5

Compensator Design

- Root-Locus Approach
- Frequency-Response Approach

5A

Root Locus Approach to Control System design

* Preliminary design consideration

⇒ In practice root-locus plot of a system may indicate that the desired performance cannot be achieved just by the adjustment of gain (or some other adjustable parameter).

- → Then it is necessary to reshape the rootlocus to meet the performance specifications.
- → The design problems, therefore, become those of improving system performance by insertion of a compensator.

The design problems, therefore become those of improving system performance by insertion of a compensator. Compensation of a control system is reduced to the design of a filter whose characteristics tends to compensate

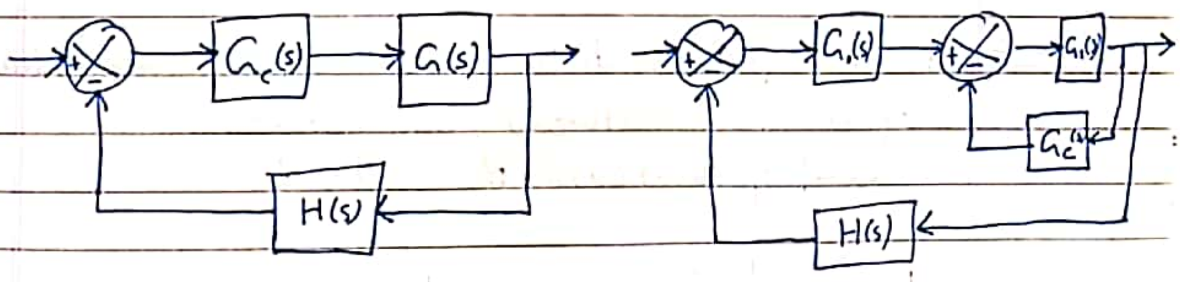
for the undesirable and unalterable characteristics of the plant.

★ **Design by Root-Locus Method**

// The design by the root locus method is based on reshaping the root locus of the System by adding poles and zeros to the System's open loop transfer function and forcing the root loci to pass through desired Closed-loop poles in the s-plane //

⇒ The characteristic of the root locus design is its being based on the assumption that the Closed loop System has a pair of dominant Closed-loop poles.

★ **Series Compensation and Parallel (or Feedback) Compensation**



Series Compensation

Parallel Compensation

$G_c(s)$ is the Compensator.

⇒ In general, Series compensation may be simpler than parallel compensation; however, Series compensation frequently requires additional amplifiers to increase the gain and/or to provide isolation.

★ Commonly used Compensators

Lead network ⇒ Output lead the Input

Lag network ⇒ output lag behind the output

Lead-Lag network ⇒ Output both lead and lag the input depending on frequency.
→ Phase lag occurs at low frequency region and; phase lead occurs at high frequency region.

⇒ A Compensation having a characteristic of a lead network, Lag network or lead-lag network is called :-

(i) Lead Compensator

(ii) Lag Compensator

(iii) Lag-Lead Compensator

Velocity feedback Compensator

★ Effect of the addition of poles (Adding Integral Control)

⇒ The addition of a pole to the openloop transfer function has the effect of pulling the root locus to the right, tending to lower the System's relative stability and to slow down the settling of the response.

★ Effect of addition of zeros (Adding derivative Control)

⇒ The addition of a zero to the openloop transfer function has the effect of pulling the root locus to the left, tending to make the System more stable and to speed up the settling of the response.

★ Lead Compensation

⇒ In carrying out a control-system design, we place a compensator in series with the unalterable transfer function $G(s)$ to obtain desirable behavior.

