4A

Designing of Control System Using Frequency Responce Method

★ Introduction

⇒ By the term Frequency response we mean the steady-state response of a system to a sinusoidal input.

↳ In frequency-response methods, we vary the frequency of the input signal over a certain range and study the resulting response.

⇒ The information we get from such analysis is different from what we get from root-locus analysis.

↳ ~~The information we get from such analysis is different from what we get from root locus analysis.~~ In Fact, the frequency response and root-locus approach compliment each other.

⇒ In many practical designs of control systems both approaches are employed.

⇒ Frequency responce method was depveloped in 1930s and 1940s by Nyquist, Bode, Nichols and many others.

⇒ The Frequency-response methods are most powerful in Classical Control theory.

↳ They are indispensable to robust Control theory.

⇒ The Nyquist stability Criterion enables us to investigate both the absolute and relative stability of linear Closed loop Systems from a Knowledge of their open loop frequency response Characteristics.

⇒ An advantage of frequency response-approach is that frequency-response test are, in general Simple and can be made accurately by use of readily available Sinusoidal Signal generators and precise measurement equipment.

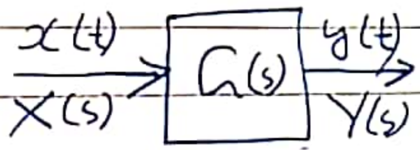
↳ TF of a Complicated Components can be determined experimentally by frequency-response tests.

⇒ In addition, the frequency-response approach has the advantages that a system may be designed so that the effects of undesired noise are neglected and that such analysis and design can be extended to certain nonlinear control system.

⇒ In designing a closed loop system, we adjust the frequency-response characteristic of the open loop ~~transient~~ transfer function by using several design criteria in order to obtain acceptable transient-response characteristic for the system.

★ Obtaining Steady State Output to Sinusoidal Input

⇒ Consider the stable, linear, time-invariant system as shown.



⇒ If Input $x(t)$ is a sinusoidal signal, the steady-state output will also be a sinusoidal signal of the same frequency, but with possibly different magnitude and phase angle.

Let $x(t) = X \sin \omega t$

Let $G(s) = \dfrac{P(s)}{q(s)} = \dfrac{P(s)}{(s+s_1)(s+s_2)(s+s_3)\cdots(s+s_n)}$

So $Y(s) = G(s) X(s) = \dfrac{P(s)}{q(s)} X(s)$

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⇒ If $Y(s)$ has only distinct poles then:

$$Y(s) = G(s) \frac{\omega X}{s^2+\omega^2}$$

$$= \frac{a}{s+j\omega} + \frac{\bar{a}}{s-j\omega} + \frac{b_1}{s+s_1} + \frac{b_2}{s+s_2} + \cdots \frac{b_n}{s+s_n}$$

Where a and b_i $(\forall i = 1, 2, \ldots n)$ are constant and $\bar{a}$ is the complex conjugate of a.

So $y(t) = a e^{j\omega t} + \bar{a} e^{j\omega t} + b_1 e^{-s_1 t} + b_2 e^{-s_2 t} + \cdots + b_n e^{-s_n t}$

$\{\forall\, t > 0\}$

⇒ For a stable system $-s_1, -s_2 \ldots -s_n$ have negative real parts. Therefore as $t \to \infty$ the term $e^{-s_1 t}$, $e^{-s_2 t}$ --- and $e^{-s_n t}$ approaches zero.

⇒ If $Y(s)$ involves multiple poles s_j of multiplicity m_j, then $y(t)$ will involve terms such as $t^{h_j} e^{-s_j t}$ $(h_j = 0, 1, 2 \ldots m_j - 1)$. For stable system, the terms $t^{h_j} e^{-s_j t}$ approaches zero as $t \to \infty$.

⇒ Thus regardless of whether the system is of the distinct-pole type or multiple-pole type, the steady state response becomes:

$$y_{ss}(t) = a e^{-j\omega t} + \bar{a} e^{j\omega t}$$

$$Y(s)\,(s+j\omega) = (s+j\omega)\left\{\frac{a}{s+j\omega} + \frac{\bar{a}}{s-j\omega} + \frac{b_1}{s+s_1} + \cdots + \frac{b_n}{s+s_n}\right\}$$

$$\Rightarrow a + \frac{\bar{a}(s+j\omega)}{s-j\omega} + b_1\left(\frac{s+j\omega}{s+s_1}\right) + \cdots b_n\left(\frac{s+j\omega}{s+s_n}\right)$$

$$Y(s)\,(s+j\omega)\Big|_{s=-j\omega} = a + 0 + 0 + \cdots + 0$$

So $a = \dfrac{G(s)\,\omega X}{s^2+\omega^2}(s+j\omega)\Big|_{s=-j\omega} = \dfrac{-X\,G(-j\omega)}{2j}$

Similarly $\bar{a} = G(s)\dfrac{\omega X}{s^2+\omega^2}(s-j\omega)\Big|_{s=j\omega} = \dfrac{X\,G(j\omega)}{2j}$

⇒ Since $G(j\omega)$ is complex quantity,

$$G(j\omega) = |G(j\omega)|\, e^{j\phi}$$

Magnitude

Angle

$$\phi = \angle G(j\omega) = \tan^{-1}\left[\frac{\text{Imaginary part of } G(j\omega)}{\text{Real part of } G(j\omega)}\right]$$

$$G(-j\omega) = |G(j\omega)| e^{-j\phi} = |G(j\omega)| e^{-j\phi}$$

$$a = \frac{-X|G(j\omega)| e^{-j\phi}}{2j}$$

$$\bar{a} = \frac{+X|G(j\omega)| e^{j\phi}}{2j}$$

$$y_{ss}(t) = X|G(j\omega)| \frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{2j}$$

$$y_{ss}(t) = X|G(j\omega)| \sin(\omega t+\phi)$$

$$\boxed{y_{ss}(t) = Y \sin(\omega t+\phi)}$$

$$\left\{ \begin{array}{l} Y = X|G(j\omega)| \\ \phi = \angle G(j\omega) \end{array} \right\}$$

$\Rightarrow$ For stable, linear, time-invariant system subjected to a sinusoidal input will, at steady state, have a sinusoidal output of the same frequency as input.

⇒ Positive phase angle is called phase lead and negative phase angle is called phase lag.

↳ A network of phase lead characteristics is called lead network

↳ While a network of phase lag characteristics is called lag network.

★ **Presenting Frequency Response Characteristics in Graphical forms**

⇒ $G(j\omega)$ is called Sinusoidal Transfer function.

↳ It is characterized by its magnitude and phase angle with frequency as the parameter.

⇒ There are three commonly used representation of Sinusoidal transfer functions:

1. Bode plot or Logarithmic plot
2. Nyquist plot or polar plot
3. Nichols plot or Log magnitude vs phase plot

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★ Bode Diagrams

⇒ A Bode diagram consists of two graphs:-

(i) Plot of log of magnitude of a Sinusoidal transfer function Vs frequency on a log Scale

(ii) Plot of phase angle Vs frequency on a logarithmic scale.

⇒ Standard representation of the logarithmic magnitude of $G(j\omega)$ is $20\log_{10}|G(j\omega)|$.

↳ The unit used in this representation of magnitude is the decibel. Usually abbreviated dB.

⇒ In the logarithmic representation, the curves are drawn on semilog paper using the log scale for frequency and linear scale for either magnitude (but in decibles) or phase angle (in degree).

⇒ The main advantage of using Bode diagram is that multiplication of magnitudes can be converted into addition.

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⇒ Note: The experimental determination of a TF can be made simple if frequency-response data are presented in the form of a Bode diagram.

★ Basic Factors of $G(j\omega)H(j\omega)$

1. Gain K
2. Integral and derivative factor
3. First-order factor
3. Quadratic factor

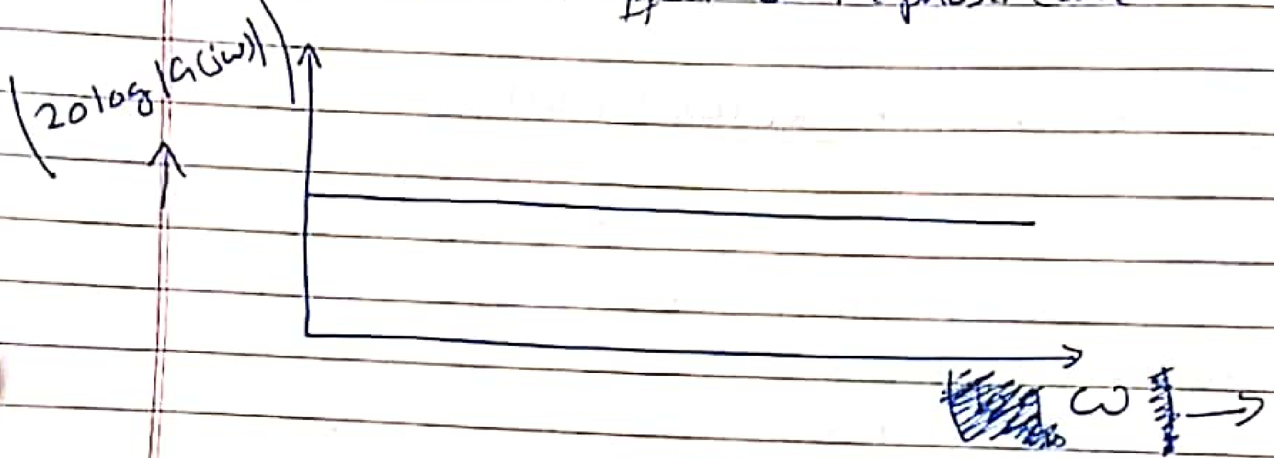
⇒ Once we become familiar with the logarithmic plots of these basic factors, it is possible to utilize them in constructing a composite logarithmic plot of any general form of $G(j\omega)H(j\omega)$.

