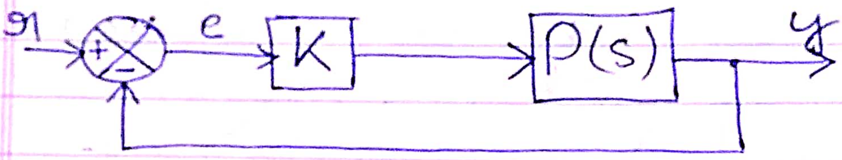
(11)

PID Control

(Proportional-integral-derivative Control)

★ **Proportional Controller**



⇒ If $P(s) = \frac{1}{s+1}$

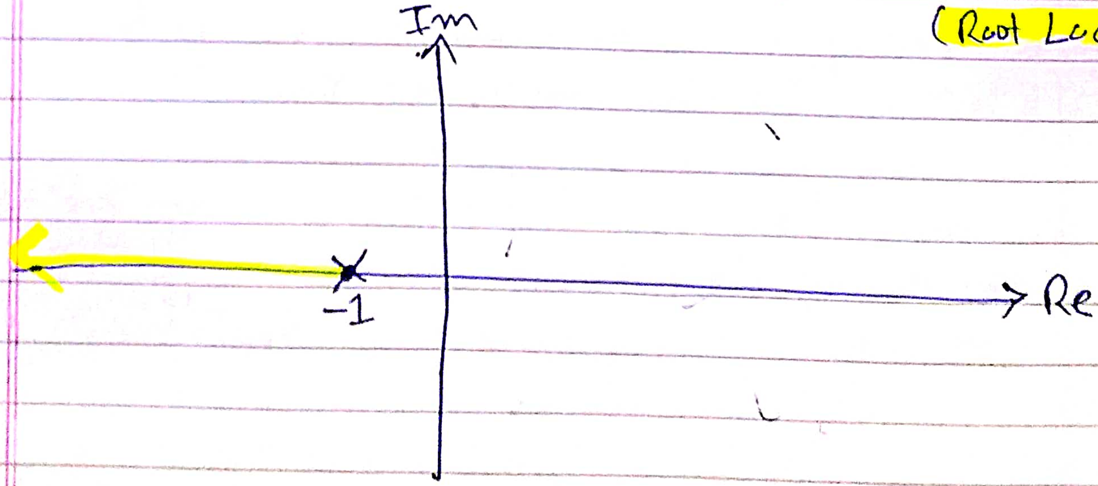
$$T(s) = \frac{L(s)}{1+L(s)} \quad \text{where, } L(s) = K\,P(s)$$

