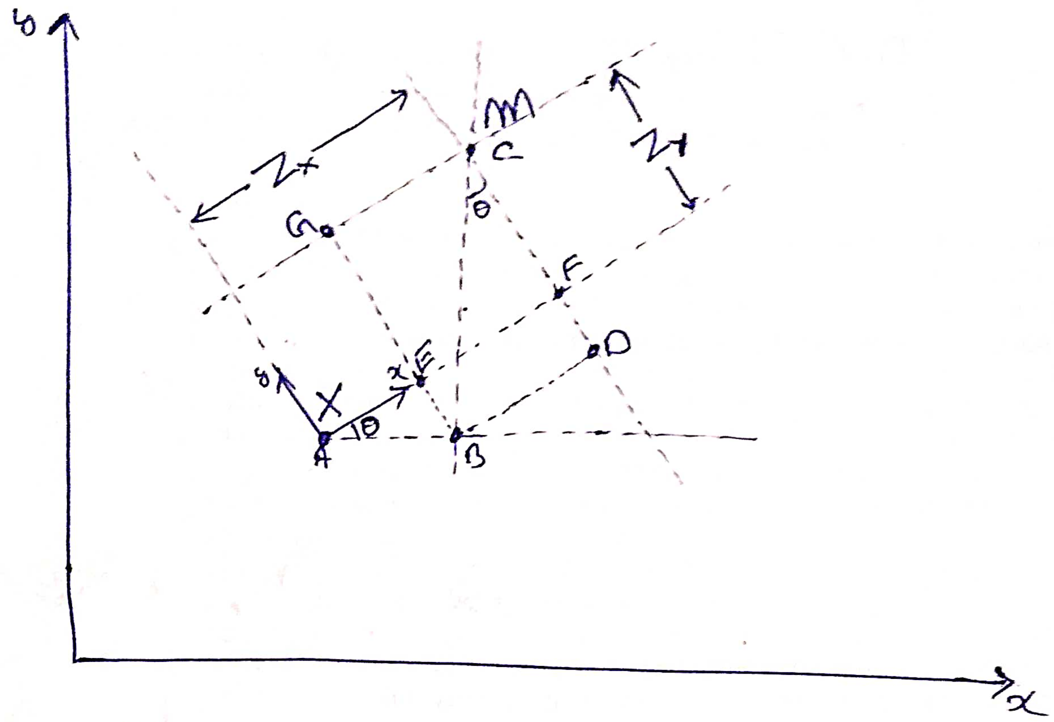
Measurement model and its Jacobian



$$AB = m_x - X_x$$
$$BC = m_y - X_y$$

$$DF = EB = AB \sin\theta = (m_x - X_x)\sin\theta$$
$$CD = BC \cos\theta = (m_y - X_y)\cos\theta$$
$$Z_y = CD - DF = (m_y - X_y)\cos\theta - (m_x - X_x)\sin\theta \quad \text{—} (1)$$

$$EF = BD = BC \sin\theta = (m_y - x_y)\sin\theta$$
$$AE = AB\cos\theta = (m_x - x_x)\cos\theta$$

$$Z_x = AE + EF = (m_x - X_x)\cos\theta + (m_y - X_y)\sin\theta \quad \text{—} (2)$$

$$Z_z = m_z \quad \text{—} (3)$$

$$\begin{bmatrix} Z_x \\ Z_y \\ Z_z \end{bmatrix} = \begin{bmatrix} (m_y - X_y)\sin\theta + (m_x - X_x)\cos\theta \\ (m_y - X_y)\cos\theta - (m_x - X_x)\sin\theta \\ m_z \end{bmatrix}$$

{Inverse ~~[illegible]~~ measurement model}

$\Rightarrow$ Let $X_x = X$, $X_y = Y$

$$\begin{bmatrix} Z_x \\ Z_y \\ Z_z \end{bmatrix} = \begin{bmatrix} (m_y - Y)\sin\theta + (m_x - X)\cos\theta \\ (m_y - Y)\cos\theta - (m_x - X)\sin\theta \\ m_z \end{bmatrix}$$

$$Z_t = h(X_t)$$

{ If there is no noise in measurement and robot state is X_t then measurement will be Z_t }

$\Rightarrow$ ~~For full size state:~~

Let $X' = \begin{bmatrix} X \\ Y \\ Z \\ m_x \\ m_y \\ m_z \end{bmatrix}$

$$Z_t = h(X'_t)$$

$\Rightarrow$ Linearization:

$$\boxed{Z_t = h(\mu'_t) + \frac{\delta h(\mu'_t)}{\delta X'_t}(X'_t - \mu'_t)}$$

★ Jacobian calculation

$$H'(\mu'_t) = \begin{pmatrix} \frac{\delta h_1}{\delta x} & \frac{\delta h_1}{\delta y} & \frac{\delta h_1}{\delta \theta} & \frac{\delta h_1}{\delta m_x} & \frac{\delta h_1}{\delta m_y} & \frac{\delta h_1}{\delta m_z} \\ \frac{\delta h_2}{\delta x} & \frac{\delta h_2}{\delta y} & \frac{\delta h_2}{\delta \theta} & \frac{\delta h_2}{\delta m_x} & \frac{\delta h_2}{\delta m_y} & \frac{\delta h_2}{\delta m_z} \\ \frac{\delta h_3}{\delta x} & \frac{\delta h_3}{\delta y} & \frac{\delta h_3}{\delta \theta} & \frac{\delta h_3}{\delta m_x} & \frac{\delta h_3}{\delta m_y} & \frac{\delta h_3}{\delta m_z} \end{pmatrix}$$

