Grid based SLAM with Rao-Blackwellized Particle Filter

(Grid-Based FastSLAM)

* Factorization of the SLAM posterior

$$\boxed{\begin{aligned} &P(x_{0:t}, m \mid z_{1:t}, u_{1:t}) \\ &= \underbrace{P(x_{0:t} \mid z_{1:t}, u_{1:t})}_{\text{Path posterior (Particle filter)}} \; \underbrace{P(m \mid x_{1:t}, z_{1:t})}_{\text{map posterior (given the path)}} \end{aligned}}$$

* Grid-Based Mapping with Rao-Balackwallized Particle Filters

→ Each particle represents a possible trajectory of the robot.

→ Each particle maintains its own map.

→ Each particle updates it upon "mapping with known poses".

* Problem

→ Too many samples are needed to sufficiently model the motion noise.

→ Increasing the number of samples is difficult as each map is quite large.

→ Idea ⇒ Improve the pose estimate before applying the particle filter.

* Pose correction with Scan Matching

Maximize the likelihood of the current pose and map relative to the previous pose and map.

$$x^*_t = \underset{x_t}{\arg\max} \left\{ p(z_t \mid x_t, m_{t-1})\, p(x_t \mid u_{t-1}, x^*_{t-1}) \right\}$$

current measurement

map constructed so far

robot motion

* Grid-Based FastSLAM with Improved Odometry (2003)

- Scan-matching provides a locally consistent pose correction.
- Pre-correct short odometry sequences using scan-matching and use them as input to FastSLAM.
- Fewer particles are needed, since the error in the input is smaller.

⇒ Can be seen as an ad-hoc solution to an improved proposal distribution.

* What's Next

⇒ Compute an improved proposal that considers the most recent observation.

$$x_t^{[k]} \sim p(x_t \mid x^{[k]}_{1:t-1}, u_{1:t}, z_{1:t})$$

Goals

1. More precise sampling.
2. More accurate maps.
3. Less particles needed.

* **The Optimal Proposal Distribution**

$$p(x_t \mid x_{t-1}^{[i]}, m^{[i]}, z_t, u_t)$$

$$= \frac{p(z_t \mid x_t, m^{[i]})\, p(x_t \mid x_{t-1}^{[i]}, u_t)}{p(z_t \mid x_{t-1}^{[i]}, m^{[i]}, u_t)}$$

Lets call it $\tau(x_t)$

$$p(z_t \mid x_{t-1}^{[i]}, m^{[i]}, u_t) = \int p(z_t \mid x_t, m^{[i]})\, p(x_t \mid x_{t-1}^{[i]}, u_t)\, dx_t$$

$$\Rightarrow p(x_t \mid x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) = \frac{\tau(x_t)}{\int \tau(x_t)\, dx_t}$$

$$= \frac{p(z_t \mid x_t, m^{[i]})\, p(x_t \mid x_{t-1}^{[i]}, u_t)}{\int \underbrace{p(z_t \mid x_t, m^{[i]})}_{\text{locally}}\, \underbrace{p(x_t \mid x_{t-1}^{[i]}, u_t)}_{\text{globally}}\, dx_t}$$

