{\omega C} \right\}$

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→ X_C plays the role of effective resistance.

→ X_C is called the reactance of the capacitor.

(iii) Phase difference between the emf and the current:

$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \;\&\; i = i_0 \sin(\omega t + \pi/2)$$

$$\Rightarrow \Delta\phi = \pi/2$$

* AC Circuit Containing only an Inductor

$\Rightarrow$ Emf across Inductor $= -L\dfrac{di}{dt}$

$$\Rightarrow L\frac{di}{dt} = \mathcal{E}_0 \sin\omega t$$

$$i = \frac{\mathcal{E}_0}{L}\int \sin\omega t \, dt$$

$$\Rightarrow -\frac{\mathcal{E}_0}{\omega L}\cos\omega t$$

$$\boxed{i = i_0 \sin(\omega t - \pi/2)}$$

$$\left\{\text{where, } i_0 = \frac{\mathcal{E}_0}{\omega L}\right\}$$

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⇒ The constant $X_L = \omega L$ plays the role of effective resistance in this circuit. It is called the reactance of the inductor

* Impedance

The peak current and the peak emf are related for any series circuit (one-loop circuit) having an AC source is given by :-

$$\boxed{i_0 = \mathcal{E}_0 / Z}$$

{where, Z is called impedance of the circuit}

* $Z = R$ ∀ Purely resistive circuit.
* $Z = 1/\omega C$ ∀ Purely capacitive circuit.
* $Z = \omega L$ ∀ Purely inductive circuit.

* Phase factor

"Phase difference between current & emf"

* $\Delta\phi = 0$ ∀ Purely resistive circuit.

* $\Delta\phi = \pi/2$ ∀ Purely capacitive circuit.

* $\Delta\phi = -\pi/2$ ∀ Purely inductive circuit.

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⇒ Vector method to find the current in an AC Circuit

⇒ Suppose an emf $\mathcal{E} = \mathcal{E}_0 \sin\omega t$ is applied in a series AC circuit which may contain a resistance, a capacitor, an inductor or any combination of these.

* Let us represent the resistance of a resistor by a vector of magnitude R along x-axis.

* Reactance of capacitor by a vector of magnitude $1/\omega C$ along +Y-axis.

* Reactance of inductor by a vector of magnitude ωL along −Y-axis.

⇒ The impedance of the circuit, Z and the phase factor ϕ are obtained by the vector sum of these three vectors.

→ The magnitude of the vector sum gives the impedance Z, and its angle with the X-axis gives the phase factor.

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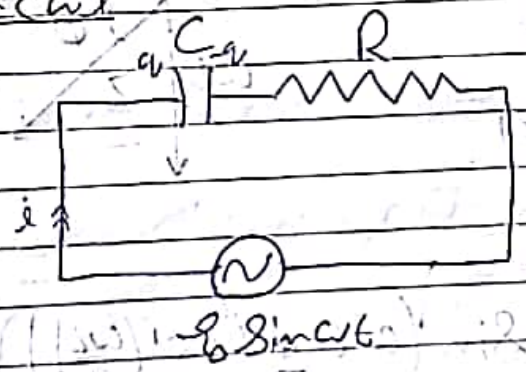
⇒ Once Z and ϕ are obtained, the current in the circuit can be easily written as:-

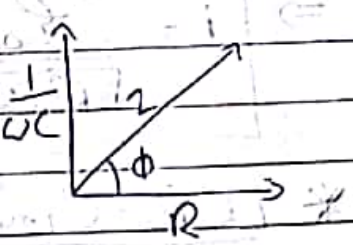
$$\boxed{i = \frac{\mathcal{E}_0}{Z} \sin(\omega t + \phi)}$$

⇒ <u>More AC Circuits:</u>

* <u>CR Circuit</u>



$$Z = \sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2} \quad \& \quad \phi = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$



$$\Rightarrow \boxed{i = \frac{\mathcal{E}}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} \sin\left\{\omega t + \tan^{-1}\left(\frac{1}{\omega C R}\right)\right\}}$$

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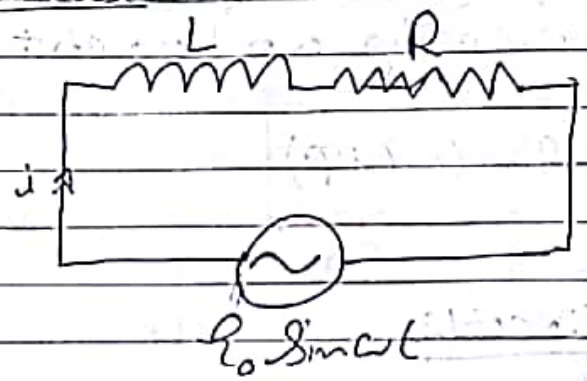
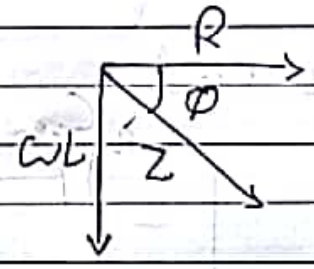
* LR Circuit