⇒ $T(s) = \frac{K}{s+1+K}$ → Has closed-loop pole at $s = -(1+K)$

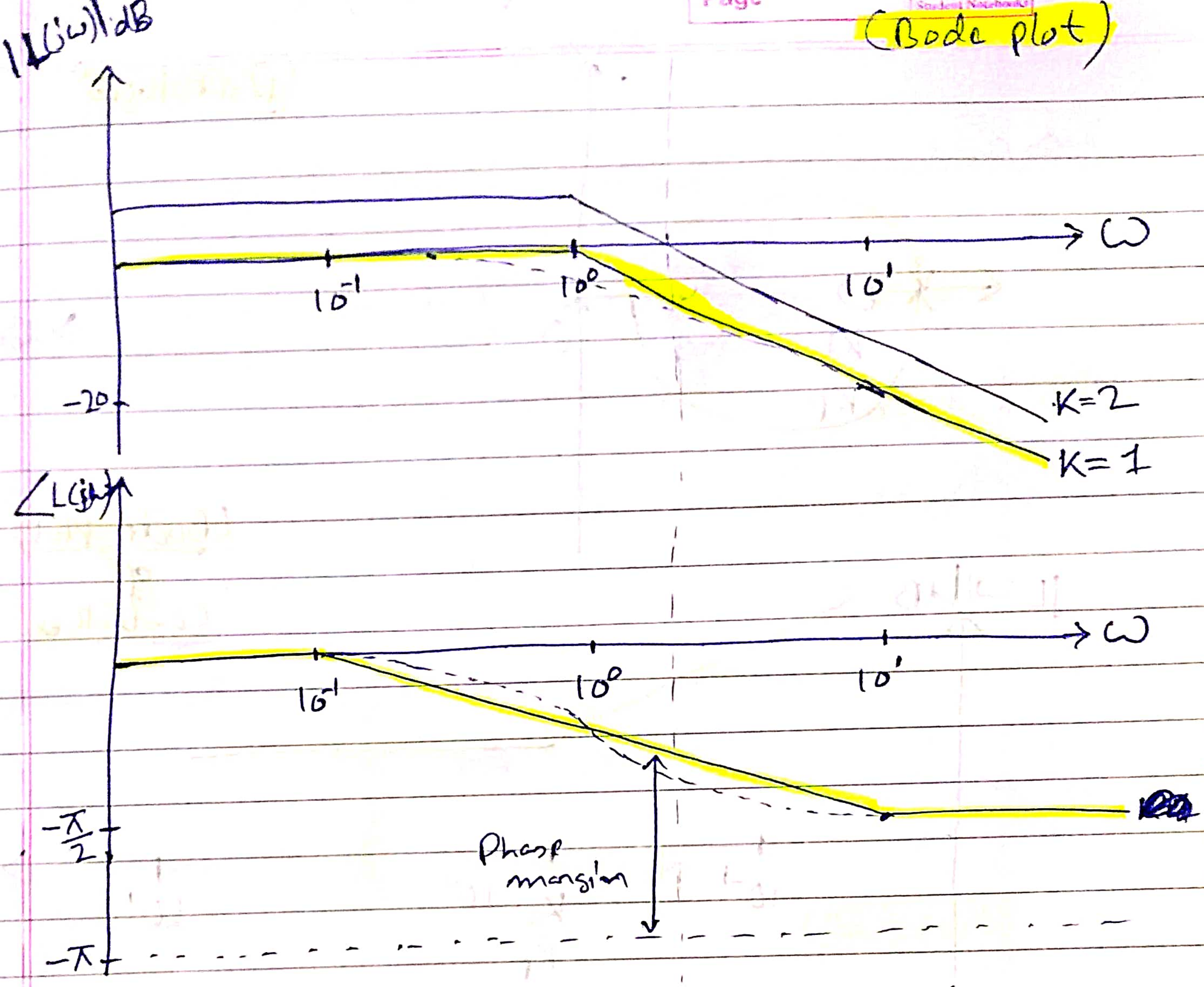
⇒ $e_{ss} = \lim_{s \to 0} \frac{1}{1+\frac{K}{1+s}} = \frac{1}{1+K}$

→ Steady state error

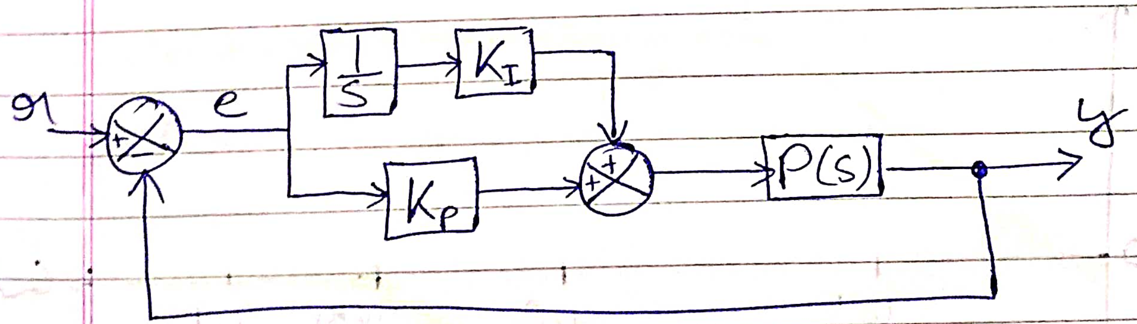
(Root Locus)



(Bode plot)



