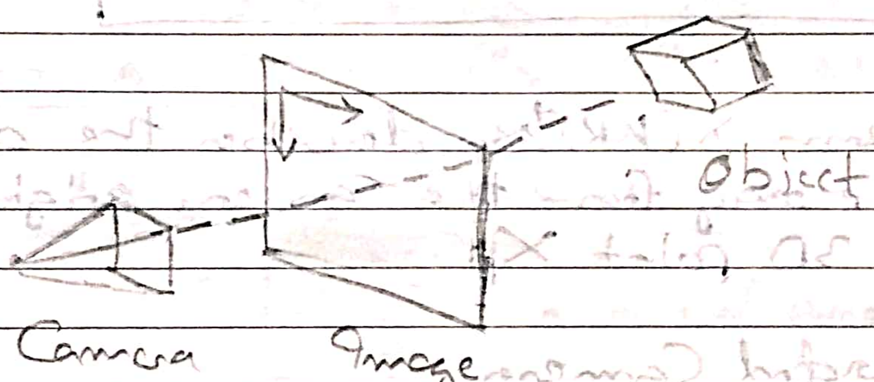
Camera Calibration : Direct Linear Transform

★ **Estimate Extrinsics and Intrinsics**

Wanted ⇒ Extrinsic and intrinsic parameters of a camera.

Given ⇒ Coordinates of object point (control point)

Observation ⇒ Coordinates (x, y) of those known 3D object points in the image.



$$x = KR[I_3 | -X_0] X$$

$$\underset{3\times1}{x} = \underset{3\times4}{P} \underset{4\times1}{X}$$

★ **How Many points are needed?**

⇒ Each point gives two observations equations, one for each image coordinate.

⇒ We want to estimate 11 parameters.

⇒ So we need atleast 6 points to estimate the 11 parameters.

★ Spatial Resection vs DLT

⇒ Calibrated camera
- → 6 unknowns
- → We need at least 3 points.
- → Problem solved by spatial resection.

⇒ Uncalibrated camera
- → 11 unknowns
- → We need at least 6 points
- → Problem solved by DLT.

★ DLT: Problem Specification

- Task: Estimate the 11 elements of P.
- Given:
  - → 3D coordinates X_i of $I \geq 6$ object points.
  - → Observed image coordinate x_i of an uncalibrated camera with the mapping

$$x_i = PX_i \qquad i = 1, \cdots I$$

  - → Data association.

Page No.
Date: / /

* Rearranging the DLT Equation

$$\Rightarrow \quad x_i = PX_i = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} X_i$$

$\Rightarrow$ Define three vectors A, B, C as follows:

$$A = \begin{bmatrix} P_{11} \\ P_{12} \\ P_{13} \\ P_{14} \end{bmatrix} \quad B = \begin{bmatrix} P_{21} \\ P_{22} \\ P_{23} \\ P_{24} \end{bmatrix} \quad C = \begin{bmatrix} P_{31} \\ P_{32} \\ P_{33} \\ P_{34} \end{bmatrix}$$

$$x_i = PX_i = \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} X_i$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} A^T X_i \\ B^T X_i \\ C^T X_i \end{bmatrix} \quad x_i = \frac{u_i}{w_i} = \frac{A^T X_i}{C^T X_i} \quad y_i = \frac{v_i}{w_i} = \frac{B^T X_i}{C^T X_i}$$

$$x_i C^T X_i - A^T X_i = 0$$

$$y_i C^T X_i - B^T X_i = 0$$

Page No.
Date: / /

⇒ Collect the elements of P within a parameter vector p

$$p = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

⇒ $\boxed{\begin{aligned} a_{x_i}^T p = 0 \\ a_{y_i}^T p = 0 \end{aligned}}$ $\left\{ \begin{aligned} a_{x_i}^T &= (-X_i^T, 0^T, x_i X_i^T) \\ a_{y_i}^T &= (0^T, -X_i^T, y_i X_i^T) \end{aligned} \right\}$

⇒ Stacking everything together:

$$\begin{bmatrix} a_{x_1}^T \\ a_{y_1}^T \\ \vdots \\ a_{x_I}^T \\ a_{y_I}^T \end{bmatrix} p = \underset{2I \times 12}{M} \underset{12 \times 1}{p} = 0$$

* Solving the Linear System

⇒ Solving a System of linear equations of the form $Ax = 0$ is equivalent to finding the null space of A.

⇒ We can apply the SVD to Solve $Mp = 0$

⇒ In case of redundant observations we will have contradiction $Mp \neq 0$

$$Mp = \omega$$

$\Rightarrow$ Find P such that it minimizes

$$\Omega = \omega^T \omega$$

$$\Rightarrow \hat{P} = \underset{P}{\operatorname{argmin}}\ \omega^T \omega$$

$$= \underset{P}{\operatorname{argmin}}\ P^T M^T M P$$

with $\|P\|_2 = \sum_{ij} P_{ij}^2 = \|P\| = 1$.

$$\Rightarrow \underset{(2I \times 12)}{M} = \underset{(12\times12)}{U}\ \underset{(12\times12)}{S}\ V^T = \sum_{i=1}^{12} s_i u_i v_i^T$$

$\Rightarrow$ Choosing $P = v_{12}$ (the singular vector belonging to the smallest singular value, s_{12}) minimizes Ω.

$\Rightarrow$ No solution if

↳ All points X_i are located on a plane.

★ From P to K, R, X_0

$$P = [KR \mid -KRX_0] = [H \mid h]$$

$$H = KR \qquad h = -KRX_0$$

$\Rightarrow$ We get the projection center through

$$\boxed{X_0 = -H^{-1}h}$$

$\Rightarrow$ Let's look to the structure $H = KR$

- $\rightarrow$ K is a triangular matrix
- $\rightarrow$ R is a rotation matrix

$\Rightarrow$ We perform QR decomposition of H^{-1} yields rotation and calibration matrix

$$H^{-1} = (KR)^{-1} = R^{-1}K^{-1} = \underbrace{R^T}_{Q}\underbrace{K^{-1}}_{R}$$

$$\boxed{K \leftarrow \frac{1}{K_{33}} K}$$

$\Rightarrow$ Decomposition $H^{-1} = R^T K^{-1}$ results in K with positive diagonal elements

$\Rightarrow$ To get negative camera constant, choose:

$$K \leftarrow K R(z,\pi) \qquad R \leftarrow R(z,\pi) R.$$

using $R(z,\pi) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ decomposition still holds: $H = KR(z,\pi)R(z,\pi)R = KR$

—— X —— X ——