Motion model and its Jacobian

Given: $X_{t-1} = \begin{bmatrix} X \\ m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix}$ $\quad U_t = \begin{bmatrix} trans\_x \\ trans\_y \\ rot \end{bmatrix}$ $\quad X = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

$m_i = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$

To find: $P(x_t \mid U_t X_{t-1})$

$\hookrightarrow$ We want this to be a gaussian with $E[X_t] = \overline{M}_t$ and $Var(X_t) = \Sigma_t$

{Deterministic Law}

$$\boxed{\begin{aligned} x_t &= x_{t-1} + \{trans\_x * \cos(\theta_{t-1})\} - \{trans\_y * \sin(\theta_{t-1})\} \\ y_t &= y_{t-1} + \{trans\_x * \sin(\theta_{t-1})\} + \{trans\_y * \cos(\theta_{t-1})\} \\ \theta_t &= \theta_{t-1} + rot \end{aligned}}$$

$\hookrightarrow$ Non Linear $\quad \{g : R^3 \to R^3\}$

$$X_t = g(X_{t-1}, U_t)$$

$$\approx g(U_t, M_{t-1}) + \frac{\delta g(U_t, M_{t-1})}{\delta x_{t-1}} (X_{t-1} - M_{t-1})$$

{First order Taylor Expantion}

$$\frac{\delta g(u_t, \mu_{t-1})}{\delta x_{t-1}} = \begin{pmatrix} \frac{\delta g_1}{\delta x_{t-1}} \\ \frac{\delta g_2}{\delta x_{t-1}} \\ \frac{\delta g_3}{\delta x_{t-1}} \end{pmatrix} = \begin{pmatrix} \frac{\delta g_1}{\delta x_{t-1,x}} & \frac{\delta g_1}{\delta x_{t-1,y}} & \frac{\delta g_1}{\delta x_{t-1,\theta}} \\ \frac{\delta g_2}{\delta x_{t-1,x}} & \frac{\delta g_2}{\delta x_{t-1,y}} & \frac{\delta g_2}{\delta x_{t-1,\theta}} \\ \frac{\delta g_3}{\delta x_{t-1,x}} & \frac{\delta g_3}{\delta x_{t-1,y}} & \frac{\delta g_3}{\delta x_{t-1,\theta}} \end{pmatrix}$$

$$G \in R^{3\times3}$$

$G_{11} = 1$ $G_{12} = 0$ $G_{13} = -\text{trans\_x} \sin(\mu\_\theta_{t-1}) - \text{trans\_y} \cos(\mu\_\theta_{t-1})$

$G_{21} = 0$ $G_{22} = 1$ $G_{23} = \text{trans\_x} \cos(\mu\_\theta_{t-1}) - \text{trans\_y} \sin(\mu\_\theta_{t-1})$

$G_{31} = 0$ $G_{32} = 0$ $G_{33} = 1$

$$\boxed{x_t = g(u_t, \mu_{t-1}) + G(u_t, \mu_{t-1})(x_{t-1} - \mu_{t-1})}$$

{Deterministic motion model Linearized}

⇒ In full state space dimention:

$$x_t = x_{t-1} + \{trans\_x * \cos(\theta_{t-1})\} - \{trans\_y * \sin(\theta_{t-1})\}$$

$$y_t = y_{t-1} + \{trans\_x * \sin(\theta_{t-1})\} + \{trans\_y * \cos(\theta_{t-1})\}$$

$$\theta_t = \theta_{t-1} + rot$$

$$m_{1,x,t} = m_{1,x,t-1}$$

$$m_{1,y,t} = m_{1,y,t-1}$$

$$m_{1,z,t} = m_{1,z,t-1}$$

$$\vdots \qquad \vdots$$

$$G_{11} = 1 \qquad G_{22} = 0 \qquad G_{13} = -(trans\_x * \sin(\mu\_\theta_{t-1}) + trans\_y * \cos(\mu\_\theta_{t-1}))$$

$$G_{14} = 0 \qquad G_{15} = 0 \cdots G_{1,3+3n} = 0$$

$$G = \begin{bmatrix} \overset{3\times3}{G} & 0 & 0 & \cdots \\ 0 & I & & \\ & & I & \\ 0 & & & \ddots \end{bmatrix}$$

$$\boxed{X_t = g + G(X_{t-1} - \mu_{t-1})}$$

$\Rightarrow$ For probablistic motion model we add a Zero mean ~~deterministic~~ random ~~vector~~ vector $\varepsilon(u_t)$ to the deterministic model.

