5B

Frequency responce approach to Control System design *

⇒ Because of difficulty in deriving the equations governing certain components, such as pneumatic & hydraulic components, the dynamic characteristics of such components are usually determined experimentally through frequency response test.

⇒ In dealing with high frequency noise, we find that the frequency-response approach is more convenient than other approaches.

⇒ There are basically two approaches in the frequency domain design.

Nyquist plot Approach

Bode plot Approach

⇒ For the design purposes, therefore it is best to work with the Bode diagram.

★ Lead Compensation

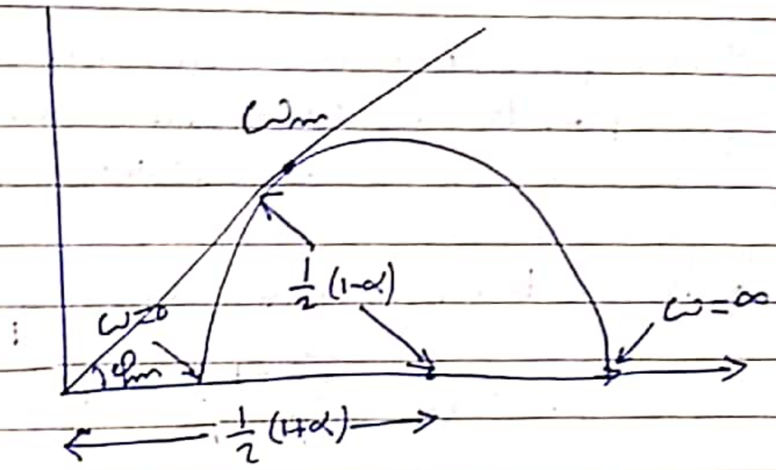
Characteristics of Lead Compensators

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}} \quad (0<\alpha<1)$$

α = attenuation factor

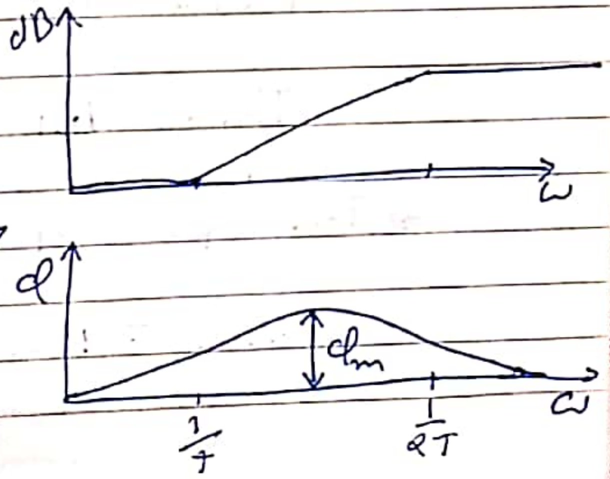
⇒ Polar plot of $K_c\alpha\frac{j\omega T+1}{j\omega\alpha T+1}$ $(0<\alpha<1)$

with $K_c = 1$



$$\sin\phi_m = \frac{1-\alpha}{1+\alpha} \qquad \omega_m = \frac{1}{\sqrt{\alpha}T}$$

{max phase lead angle}



Bode diagram

Lead Compensation Technique Based on Frequency-Response Approach

⇒ Assume that the performance specifications are given in terms of phase margin, gain margin, static velocity error constant & so on.

⇒ The primary function of the lead compensator is to reshape the frequency-response curve to provide sufficient phase lead angle to offset the excessive phase lag associated with the components of the fixed system.

⇒ The procedure for designing a lead compensator by the frequency-response approach may be stated as follows.

1. Assume the following lead compensator:-

$$G_c(s) = K_c\alpha \frac{Ts+1}{\alpha Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

$\{0 < \alpha < 1\}$

Define $K_c\alpha = K$

Then, $G_c(s) = K \dfrac{Ts+1}{\alpha Ts+1}$

⇒ The open loop TF of compensated system:-

$$G_c(s)G(s) = KG(s)\frac{Ts+1}{\alpha Ts+1} = G_1(s)\frac{Ts+1}{\alpha Ts+1}$$

$\{\text{where } G_1(s) = KG(s)\}$

1. Determine gain K to satisfy steady state requirement.

2. Using the gain K thus determined, draw a Bode diagram of $G_1(j\omega)$, the gain-adjusted but uncompensated system. Evaluate the phase margin.

