Relative Orientation: Fundamental and Essential Matrix

★ **Camera Pair**

- → A stereo camera
- → One camera that moves

{ Two configurations from which images have been taken }

★ **Orientation parameters for the camera pair & Relative Orientation**

⇒ The orientation of the camera pair can be described using independent orientations for each camera.

- → Calibrated camera: $6 + 6 = 12$ parameters
- → Uncalibrated camera: $11 + 11 = 22$ parameters.

★ **Can we estimate the camera motion without knowing the scene?**

→ Which parameters can we obtain and which not?

→ We cannot obtain the (global) translation and rotation as well as the scale.

★ What we can compute

→ The rotation R of the second camera wrt the first one (3 parameters)

→ The direction B of the line connecting the two center of projection (2 parameters)

→ We donot know their distance

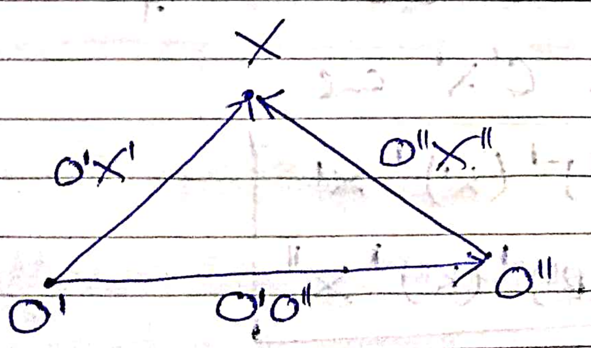
★ For Uncalibrated Cameras

⇒ Projective transform (15 parameters)

⇒ Thus, for uncalibrated cameras, we can only obtain 22-15 = 7 parameters given two images.

⇒ We need at least 5 points in 3D for the absolute orientation.

★ Coplanarity Constraint for straight-Line preserving (Uncalibrated) Cameras to Obtain the Fundamental matrix



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⇒ Consider perfect orientation and the intersection of two corresponding rays:

⇒ The rays lie within one plane in 3D:

⇒ Coplanarity can be expressed by:

$$[O'x' \quad O'O'' \quad O''x''] = 0$$

{Scalar triple Product}

{x' & x'' refers to the same point X in space}

⇒ The direction of the vectors $O'X$ and $O''X''$ can be derived from the image coordinates x', x''.

$\boxed{x' = P'X}$ $\boxed{x'' = P''X}$

$\boxed{P' = K'R'[I_3 | -X_{O'}]}$ $\boxed{P'' = K''R''[I_3 | -X_{O''}]}$

⇒ The normalized directions of the vectors $O''x''$ and $O'x'$ are

$$\boxed{\begin{aligned} {}^{n}x' &= (R')^{-1}(K')^{-1}x' \\ {}^{n}x'' &= (R'')^{-1}(K'')^{-1}x'' \end{aligned}}$$

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⇒ The base vector $O'O''$ directly results from the coordinates of the projection centers:

$$b = B = X_{O''} - X_{O'}$$

⇒ From Coplanarity Constraint we get:-

$$\Rightarrow [O'x' \ O'O'' \ O''x''] = 0$$

$$\Rightarrow [{}^{n}x' \ b \ {}^{n}x''] = 0$$

$$\Rightarrow {}^{n}x' \cdot (b \times {}^{n}x'') = 0$$

$$\Rightarrow \boxed{{}^{n}x'^{T} S_b \, {}^{n}x'' = 0}$$

Skew-Symmetric matrix

$$\Rightarrow \underbrace{\underline{x'^{T} (K')^{-T} (R')^{-T}} S_b \underline{(R'')^{-1} (K'')^{-1}}}_{F} x'' = 0$$

$$\Rightarrow F = (K')^{-T} (R')^{-T} S_b (R')^{-1} (K'')^{-1}$$

$$\Rightarrow \boxed{F = (K')^{-T} R' S_b R''^{T} (K'')^{-1}}$$

This is called fundamental matrix

⇒ It allows for expressing the coplanarity constraint by

$$\boxed{x'^{T} F x'' = 0}$$

★ Fundamental Matrix

⇒ The fundamental matrix is the matrix that fulfills the equation

$$\boxed{x'^T F x'' = 0}$$

for corresponding points.

⇒ The fundamental matrix contains all the available information about the relative orientation of two images from uncalibrated cameras.

★ Fundamental Matrix from the camera projection Matrices

⇒ If the projection matrices are given we can derive the fundamental matrix?

$$P', P'' \longrightarrow F$$

⇒ Let the projection matrices be partitioned into a left 3×3 matrix and a 3 vector as

$$P' = [A' | a']$$
$$P'' = [A'' | a']$$

$$A' = K'R' \qquad a' = -K'R'X_0$$

$$A'^{-1} a' = (K'R')^{-1} a' = -R'^T K'^{-1} K' R' X_{O'} = -X_{O'}$$

$$X_{O'} = -A'^{-1} a'$$

So $b'_{12} = A''^{-1}a'' - A'^{-1}a'$

$\Rightarrow$ So, $\boxed{F = A'^{-T} S_{b'_{12}} A'^{-1}}$

★ Essential Matrix (For Calibrated Cameras)

${}^{k}x' = K'^{-1}x'$ ${}^{k}x'' = K''^{-1}x''$

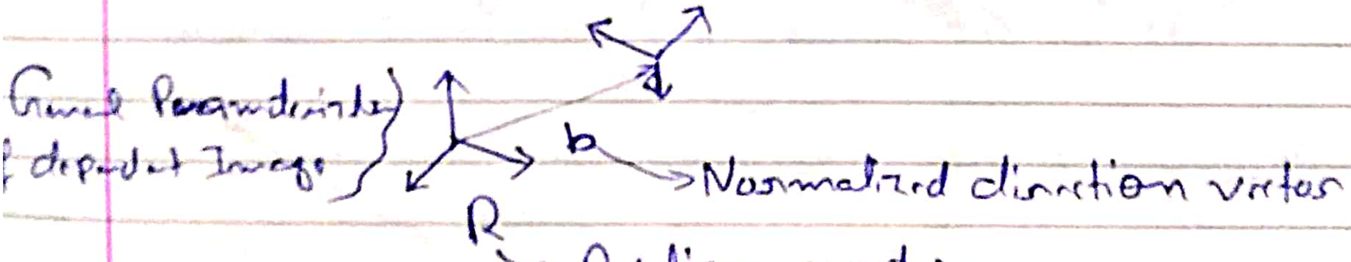
$\Rightarrow x'^{T} F x' = 0$

$\Rightarrow \underbrace{x'^{T}(K')^{-T}}_{{}^{k}x'^{T}} \underbrace{(R')^{-T} S_b (R'')^{-1}}_{E} \underbrace{(K'')^{-1}x'}_{{}^{k}x'} = 0$

$\Rightarrow \boxed{{}^{k}x'^{T} E\, {}^{k}x' = 0}$

$E \rightarrow$ Essential matrix

★ Popular parameterizations for the relative orientation



$\Rightarrow$ For first cam $R' = I_3$

$\Rightarrow$ For second cam $R'' = R$

$$\Rightarrow \quad {}^{K}x'^{T} S_b R^{T\,K} x'' = 0 \quad \text{with } |b| = 1$$

$\Rightarrow$ The resulting 5 degree of freedom are

$(\underbrace{B_x\ B_y\ B_z}_{b}\ \underbrace{\omega\ \phi,\ K}_{R})$ with $B_x^2 + B_y^2 + B_z^2 = 1$