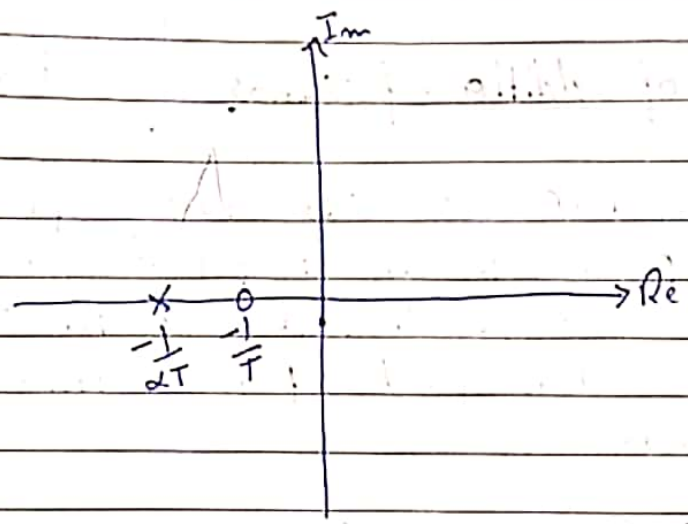
→ The main problem then involves the judicious choice of the poles & zeros of compensator $G_c(s)$ to ~~obtain~~ have the dominant closed-loop poles at the desired location in s plane.

Lead & Lag Compensator (Reff M2 end)

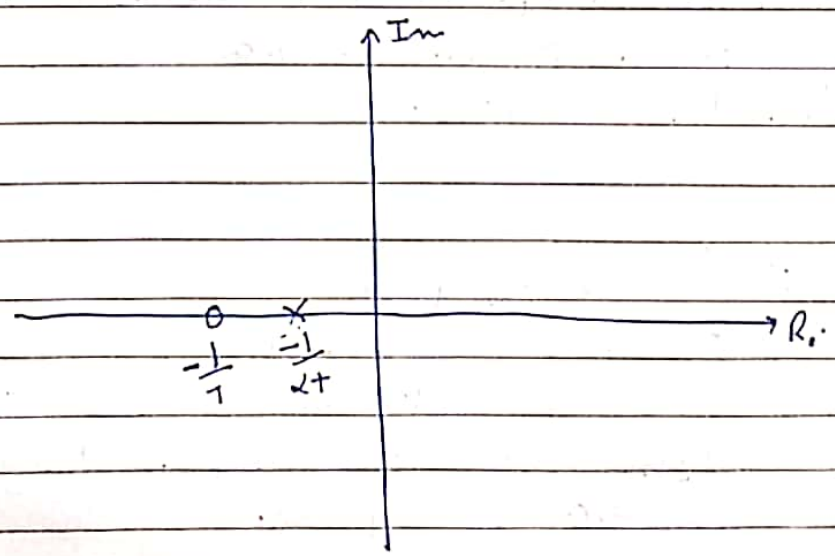
$$\frac{E_o(s)}{E_i(s)} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Lead network $\Rightarrow$ $\alpha < \phi$

Lag network $\Rightarrow$ $\alpha > \phi$



Lead network

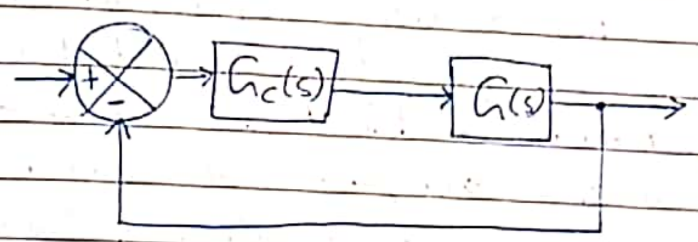


Lag network

Lead Compensation Technique based on the Root Locus Approach

⇒ Root locus approach to design is very powerful when the specifications are given in terms of time-domain quantities, such as the damping ratio and undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time & settling time.

The procedure for designing a lead compensator for the system by the root locus method may be stated as follows:-



1. From the performance specifications, determine the desired location for the dominant closed loop poles...

2. By drawing the root-locus plot of the uncompensated system ascertain whether or not the gain adjustment done can yield the desired closed-loop poles. If not calculate the angular deficiency ϕ.

   ↳ The angle must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed-loop poles.

3. Assume the lead Compensator $G_c(s)$ to be

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1} \quad (0 < \alpha < 1)$$

⇒ α & T are determined from angle deficiency.

⇒ K_c is determined from the requirement of the open loop gain.

