Modeling and Estimating the Odometry Error of a mobile robot

1D

⇒ This paper presents both:

① Error modeling of an odometry system.

② Possible procedure in order to evaluate it.

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→ Determining the odometry errors of a mobile robot is very important for:

① In order to reduce it

② In order to know the accuracy of the state configuration estimated by using encoder data.

⇒ Odometry data is actually inaccurate since the error in the position estimation integrates over the path.

↳ However the encoder data are used extensively in the localization process by fusing these data with data comming from another sensor.

⇒ Clearly any fusion architecture needs to know the accuracy of the estimate of each sensor in order to weigh all the data in a proper manner.

⇒ Accuracy is completely described by the odometry error covariance matrix Q.

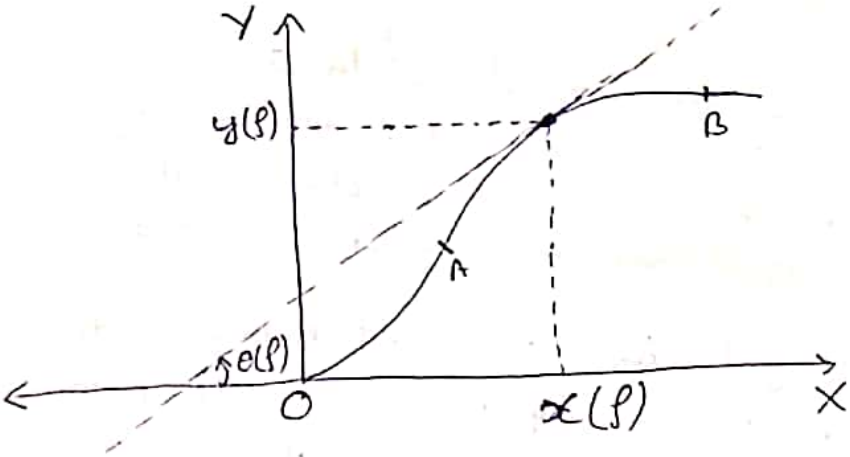
⇒ Odometry error can be both Systematic and non-Systematic.

Systematic Error → depends only on the mobile robot independently of the environment where the robot moves.

Non-Systematic Error → depends on the environment and drastically cango by changing environment.

★ The Odometry error model

⇒ Assuming a two-dimensional world, we can define the robot configuration with respect to a world coordinate frame W by a vector $X = [x, y, \theta]^T$



⇒ Robot configuration X is parameterized with respect to the curve length ρ.

⇒ Given two point A and B we have
$\hat{\delta}\rho = \rho_B - \rho_A$ && $\hat{\delta}\theta = \theta(\rho_B) - \theta(\rho_A)$

⇒ We assume that both Systematic and non-systematic errors only depends on $\hat{\delta\rho}$ and do not depend on $\hat{\delta\theta}$.

↳ In case of differential drive, this approximation holds more is trajectory is smooth.

⇒ Because of both systematic and non-systematic errors the encoder measurements $\overline{\delta\rho}$ and $\overline{\delta\theta}$ differ from real values.

$$\hat{\delta\rho} = \overline{\delta\rho} + E_T\overline{\delta\rho} + \delta\rho \quad \text{—— (1)}$$

$$\hat{\delta\theta} = \overline{\delta\theta} + E_R\overline{\delta\rho} + \delta\theta \quad \text{—— (2)}$$

↳ Non-Systematic Components

↳ Systematic Components

⇒ Actually the systematic component is proportional to the real distance traveled by the robot :-

$$\hat{\delta\rho} = \overline{\delta\rho} + E'_T\hat{\delta\rho} + \delta'\rho \quad \text{—— (3)}$$

$$\hat{\delta\theta} = \overline{\delta\theta} + E'_R\hat{\delta\rho} + \delta'\theta \quad \text{—— (4)}$$

⇒ Let us consider a general robot motion.

↳ We approximate the trajectory with N small segments.

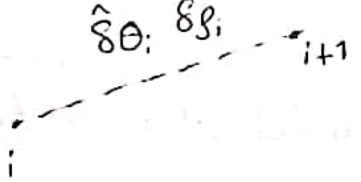
⇒ <u>Assumptions</u>:

① Robot moves in straight line along each given segment whose length, measured by the <u>encoder</u> is always the same value. $\overline{\delta\rho} = \overline{\rho}/N$.

② The angle $\hat{\delta\theta}_i$ between the orientations related to the $(i+1)^{th}$ and the i^{th} segment and the translation $\hat{\delta\rho}_i$ covered during the same step are gaussian random variables.

③ The random variable $\hat{\delta\rho}_i$ is independent of the random variable $\delta\theta_i$. Moreover $\hat{\delta\rho}_i$ is independent of $\hat{\delta\rho}_j$ $(i \neq j)$ and $\hat{\delta\theta}_i$ is independent of $\hat{\delta\theta}_j$.