3. Determine the necessary phase-lead angle to be added to the system. Add an additional 5° to 12° to the phase lead angle required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin.

4. Determine the attenuation factor α by use of following equation:

$$\sin \phi_m = \frac{1-\alpha}{1+\alpha}$$

Determine the frequency where the magnitude of the uncompensated system $G_1(j\omega)$ is equal to $-20\log(1/\sqrt{\alpha})$. Select this frequency as the new gain crossover frequency. This frequency corresponds to $\omega_m = 1/\sqrt{\alpha}T$, and the maximum phase shift ϕ_m occurs at this frequency.

5. Determine the corner frequencies of the lead compensator as follows:

Zero of lead compensator ⇒ $\omega = \frac{1}{T}$

Pole of lead compensator ⇒ $\omega = \frac{1}{\alpha T}$

6. Using the value of K determined in step 1 and that of α determined in step 4, Calculate Constant K_c from

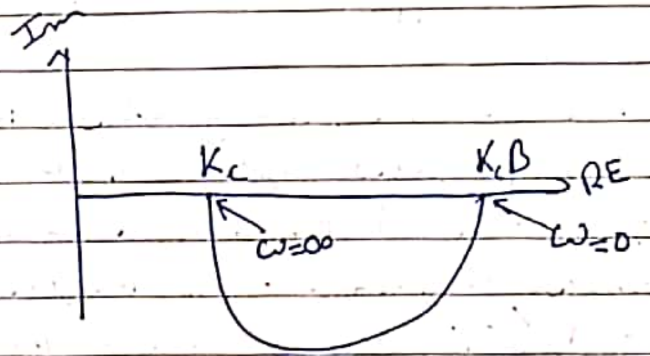
$$K_c = \frac{K}{\alpha}$$

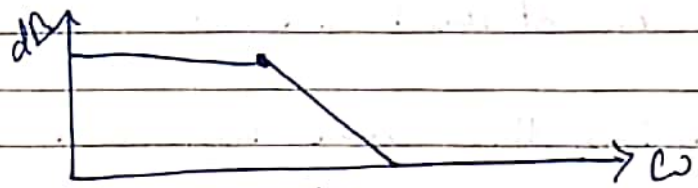
7. Check the gain margin to be sure it is satisfactory. If not, repeat the design process by modifying the pole-zero location of the Compensator until a satisfactory result is obtained.

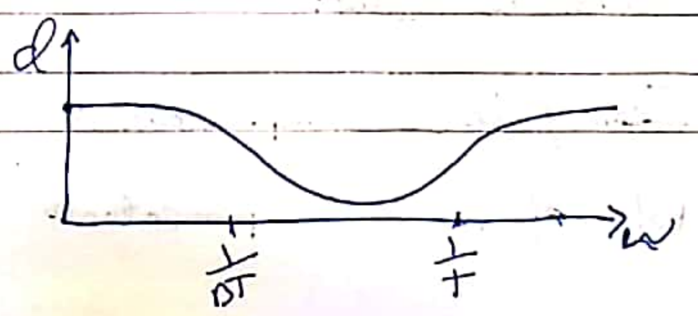
★ Lag Compensation

Characteristics of Lag Compensators

$$G_c(s) = K_c \beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad \{\beta > 1\}$$







Lag Compensation Technique based on the Frequency-Response Approach

⇒ The Primary function of a lag Compensator is to provide attenuation in the high-frequency range to give a System sufficient phase margin.

