Introduction to Discrete-Time Control System

1.1) Introduction

"The current trend toward digital rather than analog control of dynamic system is mainly due to the availability of low-cost digital computers and the advantage found in working with digital signals rather than continuous-time signal"

⇒ The process of representing a variable by a set of distinct values is called quantization, and the resulting values are called quantized value.

Analog Signal ⇒ An analog signal is a signal defined over a continuous range of time whose amplitude can assume a continuous range of values.

Discrete time Signal ⇒ A discrete time signal is a signal defined only at discrete instants of time.

Continuous time Signal ⇒ A signal defined over a continuous range of time.

Note: The amplitude may assume a continuous range of values (Analog Signal) or may assume a finite number of distinct value.

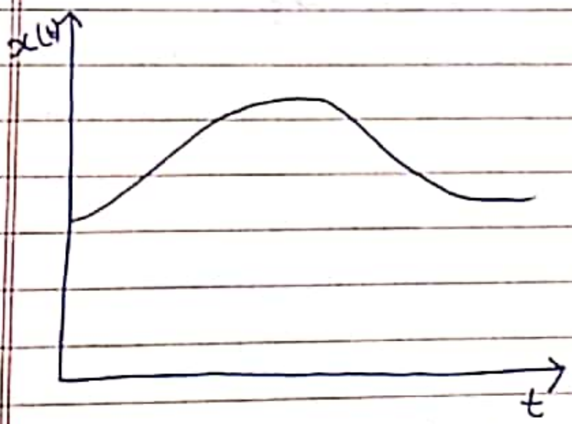
[Continuous time quantized Signal]

Sampled data Signal ⇒ In discrete time signal, if the amplitude can assume a continuous range of values, then the signal is called a sampled-data signal.

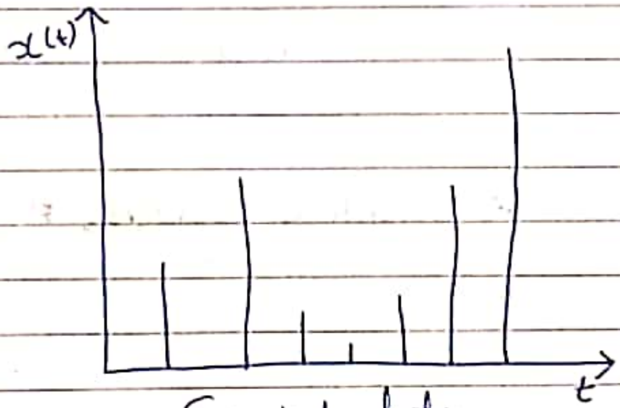
{Signals in numerically coded form}

digital signal ⇒ discrete time signal with quantized amplitude.

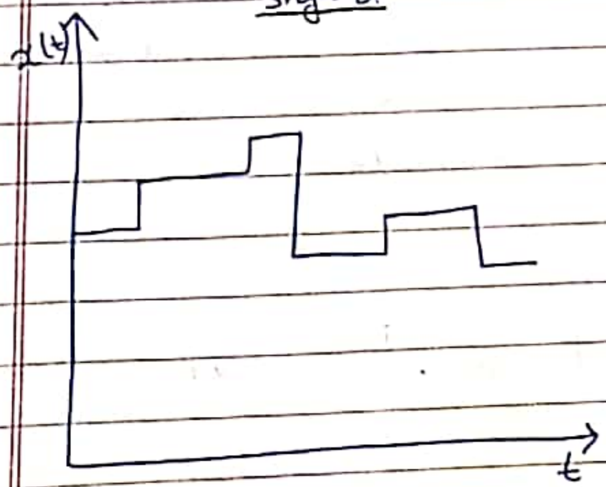
Note: The use of the digital controller requires quantization of signals both in amplitude and in time.



