Homogeneous Coordinates

⇒ H.C are a system of coordinates used in projective geometry.

→ Points at infinity can be represented using finite coordinates.

→ A single matrix can represent affine transformation

{ translation, rotation, shear & scaling }

* Defination

The representation x of a geometric object is homogeneous if x and λx represent the same object for $\lambda \neq 0$.

* From homogeneous to Euclidian Coordinates

Homogeneous

$$X = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} wx \\ wy \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Euclidian

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

* Infinitively Distant Object

⇒ It is possible to explicitly model infinitively distant points with finite coordinates.

$$X_\infty = \begin{bmatrix} u \\ v \\ 0 \end{bmatrix}$$

⇒ Similarly we can do things in 3D

★ Transformations

⇒ A projective transformation is a invertible linear mapping.

$$\boxed{x' = Mx}$$

★ Important Transformations ($\mathbb{P}^3$)

- Translation : 3 Parameters

$$M = \lambda \begin{bmatrix} I & t \\ 0^T & 1 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad t = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \qquad 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Rotation : 3 Parameters

$$M = \lambda \begin{bmatrix} R & 0 \\ 0^T & 1 \end{bmatrix}$$

R → Regular rotation matrix

$$0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- Rigid body transformation : 6 params

$$M = \lambda \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$