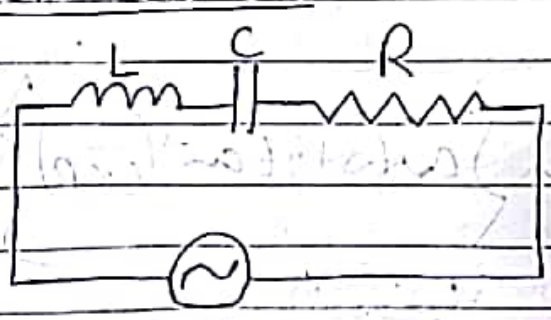
$$Z = \sqrt{R^2 + \omega^2 L^2}$$

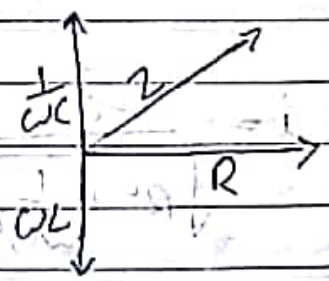
$$|\phi| = \tan^{-1}\left(\frac{\omega L}{R}\right)$$



$$\Rightarrow \boxed{i = \frac{\mathcal{E}_0}{\sqrt{R^2+\omega^2L^2}} \sin\left(\omega t \pm \tan^{-1}\left(\frac{\omega L}{R}\right)\right)}$$

* LCR-Circuit





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$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2} \qquad \tan\phi = \frac{\left(\frac{1}{\omega C} - \omega L\right)}{R}$$

$$\boxed{i = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}} \sin\left\{\omega t + \tan^{-1}\left[\frac{\frac{1}{\omega C} - \omega L}{R}\right]\right\}}$$

⇒ <u>Note</u>

⇒ If $X_C = 1/\omega C$ is greater than $X_L = \omega L$.

→ <u>Current leads emf.</u>

⇒ If $X_L > X_C$

→ <u>Current lags ~~behind~~ behind emf</u>

⇒ If $X_L = X_C$

→ <u>Current is in phase with the emf.</u>

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⇒ If we vary the angular frequency ω of the AC source, the peak current also varies.

$$\boxed{i_0 = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}}$$

⇒ i_0 is maximum when

$$\frac{1}{\omega C} - \omega L = 0$$

$$\Rightarrow \omega = \sqrt{\frac{1}{LC}}$$

$$\Rightarrow \boxed{f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}}$$

⇒ This frequency is known as the resonant frequency.

* LC-Oscillations

⇒ When a charged capacitor is connected to a pure inductor there is a current in the circuit at frequency $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$.

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⇒ The capacitor gets discharged sending a current in the inductor and induced emf in the inductor charges the capacitor again.

⇒ Thus, the energy oscillates between electric field energy in the capacitor and magnetic field energy in the inductor.

"This phenomenon is called LC oscillation"

⇒ <u>Power in AC Circuits</u>

Suppose an emf $\mathcal{E} = \mathcal{E}_0 \sin \omega t$ is applied in a circuit and a current $i = i_0 \sin(\omega t + \varphi)$ results.

The work done by the source during the time interval t to $t+dt$ is

$$dW = \mathcal{E} i \, dt$$

$$W = \frac{1}{2} \mathcal{E}_0 i_0 T \cos\varphi$$

$$\Rightarrow P = \frac{W}{T} = \left(\frac{\mathcal{E}_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right) \cos\varphi$$

$$\boxed{P = \mathcal{E}_{rms} \, i_{rms} \cos\varphi}$$

⇒ The term $\cos\varphi$ is called power factor.

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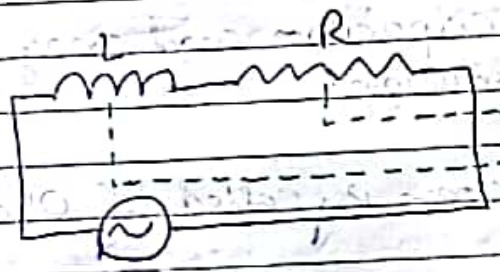
⇒ CHOKE COIL

Choke coil is simply a coil having a large inductance but small resistance.



→ fluorescent tube.

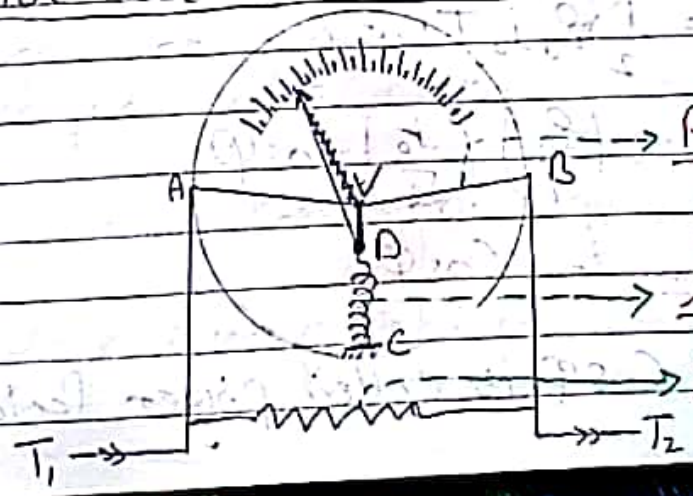
→ Choke Coil

$\mathcal{E} = \mathcal{E}_0 \sin\omega t$

⇒ The advantage of using a choke coil to reduce the voltage is that an inductor does not consume power.

⇒ Hot-Wire Instruments

* Hot-Wire Ammeter



→ Platinum-iridium wir

→ Spring

→ Shunt

Working

⇒ Alternating emf source is connected across T_1 and T_2.

