Particle Filters (Short Intro)

- **Goal:** Approach for dealing with arbitrary distributions.
- **Key Idea:** Use multiple sample to represent arbitrary distribution.

* **Particle set**

⇒ Set of weighted samples

$$X = \left\{ \left\langle \underbrace{x^{[j]}}_{\text{State hypothesis}}, \underbrace{w^{[j]}}_{\text{Importance weight}} \right\rangle \right\}_{j=1,\dots,J}$$

⇒ The sample represents the posterior

$$p(x) = \sum_{j=1}^{J} w^{[j]} \underbrace{\delta_{x^{[j]}}(x)}_{\{\text{dirac delta function}\}}$$

How to Obtain Samples?

* Closed form sampling is only possible for a few distribution

Example: Gaussian (with zero mean σ^2 variance)

$$x \longrightarrow \frac{1}{2} \sum_{i=1}^{12} rand(-\sigma, \sigma)$$

{ $rand(a,b) \rightarrow$ generates random number uniformly distributed in $[a,b]$ $\{a<b\}$ }

★ Importance Sampling Principle

⇒ If it is not possible to generate sample ~~for~~ in closed form from ~~some~~ function f.

↳ We can use different distribution g (whose closed form sampling is known) to generate sample from f.

↳ Here we account for the difference between g and f using a weight f/g.

target ⇒ f

Proposal ⇒ g

Pre-condition ⇒ $f(x) > 0 \rightarrow g(x) > 0$

★ Particle Filter

⇒ Uses sample to represent posterior and uses importance sampling principle to update the belief.

- Prediction: Draw from the proposal
- Correction: Weighting by the ratio of target and proposal.

★ Particle Filter Algorithm :

1. Sample the particle using the proposal distribution

$$x_t^{[i]} \sim \pi(x_t | \ldots)$$

2. Compute the importance weights

$$w_t^{[i]} = \frac{\text{target}(x_t^{[i]})}{\text{Proposal}(x_t^{[i]})}$$

3. Resampling : Draw sample i with probability $w_t^{[i]}$ and repeat J times.

Particle_Filter$(\mathcal{X}_{t-1}, u_t, z_t)$:

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \phi$

for $j=1$ to J do

Sample $x_t^{[j]} \sim \pi(x_t)$

$\omega_t^{[j]} = \frac{p(x_t^{[j]})}{\pi(x_t^{[j]})}$

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[j]}, \omega_t^{[j]} \rangle$

endfor

for $j=1$ to J do

draw $i \in 1 \ldots J$ with probability $\propto \omega_t^{[i]}$

add $x_t^{[i]}$ to $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

$p(x_t | u_t, x_{t-1}^{[j]})$ odometry model

{Proposal distribution}

{Target distribution}

$p(z_t | x_t^{[j]})$ sensor model

★ Monte Carlo Localization

⇒ Each particle is a pose hypothesis.

⇒ Proposal is the motion model

$$x_t^{[j]} \sim p(x_t | x_{t-1}, u_t)$$

⇒ Correction via the observation model

$$\omega_t^{[j]} = \frac{\text{target}}{\text{proposal}} \propto p(z_t | x_t, m)$$

⇒ Resampling can be implement by using Roulette wheel.

or

(Stochastic Universal sampling)