(3)

Manipulator Kinematics

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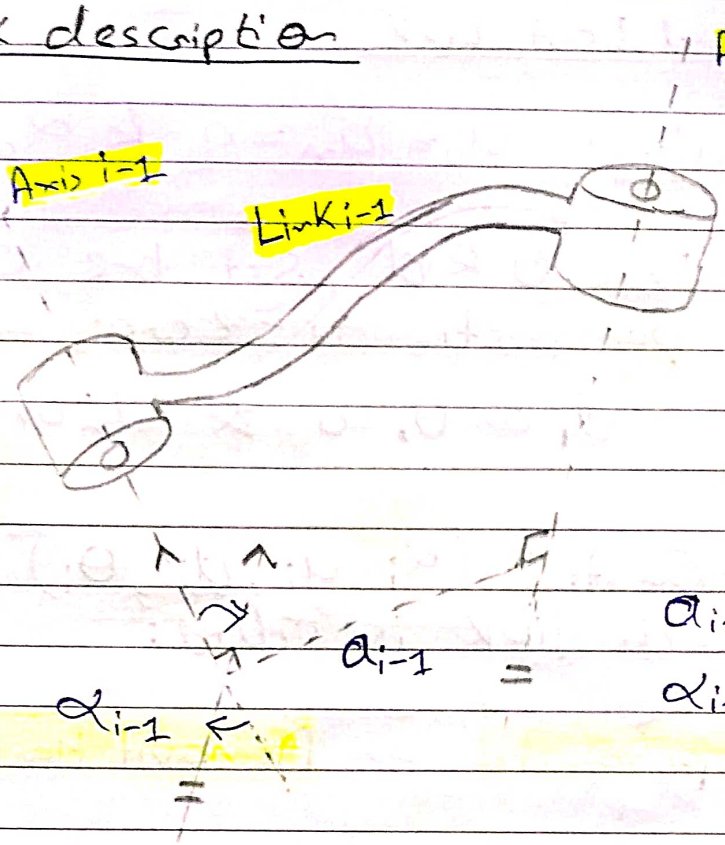
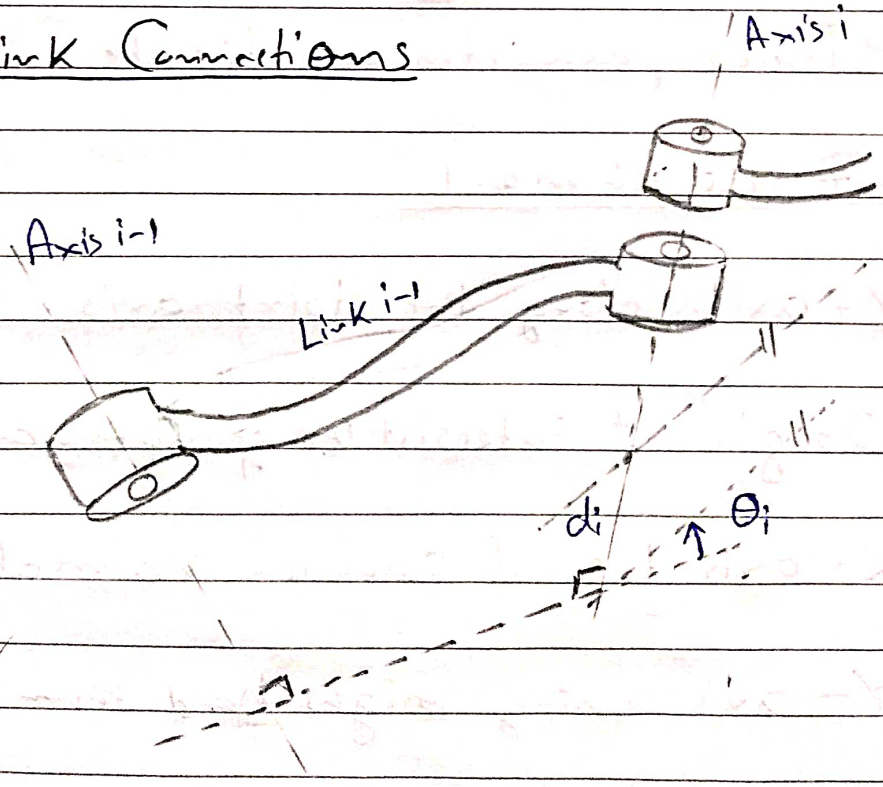
★ Link description



a_{i-1} ⇒ Link Length

α_{i-1} ⇒ Link Twist

★ Link Connections



d_i ⇒ Link offset {Variable for prismatic joint}

θ_i ⇒ Joint angle {Variable for revolute joint}

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★ First and Last Link

Convention: $a_0 = a_n = 0$ & $\alpha_0 = \alpha_n = 0$

↳ For θ & d, set the constant parameters to zero.

d_1 or $\theta_1 = 0$ & d_n or $\theta_n = 0$

⇒ The parameters $(\alpha_i, a_i, d_i, \theta_i)$ associated with each link is called:

DH parameter or Denavit Hartenberg Parameter

↳ Three parameters are fixed.

★ Frame Attachment

① Z-axis along the joint axis

② Origin at intersection of common normal.

③ X-axis toward common normal.

④ Y-axis using right hand thumb rule.

