Point Features
(Förstner Operator)

★ **Desired Properties of Points**

- Differ w.r.t the local neighborhood.
- Can be detected robustly.
- Allow for precise localization.
- Support data association.
- Invariant under geometric transformation.
- Support image interpretation.

★ **Normal Equations**

⇒ We can write the matrix N as the average squared gradients times M.

$$N = \begin{bmatrix} \sum_m f_{im}^2 & \sum_m f_{im} f_{jm} \\ \sum_m f_{jm} f_{im} & \sum_m f_{jm}^2 \end{bmatrix}$$

$$N = M \overline{\nabla f \nabla f^T} = M \begin{bmatrix} f_i^2 & f_i f_j \\ f_i f_j & f_j^2 \end{bmatrix}$$

$$= M \begin{bmatrix} G * f_i^2 & G * f_i f_j \\ G * f_i f_j & G * f_j^2 \end{bmatrix}$$

⇒ using a box filter G for smoothing.

* Position Uncertainity

$$\Sigma_{\hat{x}\hat{x}} = \hat{\sigma}_n^2 N^{-1} = \frac{\hat{\sigma}_n^2}{M}\left(\overline{\nabla f \nabla f^T}\right)^{-1} = \frac{\hat{\sigma}_n^2}{M}(\Sigma_{\nabla g \nabla g})^{-1}$$

$\Rightarrow$ Uncertainity in the position is inverse proportional to the uncertainity of gradients.

$\downarrow$

Variance of

* Resulting Approach

Step1: Estimate for every pixel position (i, j) the covariance matrix of the gradients.

$$\Sigma_{\nabla g \nabla g} = \begin{bmatrix} \sigma_{gx}^2 & \sigma_{gx}\sigma_{gy} \\ \sigma_{gx}\sigma_{gy} & \sigma_{gy}^2 \end{bmatrix} = \frac{1}{M}\begin{bmatrix} \Sigma_m f_{xm}^2 & \Sigma_m f_{xm} f_{ym} \\ \Sigma_m f_{xm} f_{ym} & \Sigma_m f_{ym}^2 \end{bmatrix}$$

Step2: Compute for every (i, j) the smallest Eigenvalue of $\Sigma_{\nabla g \nabla g}$

$$\lambda_{min}(\Sigma_{\nabla g \nabla g}) = \frac{\sigma_{gx}^2 + \sigma_{gy}^2}{2} - \frac{1}{2}\sqrt{(\sigma_{gx}^2 - \sigma_{gy}^2)^2 + 4\sigma_{gxgy}^2}$$

Step3: Test if the largest Eigenvalue is smaller than a threshold that describes the acceptable uncertainty of the location.

$$\lambda_{max}(\Sigma_{\hat{x}\hat{x}}) \leq T_{\sigma_{\hat{x}}}^2$$

Step 4: For all remaining points, Search within a local window for the minimum of $\lambda_{max}(\Sigma \hat{x}\hat{x})$.

Each region with a local minima is an interest point.