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## Visual Features: Keypoints (Harris, Shi-Tomasi, Förstner, DoG)

### Visual features

#### Keypoint

{ Locally distinct location in an image }

- Harris corners
- Shi-Tomasi corner detector
- Förstner operator
- Difference of Gaussians (DOG)

#### Descriptor

{ Summary of the local structure around the Keypoint }

- SIFT (Scale Invariant Feature Transform)
- BRIEF (Binary Robust Independent Elementary Features)
- ORB (Oriented FAST method BRIEF)

### Corners

⇒ Corners are often highly distinct points.

⇒ Corners are invariant to translation, rotation and illumination.

⇒ Corner ⇒ Two edges in roughly orthogonal directions.

⇒ Edge ⇒ A sudden brightness change.



## \* Finding Corners

⇒ To find corners we need to search for intensity changes in two directions.

⇒ Compute the sum of squared difference of neighbor pixels around  $(x, y)$ .

$$f(x, y) = \sum_{(u, v) \in W_{xy}} (I(u, v) - I(u + \delta u, v + \delta v))^2$$

Local patch  
around  $(x, y)$

⇒ Using Taylor expansion, we obtain:

$$I(u + \delta u, v + \delta v) \approx I(u, v) + [J_x \ J_y] \begin{bmatrix} \delta u \\ \delta v \end{bmatrix}$$

$$\Rightarrow f(x, y) \approx \sum_{(u, v) \in W_{xy}} \left( [J_x \ J_y] \begin{bmatrix} \delta u \\ \delta v \end{bmatrix} \right)^2$$

$$f(x, y) \approx \sum_{(u, v) \in W_{xy}} \begin{bmatrix} \delta u \\ \delta v \end{bmatrix}^T \begin{bmatrix} J_x^2 & J_x J_y \\ J_x J_y & J_y^2 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta v \end{bmatrix}$$



$$f_{\text{mag}}(x,y) = \begin{bmatrix} \delta u \\ \delta v \end{bmatrix}^T \begin{bmatrix} \sum_w J_x^2 & \sum_w J_x J_y \\ \sum_w J_y J_x & \sum_w J_y^2 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta v \end{bmatrix}$$

Structure matrix

### \* Structure Matrix

⇒ Key to finding edges and Corners

⇒ It encodes the changes in image intensity in a local area.

$$M = \begin{bmatrix} \sum_w J_x^2 & \sum_w J_x J_y \\ \sum_w J_y J_x & \sum_w J_y^2 \end{bmatrix}$$

⇒ Jacobians Computed via a Convolution with a gradient Kernel such as Scharr or Sobel:

$$J_x^2 = (D_x * I)^2$$

$$J_x J_y = (D_x * I) (D_y * I)$$

$$J_y^2 = (D_y * I)^2$$

$$D_x^{\text{Scharr}} = \frac{1}{32} \begin{bmatrix} 3 & 10 & 3 \\ 0 & 0 & 0 \\ -3 & -10 & -3 \end{bmatrix}$$

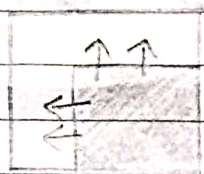
$$D_x^{\text{Sobel}} = \frac{1}{8} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$D_y^{\text{Scharr}} = \frac{1}{32} \begin{bmatrix} 3 & 0 & -3 \\ 10 & 0 & -10 \\ 3 & 0 & -3 \end{bmatrix}$$

$$D_y^{\text{Sobel}} = \frac{1}{8} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

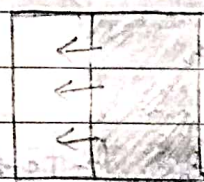


⇒ Structure matrix summarizes the dominant directions of the gradient around a point



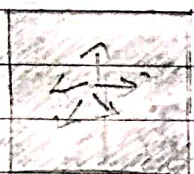
$$\Rightarrow M = \begin{bmatrix} \gg 1 & \sim 0 \\ \sim 0 & \gg 1 \end{bmatrix}$$

✓



$$\Rightarrow M = \begin{bmatrix} \sim 0 & \sim 0 \\ \sim 0 & \gg 1 \end{bmatrix}$$

✗



$$\Rightarrow M = \begin{bmatrix} \sim 0 & \sim 0 \\ \sim 0 & \sim 0 \end{bmatrix}$$

✗

⇒ Key Idea: Considers points as corners if their structure matrix has two large Eigenvalues.

