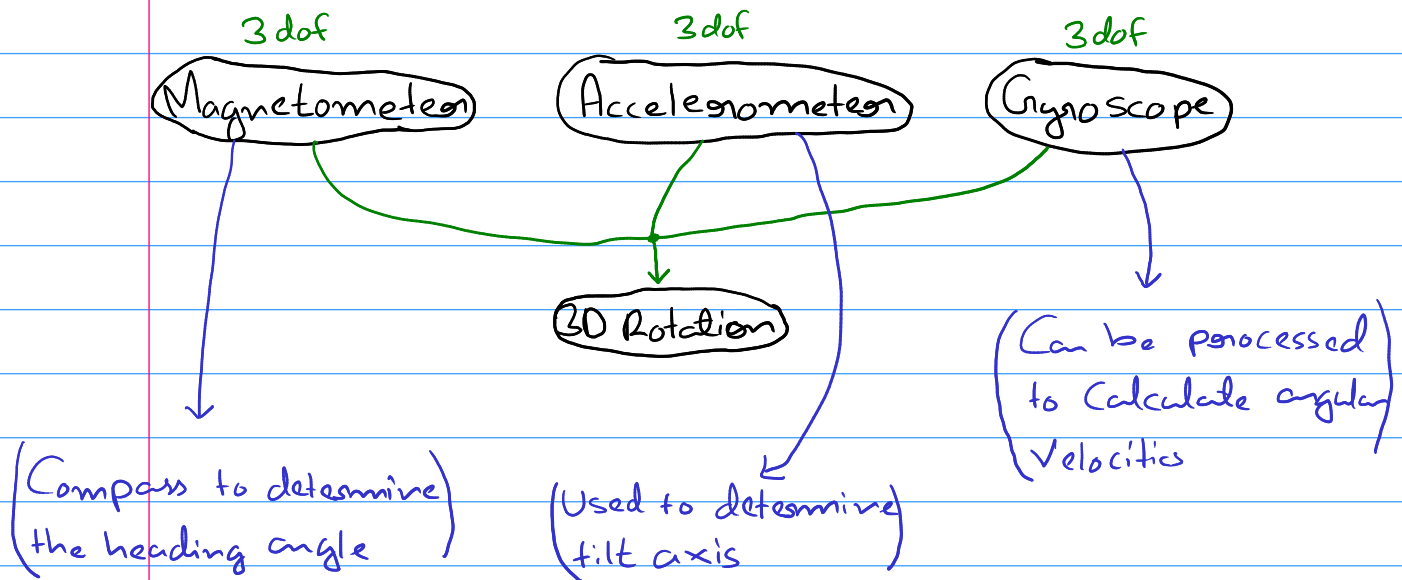


Inertial Filters

Most common filters used are:

- Complementary filter
- Kalman filter



⇒ Each sensor has its own advantage under different static and dynamic scenarios.

1. Accelerometer

⇒ Very accurate in long term.

⇒ Roll and Pitch angles can be calculated precisely when the object is stand still on earth.

⇒ However when the sensor is moving, the moving acceleration will impact the rotation calculation.

↳ However, if we apply a low pass filter to the output of the accelerometer, most of the noise and external force acceleration will be removed.

2. Magnetometer

⇒ Incorrect data when near external magnetic field source.

3. Gyroscope

↳ Yaw can be calculated easily if IMU is flat, but if it is not flat then you need accelerometer data to calculate yaw.

{otherwise good}

⇒ One problem with gyro angle estimation is that its calculation drift over time because of integration.

⇒ In addition to that, due to inertia, the gyro scale won't come back to zero when object is standstill state.

⇒ Gyro is very accurate in short term.

⇒ People usually apply highpass filter to gyroscope measurement.

★ Calculations

$\theta \rightarrow$ Rotation about x (Roll)

$\phi \rightarrow$ Rotation about y (Pitch)

