Lecture 1: Introduction

⇒ We have three main objectives:

① Modeling: Learn how to represent a dynamic control system in a way that it can be treated effectively using mathematical tools.

② Analysis: Understand the basic characteristics of a system (eg. stability, controllability, observability), and how the input effects the output.

③ Synthesis: Figure out how to change a system is such a way that it behaves in a desired way.

⇒ We will concentrate on systems that can be modeled by Ordinary differential Equations (ODE)

→ and that satisfy linearity and time-invariance condition.

→ In this course we will focus on system with a Single Input & Single Output (SISO)

★ Signals

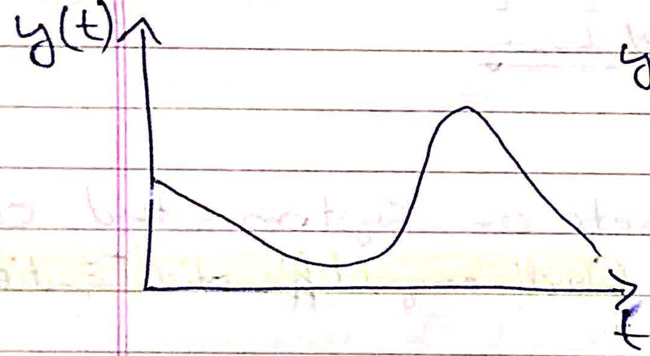
" Maps from a set T to a set W

{Time}

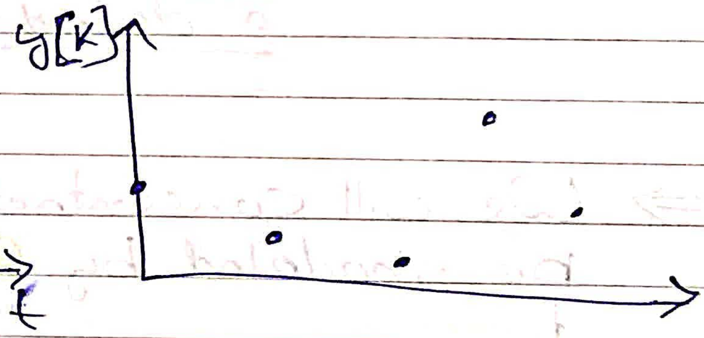
{Signal Space}

→ For single input system $W = R$

→ One can also consider vector-valued signals
$W = R^n$



Continuous time signal



Discrete time signal

★ System

"Map between signals"

↳ {Something that transform some input signal to some output signal}

⇒ Other signals that are of interest include ==disturbance== & ==noise==.

⇒ An input-output model is a map Σ from an input signal $u : t \to u(t)$ to an output signal $y : t \to y(t)$

$$y = \Sigma u \qquad\qquad u \longrightarrow \boxed{\Sigma} \longrightarrow y$$

$$\downarrow$$

$$y(t) = (\Sigma u)(t) \quad \forall\, t \in T$$

★ Memoryless (or static) System

⇒ An input-output system Σ is memoryless if there exists a function $S : W \to W$ such that $\forall\; t \in T$

$$\boxed{y(t) = (\Sigma u)(t) = S(u(t))}$$

Example

- $y(t) = 3u(t)$ Static
- $y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$ Not static

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★ Time Invariance

⇒ Let the time-shift operator σ_τ be defined as follows, for any signal u:

$$(\sigma_\tau u)(t) = u(t-\tau) \qquad \forall t \in T$$

⇒ An input-output system Σ is time-invariant if it commutes with the time-shift operator.

$$\Sigma \sigma_\tau u = \sigma_\tau \Sigma u = \sigma_\tau y \qquad \forall \tau \in T$$

★ Linearity

⇒ An input-output system Σ is linear if, for all input signals u_a, u_b and scalar $\alpha, B \in R$

$$\Sigma(\alpha u_a + B u_b) = \alpha(\Sigma u_a) + B(\Sigma u_b)$$
$$= \alpha y_a + B y_b$$

★ Causality

⇒ An input-output system Σ is causal if, for any $t \in T$, the output at time t depends only on the values of the input on $(-\infty\ t]$.

⇒ Let us define the truncation operator P as:

$$(P_T u)(t) = \begin{cases} u(t) & \forall\, t \leq T \\ 0 & \forall\, t > T \end{cases}$$

⇒ Then an input-output system Σ is causal if

$$P_T \Sigma P_T = P_T \Sigma \quad \forall\, T \in \mathbb{T}$$

⇒ An input-output system Σ is strictly causal if, for any $t \in \mathbb{T}$, the output at time t depends only on the value of input on $(-\infty, t)$.