⇒ Three similar approaches proposed in:

→ Förstner (1987)

→ Harris (1988)

→ Shi-Tomasi (1994)

⇒ All rely on the structure matrix.

⇒ Use different Criterion for deciding if a point is a corner or not.

⇒ Förstner offers subpixel estimation



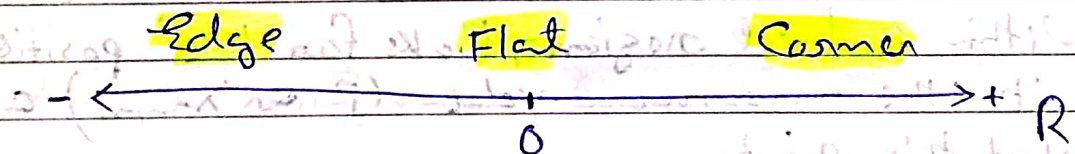
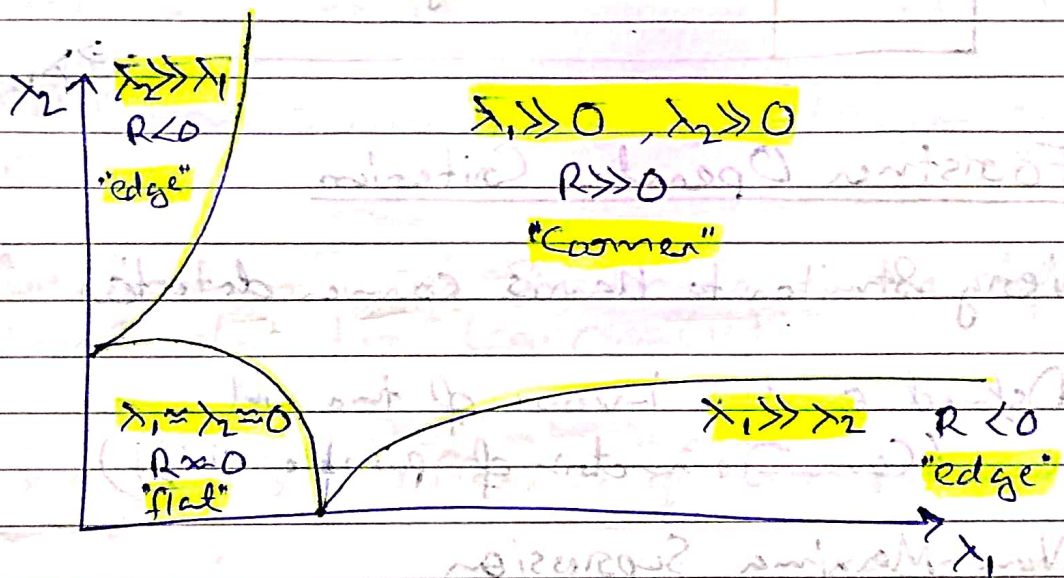
## \* Harris Corner Criterion

⇒ Criterion:

