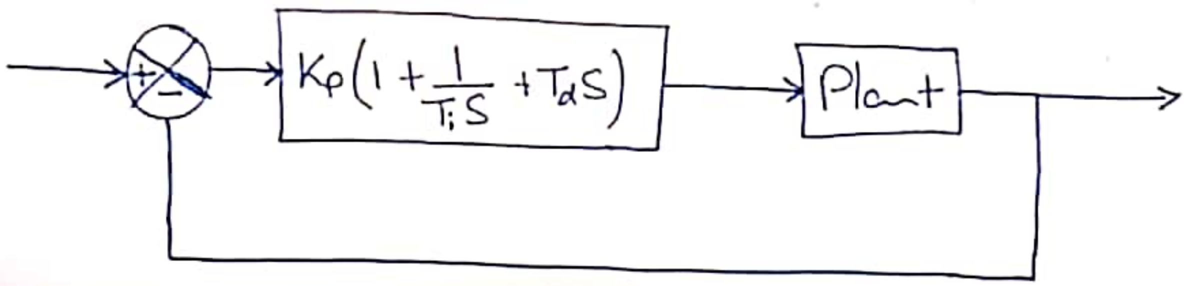
6
PID Controllers and Modified PID Controllers

★ Introduction

⇒ It is interesting to note that more than half of the industrial controllers in use today are PID Controller or modified PID Controllers.

★ Ziegler–Nichols rules for Tuning PID Controllers



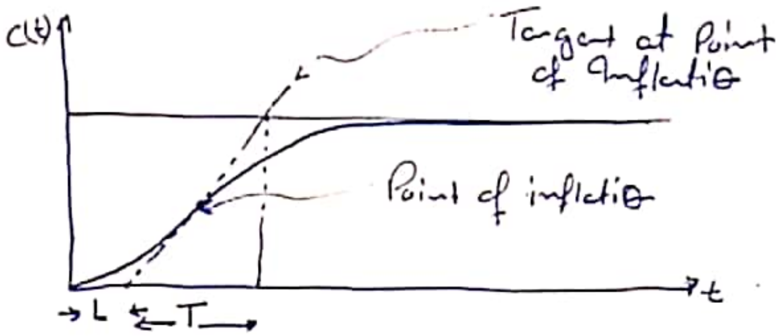
⇒ If mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining parameters of the Controller that will meet the transient and steady-state. Specification of the the closed loop system.

↳ If the plant is so complicated that its mathematical model cannot be easily obtained, then we must resort to experimental approaches to the tuning of PID Controller.

⇒ There are two methods called Ziegler–Nichols tuning rules: the first method and the second method.

First Method

⇒ First we obtain experimentally the response of the Plant to a unit-step input.



→ This method applies if the response to a step input exhibits an S-shaped Curve.

Type of Controller	K_p	T_i	T_d
P	T/L	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

$$\boxed{G_c(s) = \frac{0.6T\left(s+\frac{1}{L}\right)^2}{s}}$$

Second Method

⇒ In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using proportional control action only, increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations.

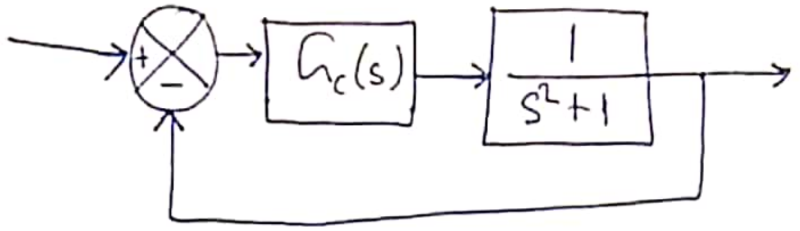
↳ If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.

⇒ Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally determined.

Type of Controller	K_p	T_i	T_d
P	$0.5\,K_{cr}$	∞	0
PI	$0.45\,K_{cr}$	$\frac{1}{1.2} P_{cr}$	0
PID	$0.6\,K_{cr}$	$0.5\,P_{cr}$	$0.125\,P_{cr}$

$$G_c(s) = 0.075\,K_{cr} P_{cr} \frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}$$

★ Design of PID Controllers With Frequency Response Approach



Using a frequency-response approach, design a PID Controller such that the static velocity error constant is 4 sec⁻¹; Phase margin is 50° or more and gain margin 10dB or more.