$\hookrightarrow$ This has a Covariance of R distributed normally.

$$\boxed{X_t = g(u_t, \mu_{t-1}) + G(u_t, \mu_{t-1})(X_{t-1} - \mu_{t-1}) + \varepsilon(u_t)}$$

{Probablistic motion model}

$$R = \left(\begin{array}{ccc|ccc}
\sigma_{xx} & 0 & 0 & 0 & 0 & \cdots \\
0 & \sigma_{yy} & 0 & 0 & 0 & \cdots \\
0 & 0 & \sigma_{\theta\theta} & 0 & 0 & \cdots \\
\hline
0 & 0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{xx}^{min} + \alpha_1 \sqrt{trans\_x^2 + trans\_y^2} + \alpha_2 |rot|$$

$$\sigma_{\theta\theta} = \sigma_{\theta\theta}^{min} + \alpha_3 \sqrt{trans\_x^2 + trans\_y^2} + \alpha_4 |rot|$$

Prediction Steps

1. $\overline{\mu}_t = g(u_t, \mu_{t-1})$

2. $\overline{\Sigma}_t = G \cdot \Sigma_{t-1} G^T + R$

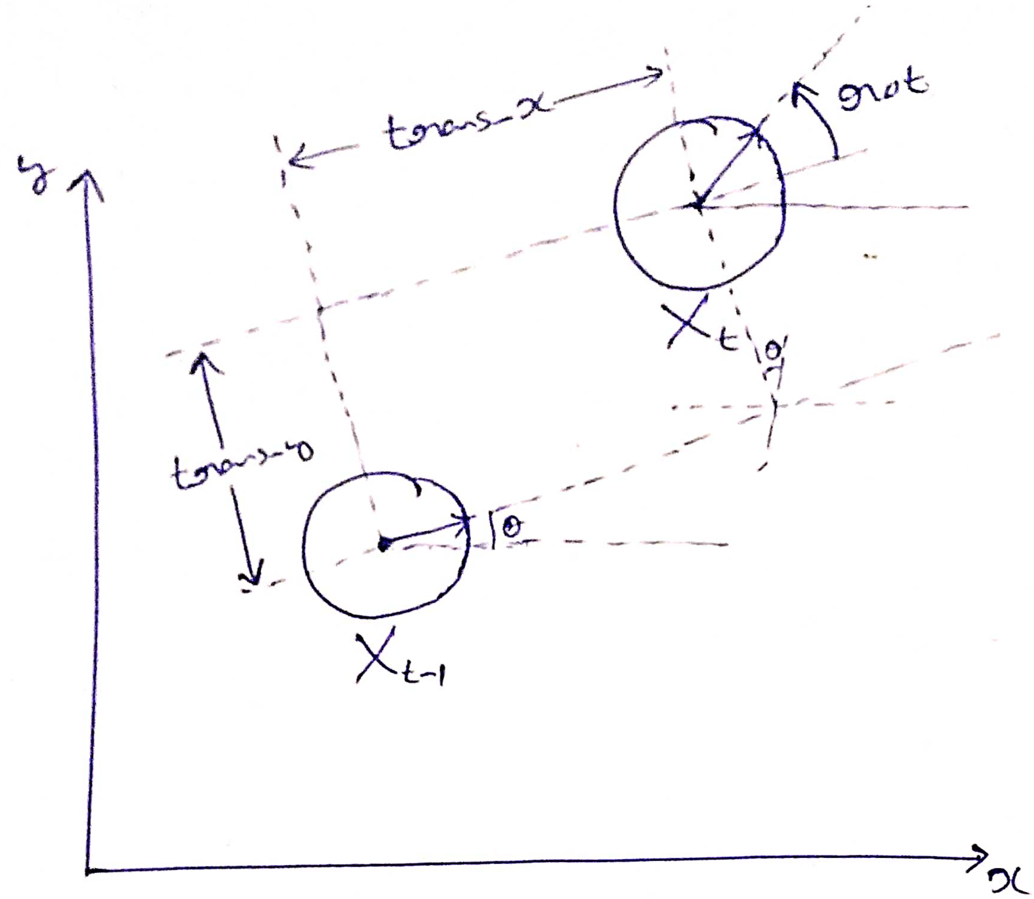
$$G = \begin{bmatrix} G_{3\times3} & O_{3\times3n} \\ O_{3n\times3} & I_{3n\times3n} \end{bmatrix} \quad G^T = \begin{bmatrix} G^T_{3\times3} & O_{3\times3n} \\ O_{3n\times3} & I_{3n\times3n} \end{bmatrix}$$

$$\Sigma_{t-1} = \begin{bmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{bmatrix} \quad R = \begin{bmatrix} R_{3\times3} & O_{3\times3n} \\ O_{3n\times3} & O_{3n\times3n} \end{bmatrix}$$

$$\overline{\Sigma}_t = \begin{bmatrix} G_{3\times3}\Sigma_{xx}G^T_{3\times3} & G_{3\times3}\Sigma_{xm} \\ \Sigma_{mx}G^T_{3\times3} & \Sigma_{mm} \end{bmatrix} + \begin{bmatrix} R_{3\times3} & O_{3\times3n} \\ O_{3n\times3} & O_{3n\times3n} \end{bmatrix}$$

$$\Sigma_{t-1}G^T = \begin{bmatrix} \Sigma_{xx}G^T_{3\times3} & \Sigma_{xm} \\ \Sigma_{mx}G^T_{3\times3} & \Sigma_{mm} \end{bmatrix}$$

$$G\Sigma_{t-1}G^T = \begin{bmatrix} G_{3\times3}\Sigma_{xx}G^T_{3\times3} & G_{3\times3}\Sigma_{xm} \\ \Sigma_{mx}G^T_{3\times3} & \Sigma_{mm} \end{bmatrix}$$



y
x
trans_x
trans_y
rot
θ
θ
X_t
X_{t-1}