⇒ As current flows through platinum-iridium wire it gets heated and expand in length.

⇒ A mechanism is set which changes its reading in proportion with length of wire.

⇒ A length is proportional to Irms then reading is also proportional to Irms.

⇒ Hence this way Hot-wire Ammeter shows its reading.

* Hot-wire Voltmeter

⇒ Hot wire Voltmeter works the same way except shunt is connected in series with the meter.

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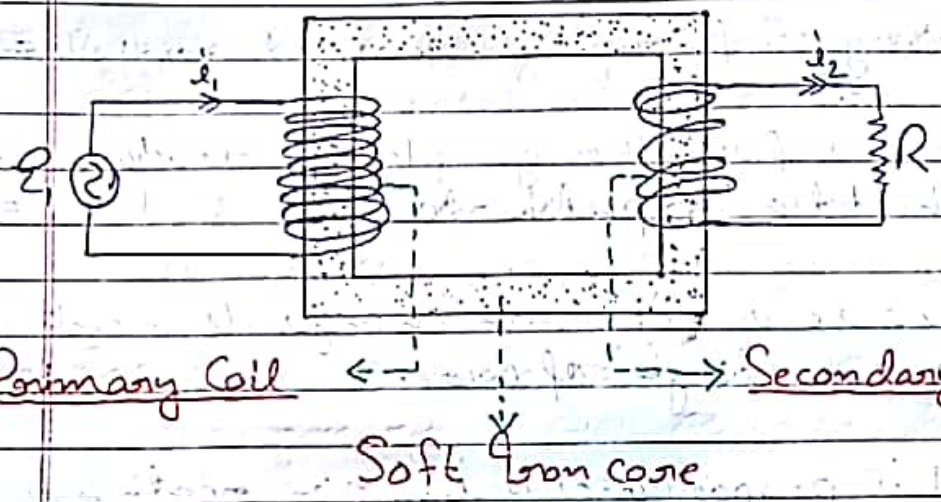
⇒ Transformer



⇒ Suppose there are N_1 turns in the primary and N_2 turns in the secondary. An alternating emf $\mathcal{E}_1$ is applied across the primary which produces a current i_1 in the primary circuit and a current i_2 in the secondary circuit.

⇒ The currents in the coils produce a magnetization in the soft-iron core and there is a corresponding magnetic field B inside the core.

⇒ We assume that the field is constant in magnitude everywhere in the core

⇒ Let flux through primary as well as secondary each turn be ϕ.

// for primary

$$\mathcal{E}_1 - N_1 \frac{d\phi}{dt} = 0 \quad \{\text{using Kirchhoff's law}\}$$

$$\Rightarrow \mathcal{E}_1 = -N_1 \frac{d\phi}{dt} \quad \text{—(I)}$$

// for Secondary

$$\mathcal{E}_2 = -N_2 \frac{d\phi}{dt} \quad \text{—(II)}$$

⇒ Using (I) & (II) we get :-

$$\Rightarrow \boxed{\frac{\mathcal{E}_1}{\mathcal{E}_2} = -\frac{N_1}{N_2}}$$

⇒ The minus sign shows that $\mathcal{E}_2$ is 180° out of phase with $\mathcal{E}_1$.

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* Power transfer

⇒ When secondary circuit is incompleate, Let i_1 be current in the primary circuit.

⇒ Let us now concider that terminals of the secondary are joined to a resistace R.

⇒ There will be an additional emf's induced in the primary as well as in the secondary due to i_2.

⇒ There will be an additional current i_1 in the primary which will cancel the emf induced due to i.

⇒ The power delivered by the AC Source is therefore $\mathcal{E}_1 i_1$. The power consumed by the resistace in the secondary circuit is $\mathcal{E}_2 i_2$.

⇒ $\mathcal{E}_1 i_1 = \mathcal{E}_2 i_2$

Hence, $\boxed{\frac{i_1}{i_2} = -\frac{N_2}{N_1}}$

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* **Step-up Transformer** ⇒ A step up transformer is one whose secondary voltage is greater than its primary voltage.

* **Step-down transformer** ⇒ A Step-down transformer is one whose secondary voltage is smaller than its primary voltage.

* **Efficiency**, $\boxed{\eta = \frac{\text{Output power}}{\text{Input power}}}$

// **Factors affecting Efficiency**

(i) Primary resistance

(ii) Hysteresis in the Core.

(iii) Eddy current

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Electromagnetic wave

⇒ Maxwell's Displacement Current

⇒ Ampere's Law $\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 i}$ is not valid if the electric field at the surface varies with time.

Verification Example

⇒ Let us consider a ll-plate capacitor with circular plates being charged by a battery.

⇒ If we place a compass needle in the space between the plates, the needle in general deflects.

⇒ This shows that their is magnetic field in this region, but no current

⇒ Hence $\oint B \cdot dl \neq \mu_0 i$

⇒ Hence, their is some other source creating magnetic field which is nothing but changing electric field.