⇒ Let consider the i^{th} segment.



$$\hat{\delta\rho}_i \sim N\left(\overline{\delta\rho}(1+E_T), \sigma^2_{\delta\rho}\right)$$

$$\hat{\delta\theta}_i \sim N\left(\overline{\delta\theta}_i + E_R\overline{\delta\rho}, \sigma^2_{\delta\theta}\right)$$

{Directly depends on rolling conditions}

⇒ The actual orientation after the ith step is:

$$\hat{\Theta}_i = \underbrace{\Theta_0}_{\{\text{Initial orientation}\}} + \underbrace{\overline{\Delta\Theta_i}}_{\{\text{Change in orientation measured by encoder}\}} + i E_R \overline{\delta\rho} + \sum_{j=1}^{i} \delta\Theta_j$$

Let $\tilde{\Theta}_i = \Theta_0 + \overline{\Delta\Theta_i} + i E_R \overline{\delta\rho}$

and
$$\Delta\Theta_i = \sum_{j=1}^{i} \delta\Theta_j$$

we obtain,
$$\hat{\Theta}_i = \tilde{\Theta}_i + \Delta\Theta_i$$

⇒ where $\Delta\Theta_i$ is still a random variable satisfying the following relation.
$$\Delta\Theta_i \sim N(0, i\,\sigma^2_{\delta\theta})$$

⇒ We introduce now two new parameters (K_θ and K_ρ) in order to characterize the non-systematic components, namely the two variance $\sigma^2_{\delta\theta}$ and $\sigma^2_{\delta\rho}$.

⇒ From the definition of $\overline{\delta\rho} = \frac{\rho}{N}$ we can write:
$$\sigma^2_{\theta} = N\sigma^2_{\delta\theta} = \rho\,\frac{\sigma^2_{\delta\theta}}{\overline{\delta\rho}}$$

$$K_\theta = \lim_{N \to \infty} \frac{\sigma^2_{\delta\theta}}{\delta\rho}$$

we therefor have,

$$\delta\theta^2 = K_\theta \bar{\rho}$$

⇒ In the same way we compute the variance σ_ρ^2 obtaining:

$$\sigma_\rho^2 = K_\rho \bar{\rho}$$

where

$$K_\rho = \lim_{N \to \infty} \frac{\sigma^2_{\delta\rho}}{\delta\rho}$$

★ The covariance matrix Q

⇒ The non-systematic error are expressed in terms of the covariance matrix Q.

⇒ The robot configuration x is a random vector whose average value $\langle x \rangle$ is given by the odometory measurements.

⇒ The covariance matrix Q is defined as follows:

$$Q = E\{[x - \langle x \rangle][x - \langle x \rangle]^T\}$$

$$= \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{xy} & \sigma_y^2 & \sigma_{y\theta} \\ \sigma_{x\theta} & \sigma_{y\theta} & \sigma_\theta^2 \end{bmatrix}$$

$\Rightarrow$ The covariance matrix can be represented as a function of the previous parameters (E_R, E_T, K_θ and K_ρ), which can be determined experimentally.

the

(ii) $\underline{\sigma_x^2}$

$$\boxed{\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2}$$

$$x = \lim_{N\to\infty} \sum_{i=1}^{N} \hat{\delta}_{\rho_i} \cos(\hat{\theta}_i).$$

$$\langle x_N \rangle = \int d\hat{\delta}_{\rho_1} \cdots d\hat{\delta}_{\rho_N}\, d\hat{\delta}_{\theta_1} \cdots d\hat{\delta}_{\theta_N}$$
$$f_G(\hat{\delta}_{\rho_1}, \sigma_{\delta\rho}) \cdots f_G(\hat{\delta}_{\rho_N}, \sigma_{\delta\rho})$$
$$f_G(\hat{\delta}_{\theta_1}, \sigma_{\delta\theta}) \cdots f_G(\hat{\delta}_{\theta_N}, \sigma_{\delta\theta})$$
$$\sum_{i=1}^{N} \hat{\delta}_{\rho_i} \cos(\hat{\theta}_i)$$

passed

Q.

$\Rightarrow$ By direct calculation we obtain:

$$\langle x_N \rangle = \bar{\delta}_\rho (1+E_T) \sum_{i=1}^{N} \cos(\tilde{\theta}_i)\, e^{-\frac{i\sigma_{\delta\theta}^2}{2}}$$

$\Rightarrow$ When $N \to \infty$

$$\langle x \rangle = (1+E_T) \int_0^{\bar{\rho}} \cos(\tilde{\theta}(s))\, e^{-\frac{K_\theta s}{2}}\, ds$$

⇒ A little bit more troblesome is the computation of the second order product average $\langle x^2 \rangle$. By a direct calculation we obtain:

$$\langle x^2 \rangle = (1+E_T)^2 \int_0^{\bar{\rho}} ds \int_0^{\bar{\rho}-s} ds' \Big\{ e^{-K_0 s'} * \Big[(1+XC(s)) \cos[\tilde{\theta}(s+s') - \tilde{\theta}(s)] - XS(s) \sin[\tilde{\theta}(s+s') - \tilde{\theta}(s)] \Big\} + \frac{K_\rho}{2} \Big[\bar{\rho} + \int_0^{\bar{\rho}} XC(s)\, ds \Big]$$

where,

$$XC(s) = \cos[2\tilde{\theta}(s)]\, e^{-2K_0 s}$$

$$XS(s) = \sin[2\tilde{\theta}(s)]\, e^{-2K_0 s}$$

⇒ Similarly for all other terms.

[Reff the original paper]
