4

Nonparametric Filters

⇒ A popular alternative to Gaussian techniques are nonparametric filters

↳ It does not rely on a fixed functional form of the posterior such as Gaussian.

↳ They approximate posteriors by a finite number of values, each roughly corresponding to a region in state space.

↳ As the number of parameters goes to infinity, nonparametric technique tends to converge uniformly to the correct posterior.

⇒ This chapter discusses two nonparametric approaches for approximating posteriors over continuous space with finitely many values:-

① Histogram filter

→ decomposes the state space into finitely many regions

→ Assigns each region a single cumulative probability

② Particle filter

→ Represents posteriors by finitely many samples.

→ Has gained immense popularity in certain robotics problem.

⇒ They are well-suited to represent complex multimodal beliefs.

↳ method of choice when a robot has to cope with phases of global uncertainty.

⇒ Representational power of these techniques comes at the price of added computational complexity.

⇒ Technique that can adapt the number of parameters to represent the posterior online are called adaptive.

⇒ If they can adapt based on computational resource, it is called resource-adaptive.

4.1) The Histogram filter

⇒ When applied to discrete spaces, such filters are known as discrete Bayes filters.

⇒ In continuous state space, they are known as histogram filters.

4.1.1) The Discrete Bayes Filter Algorithm

⇒ It apply to problems, where the random variable X_t can take on finitely many values.

Example: Occupancy grid mapping problem.

Algorithm Discrete_Bayes_filter($\{p_{k,t-1}\}, u_t, z_t$):

for all k do

$$\bar{p}_{k,t} = \sum_i p(X_t = x_t \mid u_t, X_{t-1} = x_i)\, p_{i,t-1}$$

$$p_{k,t} = \eta\, p(z_t \mid X_t = x_k)\, \bar{p}_{k,t}$$

endfor

return $\{p_{k,t}\}$

4.1.2) Continuous State

⇒ Histogram filters decompose a continuous state space into finitely many regions:

$$\text{range}(X_t) = x_{1,t} \cup x_{2,t} \cup \ldots x_{k,t}$$

⇒ Here X_t is the random variable describing the state of the robot at time t.

⇒ A straightforward decomposition of a multi-dimensional continuous state space is a grid, where each $x_{k,t}$ is a grid cell.

↳ Through the granularity of the decomposition, we can trade off accuracy and computational efficiency.

⇒ Each region $x_{k,t}$ has probability, $P_{k,t}$

$$p(x_t) = \frac{P_{k,t}}{|X_{k,t}|}$$

⇒ For cases where each region $x_{k,t}$ is small and of the same size, the densities are usually approximated by substituting $x_{k,t}$ by a representative of this region.

$$\hat{x}_{k,t} = |X_{k,t}|^{-1} \int_{X_{k,t}} x_t \, dx_t$$

⇒ One then simply replaces

$$p(z_t \mid X_{k,t}) \approx p(z_t \mid \hat{x}_{k,t})$$

$$p(X_{k,t} \mid u_t, X_{i,t-1}) \approx \frac{\eta}{|X_{k,t}|} \, p(\hat{x}_{k,t} \mid u_t, \hat{x}_{i,t-1})$$

4.1.3> Decomposition Techniques

⇒ Decomposition techniques of continuous state space come into two basic flavors:

- Static { fixed decomposition that is chosen in advance }
- Dynamic { Adapt the decomposition to the specific shape of the posterior distribution. }
  - Reduces computational complexity.

⇒ A primary of dynamic decomposition technique is the family of density trees.

{ Decomposes the state space recursively, in ways that adapt the resolution to the posterior probability mass. }

⇒ An effect similar to that of dynamic decomposition can be achieved by selective updating.

↳ Update a fraction of all grid cell only.

Topological representation

↳ Often thought of as course graph-like representations, where nodes in the graph correspond to significant features in the environment.

4.1.4) Binary Bayes Filter with Static State

⇒ Certain problem in robotics are best formulated as estimation problem with binary state that does not change over time.

⇒ Naturally, binary estimation problems of this type can be tackled using the discrete bayes filter.

4.2> The Particle Filter

4.2.1> Basic algorithm

⇒ The Key idea of the particle filter is to represent the posterior $bel(x_t)$ by a set of random state samples drawn from the posterior.

⇒ In particle filter, the samples of a posterior distribution are called particles and are denoted

$$X_t := x_t^{[1]}, x_t^{[2]}, \ldots, x_t^{[M]}$$

⇒ Each particle $x_t^{[m]}$ (with $1 \leq m \leq M$) is a Concrete Instantiation of the state at time t.

M ⇒ Number of particle.

⇒ Likelyhood for a state hypothesis x_t to be included in the particle set X_t shall be proportional to its Bayes filter posterior $bel(x_t)$

$$x_t^{[m]} \sim p(x_t \mid z_{1:t}, u_{1:t})$$

⇒ The denser a subregion of the state space is populated by samples, the more likely it is that true state falls into this region.

⇒ Particle filter construct the particle set $\mathcal{X}_t$ recursively from the set $\mathcal{X}_{t-1}$:

<u>Algorithm Particlefilter</u>$(\mathcal{X}_{t-1}, u_t, z_t)$:

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \phi$

for $m=1$ to M do

sample $x_t^{[m]} \sim p(x_t \mid u_t, x_{t-1}^{[m]})$

$w_t^{[m]} = p(z_t \mid x_t^{[m]})$

$\bar{\mathcal{X}}_t = \bar{\mathcal{X}}_t + \langle x_t^{[m]}, w_t^{[m]} \rangle$

endfor

for $m=1$ to M do

draw i with probability $\propto w_t^{[i]}$

add $x_t^{[i]}$ to $\mathcal{X}_t$

endfor

return $\mathcal{X}_t$

→ Resampling step

→ The resampling step is a probabilistic implementation of the Darwinian idea of survival of the fittest.