

## 2

# Bug Algorithms

⇒ Bug1 and Bug2 algorithms are among the earliest and simplest sensor-based planners.

⇒ These algorithms assume:

- Robot is a point ~~entity~~
- Operating in a plane
- With a contact sensor to detect obstacles.

⇒ When the robot has a finite range sensor, then the Tangent Bug Algorithm is a Bug derivative that can use the sensor information to find shorter path to the goal.

⇒ These algorithms require two behaviours:

- Move on a straight line
- Follow a boundary

{ Curve-tracing technique based  
on the implicit function  
theorem }

⇒ There success is guaranteed, when possible.

### 1) Bug1 and Bug2

⇒ Robot is assumed to have perfect positioning (no positioning error).

⇒ Workspace is assumed to be bounded.

⇒ The robot can measure distance  $d(x, y)$  between any two points  $x$  and  $y$ .

⇒ Let  $B_\sigma(x)$  denote a ball of radius  $\sigma$  centered on  $x$ .

$$B_\sigma(x) = \{y \in \mathbb{R}^2 \mid d(x, y) < \sigma\}$$

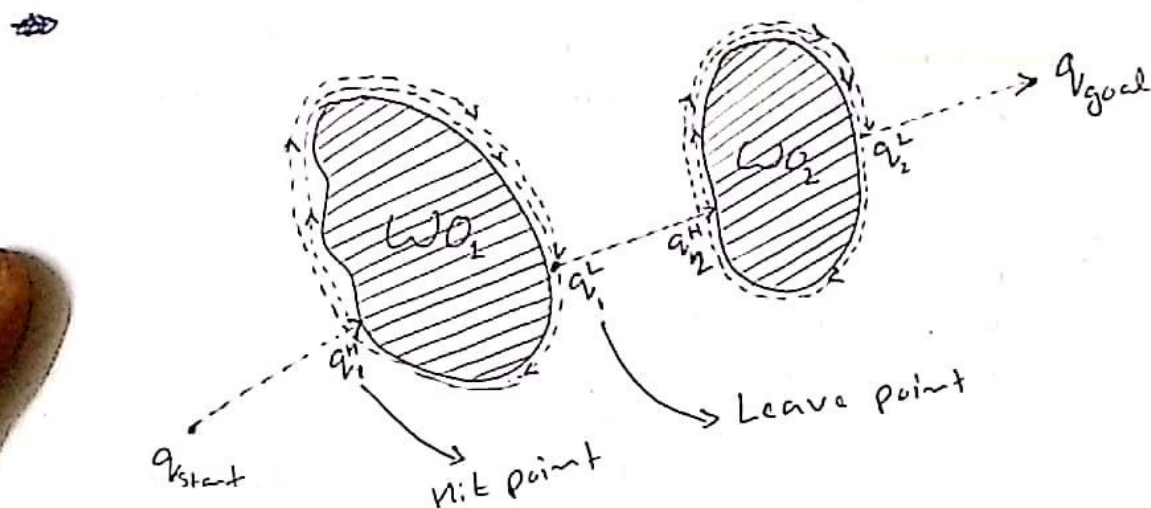
$\Rightarrow$  Workspace is bounded implies that  $\forall x \in W$ , there exists an  $n < \infty$  such that  $W \subset B_n(x)$ .

$q_{start} \Rightarrow$  Starting point position

$q_{goal} \Rightarrow$  Goal point position

$\Rightarrow$  Let  $q_0^L = q_{start}$

$\Rightarrow$  m-line be the line segment that connects  $q_0^L$  to  $q_{goal}$ .



$\Rightarrow$  During motion to goal, the robot moves along the m-line toward  $q_{goal}$  until it either encounters the goal or an obstacle.

$\Rightarrow$  If robot encounters an obstacle, the  $q_i^H$  be the point where the robot first encounters an obstacle and call this point a hit point.

$\Rightarrow$  The robot then circumnavigate the obstacle until it returns to  $q_i^H$ .

$\Rightarrow$  Then, the robot determines the closest point to the goal on the perimeter of the obstacle and traverses to this point.

Algorithm

Input:

Output:

```

1 While
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5   en
6   if
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9
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11  De
12  ha
13  Gr
14  if
15  en
16 end

