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Camera Calibration: Zhang's Method

$$\boxed{x = PX}$$

⇒ This time we only want the intrinsics.

⇒ Zhang's method for camera calibration uses a checkerboard pattern.
↳ of known size & structure.

⇒ Trick for checkerboard calibration
↳ Set the world coordinate system to the corner of the checkerboard.
↳ All points on the checkerboard lie in the XY plane, i.e. $Z=0$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

⇓

{Zero for all Point on checkerboard}

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

{Every point on checkerboard generates such equation}

Let $H = [h_1\ h_2\ h_3] = K\ [r_1, r_2, t]$

$$K = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \qquad [r_1, r_2, t] = \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}$$

⇒ For multiple observed points on the checkerboard (in the same image) we obtain

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \underset{3\times3}{H} \begin{bmatrix} X_i \\ Y_i \\ 1 \end{bmatrix} \qquad i = 1, \cdots I$$

{3x3 homography instead of 3x4 projection matrix}

{Similar steps as in DLT}

$$a_x^T h = 0$$
$$a_y^T h = 0$$

{h is vectorized H}

⇒ We need to identify at least 4 points as H has 8 DoF and each point consists of 2 observation (x & y)

⇒ We can solve the system of equation using SVD and obtain H.

★ Computing K Given H

$$H = [h_1\ h_2\ h_3] = \underbrace{\begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{\begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}}_{[r_1\ r_2\ t]}$$

⇒ Four step procedure:

1. Exploit constraints about K, r_1, r_2
2. Define a matrix $B = K^{-T}K^{-1}$
3. This B can be computed by solving another homogeneous linear system.
4. Decompose matrix B.

★ Exploiting constraints for determining the parameters

$$[h_1\ h_2\ h_3] = K[r_1\ r_2\ t].$$

$$[r_1\ r_2\ t] = K^{-1}[h_1\ h_2\ h_3]$$

$$r_1 = K^{-1}h_1 \qquad r_2 = K^{-1}h_2$$

⇒ r_1, r_2, r_3 forms an orthonormal basis.

$$r_1^T r_2 = 0 \qquad \|r_1\| = \|r_2\| = 1$$

$$r_1^T r_2 = 0$$

$$\hookrightarrow \boxed{h_1^T K^{-T} K^{-1} h_2 = 0}$$

$\|r_1\| = 1$ $\|r_2\| = 1$

$\hookrightarrow h_1^T K^{-T} K^{-1} h_1 = 1$ $\hookrightarrow h_2^T K^{-T} K^{-1} h_2 = 1$

$$\boxed{h_1^T K^{-T} K^{-1} h_1 - h_2^T K^{-T} K^{-1} h_2 = 0}$$

$\Rightarrow$ Let $B = K^{-T} K^{-1}$ {Symmetric & positive definite}

$$h_1^T B h_2 = 0$$

$$h_1^T B h_1 - h_2^T B h_2 = 0$$

$\Rightarrow$ From B, the calibration matrix can be recovered through Cholesky decomposition.

$$\text{Chol}(B) = A A^T \quad \{A = K^{-T}\}$$

$\Rightarrow$ Let $b = (b_{11}\ b_{12}\ b_{13}\ b_{22}\ b_{23}\ b_{33})$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

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⇒ Construct a system of linear equations:

$$V_{12}^T b = 0$$

$$V_{11}^T b - V_{22}^T b = 0$$

⇒ Let $V = \begin{pmatrix} V_{12}^T \\ V_{11}^T - V_{22}^T \end{pmatrix}$ 2×6

⇒ $\boxed{V b = 0}$

⇒ For one image, we obtain

$$V b = 0$$

⇒ For multiple images, we stack the matrices to a $2n \times 6$ matrix. $(n \geq 3)$

$$\begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} b = 0$$

For 1st Image ← V_1

For nth Image ← V_n

{This can be easily solved using SVD as in case of DLT.}

★ Non-Linear Parameters

⇒ General calibration matrix is obtained by combining the one of the affine camera with the general mapping

$$^{a}K(\alpha, q) = {}^{a}H_s(\alpha, q)\, K$$

$$= \begin{bmatrix} 1 & 0 & \Delta x(\alpha, q) \\ 0 & 1 & \Delta y(\alpha, q) \\ 0 & 0 & 1 \end{bmatrix} K$$

⇒ Lens distortion can be calculated by minimizing a non-linear error function

$$\min_{(K, q, R_n, t_n)} \sum_{n} \sum_{i} \| x_{ni} - \hat{x}(K, q, R_n, t_n, X_{ni}) \|^2$$

- Linearize about initial guess to obtain a quadratic function.

This can be obtained by performing Zhang's method assuming no non-linear error

- Compute derivative
- Set it to zero.
- Solve linear System
- Iterate until convergence.