Fast SLAM

(Feature based SLAM with Particle Filter)

→ Works well in low-dimensional spaces

→ 3-step procedure

- Sample from proposal
- Importance weighting
- Resampling {Survival of the fittest}

⇒ For feature-based SLAM:

$$x = (\underbrace{x_{1:t}}_{\text{poses}}, \underbrace{m_{1,x}, m_{1,y}, \ldots m_{M,x}, m_{M,y}}_{\text{Landmarks}})^T$$

⇒ Particle filters are effective in low dimensional spaces as the likely regions of the state space need to be covered with samples.

⇒ Can we exploit dependencies between the different dimensions of the state space?

⇒ If we know the poses of the Robot, Mapping is easy.

Key Idea: If we use the particle set, only to model the robot's path, each sample is a path hypothesis. For each sample, we can compute an individual map of landmarks.

★ Rao-Blackwellization Particle Filter $\left(\frac{2000}{1999}\right)$

⇒ Factorization to exploit dependencies between variables:

$$p(a,b) = p(b|a)\,p(a)$$

⇒ If $p(b|a)$ can be computed efficiently, represent only $p(a)$ with samples and compute $p(b|a)$ for every sample.

Map

Pose

⇒ Factorization of the SLAM posterior:

$$p(\underbrace{x_{0:t}}_{\text{Poses}}, \underbrace{m_{1:M}}_{\text{Map}} \mid \underbrace{z_{1:t}}_{\text{Observations}}, \underbrace{u_{1:t}}_{\text{Control input}})$$

$$= p(x_{0:t} \mid z_{1:t}, u_{1:t})\,\underline{p(m_{1:M} \mid x_{0:t}, z_{1:t})}$$

⇒ Landmark variables are independent given the robot's path.

$$= \underbrace{p(x_{0:t} \mid z_{1:t}, u_{1:t})}_{\text{Particle filter similar to MCL}} \prod_{i=1}^{M} \underbrace{p(m_i \mid x_{0:t}, z_{1:t})}_{\text{2D EKF}}$$

{Mapping with known pose}

* Modeling the Robot's Path

⇒ Sample-based representation for
$p(x_{0:t} \mid z_{1:t}, u_{1:t})$

⇒ Each sample is a path hypothesis

$x_0 \quad x_1 \quad x_2 \quad \ldots$

⇒ Past poses of a sample are not revised

↳ No need to maintain past poses in the sample set.

* Key Steps of FastSLAM 1.0

- Extend the path posterior by sampling a new pose for each sample

$$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$$

- Compute particle weight

e.g. observation

$$\omega^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]})\right\}$$

Measurement Covariance

- Update belief of observed landmarks (EKF update rule)

- Resample

Fast SLAM 1.0 _ Known_Correspondence (z_t, c_t, u_t, X_{t-1})

for $k=1$ to N do

Let $\langle x_{t-1}^{[k]}, \langle \mu_{1,t-1}^{[k]}, \Sigma_{1,t-1}^{[k]} \rangle, \dots \rangle$

$x_t^{[k]} \sim p(x_t \mid x_{t-1}^{[k]}, u_t)$

$j = c_t$

If feature j never seen before

$\mu_{j,t}^{[k]} = h^{-1}(z_t, x_t^{[k]})$

$H = h'(\mu_{j,t}^{[k]}, x_t^{[k]})$

$\Sigma_{j,t}^{[k]} = H^{-1} Q_t (H^{-1})^T$

$\omega^{[k]} = p_0$ // default importance weight

else

$\langle \mu_{j,t}^{[k]}, \Sigma_{j,t}^{[k]} \rangle = \text{EKF-Update}(\dots)$

$\omega^{[k]} = |2\pi Q|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} (z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]}) \right\}$

$\boxed{Q = H \Sigma_{j,t-1}^{[k]} H^T + Q_t}$

endif

for all unobserved features j' do

$\langle \mu_{j',t}^{[k]}, \Sigma_{j',t}^{[k]} \rangle = \langle \mu_{j',t-1}^{[k]}, \Sigma_{j',t-1}^{[k]} \rangle$

endfor

endfor

$X_t = \text{resample}\left(\langle x_t^{[k]}, \langle \mu_{1,t}^{[k]}, \Sigma_{1,t}^{[k]} \rangle, \dots, \omega^{[k]} \rangle_{k=1\dots N} \right)$

return X_t

* The Importance Weight

⇒ The target distribution is

$$p(x_{1:t} \mid z_{1:t}, u_{1:t})$$

⇒ The proposal distribution is

$$p(x_{1:t} \mid z_{1:t-1}, u_{1:t})$$

⇒ Proposal is used step by step

$$\underbrace{p(x_{1:t} \mid z_{1:t-1}, u_{1:t})}_{X_t} = p(x_t \mid x_{t-1}, u_t)\, \underbrace{p(x_{1:t-1} \mid z_{1:t-1}, u_{1:t-1})}_{X_{t-1}}$$

$$\omega^{[k]} = \frac{\text{target}\,(x^{[k]})}{\text{Proposal}\,(x^{[k]})}$$

$$= \frac{p(x_{1:t}^{[k]} \mid z_{1:t}, u_{1:t})}{p(x_t^{[k]} \mid x_{t-1}, u_t)\, p(x_{1:t-1}^{[k]} \mid z_{1:t-1}, u_{1:t-1})}$$

$$= \eta\, p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1})$$

$$= \eta \int p(z_t \mid x_{1:t}^{[k]}, z_{1:t-1}, m_j)\, p(m_j \mid x_{1:t}^{[k]}, z_{1:t-1})\, dm_j$$

$$= \eta \int \underbrace{p(z_t \mid x_t^{[k]}, m_j)}_{N(z_t;\, \hat{z}^{[k]},\, Q_t)}\, \underbrace{p(m_j \mid x_{1:t-1}^{[k]}, z_{1:t-1})}_{N(m_j:\, \mu_{j,t-1}^{[k]},\, \Sigma_{j,t-1}^{[k]})}\, dm_j$$

So $Q = \underbrace{H \Sigma^{[k]}_{j,t-1} H^T}_{\text{Pose Uncertainty of Landmark Estimate}} + \underbrace{Q_t}_{\text{Measurement noise}}$

Q: Measurement Covariance

$$w^{[k]} \simeq |2\pi Q|^{-\frac{1}{2}} \exp\left\{ -\frac{1}{2} (z_t - \hat{z}^{[k]})^T Q^{-1} (z_t - \hat{z}^{[k]}) \right\}$$

* Data Association Problem
  → {Which observation belongs to which landmark}

→ Multi-modal belief

→ Pose error is factored out of data association decisions.

→ Simple but effective data association

→ Big advantage of FastSLAM over EKF SLAM.