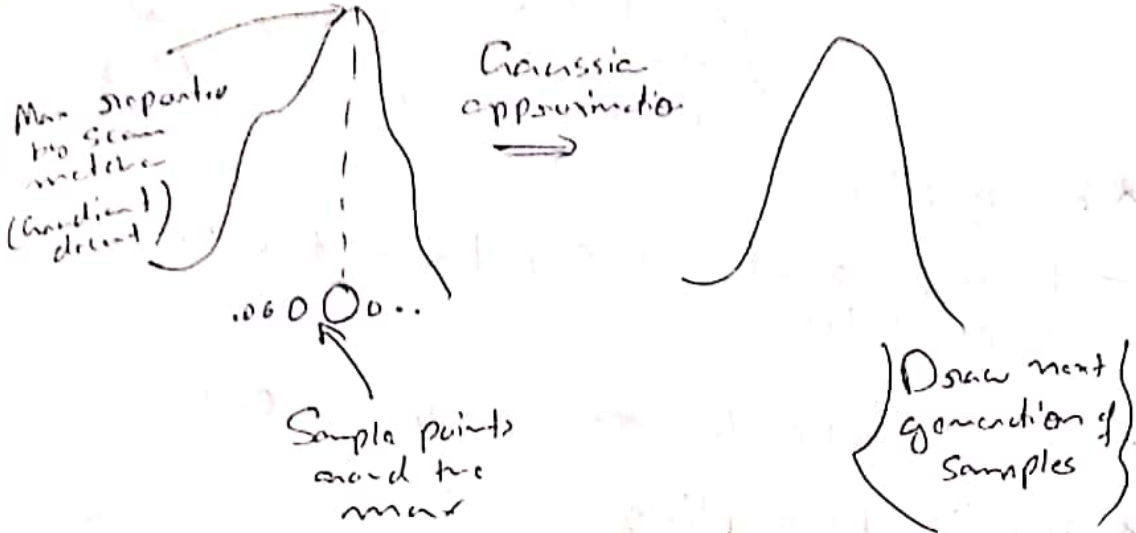
locally limits the area over which to integrate. (measurement)

globally limits the area over which to integrate. (odometry)

$$\Rightarrow p(x_t \mid x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) \simeq \frac{\tau(x_t)}{\int_{\{x_t \mid \tau(x_t) > \varepsilon\}} \tau(x_t)\, dx_t}$$

How to Sample from this term?

$$p(x_t \mid x_{t-1}^{[i]}, m^{[i]}, z_t, u_t) \simeq N(\mu^{[i]}, \Sigma^{[i]})$$



* Estimating the parameters of the Gaussian for each Particle

$$\mu^{[i]} = \frac{1}{\eta}\sum_{j=1}^{K} x_j \, \tau(x_j)$$

$$\Sigma^{[i]} = \frac{1}{\eta}\sum_{j=1}^{K} (x_j - \mu^{[i]})(x_j - \mu^{[i]})^T \tau(x_j)$$

$\Rightarrow$ x_j are a Set of points Sampled around the point x^* the Scan matching has Converged to.

* Computing the Importance Weight

$$w_t^{[i]} = w_{t-1}^{[i]} \, p(z_t \mid x_{t-1}^{[i]}, m^{[i]}, u_t)$$

$$= w_{t-1}^{[i]} \int \underbrace{p(z_t \mid x_t, m^{[i]})\, p(x_t \mid x_{t-1}^{[i]}, u_t)}_{\tau(x_t)} \, dx_t$$

$$\simeq w_{t-1}^{[i]} \int_{\{x_t \mid \tau(x_t) > \varepsilon\}} \tau(x_t)\, dx_t$$

$$w_t^{[i]} \simeq w_{t-1}^{[i]} \sum_{j=1}^{K} \tau(x_j)$$

{Sampled points around the maximum of the likelihood function found by scan matching}

★ Resampling

→ Resampling at each step limits the "memory" of our filter.

→ Goal: Reduce the resampling actions.

★ Selective Resampling

- Resampling is necessary to achieve convergence.
- Resampling is dangerous, since important samples might get lost ("Particle depletion")
- Resampling makes only sense if particle weights differ significantly.

★ Number of Effective Particles

- Emperical measurement of how well the target distribution is approximated by samples drawn from the proposal.

$$n_{eff} = \frac{1}{\sum (w_t^{[i]})^2}$$

{It describes inverse variance of the normalized particle weight}

⇒ for equal weights, the sample approximation is close to the target.

★ Resa…

⇒ If …
, no …

⇒ We …
a gi…

★ Limit…

(1) Use …
di…

→ Ef…

⇒ …

Ef…

* Resampling with N_{eff}

⇒ If our approximation is close to the target, no resampling is needed.

⇒ We only resample when N_{eff} drops below a given threshold ($N/2$)

* Limitation

(1) Use of Gaussian to approximate proposal distribution.

↳ Efficient Multi-Modal Sampling

⇒ Sample from odometry first and then use this as the start point for scan matching.

Effects

→ Allows for better modeling multi-model likelihood function (high KLD values do not occur).

→ Minimal computational overhead.