Lecture 9

The Nyquist Condition

★ The polar plot

⇒ In the polar plot, the frequency response $G(j\omega)$ is plotted on the complex plane as a parametric function of ω.

⇒ No special rules for drawing it.

⇒ In fact, it is convenient to sketch a Bode plot first, so that we can have a good idea of what the polar plot looks like.

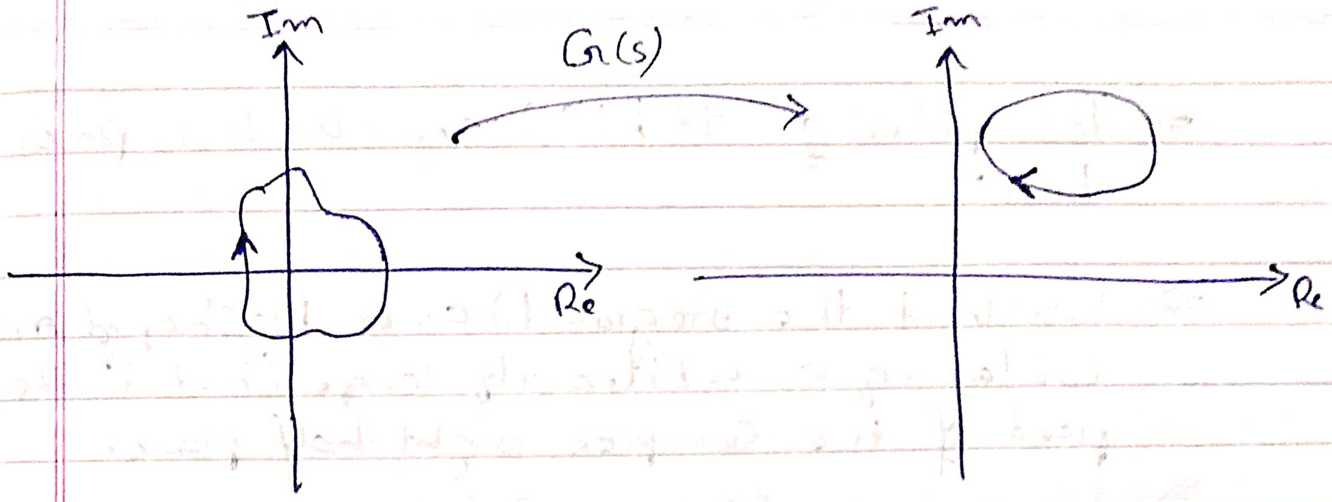
⇒ The only things that really matter in the polar plot are:

- → Where the plot intersects the unit circle $(|G(j\omega)| = 1)$
- → Where the plot crosses the real axis $(\angle G(j\omega) = I \cdot 180°)$

★ The principle of variation of the argument

⇒ Let $D \subset \mathbb{C}$ be a bounded, simply-connected region of the complex plane, and Γ be its boundary.

⇒ As s moves along the closed curve Γ, $G(s)$ describes another closed curve.



⇒ The number N of times that $G(s)$ encircles the origin of the complex plane as s moves along the boundary Γ of a bounded simply-connected region of the plane satisfies

$$\boxed{N = Z - P}$$

Where Z and P are the numbers of zeros and poles of $G(s)$ in D, respectively.

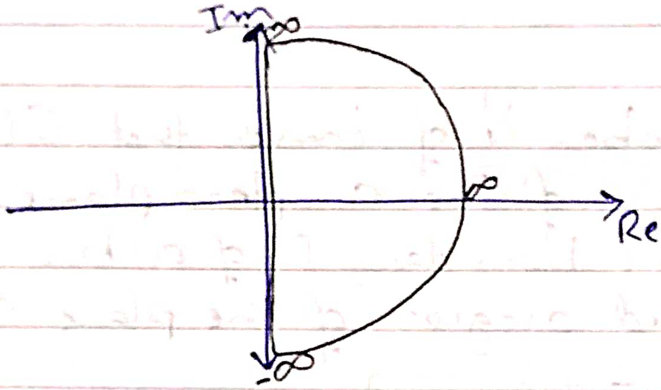
<u>Note</u>:: Encirclements are counted positive if in the same direction as s moves along Γ, and negative otherwise.

