5.

Kinetostatics of Serial Robot

5.1) Introduction

⇒ We derive first the relation between the twist of the robot EE and the set of joint rates.
  ↳ Which is given by a linear transformation induced by the robot Jacobian matrix.

⇒ Once the above relation is established for a general six-joint robot, the relation between the static wrench exerted by the environment on the EE and the balancing joint torques is derived by duality

⇒ Three-dimensional workspace is derived.

⇒ An algorithm is proposed for the display of this workspace as pertaining to general orogiand structure whose inverse displacement analysis leads to a quartic polynomial.

⇒ Chapter closes with Kinetostatic performance indices. Their purpose are:-
  ↳ Needed in robot design to help the designer best dimension the links of the robot in the early stage of design process, prior to the elastostatic & the electrodynamic design stage

→ Also needed in the Control of a given robot to ensure an acceptable kinetostatic Performance under feedback Control.

→ Comparison of various Candidate robot when a robotic facility is being planned.

Elastostatic design pertains to the Structural design of robot to ensure that the links and the joint mechanical transmissions will be able to withstand the static load that arise when the robot is in operation.

Elastodynamic design Considers the inertia load of the Structural elements while accounting for link flexibility, which gives rise to mechanical Vibration.

5.2> Velocity analysis of Serial Manipulator

⇒ First, a Serial n-axis manipulator Containing only revolute pair is Considered.

⇒ Then, relations associated with prismatic Pair are introduced.

⇒ Finally, the joint rates of Six-axis manipulators are Calculated in terms of the EE twist.

⇒ We consider manipulator with joint coordinate θ_i, joint rate $\dot{\theta}_i$, and a unit vector $\bar{e}_i$ are associated with each revolute axis.

↳ The $\{X_i\, Y_i\, Z_i\}$ coordinate frame, attached to the $(i-1)^{st}$ link, with origin O_i

⇒ If the angular-velocity vector of i^{th} link is denoted by $\bar{\omega}_i$, then we have,

$$\bar{\omega}_0 = \bar{0}$$

$$\bar{\omega}_1 = \dot{\theta}_1 \bar{e}_1$$

$$\bar{\omega}_2 = \dot{\theta}_1 \bar{e}_1 + \dot{\theta}_2 \bar{e}_2$$

$$\vdots$$

$$\bar{\omega}_n = \dot{\theta}_1 \bar{e}_1 + \dot{\theta}_2 \bar{e}_2 + \cdots + \dot{\theta}_n \bar{e}_n$$

⇒ and if the angular velocity of the EE is denoted by $\bar{\omega}$ then,

$$\bar{\omega} = \bar{\omega}_n = \sum_{i=1}^{n} \dot{\theta}_i \bar{e}_i \quad \text{---} \ (1)$$

⇒ Position vector of point on EE is readily derived as:-

$$\bar{p} = \bar{a}_1 + \bar{a}_2 + \cdots + \bar{a}_n \quad \text{---} \ (2)$$

⇒ Upon differentiating both sid of eq (1) we get

$$\dot{\bar{p}} = \dot{\bar{a}}_1 + \dot{\bar{a}}_2 + \cdots + \dot{\bar{a}}_n \quad \text{---} \ (3)$$

⇒ Since Vector a_i is fixed ~~vector~~ to the i^{th} link,

$$\dot{\bar{a}}_i = \bar{\omega}_i \times \bar{a}_i \quad i = 1, 2 \ldots n \quad \text{— (4)}$$

⇒ Furthermore, Substitution of Eq (1) & (4) into eq (3) yields:-

$$\dot{\bar{p}} = \dot{\theta}_1 \bar{e}_1 \times \bar{a}_1 + (\dot{\theta}_1 \bar{e}_1 + \dot{\theta}_2 \bar{e}_2) \times \bar{a}_2 + \cdots + (\dot{\theta}_1 \bar{e}_1 + \dot{\theta}_2 \bar{e}_2 + \cdots + \dot{\theta}_n \bar{e}_n) \times \bar{a}_n \quad \text{— (5)}$$

⇒ Above Can be rearranged as:-

$$\dot{\bar{p}} = \dot{\theta}_1 \bar{e}_1 \times (\bar{a}_1 + \bar{a}_2 + \cdots + \bar{a}_n) + \dot{\theta}_2 \bar{e}_2 \times (\bar{a}_2 + \bar{a}_3 + \cdots \bar{a}_n) + \cdots + \dot{\theta}_n \bar{e}_n \times \bar{a}_n$$

⇒ Now Vector $\bar{r}_i$ is defined as that joining O_i with P, directed from the former to the latter.

$$\bar{r}_i = \bar{a}_i + \bar{a}_{i+1} + \cdots + \bar{a}_n \quad \text{— (6)}$$