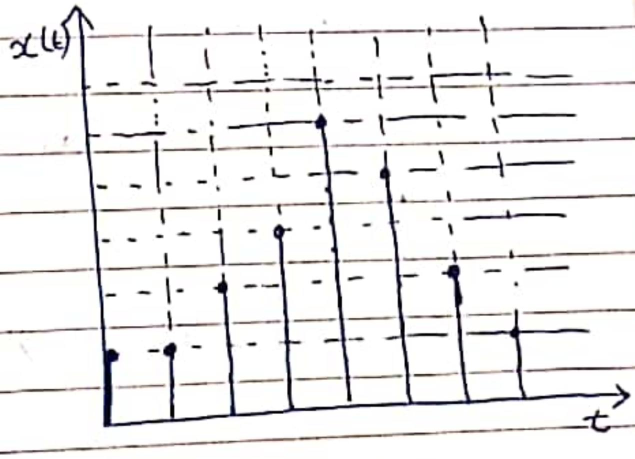
Continuous time Analog Signal



Sampled-data signal



Continuous time quantized Signal



digital Signal

⇒ A time-invariant system is one in which the coefficients in the differential equation or difference equation do not vary with time.

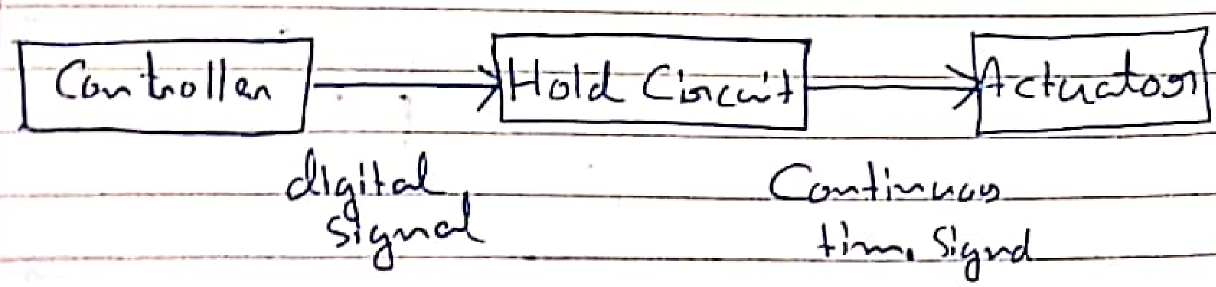
Discrete Time Control System ⇒ Control system in which one or more variables can change at discrete instants of time.

↳ These instants, which we shall denote by KT or t_K $(K=0,1,2,\cdots)$

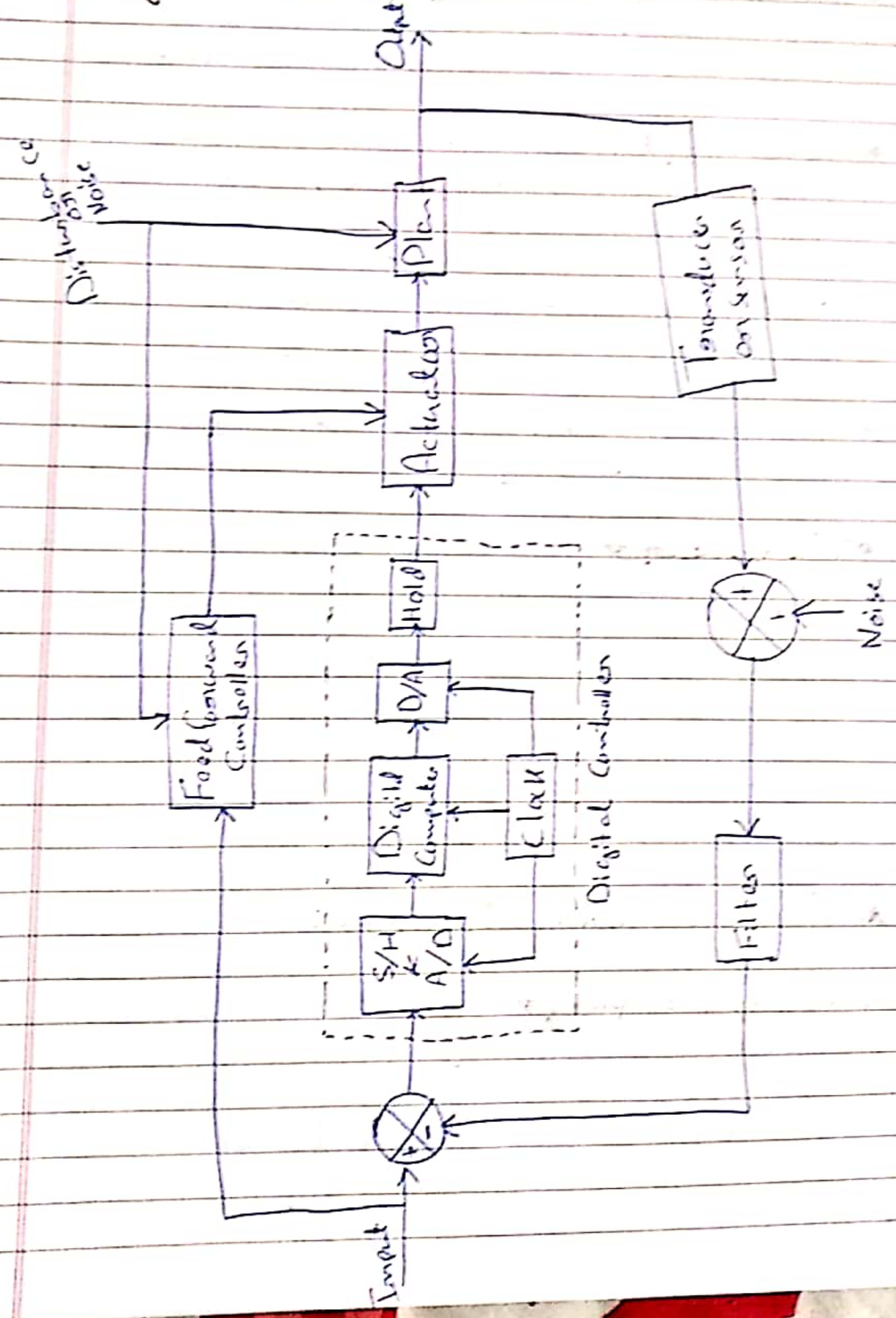
↳ The time interval between two discrete instants is taken to be sufficiently short that the data for the time between them can be approximated by simple interpolation.

