Camera Extrinsics & Intrinsics

Goal: Describe how a point is mapped to a pixel coordinate

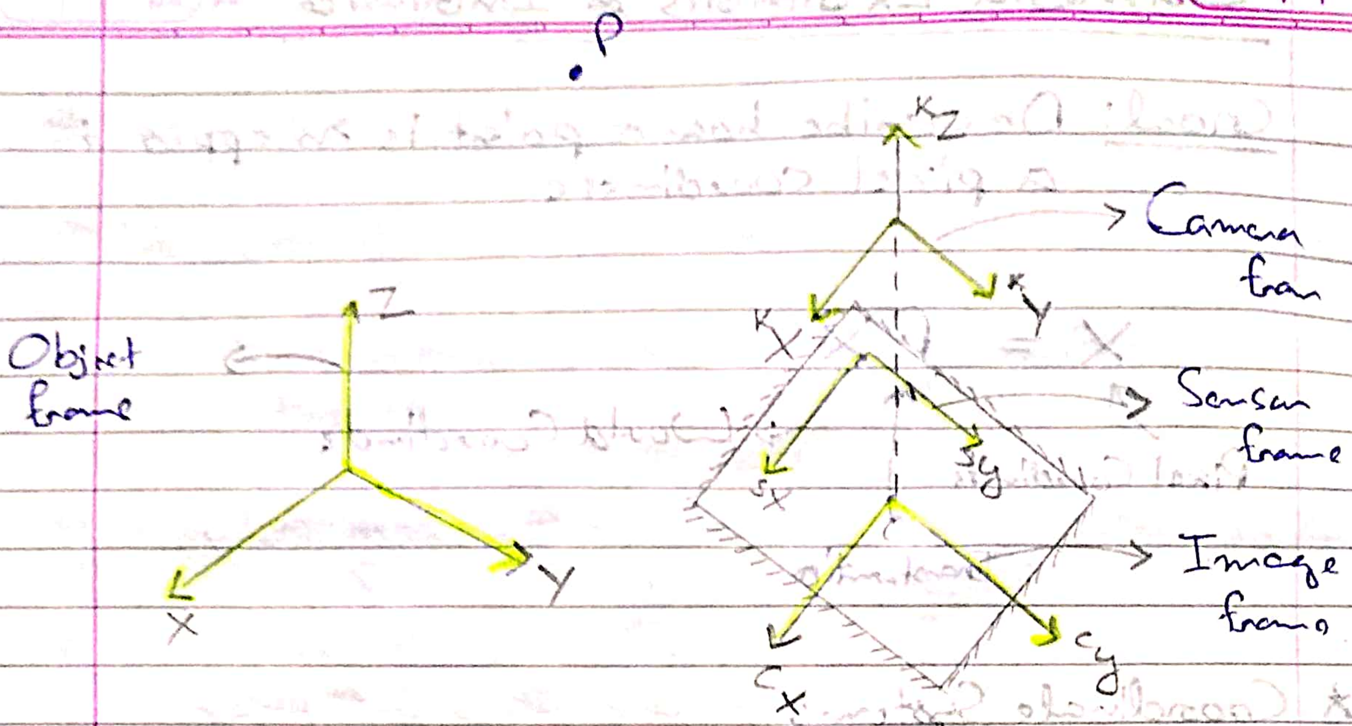
$$\underbrace{x}_{\text{Pixel Coordinate}} = \underbrace{P}_{\text{Transformation}} \underbrace{X}_{\text{World Coordinate}}$$

★ Coordinate Systems

1. World / Object coordinate system (S_o)
   $[X, Y, Z]^T$
2. Camera coordinate system (S_k)
   $[{}^kX, {}^kY, {}^kZ]^T$
3. Image (Plane) coordinate system (S_c)
   $[{}^cx, {}^cy]^T$
4. Sensor coordinate system (S_s)
   $[{}^sx, {}^sy]^T$

