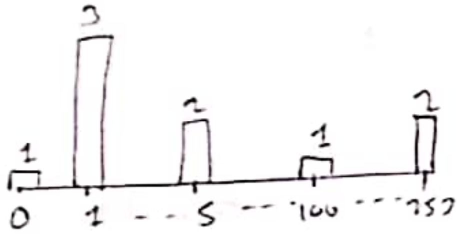
4a

Histogram and Point Operators

★ Histogram

1	5	255
1	100	255
5	1	0

Gray Scale Image



Histogram

⇒ A histogram is a frequently used tool for analyzing images.

$h(g) \Rightarrow$ Number of pixel that take value g.

0 ---- 255

⇒ Probability that a random pixel has value g.

$$P(g) = \frac{h(g)}{N} \qquad \{ N = I \times J)$$

⇒ Histogram can be calculated with linear complexity. $O(N)$

★ Cumulative Histogram

$$\boxed{H(g) = \sum_{x=0}^{g} h(x)} = \text{Number of pixel with } g(i,j) \leq g$$

$$\boxed{P(g) = \frac{h(g)}{N}} \qquad \boxed{F(g) = \frac{H(g)}{N}}$$

Mean, Variance, Median of histogram

⇒ All three values are a characteristic of the image

⇒ Mean describes the brightness.

⇒ Variance describes contrast.

⇒ Median is a robust description of the brightness.

$$\mu_g = \frac{1}{N}\sum_i \sum_j g(i,j) \longrightarrow O(N)$$

$$\mu_g = \frac{1}{N}\sum_g g\,h(g) \longrightarrow O(1)$$

$$\sigma_g^2 = \frac{1}{N-1}\sum_i \sum_j (g(i,j)-\mu_g)^2 \longrightarrow O(N)$$

$$\sigma_g^2 = \frac{1}{N-1}\sum_g (g-\mu_g)^2 h(g) \longrightarrow O(1)$$

$$\boxed{med(g) = F^{-1}(0.5)}$$

Types of Operators

① Global Operator

② Local operator

③ Point operator

★ Point Operator

⇒ The point operator maps the pixel value to a new value only based on the value and the location of the input pixel in the image.

$$\underbrace{b(i,j)}_{\text{Output}} = \underbrace{f}_{\text{Operator}}(\underbrace{a(i,j)}_{\text{Input}}, \underbrace{P}_{\text{Parameter}})$$

★ Linear Transformation

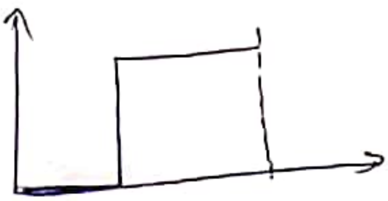
$$\boxed{b(i,j) = K + m\,a(i,j)} \qquad \{ P = [K, m]^T \}$$

$$\boxed{\mu_b = K + m\mu_a} \qquad \boxed{\sigma_b = |m|\sigma_a}$$

★ Nonlinear Transformations

⇒ Example: thresholding

$$b(a) = \begin{cases} b_0 & \text{if } a < T \\ b_1 & \text{otherwise} \end{cases}$$

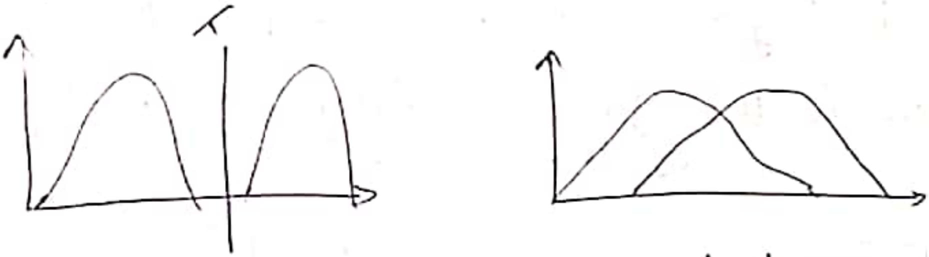


{Binary Image}

Application

⇒ Foreground/background separation.

★ Foreground and Background Separation



⇒ Often, the ratio is unknown but we might can assume a bi-modal distribution of gray values.

⇒ We can use a sum of two Gaussians to represent the bi-modal distribution.

$$P(g) = \alpha\, G(g : \mu_1\ \sigma_1^2) + (1-\alpha)\, G(g : \mu_2\ \sigma_2^2)$$

⇒ The parameters α, μ_1, σ_1, μ_2, σ_2 are estimated from the histogram.

4b

Histogram and Point Operation

★ How do the Operators Affect the Histogram of the Image?

