Discrete time system (cont.) and introduction to MIMO systems

① Discrete time System (cont.)

★ **Stability: distinct eigenvalues**

$$x(k+1) = A_d x(k) + B_d u(k)$$
$$y(k) = C_d x(k) + D_d u(k)$$

⇒ The above system is asymptotic stable when eigenvalues of A_d are inside the unit circle.

⇒ If input u is 0 $\forall K$

$$x(K) = A^K x(0)$$

$$A = T \Lambda T^{-1} \qquad A^K = T \Lambda^K T^{-1}$$

Λ → diagonal $\begin{pmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ & & \ddots \\ 0 & & & \lambda_n \end{pmatrix}$

★ **Z-transform fundamentals**

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{Continuous} & f(t) \arrow[r, shift left, "\text{Laplace transform}"] \arrow[d, shift left, "\text{ADC}"] & F(s) \arrow[l, shift left, "\text{Inverse Laplace}"] \arrow[d, "\substack{\text{Approximation} \\ \text{(eg. Tustin)}}"] \\
\text{Discrete} & f(k) \arrow[r, shift left, "\text{Z-transform}"] \arrow[u, shift left, "\text{DAC}"] & F(z) \arrow[l, shift left, "\text{Inverse Z-transform}"]
\end{tikzcd}$$

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⇒ Definition

$$F(z) = Z[f(k)](z) = \sum_{k=-\infty}^{\infty} f(k) z^{-k}$$

⇒ Linearity

$$Z[af(k) + bg(k)] = aF(z) + bG(z)$$

⇒ Time shift

$$Z[f(k-k_0)] = z^{-k_0} F(z)$$

Very useful for implementation of a discrete time controller

② Introduction to MIMO system

⇒ MIMO systems:

- have transfer function matrices not just function
- are different from SISO system for the concept of directionality.

★ MIMO state space representation

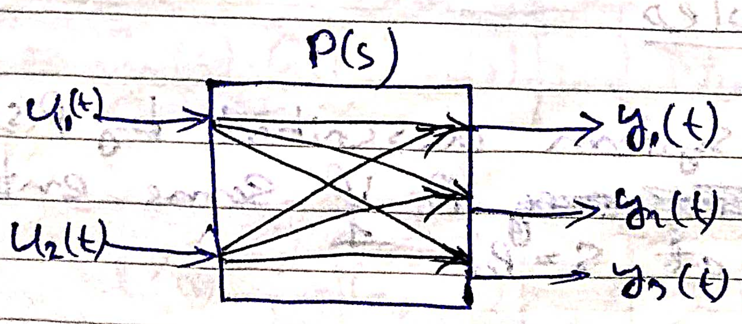
$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

$$\boxed{\begin{aligned} & A \in R^{n\times n} \\ & x \in R^{n\times 1} \\ & u \in R^{m\times 1} \\ & B \in R^{n\times m} \\ & y \in R^{l\times 1} \\ & C \in R^{l\times n} \\ & D \in R^{l\times m} \end{aligned}}$$

★ MIMO transfer function matrix

$$P(s) = C(sI-A)^{-1}B \in R^{l\times m}$$

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) & \cdots & P_{1m}(s) \\ P_{21}(s) & P_{22}(s) & \cdots & P_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ P_{l1}(s) & P_{l2}(s) & \cdots & P_{lm}(s) \end{bmatrix}$$



* ~~When each input effects many outputs~~

* Interactive system
  - When each input effects many outputs, the system is called interactive.

* Non-Interactive system
  - When each input effects only one output, the system is called non-interactive.

⇒ When a system has more outputs than inputs there are output 'directions' that are not effected by the input.

⇒ When a system has more input than outputs there are input "directions" that do not effect the output.

⇒ A system is functionally controllable if the rank of transfer matrix is equal to number of output.

* MIMO poles

⇒ A MIMO system described by $P(s)$ has a pole at ~~[illegible]~~ P_0 if some entries of $P(s)$ has pole at $s = P_0$.

⇒ Equivalently: The pole of a MIMO system are the roots of the characteristic polynomial of A.

$$P(S) = C(SI-A)^{-1}B + D = \frac{N(S)}{\det(SI-A)}$$

★ MIMO Zeros

⇒ P(s) has a (transmission) zero at Z_0 if it drops rank at $S = Z_0$.

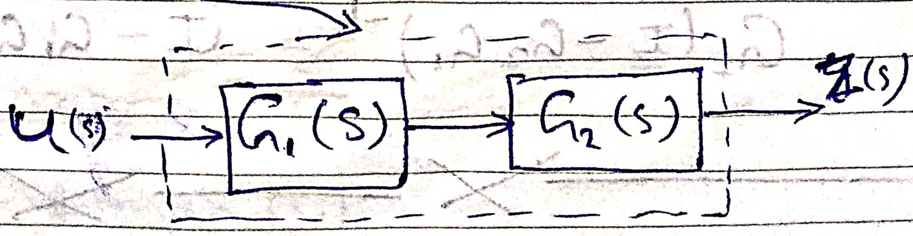
⇒ A rational matrix P(s) ~~of~~ ~~h~~as a zero at $S = Z_0$ if there is a rational u(s) vector such that $u(Z_0)$ is finite and different from zero, and:

$$\lim_{S \to Z_0} P(S)\,u(S) = 0$$

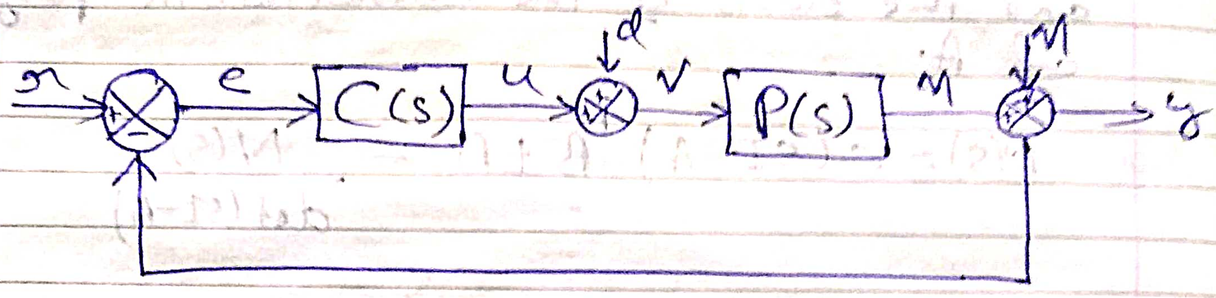
⇒ Poles and Zeros have (directions) associated with them.

★ MIMO Interconnections

$$\boxed{Z(s) = G_2(s)\,G_1(s)\,u(s)}$$ {Not Commutative}



★ Output Sensitivity function



$L_O(s) = P(s)\,C(s)$ {Loop transfer function}

$(r \to y)$ $S_O(s) = (I + L_O(s))^{-1}$ {Sensitivity function}

$(r \to y)$ $T_O(s) = (I + L_O(s))^{-1} L_O(s)$ {Complementary Sensitivity function}

★ Input Sensitivity function

$L_I(s) = C(s)\,P(s)$

$S_I(s) = (I + L_I(s))^{-1}$

$T_I(s) = (I + L_I(s))^{-1} L_I(s)$

★ Observations

⇒ The following relationship ("Push-through rule") holds for all matrices of appropriate dimensions:

$$G_1 (I - G_2 G_1)^{-1} = (I - G_1 G_2)^{-1} G_1$$