Linear Quadratic (LQ) Optimal Control problem

⇒ The Controlled system is described by

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \quad x(t_0) = x_0$$

⇒ The optimal control is sought to minimize the quadratic performance index:

$$J = \frac{1}{2}x^T(t_f)\,S\,x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\,dt$$

where S and $Q(t) = C^T(t)C(t)$ are positive semidefinite matrix and $R(t)$ is positive definite.

★ **Deriving of Solution**

The Solution of the continuous time LQ problem is given by the state feedback control law,

$$u(t) = -R^{-1}(t)B^T(t)P(t)x(t)$$

where $P(t)$ is the positive semidefinite solution of the Riccati differential equation

$$-\frac{dP(t)}{dt} = A^T(t)P(t) + P(t)A(t) - P(t)B(t)R^{-1}(t)B^T(t)P(t) + C^T(t)C(t)$$

$$P(t_f) = S$$

and the minimum value of the quadratic performance index is given by

$$J^0(x(t_0)) = \frac{1}{2}x^T(t_0)P(t_0)x(t_0)$$

④ Stationary Case

⇒ We will let $t_f \to \infty$. In this case, we introduce the following two key assumptions.

Assumption 1: The System is controllable or Stabilizable

Assumption 2: (A, C) is observable or detectable.

⇒ The stationary solution is obtained by solving the algebraic Riccati equation (ARE)

$$\boxed{A^T P + PA - PBR^{-1}B^T P + C^T C = 0}$$