1. Gain K

⇒ A number greater than unity has a positive value in decibels, while a number smaller than unity has a negative value.

⇒ The phase angle of gain K is zero.

"The effect of varying the gain K in the TF is that it raises or lowers the log-magnitude curve of the TF ~~[illegible]~~ by the corresponding constant amount, but it has no effect on the phase curve"



2. <u>Integral and Derivative Factors $(j\omega)^{\mp}$</u>

$\frac{1}{j\omega}$ → Magnitude $\frac{1}{\omega}$

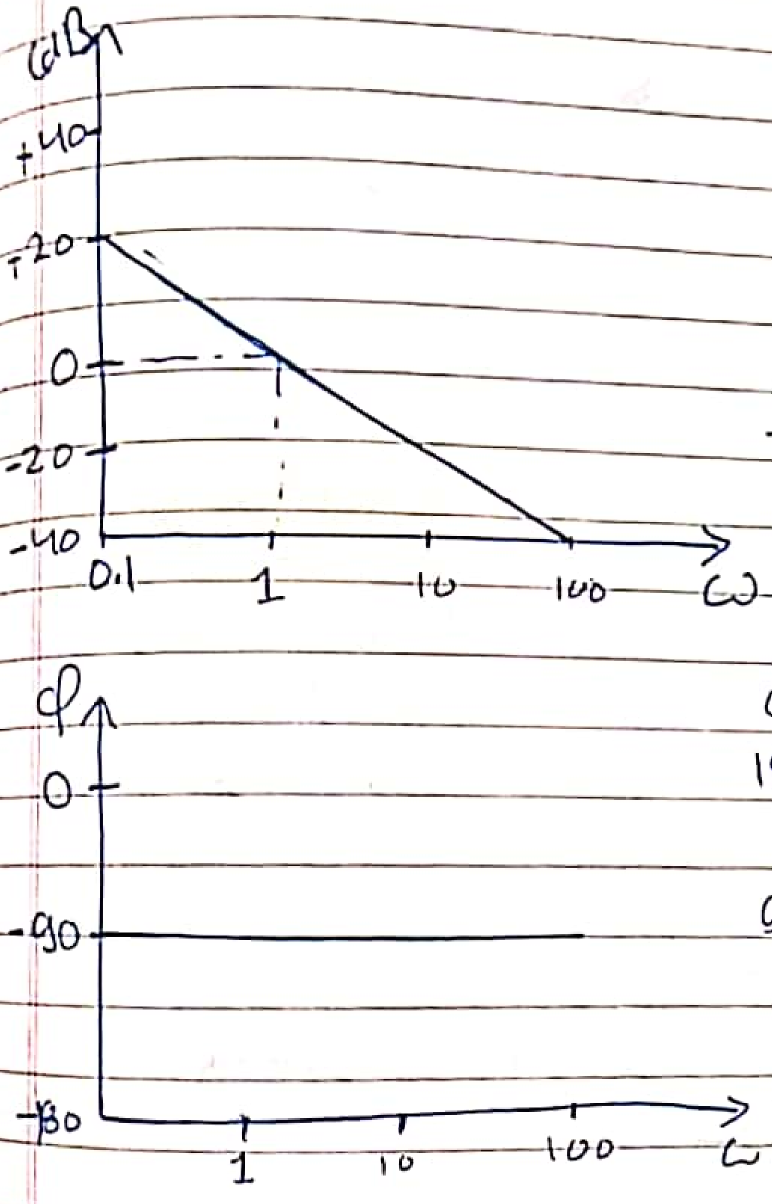
→ Angle $-\pi/2$

$$\left(20\log\frac{1}{\omega}\right) = -20\log\omega \text{ dB}$$

$j\omega$ → Magnitude ω

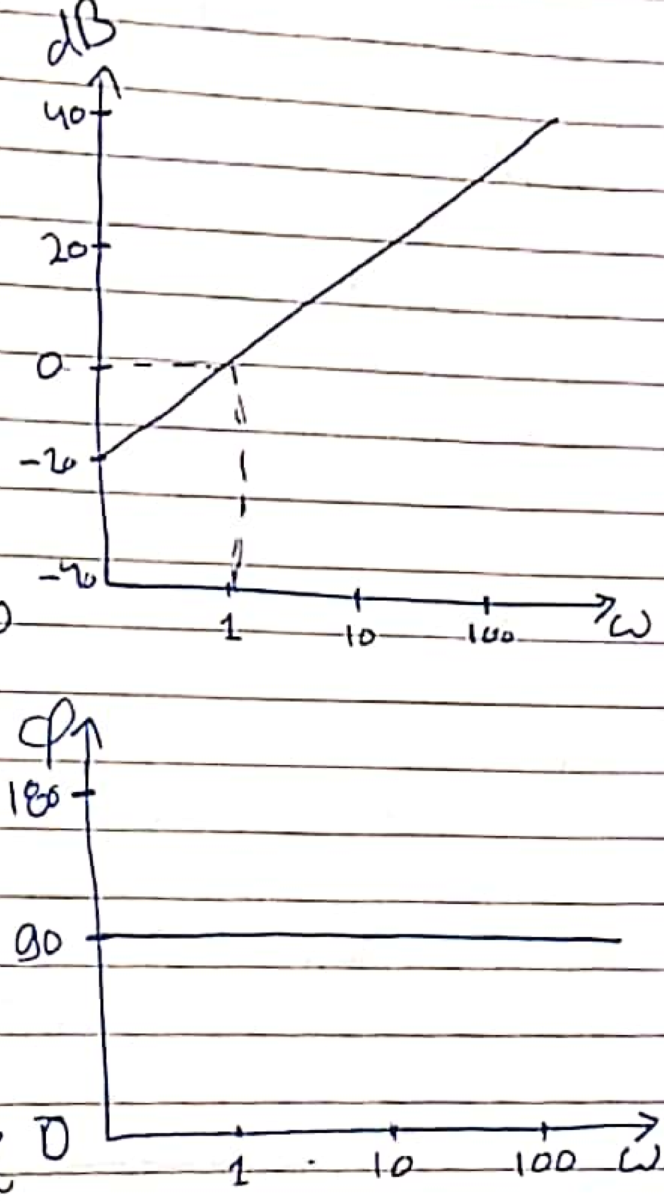
→ Angle $\pi/2$

$|20 \log \omega|$



Bode plot of

$G(j\omega) = \frac{1}{j\omega}$



Bode plot of

$G(j\omega) = j\omega$

3. First-Order Factors

$$\left|\frac{1}{1+j\omega T}\right| \Rightarrow 20\log\left|\frac{1}{1+j\omega T}\right| = -20\log\sqrt{1+\omega^2T^2}$$

// For low frequencies $\omega \ll 1/T$

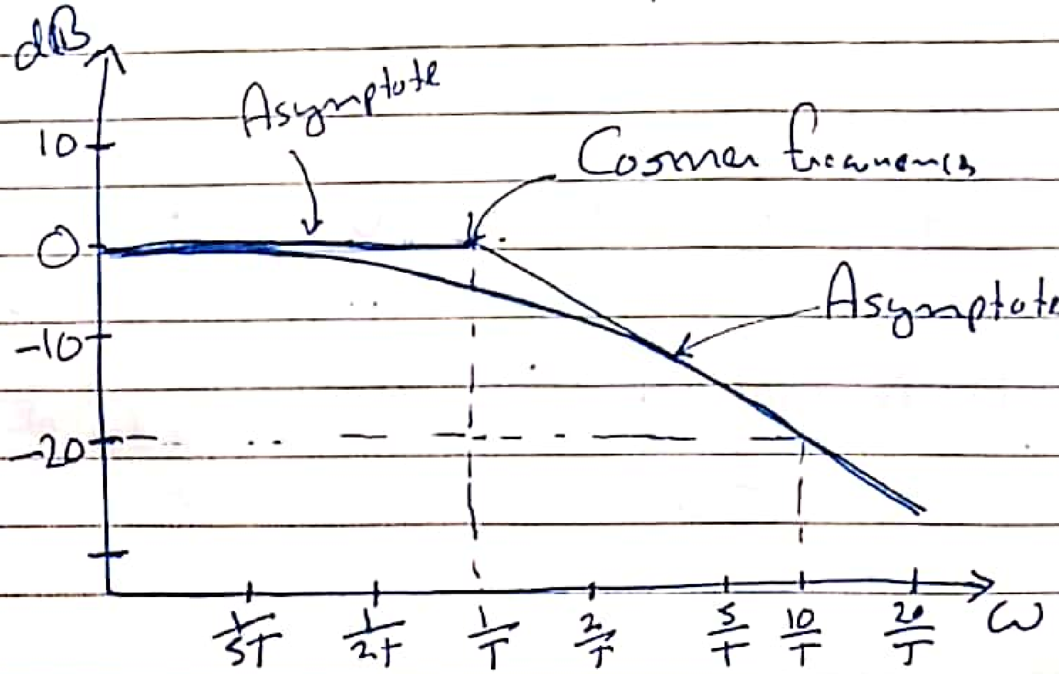
$$\Rightarrow -20\log\sqrt{1+\omega^2T^2} = -20\log 1 = 0\,dB$$

// For high frequencies $\omega \gg 1/T$

$$-20\log\sqrt{1+\omega^2T^2} = -20\log \omega T$$

⇒ At $\omega = 1/T$ log magnitude is equal to zero at $\omega = 10/T$ the log magnitude is $-20dB$.

