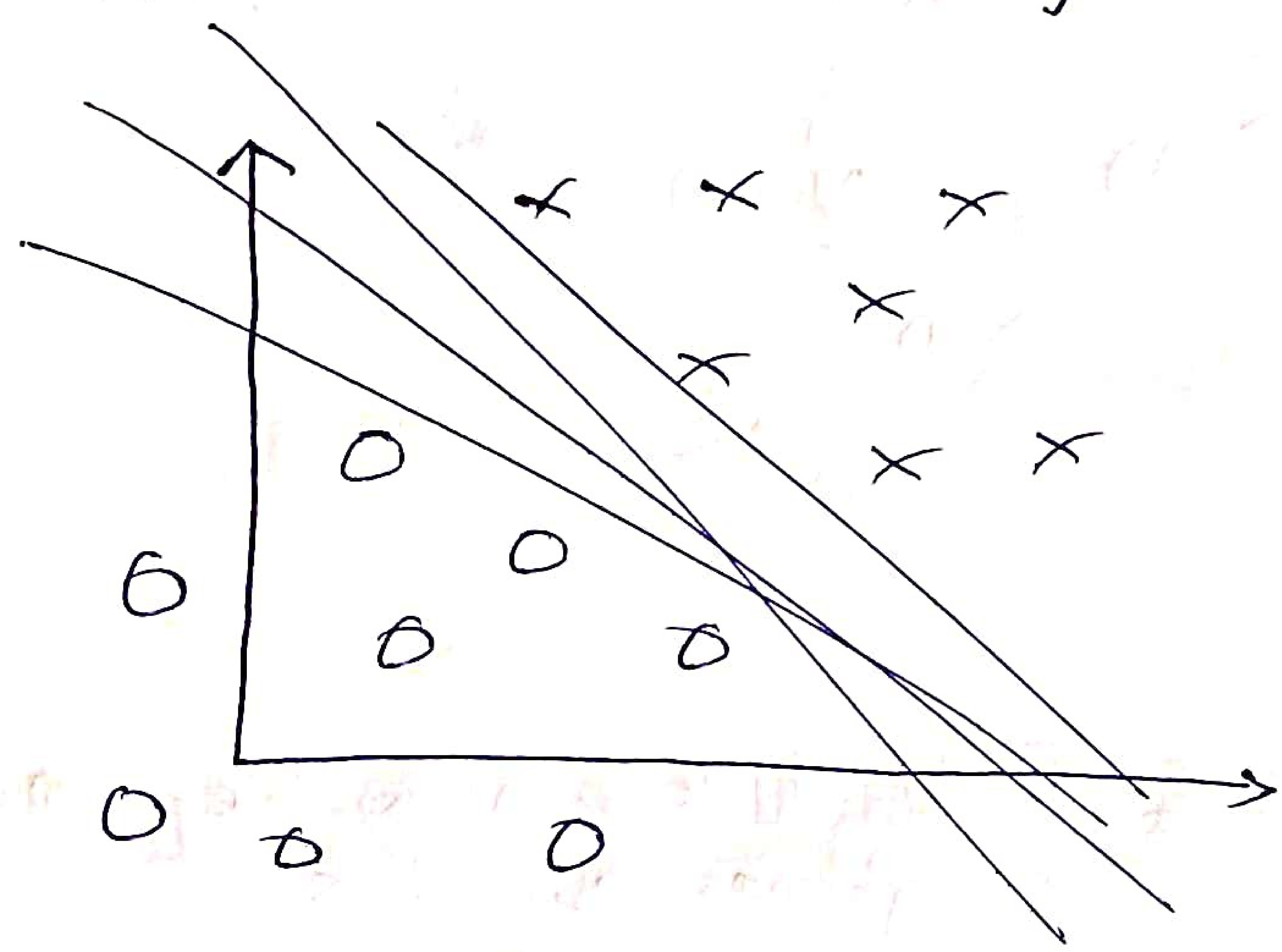
SVM

(Support Vector Machine)



⇒ It is an algorithm for performing <u>binary classification</u>.

↳ We take feature vector X map it to high dimension vector.

