# Lecture 2: Modeling

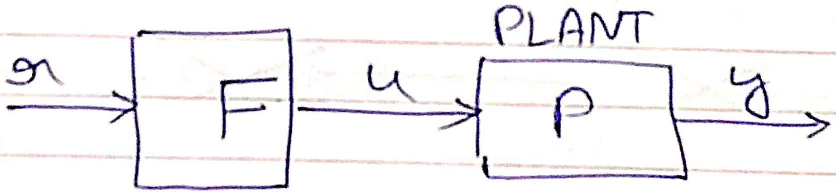
## ★ Control objective

① **Stabilization**: Make sure the system stays within "normal" operating conditions.

② **Regulation**: Maintain a desired operating point in spite of disturbance.

③ **Tracking**: Follow a reference trajectory that changes over time, as closely as possible.

## ★ Basic control architectures: Feed Forward



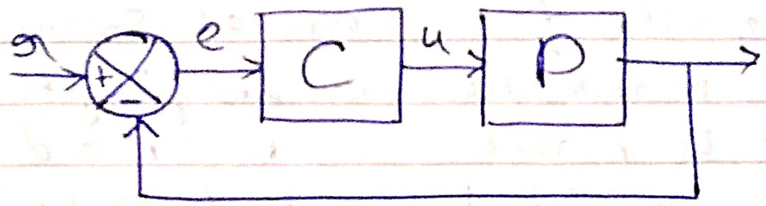

⇒ Basic Idea: Given $r$, attempt to compute what should be the control input $u$ that would make $y = r$.

⇒ Here $F$ is 'Inverse' of $P$.

⇒ This method relies on good Knowledge of $P$.

↳ Very sensitive to modeling error.

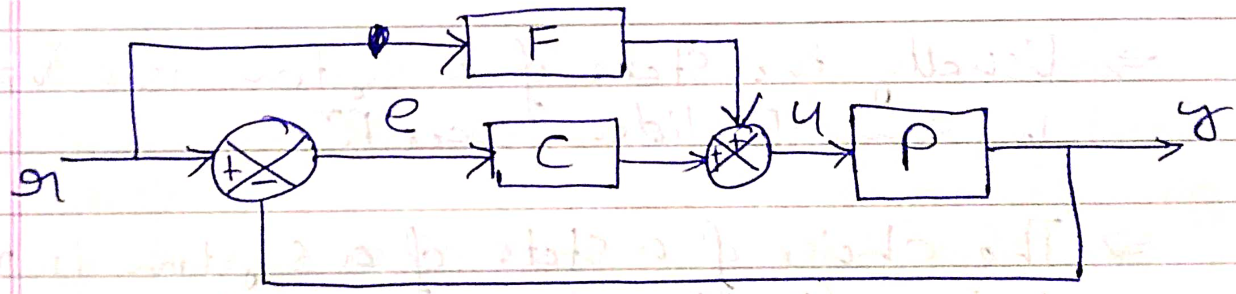
## ★ Basic Control architecture : Feedback




⇒ <u>Basic Idea:</u> Given the error $r-y$, compute $u$ such that the error is small.

⇒ This method does not require a precise knowledge of $P$ — robust to modeling errors.

## ★ Basic control architecture : Feed Forward & Feedback




⇒ <u>Basic Idea:</u> Given $r$, compute a guess for the control input $u$ that would make $y=r$; Correct the guess based on the measurement of the error $e$, so that the error is minimized.

⇒ Combines the main advantages of feed-forward & feedback architectures.

- Ensures stability & robustness
- Speed up tracking / regulation.

## ★ State of a System

"The state $x(t_1)$ of a causal system at time $t_1$ is the information needed, together with the input $u$ between times $t_1$ and $t_2$ to uniquely predict the output at time $t_2$ for all $t_2 \geq t_1$"

⇒ In other words, the state of the system at a given time $t$ summarizes the whole history of the past input over $(-\infty, t)$, for the purpose of predicting the output at future times.

⇒ Usually, the state of a system is a vector in some Euclidean space $R^n$.

⇒ The choice of a state of a system is not unique (In fact, there are infinite choices)

## ★ Dimension of a system (order of the system)

⇒ We define the dimension of a causal system as the minimal number of variables sufficient to describe the system's state.

⇒ We will mostly deal with finite-dimensional system.

Date
Page

## ★ LTI State-space model

$$\boxed{\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}}$$

⇒ The state x represents the memory of the system.

⇒ The state x is an internal variable that cannot be accessed directly, but only controlled through input ~~u~~ and observed through the output y.

## ★ Nonlinear systems

⇒ Finite-dimentional, time invariant, causal nonlinear control systems, with input u and output y can typically be modeled using a set of differential equations as follows:

$$\boxed{\begin{aligned}\dot{x}(t) &= f(x(t), u(t))\\ y(t) &= g(x(t), u(t))\end{aligned}}$$

## ★ Equilibrium point

A system described by an ODE $\dot{x}(t) = f(x(t), u(t))$ has an equilibrium point $(x_e, u_e)$ if $f(x_e, u_e) = 0$.

## ★ Jacobian Linearization Procedure

⇒ Given an equilibrium point $(x_e, u_e)$, with output $y_e = g(x_e, u_e)$, a linearized model is obtained by setting

$$x \leftarrow x_e + \delta x$$
$$u \leftarrow u_e + \delta u$$
$$y \leftarrow y_e + \delta y$$

and then neglecting all terms of second (or higher) order in $\delta x$, $\delta u$ and $\delta y$ in the (nonlinear) dynamics model.