Iterative Closest point (ICP)
(Point Cloud Alignment)

★ **Alignment of 3D Data Points**

⇒ Find the parameters of the transformation that best align corresponding data points.

⇒ Optimization / Search for parameters

- → Least squares and robust least squares
- → Iterative closest point (ICP)

★ **The problem:**

⇒ Given two point sets:

$Q = \{q_1, \dots q_N\}$ $P = \{P_1, \dots P_M\}$

with correspondences $C = \{(i,j)\}$

⇒ Wanted: Translation t and Rotation R that minimize the sum of the squared error.

$$E(R,t) = \sum_{(i,j) \in C} \| q_i - (R P_j + t) \|^2$$

★ **Key Idea**

If the correct correspondences are known, the correct relative rotation/translation can be computed directly.

1. Shift via the center of mass
2. Rotational alignment.

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★ Center of Mass

⇒ The center of mass of the corresponding points in both sets.

$$\mu_Q = \frac{1}{|C|} \sum_{(i,j)\in C} q_i$$

$$\mu_P = \frac{1}{|C|} \sum_{(i,j)\in C} p_j$$

⇒ Substract the corresponding center of mass from every point

$$Q' = \{ q_i - \mu_Q \} = \{ q_i' \}$$

$$P' = \{ p_j - \mu_P \} = \{ p_j' \}$$

★ Orthogonal Procrustes Problem

⇒ Minimizing: $E(R,t) = \sum_{(i,j)\in C} \| q_i - (R p_j - t) \|^2$

⇒ Equivalent to minimizing:

$$E'(R) = \| [q_1' \cdots q_n'] - R\,[p_1' \cdots p_n'] \|_F^2$$

{Frobenius Norm}

Frobenius Norm

Let A be a matrix

$$\|A\|_F = \sqrt{\sum_{i,j} (A_{ij})^2}$$

⇒ Called **Orthogonal Procrustes problem.**

⇒ Can be solved through SVD.

★ **Singular Value Decomposition**

⇒ Compute the cross-covariance matrix

$$W = \sum_{(i,j)\in C} q'_i \, p'^T_j$$

⇒ Use the SVD to decompose

$$W = UDV^T$$

→ 3x3 rotation matrix

→ Diagonal $(\sigma_1, \sigma_2, \sigma_3)$

⇒ If rank(W) = 3, the parameters minimizing E(R,t) are unique and given by:

$$\boxed{\begin{aligned} R &= UV^T \\ t &= \mu_Q - R\mu_P \end{aligned}}$$

★ ICP with Unknown Data Association

⇒ If the correct correspondences are not known, it is generally impossible to determine the optimal relative rotation and translation in one step.

Idea: Iterate to find alignment

⇒ Converges if starting position are close enough.

★ Basic ICP Algorithm

1. error = inf
2. While (error decreased and error > threshold)
3. Determine corresponding points.
4. Compute rotation R, translation via SVD
5. Apply R & t to the points of the set to be registered.
6. error = E(R, t)

★ ICP Variants

⇒ Variants on the following stages of ICP have been proposed:

1. Point subset (from one or both point sets)

2. Weighting the Correspondences

3. Data association

4. Rejecting certain (outlier) point pairs.

1. Point Subset

★ Selecting source points

- Use all points
- Uniform sub-sampling
- Random sampling
- Feature based Sampling
- Normal-space Sampling

{Ensure that samples have normals distributed as uniformly as possible}

2. Weighting the Correspondences

⇒ Noise: Weighting based on sensor uncertainity.

⇒ Example: In case of Stereo camera we may want to give more weitage to the point closer, as error is small.

⇒ Determine transformation that minimizes the weighted error function.

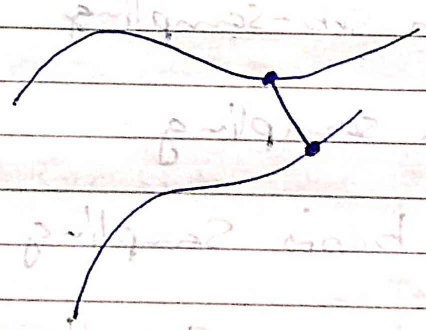
3. <u>Data association</u>

⇒ Has greatest effect on convergence and speed.

⇒ Matching methods:

- <u>Closest point</u>

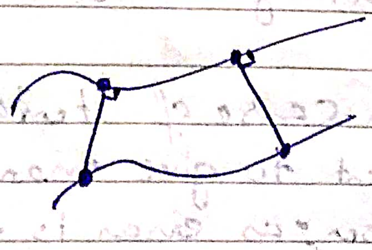
  ⇒ Find closest point in other the point set (using Kd-tree).



  ⇒ Generally stable, but slow convergence and requires preprocessing.

- <u>Normal shooting</u>

  ⇒ Project along normal, intersect other point set.



⇒ Slightly better convergence results than closest point for smooth structures, worse for noisy or complex structures.

- <u>Closest Compatible point</u>

⇒ Only match compatible points

→ Compatibility can be based on

- → Normals
- → Colors
- → Curvature
- → Higher-order derivatives
- → Other local features.

- <u>Point to Plane Error Metric</u>

⇒ Minimize the sum of the squared distance between a point and the tangent plane at its correspondence point.

4. <u>Rejecting certain (outlier) point pairs</u>

- Rejecting point-to-point distance larger than a given threshold.

- Rejection of pairs that are not consistent with their neighboring pairs.

- <u>Trimmed ICP</u>: Sort correspondences wrt their error, ignore the worst t%.