Sampling process ⇒ The sampling of a continuous-time signal replaces the original continuous-time signal by a sequence of values at discrete time points.

⇒ The sampling process is usually followed by a quantization process. In the quantization process the sampled analog amplitude is replaced by a digital amplitude.



1.2) Digital Control System



Sampling or discretization ⇒ Operation that transforms continuous-time signal to discrete time data is called discretization or sampling.

data hold ⇒ Operation that transforms discrete-time data in a continuous time signal is called data hold.

⇒ Analog to digital (A/D) conversion process is called Coding or encoding.

⇒ Digital to analog (D/A) conversion process is called decoding.

Transducer ⇒ It is a device that converts an input signal into an output signal of another form.

eg ⇒ Pressure signal into voltage output

- Analog Transducer
- Sampled-data transducer
- digital transducer

★ Type of Sampling Operation :-

1) Periodic Sampling ⇒ The sampling instants are equally spaced or $t_k = KT$ $(K = 0, 1, 2 - -)$

→ It is most conventional type.

2) <u>Multiple-order Sampling</u> ⇒ The pattern of the t_k's is repeated periodically; that is $t_{k+r} - t_k$ is Constant $\forall K$.

3) <u>Multiple-rate Sampling</u> ⇒ In a Control system having multiple loops, the largest time Constant involved in one loop may be quite different from that in other loops.

⇒ It may be advisable to sample slowly in a loop involving a large time Constant, while in a loop involving only small time Constants, the Sampling rate must be fast.

⇒ Thus, a digital control system may have different Sampling periods in different feedback paths or may have multiple Sampling rate.

4) <u>Random Sampling</u> ⇒ The Sampling Instants are random or t_k is a random variable.

1.3) Quantizing & Quantization error

⇒ The main functions involved in analog to digital Conversion are <u>Sampling</u>, <u>amplitude quantizing</u> and <u>Coding</u>.

⇒ The Sampling period and quantizing levels affect the performance of digital control System. So they must be determined Carefully.

Quantization level (Q) ⇒ It is defined as the range between two adjacent decision points and is given by:

$$Q = \frac{FSD}{2^n}$$

FSD = Full scale deflection
n = number of bits

Quantization Error ⇒ Digital output can assume only a finite number of levels, so any A/D Conversion involve Quantization error.

→ Quantization error varies between 0 & $\pm \frac{1}{2} Q$.