⇒ Thus the value of $-20\log\omega T$ dB decreases by 20dB for every decade of ω.



⇒ The Corner frequency divides the frequency-response curve into two regions :-

→ Curve for low frequency region

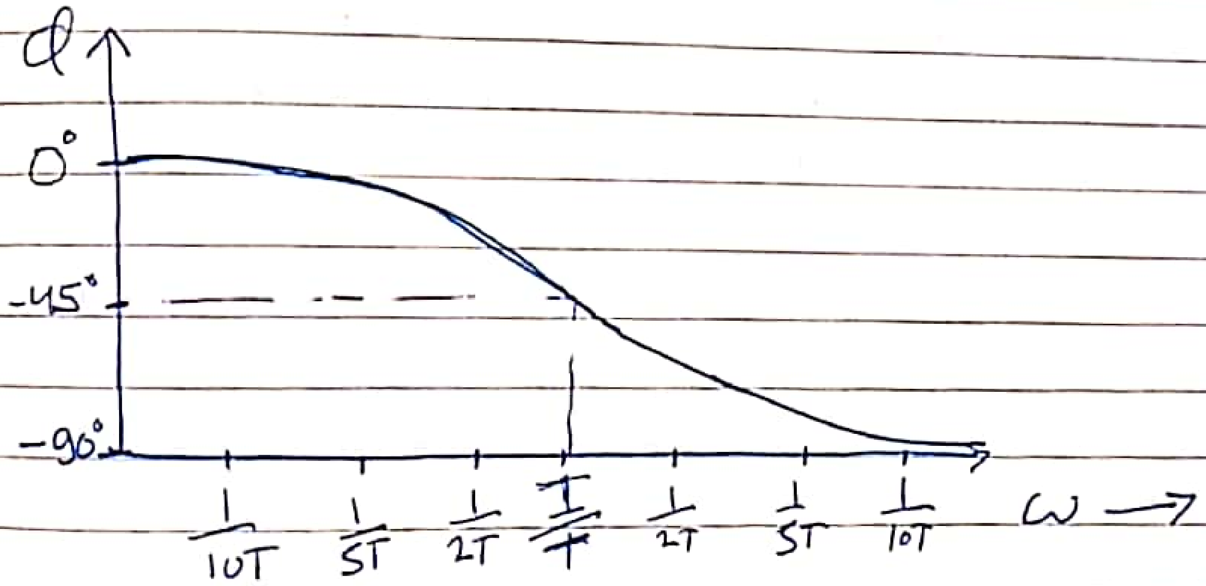
→ Curve for high frequency region.

$$\phi = -\tan^{-1}(\omega T)$$

$\omega = 0$ $\quad \phi = 0$

$\omega = 1/T$ $\quad \phi = -\pi/4$

$\omega = \infty$ $\quad \phi = -\pi/2$



⇒ The transfer function $1/(1+j\omega T)$ has the characteristic of a low-pass filter.

⇒ Bode plot of $(1+j\omega T)^n$ can be easily found out.

4) Quadratic Factor

$$G(j\omega) = \frac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$$

⇒ If $\xi > 1$, this quadratic factor can be expressed as a product of two first-order factors; with real poles.

⇒ If $0 < \xi < 1$, this quadratic factor is the product of two complex-conjugate factors.

⇒ Asymptotic approximations to the frequency-response curves are not accurate for a factor with low values of ξ.

↳ This is because the magnitude and phase of quadratic factor depends on both the corner frequency and the damping ratio ξ.

$$\Rightarrow 20\log\left|\frac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}\right|$$

$$\Rightarrow -20\log\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

$\forall$ low frequency $\omega << \omega_n$

$-20\log 1 = 0 \Rightarrow$ Horizontal line at 0dB

$\forall$ high frequency $\omega >> \omega_n$

$$-20\log\frac{\omega^2}{\omega_n^2} = -40\log\frac{\omega}{\omega_n}$$

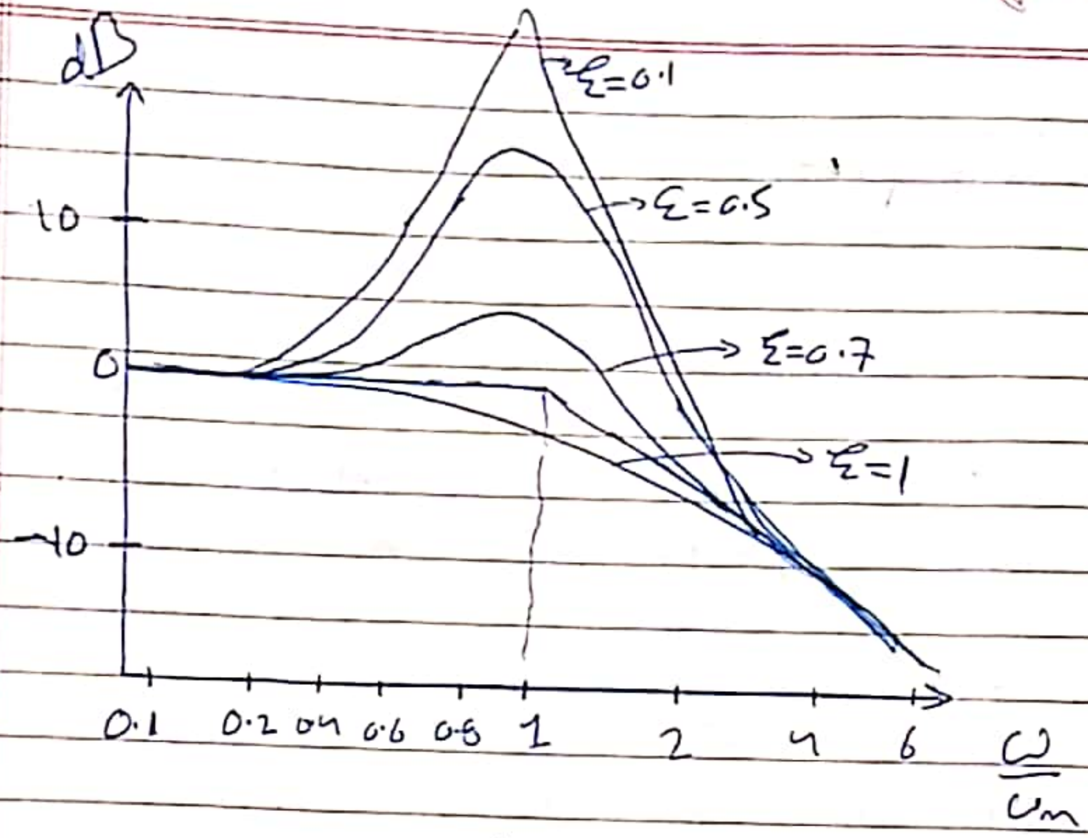
$\Rightarrow$ Straight line with slope -40dB

$\Rightarrow$ The high-frequency asymptote intersect the low frequency one at $\omega = \omega_n$.

ω_n = Corner frequency.

$\Rightarrow$ Near the frequency $\omega = \omega_n$, a resonant peak occurs, ~~as may b~~ and damping ratio ξ determines the magnitude of this resonant peak.

$\hookrightarrow$ Resonance peak is large for small value of ξ.



$$\phi = \angle \frac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$$

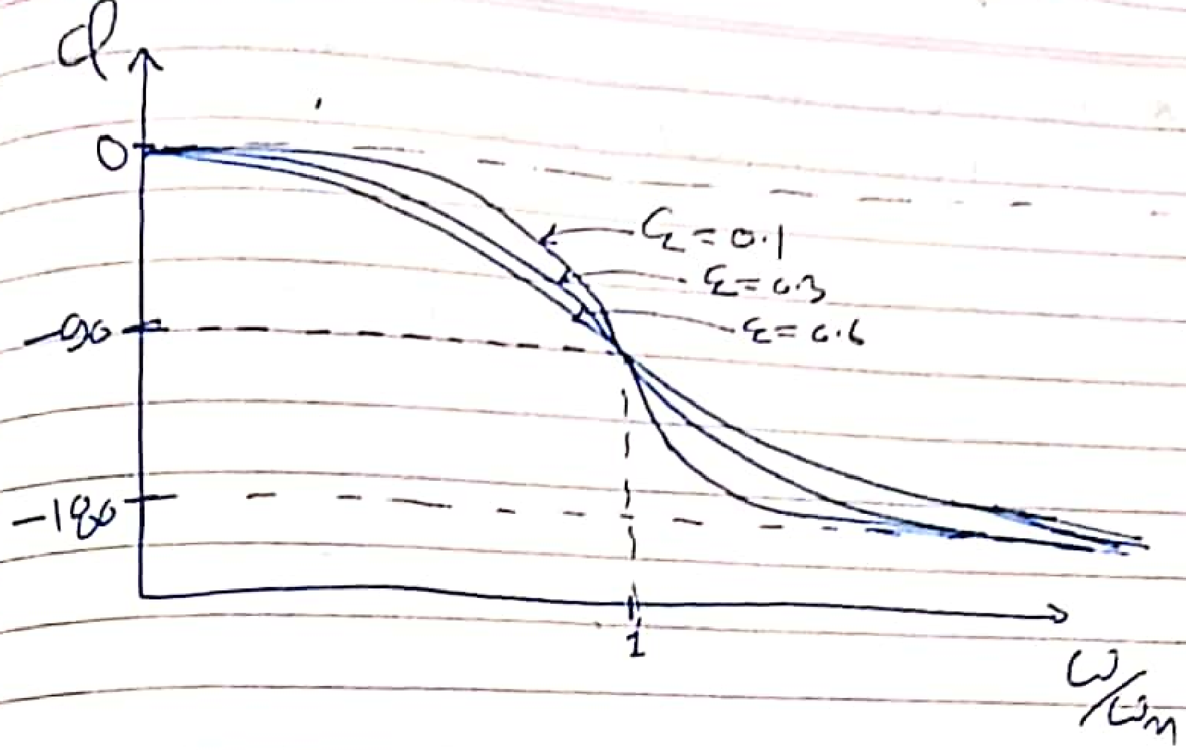
$$= -\tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

⇒ The phase angle is a function of both ω & ξ.

