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Observers

⇒ The state feedback control law is not implementable if we do not have direct access to all state variables.

⇒ Deterministic state estimators are called observers.

⇒ In stochastic context, the most frequently appearing estimator in the control theory is the Kalman filter.

★ Open loop Observer

⇒ The conceptually simplest scheme for estimating $x(t)$ is to utilize an available mathematical model of the system and let it run on a computer.

⇒ For example the state vector $x(t)$ is estimated by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) \qquad \hat{x}(k+1) = A\hat{x}(k) + Bu(k)$$

where $\hat{x}$ is the observer state vector (i.e. the state estimate)

⇒ The idea of open loop observer looks simple, but it does not work in any practical sense.

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★ Luenberger Observer (Closed loop observer)

⊕ Continuous time full state Observer

⇒ We consider the n-dim system

$$\dot{X}(t) = AX(t) + BU(t) \qquad X(0) = X_0$$
$$Y(t) = CX(t)$$

$X \in R^{n\times 1}$
$U \in R^{r\times 1}$
$Y \in R^{m\times 1}$

⇒ An obvious weakness of the open loop observer is that it does not utilize the system output which includes rich information on the system state vector.

⇒ The difference between the measured output $y(t) = Cx(t)$, and the predicted output based on the observer state $\hat{y}(t) = C\hat{x}(t)$, should be utilized to improve the estimate.

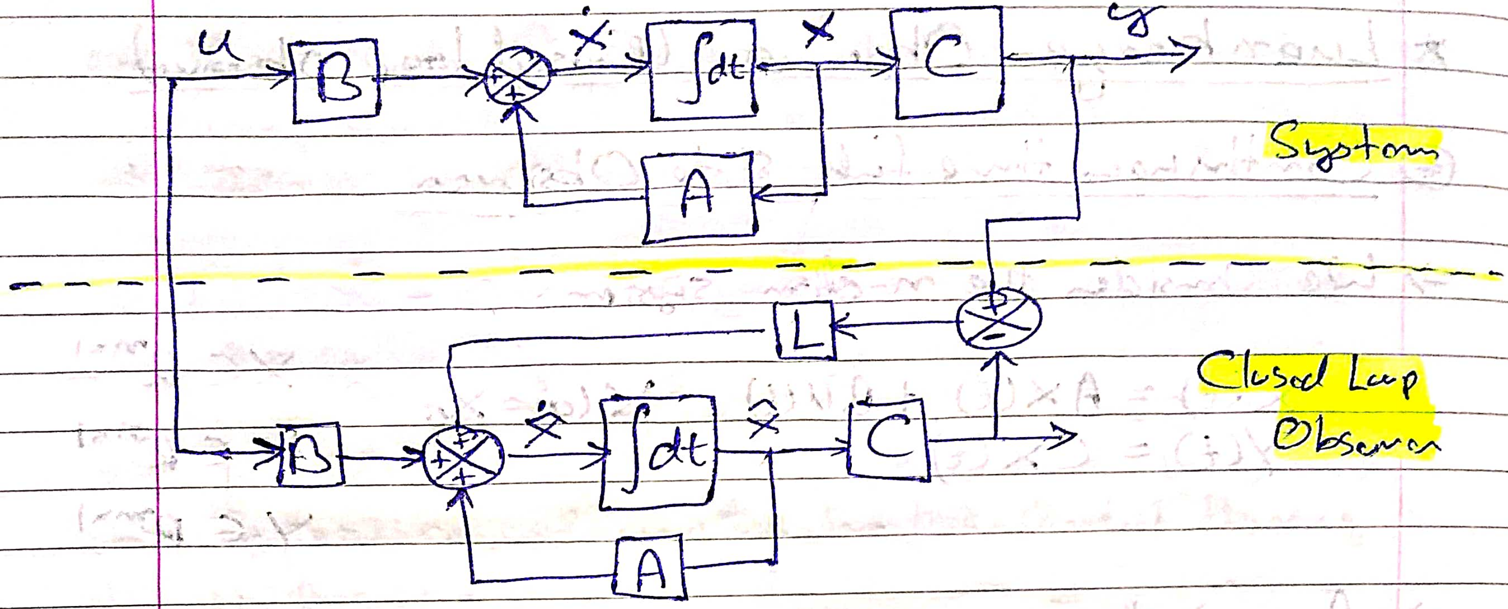
⇒ This idea leads us to the closed loop observer:

$$\dot{\hat{x}} = A\hat{x}(t) + Bu(t) + L[y(t) - C\hat{x}(t)] \qquad \hat{x}(0) = 0$$

$$\boxed{\dot{\hat{x}}(t) = (A - LC)\hat{x}(t) + Ly(t) + Bu(t)}$$

{L is n×m observer gain}

⇒ This observer has the same order as the system and is a full order observer.



⇒ Defining $e(t) = x(t) - \hat{x}(t)$, the error equation is

$$\dot{e}(t) = (A - LC)e(t) = A_c e(t) \quad e(0) = x(0)$$

⇒ If all the eigenvalues of A_c have negative real parts, the error equation can be made asymptotically stable.

<u>Theorem 01</u>: If the continuous time-invariant system is observable, all the eigenvalues of $A - LC$ can be arbitrarily assigned.

Discrete time full state Observer

⇒ For the n-dimensional discrete-time system described by:

$$X(K+1) = AX(K) + Bu(K), \quad X(0) = X_0$$
$$Y(K) = CX(K)$$

⇒ A full state order observer is

$$\hat{x}(K+1) = A\hat{x}(K) + Bu(K) + L[y(K) - C\hat{x}(K)], \hat{x}(0)=0$$

⇒ The estimation error equation is

$$e(K+1) = (A-LC)e(K)$$

$$e(K) = x(K) - \hat{x}(K) \quad e(0) = x(0)$$

⇒ The eigenvalue of A-LC can be arbitrarily assigned if the system is observable.