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// Relation between changing electric field creating magnetic field :-

$$\boxed{\int \vec{B} \cdot dl = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}} \quad \left\{ \text{where } \phi_E = \text{Electric flux} \right\}$$

⇒ It was James Clerk Maxwell who generalised Ampere's Law as :-

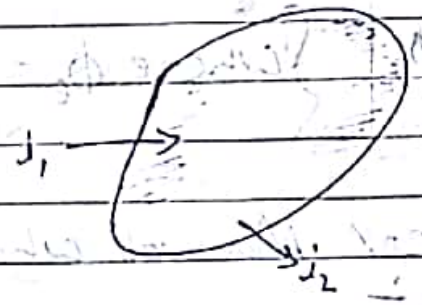
$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \varepsilon_0 \left(\frac{d\phi_E}{dt} \right)}$$

$$\Rightarrow \mu_0 (i + i_d) \quad \left\{ \text{where } i_d = \varepsilon_0 \frac{d\phi_E}{dt} \right\}$$

⇒ Maxwell named the term $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$ as <u>displacement current</u>.

⇒ The current due to flow of charge is often called <u>conduction current</u> and is denoted by i.

⇒ Continuity of Electric current



⇒ Suppose a total conduction current j_1 goes into the volume and a total conduction current j_2 goes out of it.

⇒ The charge going into the volume in a time dt is $j_1 dt$ and that coming out is $j_2 dt$.

⇒ The charge accumulated inside the volume is:-

$$d(q_{in}) = j_1 dt - j_2 dt$$

$$\frac{d(q_{in})}{dt} = j_1 - j_2 \quad \text{—(1)}$$

// From Gauss's law

$$\phi_E = \oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

$$\varepsilon_0 \frac{d\phi_E}{dt} = \frac{d(q_{in})}{dt}$$

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$$\Rightarrow j_d = \frac{d(q_{in})}{dt} \quad \text{--- (II)}$$

// Comparing (I) & (II) we get :-

$$\boxed{j_1 = j_2 + j_d}$$

"Thus, the total current (conduction + displacement) going into the volume is equal to the total current coming out of it"

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⇒ Maxwell's Equations and Plane Electromagnetic waves

⇒ The whole subjet of electricity and magnetism may be described mathematically with the help of four fundamental equations

① $\oint \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}$ {Gauss's Law for Electricity}

② $\oint \vec{B} \cdot d\vec{s} = 0$ {Gauss's Law for Magnetism}

③ $\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$ {Faraday's Law}

④ $\oint \vec{B} \cdot d\vec{l} = \mu_0 i + \varepsilon_0 \mu_0 \frac{d\phi_E}{dt}$ {Ampere's Law}

"These equations are collectively known as Maxwell's Equations"

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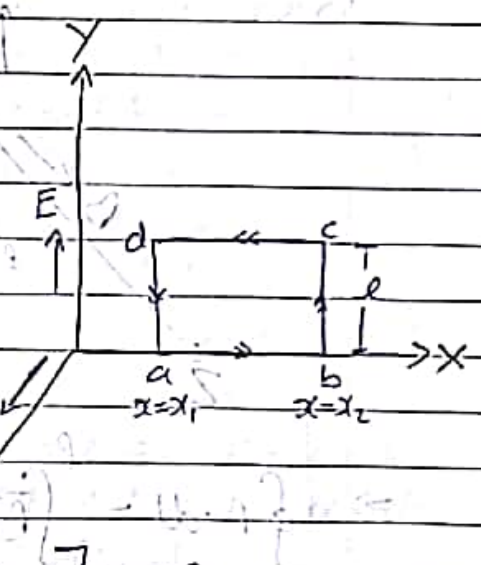
⇒ Let us check if these equations are satisfied by a plane electromagnetic wave given by :-

$$E_y = E_0 \sin \omega(t - x/c)$$

& $$B_z = B_0 \sin \omega(t - x/c)$$

→ The wave described above propagates along the +x-direction, the electric field remains along the y-direction and the magnetic field along z-direction.

* <u>Faraday law</u>



$$\Rightarrow \oint \vec{E} \cdot d\vec{l} = \int_a^b \vec{E} \cdot dl + \int_b^c \vec{E} \cdot d\vec{l} + \int_c^d \vec{E} \cdot dl + \int_d^a \vec{E} \cdot d\vec{l}$$

$$\Rightarrow 0 + E(x_2)l + 0 + E(x_1)(-l)$$

$$\Rightarrow E_0 l \left[\sin \omega(t - x_2/c) - \sin \omega(t - x_1/c)\right] \quad \text{—(I)}$$

$$\Phi_B = \oint \vec{B} \cdot d\vec{S} = \int_{x_1}^{x_2} B(x) \cdot l \cdot dx = \int_{x_1}^{x_2} B_0 l \sin \omega(t - x/c) \cdot dx$$

$$\Rightarrow \frac{d\Phi_B}{dt} = -c B_0 l \left[\sin \omega(t - x_2/c) - \sin \omega(t - x_1/c)\right] \quad \text{—(II)}$$

