(11)

Probability

→ Random variable
→ Joint and Marginal Distributions
→ Conditional Distributions
→ Product Rule, Chain Rule, Bayes' Rule
→ Inference
→ Independence

Observed variables (evidence)
↳ Agent knows certain things about the state of the world (e.g. sensor reading)

Unobserved variables
↳ Agent needs to reason about other aspects

Model
↳ Agent knows something about how the known variables relate to the unknown variable.

⇒ Probabilistic reasoning gives us a framework for managing our beliefs & knowledge.

★ Random Variables

⇒ A random variable is some aspect of the world about which we (may) have uncertainity.

↳ We denote random variable with Capital letters.

★ Probability Distribution

⇒ Associate a probability with each value.

⇒ Unobserved random variable have distributions.

★ Joint Distributions

⇒ A joint distribution over a set of random variables $X_1, X_2 \dots X_n$ specifies a real number for each assignment (or outcome)

$$P(X_1 = x_1, X_2 = x_2 \dots X_n = x_n)$$

or

$$P(x_1, x_2 \dots x_n)$$

★ Probabilistic Models

⇒ A probabilistic model is a joint distribution over a set of random variables.

★ Events

⇒ A event is a set E of outcomes

$$P(E) = \sum_{(x_1, \dots x_n) \in E} P(x_1, \dots x_n)$$

⇒ Typically, the events we care about are partial assignments.

★ Marginal Distributions

⇒ Marginal distributions are sub-table ~~the~~ which eliminate variables.

⇒ Marginalization (Summing up)

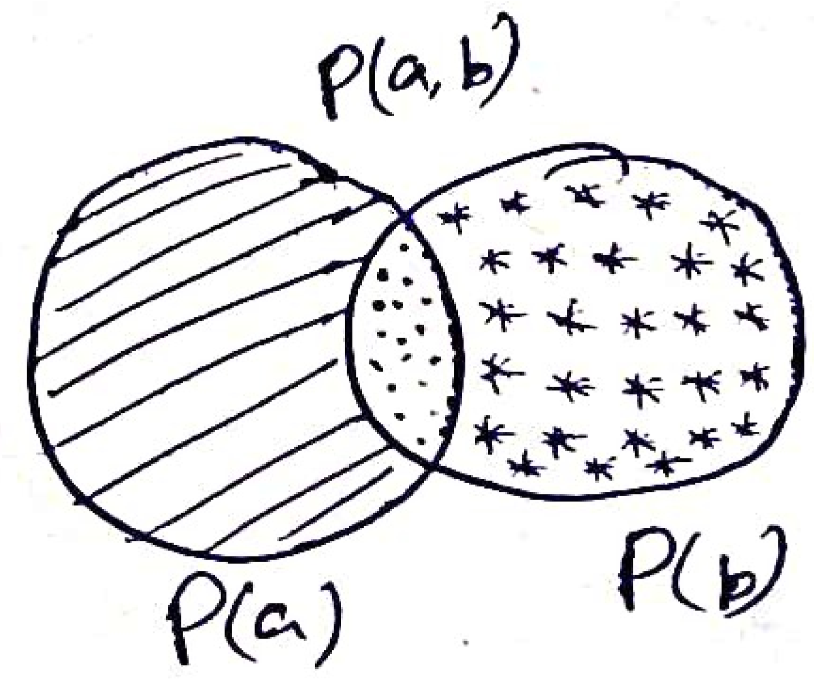
↳ Combine Collapsed rows by adding

$$P(t) = \sum_{s} P(t, s)$$

★ Conditional Probabilities

⇒ A simple relation between joint & conditional probability:

$$P(a|b) = \frac{P(a,b)}{P(b)}$$



{In fact, this is taken as the defination of a Conditional probability}

★ Conditional Distributions

⇒ Conditional distributions are probability distributions over some variable given fixed values of others.

★ Normalization Trick

1. Select the joint probabilities matching the evidence
2. Normalize the selection (make it sum to 1)

* Probablistic Inference

↳ Compute a desired probability from other known probabilities.

→ We generally compute Conditional probablities

→ Probabilities change with new evidence.

(11) Inference by Enumeration

- Evidence Variable $E_1, \dots E_k = e_1, \dots e_k$
- Query* Variable Q
- Hidden variable $H_1, \dots H_n$

$\Big\}$ $X_1, X_2 \dots X_n$ All Variables

⇒ We want:

$$P(Q \mid e_1 \dots e_k)$$

Step 1: Select the entries consistent with evidence

Step 2: Sum out H to get joint of Query & evidence

Step 3: Normalize

* The Product Rule

→ Sometime we have Conditional distributions but want the joint

$$P(x, y) = P(y) \, P(x \mid y)$$

* The chain Rule

⇒ More generally, can always write any joint distribution as an incremental product of conditional distributions

$$P(x_1, x_2 \dots x_3) = P(x_1)\, P(x_2 \mid x_1)\, P(x_3 \mid x_1, x_2)$$

$$P(x_1; x_2 \dots x_n) = \prod_i P(x_i \mid x_1 \dots x_{i-1})$$

* Bayes Rule

$$\boxed{P(x \mid y) = \frac{P(y \mid x)}{P(y)}\, P(x)}$$