→ The uncertainty present in the quantization process is called quantization noise.

→ The quantization error e(t) is the difference between the Input Signal and the quantized output:

$$e(t) = x(t) - y(t)$$

⇒ For a small quantization level Q, the nature of the quantization is similar to that of random noise. An, in effect, the quantization process acts as a source of random noise.

⇒ Suppose that the quantization level Q is small and we assume that the quantization error e(t) is distributed uniformly between $-\frac{1}{2}Q$ and $\frac{1}{2}Q$ and that this error acts as a white noise.

⇒ Then the variance σ^2 of the quantization noise is

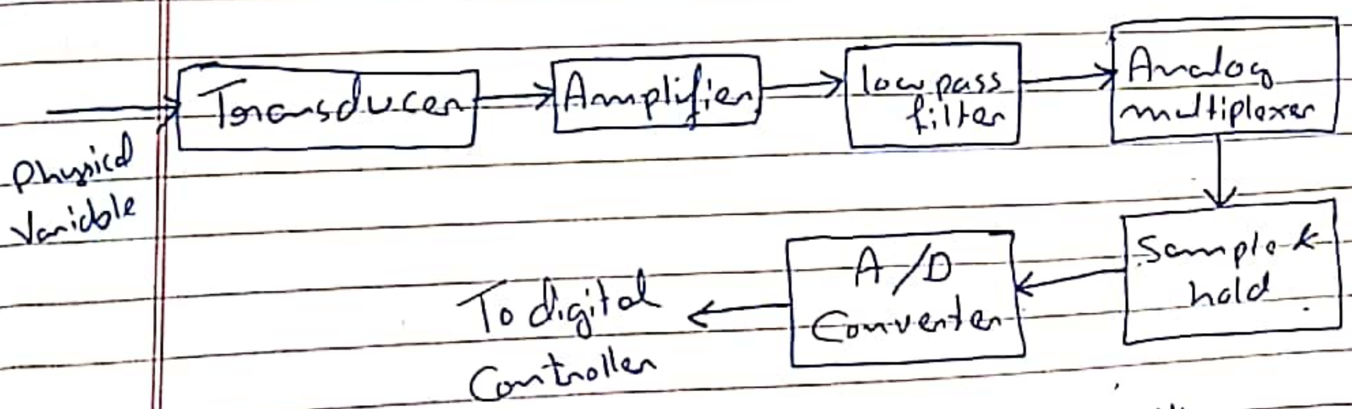
$$\sigma^2 = E\left[e(t) - \overline{e(t)}\right]^2 = \frac{1}{Q}\int_{-Q/2}^{Q/2} \varepsilon^2 d\varepsilon = \frac{Q^2}{12}$$

{Assuming Uniform Probability distribution}

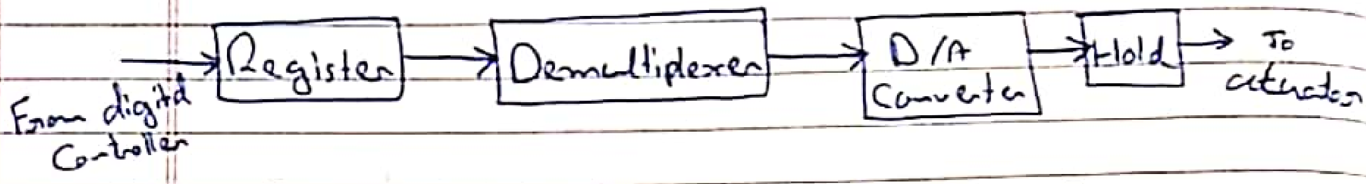
1.4 Data Acquisition, Conversion, and Distribution System

⇒ The signal conversion that takes place in the digital control system involves the following operations:-

1. Multiplexing and demultiplexing
2. Sample and hold
3. Analog to digital conversion (quantizing & encoding)
4. Digital to analog conversion (decoding)