Let us choose the PID Controller to be

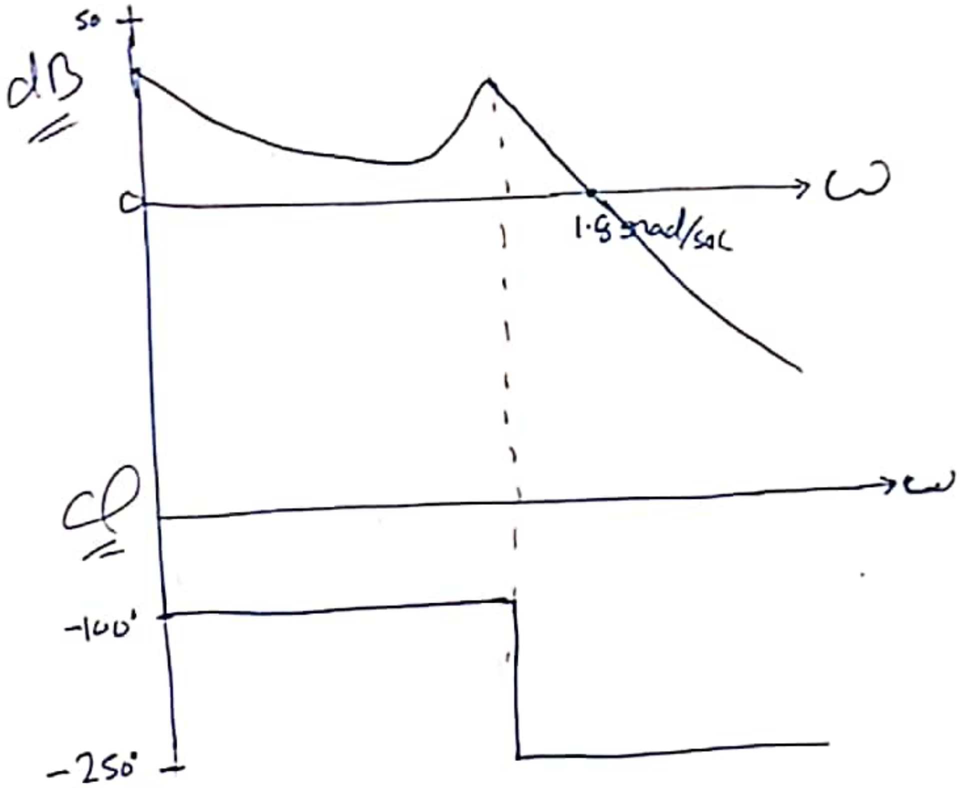
$$G_c = \frac{K(as+1)(bs+1)}{s}$$

$$K_v = \lim_{s\to 0} G_c(s)\frac{1}{s^2+1} = \lim_{s\to 0} s\,\frac{K(as+1)(bs+1)}{s(s^2+1)} = K = 4$$

