(4)

Basic Pick and Place (Part 2)

(1) Differential Kinematics

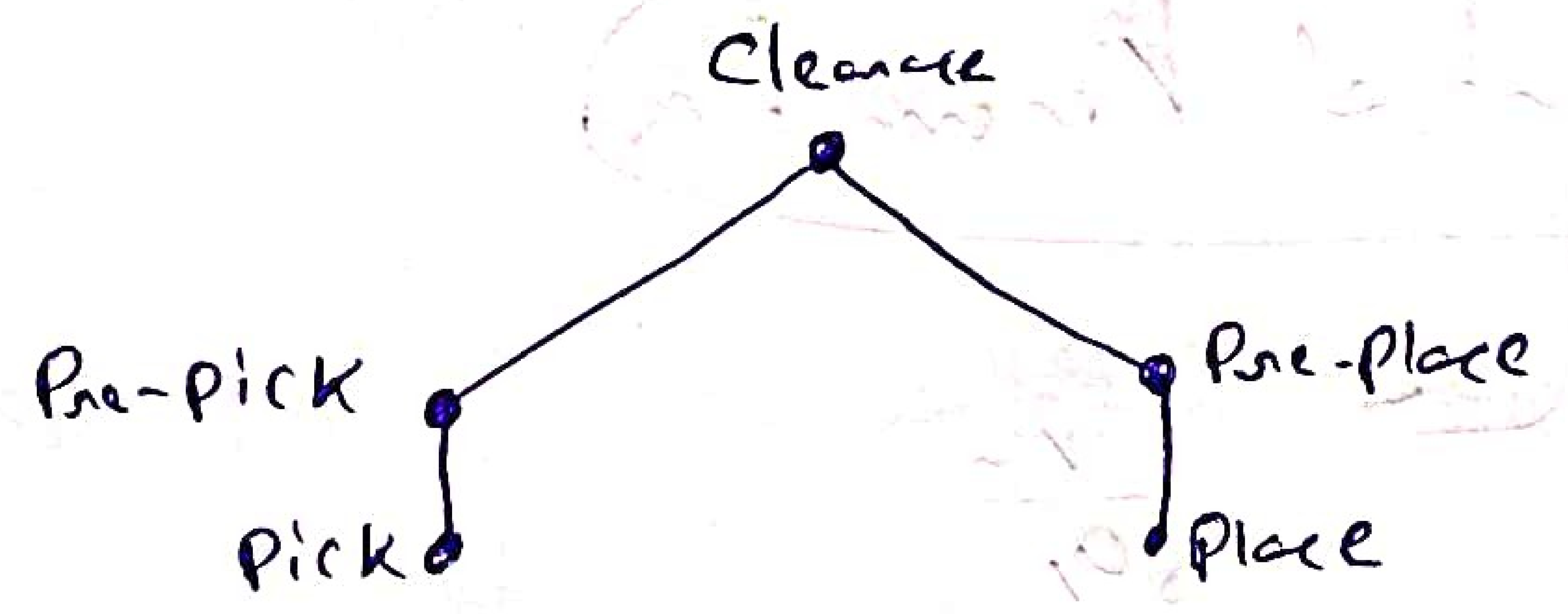
→ How do changes in q relate to changes in X^G

→ Spatial Velocity

(2) From Keyframes to trajectories.

$X^{G_{pick}}$ $X^{G_{pre\text{-}pick}}$ .....

t_{pick} $t_{pre\text{-}pick}$



★ Differential Kinematics

$$X^G = f^G_{kin}(q)$$

$$\frac{d}{dt} X^G = \underbrace{\frac{\partial f^G_{kin}(q)}{\partial q}}_{\text{Jacobian}} \frac{dq}{dt} = J^G(q) \frac{dq}{dt}$$

or

Kinematic Jacobian

→ {There are many different Jacobian depending upon the choice of representation of orientation on both side}

Key Idea

→ For orientations, there are many representations and there, and there is no "best one". It depends on the use case.

→ For rate of change of orientations, you can pick one representation and be happy with it.

→ Most common representation is angular Velocity.

$$\begin{bmatrix} \dot{\theta}\, x \\ \dot{\theta}\, y \\ \dot{\theta}\, z \end{bmatrix}$$

(?) Why are 3 numbers insufficient for rotation but sufficient for rate of rotation?

⇒ θ wraps on 2π, but $\dot{\theta}$ dosent.

★ Angular Velocity

$$^{B}\omega^{A}_{C} \in \mathbb{R}^3$$

$$^{B}\omega^{A}_{C} = \begin{bmatrix} \dot{\theta}\, x \\ \dot{\theta}\, y \\ \dot{\theta}\, z \end{bmatrix} \rightarrow \boxed{^{C}\omega^{A}_{F} = {}^{C}\omega^{B}_{F} + {}^{B}\omega^{A}_{F}}$$

$$\boxed{^{B}\omega^{A}_{G} = {}^{G}R^{F}\, {}^{B}\omega^{A}_{F} \quad \text{if } {}^{G}\dot{R}^{F} = 0}$$

★ Translational Velocity

$$\boxed{{}^{B}v_{C}^{A} \in \mathbb{R}^3 = {}^{B}\dot{p}_{C}^{A}}$$

★ Spatial Velocity

$$\boxed{{}^{B}V_{C}^{A} = \begin{bmatrix} {}^{B}\omega_{C}^{A} \\ {}^{B}v_{C}^{A} \end{bmatrix} \in \mathbb{R}^6}$$

$$\frac{d}{dt} X^{a} = \frac{\delta f_{Kin}^{a}(q)}{\delta q} \cdot \frac{dq}{dt}$$

$$V^{a} = J^{a}(q)\,\dot{q}$$