6

Uncertainty & Utilities

Idea: Uncertain outcomes controlled by chance not an adversary!

* Expectimax Search

△	▽	○
Max	Min	Chance

⇒ Value should now reflect average-case (expectimax) outcome, not worst-case (minmax) outcomes.

⇒ Expectimax Search: Compute the average score under optimal play.

- → Max nodes as in minmax search
- → Chance nodes are like min nodes but the outcome is uncertain.
- → Calculate their expected utilities (i.e. take weighted average (expectation) of children)

★ Expectimax Pseudo code

```
def value (state):
    if the state is a terminal state
        ↳ return the state's utility

    if the next agent is MAX:
        ↳ return max-value(state)
    if the next agent is EXP:
        ↳ return exp-value(state).
```

```
def max-value (state)
    initialize v = -∞
    for each successor of state
        v = max(v, value(successor))
    return v
```

```
def exp-value (state)
    initialize v = 0
    for each successor of state:
        p = Probability(successor)
        v += p* value(successor)
    return v
```

★ Reminder: Probabilities

Random variable
↳ Represents an event whose outcome is unknown

Probability distribution
↳ Assignments of weights to outcomes.

⇒ The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes.

★ Utilities

↳ Function from outcomes (states of the world) to real number that describes an agent performance

⇒ An agent must preferences among:

- Prizes: A, B etc
- Lotteries: Situation with uncertain prizes

$$L = [P, A; (1-P), B]$$

⇒ Notation:

- Preference: $A \succ B$
- Indifference: $A \sim B$

★ Rational Preferences

⇒ We want some constraints on preferences before we call them rational, such as:

- Axiom of Transitivity:

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- Orderability

$$\cancel{(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)}$$

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

- Continuity

$$A \succ B \succ C \Rightarrow \exists P \; [P, A; 1-P, C] \sim B$$

- Substitutability

$$A \sim B \Rightarrow [P, A; 1-P, C] \sim [P, B; 1-P, C]$$

- Monotonicity

$$A \succ B \Rightarrow (P \geq q \Leftrightarrow [P, A; 1-P, B] \succ [q, A; 1-q, B])$$

Theorem [Ramsey, 1931; Von Neumann & Morgenstern, 1944]

Given any preference satisfying these constraints, there exists a real-valued function U such that

$$U(A) \geq U(B) \Leftrightarrow A \succeq B$$

$$U([P_1, S_1; \cdots; P_n, S_n]) = \sum_i P_i U(S_i)$$

$\Rightarrow$ Maximum expected utility (MEU) principle:

↳ Choose the action that maximizes expected utility.