$\psi \rightarrow$ Rotation about z (Yaw)

$$\tan(\theta) = \frac{a_y}{a_z}$$

$$\tan(\phi) = -\frac{a_x}{\sqrt{(a_y^2 + a_z^2)}} \quad \{ \text{Accelerometer} \}$$

$$M_x = m_x \cdot \cos(\phi) + m_z \cdot \sin(\phi)$$

$$M_y = m_x \cdot \sin(\theta) \cdot \sin(\phi) + m_y \cdot \cos(\theta) - m_z \cdot \sin(\theta) \cdot \cos(\phi)$$

$$\psi = \tan^{-1} \frac{M_y}{M_x}$$

{Magnetometer
with accelerometer
based tilt correction}

$$\theta = \omega_\theta * \Delta t$$

$$\phi = \omega_\phi * \Delta t$$

$$\psi = \omega_\psi * \Delta t$$

{Gyroscope}

★ Fusion method

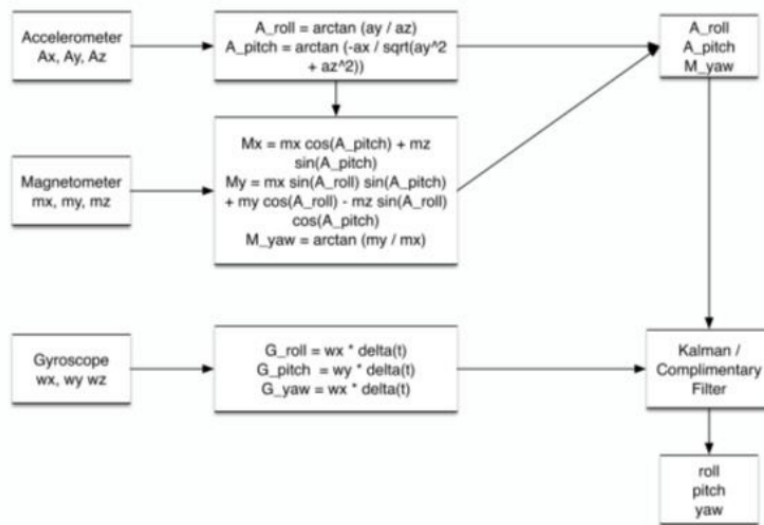


Figure 9: Procedure's block diagram

1. Complimentary Filter

⇒ It basically take advantage of each sensor and compensate for the disadvantages the other sensor have.

⇒ Accelerometer performs best with low frequency while gyroscope performs the best with high frequency.

⇒ Complimentary filter is simple and easy to use, it contain a fixed value for low pass and high pass weights.

$$CF_angle = HP_weight * (CF_angle + gyro_data * \Delta t) + LP_weight * acc_data$$

$$\{ \text{Where } HP_weight + LP_weight = 1 \}$$

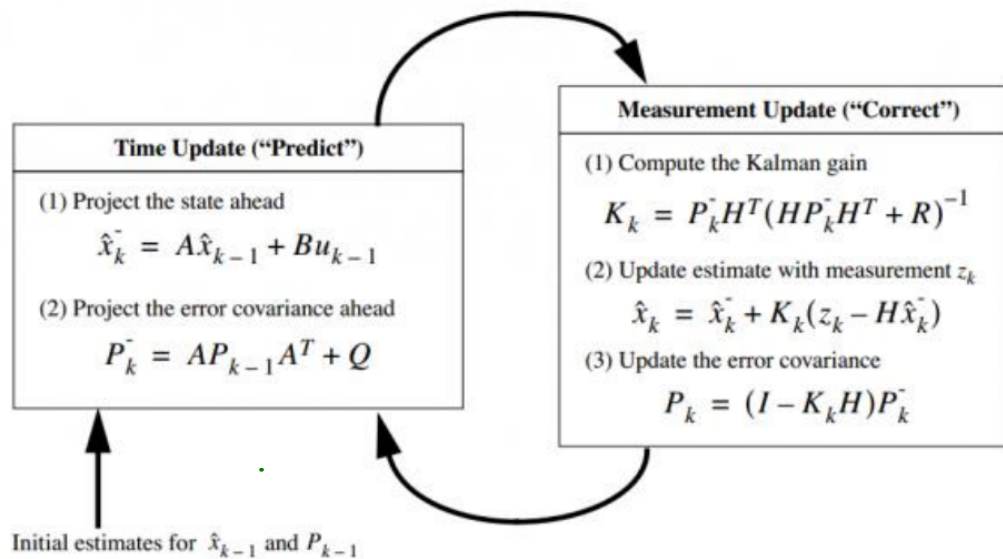
{ HP_weight should be very close to 1 and LP_weight should be very close to 0.

$$\{ \text{Ex: } HP_weight = 0.98 \quad LP_weight = 0.02$$

{ This is actually accelerometer + Magnetometer data

★ Kalman Filter

- ⇒ Kalman filter takes into account a series of measurements over time, and dynamically calculate and update the weight (Kalman gain).
- ⇒ Kalman filter consists of two main steps:
- ① Prediction.
 - ② Correction.
- ⇒ The three roll, pitch and yaw angle are independent; therefore each α_i could be tracked separately.