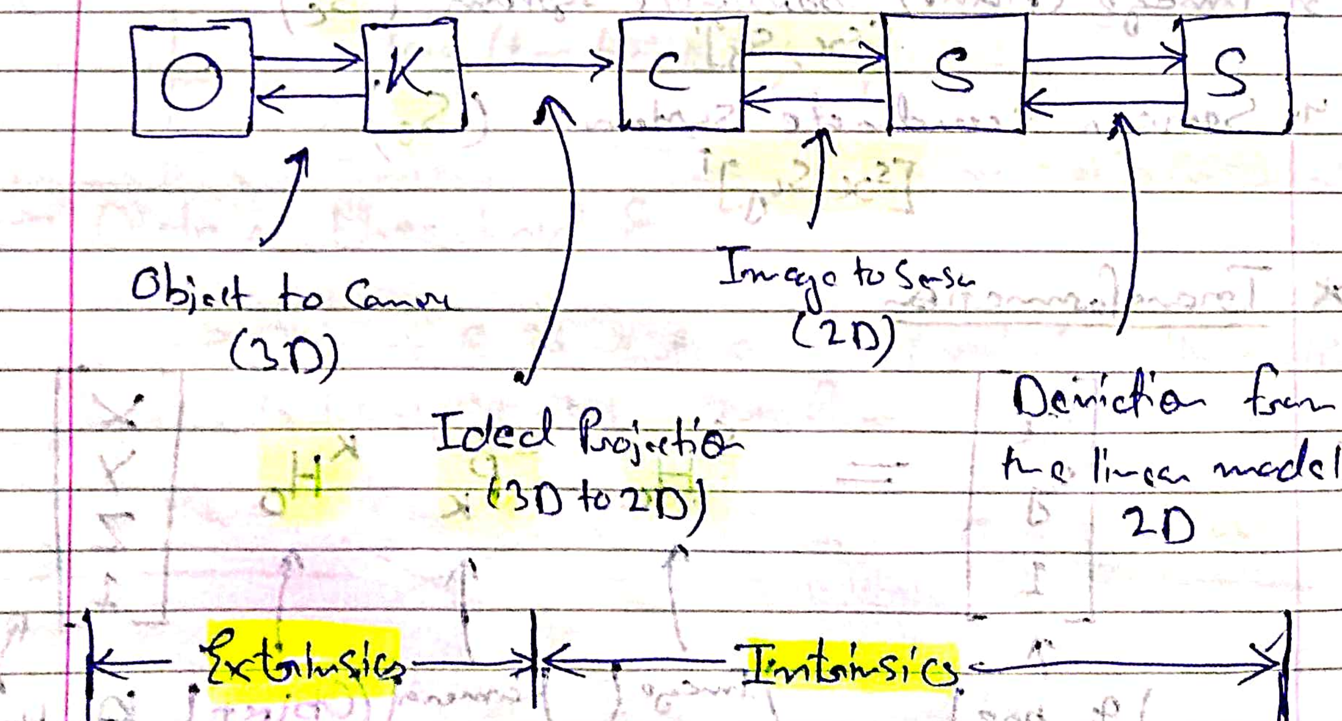
★ Transformation

$$\underbrace{\begin{bmatrix} {}^sx \\ {}^sy \\ 1 \end{bmatrix}}_{\text{In the Sensor System}} = \underbrace{{}^sH_c}_{\text{Image Plane to Sensor}} \; \underbrace{{}^cP_k}_{\text{Camera to Image}} \; \underbrace{{}^kH_o}_{\text{Object to Camera}} \underbrace{\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}}_{\text{On the object System}}$$



$$^{K}O_c = [0,0,c]^T, \text{ with } c<0$$

★ <u>From the World to the Sensor</u>



Page No.
Date:

⇒ Extrinsic parameters describes the pose of the camera in the world.

⇒ Intrinsic parameters describes the mapping of the scene infront of the camera to the pixels in the find image (sensor).

★ <u>Extrinsic Parameters</u> (6 parameters)

⇒ Describe the Pose of the Camera with respect to the world.

⇒ Point P with Coordinates in world coordinates

$$X_P = [X_P, Y_P, Z_P]^T$$

⇒ The Center O of the projection:

$$X_O = [X_O, Y_O, Z_O]^T$$

⇒ $$^{K}X_P = R(X_P - X_O)$$

⇒ $$^{K}X_P = {}^{K}H_O \, {}^{O}X_P \quad \text{with} \quad {}^{K}H_O = \begin{bmatrix} R & -RX_O \\ 0^T & 1 \end{bmatrix}$$

In homogeneous Coordinate

★ Intrinsic Parameters

⇒ We split up the mapping into 3 steps:

1. Ideal perspective projection to the image plane.

2. Mapping to the sensor coordinate system.

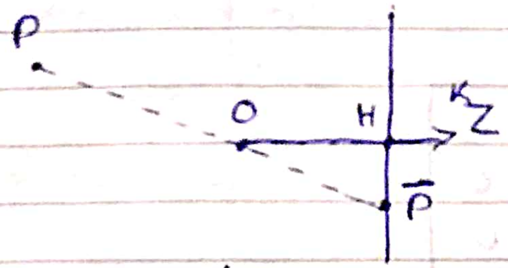
3. Compensation for the fact that the two previous mappings are idealized.