★ <u>Introducing an integrator</u>

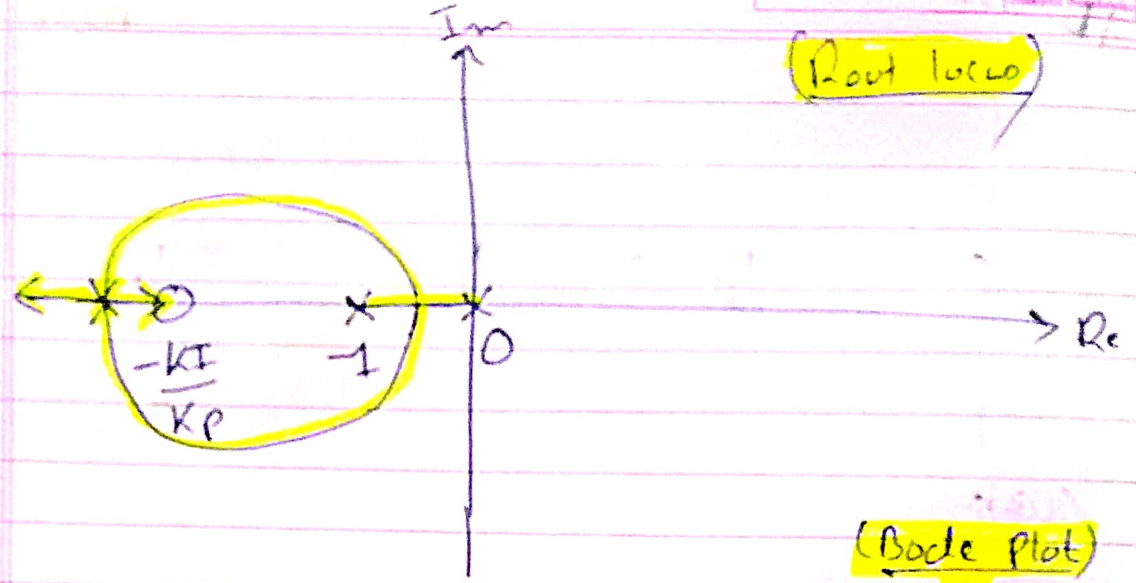


$$u(t) = K_p e(t) + K_I \int_0^t e(\tau)\,d\tau$$

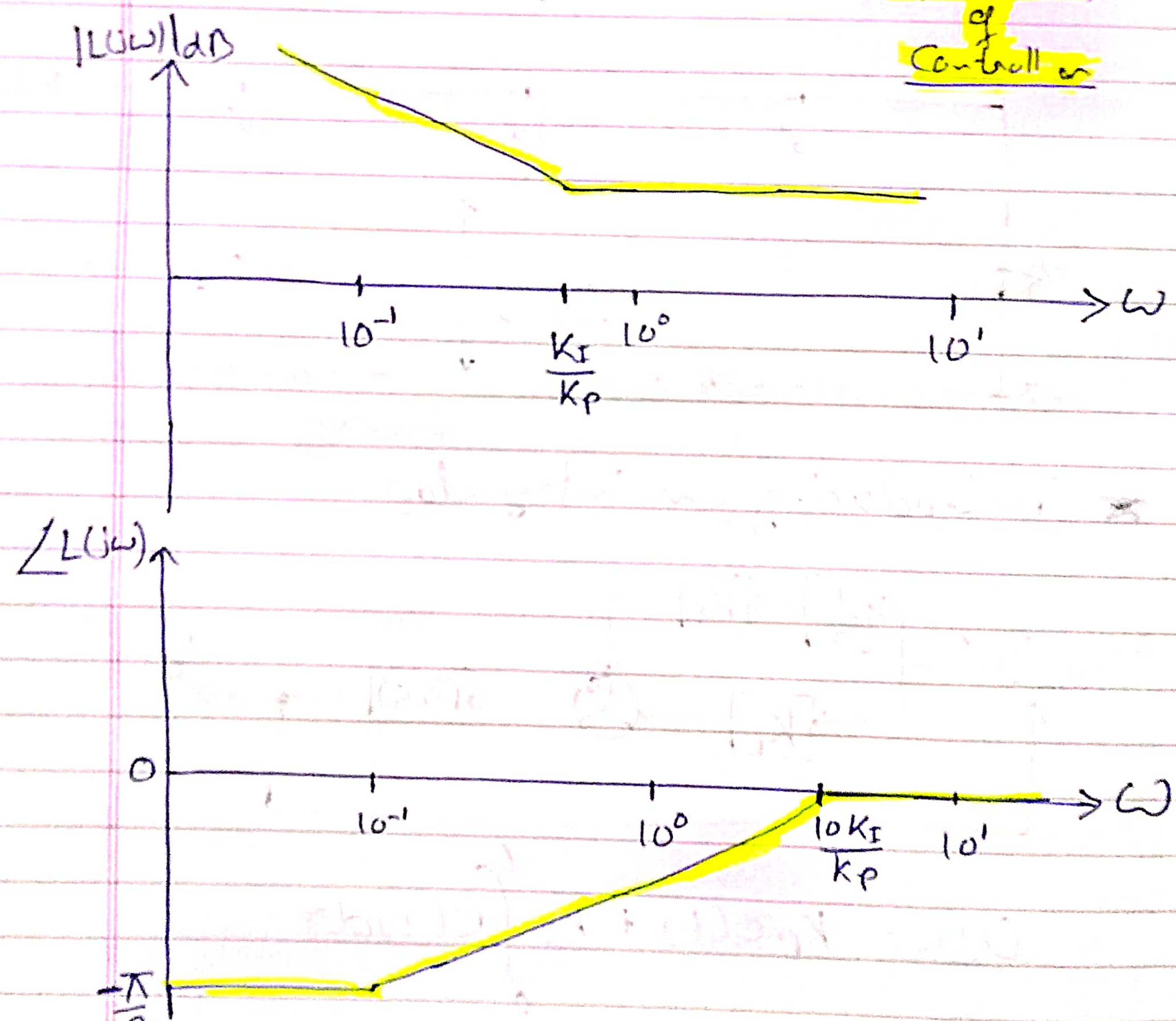
$$C(s) = K_p + \frac{K_I}{s} = \frac{K_p s + K_I}{s} = K_I\left(\frac{K_p/K_I\, s + 1}{s}\right)$$

