5

Output Controllability

Let the System be described by:-

$$\dot{\bar{x}} = \bar{\bar{A}}\bar{x} + \bar{\bar{B}}\bar{u}$$
$$\bar{y} = \bar{\bar{C}}\bar{x}$$

$\bar{x} \Rightarrow n$-Vector
$\bar{u} \Rightarrow r$-Vector
$\bar{y} \Rightarrow m$-Vector

It is completely output controllable if and only if the composite $m \times nr$ matrix $\bar{P}$, where

$$\bar{P} = [\bar{\bar{C}}\bar{\bar{B}} \,\vdots\, \bar{\bar{C}}\bar{\bar{A}}\bar{\bar{B}} \,\vdots\, \cdots \,\vdots\, \bar{\bar{C}}\bar{\bar{A}}^{n-1}\bar{\bar{B}}]$$

is of rank m.

Notice: Complete state controllability is neither necessary nor sufficient for complete output controllability

⇒ Suppose the System is output controllable and the output $y(t)$ starting from any $y(0)$ the initial output, can be transferred to the origin of the output space in finite ~~ins~~ time interval $0 \leq t \leq T$.

$$\bar{y}(T) = \bar{\bar{C}}\bar{x}(T) = 0$$

$$\bar{x}(t) = e^{\bar{A}t}\left[\bar{x}(0) + \int_0^t e^{-\bar{A}\tau}\bar{B}\bar{u}(\tau)\,d\tau\right]$$

at $t = T$

$$\bar{x}(T) = e^{\bar{A}T}\left[\bar{x}(0) + \int_0^T e^{-\bar{A}\tau}\bar{B}\bar{u}(\tau)\,d\tau\right]$$

So, $\bar{y}(T) = \bar{C}\bar{x}(T) = \bar{C}e^{\bar{A}T}\left[\bar{x}(0) + \int_0^T e^{-\bar{A}\tau}\bar{B}\bar{u}(\tau)\,d\tau\right] = 0$ Let

$$\Rightarrow \bar{C}e^{\bar{A}T}\bar{x}(0) = -\bar{C}e^{\bar{A}T}\int_0^T e^{-\bar{A}\tau}\bar{B}\bar{u}(\tau)\,d\tau$$

$$= -\bar{C}\int_0^T e^{\bar{A}(T-\tau)}\bar{B}\bar{u}(\tau)\,d\tau$$

$$= -\bar{C}\int_0^T e^{\bar{A}\tau}\bar{B}\bar{u}(T-\tau)\,d\tau$$

$$e^{\bar{A}\tau} = \sum_{i=0}^{P-1}\alpha_i(\tau)A^i \quad \left\{P \Rightarrow \text{degree of the minimal Polynomial of } A\right\}$$

$$\bar{C}e^{\bar{A}T}\bar{x}(0) = -\bar{C}\int_0^T \sum_{i=0}^{P-1}\left(\alpha_i(\tau)\bar{A}^i\right)\bar{B}\bar{u}(T-\tau)\,d\tau$$

$$= -\bar{C}\sum_{i=0}^{P-1}\bar{A}^i\bar{B}\int_0^T \alpha_i(\tau)\bar{u}(T-\tau)\,d\tau$$

$$= -\sum_{i=0}^{P-1}\bar{C}\bar{A}^i\bar{B}\int_0^T \alpha_i(\tau)\bar{u}(T-\tau)\,d\tau$$

~~Let $Y_{ij} = \int_0^T \alpha_i(\tau)\,u_j(T-\tau)\,d\tau$~~

~~j^{th} component of vector u~~

~~Let $\bar{B}_j$ be j^{th} column of $\bar{B}$~~

~~So $\bar{C}e^{\bar{A}T}\bar{x}(0) = -\sum_{i=0}^{P-1}\bar{C}\bar{A}^i\sum_{j=1}^{m}Y_{ij}\bar{B}_j$~~

Scanned by CamScanner

Let $\bar{Y}_i = \int_0^T \alpha_i(\tau)\,\bar{u}(T-\tau)\,d\tau$

$$\Rightarrow \bar{C}e^{\bar{A}T}\bar{x}(0) = -\sum_{i=0}^{p-1} \bar{C}\bar{A}^i\bar{B}\,\bar{Y}_i$$

$$= -\left[\bar{C}\bar{B} \,\vdots\, \bar{C}\bar{A}\bar{B} \,\vdots\, \bar{C}\bar{A}^2\bar{B} \,\vdots\, \cdots \,\vdots\, \bar{C}\bar{A}^{p-1}\bar{B}\right] \cdot \begin{bmatrix} \bar{Y}_1 \\ \bar{Y}_2 \\ \vdots \\ \bar{Y}_{p-1} \end{bmatrix}$$

$$\bar{C}e^{\bar{A}T}\bar{x}(0) = -\bar{\bar{Q}}\,\bar{Y}$$

If $\bar{\bar{Q}}$ is of rank m then $\bar{C}e^{\bar{A}T}\bar{x}(0)$ Spans the m-dimensional Output Space.

↓

This means that if the rank of Q is m the $\bar{C}\bar{x}(0)$ also Spans the m-dimensional output Space and the system is Completely output Controllable.