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Designing of a Control System

- Root-Locus Method
- Frequency Response method

★ Routh's stability Criterion

⇒ Routh's stability Criterion tells us whether or not there are unstable roots in a polynomial equation without actually solving them.

Procedure

1. Write the polynomial in s in the following form:

$$a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n = 0$$

$$\left\{ \begin{array}{l} \text{Cofficients are real} \\ a_n \ \& \ a_0 \neq 0 \end{array} \right\}$$

2. If any of the cofficient are zero or negative in the presence of atleast one positive cofficient

⇓

{ a root or root's exist that are imaginary and that have a positive real part }

↳ {Therefor system is not stable}

"A polynomial in s having real coefficients can always be factored into linear and quadratic factor, such as $(s+a)$ and (s^2+bs+c) where a, b, c are real"

Proof: (a) for $(s+a)$ to yield negative roots a must be positive.

(b) The factor (s^2+bs+c) to yield roots having negative real parts only if b and c are both positive.

⇒ For all roots to have negative real part, the constant a, b, c and soon in all factor must be positive.

⇒ The product of any number of linear and quadratic factors containing only positive coefficients always yields a polynomial with positive coefficient.

⇒ It is important to note that the condition that all the coefficients be positive is not sufficient to assume stability. (It is necessary but not necessary).

Ex:- $\cancel{(s+8)(s-3)(s-2)}$ $(s^2-2s+10)(s+3)$

$\Rightarrow \cancel{(s+6)(s^2-5s+6)}$ $\Rightarrow s^3+(3-2)s^2+(10-6)s+30$

$\Rightarrow \cancel{s^3}$ $\Rightarrow s^3+s^2+4s+30$

3. If all coefficients are positive, arrange the coefficients of the polynomial in rows and columns according to the following pattern

$$\begin{array}{lllll}
s^n & a_0 & a_2 & a_4 & a_6 \cdots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 \cdots \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
s^2 & e_1 & e_2 & & \\
s^1 & f_1 & & & \\
s^0 & g_1 & & &
\end{array}$$

⇒ The process of forming rows continues until we run out of element.

⇒ The coefficients $b_1, b_2 \cdots b_3$ and so on are evaluated as follows:-

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

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⇒ Routh's stability criterion states that the number of roots of equation above with positive real parts is equal to number of change in sign of the coefficients of the first column of the array.

Example

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

$$\begin{array}{c|c|cc}
s^4 & 1 & 3 & 5 \\
s^3 & 2 & 4 & 0 \\
s^2 & 1 & 5 & \\
s^1 & -6 & 0 & \\
s^0 & 5 & &
\end{array}$$

⇒ There are two roots with positive real part.

★ Special Case

⇒ If a first-column term in any row is zero, but the remaining terms are not zero or there is no remaining term, then the zero term is replaced by a very small positive number ε.

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Example

$$s^3 + 2s^2 + s + 2 = 0$$

s^3	1	1
s^2	2	2
s^1	ϵ	
s^0	2	

⇒ If the sign of the coefficient above the zero (ϵ) is same as that below it, it indicates that there is a pair of imaginary roots at $s = \pm j$.

⇒ If, however, the sign of the coefficients above the zero (ϵ) is opposite that below it, it indicates that there is one sign change.

If all the coefficients in any desired row are zero, it indicates that there are roots of equal magnitude lying radially opposite in s-plane.

- Two real roots with equal magnitude and opposite sign
  and/or
- Two conjugate imaginary roots

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Example

$$s^5 + 2s^4 + 24s^3 + 48s^2 - 25s - 50 = 0$$

s^5	1	24	-25
s^4	2	48	-50
s^3	0	0	

← Auxiliary Polynomial. P(s)

$P(s) = 2s^4 + 48s^2 - 50$

$P'(s) = 8s^3 + 96s$

Indicates two pair of roots with equal and opp sign

⇒ s^5	1	24	-25
s^4	2	48	-50
s^3	8	96	
s^2	24	-50	
s^1	112.7	0	
s^0	-50		

⇒ Clearly, the Original equation have one root with a positive real part.

★ Relative stability analysis

⇒ Routh's stability criterion provides the answer to the question of absolute stability. This in many practical cases, is not sufficient.

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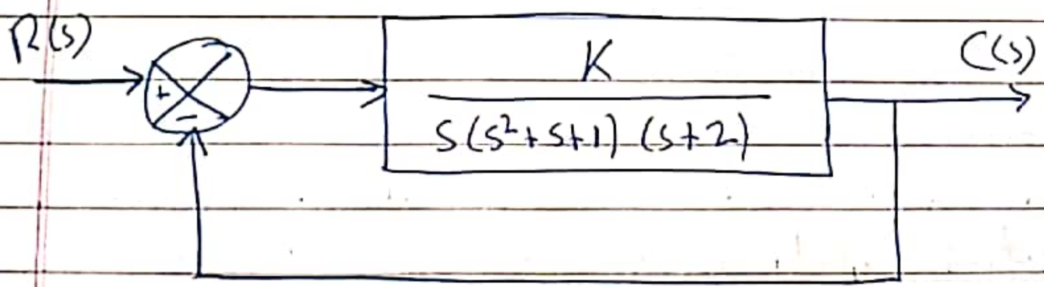
⇒ A useful approach for examining relative stability is to shift the s-plane axis and apply Routh's Stability Criterion.

$$S = \hat{S} - \sigma$$

⇒ Write polynomial in terms of $\hat{s}$.

⇒ One applying Routh's Stability Criterion. The number of change of sign indicates number of roots that are located to the right of the vertical line $s = -\sigma$.

★ Application of Routh's Stability Criterion to Control-System Analysis



$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2+s+1)(s+2) + K}$$

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⇒ The Characteristic equation is

$$s^4 + 3s^3 + 3s^2 + 2s + K = 0$$

$$\begin{array}{llll} s^4 & 1 & 3 & K \\ s^3 & 3 & 2 & 0 \\ s^2 & 7/3 & K & \\ s^1 & 2-\frac{9}{7}K & & \\ s^0 & K & & \end{array}$$

For stability $K > 0$ & $2 - \frac{9}{7}K > 0$

$$\Rightarrow \boxed{0 < K < \frac{14}{9}} \quad \{\text{for stability}\}$$

★ Effects of Integral and Derivative Control action on System peformance

★ Integral Control action

⇒ In proportional Control of a plant whose TF does no possess an integrator (1/s), there is a steady-state error, or offset, in the responce to a step input.

↳ Such an offset can be elimineded if the integral Control action is included in the controller.

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⇒ Integral control action, while removing offset or steady-state error, may lead to oscillatory response of slowly decreasing amplitude or even increasing amplitude, both of which are usually undesirable.

★ Proportional Control Systems

⇒ Proportional Control of System without an integrator will result in a steady-state error with a step input.

★ Derivative Control Action

⇒ Derivative Control when added to a proportional controller, provides a means of obtaining a controller with high sensitivity.

⇒ Derivative control thus anticipates the actuating error, initiates an early corrective action and tends to increase the stability of the system.

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