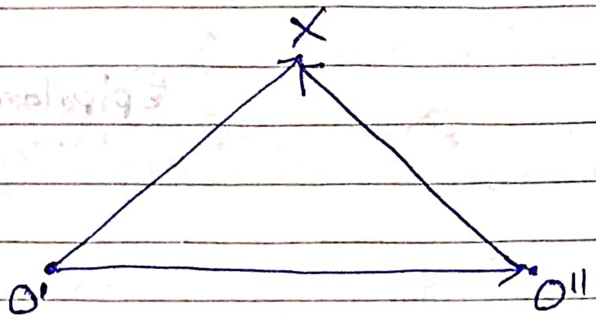
(16)

Epipolar Geometry



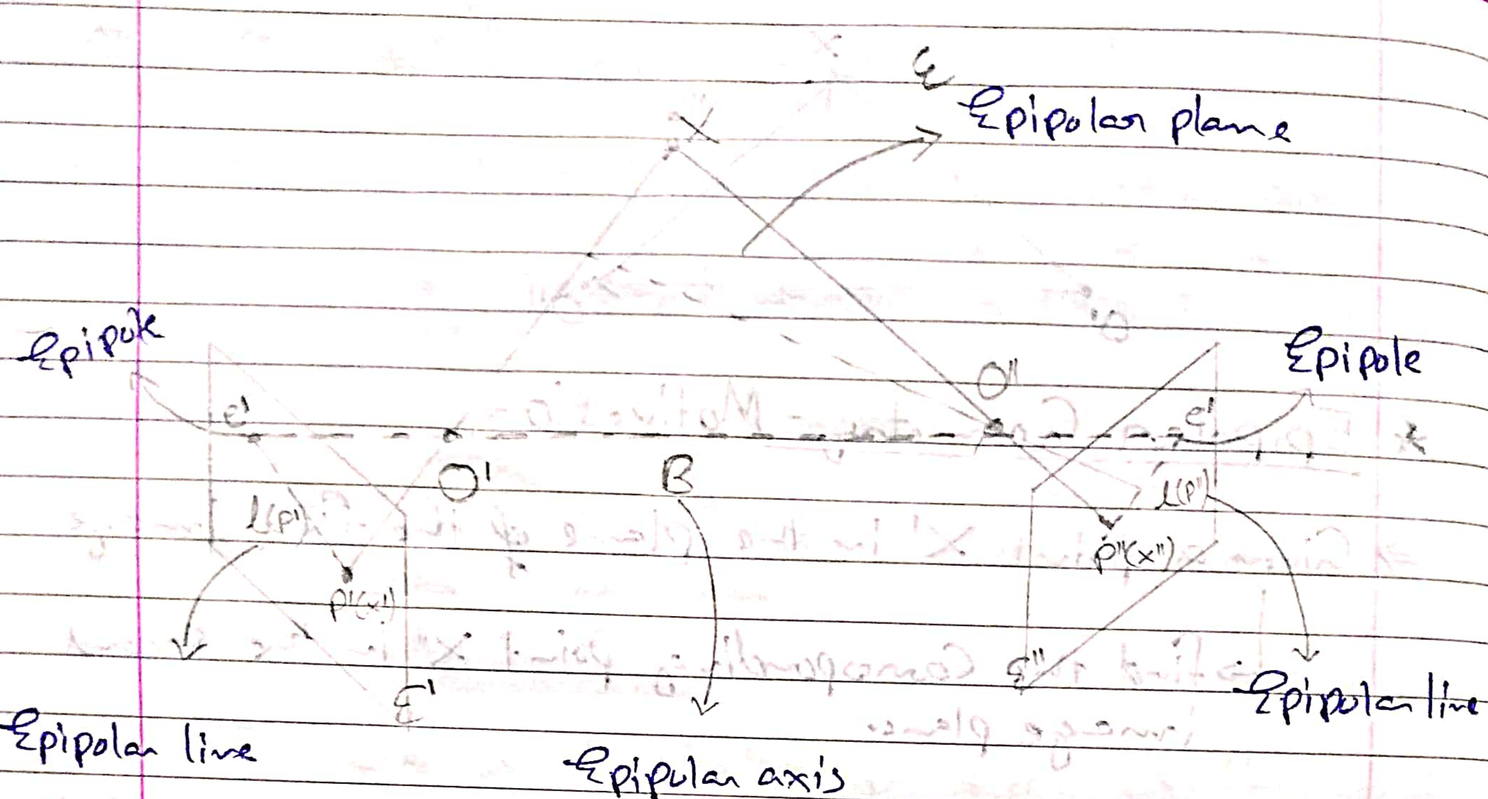
★ Epipolar Geometry - Motivation

⇒ Given a point X' in the plane of the first image
↳ find the corresponding point X'' in the second image plane.

⇒ Epipolar geometry is used to describe geometric relations in image pairs.

⇒ Enables efficient search for and prediction of corresponding points.

⇒ Given a straight-line preserving mapping, the search space reduces from 2D to 1D.



★ Important Elements

- Epipolar axis ⇒ $B = (O'O'')$ is the line through the two projection centers.
- Epipolar plane ⇒ $\varepsilon = (O'O''X)$ depends on the projection centers & the point.
- Epipoles ⇒ $e' = (O'')'$, $e'' = (O')''$ are the images of the projection centers.
- Epipolar line ⇒ $l'(X) = (O''X)'$ $l''(X) = (O'X)''$ are the image of the rays $O''X$ and $O'X$ in the other image respectively.

⇒ Epipoles can also be written as:

$$e' = (O'O'') \wedge \varepsilon' \qquad e'' = (O'O'') \wedge \varepsilon''$$

⇒ Epipolar lines can be written as:

$$l'(x) = \varepsilon \wedge \varepsilon' \qquad l''(x) = \varepsilon \wedge \varepsilon''$$

★ **Predicting the Location of Corresponding points**

⇒ Predict the location of x'' given x'.

⇒ The corresponding point x'' lies on the epipolar line $l''(x)$.

★ **Computing the Key Elements of the Epipolar Geometry**

① **Epipolar Axis**

$$\boxed{b = X_{O''} - X_{O'}}$$

② **Epipolar line**

⇒ The image points lie on the epipolar lines.

$$x' \in l' \ \& \ x' \in l''.$$

$$x'^T l' = 0 \quad \{\text{Condition if } x' \text{ lies of } l'\}$$

⇒ We can exploite the Coplanarity Constraint for both points x', x''.

$$x'^T \underbrace{F x'}_{l'} = 0 \rightarrow \boxed{l' = F x''}$$

⇒ The same for the point x''

$$l''^T x'' = 0$$

$$\underbrace{x'^T F}_{l''^T} x'' = 0$$

$$\Rightarrow l''^T = x'^T F$$

$$\Rightarrow \boxed{l'' = F^T x'}$$

★ Epipoles

$$e' = P' X_{0''}$$

$$e'' = P'' X_{01}$$

⇒ Epipole is the intersection of all epipolar lines in an image.

$$e'^T l' = 0 \quad \forall l'$$

$$e''^T l'' = 0 \quad \forall l''$$

⇒ $e'^T F x'' = 0$

↳ The epipole is the null space of F^T

$null(F^T) = e'$, $null(F) = e''$

⇒ The epipoles are the left & right eigenvectors of the fundamental matrix corresponds to an eigenvalue of zero.