(12)

Date

Page

Loop Shaping

* **Proportional (Static) Compensation**

Choose a proportional controller with transfer function $C(s) = K$.

* **Dynamic compensation**

Choosing a controller (compensator) with transfer function $C(s)$ so that $L(s) = P(s)\,C(s)$ satisfies the requirements.

⇒ The Basic ~~idea~~ approach to designing a dynamic compensator is to choose one or more of the following elements.

→ Gain : K

→ Lead : $\left(\dfrac{\frac{s}{a}+1}{\frac{s}{b}+1}\right)$ with, $0 < a < b$

→ Lag : $\left(\dfrac{\frac{s}{a}+1}{\frac{s}{b}+1}\right)$ with, $0 < b < a$

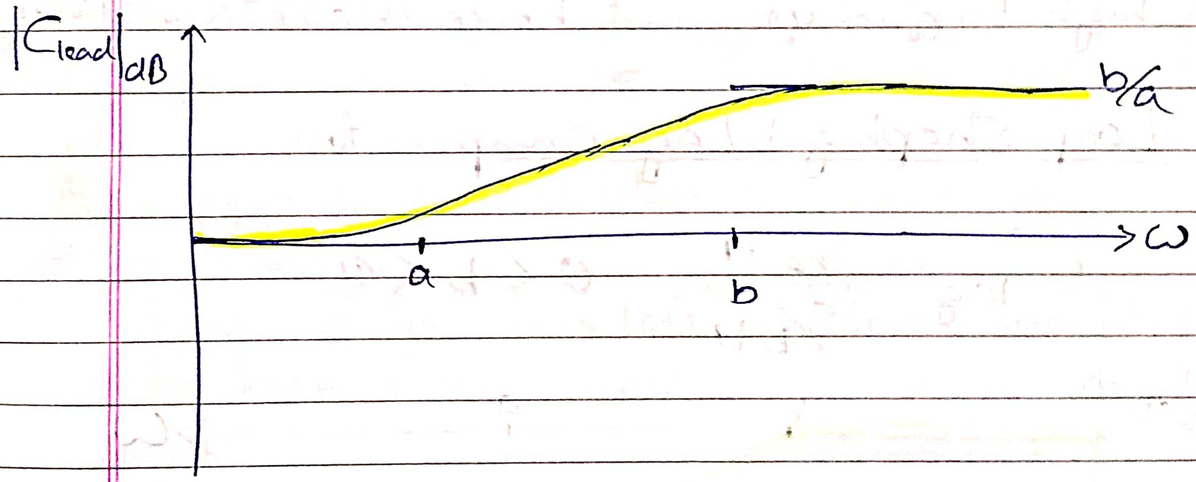
★ Loop Shaping: proportional control

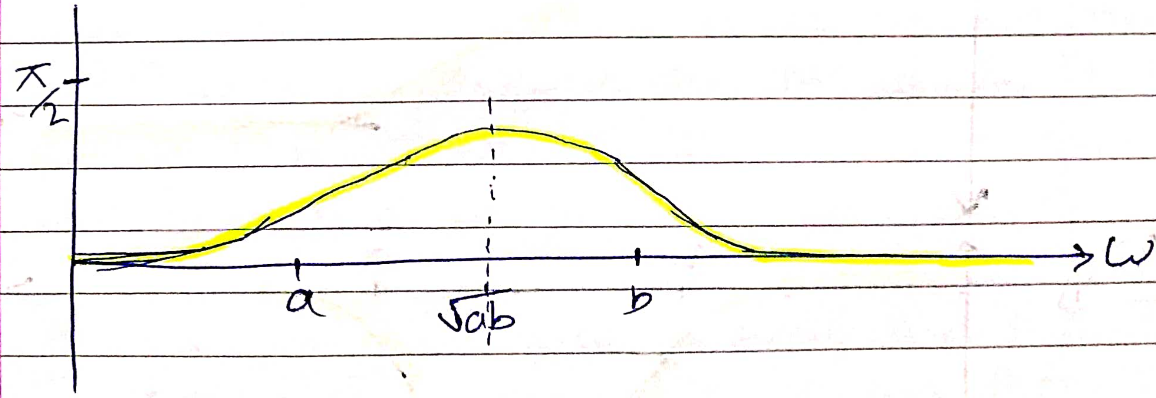
⇒ The effects of proportional control are to shift the magnitude plot of the loop transfer function up and down.

↳ The phase plot is not affected.

