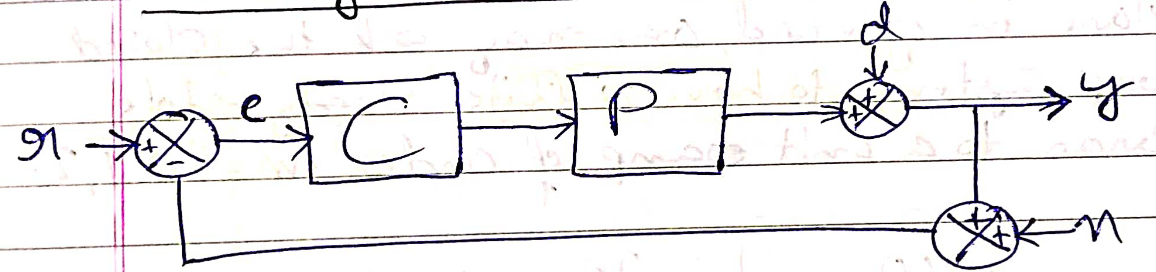
(10)

Specifications for feedback system

⇒ Specifications on the closed-loop behavior are typically given using two main paradigms, plus one that can be seen both ways:

- → Steady state error
- → Time-domain specification
- → Frequency-domain specification.

★ Steady-state error to step inputs



⇒ Let $L(s) = P(s)\,C(s)$

$$S(s) = \frac{1}{1+L(s)} \quad \left\{ \frac{E(s)}{R(s)} \right\}$$

⇒ If the input is a unit step $r(t) = 1 = e^{0t} \;\forall t \geq 0$

↳ The steady state output will be

$$e_{ss}(t) = S(0)\,e^{0t} = \frac{1}{1+L(0)} \quad \forall t \geq 0$$

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⇒ So if $\lim_{s \to 0} L(s) = K_{Bode} = K_{DC}$ is finite then the steady-state error to a unit step is $\frac{1}{1+K_{DC}}$.

⇒ If the limit is infinite (i.e. there is a pole at $s=0$, $L(s)$ contains an integrator), the steady-state error is zero.

★ Steady-state error to higher-order ramps

⇒ More in general, one may ask the closed loop system to have a finite steady-state error to a unit ramp of order $m = \{0, 1, 2 \ldots\}$

$$r(t) = \frac{1}{m!} t^m \quad t \geq 0$$

⇒ The steady-state error can be computed as:

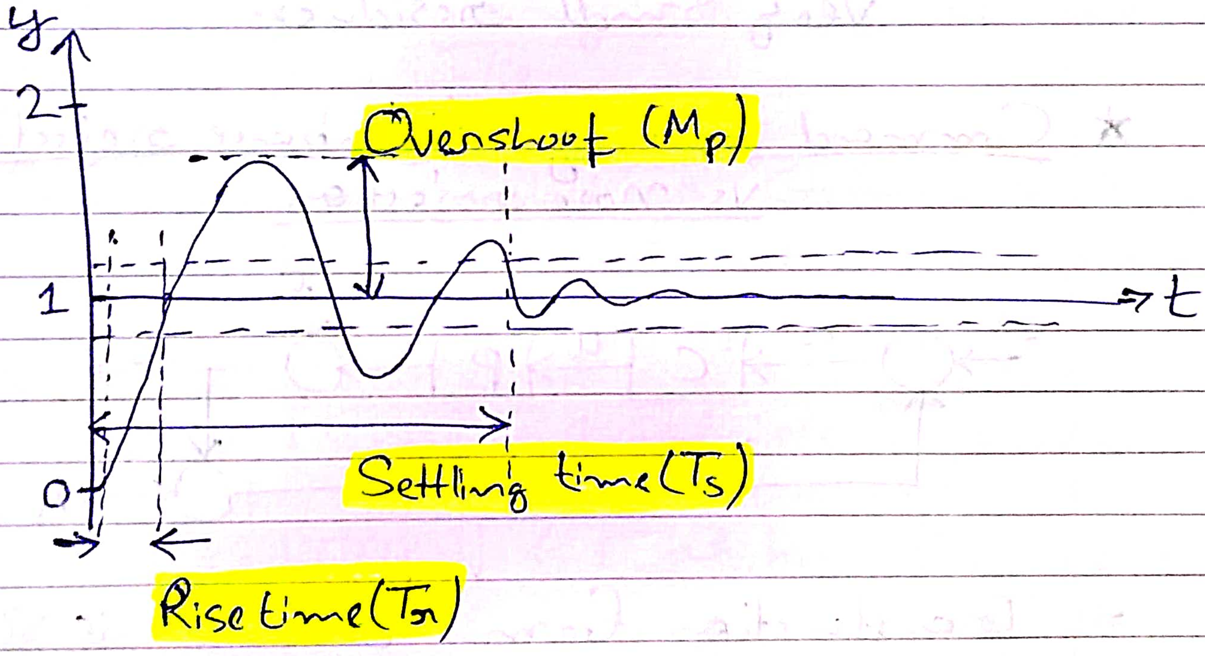
$$e_{ss}(t) = \lim_{s \to 0} \left(\frac{1}{1+L(s)} \frac{e^{st}}{s^m} \right)$$

