Bayes Filters and Related models

$$P(x \mid z, u)$$

State

Control

Observation

$$bel(x_t) = P(x_t \mid z_{1:t}, u_{1:t})$$

$$= P(x_t \mid z_t, z_{1:t-1}, u_{1:t})$$

$$= \frac{P(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, P(x_t \mid z_{1:t-1}, u_{1:t})}{P(z_t)}$$

$$\Rightarrow \eta\, P(z_t \mid x_t, z_{1:t-1}, u_{1:t})\, P(x_t \mid z_{1:t-1}, u_{1:t})$$

$$\Rightarrow \eta\, P(z_t \mid x_t)\, P(x_t \mid z_{1:t-1}, u_{1:t})$$

{Markov assumptions}

$$\int_{x_{t-1}} P(x_{t-1} \mid z_{1:t-1}, u_{1:t})\, P(x_t \mid x_{t-1}, z_{1:t-1}, u_{1:t})\, dx_{t-1}$$

$$P(x_{t-1} \mid z_{1:t-1}, u_{1:t-1}) \qquad P(x_t \mid x_{t-1}, u_t)$$

$$bel(x_t) = \eta\, P(z_t \mid x_t) \int_{x_{t-1}} P(x_t \mid x_{t-1}, u_t)\, bel(x_{t-1})\, dx_{t-1}$$

① Prediction Step

Motion model

$$\overline{bel}(x_t) = \int P(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$$

② Correction Step

$$bel(x_t) = \eta\, P(z_t \mid x_t)\, \overline{bel}(x_t)$$

Observation model