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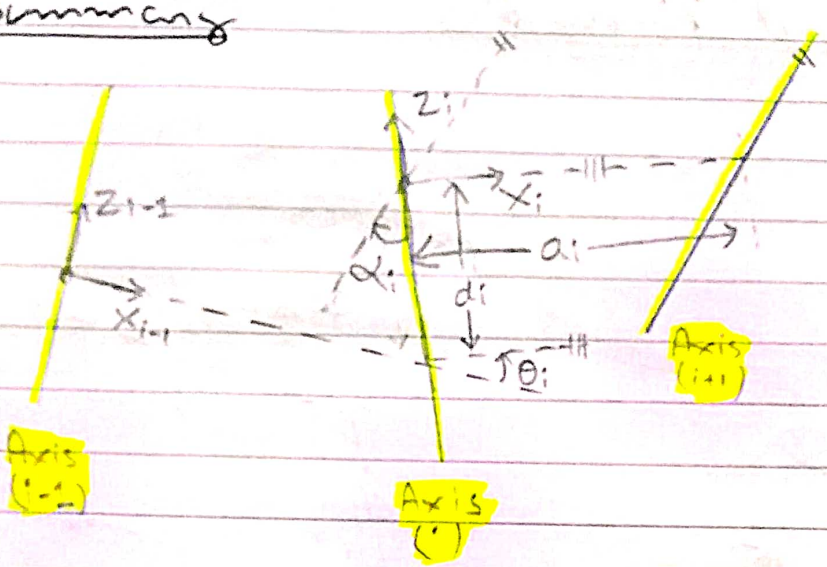
Student Notebooks

* Summary



a_i ⇒ distance (Z_i, Z_{i+1}) along X_i

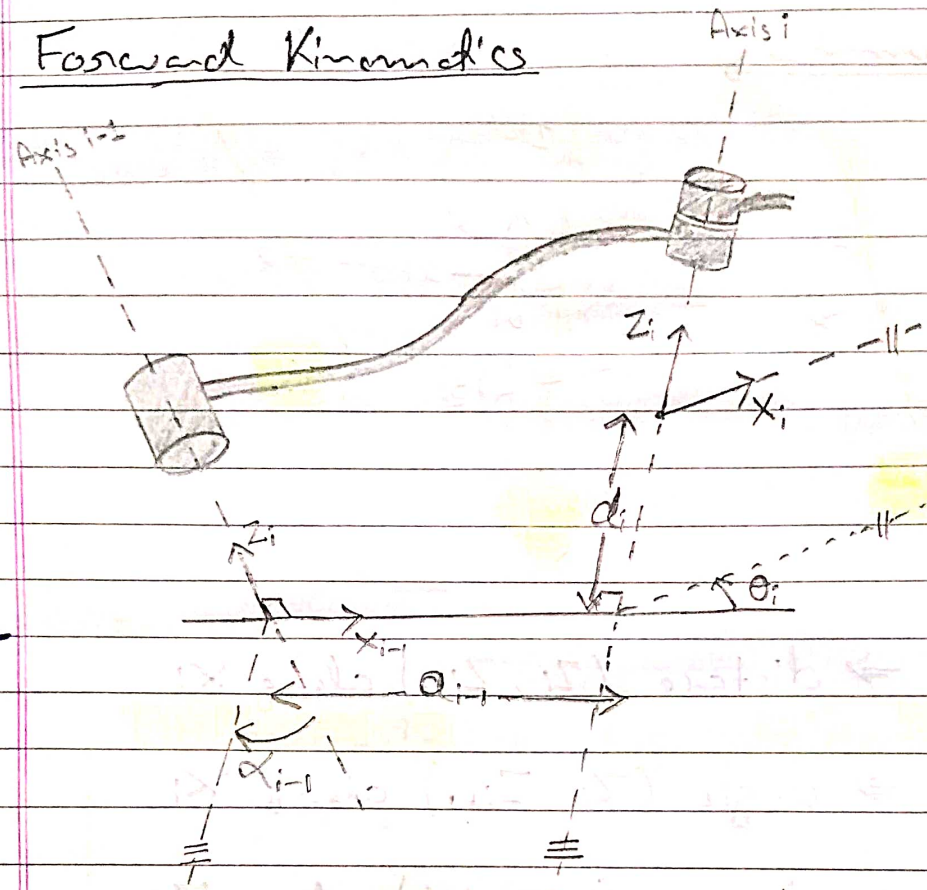
α_i ⇒ angle (Z_i, Z_{i+1}) about X_i

d_i ⇒ distance (X_{i-1}, X_i) along Z_i

θ_i ⇒ angle (X_{i-1}, X_i) about Z_i

i	α_{i-1}	a_{i-1}	d_i	θ_i
–	–	–	–	–

★ Forward Kinematics



$$^{i-1}T_i = {}^{i-1}T_R \; {}^{R}T_Q \; {}^{Q}T_P \; {}^{P}T_i$$

$^{P}T_i \rightarrow D_z(d)$

$^{R}T_Q \rightarrow D_x(a_{i-1})$

$^{Q}T_P \rightarrow R_z(\theta_i)$

$^{i-1}T_R \rightarrow R_x(\alpha_{i-1})$

R ⇒ Rotation
D ⇒ Displacement

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$$
{}^{i-1}T_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & d_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -S\alpha_{i-1} d_i \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & C\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

⇒ Forward Kinematics:

$$
{}^{0}T_N = {}^{0}T_1 \, {}^{1}T_2 \cdots {}^{N-2}T_{N-1} \, {}^{N-1}T_N
$$