★ System type

⇒ System type indicate number of integrator in $L(s)$. Type 0 ⇒ 0 integrator, Type 1 ⇒ 1 integrator etc...

e_{ss}	$m=0$	$m=1$	$m=2$
Type 0	$\frac{1}{1+K_{Bode}}$	∞	∞
Type 1	0	$\frac{1}{K_{Bode}}$	∞
Type 2	0	0	$\frac{1}{K_{Bode}}$

★ Time domain: Step response of a 2nd order system



⇒ Time domain specifications are usually given in terms of the step response of a 2nd order system.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\boxed{y(t) = (1 - e^{-\sigma t}\cos(\omega t))}$$

⇒ Poles are at $s = -\sigma \pm j\omega$

$\omega_n^2 = \sigma^2 + \omega^2$

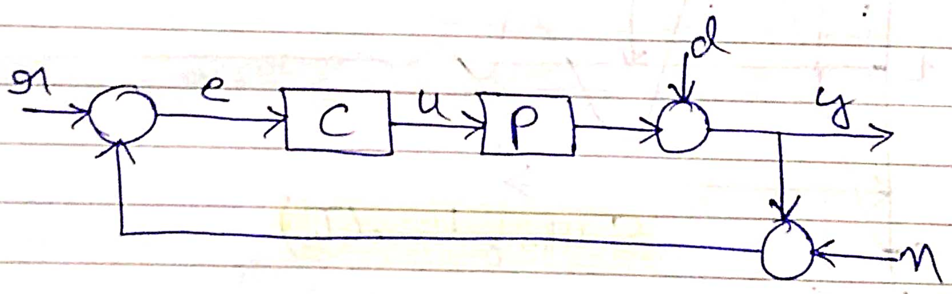
$\xi = \sigma/\omega$

★ Dominant pole approximation

⇒ The approximation is based on the concept of dominant poles.

- → Dominant poles are typically those with large real part
- → Exceptions are made when the poles with the largest real part also have very small residues.

★ Command tracking / disturbance rejection vs noise rejection



⇒ Transfunction from $d \to e$ is $S(s) = \frac{1}{1+L(s)}$

⇒ Transfer function from $n \to y$ is $T(s) = \frac{L(s)}{1+L(s)}$

⇒ To reject disturbance, we need $S(s)$ to be small.

⇒ If we do not want the effect of noise to be observed at the output, then we need $T(s)$ to be small.

⇒ But $\boxed{T(s) + S(s) = 1}$

★ Frequency-domain specification

⇒ Typically command and disturbance act at "low" frequency. eg. no more than 10 Hz.

⇒ Noise is typically a high frequency phenomenon e.g more than 100 Hz.

⇒ So we can reconcile both command tracking /disturbance rejection and noise rejection by separating them frequency-wise!

→ Make $|S(j\omega)| \ll 1$ at low frequencies

→ Make $|T(j\omega)| \ll 1$ at high frequencies.

★ Frequency-domain specifications on the Bode plot

⇒ For good command tracking / disturbance rejection, we want $|S(j\omega)| = |1 + L(j\omega)|^{-1}$ to be small at low frequencies.