★ <u>The Nyquist or D Contour</u>

⇒ For closed-loop stability, the closed-loop poles, which corresponds to the roots (i.e. zeros) of the characteristic polynomial $1 + kL(s)$ must has negative real part.

⇒ The poles of $1+KL(s)$ are also the poles of $L(s)$.

⇒ Construct the region D as a D-shaped region containing an arbitrarily large (but finite) part of the complex right-half plane.



⇒ As s moves along the boundary of this region $1+KL(s)$ encircles the origin $N=Z-P$ times:

* Z is the number of unstable closed-loop poles
* P is the number of unstable open loop poles

OR

⇒ As s moves along the boundary of this region, $L(s)$ encircles the $-1/K$ point $N=Z-P$ times where,

* Z is the number of unstable closed-loop poles.
* P is the number of unstable open-loop poles

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⇒ Symmetry of poles / zeros about the real axis implies that

$$\angle L(-j\omega) = -\angle L(j\omega)$$

hence the plot of $L(s)$ when s moves on the boundary of the Nyquist Contour is just the polar plot + its symmetric plot about the real axis.

↳ This is what is called Nyquist plot.

⇒ Theorem: The Nyquist condition

"Consider a closed-loop system with loop transfer function $KL(s)$, which has P poles in the region enclosed by the Nyquist contour. Let N be the net number of clockwise encirclements of $-1/K$ by $L(s)$ when s moves along the Nyquist contour in the clockwise direction. The closed loop system has $Z = N + P$ poles in the Nyquist contour"

↳ If the open-loop system is stable, the closed loop system is stable as long as the Nyquist plot of $L(s)$ does NOT encircle the $-1/K$ point.

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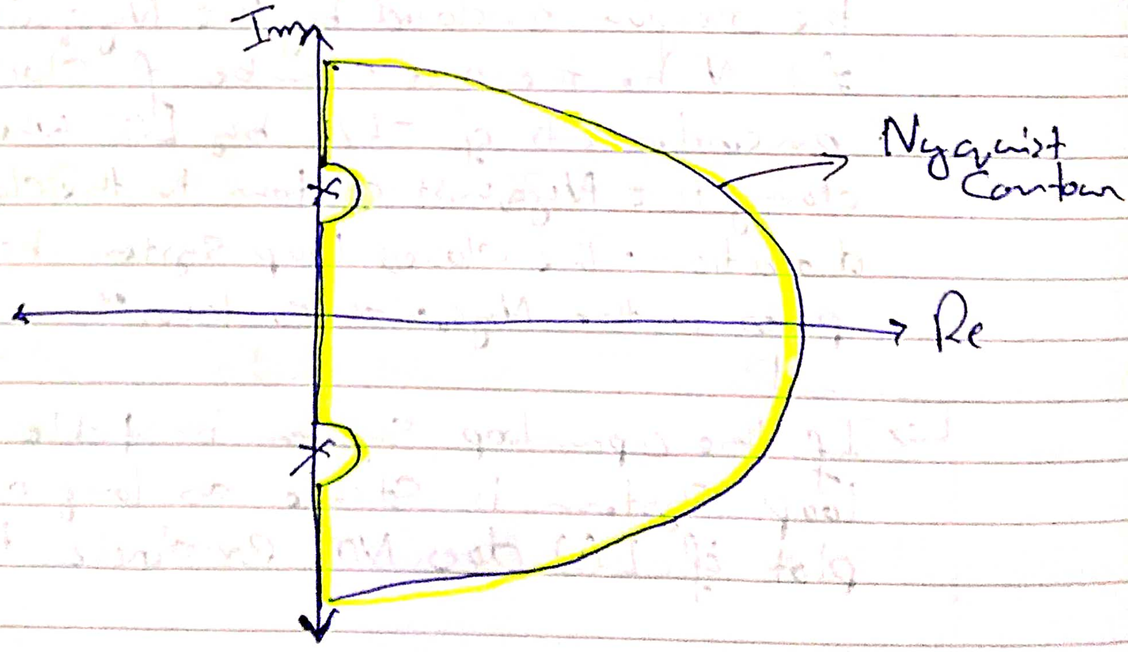
⇒ If the open loop system has P poles, the closed-loop system is stable as long as the Nyquist plot of L(s) encircles the $-1/K$ point P times in the negative (counter-clockwise) direction