★ Ideal Perspective Projection

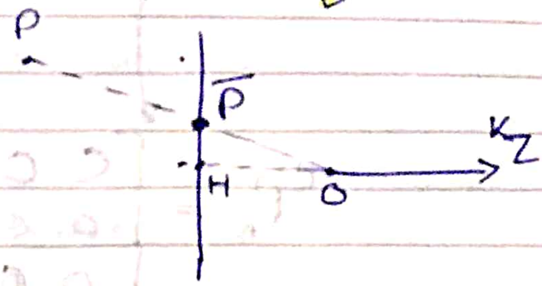
⇒ Assumptions:

- Distortion-free lens.
- All rays are straight lines and pass through the projection center.
  - This point is the origin of the camera coordinate system S_K.
- Focal point and principal point lie on the optical axis.
- The distance from the camera origin to the image plane is the constant c.

* Image Coordinate System



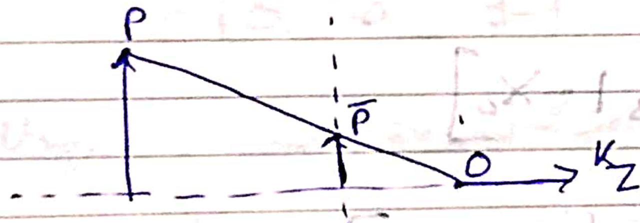
C>0



C<0

Physically motivated coordinate system: C>0

Most popular image coordinate system: C<0



$$^c x_{\bar{P}} = {}^k X_{\bar{P}} = c\,\frac{^k X_P}{^k Z_P}$$

$$^c y_{\bar{P}} = {}^k Y_{\bar{P}} = c\,\frac{^k Y_P}{^k Z_P}$$

$$\begin{bmatrix} ^c x_{\bar{P}} \\ ^c y_{\bar{P}} \\ 1 \end{bmatrix} = {}^c X_{\bar{P}} = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} ^k X_P \\ ^k Y_P \\ ^k Z_P \\ 1 \end{bmatrix}$$

$\Rightarrow \quad {}^{c}x_{\bar{p}} = {}^{c}P_{K}\, {}^{K}x_{p}$

$${}^{c}P_{K} = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

★ Calibration matrix (Ideal Camera)

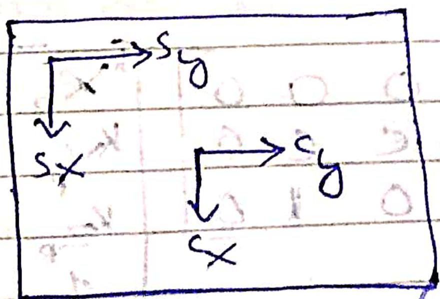
$${}^{c}P = {}^{c}P_{K}\, {}^{K}H_{0} = \begin{bmatrix} c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R & -RX_0 \\ 0^T & 1 \end{bmatrix}$$

$${}^{c}P = {}^{c}K R [I_3 | -X_0]$$

where, $${}^{c}K = \begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ is Calibration matrix of ideal camera.

★ Mapping to the Sensor (assuming linear error)

⇒ The origin of the sensor system is not at the principal point.



40:00

Page No.

Date: | |

$$
{}^{S}H_c = \begin{bmatrix} 1 & 0 & x_H \\ 0 & 1 & y_H \\ 0 & 0 & 1 \end{bmatrix}
$$

★ Sheer and Scale Difference

⇒ Scale difference m in X and Y.

