

③

## Loss function and Optimization

→ Todo:

① Determine a loss function

{ quantifies our unhappiness }  
 { with the scores across }  
 { the training data }

② Come up with a way of efficiently finding the Parameters that minimize the loss function.

{ Optimization }

⇒ Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^N$$

where,  $x_i$  is image  $k$

$y_i$  is (integer) label

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

### ★ Multi-class SVM loss

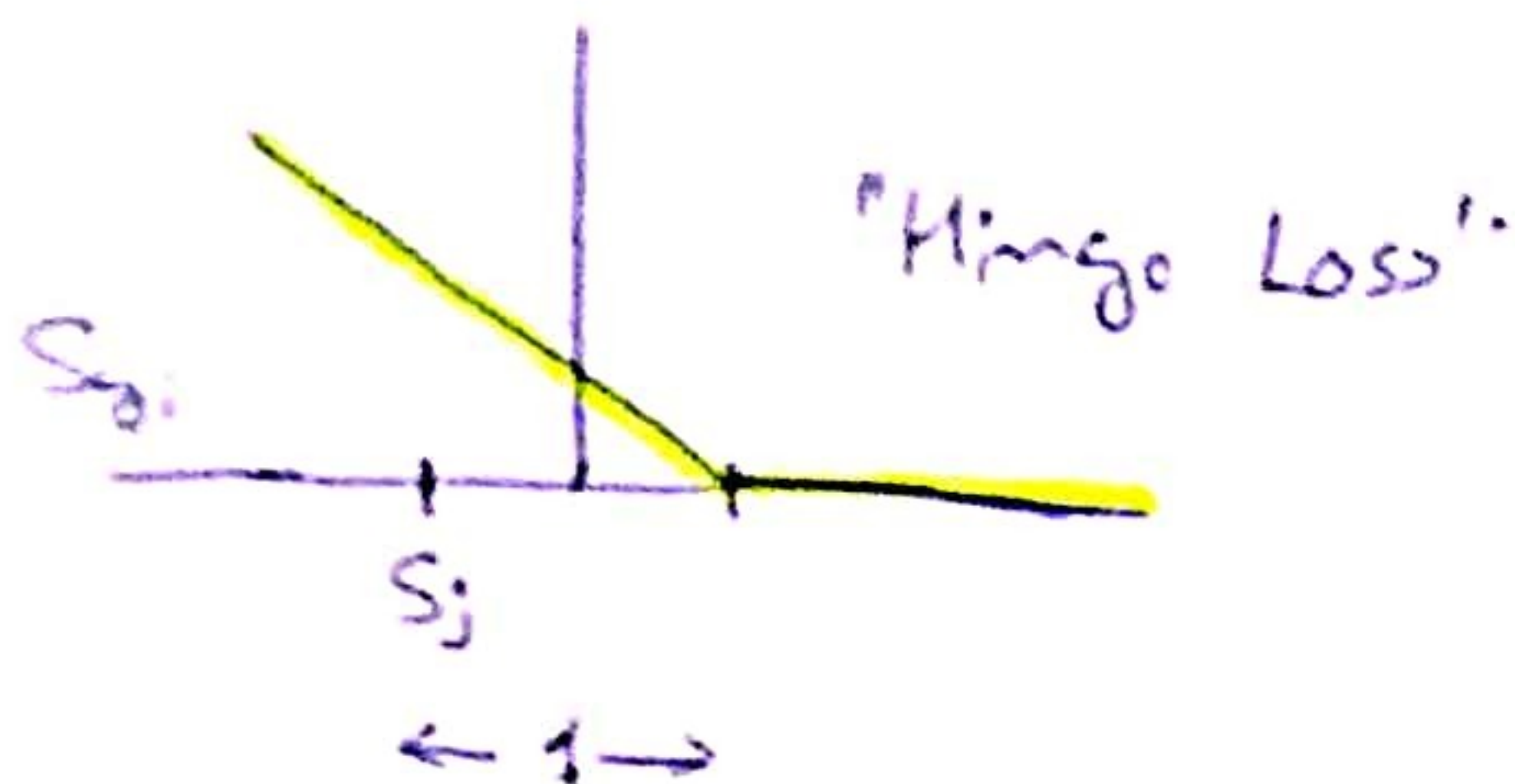
⇒ Let  $S_i = f(x_i, W)$  be the shorthand for the Score vector.