★ <u>Dealing with open-loop poles on the imaginary axis</u>

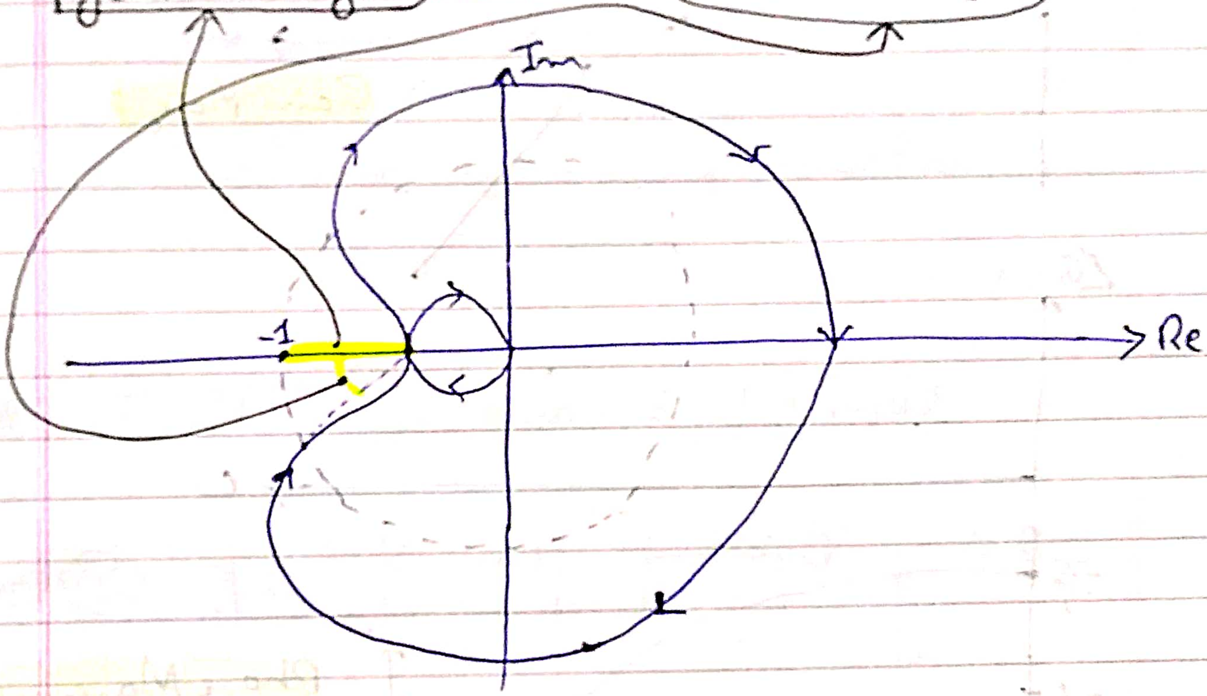
⇒ If there are open-loop poles on the imaginary axis make small "indentations" in the Nyquist contour, e.g. leaving the imaginary poles on the left.

⇒ Be careful on how you close the Nyquist plot "at infinity". If moving CCW around the poles, then close the plot CW.



★ The Nyquist condition and robustness margins

① Gain margin ② Phase margin



★ The Nyquist condition and Bode plots

⇒ If the open-loop is stable, then we know that in order for the closed-loop to be stable the Nyquist plot of $L(s)$ should Not encircle the -1 point.

↳ Or $|L(j\omega)| < 1$ whenever $\angle L(j\omega) = 180$

⇒ On the Bode plot, this means that the magnitude plot should be below the 0 dB line if/when the phase plot crosses the -180 line.

↳ Valid only if the open loop is stable.

* Gain and Phase margin