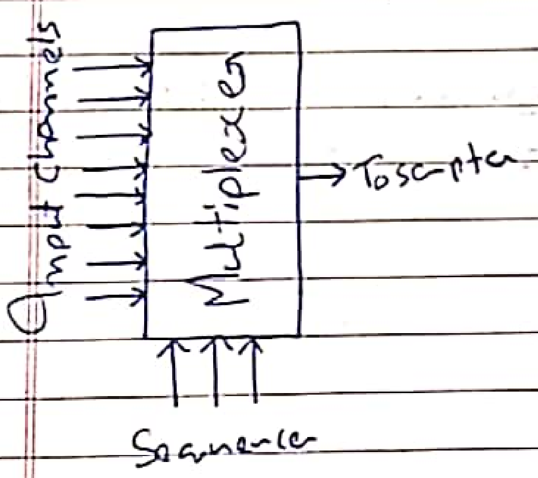
Block diagram of a data-acquisition system

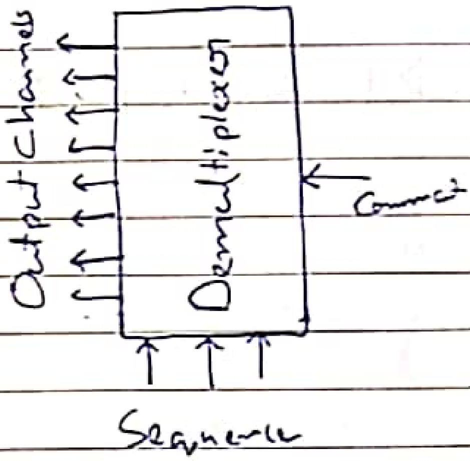


Block diagram of data distribution

The low pass filter that follows the amplifier attenuates the high-frequency signal components, such as noise signals.

★ **Analog Multiplexer** ★ **Demultiplexer**



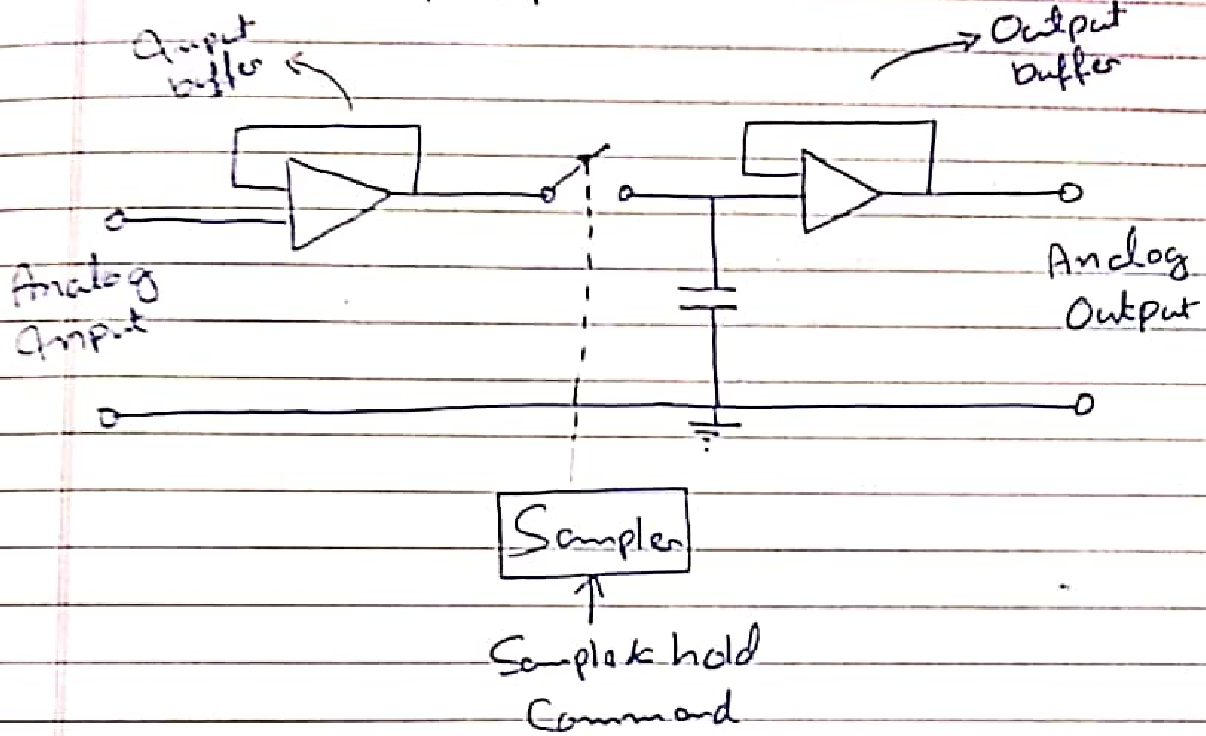


★ **Sample and Hold Circuit**

A Sampler in a digital System converts an analog signal into a train of amplitude-modulated pulses. The hold circuit holds the value of the sampled pulse signal over a specified period of time.