- ⇒ This is a recursive state estimation filter which takes inputs:

$$x_{k-1} = \begin{bmatrix} \theta_k \\ \dot{\theta}_k \end{bmatrix} \begin{matrix} \rightarrow \text{angle} \\ \rightarrow \text{bias angular velocity.} \end{matrix} \quad \{ \text{Mean} \}$$

$$P_{k-1} = \begin{bmatrix} P_{0,0} & P_{0,1} \\ P_{1,0} & P_{1,1} \end{bmatrix} \rightarrow \underline{\text{Covariance}} \text{ of } x_{k-1}$$

$$u_{k-1} = \dot{\theta} \rightarrow \text{angular velocity.}$$

$Z_k = \theta_k^2 \rightarrow$ angle from sensor measurement

\Rightarrow Model parameters:

$$A = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

\Rightarrow Noise parameters:

$$Q = \text{Covariance of process noise} = \begin{bmatrix} Q_\theta & 0 \\ 0 & \dot{Q}_b \end{bmatrix}$$

$$R = \text{Covariance of observation noise} = R_{\text{measure}}$$

\Rightarrow Typical noise parameters:

$$\theta_0 = 0.001$$

$$\dot{\theta}_b = 0.003$$

$$R_{\text{measure}} = 0.03$$

★ Experimental Results

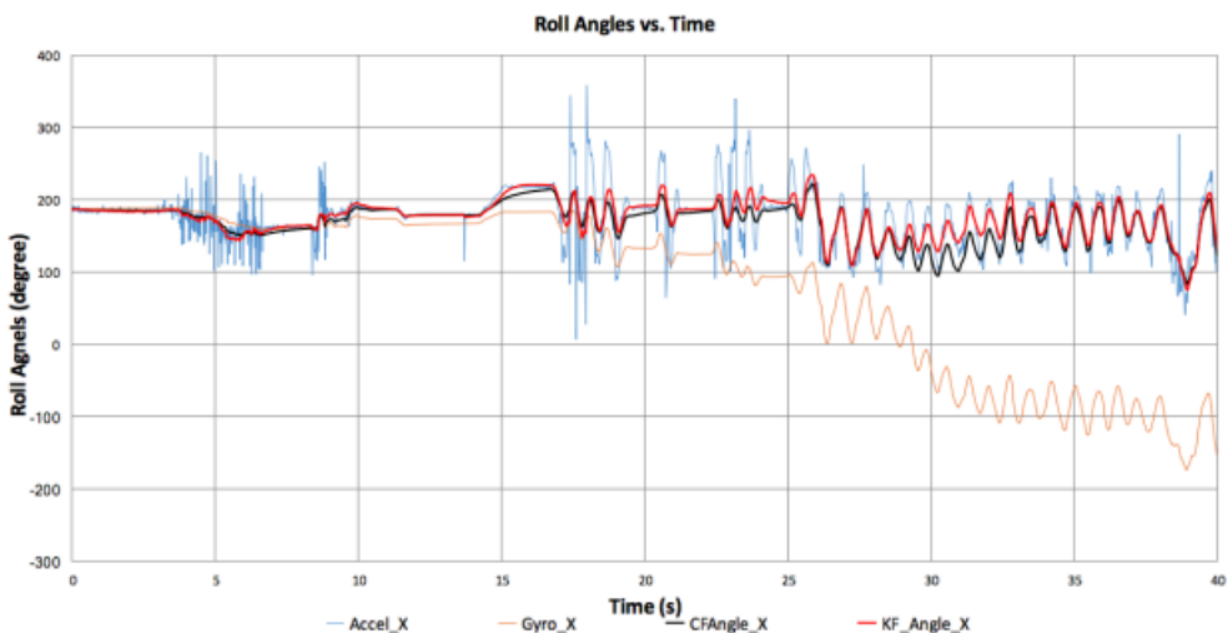


Figure 11: Roll Angles vs. Time

★ Imu data sheet

0. Common

⇒ Size of output data (ie. 16 bit / 8 bit output)
↳ This is resolution of output data.

1. Gyroscope

DPS (Degree per sec)

↳ Value on full scale deflection will be given. (Example ± 2000 dps)

temperature range

{ Example -40°C to $+85^{\circ}\text{C}$ }

data output rate

{ Example 50 Hz }

2. Accelerometer

Range

↳ Maximum possible value to measure { Ex: $\pm 8g$ }
↳ Earth's gravitational acceleration

3. Magnetometer

Range

↳ Maximum possible value to measure { Ex: ± 8.1 gauss }