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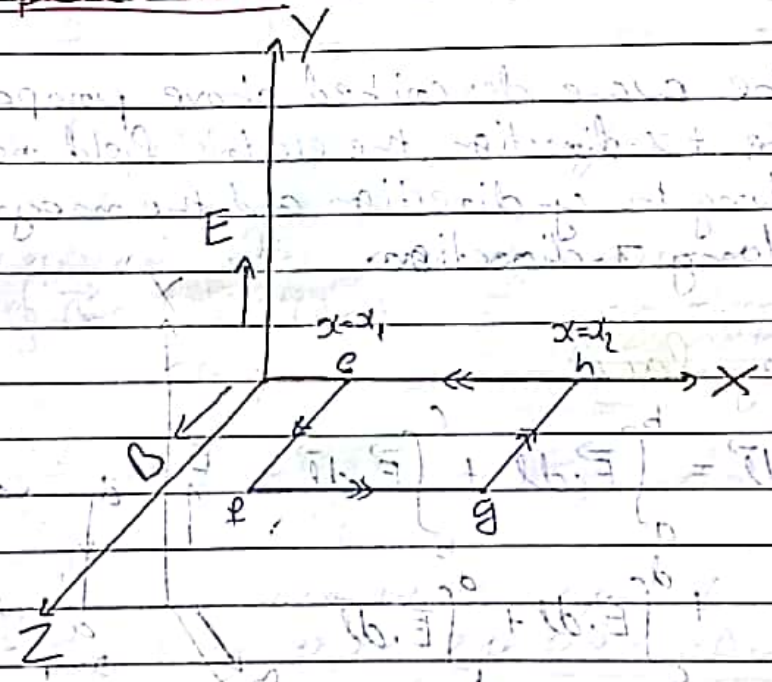
// Relating eq ① & ⓘ with Faradays law we get

$\Rightarrow \boxed{E_0 = c B_0}$ — ⓐ

* Ampere's Law



$$\Rightarrow \oint B \cdot dl = \int_e^f \vec{B} \cdot d\vec{l} + \int_f^g \vec{B} \cdot d\vec{l} + \int_g^h \vec{B} \cdot d\vec{l} + \int_h^e \vec{B} \cdot d\vec{l}$$

$$\Rightarrow B(x_1) l + 0 + B(x_2)(-l) + 0$$

$$\Rightarrow B_0 l' [\sin\omega(t - x_1/c) - \sin\omega(t - x_2/c)] \quad \text{— (iii)}$$

$$\Rightarrow \Phi_E = \int \vec{E} \cdot d\vec{s} = \int_{x_1}^{x_2} E(x)\, l\, dx = E_0 l \int_{x_1}^{x_2} \sin\omega(t - x/c)\, dx$$

$$\Rightarrow \frac{d\Phi_E}{dt} = -c E_0 l \left[\sin\omega(t - x_2/c) - \sin\omega(t - x_1/c)\right] \quad \text{— (IV)}$$

// Relating eq (III) & (IV) with Ampere's law we get:-

$$\boxed{\mu_0 \varepsilon_0 = \frac{B_0}{E_0 c}} \quad \text{— (b)}$$

$\Rightarrow$ Relating eq (a) & (b) we get:-

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$$

$\Rightarrow$ The speed of electromagnet wave was exactly equal to that of light which provided a confirmation proof that light is an <u>electromagnetic wave.</u>

⇒ Energy density and Intensity

⇒ Energy density in a region containing both electric & magnetic field.

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \sin^2\omega(t - x/c) + \frac{1}{2\mu_0} B_0^2 \sin^2\omega(t - x/c)$$

⇒ If we take the average over a long time, the $\sin^2$ terms have an average value of $1/2$. Thus,

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2$$

Relating above with equation ⓐ & ⓑ :

$$\boxed{u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2}$$

* **Intensity** ⇒ The energy crossing per unit area per unit time ⊥ to the direction of propagation is called the intensity of a wave.

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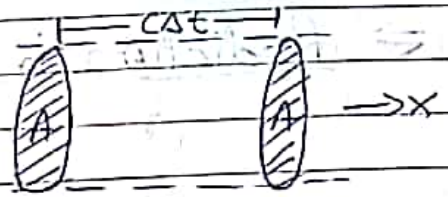
⇒ Let us consider a cylindrical volume with area of cross-section A and length $c\Delta t$ along x-axis.



⇒ $U = u_{av}(c\Delta t)A$

⇒ $I = \dfrac{U}{A\Delta t}$

⇒ $\boxed{I = u_{av}c = \frac{1}{2}\varepsilon_0 E_0^2 c}$

⇒ Momentum

⇒ The linear momentum caried by the portion of wave having energy U is given by:-

