

EKF SLAM

★ Given

① The robot's Controls

$$U_{1:T} = \{u_1, u_2, \dots, u_T\}$$

② Observations

$$Z_{1:T} = \{z_1, z_2, \dots, z_T\}$$

★ Wanted

① Map of the environment

m

② Path of the robot

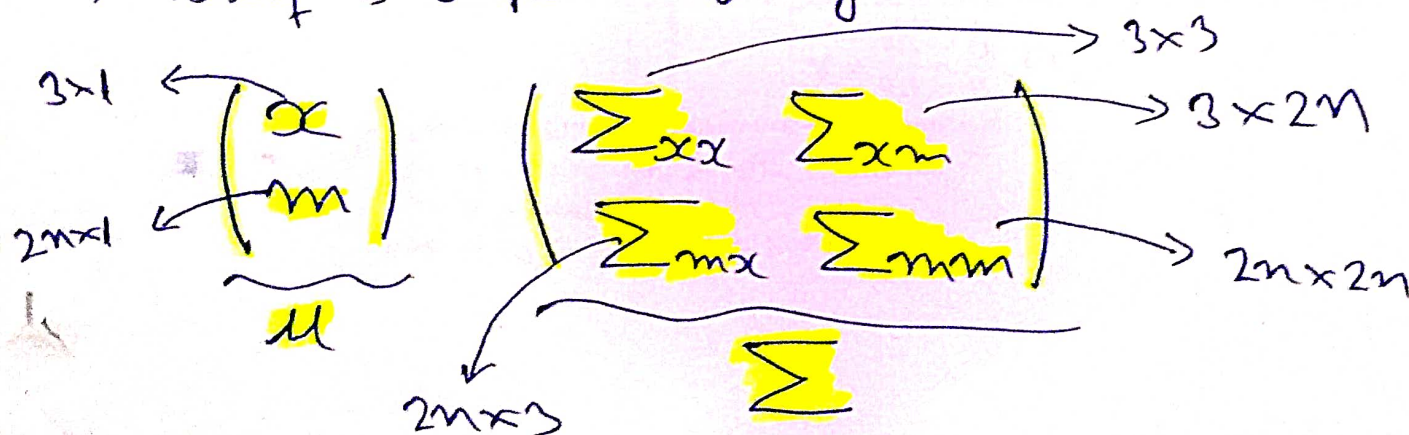
$$x_{0:T} = \{x_0, x_1, \dots, x_T\}$$

★ State Space

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1x}, m_{1y}}_{\text{landmark 1}}, \dots, \underbrace{m_{nx}, m_{ny}}_{\text{landmark } n})^T$$

⇒ Map with n landmarks: $(3+2n)$ -dimensional Gaussian

⇒ Belief is represented by:-



x	σ_x	σ_y	σ_z	$\sigma_{x_{max}}$	$\sigma_{x_{min}}$...	$\sigma_{x_{max}}$	$\sigma_{x_{min}}$
y	σ_{yx}	σ_{yy}	σ_{yz}	$\sigma_{y_{max}}$	$\sigma_{y_{min}}$...	$\sigma_{y_{max}}$	$\sigma_{y_{min}}$
z	σ_{zx}	σ_{zy}	σ_{zz}	$\sigma_{z_{max}}$	$\sigma_{z_{min}}$...	$\sigma_{z_{max}}$	$\sigma_{z_{min}}$
m_x				$\sigma_{m_x m_x}$	$\sigma_{m_x m_y}$...		
m_y				$\sigma_{m_y m_x}$	$\sigma_{m_y m_y}$...		
\vdots								
m_z								
m_{xy}								

* Initialization

⇒ Robot starts in its own reference frame
(all landmarks unknown)

$$\mu_0 = (0 \ 0 \ 0 \ \dots \ 0)^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & & \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & & \\ \hline & & & \infty & \\ & & & & \infty \end{pmatrix}$$

* Prediction Step

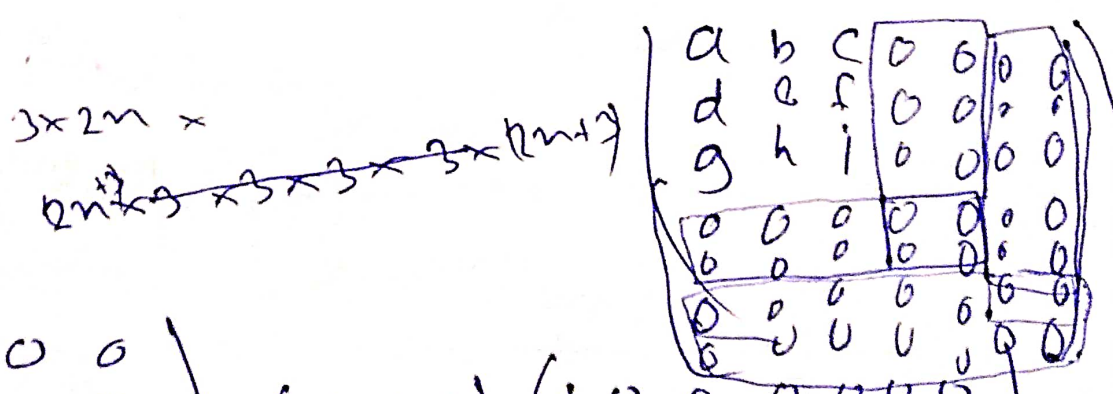


$$x_{t,x,y_0} = g_{x,y_0}(u_t, x_{t-1,x,y_0})$$

} Non linear motion model

$$x_t = g(u_t, x_{t-1})$$

$$G_t = \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix}$$



$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b & c & 0 & 0 & 0 & 0 \\ d & e & f & 0 & 0 & 0 & 0 \\ g & h & i & 0 & 0 & 0 & 0 \end{pmatrix}$$

* Observation Step

⇒ Assumption: Known data association

$$C_t^i = j$$

→ i th measurement at time t observes the landmark with index j

⇒ Range-Bearing observation

$$Z_t^i = (r_t^i, \phi_t^i)^T$$

⇒ If landmark has not be observed

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$$

⇒ Expected observation:

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$$q = \delta^T \delta$$

$$Z_t^i = \begin{pmatrix} \sqrt{q} \\ a \tan 2(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix} = h(\bar{\mu}_t)$$

⇒ Compute the Jacobian

$$\overset{\text{Low}}{\uparrow} H_t^i = \frac{\delta h(\bar{\mu}_t)}{\delta \bar{\mu}_t}$$

low-dim space $(\alpha, \gamma, \theta, m; \alpha, m; \gamma)$

$$H_t^i = {}^{\text{low}}H_t^i F_{\alpha, j}$$