$$P(s) = \frac{1}{s+1} \qquad C(s) = K_I\left(\frac{\frac{K_P}{K_I}s + 1}{s}\right)$$

Root locus



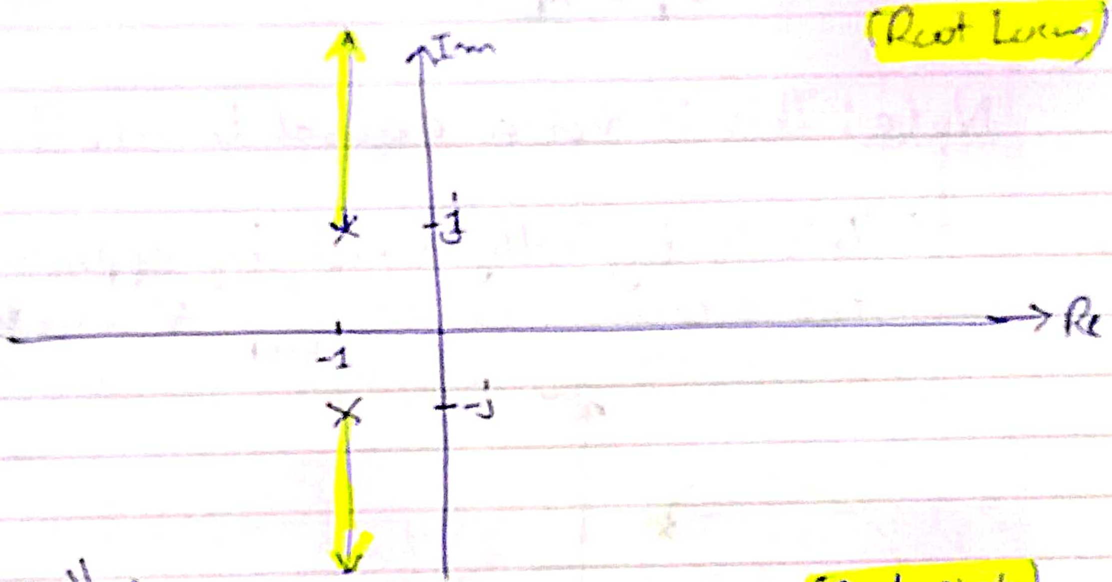
Bode plot of Controller



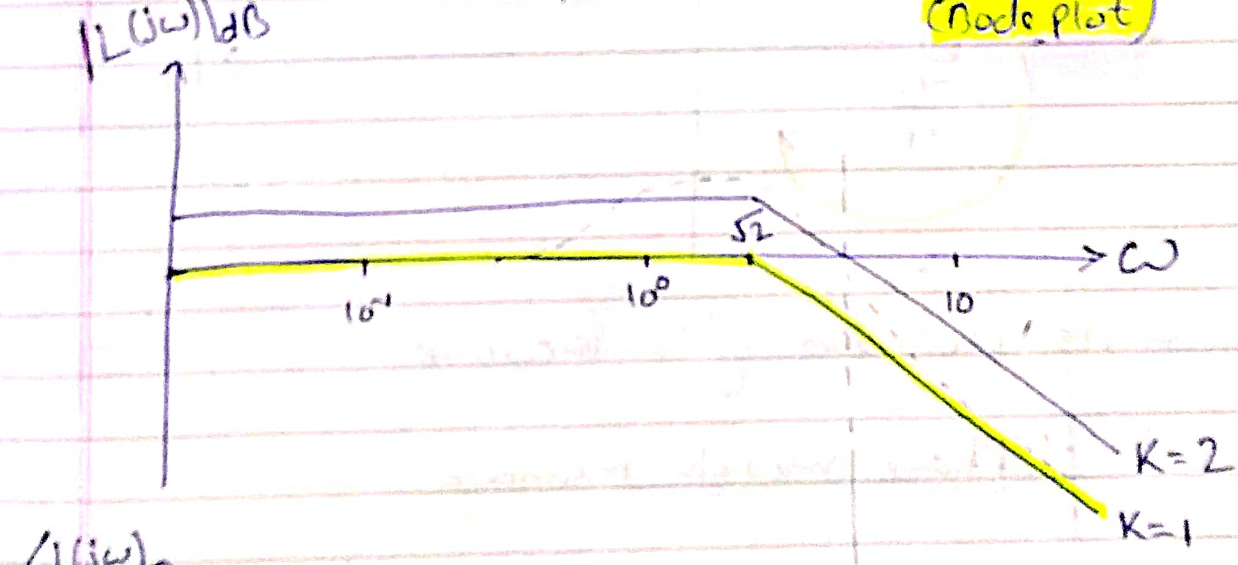