⇒ Commercially, sample & hold circuits are available in a single unit, known as a Sample and hold (S/H).

⇒ In practice, sampling duration is very short compared with the sampling period T.



→ Input voltage is acquired and then stored on a high quality capacitor with low leakage and low dielectric absorption characteristics.

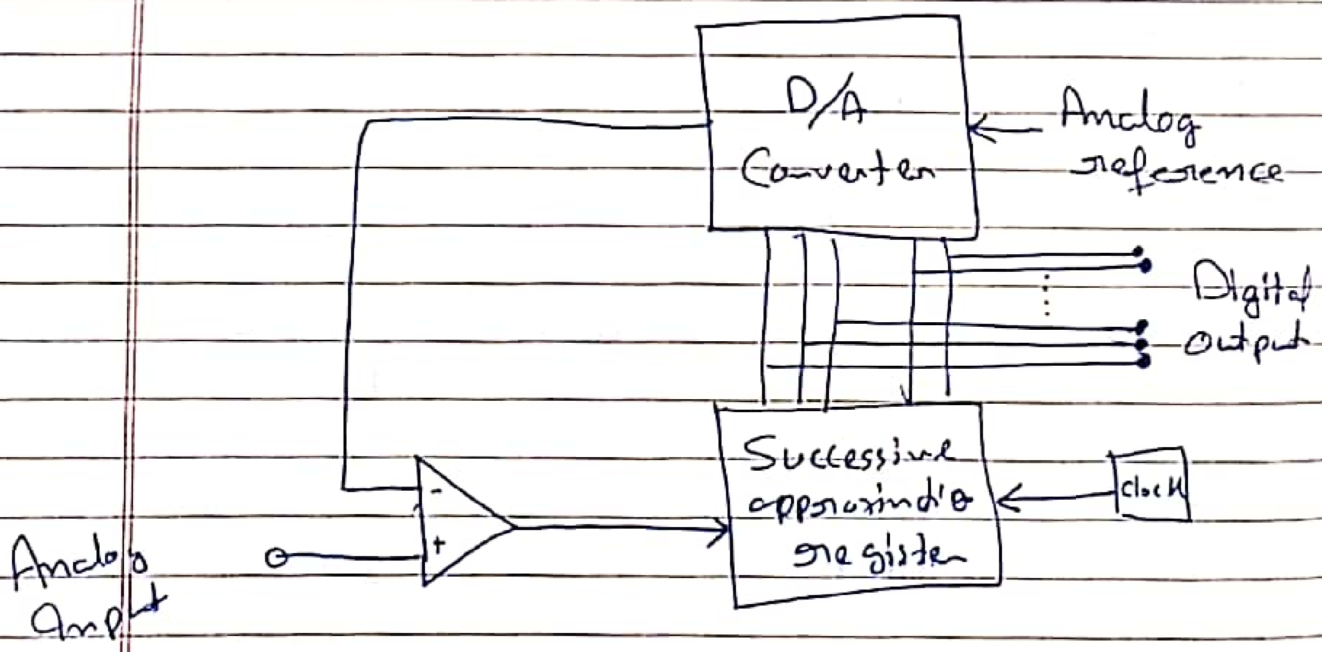
→ The time interval during which the switching takes place is called aperture time.

→ Input & Output buffer must have high impedance.

* Types of Analog to Digital (A/D) Converters

- → Successive-approximation type ← Fast & most frequently used
- → Integrating type
- → Counter type ← Simplest
- → Parallel type

⇒ In any particular application, the conversion speed, accuracy, size and cost are the main factor to be considered in choosing the type of A/D converter.

