Jordan Canonical Form

Consider the System defined by :-

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots b_{n-1} s + b_n}{(s+P_1)^3 (s+P_4)(s+P_5)\cdots(s+P_n)} \quad \text{--- (0)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \left[\begin{array}{ccc|ccc} -P_1 & 1 & 0 & 0 & \cdots & 0 \\ 0 & -P_1 & 1 & & & 0 \\ 0 & 0 & -P_1 & 0 & \cdots & 0 \\ \hline 0 & \cdots & 0 & -P_4 & & 0 \\ \vdots & & \vdots & & \ddots & \\ 0 & \cdots & 0 & 0 & & -P_n \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u \quad \text{--- (1)}$$

$$y = [C_1 \; C_2 \; \cdots \; C_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u \quad \text{--- (2)}$$

Solution

⇒ Partial-fraction expansion of Eq (0) :-

$$\frac{Y(s)}{U(s)} = b_0 + \frac{C_1}{(s+P_1)^3} + \frac{C_2}{(s+P_1)^2} + \frac{C_3}{(s+P_1)} + \frac{C_4}{s+P_4} + \cdots + \frac{C_n}{s+P_n}$$

$$\Rightarrow Y(s) = b_0 U(s) + \frac{C_1}{(s+P_1)^3} U(s) + \frac{C_2}{(s+P_1)^2} U(s) + \frac{C_3}{s+P_1} U(s) + \frac{C_4}{s+P_4} U(s) + \cdots + \frac{C_n}{s+P_n}$$

Let us define State Variable as :-

$$X_1(s) = \frac{1}{(s+P_1)^3} U(s)$$

$$X_2(s) = \frac{1}{(s+P_1)^2} U(s)$$

$$X_3(s) = \frac{1}{s+P_1} U(s) \Longrightarrow (a)\ sX_3(s) = -P_1 X_3(s) + U(s)$$

$$X_4(s) = \frac{1}{s+P_4} U(s) \longrightarrow (b)\ sX_4(s) = -P_4 X_4(s) + U(s)$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$X_n(s) = \frac{1}{s+P_n} U(s) \longrightarrow (f)\ sX_n(s) = -P_n X_n(s) + U(s)$$

⇒ Notice the following relationships between $X_1(s), X_2(s)$ & $X_3(s)$:

$$\frac{X_1(s)}{X_2(s)} = \frac{1}{s+P_1} \Rightarrow sX_1 = -P_1 X_1(s) + X_2(s) \quad (g)$$

$$\frac{X_2(s)}{X_3(s)} = \frac{1}{s+P_1} \Rightarrow sX_2 = -P_1 X_2(s) + X_3(s) \quad (h)$$

⇒ Converting eq (a), (b) - - - (f) (g) & (h) in time domain we get:-

$$\dot{x}_1 = -P_1 x_1 + x_2$$
$$\dot{x}_2 = -P_1 x_2 + x_3$$
$$\dot{x}_3 = -P_1 x_3 + u$$
$$\dot{x}_4 = -P_4 x_4 + u$$
$$\vdots$$
$$\dot{x}_n = -P_n x_n + u$$

From this we get eq (1) {first part of solution}

$$Y(s) = b_0 U(s) + C_1 X_1(s) + C_2 X_2(s) + \cdots + C_n X_n(s)$$

$\Downarrow$ {Time domain}

$$\boxed{y = C_1 x_1 + C_2 x_2 + \cdots C_n x_n + b_0 u}$$

{ from this we get eq ①
{Second part of the solution} }