→ i.e. we want $|L(j\omega)|$ to be large at low frequency.

→ This can be seen as Low frequency obstacle.

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⇒ For good noise rejection, we want $|T(j\omega)| = |L(j\omega)| / |1+L(j\omega)|$ to be small at high frequencies.

↳ If $|T(j\omega)|$ is small, then $|L(j\omega)|$ has to be small.

↳ This can be seen as high-frequency obstacle.

★ <u>Closed-loop bandwidth and (open loop) Crossover</u>

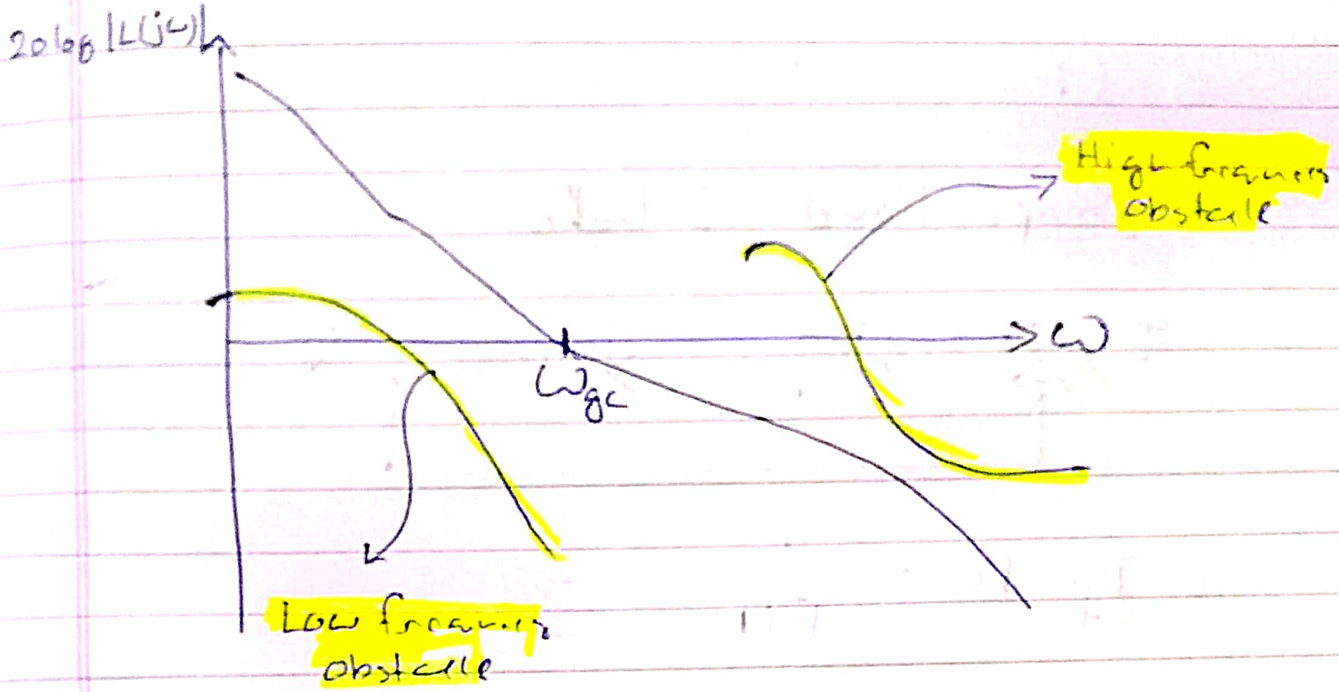
⇒ The bandwidth of the closed-loop system is defined as the maximum frequency ω for which $|T(j\omega)| > 1/\sqrt{2}$.

↳ The output can track the command to within a factor of $\simeq 0.7$.

⇒ Let ω_c be the crossover frequency, such that $|L(j\omega_c)| = 1$.

↳ If we assume that the phase margin is about $90°$, then $L(j\omega_c) = -j$ and $T(j\omega_c) = 1/\sqrt{2}$.

⇒ In other word, the (open loop) cross over frequency is approximately equal to the bandwidth of the closed-loop system.



20log|L(jω)|
ω
ω_{gc}
High frequency obstacle
Low frequency obstacle