So $G_c(s) = \dfrac{4(as+1)(bs+1)}{s}$

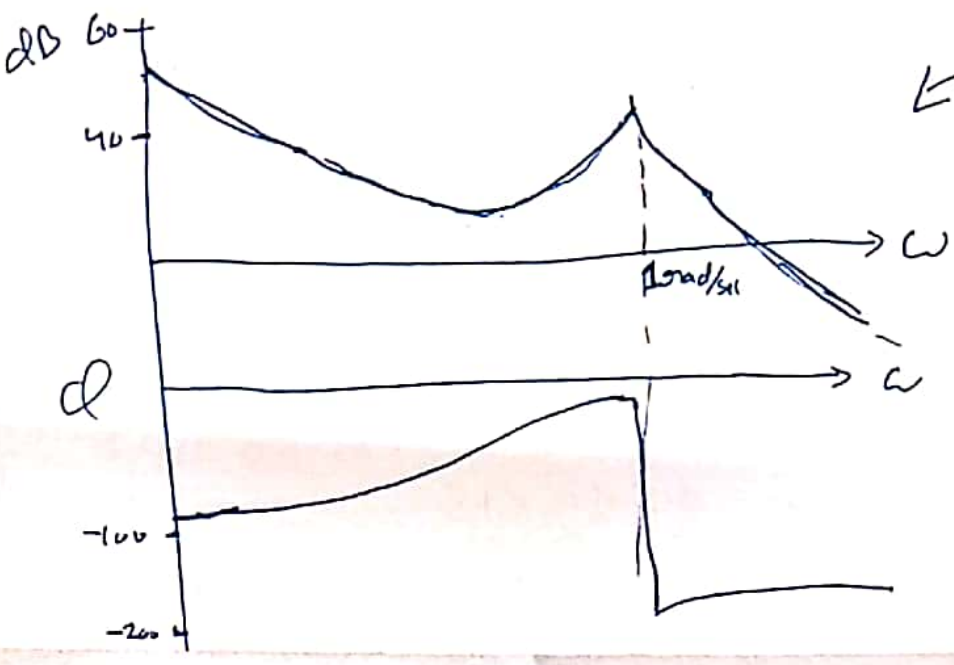
Let us plot a Bode diagram of

$$G(s) = \frac{4}{s(s^2+1)}$$



⇒ Let us assume the gain crossover frequency of the compensated system to be somewhere between $\omega = 1$ and $\omega = 10$ rad/s.

↳ We choose $a = 5$. Then $(as+1)$ will contribute up to $90°$ phase lead in the high frequency region



$$\left|\frac{4(5s+1)}{s(s^2+1)}\right|$$

⇒ The term $(bs+1)$ needs to give phase margin of at least $50°$. We find $b=0.25$ give phase margin $50°$ & gain margin $+\infty$.

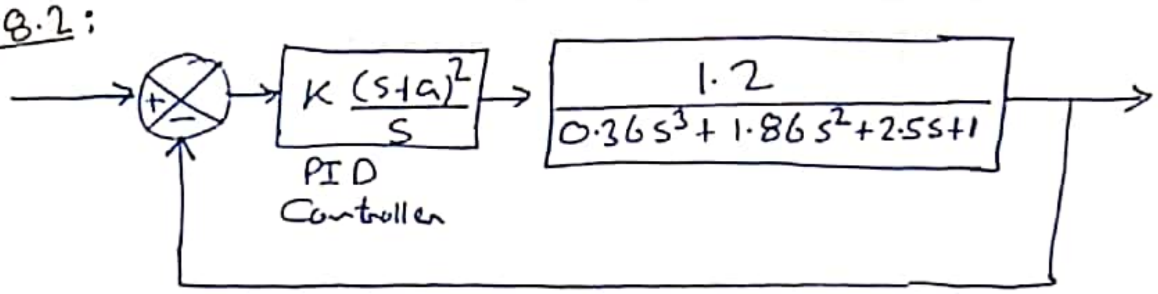
So $G_c(s) = \dfrac{4(5s+1)(0.25s+1)}{s}$

Open loop TF of designed System $= \dfrac{4(5s+1)(0.25s+1)}{s} \times \dfrac{1}{s^2+1}$

$$= \frac{5s^2+21s+4}{s^3+s}$$

★ Design of PID Controllers with Computational Optimization Approach

Example 8.2:



It is desired to find a Combination of K & a Such that the Closed-loop System will have 10% maximum overshoot in the unit step response.

⇒ To Solve this problem, we first specify the region to Search for appropriate K and a.

↳ Assume that the region to search for K & a is

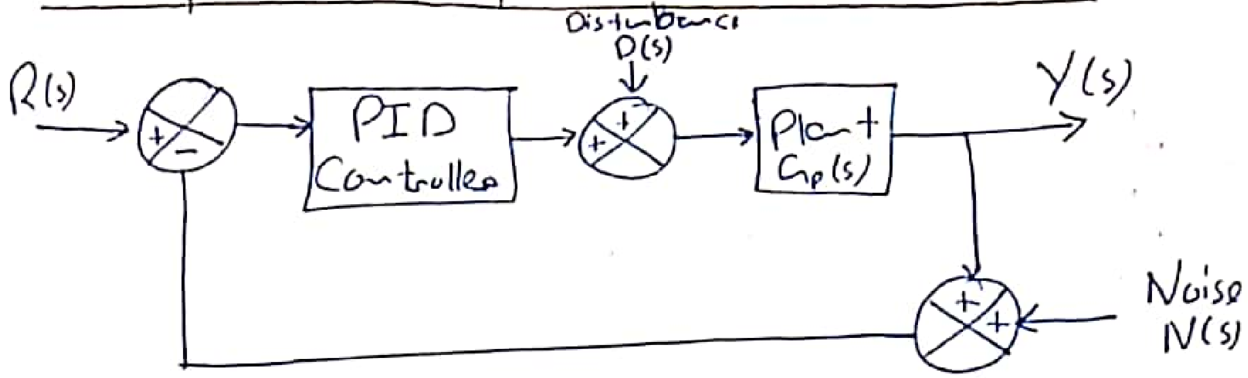
$$2 \leq K \leq 3 \quad \& \quad 0.5 \leq a \leq 1.5$$

⇒ If solution does not exist in the region, then we need to expand it.