⇒ hence $\dot{\bar{p}}$ can be rewritten as:-

$$\dot{\bar{p}} = \sum_{i=1}^{n} \dot{\theta}_i \bar{e}_i \times \bar{r}_i$$

⇒ Further, let A and B denote the $3 \times n$ matrices defined as:-

$$A = [\bar{e}_1 \, \bar{e}_2 \ldots \bar{e}_n] \quad \text{— (7)}$$

$$B = [\bar{e}_1 \times \bar{r}_1, \bar{e}_2 \times \bar{r}_2, \ldots, \bar{e}_n \times \bar{r}_n]$$

the n-dimensional joint-rate vector $\dot{\bar{\theta}}$ being defined as:-

$$\dot{\bar{\theta}} = [\dot{\theta}_1\ \dot{\theta}_2 \cdots \dot{\theta}_n]^T$$

⇒ Thus $\bar{\omega}$ and $\dot{\bar{p}}$ can be expressed in a more compact form as:-

$$\bar{\omega} = \bar{\bar{A}}\dot{\theta}$$

$$\dot{\bar{p}} = \bar{\bar{B}}\dot{\theta}$$

⇒ the twist of the EE being defined, in turn as:-

$$\bar{t} = \begin{bmatrix} \bar{\omega} \\ \dot{\bar{p}} \end{bmatrix} \quad \text{—— (8)}$$

⇒ The EE twist is thus linearly related to the joint-rate vector $\dot{\bar{\theta}}$

$$\boxed{\bar{\bar{J}}\dot{\bar{\theta}} = \bar{t}} \quad \text{—— (9)}$$

⇒ where $\bar{\bar{J}}$ is the <u>jacobian matrix</u>.

→ First introduced by Whitney (1972)

→ It is $6 \times n$ matrix.

$$\bar{\bar{J}} = \begin{bmatrix} \bar{\bar{A}} \\ \bar{\bar{B}} \end{bmatrix}$$

$$\bar{\bar{J}} = \begin{bmatrix} \bar{e}_1 & \bar{e}_2 & \cdots & \bar{e}_n \\ \bar{e}_1 \times \bar{r}_1 & \bar{e}_2 \times \bar{r}_2 & \cdots & \bar{e}_n \times \bar{r}_n \end{bmatrix} \quad \text{—— (10)}$$

⇒ Apparently, an alternative definition of the foregoing Jacobian matrix can be given as :-

$$\bar{\bar{J}} = \frac{\delta \bar{t}}{\delta \dot{\bar{\theta}}} \quad \text{—— (11)}$$

If $\bar{j}_i$ denotes i^{th} column of $\bar{\bar{J}}$ then,

$$\bar{j}_i = \begin{bmatrix} \bar{e}_i \\ \bar{e}_i \times \bar{r}_i \end{bmatrix} \quad \text{—— (12)}$$

⇒ It is noteworthy that if the axis of the i^{th} revolute is denoted by R_i then $\bar{j}_i$ is nothing but the Plücker array of that line, with the moment of R_i being taken with respect to the operator point P of the EE.

⇒ If i^{th} pair is prismatic, then the $(i-1)^{st}$ and i^{th} links have the same angular velocity.

Hence,

$$\bar{\omega}_i = \bar{\omega}_{i-1}, \qquad \dot{\bar{a}}_i = \bar{\omega}_{i-1} \times \bar{a}_i + b_i \bar{e}_i$$

⇒ One can readily prove in this case that :-

$$\bar{\omega} = \dot{\theta}_1\bar{e}_1 + \dot{\theta}_2\bar{e}_2 + \cdots \dot{\theta}_{i-1}\bar{e}_{i-1} + \dot{\theta}_{i+1}\bar{e}_{i+1} + \cdots + \dot{\theta}_n\bar{e}_n$$

$$\dot{P} = \dot{\theta}_1\bar{e}_1\times\bar{r}_1 + \dot{\theta}_2\bar{e}_2\times\bar{r}_2 + \dot{\theta}_{i-1}\bar{e}_{i-1}\times\bar{r}_{i-1} + \dot{b}_i\bar{e}_i + \dot{\theta}_{i+1}\bar{e}_{i+1}\times\bar{r}_{i+1} + \cdots + \dot{\theta}_n\bar{e}_n\times\bar{r}_n$$

⇒ If Vector $\dot{\bar{\theta}}$ is now defined as

$$\dot{\bar{\theta}} = [\dot{\theta}_1 \ \dot{\theta}_2 \ \cdots \ \dot{\theta}_{i-1} \ \dot{b}_i \ \dot{\theta}_{i+1} \cdots \dot{\theta}_n]$$

the i^{th} Column of J then changing to

$$\bar{J}_i = \begin{bmatrix} \bar{0} \\ \bar{e}_i \end{bmatrix}$$

Plücker array of the axis of the i^{th} joint, is that of a line at infinity lying in a plane normal to the unit Vector $\bar{e}_i$

