(c) Diagonal Canonical form

Consider the transfer-function of a System defined by:-

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{(s+P_1)(s+P_2) \cdots (s+P_n)} = b_0 + \frac{C_1}{s+P_1} + \frac{C_2}{s+P_2} + \cdots \frac{C_n}{s+P_n} \quad \text{--- (1)}$$

where $P_i \neq P_j$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} -P_1 & & & 0 \\ & -P_2 & & \\ & & \ddots & \\ 0 & & & -P_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u \quad \text{--- (2)}$$

$$y = \begin{bmatrix} C_1 & C_2 & \cdots & C_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u \quad \text{--- (3)}$$

Solution

$$Y(s) = b_0 U(s) + \frac{C_1}{s+P_1} U(s) + \frac{C_2}{s+P_2} U(s) + \cdots \frac{C_n}{s+P_n} U(s)$$

Lets define state variable as follows:

$$X_1(s) = \frac{U(s)}{s+P_1}$$

$$X_2(s) = \frac{1}{s+P_2} U(s)$$

$$\vdots \qquad \vdots$$

$$X_n(s) = \frac{1}{s+P_n} U(s)$$

⇒ The above equations may be re-written as :-

$$sX_1(s) = -P_1 X_1(s) + U(s)$$
$$sX_2(s) = -P_2 X_2(s) + U(s)$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$sX_n(s) = -P_n X_n(s) + U(s)$$

$\Longrightarrow$ {Time domain}

$$\dot{x}_1 = -P_1 x_1 + u$$
$$\dot{x}_2 = -P_2 x_2 + u$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$
$$\dot{x}_n = -P_n x_n + u$$

{from this we can get eq 2 (first part of the solution)}

$$\Rightarrow Y(s) = b_0 U(s) + C_1 X_1(s) + C_2 X_2(s) + \cdots C_n X_n(s)$$

$\Downarrow$ {Time domain}

$$\boxed{y = b_0 u + C_1 x_1 + C_2 x_2 + \cdots C_n x_n}$$

{from this we can get eq 3 (second part of solution)}