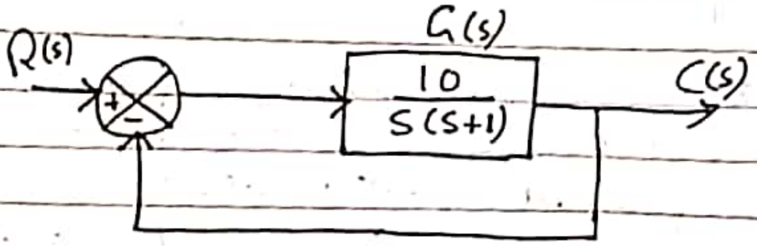
4. If static error constants are not specified, determine the location of the poles and zero of the lead compensator so that the lead compensator will contribute the necessary angle φ. If no other requirements are imposed on the system, try to make the value of α as large as possible.

↳ A larger value of α generally results in a large value of K_v, which is desirable.

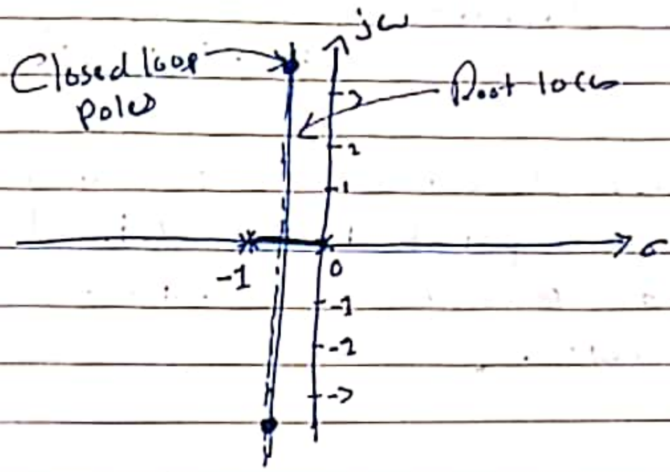
$$K_v = \lim_{s \to 0} s\, G_c(s) G(s) = K_c \alpha \lim_{s \to 0} s\, G_0(s)$$

5. Determine the value of K_c of the lead compensator from the magnitude condition.

Example 6.6:



$$\frac{C(s)}{R(s)} = \frac{10}{s^2+s+10} = \frac{10}{(s+0.5+j3.1225)(s+0.5-j3.1225)}$$

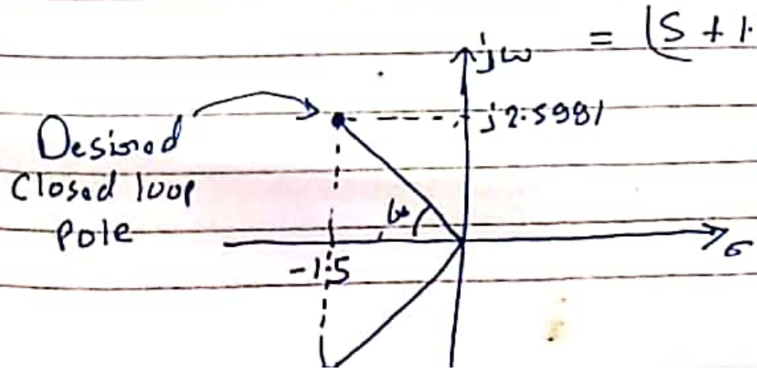


$$\xi = \frac{1}{2}\sqrt{10} = 0.158$$
$$\omega_n = \sqrt{10} = 3.162 \text{ rad/sec}$$
{Present Values}

$$\xi = 0.5$$
$$\omega_n = 3$$
{desired Values}

⇒ The desired location of the dominant closed loop poles can be determined from

$$s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 3s + 9$$
$$= (s+1.5+j2.5981)(s+1.5-j2.5981)$$