★ Proportional Control - Higher order system

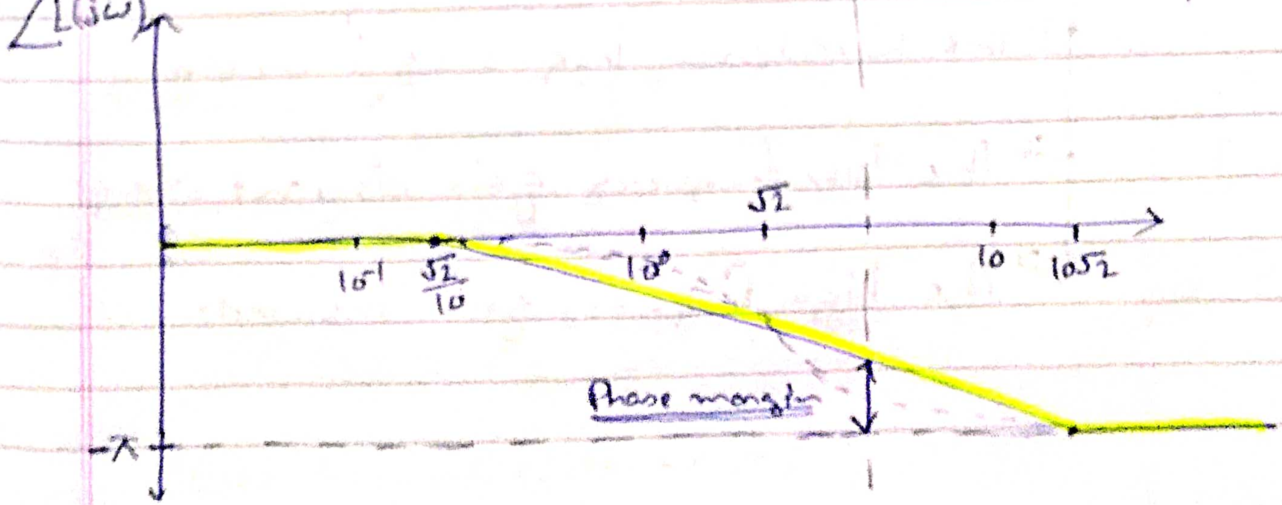
Let $P(s) = \dfrac{2}{s^2+2s+2} = \dfrac{1}{\dfrac{s^2}{(\sqrt{2})^2} + \dfrac{2 \cdot \frac{\sqrt{2}}{2}}{\sqrt{2}} s + 1}$

(Root Locus)



(Bode plot)





