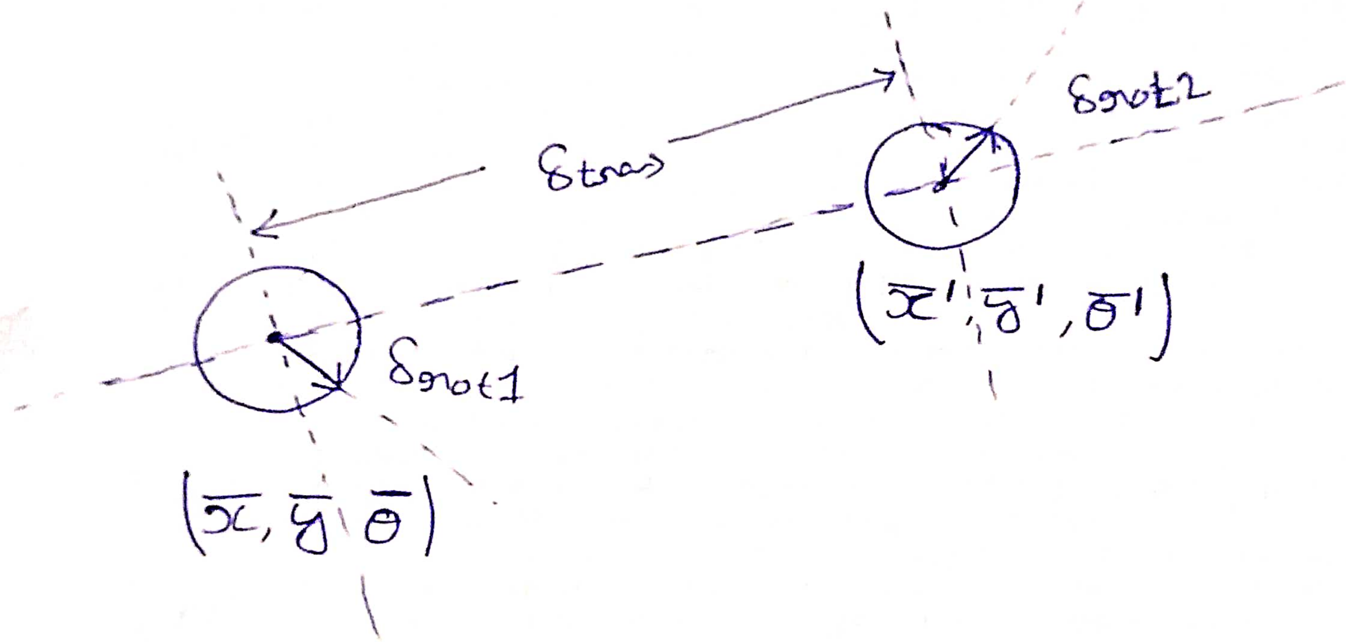
Kalman Filter

1 Motion model (Odometry based)

$p(x_t \mid u_t, x_{t-1})$



Odometry Information $u = (\delta_{rot1}, \delta_{trans}, \delta_{rot2})$

$$\delta_{trans} = \sqrt{(\bar{x}' - \bar{x})^2 + (\bar{y}' - \bar{y})^2}$$

$$\delta_{rot1} = \operatorname{atan2}(\bar{y}' - \bar{y}, \bar{x}' - \bar{x}) - \bar{\theta}$$

$$\delta_{rot2} = \bar{\theta}' - \bar{\theta} - \delta_{rot1}$$

2 Sensor model (Landmarks with Range-Bearing Sensors)

$P(Z_t \mid x_t)$

Range-bearing: $Z_t^i = (r_t^i, \phi_t^i)^T$

Robot's pose: $(x, y, \theta)^T$

Observation of feature j at location $(m_{jx}, m_{jy})^T$

$$\begin{pmatrix} r_t^i \\ \theta_t^i \end{pmatrix} = \begin{pmatrix} \sqrt{(m_{jx} - x)^2 + (m_{jy} - y)^2} \\ \operatorname{atan2}(m_{jy} - y, m_{jx} - x) - \theta \end{pmatrix} + Q_t$$

★ **Assumptions**

→ Models are Linear.
→ Distributions are Gaussian

{EKF is optimal estimator in this case}

★ **Gaussian distribution**

$$P(x) = \det(2\pi\Sigma)^{-1/2} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

Mean

Covariance

Porperties

Given $x = \begin{pmatrix} x_a \\ x_b \end{pmatrix}$ $P(x) = N$

$\Rightarrow$ The marginal are Gaussians

$$P(x_a) = N \quad P(x_b) = N$$

$\rightarrow$ as well as the conditional

$$P(x_a \mid x_b) = N \quad P(x_b \mid x_a) = N$$

Marginalization

$\Rightarrow$ Given $P(x) = P(x_a, x_b) = N(\mu, \Sigma)$

with $\mu = \begin{pmatrix} \mu_a \\ \mu_b \end{pmatrix}$ $\Sigma = \begin{pmatrix} \Sigma_{aa} & \Sigma_{ab} \\ \Sigma_{ba} & \Sigma_{bb} \end{pmatrix}$

$\Rightarrow$ The marginal distribution is

$$P(x_a) = \int P(x_a, x_b)\, dx_b = N(\mu, \Sigma)$$

with $\underline{\underline{\mu = \mu_a}}$, $\underline{\underline{\Sigma = \Sigma_{aa}}}$

Conditioning

$\Rightarrow$ The conditional distribution is

$$P(x_a \mid x_b) = \frac{P(x_a, x_b)}{P(x_b)} = N(\mu, \Sigma)$$

$$\boxed{\begin{aligned} \mu &= \mu_a + \Sigma_{ab} \Sigma_{bb}^{-1} (b - \mu_b) \\ \Sigma &= \Sigma_{aa} - \Sigma_{ab} \Sigma_{bb}^{-1} \Sigma_{ba} \end{aligned}}$$

★ Linear model

$$x_t = A_t x_{t-1} + B_t u_t + \epsilon_t$$

$$Z_t = C_t x_t + \delta_t$$

mean $= 0$
Covariance $= R_t$

mean $= 0$
Covariance $= Q_t$

① Linear motion model

$$p(x_t \mid u_t, x_{t-1})$$

$$= \det(2\pi R_t)^{-1} \exp\left(-\frac{1}{2}(x_t - A_t x_{t-1} - B_t u_t)^T R_t^{-1} (x_t - A_t x_{t-1} - B_t u_t)\right)$$

② Linear observation model

$$p(z_t \mid x_t)$$

$$= \det(2\pi Q_t)^{-1/2} \exp\left(-\frac{1}{2}(z_t - C_t x_t)^T Q_t^{-1} (z_t - C_t x_t)\right)$$

★ Kalman Filter Algorithm

1. Kalman_filter ($\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$)

2. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$

3. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$

4. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$

5. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$

6. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$

7. return μ_t, Σ_t