Extended Kalman Filter (EKF)

⇒ In most realistic problem, motion model and sensor model are not linear.

⇒ Let us consider a non linear motion model and sensor model:

$\cancel{x_t}$

$$\boxed{\begin{aligned} x_t &= g(u_t, x_{t-1}) + \epsilon_t \\ z_t &= h(x_t) + \delta_t \end{aligned}}$$

★ **Linearization of motion & sensor model using First order Taylor Expation**

① $$g(u_t, x_{t-1}) \approx g(u_t, \mu_{t-1}) + \underbrace{\frac{\delta g(u_t, \mu_{t-1})}{\delta x_{t-1}}}_{G_t} (x_{t-1} - \mu_{t-1})$$

② $$h(x_t) \simeq h(\bar{\mu}_t) + \underbrace{\frac{\delta h(\bar{\mu}_t)}{\delta x_t}}_{H_t} (x_t - \bar{\mu}_t)$$

(Jacobian Matrices)

★ Linearized Motion Model

$$p(x_t \mid u_t, x_{t-1})$$

$$= \det(2\pi R_t)^{-1} \exp\Big(-\frac{1}{2}\big(x_t - g(u_t, \mu_{t-1}) - G_t(x_{t-1} - \mu_{t-1})\big)^T$$

$$R_t^{-1}\big(x_t - g(u_2, \mu_{t-1}) - G_t(x_{t-1} - \mu_{t-1})\big)\Big)$$

★ Linearized Observation Model

$$p(z_t \mid x_t) = \det(2\pi Q_t)^{-\frac{1}{2}}$$

$$\exp\Big(-\frac{1}{2}\big(z_t - h(\bar{\mu}_t) - H_t(x_t - \bar{\mu}_t)\big)^T$$

$$Q_t^{-1}\big(z_t - h(\bar{\mu}_t) - H_t(x_t - \bar{\mu}_t)\big)\Big)$$

★ Extended Kalman Filter Algorithm

1. Extended_Kalman_Filter$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:
2. $\bar{\mu}_t = g(u_t, \mu_{t-1})$
3. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
4. $K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$
5. $\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t))$
6. $\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$
7. return μ_t, Σ_t