⇒ The SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } S_{y_i} \geq S_j + 1 \\ S_j - S_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, S_j - S_{y_i} + 1)$$





$$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(f(x_i, W), y_i) + \lambda R(W)$$

Occam's Razor

→ Among competing hypotheses  
the simplest is the best

↑  
Regularization

{ Model should be  
"Simple", so it works  
on test data. }

↓  
{ L2 regularization }  
is most common



①  
\* Softmax Classifier (Multinomial Logistic Regression)  
↳ Also called Cross-entropy loss  
Score = Unnormalized log probability of the classes

$$P(Y=k | X=x_i) = \frac{e^{S_k}}{\sum_j e^{S_j}}$$

where  $S = f(x_i; w)$

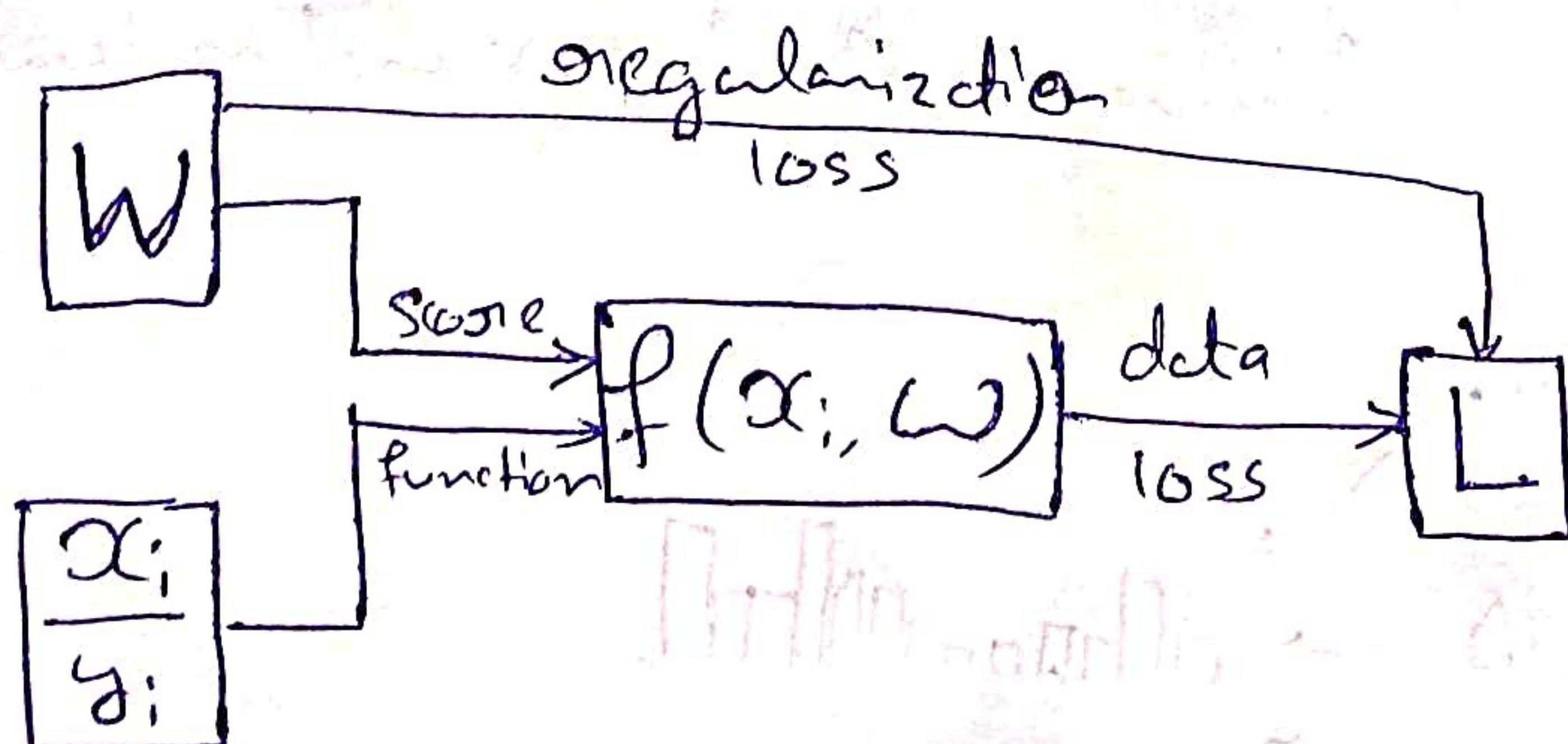
⇒ We want to maximize the log likelihood or to minimize negative log likelihood of the correct class.

$$L_i = -\log P(Y=y_i | X=x_i)$$

$$L_i = -\log \left( \frac{e^{S_k}}{\sum_j e^{S_j}} \right)$$

Minimum loss = 0

Maximum loss =  $\infty$





# ★ Optimization

## Strategy #1: Random Search

- Very bad Solution.
- Sample  $W$  randomly and evaluate all of these with loss function, & select the one with minimum loss.