★ Loop Shaping: Lead Compensator

$$C_{lead} = \frac{s/a + 1}{s/b + 1} \qquad 0 < a < b$$





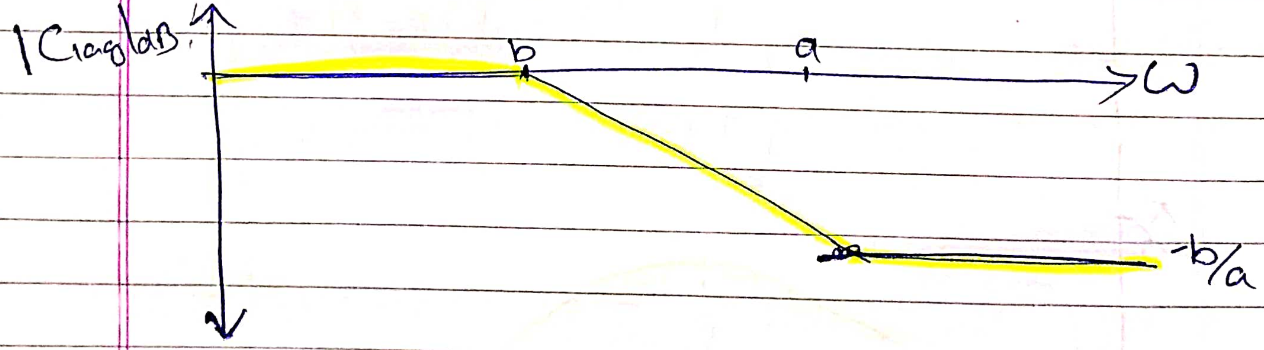
⇒ The typical use of a lead compensator is to increase phase margin.

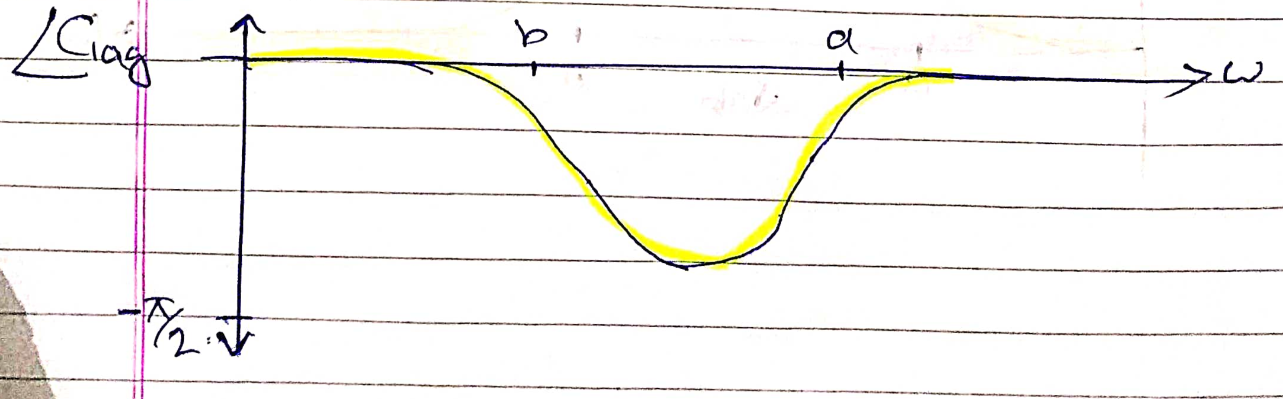
- → Pick $\sqrt{ab}$ at the desired crossover frequency
- → Pick b/a depending on the desired phase increase
- → Adjust K to put the crossover at the desired frequency.

⇒ <u>Possible side effect</u>: Increase magnitude at high frequencies, and hence noise sensitivity.

★ <u>Loop shaping: Lag compensator</u>

$$C_{lag} = \frac{s/a+1}{s/b+1} \qquad 0<b<a$$





Date
Page

⇒ The typical use of a lag compensator is to improve command tracking / disturbance rejection

→ Pick a/b as the desired increase in the magnitude at low frequencies.

→ Pick a so that it is sufficiently small than the crossover frequency, no to affect crossover frequency & phase margin.

→ Increase the gain K by a/b

⇒ Possible side effect: phase lag at low frequencies, potential reduction of phase margin.

★ A general procedure for open-loop stable system

⇒ Proceed from "the left" i.e. from low frequency to high frequencies.

1. Figure out how many integrators are needed in C(s). This depends on the order of the ramp that must be tracked with zero steady state error.

2. Fix the gain in such a way that the low frequency asymptote clears the command-tracking / disturbance specification (low frequency Bode obstacle).

3. Add terms of the form $(\tau s+1)$ at the numerator or denominator in such a way that the Bode magnitude plot intersect the 0dB line with a slope of about 20dB/s.

→ Poles steer "down".
→ Zeros steer "up".

4. Past the crossover, add poles as needed to clear the high frequency Bode obstacle. (noise reduction / uncertainity).

★ <u>Loop shaping for non-minimum-phase / Unstable system</u>

⇒ Factor the plant transfer function as follows:

$$P(s) = P_{mp}(s)\, D(s)$$

Where,

$P_{mp}(s)$ is obtained by replacing all poles / zeros of $P(s)$ in the right half plane with their mirror image wrt the imaginary axis.

$D(s)$ contains all the poles/zeros of $P(s)$ in the right half plane, times the inverse all the mirror image introduced in $P_{mp}(s)$.

→ $|D(j\omega)| = 1 \ \forall \omega$ {all pass filter}

→ Choose the sign of $D(s)$ so that the phase of $D(j\omega)$ is negative.