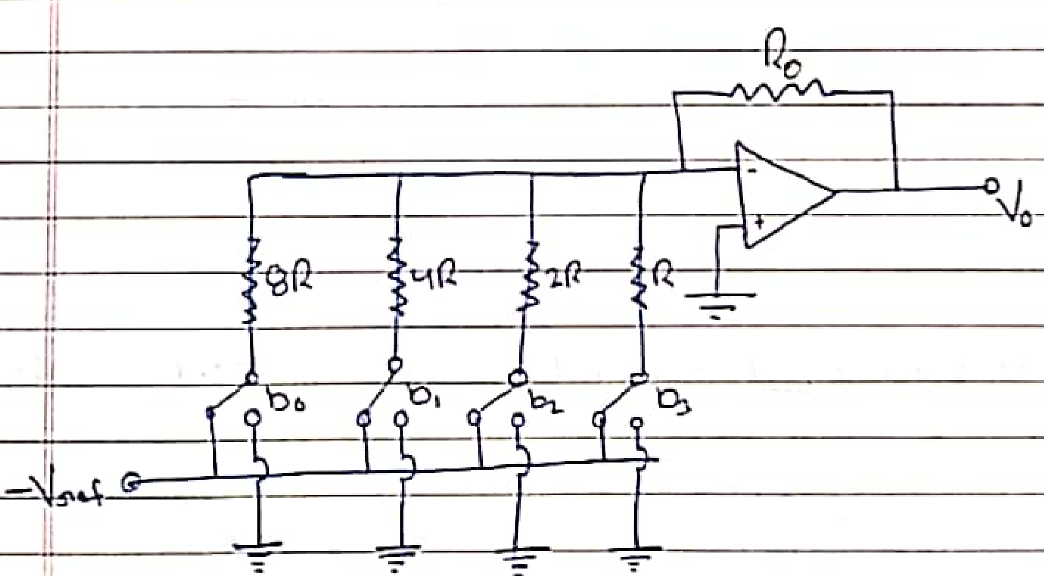


Successive-approximation type A/D Converter

★ Digital to analog Converter

⇒ Two method are commonly used for digital to analog Conversion:-

- → Weighted resistor. → [Simple but accuracy is not good]
- → R-2R Ladder network. → [Little more Complicated but is more accurate]

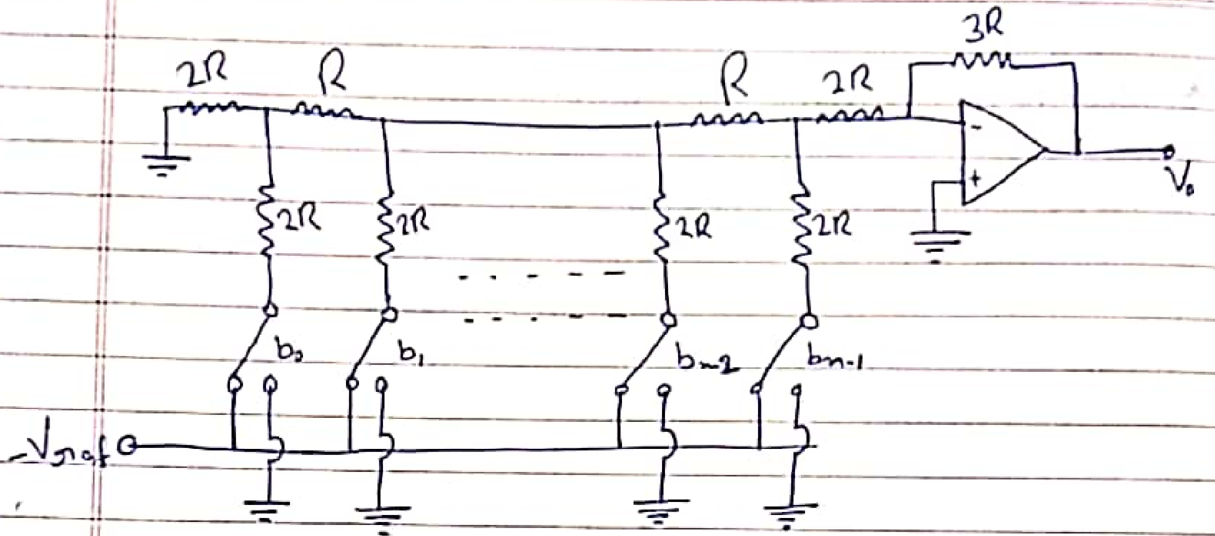


D/A Converter using Weighted resistors

⇒ b_0, b_1, b_2 & b_3 Can either be zero or one.

