★ Steady-State Error In Unity feedback Control System

⇒ Any physical Control System inherently suffers steady-state error in response to certain type of input.

↳ A System may have no steady state error to a step input, but the same System may exhibit non zero steady-state error to a ramp input.

Classification of Control System

⇒ Control System may be classified according to their ability to follow step Input, ramp input, parabolic inputs, and so on.

↳ The magnitudes of the steady-state errors due to these Individual inputs are indicative of the goodness of the system.

⇒ Consider the unity-feed Control System with the following open loop transfer function $G(s)$:-

$$G(s) = \frac{K\,(T_a s+1)(T_b s+1)\cdots(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)\cdots(T_p s+1)}$$

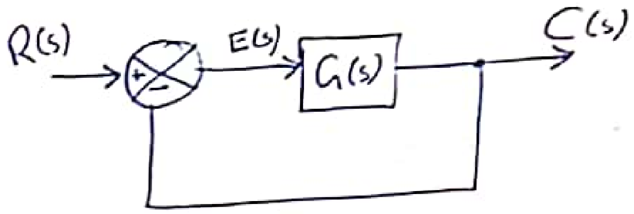
→ s^N in the denominator, representing a pole of multiplicity N at the origin.

→ A System is called type 0, type 1, type 2 if $N=0$, $N=1$ & $N=2$ respectively.

⇒ As the type number is increased, accuracy is improved, however increasing the type number aggravates the stability problem. Static

↳ Compromise between steady state accuracy and relative stability is always necessary.

Steady State Error



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} \qquad \frac{E(s)}{R(s)} = 1 - \frac{C(s)}{R(s)} = \frac{1}{1+G(s)}$$

⇒ The Final Value theorem provides a convenient way to find the steady state performance of a stable system.

$$E(s) = \frac{1}{1+G(s)} R(s)$$

$$e_{ss} = \boxed{\lim_{t\to\infty} e(t) = \lim_{s\to 0} s E(s)} = \lim_{s\to 0} \frac{s R(s)}{1+G(s)}$$

⇒ In a given system, the output may be the position, velocity, pressure, temperature etc...

⇒ The physical form of output, however is immaterial to the present analysis. Therefore what follows, we shall call the output "position", the rate of change of output "velocity" and so on.

Static position error Constant K_p

$$e_{ss} = \lim_{s\to 0} \frac{s}{1+G(s)} \cdot \frac{1}{s}$$ {Steady-State error of unit Step input.}

$$= \frac{1}{1+G(0)}$$

⇒ The static position error constant K_p is defined by ÷

$$K_p = \lim_{s\to 0} G(s) = G(0)$$

So $\boxed{e_{ss} = \frac{1}{1+K_p}}$

// For a type 0 System

$$K_p = \lim_{s\to 0} \frac{K(T_a s+1)(T_b s+1)---}{(T_1 s+1)(T_2 s+1)---} = K$$

$$e_{ss} = \frac{1}{1+K}$$ //

// For higher order System

$$K_p = \infty$$

$$e_{ss} = 0$$ //

Static Velocity Error Constant K_v

⇒ The Steady-state error of the System with a unit-ramp input is given by

$$e_{ss} = \lim_{s\to 0} \frac{s}{1+G(s)} \frac{1}{s^2} = \lim_{s\to 0} \frac{1}{sG(s)}$$

So, Static velocity error Constant K_p is defined by

$$K_v = \lim_{s\to 0} sG(s) \Longrightarrow \boxed{e_{ss} = \frac{1}{K_v}}$$

⇒ The term velocity error is used here to express the steady-state error for a ramp input.

System type	K_v	e_{ss}
0^{th} type	0	∞
1^{st} type	K	$1/K$
2^{nd} type & higher	∞	0

Similarly for Static Acceleration Error Constant K_a