$X \longrightarrow X^*$

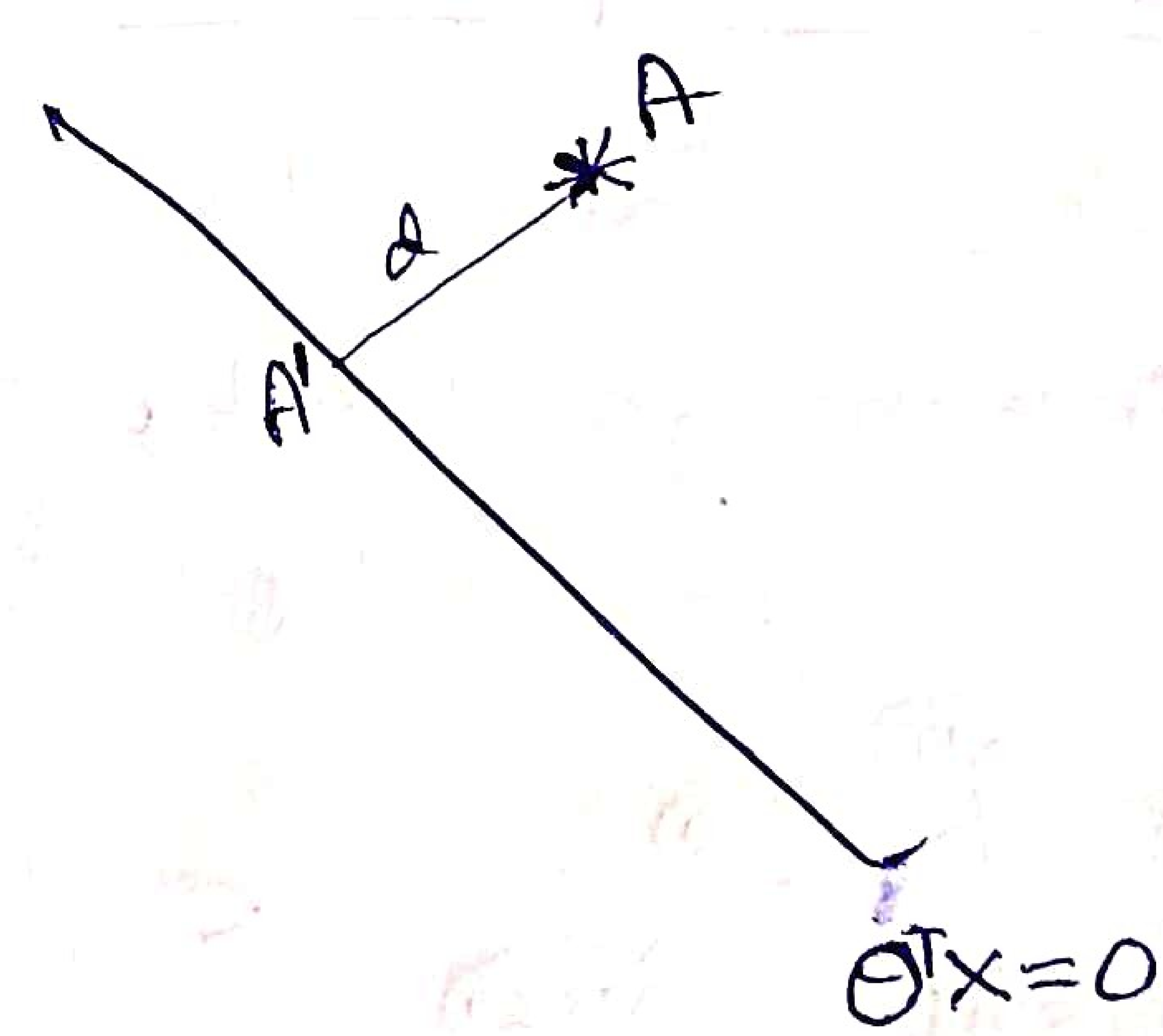
Optimal Margin Classifier

⇒ <u>Assumption</u>: Inputs can be separated into different classes by a linear decision boundary.

margin

↳ Functional Margin $\theta^T A$

↳ Geometric Margin



$\frac{w}{\|w\|}$

$\Rightarrow$ Let $\{(x^{(i)}, y^{(i)}) \mid x^{(i)} \in R^n, y^{(i)} \in \{-1, 1\}, i \in \{1, \dots m\}\}$ be the training set.

Objective: Find linear decision boundary ~~...~~ separating +ve & and -ve examples.

Assumption: Inputs are Linearly Seperable.