\# $\omega = 0$ $\phi = 0$

\# $\omega = \omega_n$ $\phi = -\pi/2$

\# $\omega = \infty$ $\phi = -\pi$



⇒ The frequency-response curves for the factor:

$$1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2$$

Can easily be obtained by merely reversing the sign of the log magnitude and that of the phase angle of the factor.

$$\frac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$$

★ The resonance frequency (ω_r) and the Resonant peak Value M_r

$$G(j\omega) = \frac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$|G(j\omega)|$ will be maximum if $g(\omega)$ is minimum.

$$g(\omega) = \left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2$$

$$g(\omega) = \left[\frac{\omega^2 - \omega_n^2(1-2\xi^2)}{\omega_n^2}\right]^2 + 4\xi^2(1-\xi^2)$$

$g(\omega)$ is minimum when $\boxed{\omega = \omega_n\sqrt{1-2\xi^2}}$

So $\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$ $\{\forall\ 0 \leq \xi \leq \sqrt{2}\}$

$\Rightarrow$ for $\xi > \sqrt{2}$ there is no resonant peak.

$$\boxed{M_r = |G(j\omega)|_{max} = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$

$\Rightarrow$ As $\xi \to 0 \Rightarrow M_r \to \infty$

$\Rightarrow$ Phase angle at resonance frequency,

$$\boxed{\phi = -\tan^{-1}\frac{\sqrt{1-2\xi^2}}{\xi} = -\frac{\pi}{2} + \sin^{-1}\left(\frac{\xi}{\sqrt{1-\xi^2}}\right)}$$

★ General Procedure of Plotting bode diagram

1) first write the Sinusoidal TF $G(j\omega)H(j\omega)$ as a product of basic factors.

2) Then identify Corner frequency associated with these basic factors.

3) Finally draw the asymptotic log magnitude curve with proper slope between the Corner frequencies.

4) Then exact curve that lies close to the asymptotic Curve, can be obtained by adding proper Corrections.

5) The phase-angle curve of $G(j\omega)H(j\omega)$ can be drawn by adding the phase angle curves of individual factors.

★ Minimum - Phase System and Nonminimum phase System

Minimum-phase System

⇒ System with minimum-phase transfer function.

⇒ TF having neither poles nor zero in the right half of S-plane are minimum phase TF.

Non Minimum-phase system.

⇒ System with non-minimum phase TF.

⇒ TF having poles or/and zeros in the right half of S-plane are Non-Minimum-Phase TF.

// For a System with the Same magnitude characteristic, the range in phase angle of the minimum-phase TF is minimum among all such System while the range in phase angle of any non-minimum phase TF is greater than this minimum.

★ Transport Lag (dead time)

↳ It is of nonminimum phase behavior and has an excessive phase lag with no attenuation at high frequencies.

⇒ Transport lag normally exist in thermal, hydraulic and pneumatic systems.

⇒ Consider a transport lag given by

$$G(j\omega) = e^{-j\omega T}$$

→ Magnitude is always equal to 1.

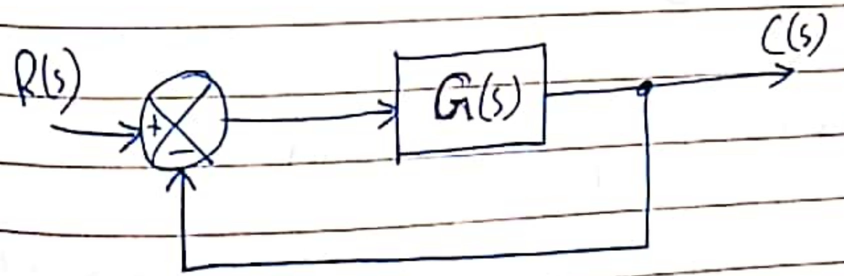
$$|e^{-j\omega T}| = 1$$

→ The phase angle,

$$\angle e^{-j\omega T} = -\omega T \quad (\text{radians})$$

★ Relationship between System Type & Log-Magnitude Curve

i) Determination of Static position Error Constant



Let $G(s) = \dfrac{K(T_a s+1)(T_b s+1)\cdots(T_m s+1)}{s^N (T_1 s+1)(T_2 s+1)\cdots(T_p s+1)}$

$$G(j\omega) = \frac{K(T_a j\omega+1)(T_b j\omega+1)\cdots(T_m j\omega+1)}{(j\omega)^N (T_1 j\omega+1)(T_2 j\omega+1)\cdots(T_p j\omega+1)}$$

$$\lim_{\omega\to 0} G(j\omega) = K = K_p \quad \{\text{for type zero system } N=0\}$$

ii) **Determination of Static Velocity Error Constant**

K_v

(iii) **Determination of static acceleration error Constant**

$$K_a = \omega_a^2 \quad \{\text{for intersection}\}$$

For type 1 System

$$G(j\omega) = \frac{K_v}{j\omega} \quad \forall\ \omega \ll 1$$

$$20\log\left|\frac{K_v}{j\omega}\right|_{\omega=1} = 20\log K_v$$

for intersection $\left|\dfrac{K_v}{j\omega}\right| = 1 \Rightarrow K_v = \omega_1$

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* Polar plot {Nyquist plot}

"The polar plot of a sinusoidal transfer function $G(j\omega)$ is plot of the magnitude of $G(j\omega)$ Vs phase angle of $G(j\omega)$ on polar coordinate. as ω is varied from zero to infinity"

→ Thus polar plot is the locus of vector $|G(j\omega)| \angle G(j\omega)$ as ω is varied from zero to infinity.

Note
- → Negative phase angle is measured clock wise.
- → Positive phase angle as anti-clockwise.

⇒ In the polar plot it is important to show the frequency graduation of the locus.

Advantage: It depicts the frequency-response characteristics of a system over the entire frequency range in a single plot.

Disadvantage: Dose not clearly indicates the contribution of each individual factor of the open-loop TF

Integral and Derivative factor

$\Rightarrow \frac{1}{j\omega} = -\frac{1}{\omega} j = \frac{1}{\omega} \angle -90$

So polar plot is negative imaginary axis.

$\Rightarrow j\omega = \omega \angle 90$

So polar plot is positive imaginary axis.

First Order factor $(1+j\omega T)^{\mp 1}$

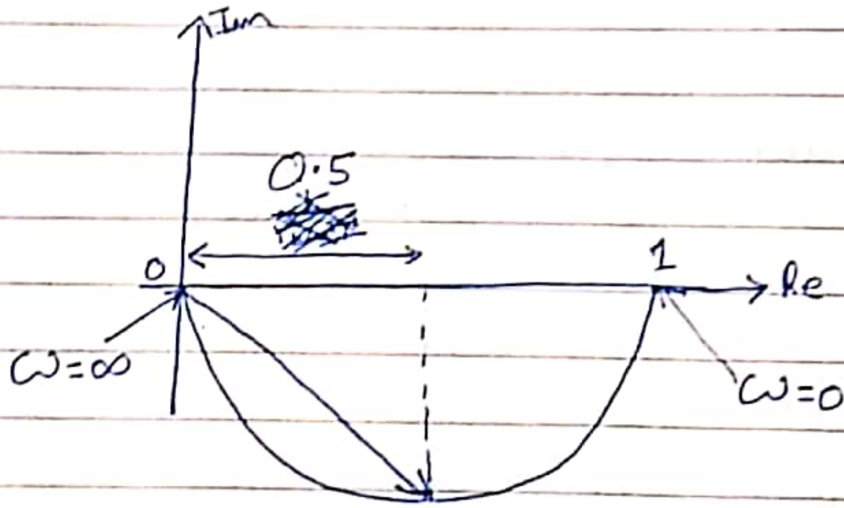
$\Rightarrow G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2 T^2}} \angle -\tan^{-1}\omega T$

1) $\omega = 0$

$1 \angle 0$

2) $\omega = \frac{1}{T}$

$\frac{1}{\sqrt{2}} \angle -45$



$\Rightarrow$ The polar plot of this Transfer function is a Semicircular as the frequency ω Varies from zero to infinity.

