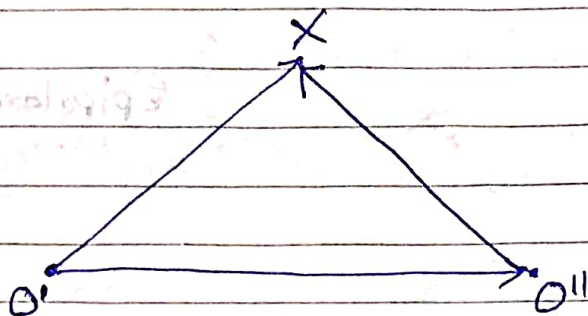


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Page No.

Date: | |

Epipolar Geometry★ Epipolar Geometry - Motivation

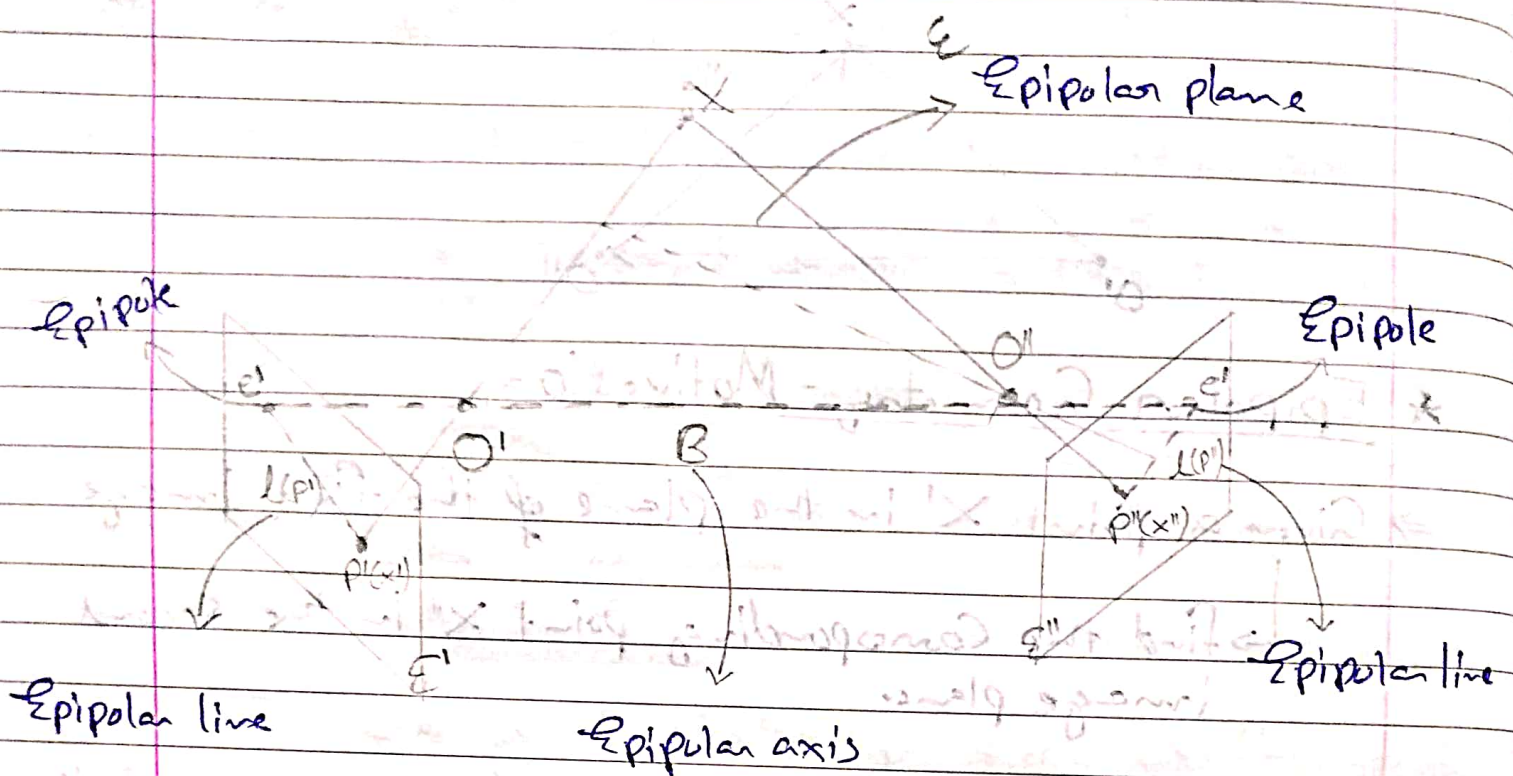
⇒ Given a point X' in the plane of the first image

↳ find the corresponding point X'' in the second image plane.