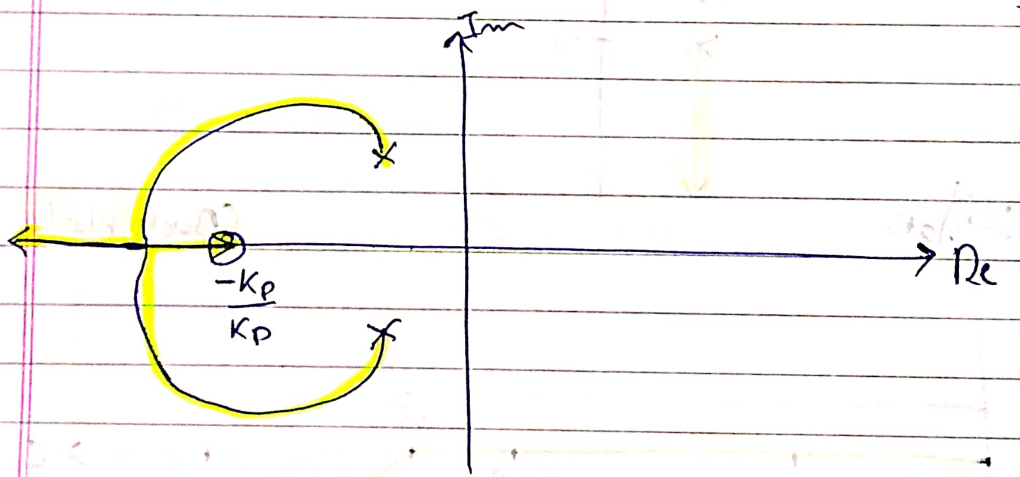
★ **Introducing a differentiator**

$$u(t) = K_p e(t) + K_D \dot{e}(t)$$

$$C(s) = K_p + K_D s$$

Note: This is not a causal transfer function.

→ This is typically fixed by approximating the derivative as $s \approx \frac{s}{cs+1}$ $\forall$ some ~~[illegible]~~ small c.



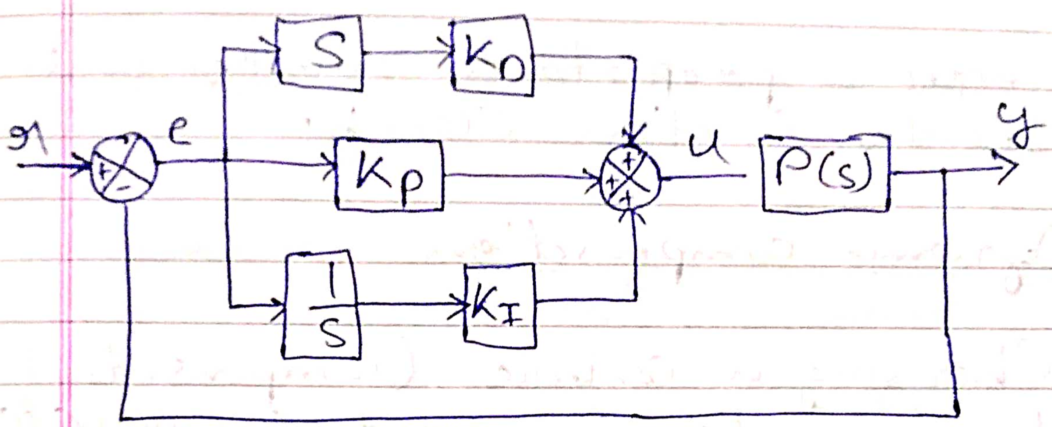
⇒ As derivative gain increase

- → Phase margin increase
- → The crossover frequency increase
- → The low frequency gain does not change
- → The high frequency gain increase.

Date

Page

★ PID Control



$$u(t) = K_P e(t) + K_I \int_0^t e(\tau)\,d\tau + K_D \dot{e}(t)$$

$$C(s) = K_P + \frac{K_I}{S} + K_D S = \frac{K_D S^2 + K_P S + K_I}{S}$$

★ PID Tuning

"Choosing the parameters K_P, K_I and K_D to reach the feedback control design specification"

→ Random guess a start point and iteratively correct it by hit & trial.

→ Use Matlab (This require system model)

⇒ Think of a PID as

$$C(s) = K_{RL} \frac{(s-z_1)(s-z_2)}{S}$$

"two zeros & a pole at Origin"