⇒ To prove that polar plot of the first-order factor $G(j\omega) = 1/(1+j\omega T)$ is a semicircle define,

$$G(j\omega) = X + jY$$

$$X = \frac{1}{1+\omega^2T^2} \qquad Y = \frac{-\omega T}{1+\omega^2T^2}$$

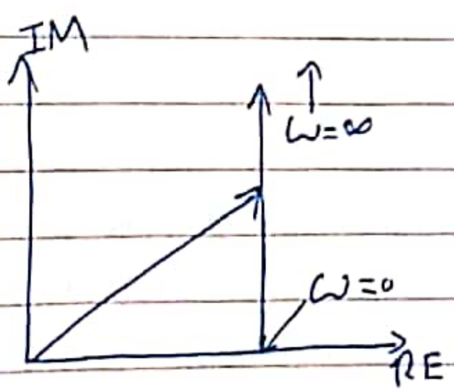
$$\left(X - \frac{1}{2}\right)^2 + Y^2 = \left(\frac{1}{1+\omega^2T^2} - \frac{1}{2}\right)^2 + \left(\frac{-\omega T}{1+\omega^2T^2}\right)^2 = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \boxed{\left(X - \frac{1}{2}\right)^2 + Y^2 = \left(\frac{1}{2}\right)^2}$$ {Hence semicircular at center $x = \frac{1}{2}$ and radius $\frac{1}{2}$}

⇒ The lower semicircle corresponds to $0 \leq \omega \leq \infty$ and the upper semicircle corresponds to $-\infty \leq \omega \leq 0$.

$$1 + j\omega T = \sqrt{1+\omega^2T^2} \angle \tan^{-1}\omega T$$

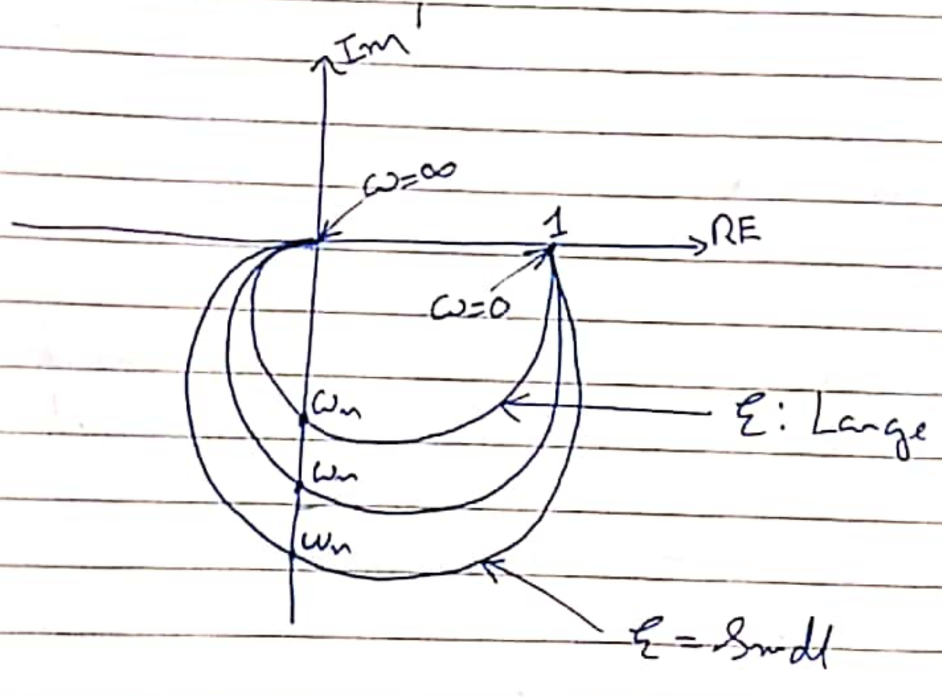
⇒ The polar plot of the transfer function $1+j\omega T$ is simply the upper half of the straight line passing through point $(1,0)$ in the complex plane and ∥ to imaginary axis as shown.



Quadratic Factor $\left[1 + 2\xi(j\omega/\omega_n) + (j\omega/\omega_n)^2\right]^{\mp 1}$

$G(j\omega) = \dfrac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$ $\quad \forall\ \xi > 0$

$\lim_{\omega \to \infty} G(j\omega) = 1\angle 0 \qquad \lim_{\omega \to \infty} G(j\omega) = 0\ \angle -180$



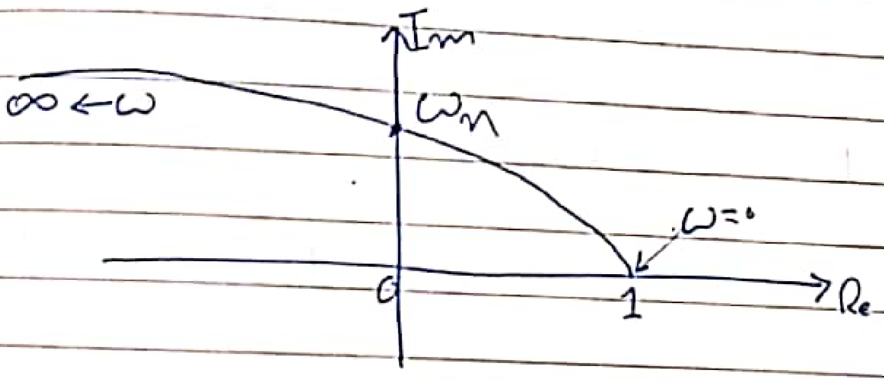
⇒ The exact shape of the polar plot depends on the value of ξ but the general shape of the plot is the same for both the underdamped case $(1 > \xi > 0)$ and over damped case $(\xi > 1)$.

⇒ In the polar plot, the frequency point whose distance from the origin is maximum corresponds to the resonant frequency ω_r.

⇒ For the overdamped case, as ξ increases well beyond unity, the $G(j\omega)$ locus approaches a semicircle.

$$G(j\omega) = 1 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2$$

$$\lim_{\omega \to 0} G(j\omega) = 1\angle 0 \qquad \lim_{\omega \to \infty} G(j\omega) = \infty \angle 180$$



★ <u>General Shape of Polar plot</u>

$$G(j\omega) = \frac{K(1+j\omega T_a)(1+j\omega T_b)\cdots}{(j\omega)^\lambda(1+j\omega T_1)(1+j\omega T_2)\cdots}$$

$$= \frac{b_0(j\omega)^m + b_1(j\omega)^{m-1} + \cdots}{a_0(j\omega)^n + a_1(j\omega)^{n-1} + \cdots}$$

{where $n > m$}

1. For $\lambda = 0$ or type 0 System

→ Starting point finite k is on positive real axis.

→ Tangent to polar plot at $\omega = 0$ is ⊥ to real axis.

→ The terminal point, which corresponds to $\omega = \infty$, is at origin, and the curve is tangent to one of the axis.

2. For $\lambda = 1$ or type 1 System.

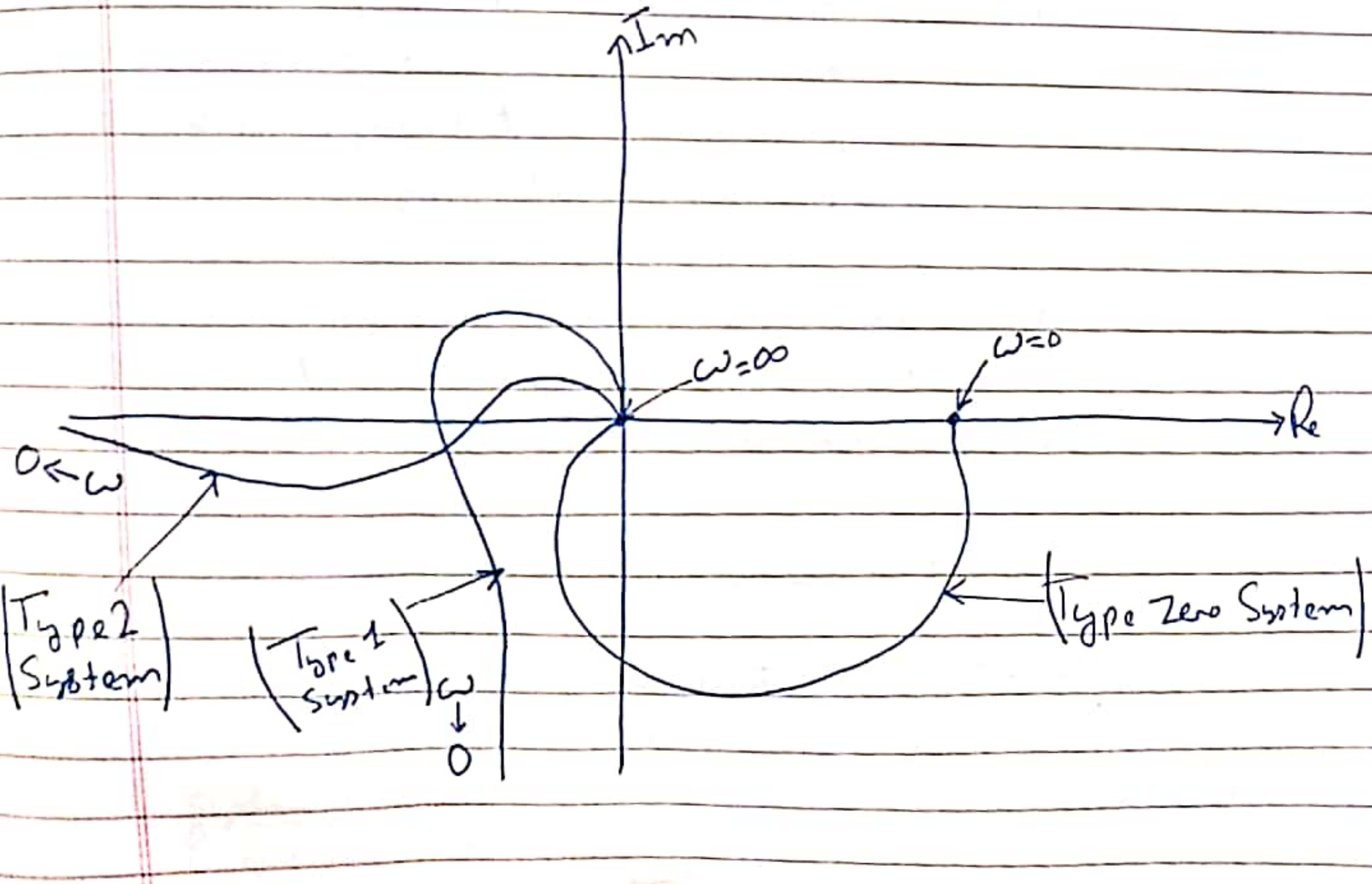
→ At $\omega = 0$ the magnitude of $G(j\omega)$ is ∞ and the phase angle become $-90°$.