$\Rightarrow$ Let the seperating hyperplane be parametrized as

$$\omega^T x + b = 0 \quad , \quad \omega \in R^n \ \& \ b \in R$$

Let us define

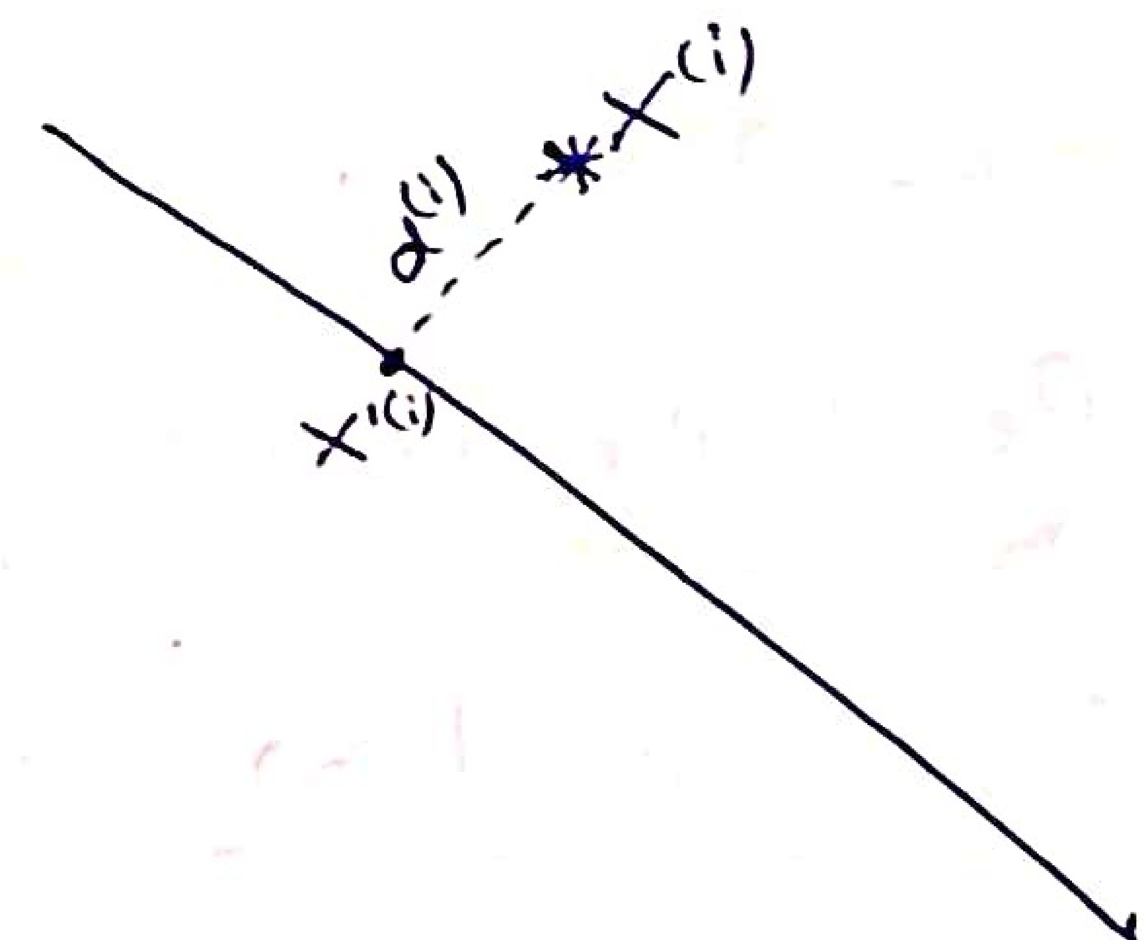
$\Rightarrow$ Margin for a data point $(x^{(i)}, y^{(i)})$:

(1) $\to$ Functional Margin

$$y^{(i)} \theta^T x^{(i)}$$

or $\quad \theta = \begin{bmatrix} b \\ \omega \end{bmatrix}$

$$\boxed{y^{(i)} [\omega^T x^{(i)} + b]}$$



(2) $\to$ Geometric Margin

Margin

$\to$ Gives a sense of how far a point is from separating line. (decision boundary) (No direction)

$$x'^{(i)} = x^{(i)} - d^{(i)} y^{(i)} \frac{\omega}{\|\omega\|_2}$$

$$\omega^T x'^{(i)} + b = 0$$

$$\omega^T x^{(i)} - d^{(i)} y^{(i)} \frac{\omega^T \omega}{\|\omega\|_2} + b = 0$$

$\|\omega\|_2^2$

$\|\omega\|_2$

$$d^{(i)} y^{(i)} \|w\|_2 = w^T x^{(i)} + b$$

$$\Rightarrow \boxed{d^{(i)} = y^{(i)} \left(\left(\frac{w}{\|w\|_2} \right)^T x^{(i)} + \frac{b}{\|w\|_2} \right)}$$

$\Rightarrow$ Geometric margin is the absolute distance (Euclidean) of a data point from the decision boundary.

★ <u>Geometric Margin of training Set</u>

$$\min_i d^{(i)}$$

or

$$\boxed{\min_i y^{(i)} \left(\left(\frac{w}{\|w\|_2} \right)^T x^{(i)} + \frac{b}{\|w\|_2} \right)}$$

$\Rightarrow$ Optimal margin classifier, finds the decision boundary that maximizes the geometric margin.

$\Rightarrow$ The above can be formulated as the following optimization Problem :

$$\cancel{\max_{w,b} \min_i \frac{y^{(i)} (w^T x^{(i)} + b)}{\|w\|_2}} \quad \forall\, i = \{1, \dots m\}$$

$$\boxed{\begin{aligned} &\max_{w,b,\gamma} \gamma \\ &\text{st} \quad \frac{y^{(i)} (w^T x^{(i)} + b)}{\|w\|_2} \geq \gamma \quad \forall\, i = \{1, \dots m\} \end{aligned}}$$