⇒ In the Computational approach we need to determine the step size for each of K and a.

$$\boxed{\text{Solution found } K=2 \quad a=0.9}$$

★ Modifications of PID Control Schemes



PID - Controlled System

⇒ If the reference input is a step function, then; because of the presence of the derivative term in the Control action, the manipulated variable u(t) will involve impulse function.

⇒ In an actual PID Controller, instead of the pure derivative term $T_d s$, we employ

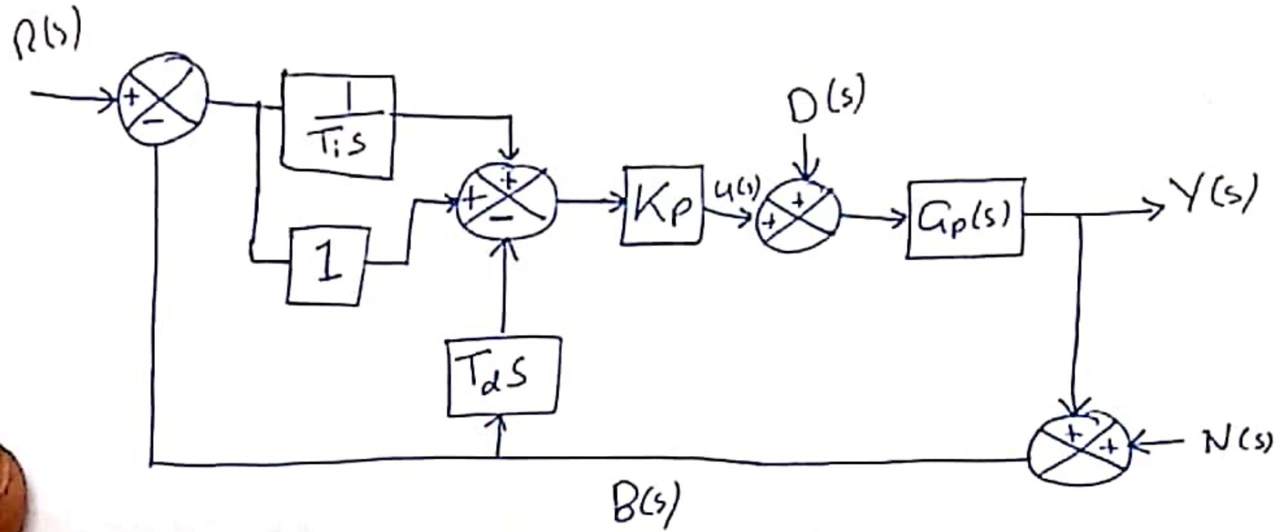
$$\frac{T_d s}{1+\gamma T_d s} \quad \{\text{where } \gamma \text{ is around } 0.1\}$$

⇒ Now, when the reference input is a step function the manipulated variable u(t) will not involve an impulse function, but will involve a sharp pulse function.

↳ Such a phenomenon is called Set-point Kick.