$$\boxed{p = \frac{U}{c}}$$

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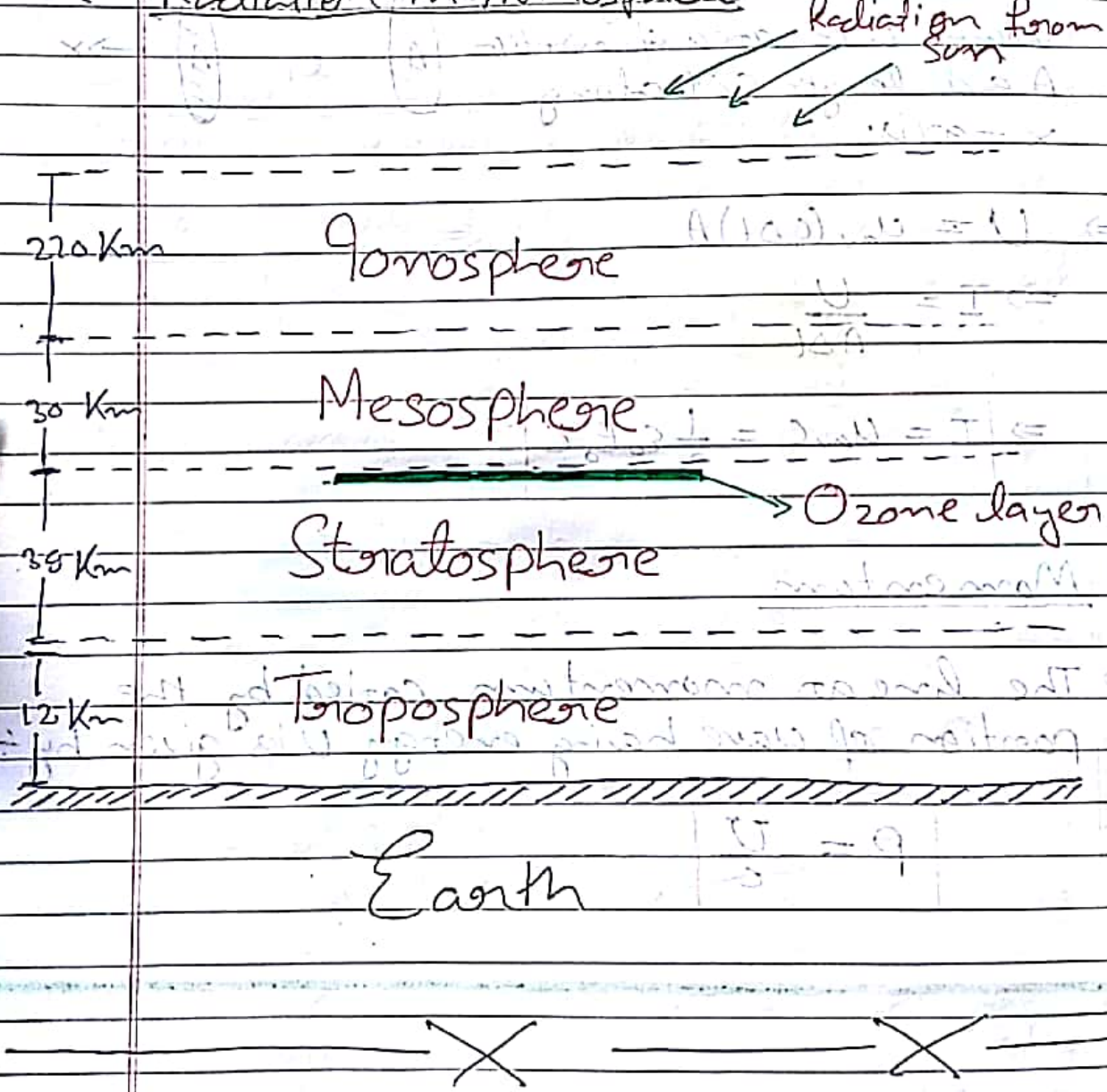
⇌ Radiation in Atmosphere



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Magnetism and matter

① Magnetic poles and bar magnets

The behaviour of a current loop can be described by the following hypothetical model :-

ⓐ There are two types of magnetic charges, positive magnetic charge and negative magnetic charge. A magnetic charge m placed in a magnetic field $\vec{B}$ experiences a force.

$$\boxed{\vec{F} = m\vec{B}}$$

ⓑ A magnetic charge m produces a magnetic field

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{m}{r^2}\hat{r}}$$

ⓒ A magnetic dipole is formed when a negative magnetic charge $-m$ and a positive magnetic charge $+m$ are placed at a small separation d.

→ The magnetic dipole moment is $\mu = md$ and its direction is from $-m$ to $+m$.

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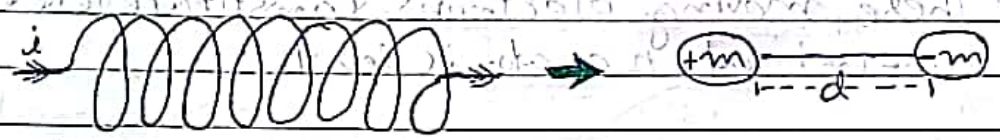
Note

→ It is customary to call a positive magnetic charge a north pole and a negative magnetic charge a South pole.

→ The quantity m is called pole strength.



⇒ In nature, we find certain objects whose magnetic behaviour may be described by assuming that there is a South pole placed at a certain points in the object and a north pole placed at a different point. Such an object is called a magnet.

* Bar magnet ⇒ A magnet in the shape of a rod or a bar is called a bar magnet.

→ Poles appear at points which are slightly inside the two ends.

→ The line joining the positions of the assumed poles is called the magnetic axis of the bar magnet.

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* How can a rod produce magnetic field when no current is passed through it?

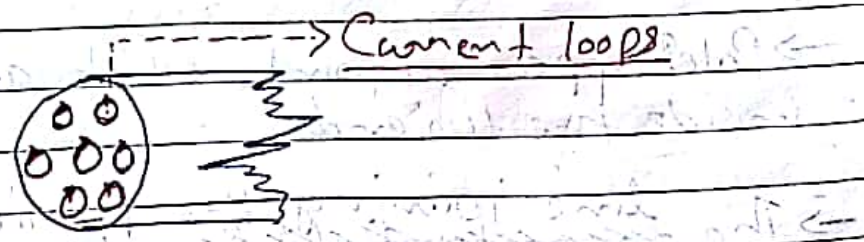
⇒ A simple model tells us that matter is made of atoms and each atom contains electrons circulating around its nucleus.

⇒ These moving electrons constitute electric current at the atomic level.

⇒ The actual description of these atomic currents is quit complicated but we can assume that these atomic currents are equivalent to small, circular current-loops.