⇒ Sheer compensation S (for digital cameras we typically has $S \simeq 0$)

$$
{}^{S}H_c = \begin{bmatrix} 1 & S & x_H \\ 0 & 1+m & y_H \\ 0 & 0 & 1 \end{bmatrix}
$$

⇒ Finally,

$$
{}^{S}X = \underbrace{{}^{S}H_c\,{}^{c}K}_{K}\, R[I_3 | -X_0]\, X
$$

$$
K = {}^{S}H_c\,{}^{c}K = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}
$$

★ Calibration Matrix

⇒ The calibration matrix of an affine camera:

$$
K = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix} \left\{ \text{5 parameters} \right\}
$$

Page No.
Date: / /

★ DLT : Direct Linear Transform

⇒ The mapping $x = PX$ with $P = KR[I_3 | -X_0]$ and

$$K = \begin{bmatrix} c & cs & x_H \\ 0 & c(1+m) & y_H \\ 0 & 0 & 1 \end{bmatrix}$$ is called direct linear transform

⇒ It is the model of the affine camera

Camera with an affine mapping to the sensor (after the central projection is applied)

⇒ DLT contain $5 + 6 = 11$ parameters.

★ Non-linear Error

⇒ Reasons for non-linear error

- Imperfect lens
- Planarity of the Sensor

★ General Mapping

⇒ Idea: Add a last step that covers the non linear effect

⇒ Location-dependent shift in the sensor coordinate system.

Page No.
Date: / /

$$\boxed{\begin{aligned} {}^a x &= {}^s x + \Delta x(x, q) \\ {}^a y &= {}^s y + \Delta y(x, q) \end{aligned}}$$

⇒ General mapping yields

$${}^a X = {}^a H_s(x)\, {}^s X$$

$${}^a H_s(x) = \begin{bmatrix} 1 & 0 & \Delta x(x, q) \\ 0 & 1 & \Delta y(x, q) \\ 0 & 0 & 1 \end{bmatrix}$$

⇒ So that the overall mapping becomes

$${}^a X = {}^a H_s(x)\, KR[I_3 | -X_0] X$$

★ <u>General Calibration Matrix</u>

$${}^a K(x, q) = \begin{bmatrix} c & cs & x_H + \Delta x(x, q) \\ 0 & c(1+m) & y_H + \Delta y(x, q) \\ 0 & 0 & 1 \end{bmatrix}$$

★ <u>Barrel Distortion</u>

⇒ A standard approach for wide angle lenses is to model the barrel distortion

$${}^a x = x(1 + q_1 r^2 + q_2 r^4)$$

$${}^a y = y(1 + q_1 r^2 + q_2 r^4)$$

$\Rightarrow$ $r \rightarrow$ distance of the pixel in the image to the principal point.

$\Rightarrow$ The terms q_1, q_2 are the additional parameters of the general mapping.

★ Inversion of the Mapping

① 1st Step: ${}^{a}x \rightarrow {}^{s}x$

② 2nd Step: ${}^{s}x \rightarrow x$

★ Inversion Step 1 $\left({}^{a}x = {}^{a}H_s(x, q)\,{}^{s}x\right)$

$\Rightarrow$ Iteration due to unknown x in ${}^{a}H_s(x)$

$\Rightarrow$ Start with ${}^{a}x$ as the initial guess

$$x^{(1)} = \left[{}^{a}H_s({}^{a}x)\right]^{-1} {}^{a}x$$

$\Rightarrow$ and iterate

$$x^{(\nu+1)} = \left[{}^{a}H_s(x^{(\nu)})\right]^{-1} {}^{a}x$$

$\Rightarrow$ As ${}^{a}x$ is often a good initial guess, this procedure converges quickly.

★ Inversion Step 2

$\Rightarrow$ Inversion of the projective mapping.

Page No.
Date: / /

$$\Rightarrow \lambda\, {}^{s}X_{\bar{p}} = P X_p$$

$$= KR[I_3 | -X_o] X_p$$

$$= [KR | -KRX_o] \begin{bmatrix} X_p \\ 1 \end{bmatrix}$$

$$= KRX_p - KRX_o$$

$$\Rightarrow \boxed{X_p = X_o + \lambda (KR)^{-1}\, {}^{s}X_{\bar{p}}}$$

⇒ The term $\lambda (KR)^{-1}\, {}^{s}X_{\bar{p}}$ describes the direction of the ray from the camera origin X_o to the 3D point X_p

★ Calibrated Camera

⇒ If the Intrinsics are unknown, we call the camera uncalibrated.

⇒ If the intrinsics are known, we call the camera calibrated.

⇒ If the intrinsics are known and do not change, the ~~camera~~ camera is called metric camera.

⇒ The process of obtaining the intrinsics is called camera calibration.