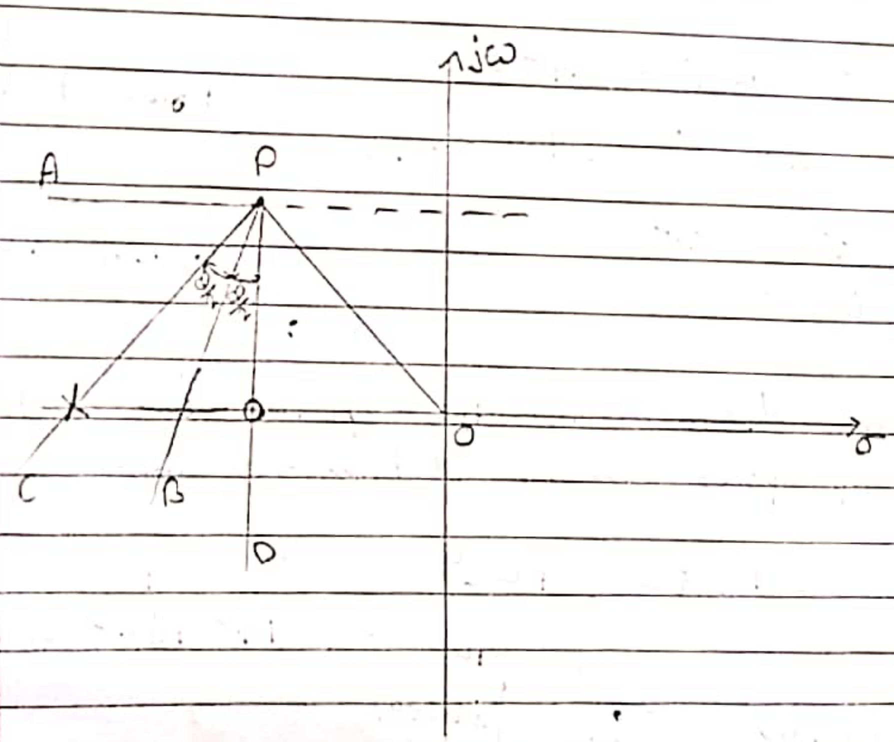


⇒ The angle from the pole at the origin to the desired dominant closed-loop pole at $s = -1.5 + j2.5981$ is $120°$. The angle from the pole at $s = -1$ to the desired closed loop pole is 100.894.

Hence, Angle of deficiency $= 180 - 120 - 100.894$

$= -40.894°$

<u>Method 1:</u> # First draw horizontal line passing through point P, ~~the~~ the desired location for one of the dominant closed-loop poles.



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Draw also a line connecting point P & the origin.
Bisect angle between line PA & PO by PB.
Draw two line PC & PD that makes angle $\pm \phi/2$ with the bisector PB.

The intersections of PC and PD with the negative real axis give the necessary locations for the poles & zero of the lead network.

⇒ The Compensator thus designed will make point P a point on the root locus of the compensated System.

⇒ The locations of the zeros and poles are found as follows.

$$\text{Zero at } s = -1.9432$$
$$\text{Pole at } s = -4.6458$$

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = K_c \frac{s + 1.9432}{s + 4.6458}$$