⇒ The Procedure for designing lag Compensators for the System Shown by the frequency-response approach may be stated as follows:-

1. Assume the following lag Compensator:-

$$G_c(s) = K_c \beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \quad (\beta > 1)$$

Define $K_c \beta = K$

Then, $G_c(s) = K \frac{Ts+1}{\beta Ts+1}$

⇒ The open loop TF of the Compensated System is

$$G_c(s)\, G(s) = K \frac{Ts+1}{\beta Ts+1} G(s) = \frac{Ts+1}{\beta Ts+1} K G(s)$$

$$= \frac{Ts+1}{\beta Ts+1} G_1(s) \quad \left\{ \text{where } G_1(s) = KG(s) \right\}$$

Determine gain K to satisfy the requirement on the given static velocity error constant.

2. If the gain adjusted but uncompensated System $G_1(j\omega) = KG(j\omega)$ does not satisfy the Specifications on the phase and gain margins, then find the frequency point where the phase angle of the open loop TF is equal to $-180°$, plus the required phase margin.

↳ The required phase margin is the Specified phase margin plus $5°$ to $12°$.

⇒ Choose this frequency as the new gain cross over frequency.

3. To prevent detrimental effects of phase lag due to the lag compensation, the pole and zero of the lag compensator must be located substantially lower than the new gain crossover frequency.

↳ Choose the Compensator pole and Zero sufficiently small. Thus the phase lag occurs at the low-frequency region so that it will not affect the phase margin.

4. Determine the attenuation necessary to bring the magnitude Curve down to 0dB at the new gain Crossover frequency. Noting that this attenuation is $-20\log\beta$, determine the value of β. Then the other Corner frequency is determined from $\omega = \frac{1}{\beta T}$.

5. Using the value of K determined in Step 1 and that of B determined in Step 4, Calculate Constant K_c from

$$K_c = \frac{K}{B}$$

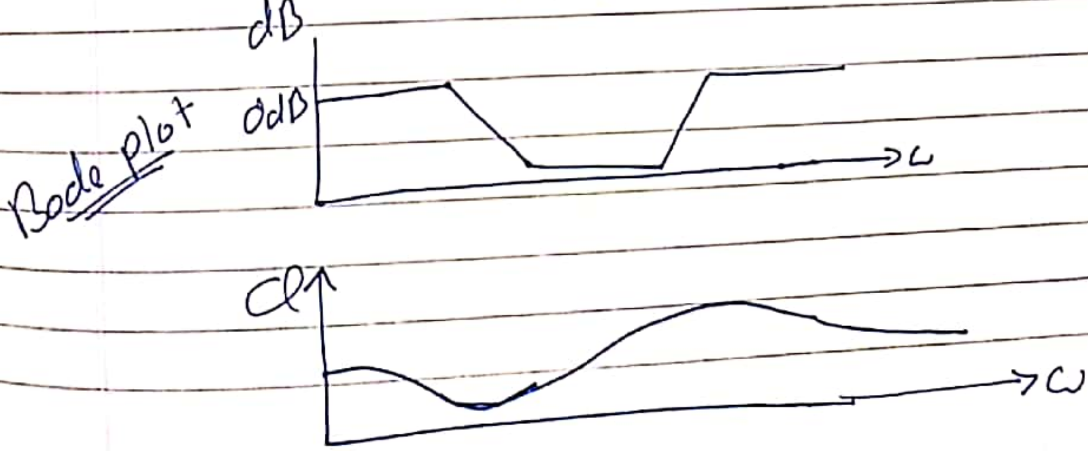
★ Lead Lag Compensation

Charateristic of Lead-Lag Compensator

$$G_c(s) = K_c \left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{BT_2}}\right)$$

When $\gamma > 1$ & $B > 1$

$$\omega_1 = \frac{1}{\sqrt{T_1 T_2}}$$



⇒ The design of a lag-lead Compensator by the frequency-response approach is based on the Combination of the design techniques discussed under lead Compensation and lag compensation.

The phase-lead portion of the lag-lag Compensator alters the frequency-response Curve by adding phase-lead angle and increasing the phase margin at the gain Crossover frequency.

The phase-Lag portion provides attenuation near and above the gain Cross over frequency and there be allows an increase of gain at the low frequency range to improve the Steady state performance.