⇒ In general, $\bar{\bar{J}}_A$ denotes the Jacobian defined for a point A of the EE and $\bar{\bar{J}}_B$ that defined for another point B, then the relation between $\bar{\bar{J}}_A$ and $\bar{\bar{J}}_B$ is

$$\boxed{\bar{\bar{J}}_B = \bar{\bar{U}}\,\bar{\bar{J}}_A} \quad \text{—— (13)}$$

Where the 6×6 matrix $\bar{\bar{U}}$ is defined as:-

$$\bar{\bar{U}} = \begin{bmatrix} \bar{\bar{I}} & \bar{\bar{0}} \\ \bar{A}-\bar{B} & \bar{\bar{I}} \end{bmatrix}$$

Scanned by CamScanner

Where, $\bar{\bar{A}}$ & $\bar{\bar{B}}$ are Cross-product matrices of the position vectors $\bar{a}$ and $\bar{b}$ of points A and B respectively.

<u>Theorem 5.2.1</u>: The determinant of the Jacobian matrix of a six-axis manipulator is not affected under a change of operation point of the EE.

$$\det(\bar{\bar{J}}_B) = \det(\bar{\bar{J}}_A)$$

⇒ Equation (13) is called <u>Jacobian transfer Matrix</u>

⇒ In particular, for six-axis manipulators, $\bar{\bar{J}}$ is a 6×6 matrix. Whenever this matrix is non-singular eq (5) can be solved for $\dot{\Theta}$.

$$\boxed{\dot{\Theta} = \bar{\bar{J}}^{-1} E} \quad \text{— (14)}$$

This is solved using Gauss-elimination algorithm or LU Decomposition.

⇒ Gaussian elimination produces the solution by recognizing that a system of linear equations is most easily solved when it is in either <u>upper or lower-triangular form.</u>

Scanned by CamScanner

⇒ To exploit this fact, matrix $\bar{\bar{J}}$ is decomposed into the unique $\bar{L}$ & $\bar{U}$ factor in the form :-

$$\bar{\bar{J}} = \bar{L}\,\bar{U}$$

Upper-triangular

Lower-triangular

⇒ Thus, the unknown vectors of joint rates can now be computed from two triangular systems, namely,

$$\boxed{\bar{L}\bar{y} = \bar{t}} \quad — (15)$$

$$\boxed{\bar{U}\dot{\bar{\theta}} = \bar{y}} \quad — (16)$$

⇒ The Solution of a system of n linear equation in n unknowns, using the LU-decomposition method, can be accomplished with M_n multiplications & A_n additions :-

$$M_n = \frac{n}{6}(2n^2 + 9n + 1)$$

$$A_n = \frac{n}{3}(n^2 + 3n - 4)$$

$\left\{\dfrac{\text{Dahlquist \& Bjorck}}{1974}\right\}$

eg $\left\{ \begin{array}{l} M_6 = 127 \\ A_6 = 100 \end{array} \right\}$

5.2.1) Decoupled Manipulator

⇒ For manipulator of this type of architecture, it is more convenient to deal with the velocity of the center C of the wrist.
↳ Than with that of the operation point P.

⇒ Thus,

$$\bar{t}_c = \bar{\bar{J}}\,\dot{\bar{\theta}}$$

where, $\bar{t}_c$ is defined as

$$\bar{t}_c = \begin{bmatrix} \bar{\omega} \\ \dot{\bar{c}} \end{bmatrix}$$

⇒ and can be obtained from $\bar{t}_p = [\bar{\omega}^T\ \dot{\bar{p}}^T]^T$ using the twist-transfer formula :-

$$\bar{t}_c = \begin{bmatrix} \bar{\bar{I}} & \bar{\bar{O}} \\ \bar{\bar{P}} - \bar{\bar{C}} & \bar{\bar{I}} \end{bmatrix} \bar{t}_p$$

with $\bar{C}$ & $\bar{P}$ defined as the cross-product matrices of the position vectors $\bar{c}$ & $\bar{p}$ respectively.

⇒ Since C is on the last three joint axes, its velocity is not affected by the motion of the last three joints, and hence, we can write :-

$$\bar{C} = \dot{\theta}_1 \bar{e}_1 \times \bar{r}_1 + \dot{\theta}_2 \bar{e}_2 \times \bar{r}_2 + \dot{\theta}_3 \bar{e}_3 \times \bar{r}_3$$

{ $\bar{r}_i$ is defined as that directed from O_i to C }

$\Rightarrow$ On the