$H'_{11} = -\cos(\mu'_\theta)$

$H'_{12} = -\sin(\mu'_\theta)$

$H'_{13} = (\mu'_{m_y} - \mu'_y)\cos(\mu'_\theta) - (\mu'_{m_x} - \mu'_x)\sin(\mu'_\theta)$

$H'_{14} = \cos(\mu'_\theta)$

$H'_{15} = \sin(\mu'_\theta)$

$H'_{16} = 0$

$H'_{21} = \sin(\mu'_\theta)$

$H'_{22} = -\cos(\mu'_\theta)$

$H'_{23} = -(\mu'_{m_x} - \mu'_y)\sin(\mu'_\theta) - (\mu'_{m_x} - \mu'_x)\cos(\mu'_\theta)$

$H'_{24} = -\sin(\mu'_\theta)$

$H'_{25} = \cos(\mu'_\theta)$

$H'_{26} = 0$

$H'_{31} = 0 \quad H'_{32} = 0 \quad H'_{33} = 0 \quad H'_{34} = 0 \quad H'_{35} = 0 \quad H'_{36} = 1$

⇒ For probabilistic measurement model, let add a random variable δ to represent measurement noise..

- → mean = 0
- → covariance = Q

$$Q = \begin{pmatrix} \sigma_{z_x z_x} & 0 & 0 \\ 0 & \sigma_{z_y z_y} & 0 \\ 0 & 0 & \sigma_{z_z z_z} \end{pmatrix}$$

$$\boxed{Z_t = h(\mu'_t) + H(\mu'_t)(X'_t - \mu'_t) + \delta}$$

⇒ For full size state:

Let $X = [x, y, z, m_{1x}, m_{1y}, m_{1z} \cdots m_{jx}\, m_{jy}\, m_{jz} \cdots]^T$

be the state vector

$$Z = [Z_x, Z_y, Z_z]^T$$

Let

$$H' = (H'_x \;\; H'_m)$$

$H' \in R^{3 \times 6}$

$H'_x \in R^{3 \times 3}$

$H'_m \in R^{3 \times 3}$

$$H = (H'_x \;\; O_1 \;\; O_2 \;\cdots\; O_{j-1} \;\; H'_m \;\cdots)$$

$H \in R^{3 \times 3+3n}$

★ Correction Step

1. $K_t = \bar{\Sigma}_t H^T \left(H \bar{\Sigma}_t H^T + Q_t\right)^{-1}$

2. $\mu_t = \bar{\mu}_t + K_t\left(z_t - h(\bar{\mu}_t)\right)$

3. $\Sigma_t = \left(I - K_t H\right) \bar{\Sigma}_t$

Let, $$\bar{\Sigma}_t = \begin{pmatrix} \bar{\Sigma}_{xx} & \bar{\Sigma}_{xm} \\ \bar{\Sigma}_{mx} & \bar{\Sigma}_{mm} \end{pmatrix}$$

$$= \begin{pmatrix} \bar{\Sigma}_{xx} & \bar{\Sigma}_{xm_1} & \cdots & \bar{\Sigma}_{xm_n} \\ \bar{\Sigma}_{m_1x} & \bar{\Sigma}_{m_1m_1} & & \vdots \\ \vdots & & \ddots & \\ \bar{\Sigma}_{m_nx} & \cdots & & \bar{\Sigma}_{m_nm_n} \end{pmatrix}$$

$$= \begin{pmatrix} \bar{\Sigma}_{*x} & \bar{\Sigma}_{*m_1} & \cdots & \bar{\Sigma}_{*m_n} \end{pmatrix}$$

$$= \begin{pmatrix} \bar{\Sigma}_{x*} & \bar{\Sigma}_{m_1*} & \cdots & \bar{\Sigma}_{m_n*} \end{pmatrix}^T$$

(a) $\overline{\Sigma}_t H^T = \overline{\Sigma}_{*x} H_x^T + \overline{\Sigma}_{*m_j} H_m^T$

(b) $H \overline{\Sigma}_t H^T = H_x \left(\overline{\Sigma}_{xx} H_x^T + \overline{\Sigma}_{xm_j} H_m^T \right)$

$$+ H_m \left(\overline{\Sigma}_{m_j x} H_x^T + \overline{\Sigma}_{m_j m_j} H_m^T \right)$$

(c) $K H \overline{\Sigma}_t = K \left(H_x \overline{\Sigma}_{x*} + H_m \overline{\Sigma}_{m_j *} \right)$

let $\overline{\mu}_t = \begin{bmatrix} \overline{\mu}_x \\ \overline{\mu}_{m_1} \\ \vdots \\ \overline{\mu}_{m_n} \end{bmatrix}$

★ Forward measurement model

$$m_x = x + Z_x \cos\theta - Z_y \sin\theta$$
$$m_y = y + Z_x \sin\theta + Z_y \cos\theta$$
$$m_z = Z_z$$