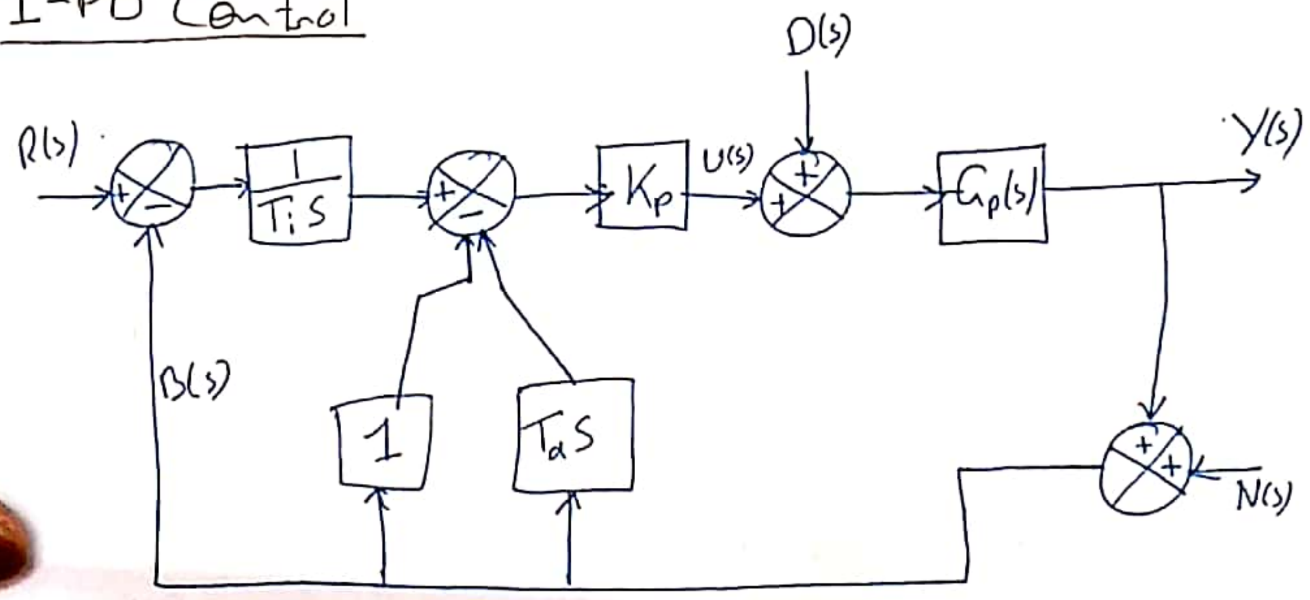
PI-D Control

To avoide the set-point kick phenomenon, we may wish to operate the derivative action only in the feedback path so the differentiation occurs only on the feedback signal and not on the reference signal.



$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

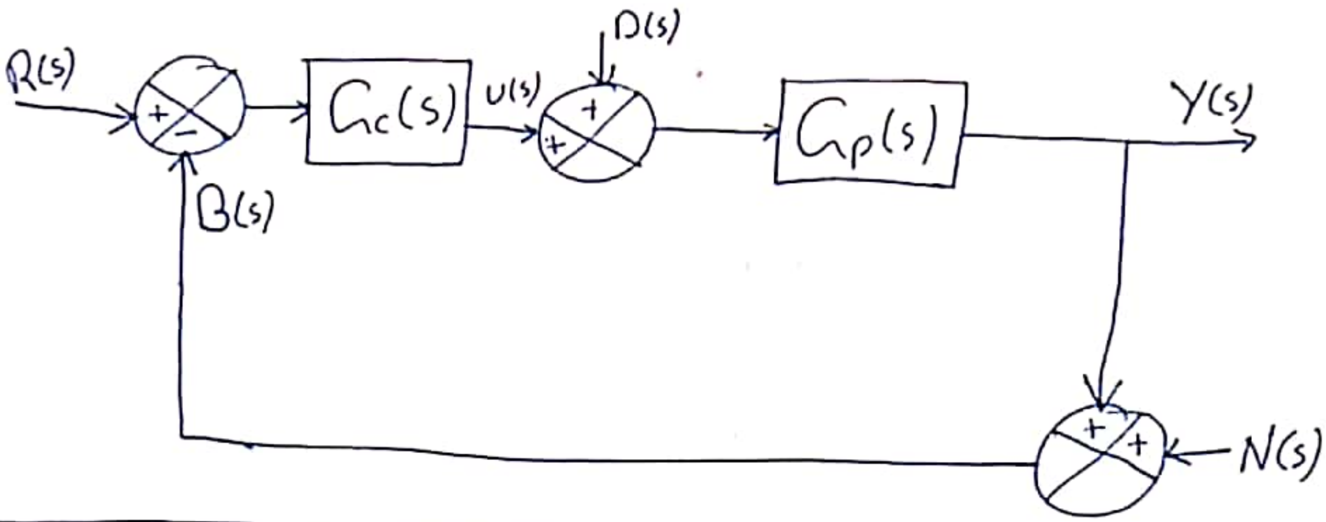
I-PD Control



$$U(s) = \frac{K_p}{T_i s}R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

★ Two degrees of Freedom Control

⇒ Consider the System, where the System is Subjected to the disturbance input $D(s)$ and noise input $N(s)$, in addition to the reference input $R(s)$.



