Page No.
Date: / /

Projective 3-Point (P3P) Algorithm or Spatial Resection

★ Camera Localization

Given
↳ 3D coordinates of object points X_i

Observed
↳ 2D image coordinates x_i of the object

Wanted
↳ Extrinsic parameters R, X_0 of the calibrated camera.

$$\boxed{x = KR[I_3 | -X_0]X = PX} \quad \{DLT\}$$

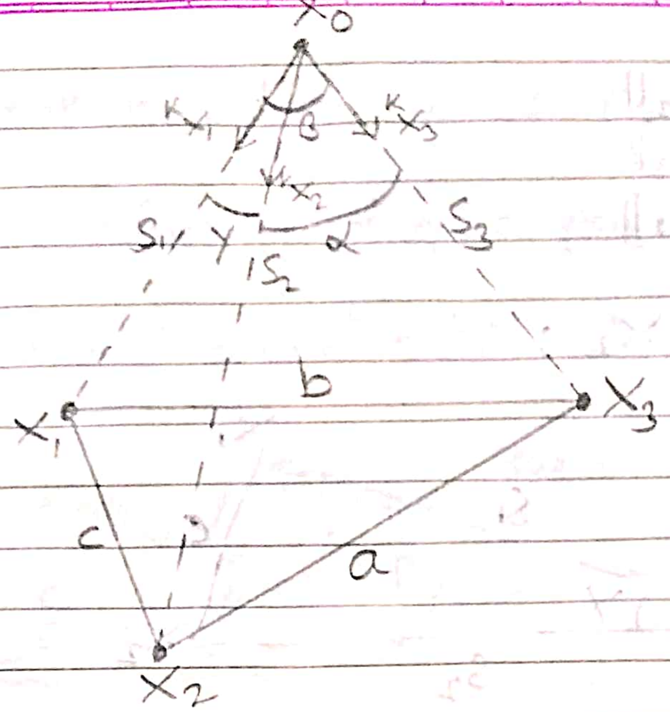
⇒ 6 unknown (3 rotation, 3 translation), so we need at least 3 points.

⇒ 2-Step process: (Grunert 1841)

① ↳ Estimate length of projection rays

② ↳ Estimate the orientation.

★ Step 1: Estimating length of projection rays,



⇒ Coordinates of object points within the camera system are given by

$$s_i \, {}^kX_i^s = R(X_i - X_0)$$

⇒ From image coordinates, we obtain the directional vectors of projection ray

$${}^kX_i^s = -\mathrm{Sign}(c)\, N(K^{-1}x_i)$$

$$\cos\gamma = \frac{(X_1 - X_0)\cdot(X_2 - X_0)}{\|X_1 - X_0\|\,\|X_2 - X_0\|}$$

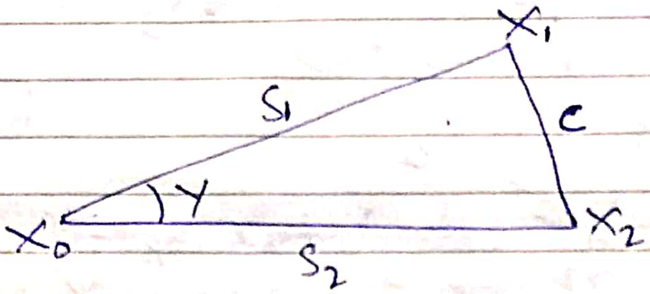
$$\alpha = \arccos({}^kX_2^s, {}^kX_3^s)$$
$$\beta = \arccos({}^kX_3^s, {}^kX_1^s)$$
$$\gamma = \arccos({}^kX_1^s, {}^kX_2^s)$$

$a = \|X_3 - X_2\|$

$b = \|X_1 - X_3\|$

$c = \|X_2 - X_1\|$

$\Rightarrow$ For $\Delta X_0 X_1 X_2$:



$$S_1^2 + S_2^2 - 2S_1S_2 \cos\gamma = c^2$$

Similarly, $$S_2^2 + S_3^2 - 2S_2S_3 \cos\alpha = a^2$$

$$S_1^2 + S_3^2 - 2S_1S_3 \cos\beta = b^2$$

Let $u = \frac{S_2}{S_1}$ $\quad v = \frac{S_3}{S_1}$

$$a^2 = S_1^2(u^2 + v^2 - 2uv\cos\alpha)$$

$$S_1^2 = \frac{a^2}{u^2 + v^2 - 2uv\cos\alpha}$$

$$S_1^2 = \frac{b^2}{1 + v^2 - 2v\cos\beta}$$

$$S_1^2 = \frac{c^2}{1 + u^2 - 2u\cos\gamma}$$

Page No.
Date: / /

⇒ Solve one equation for u put into the other:

⇒ Results in a fourth degree polynomial:

$$A_4 V^4 + A_3 V^3 + A_2 V^2 + A_1 V + A_0 = 0$$

From this we can get

S_1, S_2 & S_3 {we get 4 solution}

⇒ How to Eliminate this ambiguity?

→ Known approximate solution

→ Use 4^{th} point to confirm the right solution.

★ Step 2: Orientation of the Camera

$$^{K}X_i = R(X_i - X_0) \quad \left\{ {}^{K}X_i = S_i \, {}^{K}X_i^{S} \right\}_{i=1,2,3}$$