$$R = \det(M) - k(\text{trace}(M))^2$$

$$= \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$$

$$k \in [0.04, 0.06]$$



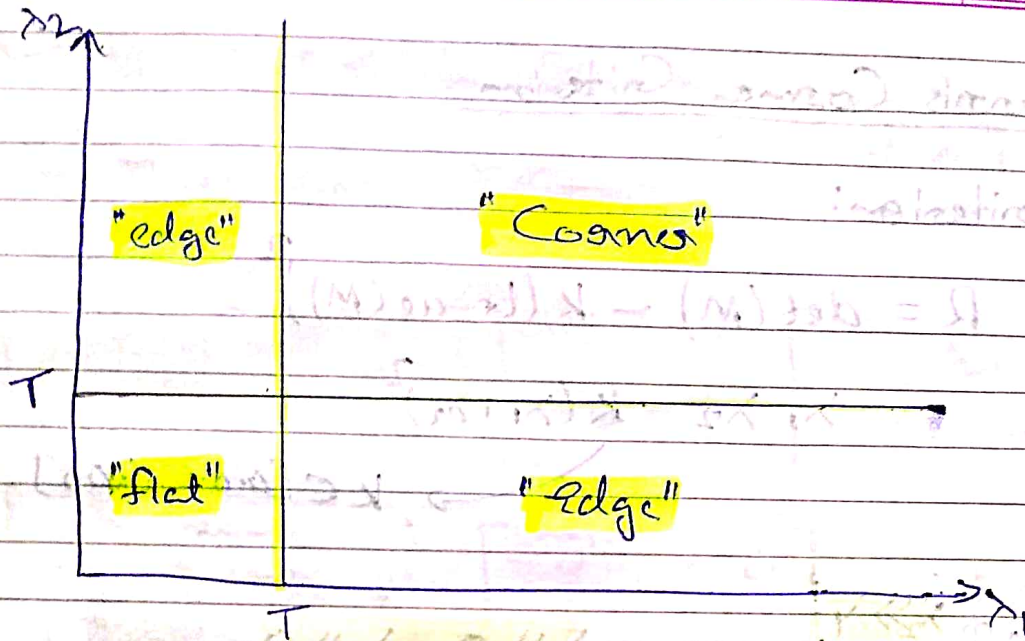
## \* Shi-Tomasi Corner Detector

⇒ Criterion: Threshold smallest Eigenvalue

$$\lambda_{\min}(M) = \frac{\text{trace}(M)}{2} - \frac{1}{2} \sqrt{\text{trace}(M)^2 - 4 \det(M)}$$

$$\lambda_{\min}(M) \geq T \Rightarrow \text{Corner.}$$





### \* Förstner Operator Criterion

- ⇒ Very similar to Harris corner detection
- ⇒ Defined on the inverse of the  $M$   
(Covariance matrix of possible shift)

### \* Non-Maxima Suppression

- ⇒ Within a local region, looks for the position with the maximum value (R on  $\lambda_{min}$ ) and select this point.

### \* Implementation Remarks

- ⇒ RGB to gray-scale conversion first.
- ⇒ Real images are affected by noise, smoothing of the input is suggested.



## \* Corner Detectors Comparison

- ⇒ All three detectors perform similarly.
- ⇒ Forstner was the first one and additionally described subpixel estim.
- ⇒ Harris became the most famous corner detector in the past.
- ⇒ Shi-Tomasi seems to slightly outperform Harris corner.
- ⇒ Most libraries use Shi-Tomasi as the default corner detector. (eg. OpenCV)

## Keypoints: Difference of Gaussians

⇒ A variant of corner detection

⇒ Provides responses at:

Corners, edges & blobs

Mainly constant regions  
but different to its  
surrounding



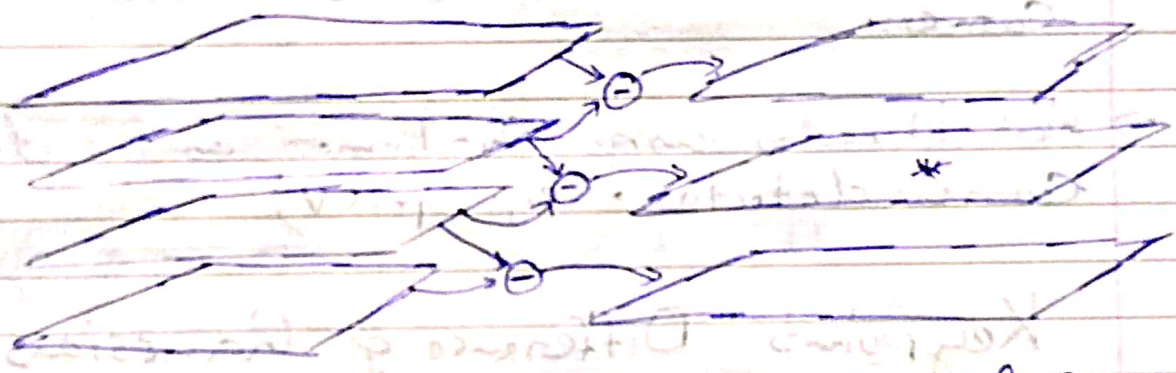
⇒ Difference of Gaussians over scale-space pyramid

⇒ Procedure: (over different image pyramid levels)

① Gaussian Smoothing (differently blurred image)

② Difference of Gaussians: find extrema

③ Maxima suppression of edges.



Gaussians

Difference of Gaussians  
(DOG)

### \* Extrema Suppression

⇒ The DOG finds blob-like and corner like image structures but also leads to strong responses along edges.

⇒ Edges are bad for matching.

⇒ Eliminate edges via Eigenvalue test.

