(12)
Bayes Nets

* Independence

⇒ Two variables are independent if:

$$\forall x, y: \quad P(x, y) = P(x)\,P(y)$$

- Another form:

$$\forall x, y: \quad P(x \mid y) = P(x)$$

- We write:

$$X \perp\!\!\!\perp Y$$

⇒ Independence is a simplifying modeling assumption.

* Conditional Independence

$$P(A \mid B, C) = P(A \mid C)$$

⇒ A is conditionally independent of B given C.

⇒ Written as:

$$A \perp\!\!\!\perp B \mid C$$

* Bayes net

⇒ Graphical models help us express conditional independence assumptions.

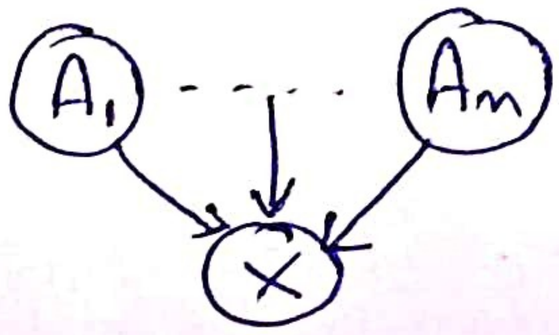
⇒ A technique for describing complex joint distributions (models) using simple, local distributions (conditional probabilities)

★ Graphical Model Notation

- Nodes : Variables (with domain)
  - ↳ Can be assigned (observed)
    or Unassigned (Unobserved)
- Arcs : Interactions
  - ↳ Indicate "direct influence" between variable
  - ↳ Encode Conditional Independence

★ Bayes Net Semantics

- A set of nodes, one per variable X.
- A directed, acyclic graph.
- A Conditional distribution for each node.



$P(X \mid A_1, \dots A_n)$

$$\text{Bayes Net} = \text{Topology (Graph)} + \text{Local Conditional Probabilities}$$

★ Probabilities in BNs

⇒ Bayes nets implicitly encode joint distributions.

$$P(x_1, x_2 \dots x_n) = \prod_{i=1}^{n} P(x_i \mid \text{Parents}(X_i))$$

⇒ When Bayes's nets reflect true causal patterns:

↳ Often easier to think about.

⇒ Bayes's nets need not actually be causal.