⇒ In magnets these loops are arranged nearly parallel to each other and have current in the same sense.

⇒ Hence it behaves like a magnet.



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Note

(i) $\frac{\text{Magnetic length}}{\text{Geometrical length}} = 0.84$

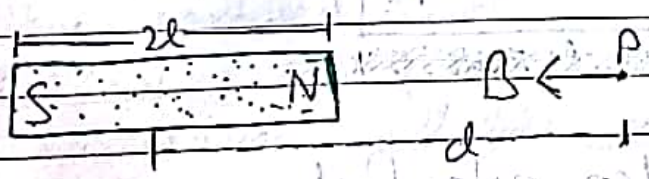
(ii) The magnetic moment of a bar magnet is conventionally denoted by M and the magnetic length of a bar magnet is written as 2l.

② Torque on a bar magnet placed in a Magnetic field

$$\vec{\tau} = \vec{M} \times \vec{B}$$

③ Magnetic field due to bar magnet

ⓐ End-on position



$$\boxed{B = \frac{\mu_0}{4\pi} \times \frac{2Md}{(d^2 - l^2)^2}}$$

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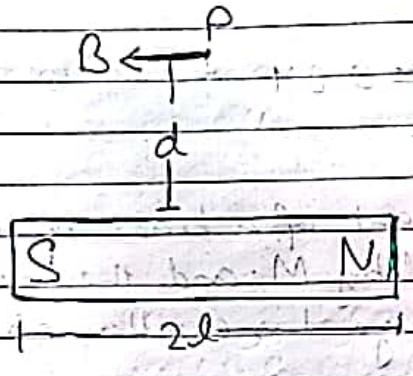
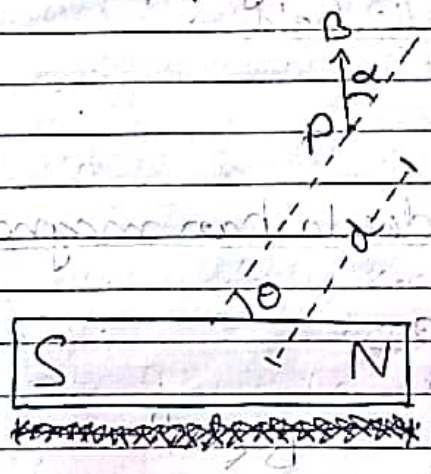
(b) Broadside-on Position



$$B = \frac{\mu_0}{4\pi} \times \frac{M}{(d^2+l^2)^{3/2}}$$

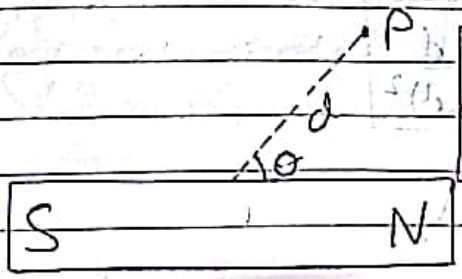
(c) At a general point



$$B = \frac{\mu_0}{4\pi} \times \frac{M}{d^2}\sqrt{1+3\cos^2\theta}$$

$$\tan\alpha = \frac{\tan\theta}{2}$$

(d) Magnetic scalar potential



$$V = \frac{\mu_0}{4\pi} \times \frac{m(2l\cos\theta)}{(r^2 - l^2\cos^2\theta)}$$

(5) Terr

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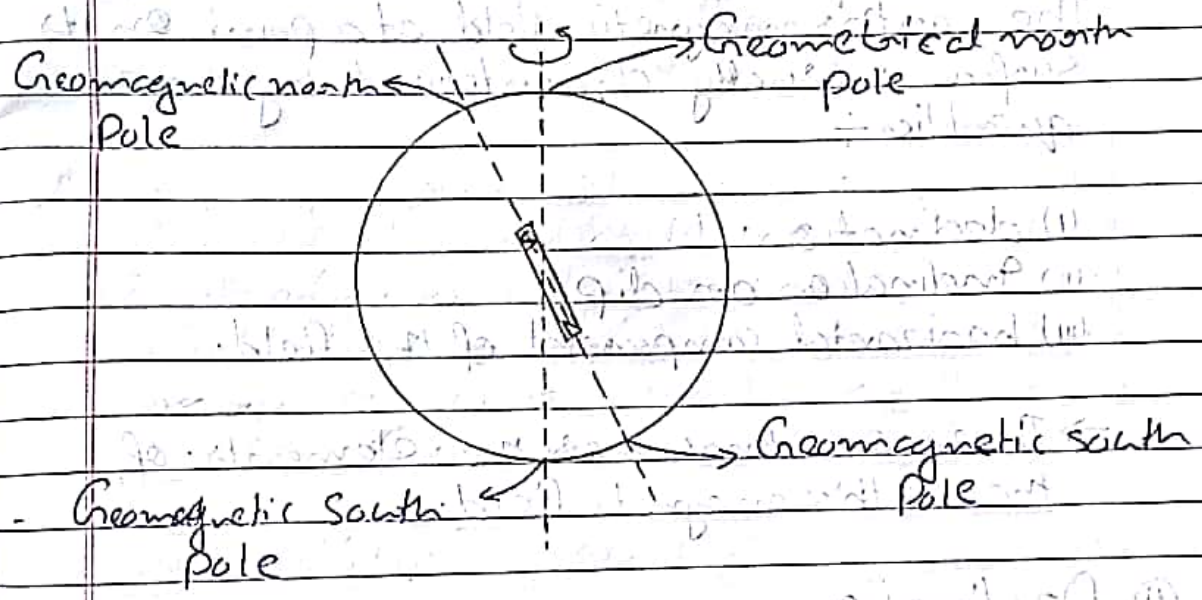
⑤ Terrestrial magnetism



⇒ The theory of earth's magnetic field is not yet well understood.