→ At low frequency, the polar plot is asymptotic to a line parallel to the negative imaginary axis.

→ At $\omega = \infty$, the magnitude becomes zero, and the curve converges to the origin and is tangent to one of the axis.

3. For $\lambda = 2$ or type 2 System

- At $\omega = 0$, the magnitude of $G(j\omega)$ is ∞ and the phase angle is equal to $-180°$.
- At low frequency, the polar plot may be asymptotic to the negative real axis.
- At $\omega = \infty$, the magnitude becomes zero, and the Curve is tangent to one of the axes.



★ Log Magnitude-Versus Phase plot (Nichols plot)

↳ Plot of the logarithm magnitude in decibels Verses the phase angle or phase margin for a frequency range of interest.

⇒ Phase margin is the difference between actual phase angle φ and $-180°$; that is $\varphi - (-180) = 180 + \varphi$.

⇒ The Curve is graducted in terms of the frequency ω.

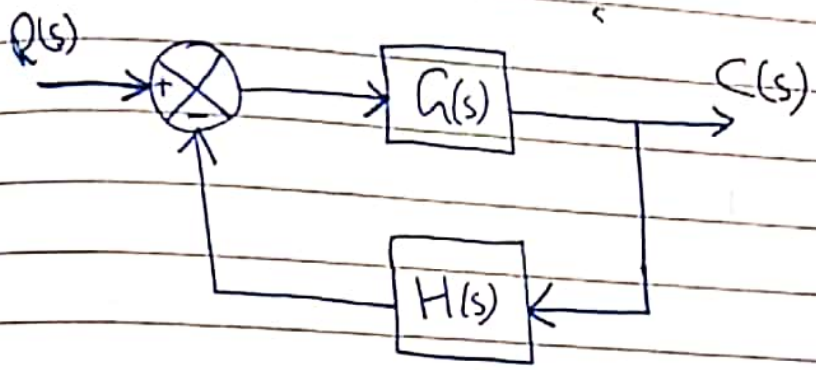
Advantage ⇒ The relative stability of the closed loop System can be determined quickly and that Compensation can be worked out easily.

⇒ The log-magnitude-Versus phase plot for the Sinusoidal transfer function $G(j\omega)$ and that for $1/G(j\omega)$ are skew symmetrical about the origin

$$\left|\frac{1}{G(j\omega)}\right| = -|G(j\omega)|$$

★ Nyquist Stability Criterion

⇒ The Nyquist Stability Criterion determines the stability of a closed-loop system from its frequency responce and open loop poles.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

For stability all roots of $\{1 + G(s)H(s) = 0\}$ must lie in the left-half s plane.

⇒ The Nyquist Stability Criterion relates the open-loop frequency responce $G(j\omega)H(j\omega)$ to the number of zeros and poles of $1 + G(s)H(s)$ that lie in the right half s plane.

⇒ This criterion is useful in control engineering because the absolute stability of the closed-loop system can be determined graphically from open-loop frequency response curve, and there is no need for actually determining the closed-loop poles.

⇒ The Nyquist stability criterion is based on a theorem from the theory of complex variable. To understand the criterion, we shall first discuss mappings of contours in the complex plane.

★ <u>Preliminary Study</u>

$$F(s) = 1 + G(s)H(s) = 0$$

⇒ For a given continuous closed path in the s plane that does not go through any singular points, there corresponds a closed curve in F(s) plane.

⇒ The number and direction of encirclement of the origin of ~~the origin of~~ the F(s) plane by the closed curve plays an important role in determining stability of system

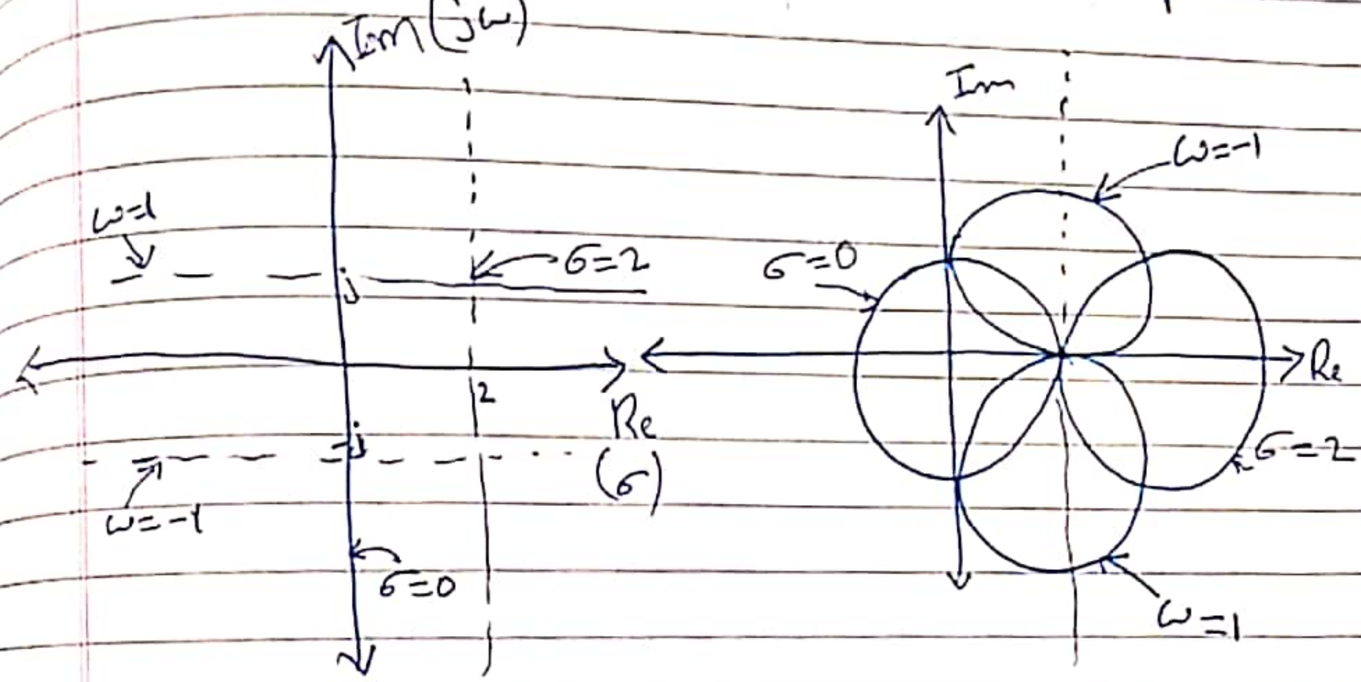
⇒ Consider, for example, the following open loop transfer function:

$$G(s)H(s) = \frac{2}{s-1}$$

So $F(s) = 1 + \frac{2}{s-1} = \frac{s+1}{s-1} = 0$

⇒ The function $F(s)$ is analytic everywhere in the s plane except at its singular points.

↳ For each point of analyticity in the S plane, there corresponds a point in the $F(s)$ plane.



S plane F(s) plane

$$\text{For } F(s) = \frac{s+1}{s-1}$$

⇒ ~~If the~~ Suppose that representative point S trace out a contour in the S plane in the <u>clockwise direction</u>.

→ If the contour in the s plane encloses the <u>pole of $F(s)$</u> ~~in the counterclockwise direction~~, there is one encirclement of the origin of the $F(s)$ plane by the locus of $F(s)$ in <u>counterclockwise direction</u>.

Mapping Theorem

- If the Contour in the S plane encloses the zero of F(s), there is one encirclement of the origin of the F(s) plane by the locus of F(s) in the clockwise direction.
- If the contour in the S plane encloses both the zero and the pole or if the Contour encloses neither the zero nor the pole, then there is no encirclement of the origin of F(s) plane by the locus of F(s).

Note: Location of a pole or zero in the S plane, whether in the right-half or left half s plane, does not make any difference, but the enclosure of a pole or zero does.

★ **Mapping Theorem** {Cauchy's argument principle}

"Let F(s) be a ratio of two polynomials in s. Let P be the number of poles and Z be the number of zeros of F(s) that lie inside some closed contour in the s plane, with multiplicity of poles and zeros accounted for. Let the contour be such that it does not pass through any poles or zeros of F(s).