## Strategy #2: Follow the Slope (Gradient descent)

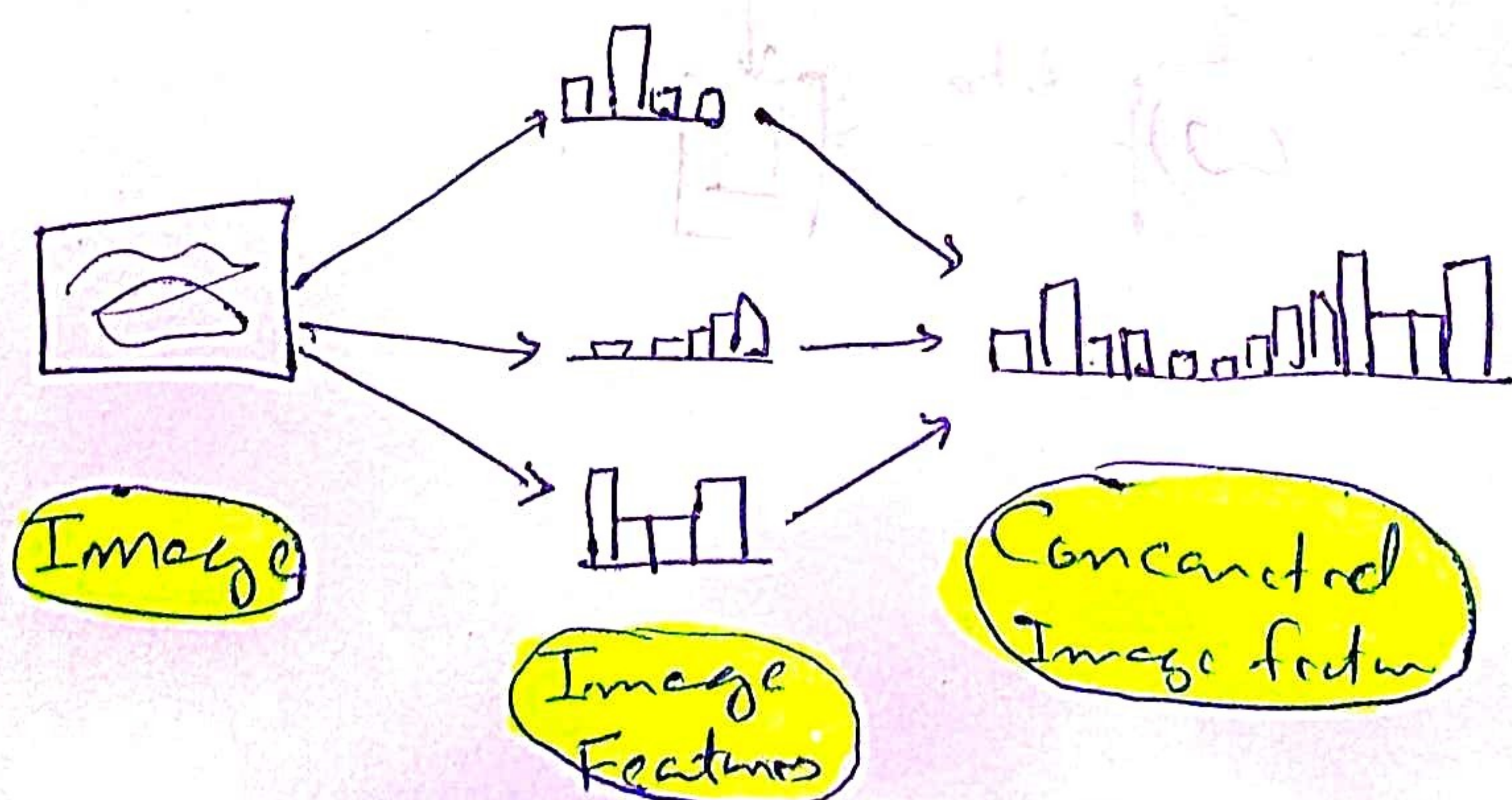
- One way to calculate gradient is by finite difference method.  
(Not a good Idea, it is very slow!)
- Best is to use calculus to compute an analytic gradient.

⇒ In practice always use analytic gradient, but check implementation with numerical gradient.

↳ This is called a gradient check

# ★ Image Features

⇒ Common before the dominance of deep neural network.





⇒ **Color Histogram** can be a feature vector.

⇒ **Histogram of Oriented Gradient (HOG)** can be a feature vector.

- Divide image into  $8 \times 8$  pixel regions.
- Within each region quantize edge direction into 9 bins.

⇒ **Bag of Words** feature vector.

- Takes inspiration from Natural Language Processing.

⇒ In CNN, instead of writing down the features ahead of ~~the~~ time, we will learn the features.