⇒ Epipolar geometry is used to describe geometric relations in image pairs.

⇒ Enables efficient search for and prediction of corresponding points.

⇒ Given a straight-line preserving mapping, the search space reduces from 2D to 1D.



* Important Elements

- Epipolar axis $\Rightarrow B = (O'O'')$ is the line through the two projection centers.
- Epipolar plane $\Rightarrow E = (O'O''X)$ depends on the projection centers & the point.
- Epipoles $\Rightarrow e' = (O'')'$, $e'' = (O')''$ are the images of the projection centers.
- Epipolar line $\Rightarrow l'(x) = (O''X)'$, $l''(x) = (O'X)''$ are the image of the rays $O''X$ and $O'X$ in the other image respectively.

⇒ Epipoles can also be written as:

$$e' = (o'o'') \wedge E' \quad e'' = (o'o'') \wedge E''$$

⇒ Epipolar lines can be written as:

$$l'(x) = E \wedge E' \quad l''(x) = E \wedge E''$$

★ Predicting the Location of Corresponding points

⇒ Predict the location of x'' given x' .

⇒ The corresponding point x'' lies on the epipolar line $l''(x)$.

★ Computing the Key Elements of the Epipolar Geometry

① Epipolar Axis

$$b = x_{o''} - x_{o'}$$

② Epipolar line

⇒ The image points lie on the epipolar lines.

$$x' \in l' \text{ \& \& } x'' \in l''.$$

$$x'^T l' = 0 \quad \left\{ \text{Condition if } x' \text{ lies of } l' \right\}$$

⇒ We can exploit the Coplanarity Constraint for both points x', x'' .

$$x'^T F x'' = 0$$

$$l' = F x''$$

\Rightarrow The same for the point x''

$$l''^T x'' = 0$$

$$\underline{x'^T F x'' = 0}$$

$$l''^T$$

$$\Rightarrow l''^T = x'^T F$$

$$\Rightarrow \boxed{l'' = F^T x'}$$

* Epipole

$$e' = p' x_{o''}$$

$$e'' = p'' x_{o'}$$

\Rightarrow Epipole is the intersection of all epipolar lines in an image.

$$e'^T l' = 0 \quad \forall l'$$

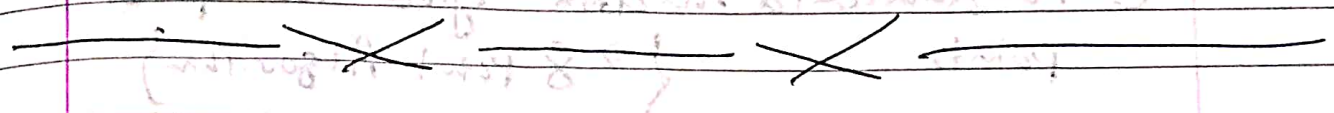
$$e''^T l'' = 0 \quad \forall l''$$

$$\Rightarrow e'^T F x'' = 0$$

\hookrightarrow The epipole is the null space of F^T

$$\text{null}(F^T) = e' \quad \text{null}(F) = e''$$

⇒ The epipoles are the left & right eigenvectors of the fundamental matrix: corresponds to an eigenvalue of zero.



① Fundamental matrix F is a 3×3 matrix of rank 2. It maps points in one image to epipoles in the other image.

② Epipoles are the points in the image plane where the epipolar lines intersect. They are the projections of the other camera's center.

③ Epipoles are not visible in the image.

⇒ Allow us to estimate the camera centers (except for the scale)

Visual odometry can be performed by integrating it into other pose estimation algorithms like SLAM.