This closed contour in the s plane is then mapped into the F(s) plane as a closed curve. The total number N of clockwise encirclements of the origin of the F(s) plane, as a representative point s traces out the entire contour in the clockwise direction is equal to Z−P //

⇒ In mapping theorem, the numbers of zeros and of poles cannot be found − only their difference //.

★ <u>Application of the Mapping Theorem to the Stability Analysis of Closed loop System</u>

⇒ For analyzing the stability of linear control systems, we let the closed contour in the s plane enclosing the entire right half s-plane.

↳ The contour consists of the entire jω axis from ω = −∞ to +∞ and a semicircular path of infinite radius in the right half s-plane.

↳ Such contour is called Nyquist path.

⇒ It is necessary that the closed contour on the Nyquist path, not pass through any zero and poles.

$$\lim_{s \to \infty} [1 + G(s)H(s)] = \text{Constant.}$$

⇒ The function $1 + G(s)H(s)$ remains constant as 'S' traverse the semicircle of infinite radius.

⇒ Because of this whether the locus of $1 + G(s)H(s)$ encircles the origin of the $1 + G(s)H(s)$ plane can be determined by considering only a part of the closed contour in the S plane — that is the $j\omega$ axis.

↳ Encirclement of the origin, if there are any occur only while a representative point moves from $-j\infty$ to $+j\infty$ along the $j\omega$ axis, provided that no zeros or poles lie on the $j\omega$ axis.

⇒ Note that the portion of the $1 + G(s)H(s)$ contour from $\omega = -\infty$ to $\omega = \infty$ is simply $1 + G(j\omega)H(j\omega)$

⇒ Since $1 + G(j\omega)H(j\omega)$ is the vector sum of the unit vector and vector $G(j\omega)H(j\omega)$, $1 + G(j\omega)H(j\omega)$ is identical to the vector drawn from the $-1 + j0$ point to the terminal point of the vector $G(j\omega)H(j\omega)$.

⇒ Encirclement of the origin by the graph of $1 + G(j\omega)H(j\omega)$ is equivalent to encirclement of the $-1+0j$ point by just the $G(j\omega)H(j\omega)$ locus.

↳ Thus, the stability of a closed loop system can be investigated by examining encirclement of the $-1+0j$ point by the locus of $G(j\omega)H(j\omega)$

⇒ Plotting $G(j\omega)H(j\omega)$ for the Nyquist path is straight forward.

↳ Plot of $G(j\omega)H(j\omega)$ and the plot of $G(-j\omega)H(-j\omega)$ are symmetrical with each other about the real axis.

* <u>Nyquist Stability Criterion</u> {when $G(s)H(s)$ has neither poles nor zeros on $j\omega$ axis}

"In a system if the open loop transfer function $G(s)H(s)$ has K poles in the right-half 's'-plane and $\lim_{s \to \infty} G(s)H(s) = \text{constant}$, then for stability, the $G(s)H(s)$ locus, as ω varies from $-\infty$ to ∞, must encircle the $-1+j0$ point K times in the counterclockwise direction"

★ Remarks on Nyquist Stability Criterion

1. The Criterion Can be expressed as

$$Z = N + P$$

where Z = Number of zeros of $1+G(s)H(s)$ in Right half s plane
N = number of clockwise encirclements of the $-1+0j$ point
P = number of poles of $G(s)H(s)$ in the right half s plane.

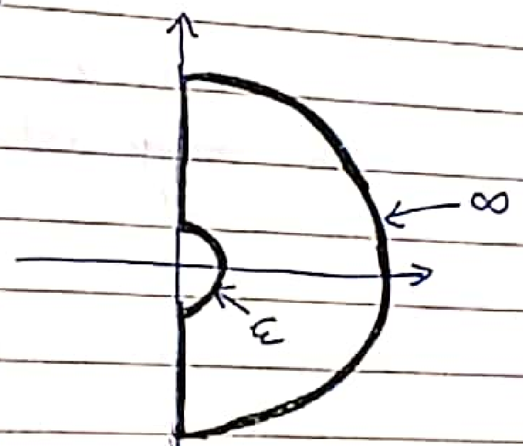
If P is not zero, for a stable control system, we must have $Z=0$, or $N=-P$, which means that we must have P counterclockwise encirclement of the $-1+0j$ point.

2. If the locus of $G(j\omega)H(j\omega)$ passes through the $-1+j0$ point, then zeros of the characteristics equation, or closed-loop poles, are located on the $j\omega$ axis. This is not desirable for practical control system. For a well-designed closed-loop system none of the roots of the characteristics equation should lie on the $j\omega$ axis.

★ Special Case when G(s)H(s) Involves Poles and/or Zeros on the jω Axis

⇒ If function G(s)H(s) has poles or zeros at the origin (or on jω axis at points other than origin), the contour in the S plane must be modified.

↳ The usual way of modifying the contour near the origin is to use a semicircle with the infinitesimal radius ε.



⇒ The semicircle may lie in the right half S plane or in the left half S-plane.

⇒ Consider for example, a closed-loop system whose open-loop transfer function is given by

$$G(s)H(s) = \frac{K}{S(TS+1)}$$

⇒ The points corresponding to $S=j0_+$ and $S=j0_-$ on the locus of G(s)H(s) in the G(s)H(s) plane are $-j\infty$ and $+\infty j$ respectively.

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⇒ On the Semicircular path with radius ε (where ε≪1), the complex variable s can be written

$$S = \varepsilon e^{j\theta} \quad \left\{ \text{where } \theta \text{ varies from } -\frac{\pi}{2} \text{ to } +\frac{\pi}{2}. \right\}$$

⇒ Then G(s) H(s) become

$$G(\varepsilon e^{j\theta}) H(\varepsilon e^{j\theta}) = \frac{K}{\varepsilon e^{j\theta}} = \frac{K}{\varepsilon} e^{-j\theta}$$

⇒ The infinitesimal Semicircle detour around the origin in the S plane maps into the GH plane as a semicircle of infinite radius.

⇒ For an open-loop transfer function G(s) H(s) involving $1/s^n$ factor, the plot of G(s) H(s) has n clockwise semicircle of infinite radius about the origin as a representative point S moves along the semicircle of radius ε.

★ <u>Nyquist Stability Criterion</u> {for general case when G(s) H(s) has poles &/or zeros on jω axis}

// In a system if the open-loop transfer function G(s) H(s) has K poles in the right half s plane, then for stability the G(s) H(s) locus, as a representative point s traces on the modified Nyquist path in the clockwise direction must encircle the $-1+j0$ point K times in the Counter clockwise direction//

* Stability Analysis

⇒ In examining the Stability of linear Control System using the Nyquist stability Criterion, we see that three possibilities can occure:-

1. There is no encirclement of the $-1+j0$ point. This implies that the System is stable if there are no poles of $G(s)H(s)$ in the Right half s-plane; otherwise, the System is unstable.

2. There is One or more Counterclockwise encirclements of the $-1+j0$ point. In this case the System is Stable if the number of Counterclockwise encirclements is the Same as the number of poles of $G(s)H(s)$ in the right-half s plane; otherwise the System is unstable.

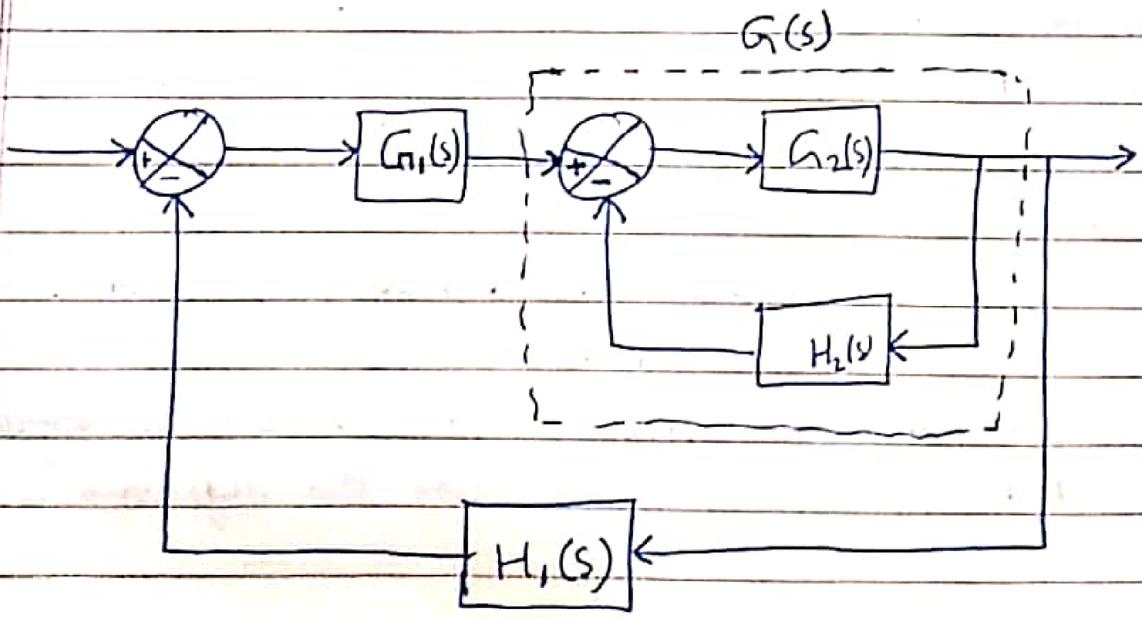
3. There are one or more clockwise encirclements of the $-1+0j$ point. In this case the system is unstable

★ Conditionally Stable System

⇒ A conditionally stable system is stable for the value of the open-loop gain lying between critical values, but it is unstable if the open loop gain is either increased or decreased sufficiently.