```

$\Rightarrow$  Bug2  
exc  
, and

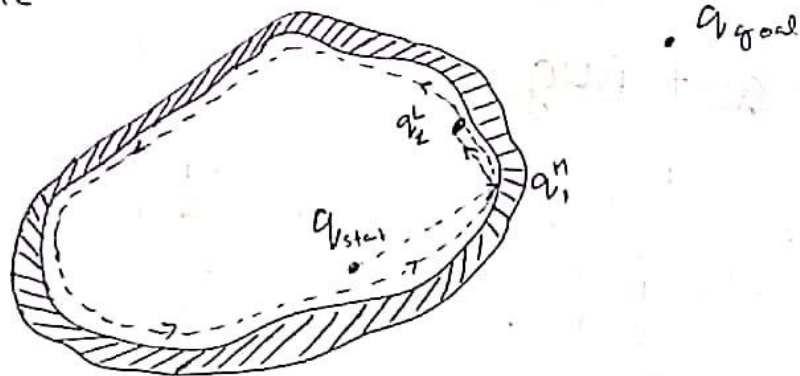


### Algorithm 1: Bug1 Algorithm

Input: A point robot with a tactile sensor

Output: A path to the goal or a conclusion no such path exists

- 1 While Forever do
- 2   repeat
- 3     From  $q_{i-1}^L$ , move toward  $q_{goal}$ .
- 4     Until  $q_{goal}$  is reached or an obstacle is encountered at  $q_i^H$
- 5     if Goal is reached then
- 6       Exit
- 7     end if
- 8   repeat
- 9     Follow the obstacle boundary
- 10    Until  $q_{goal}$  is reached or  $q_i^H$  is re-encountered.
- 11    Determine the point  $q_i^L$  on the perimeter that has the shortest distance to the goal.
- 12    Go to  $q_i^L$
- 13    if the robot were to move toward the goal then
- 14      Conclude  $q_{goal}$  is not reachable and exit
- 15    end if
- 16 end while



{ Bug1 algorithm will report  
goal is unreachable }

$\Rightarrow$  Bug2 algorithm is similar to Bug1 algorithm except on-line connect  $q_{start}$  and  $q_{goal}$ , and thus remain fixed.

⇒ Bug2 algorithm is also called greedy, since it opts for the first promising option that is found.

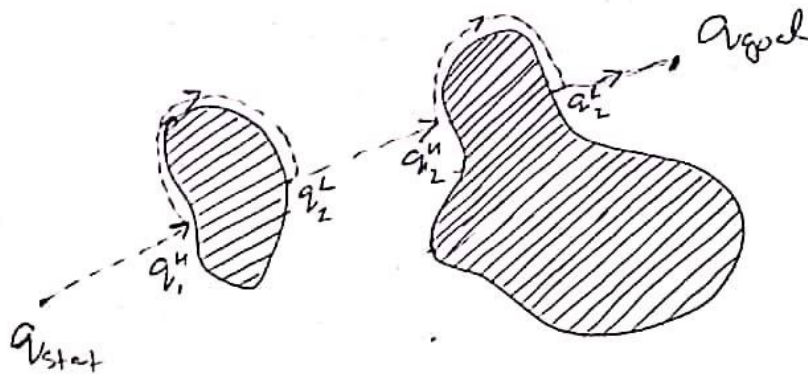
⇒ Bug1 algorithm performs an exhaustive search to find the optimal leave point.

Bug1

{ Performance is good }  
when obstacle  
is complex

Bug2

{ Performance is good }  
when obstacle  
is simple



## Bug2 Algorithm

### 2) Tangent Bug

⇒ Tangent Bug serves as an improvement to the Bug2 algorithm in that it determines a shorter path to the goal using a range sensor with 360 degree infinite orientation resolution.

⇒ We model this range sensor with the ray distance function  $f: \mathbb{R}^2 \times S^1 \rightarrow \mathbb{R}$

⇒ The value  $f(x, \theta)$  is the distance to the closest obstacle along the ray from  $x$  at an angle  $\theta$ .



Since found. Search

$$f(x, \theta) = \min_{\lambda \in [0, \infty]} d(x, x + \lambda [\cos \theta, \sin \theta]^T),$$

such that  $x + \lambda [\cos \theta, \sin \theta] \in \bigcup O_i$

good? le

$\Rightarrow$  Since real sensors have limited range, we define the saturated saw distance function, denoted  $f_R: \mathbb{R}^2 \times S^1 \rightarrow \mathbb{R}$ , which takes on the same values as  $f$  when the obstacle is within sensing range, and has a value of infinity when the ray lengths are greater than the sensing range,  $R$ .

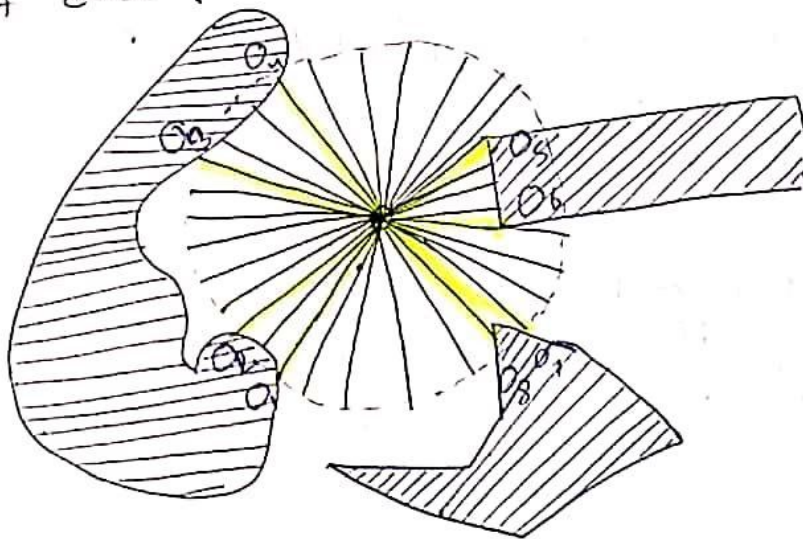
$$f_R(x, \theta) = \begin{cases} f(x, \theta), & \text{if } f(x, \theta) < R \\ \infty, & \text{otherwise} \end{cases}$$

$\Rightarrow$  The Tangent Bug planner assumes that the robot can detect discontinuities in  $f_R$ .