$$\left.\begin{aligned} \alpha &= 0.418 \\ T &= 0.5146 \end{aligned}\right\}$$

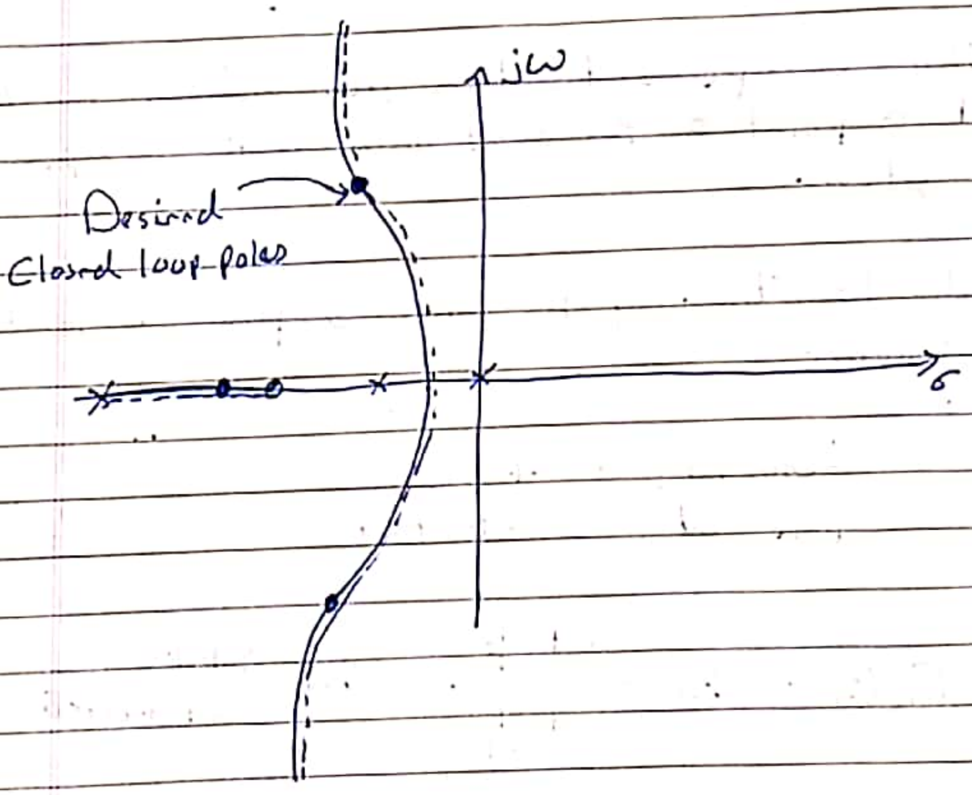
⇒ The value of K_c can be determined by use of the magnitude condition ÷

$$\left| K_c \frac{s+1.9432}{s+4.6458} \times \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5981} = 1$$

$$K_c = 1.2287$$

Hence lead Compensator $G_c(s)$ is given by :-

$$G_c(s) = 1.2287 \frac{s+1.9432}{s+4.6458}$$



⇒ It is worthwhile to check the static Velocity error constant K_v for the System just designed.

$$K_v = \lim_{s \to 0} s\, G_c(s) G(s) = 5.139$$

Method 2: If we choose the zero of the lead compensator at $s=-1$, so that it will cancel the plant pole at $s=-1$, then the compensator pole must be located at $s=-3$.

$$G_c(s) = K_c \frac{s+1}{s+3}$$

$$\left| K_c \frac{s+1}{s+3} \frac{10}{s(s+1)} \right| = 1 \quad \Rightarrow K_c = 0.9$$

$$\Rightarrow \boxed{G_c = 0.9 \frac{s+1}{s+3}}$$

⇒ The static velocity error constant for the present case is obtained as follows.

$$K_v = \lim_{s \to 0} s G_c(s) G(s) = 3$$

For different combination of a zero and pole of the compensator that contributes 40.894°, the value of K_v will be different.

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★ Lag Compensation

$$\frac{E_o(s)}{E_i(s)} = \hat{K}_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad \{\beta > 1\}$$

⇒ Consider the problem of finding a suitable Compensation network for the case where the system exhibits satisfactory transient response characteristics but unsatisfactory steady-state characteristics.

⇒ Compensation in this case essentially consists of increasing the open loop gain without appreciably changing the transient response characteristic.

↳ This can be accomplished by Lag Compensator.

⇒ To avoid an appreciable change in the root loci, the angle contribution of the lag network should be limited to a small amount, say less than 5°.

↳ To assure this, we place the pole & zero of the lag network relatively close together and near the origin of s plane.

↳ Then the closed-loop poles of the compensated system will be shifted only slightly from original location.

→ Hence the transient-response characteristics will be changed only slightly.

