# EKF SLAM

★ **Given**

① The robot's Controls

$$u_{1:T} = \{u_1, u_2, \dots, u_T\}$$

② Observations

$$z_{1:T} = \{z_1, z_2, \dots, z_T\}$$

★ **Wanted**

① Map of the environment

$$m$$

② Path of the robot

$$x_{0:T} = \{x_0, x_1, \dots, x_T\}$$

★ **State Space**

$$x_t = (\underbrace{x, y, \theta}_{\text{robot's pose}}, \underbrace{m_{1x}, m_{1y}}_{\text{landmark1}}, \dots, \underbrace{m_{nx}, m_{ny}}_{\text{landmark n}})^T$$

⇒ Map with n landmarks: (3+2n)-dimensional Gaussian

⇒ Belief is represented by:-

$$\begin{array}{r} 3\times1 \\ 2n\times1 \end{array} \underbrace{\begin{pmatrix} x \\ m \end{pmatrix}}_{\mu} \qquad \underbrace{\begin{pmatrix} \underset{3\times3}{\Sigma_{xx}} & \underset{3\times2n}{\Sigma_{xm}} \\ \underset{2n\times3}{\Sigma_{mx}} & \underset{2n\times2n}{\Sigma_{mm}} \end{pmatrix}}_{\Sigma}$$

$$\begin{pmatrix} x \\ y \\ \theta \\ m_{1,x} \\ m_{1,y} \\ \vdots \\ m_{n,x} \\ m_{n,y} \end{pmatrix} \qquad \left(\begin{array}{ccc|cc|c|cc} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1,x}} & \sigma_{xm_{1,y}} & \cdots & \sigma_{xm_{n,x}} & \sigma_{xm_{n,y}} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1,x}} & \sigma_{ym_{1,y}} & \cdots & \sigma_{ym_{n,x}} & \sigma_{ym_{n,y}} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1,x}} & \sigma_{\theta m_{1,y}} & \cdots & \sigma_{\theta m_{n,x}} & \sigma_{\theta m_{n,y}} \\ \hline & & & \sigma_{m_{1,x} m_{1,x}} & \sigma_{m_{1,x} m_{1,y}} & \cdots & & \\ & & & \sigma_{m_{1,y} m_{1,x}} & \sigma_{m_{1,y} m_{1,y}} & \cdots & & \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline & & & & & \cdots & & \\ & & & & & \cdots & & \end{array}\right)$$

★ <u>Initialization</u>

⇒ Robot starts in its own reference frame (all landmarks unknown)

$$\mu_0 = (0 \; 0 \; 0 \; \cdots \; 0)^T$$

$$\Sigma_0 = \left(\begin{array}{ccc|cccc} 0 & 0 & 0 & & & & \\ 0 & 0 & 0 & & 0 & & \\ 0 & 0 & 0 & & & & \\ \hline & & & \infty & & & 0 \\ & 0 & & & \infty & & \\ & & & & & \ddots & \\ & & & 0 & & & \infty \end{array}\right)$$

## * Prediction Step

$$\cancel{x_{t,x,y,\theta} = x_{t-1,x,y,\theta} + g}$$

$$\cancel{x_{t,x,y,\theta} = g_{x,y,\theta}}$$

$$\boxed{x_{t,x,y,\theta} = g_{x,y,\theta}(u_t, x_{t-1,x,y,\theta})}$$

{Non linear motion model}

$$\boxed{x_t = g(u_t, x_{t-1})} \rightarrow G_t = \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix}$$

$3 \times 2n \times$

$\cancel{(2n+3) \times 3 \times 3 \times 3 \times 3 \times (2n+3)}$

$$\begin{pmatrix} a & b & c & 0 & 0 & 0 & 0 \\ d & e & f & 0 & 0 & 0 & 0 \\ g & h & i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b & c & 0 & 0 & 0 & 0 \\ d & e & f & 0 & 0 & 0 & 0 \\ g & h & i & 0 & 0 & 0 & 0 \end{pmatrix}$$

## ★ Observation Step

⇒ Assumption: Known data association

$$\boxed{c_t^i = j}$$

↳ ith measurement at time t observe the landmark with index j

⇒ Range-Bearing observation

$$z_t^i = (r_t^i, \phi_t^i)^T$$

⇒ If landmark has not be observed

$$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \mu_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$$

⇒ Expected observation:

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

$$q = \delta^T \delta$$

$$z_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix}$$

$$= h(\bar{\mu}_t)$$

$\Rightarrow$ Compute the Jacobian

$$^{low}H_t^i = \frac{\delta h(\bar{\mu}_t)}{\delta \bar{\mu}_t}$$

low-dim space $(x, y, \theta, m_{j,x}, m_{j,y})$

$$H_t^i = {}^{low}H_t^i \, F_{x,j}$$