★ Correlation Between Transfer functions & State-Space Equation

Let us Consider the System whose TF is given by:-

$$G(s) = \frac{Y(s)}{U(s)}$$

⇒ The System may be represented in state space by the following equations:-

$$\dot{x} = Ax + Bu \quad \text{——} ①$$
$$y = Cx + Du \quad \text{——} ②$$

⇒ Taking Laplace transform of eq ① & ② :-

$$sX(s) = AX(s) + BU(s)$$
$$Y(s) = CX(s) + DU(s)$$

$$(sI - A)X(s) = BU(s)$$
$$X(s) = (sI-A)^{-1}BU(s)$$
$$Y(s) = C(sI-A)^{-1}BU(s) + DU(s)$$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = C(sI-A)^{-1}B + D}$$

$$\boxed{G(s) = \frac{Q(s)}{|sI-A|}}$$

Q(s) ← Same Polynomial in s.

|sI−A| → Characteristic Polynomial of G(s)

⇒ Eigen Values of A are identical to the Poles of G(s).

* Transfer Matrix

⇒ Consider a multiple input - Multiple Output System.

⇒ Assume that there are r inputs & m outputs.

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_r \end{bmatrix}$$

$$\boxed{G(s) = \frac{Y(s)}{U(s)} = C(SI-A)^{-1}B + D.}$$

→ Transfer matrix