⇒ If we place the zero & pole of the lag compensator very close to each other then at $s = s_1$ (one of the dominant closed-loop poles)

$$s_1 + \left(\frac{1}{T}\right) \approx s_1 + \frac{1}{\beta T}$$

So, $$|G_c(s_1)| = \left| \hat{K}_c \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\beta T}} \right| = \hat{K}_c$$

⇒ To make the angle contribution of the lag portion of the compensator small, we require:

$$-5^\circ < \angle \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\beta T}} < 0$$

⇒ If gain $\hat{K}_c$ of lag compensator is set equal to 1, the alteration in the transient-response characteristics will be very small.

⇒ The static velocity error constant $\hat{K}_v$:

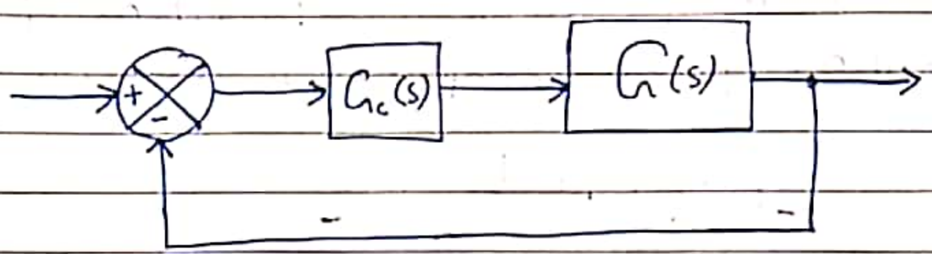
$$\hat{K}_v = \lim_{s \to 0} s G_c(s) G(s) = \lim_{s \to 0} G_c(s) K_v = \beta K_v \hat{K}_v$$

⇒ β should be chosen as high as possible.

⇒ The main negative effect of the lag Compensation is that the Compensator Zero that will be generated near the origin Creates a closed loop pole near the origin.

↳ This closed loop pole & Compensator Zero will generate a long tail of small amplitude in the step response, thus increasing the settling time.

Design Procedures for lag Compensation by root Locus Method



⇒ We assume that the uncompensated System meets the transient-response Specification by simple gain adjustment.

1. Draw the root-locus plot for the Uncompensated System whose open-loop transfer function is $G(s)$. Based on the transient-response Specifications, locate the dominant closed loop poles on the root locus.

2. Assume the transfer function of the lag compensator to be given by:

$$G_c(s) = \hat{K}_c \beta \frac{Ts+1}{\beta Ts+1} = \hat{K}_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

→ Then the openloop transfer function of the compensated system becomes: $G_c(s)\,G(s)$.

3. Evaluate the particular static error constant specified in the problem.

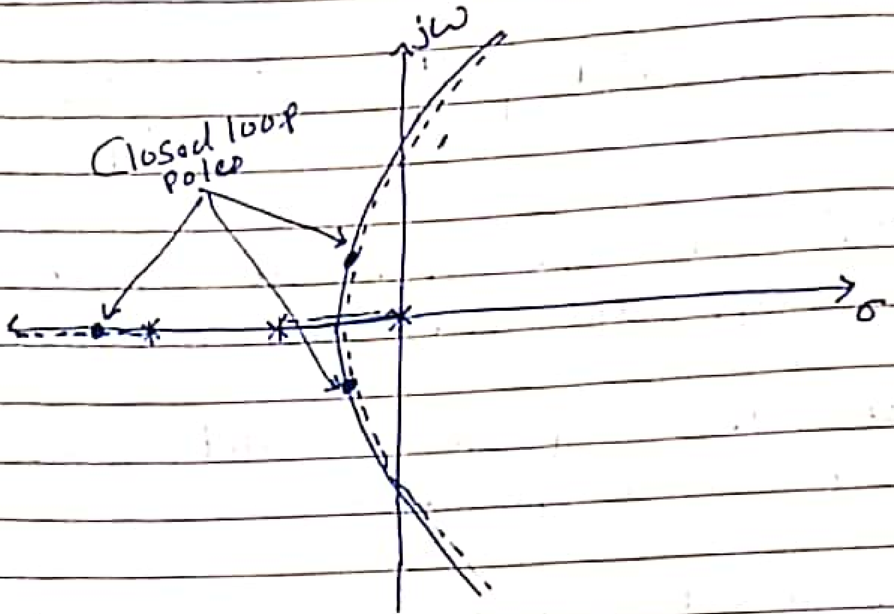
4. Determine the amount of increase in the static error constant necessary to satisfy the specification.