"At present it seems that the field results mainly due to circulating electric currents induced in the molten liquid and other conducting material inside the earth"

* Elements of the Earth's magnetic field

The earth's magnetic field at a point on its surface is usually characterised by three quantities :-

(i) declination.
(ii) Inclination or dip.
(iii) horizontal component of the field.

⇒ These are known as the elements of the earth's magnetic field.

(1) Declination

"The angle made by the magnetic meridian at a point with the geographical meridian is called the declination at that point."

→ Geographical meridian ⇒ A plane passing through the geographical poles and a given point P on the earth surface is called the geographical meridian at the point P.

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→ Magnetic meridian ⇒ A plane passing through the geomagnetic poles on the point P is called magnetic meridian at point P.

(5) Dip angle (δ)

"The angle made by the earth's magnetic field with the horizontal direction in the magnetic meridian is called the inclination or dip at that point."

(6) Horizontal component of earths magnetic field

"Earths magnetic field in the horizontal direction in the magnetic meridian.

$$\boxed{B_H = B\cos\delta} \quad \{\delta = \text{dip angle}\}$$

(7) Gauss's Law for magnetism

$$\boxed{\oint \vec{B}\cdot d\vec{s} = 0}$$

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(7) Magnetization of materials :- Intensity of magnetization

⇒ Magnetic moment in an atom is mostly due to electrons :-

(i) magnetic moment due to electron revolving around the nucleus.

(ii) Permanent magnetic moment due to spin.

⇒ The magnetic moments of the electrons of an atom have a tendency to cancel in pairs.

⇒ In some cases, the magnetic moment of an atom is not zero. Such an atom may be represented by a magnetic dipole having a permanent magnetic moment.

⇒ Any object of finite size contains a large number of atoms. In general, the magnetic moments of these atoms are randomly oriented and hence there is no net magnetic moment in any volume of the material that contains more than several thousand atoms

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⇒ The alignment is only partial, the degree of ~~att~~ alignment increases is the strength of the applied field is increased and also if the temperature is decreased.

* Magnetic saturation ⇒ When sufficiently strong field is applied, the ~~att~~ alignment is nearly perfect. We then say that the material is magnetically saturated.

* Intensity of magnetization ⇒ It is defined as magnetization

as magnetic moment per unit volume.

$$\boxed{\vec{I} = \frac{\text{magnetic moment}}{\text{Volume}} = \frac{\vec{M}}{V}}$$

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Strength of attraction:-

Ferromagnetic > Paramagnetic

→ Repel weakly in case of paramagnetism.

⑧ Paramagnetism, Ferromagnetism and Diamagnetism

* Paramagnetism ⇒ Paramagnetism is a form of magnetism whereby certain materials are attracted by an externally applied magnetic field.

* Diamagnetism ⇒ Diamagnetic materials creates a magnetic field in opposition to an externally applied magnetic field.

* Ferromagnetism ⇒ Ferromagnetism is the basic mechanism by which certain material form permanent magnets.

⑨ Magnetic Intensity (H)

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{I}}$$ {definition}

{ $\vec{B}$ = Resultant magnetic field
$\vec{I}$ = Intensity of Magnetization }

(10) Magnetic susceptibility

⇒ For paramagnetic and diamagnetic substances, the intensity of magnetization of a material is directly proportional to the magnetic intensity.

$$\boxed{\vec{I} = \chi \cdot \vec{H}}$$

→ The proportionality constant χ is called the susceptibility of the material.

⇒ The materials with positive value of χ are paramagnetic and those with negative value of χ are diamagnetic.

(11) Permeability

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{I} \Rightarrow \boxed{\vec{B} = \mu_0(1+\chi)\vec{H}}$$ {from above equation}

$$\Rightarrow \boxed{\vec{B} = \mu \vec{H}}$$ {where, $\mu = \mu_0(1+\chi)$ is called Permeability of the material}

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⇒ $\boxed{\mu_r = \frac{\mu}{\mu_0} = (1+\chi)}$ is called the relative permeability of the material.

(12) Curie's law

⇒ Curie's law states that far away from saturation, the susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature.

$$\boxed{\chi = \frac{C}{T}}$$ {where, C is Curie Constant}

* Curie point ⇒ The temperature at which ferromagnetic material becomes paramagnetic is called curie point.

⇒ Curie Law in case of ferromagnetic substance after curie point.

$$\boxed{\chi = \frac{C'}{T - T_c}}$$ $\left\{ \begin{array}{l} T_c = \text{Curie point} \\ C' = \text{Constant} \end{array} \right\}$

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(13) Hysteresis ⇒ Hysteresis is the dependence of a system not only on its current environment but also on its past environment.

⇒ The magnetization in a ferromagnetic material not only depends on the magnetic intensity H but also on the ~~penivious~~ Previous history.

(14) Soft iron ⇒ Soft iron are material which are easly magnetized by a magnetizing field but retains only a small magnitization.

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