$$\boxed{V_0 = \frac{R_0}{R}\left(b_3 + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8}\right)V_{ref}}$$

⇒ as the number of bits is increased the range of resistance values becomes large and Consequently the accuracy becomes poor.



n-Bit D/A Converter using an R-2R ladder

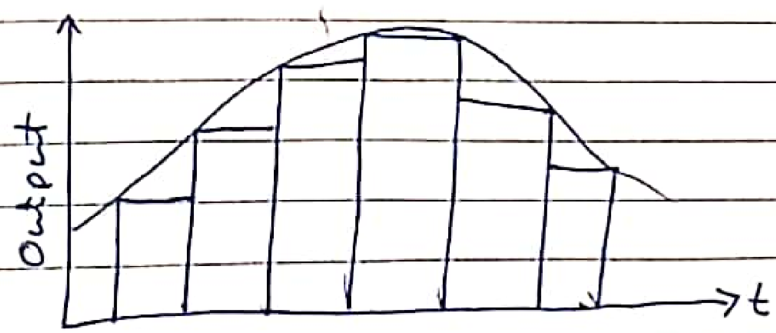
⇒ All resistors involved are either R or 2R. This means that a high level of accuracy can be achieved.

$$V_0 = \frac{1}{2}\left(b_{n-1} + \frac{1}{2} b_{n-2} + \text{---} \frac{1}{2^{n-1}} b_0\right) V_{ref}$$

★ Reconstructing the input Signal by Hold Circuits

⇒ The Sampling operation produces an amplitude-modulated PAM signal. The function of hold ~~signal~~ circuit is to reconstruct the analog signal by filling the space between.

\# Zero-order hold ⇒ The hold circuit that produces such a staircase waveform is called a zero order hold.



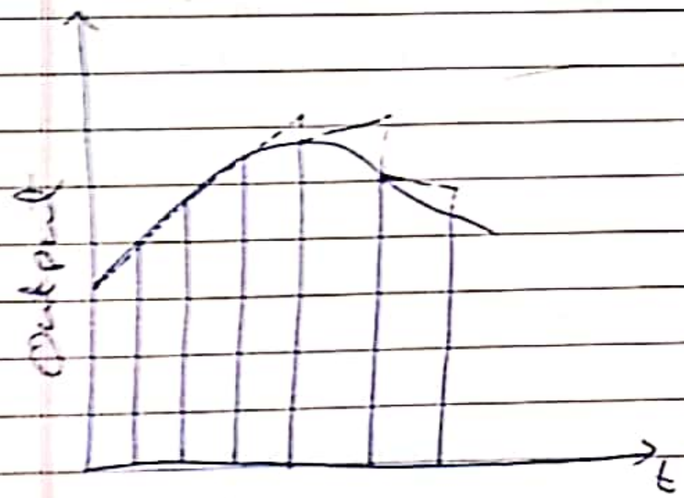
Higher Order hold Circuit

→ First order hold Circuit

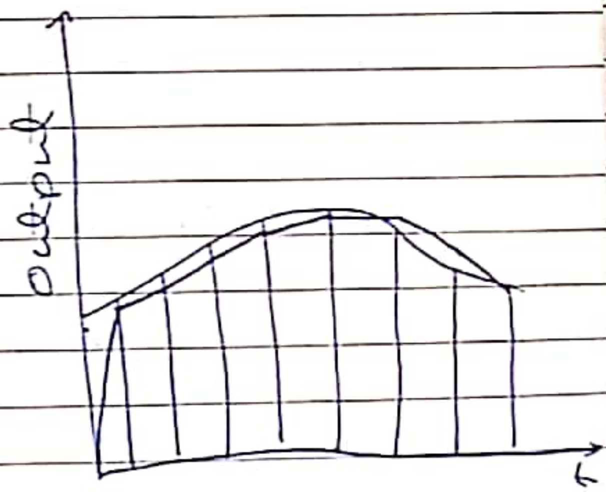
→ Second order hold Circuit

Interpolative first order hold

→ It reconstructs the original signal much more accurately but there is delay of 1 sampling period.



First Order hold



Interpolative First order hold