5. Determine the pole and zero of the lag compensator that produce the necessary increase in the particular static error constant without appreciably altering the original root loci.

6. Draw a new root locus plot for the compensated system. Locate the desired dominant closed-loop poles on the root locus.

7. Adjust gain $\hat{K}_c$ of the compensator from the magnitude condition so that the dominant closed-loop poles lie at the desired location.

Example 6.7: $G(s) = \frac{1.06}{s(s+1)(s+2)}$



⇒ The Closed loop transfer function becomes:

$$\frac{C(s)}{R(s)} = \frac{1.06}{s(s+1)(s+2) + 1.06}$$

$$= \frac{1.06}{(s+0.3307-j0.5864)(s+0.3307+j0.5864)(s+2.3386)}$$

⇒ The dominant, closed-loop poles are

$$s = -0.3307 \pm j0.5864$$

$\xi = 0.491$

$\omega_n = 0.673$ rad/sec

$K_v = 0.53$ 1/sec

⇒ It is desired to increase the static velocity error constant K_v to about $5\ sec^{-1}$ without appreciably changing the location of the dominant closed-loop poles.

⇒ To increase the static velocity error constant by a factor of about 10, let us choose $\beta = 10$ and place the zero and pole of the lag compensator at $s = -0.05$ and $s = -0.005$ respectively.

↳ The transfer function of the lag compensator becomes

$$G_c(s) = \hat{K}_c \frac{s+0.05}{s+0.005}$$

⇒ The angle contribution of this lag network near a dominant closed-loop pole is about 4°.

⇒ The open-loop transfer function of the compensated system then become.

$$G_c(s)G(s) = \frac{K(s+0.05)}{s(s+0.005)(s+1)(s+2)} \quad \left\{ K = 1.06\hat{K}_c \right\}$$

⇒ If the damping ratio of the new dominant closed loop poles is kept the same, then these poles are obtained from the new root locus plot as follows.

$$s = -0.31 \pm j0.55$$

⇒ The open loop gain K is determi. by magnitude condition ÷

$$K = \left| \frac{s(s+0.005)(s+1)(s+2)}{(s+0.005)} \right|_{s=-0.31+j0.55}$$

$$K = 1.0235$$

$$\hat{K}_c = \frac{K}{1.06} = 0.9656$$

$$\text{So } G_c(s) = 0.9656 \, \frac{s+0.05}{s+0.005}$$

⇒ The static velocity error constant K_v is ÷

$$K_v = \lim_{s \to 0} s\,G_c(s)\,G(s) = 5.12 \text{ sec}^{-1}$$

⇒ The steady-state error with ramp input has decreased to about 10% of the original system.

★ Lead-Lag Compensation

Lead Compensation basically speeds up the response and increases the stability of the system.

Lag Compensation improves the steady-state accuracy of the system, but reduces the speed of the response.

⇒ If improvements in both transient response and steady state response are desired, then both a lead compensator and a lag compensator may be used simultaneously.

↳ Rather than introducing both a lead compensator and a lag compensator as separate units, however it is economical to use a single lead-lag compensator.

$$\frac{E_o(s)}{E_i(s)} = K_c \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{\gamma}{T_1}\right)\left(s+\frac{1}{\beta T_2}\right)} \quad \{\beta > 1 \quad \gamma > 1\}$$

LEAD

LAG

⇒ In designing lag-lead compensators, we consider two cases where $\gamma \neq \beta$ & $\gamma = \beta$.

Case 1: ($\gamma \neq \beta$) The design process is combination of the design of the lead compensator and that of the lag compensator.

1. From the given performance specifications, determine the desired location of the dominant closed loop poles.

2. Using the uncompensated open loop transfer function $G(s)$, determine the angle deficiency ϕ if the dominant closed-loop poles are to be at the desired location. The phase-lead portion of the lag-lead compensator must contribute this angle ϕ.