$$\boxed{Y(s) = \frac{G_cG_p}{1+G_cG_p}R(s) + \frac{G_p}{1+G_cG_p}D(s) + \frac{G_cG_p}{1+G_cG_p}N(s)}$$

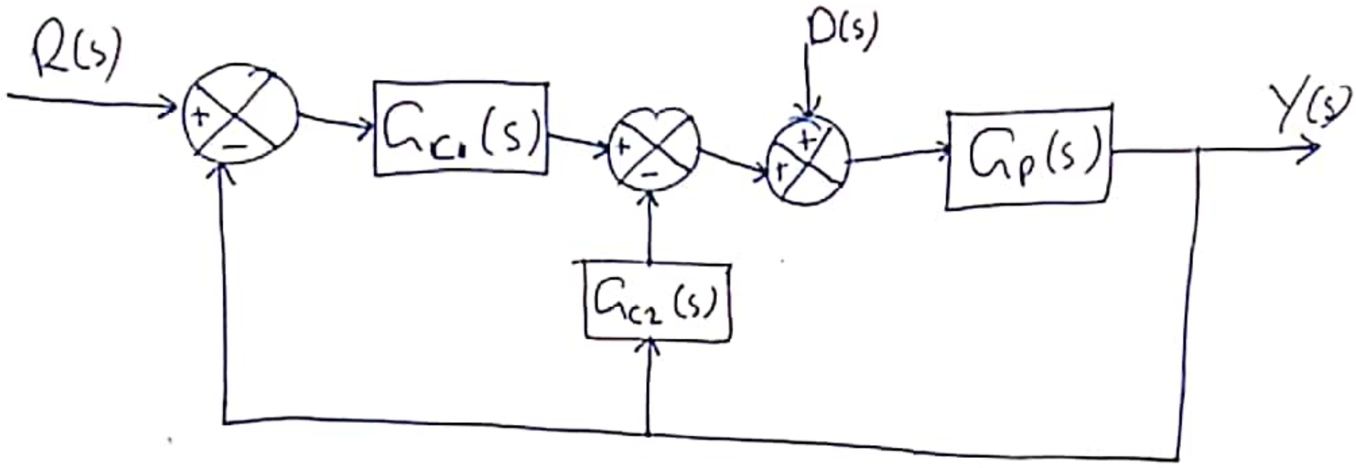
★ Zero-Placement Approach to Improve Response Characteristics

⇒ In high-performance Control System it is always desired that the System output follow the changing input with minimum error.

↳ For Step, ramp and acceleration input, it is desired that the System output exhibit no steady-state error.

⇒ Consider the two DOF Control System Shown:

$$G_p = K\frac{A(s)}{B(s)}$$



Zeso

$$A(s) = (S+Z_1)(s+Z_2)\cdots(S+Z_m)$$
$$B(s) = S^N(s+P_{N+1})(S+P_{N+2})\cdots(S+P_m)$$

⇒ Let us Assume G_{c1} is a PID Controller followed by a filter $1/A(s)$.

$$G_{c1}(s) = \frac{\alpha_1 S + \beta_1 + \gamma_1 S^2}{S}\,\frac{1}{A(s)}$$

and G_{c2} is a PID, PI, PD, I, D or P Controller followed by a $1/A(s)$ filter.

$$G_{c2}(s) = \frac{\alpha_2 S + \beta_2 + \gamma_2 S^2}{S}\,\frac{1}{A(s)}$$

⇒ Then $G_{c1}(s) + G_{c2}(s) = \dfrac{\alpha S + \beta + \gamma S^2}{S}\,\dfrac{1}{A(S)}$

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+(G_{c1}+G_{c2})G_p} = \frac{SKA(s)}{SB(s) + (\alpha S+\beta+\gamma S^2)K}$$

$$\frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1+(G_{c1}+G_{c2})G_p}$$

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Zero placement

Consider the System

$$\frac{Y(s)}{R(s)} = \frac{P(s)}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

If we choose $P(s)$ as

$$P(s) = a_2 s^2 + a_1 s + a_0 = a_2 (s + s_1)(s + s_2)$$

→ The numerator polynomial $P(s)$ is equal to the sum of the last three terms of the denominator polynomial - then the system will exhibit no steady state error in response to the step input, ramp input & acceleration input.