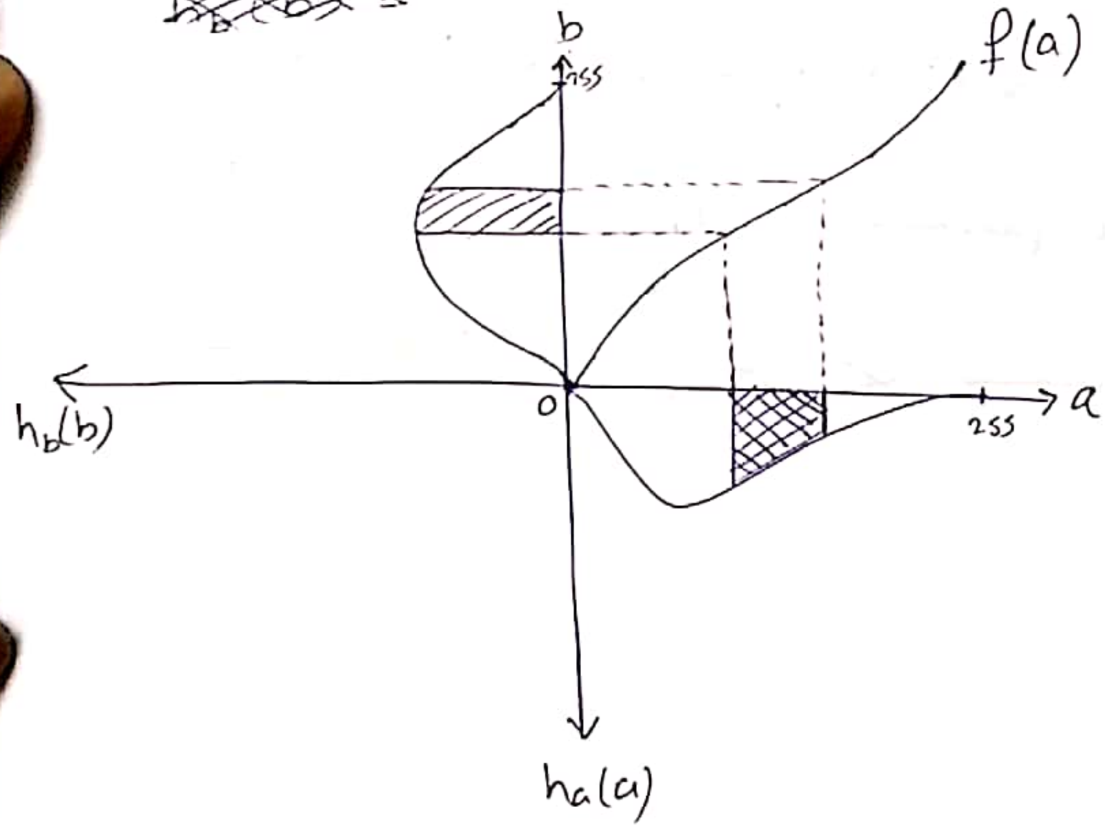
→ Monotonous function $b = f(a)$

→ Histogram of image $a(i,j)$ is $h_a(a)$

→ Compute the histogram $h_b(b)$ of image b.

Goal

~~$h_b(b) =$~~



⇒ (cross-hatched) and (hatched) should have same areas as function is monotonic.

→ If slop of $f(a)$ is >1, histogram of b is being stretched and if slop of $f(a)$ <1, then histogram of b is being compressed.

$h_a(a)\,da = h_b(b)\,db$

$$\boxed{h_b(b) = \frac{h_a(a)}{\left|\frac{db}{da}\right|} = \frac{h_a(a)}{|f'(a)|}}$$

$$\boxed{h_b(b) = \frac{h_a(f^{-1}(b))}{|f'(f^{-1}(b))|}}$$

★ Histogram Equalization

⇒ For this to happen,

$$h_b(b) = \text{Const} = \frac{h_a(a)}{\left|\frac{db}{da}\right|}$$

let $$\frac{1}{K} = \frac{h_a(a)}{\left|\frac{db}{da}\right|}$$

$$\Rightarrow \quad db = K h_a(a)\,da$$

$$\Rightarrow \quad \boxed{b = K \underbrace{\int_{\alpha=0}^{\alpha=a} h_a(\alpha)\,d\alpha}_{H(a)\ \{\text{Cummulative histogram}\}} + C}$$

$$\Rightarrow \quad \boxed{b(a) = K H_a(a) + C}$$

$b(0) = 0$

$b(255) = 255$ ⟶ $\boxed{K = \frac{255}{N - H(0)}}$ $\boxed{C = -H(0)\frac{255}{N - H(0)}}$

$$\boxed{b(a) = \frac{255}{N - H(0)} (H(a) - H(0))}$$

★ Noise Variance Equalization

→ Goal: adjust the variance of the intensities to a fixed value.

→ Useful for statistical analysis of images
→ Realized by a monotonic point operator.

$$\sigma_a^2 = ma$$

Goal: $\sigma_b = \sigma_0$ ∀ all intensities

⟹ Variance propagation yields

$$\sigma_b = \left|\frac{db}{da}\right| \sigma_a = \sigma_0$$

$$\Rightarrow \frac{db}{da} \sigma_a = \sigma_0 \quad \{\text{Monotonic function}\}$$

$$db = \frac{\sigma_0}{\sigma_a} da = \frac{\sigma_0}{\sqrt{m}} a^{-1/2} da \quad \{\text{as } \sigma_a^2 = ma\}$$

$$\Rightarrow \boxed{b = \frac{2\sigma_0}{\sqrt{m}} \sqrt{a} + C}$$

$$\Rightarrow a = 0 \longrightarrow b = 0$$
$$a = 255 \longrightarrow b = 255$$

$$\Rightarrow C = 0$$

$$\sigma_0 = \frac{255}{\sqrt{255}} \frac{\sqrt{m}}{2} = \frac{1}{2}\sqrt{255m}$$

* Point Operation on Multiple Image

$\Rightarrow b = f(a_1, \ldots a_m)$

* Often two input images

$$b = f(a_1, a_2)$$

* Masking

$$b(i,j) = \begin{cases} a_2(i,j) & \text{if } a_1(i,j) = 1 \\ 0 & \text{otherwise} \end{cases}$$

* Montage of Image

$$b(i,j) = \begin{cases} a_2(i,j) & \text{if } a_1(i,j) = 1 \\ a_3(i,j) & \text{otherwise} \end{cases}$$

* Point Operators

$\Rightarrow$ Point operators can be efficiently computed using look-up table.