3. Assume that we later choose T_2 sufficiently large so that the magnitude of the lag portion is approximately unity, where $s = s_1$ is one of the dominant closed-loop poles, choose the values of T_1 & γ from the requirement that

$$\angle \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} = \phi$$

The choice of T_1 & γ is not unique. Then determine the value of K_c from the magnitude condition.

$$\left| K_c \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} G(s_1) \right| = 1$$

4. If the static velocity error constant K_v is specified, determine the value of β to satisfy the requirement for K_v.

$$K_v = \lim_{s \to 0} s\, G_c(s)\, G(s)$$

$$K_v = \lim_{s \to 0} s K_c \frac{\beta}{\gamma} G(s)$$

Hence given the value of K_v, the value of β can be determined. Then, using the value of β thus determined, choose the value of T_2 such that.

$$\left| \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right| = 1 \qquad -5^\circ < \angle \frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} < 0^\circ$$

Case 2: $(\gamma = \beta)$

1. From the given performance specification, determine the desired location for the dominant closed-loop poles.

2. If static velocity error constant K_v is specified, determine the value of constant K_c from the following equation:-

$$K_v = \lim_{s \to 0} s\, G_c(s)\, G(s)$$

$$= \lim_{s \to 0} s K_c G(s)$$

3. To have the dominant closed-loop poles at the desired location, calculate the angle contribution ϕ needed from the phase lead portion of the lag lead-compensator.

4. For the lag-lead compensator we later choose T_2 sufficiently large so that

$$\left| \frac{s + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right|$$

is approximately unity, when $s = s_1$ is one of the dominant closed loop poles. Determine the values of T_1 & β from the magnitude and angle condition.

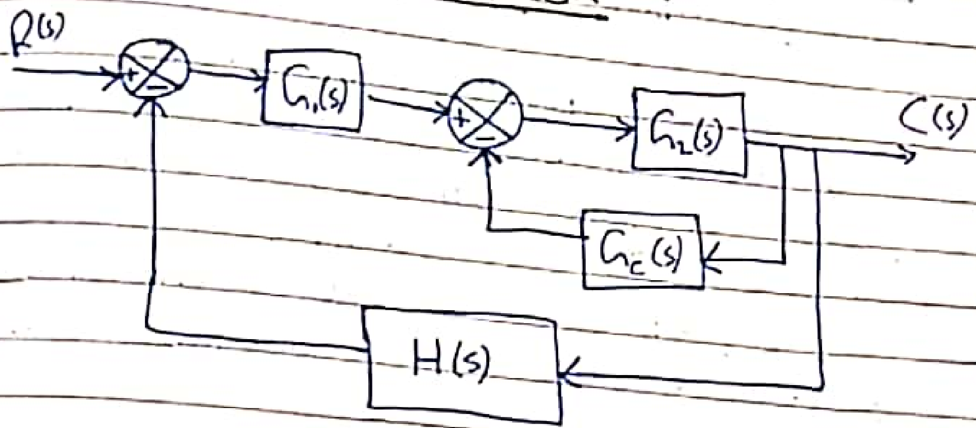
$$\left| K \left(\frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\beta}{T_1}} \right) G(s_1) \right| = 1$$

$$\angle \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\beta}{T_1}} = \phi$$

5. Usually the value of β just determined, choose T_2 so that

$$\left| \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right| \doteq 1 \qquad -5^\circ < \angle \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} < 0^\circ$$

★ Parallel Compensation



$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_2 G_c + G_1 G_2 H}$$

⇒ The characteristic equation is

$$1 + G_1 G_2 H + G_2 G_c = 0$$

$$\Rightarrow \quad 1 + \frac{G_c G_2}{1 + G_1 G_2 H} = 0$$

Let $G_f = \dfrac{G_2}{1 + G_1 G_2 H}$

the equation become

$$1 + G_c G_f = 0$$

fixed TF

⇒ Hence the same design approach applies to the parallel compensated system.

ex. ⇒ Velocity feedback system.