$\Rightarrow$  For a fixed  $x \in \mathbb{R}^2$ , an interval of continuity is defined to be a connected set of points  $x + f(x, \theta) [\cos \theta, \sin \theta]^T$

finite

$\Rightarrow$  Let end point be denoted by  $O_i$ .





⇒ Just like the other Bugs, Tangent Bug iterates between two behaviours:

→ motion-to-goal  
→ Boundary following.

⇒ First, the robot moves in a straight line toward the goal until it senses an obstacle  $R$  units away and directly between it and the goal.

⇒ When the robot initially senses an obstacle, the circle of radius  $R$  becomes tangent to the obstacle.

⇒ Immediately after this, tangent point splits into two  $O_i$ 's which are the endpoints of the interval.

⇒ The robot then moves toward the  $O_i$  that ~~maximizes~~ minimizes a heuristic distance to the goal.

$$(d(x, O_i) + d(O_i, q_{goal}))$$

Can be more complicated when factoring in available info with regard to the obstacles

⇒ When the robot switches to Boundary-following, it finds a point  $M$  on the sensed portion of the obstacle which has the shortest distance on the obstacle to the goal.

⇒ If as

Sen  
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⇒ Quit  
Obs.

⇒ Now  
as

⇒ While  
also

Distance between  
goal and me  
on the follow  
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sight of the

⇒ Let  
 $d$   
Obst

$\Lambda =$

done

⇒ When  
term  
behav

$\Rightarrow$  If sensor range is 0, then  $M$  is the same as the hit point from the Bug1 and Bug2 Algorithm.

Followed obstacle

blocking Obstacle

Sensed Obstacle is called followed obstacle  $\left\{ \begin{array}{l} \text{Closest obstacle within} \\ \text{Sensor range that intersects} \\ \text{the Segment.} \\ (1-\lambda)x + \lambda q_{goal} \quad \forall \lambda \in [0,1] \end{array} \right\}$

$\Rightarrow$  Initially, the blocking Obstacle and the followed Obstacle are the same.

$\Rightarrow$  Now the robot moves in the same direction as if it were in the motion-to-goal behavior.

$\Rightarrow$  While undergoing this motion, the planner also updates two values:  $d_{followed}$  and  $d_{reach}$

$\swarrow \searrow$   
 $\left\{ \begin{array}{l} \text{Distance between the} \\ \text{goal and the closest point} \\ \text{on the followed obstacle} \\ \text{that is within line of} \\ \text{sight of the robot} \end{array} \right\} \left\{ \begin{array}{l} \text{Shortest distance between the} \\ \text{boundary which had been} \\ \text{sensed and the goal.} \end{array} \right\}$

$\Rightarrow$  Let  $\Lambda$  be all the points within line of sight of  $d$  with sense  $R$  that are on the followed obstacle  $\cup Q_f$ .

$$\Lambda = \{y \in \partial \cup Q_f : \lambda x + (1-\lambda)y \in Q_{free} \quad \forall \lambda \in [0,1]\}$$

$$d_{reach} = \min_{C \in \Lambda} d(q_{goal}, C)$$

$\Rightarrow$  When  $d_{reach} < d_{followed}$ , the robot terminates the boundary-following behavior.



⇒ Let  $T$  be the point where a circle centered at  $x$  of radius  $R$  intersects the segment that connects  $x$  and  $g_{\text{goal}}$ .

### Tangent Bug Algorithm

Input: A point robot with range sensor.

Output: A path to the goal or a conclusion no such path exists.

- 1 While True do
- 2   repeat
- 3     Continuously move toward the point  $n \in \{T, O_i\}$  which minimizes  $d(x, n) + d(n, g_{\text{goal}})$
- 4   Until
- 5     the goal is encountered or
- 6     the direction that minimizes  $d(x, n)$  begins to increase  $d(x, g_{\text{goal}})$  i.e. the robot "detects a local minimum" of  $d(\cdot, g_{\text{goal}})$
- 7   Choose a boundary following direction which continues in the same direction as the most recent motion-to-goal direction.
- 8   repeat
- 9     Continuously update  $d_{\text{obst}}$ ,  $d_{\text{followed}}$  &  $\{O_i\}$
- 10    Continuously move toward  $n \in \{O_i\}$  that is in the chosen boundary direction.
- 11   Until
- 12     the goal is reached
- 13     the robot completes a cycle around the obstacle in which case the goal cannot be achieved.
- 14      $d_{\text{obst}} < d_{\text{followed}}$
- 15 end while



### 3) Implementation

⇒ If the obstacle were flat, such as a long wall in a corridor, then following the obstacle is trivial.

↳ Simply move parallel to the obstacle

⇒ Driving toward a point is simple gradient descent.