↳ Such system becomes unstable when large input signals are applied, since a large signal may cause saturation, which in turn reduces the open loop gain of the system. It is advisable to avoid such a situation.

★ Multiple-Loop System



$$G(s) = \frac{G_2(s)}{1 + G_2(s)H_2(s)}$$

⇒ Since the open-loop transfer function of the entire system is given by $G_1(s)\,G(s)\,H_1(s)$, the stability of this closed loop system can be found from the Nyquist plot of $G_1(s)\,G(s)\,H_1(s)$ and knowledge of the right-half plane poles of $G_1(s)\,G(s)\,H_1(s)$.

★ Nyquist Stability Criterion Applied to Inverse Polar plot

⇒ In analysing multiple-loop system, the inverse transfer function may sometimes be used in order to permit graphical analysis; this avoids much of the numerical calculation.

⇒ The inverse polar plot of $G(j\omega)H(j\omega)$ is a graph of $1/[G(j\omega)H(j\omega)]$ as a function of ω.

Nyquist Stability Criterion {Applied to inverse plots}

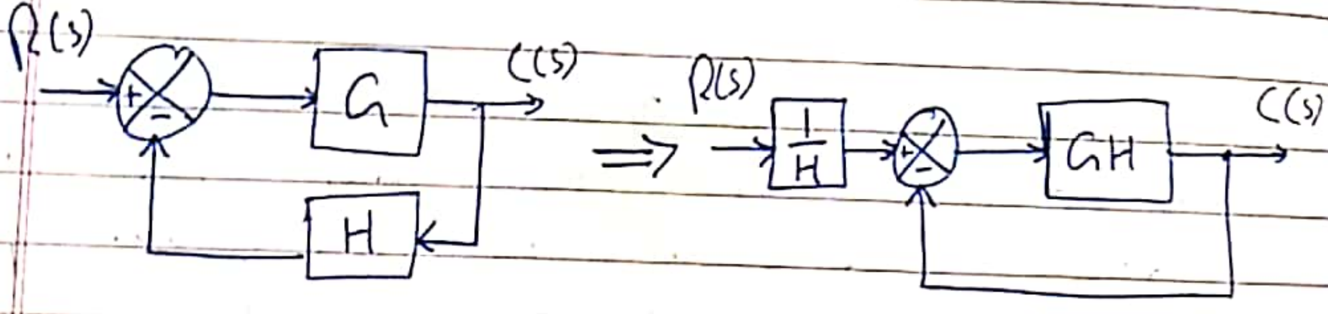
" For a closed-loop system to be stable, the encirclement, if any of the $-1+0j$ point by the $1/G(s)H(s)$ locus must be counterclockwise and the number of such encirclements must be equal to number of poles of $1/G(s)H(s)$ that lie in the right-half s plane."

★ Relative Stability Analysis

Relative stability ⇒ In designing a Control System, we require that the system be stable. Furthermore, it is necessary that the system have adequate relative stability !!

⇒ In analysis we shall assume that the systems considered have unity feedback.

↳ It is always possible to reduce a system with feedback element to a unit feedback system.



⇒ We shall also assume that, unless otherwise stated, the systems are minimum-phase systems; that is, the open-loop transfer function have neither poles nor zeros in the right half s plane.

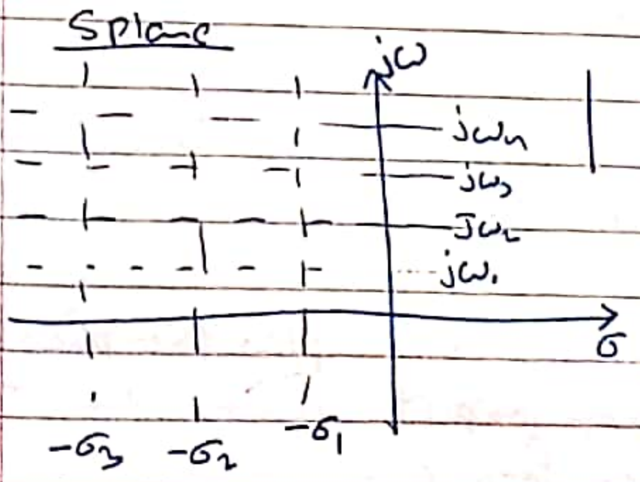
★ Relativity stability ~~and~~ analysis by Conformal Mapping

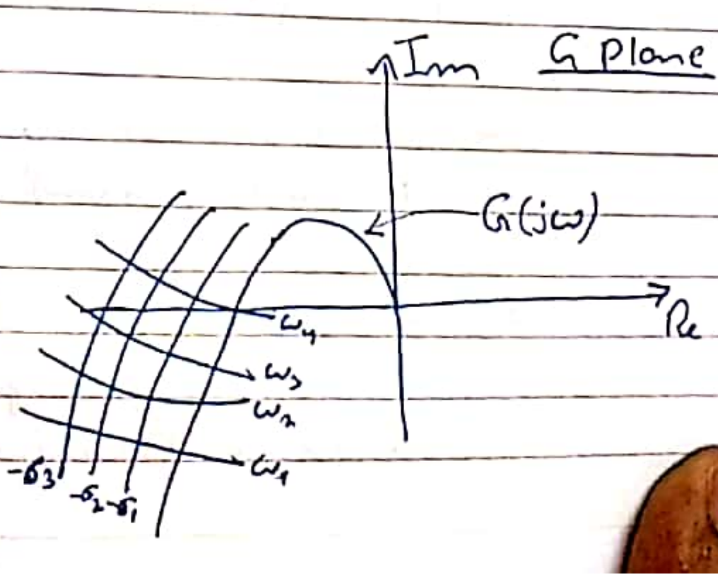
⇒ One of the important problems in analysing a control System is to find all closed-loop poles or atleast those closest to the jω axis.

⇒ If the open loop frequency response characteristics of a system are known, it may be possible to estimate the closed loop poles closest to the jω axis.

⇒ Consider the Conformal mapping of constant-σ lines (lines σ+jω, where σ is constant and ω varies) and constant-ω lines (lines s = σ+jω, where ω = constant and σ varies) in the s plane.

⇒ The constant σ lines in the s plane map into curves that are similar to the Nyquist plot and in a sense parallel to the Nyquist plot.





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⇒ The Closeness of approach of the $G(j\omega)$ locus to the $-1+j0$ point is an indication of the relative stability of a stable system.

↳ The closer the $G(j\omega)$ locus is to the $-1+j0$ point, the larger the maximum overshoot is in the step transient response and the longer it takes to damp out.

★ Phase and Gain Margins

⇒ In general, the closer the $G(j\omega)$ locus comes to encircling the $-1+j0$ point, the more oscillatory is the system responce.

Phase margin: The phase margin is that amount of additional phase lag at the gain crossover frequency required to bring the system to the verge of instability.

⇒ The gain crossover frequency is the frequency at which $|G(j\omega)|$ is unity.

⇒ The phase margin γ is $180°$ plus the phase angle ϕ of the open loop transfer function at the gain crossover frequency.

$$\boxed{\gamma = 180^\circ + \phi}$$

⇒ For a minimum phase system to be stable, the phase margin must be positive.

Gain margin: The gain margin is the reciprocal of the magnitude of the frequency at which phase angle is -180°.

$$K_g = \frac{1}{|G(j\omega_1)|}$$ → Phase Crossover frequency

⇒ In terms of decibels,

$$K_g\,dB = 20 \log K_g = -20 \log |G(j\omega_1)|$$

⇒ Positive gain margin (in decibels) means that the system is stable, and a negative gain margin (in decibels) means that the system is unstable.

⇒ For a stable Minimum-phase system, the gain margin indicates how much the gain can be increased before the system becomes unstable.

⇒ For an unstable system the gain margin indicates how much gain must be decreased to make the system stable.

⇒ The gain margin of a first or second order system is infinite since the polar plot for such system do not cross the negative real axis.

For a nonminimum-phase system with unstable open loop the stability condition will not be satisfied unless the $G(j\omega)$ plot encircles the $-1+0j$ point.

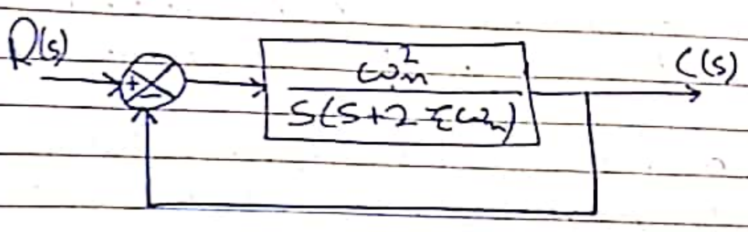
↳ Hence, such a stable nonminimal-phase system will have negative phase & gain margins.

Conditionally stable system and some higher-order system with complicated numerator dynamics may also have two or more gain crossover frequencies, the phase margin is measured at the highest gain crossover frequency.

⇒ Either the gain margin alone or the phase margin alone does not give a sufficient indication of the relative stability. Both should be given in the determination of relative stability.

⇒ For satisfactory performance, the phase margin should be between 30° and 60° and the gain margin should be greater than 6dB.

★ Resonant Peak Magnitude M_r and Resonant Frequency ω_r



$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = Me^{j\alpha}$$

$$M = \frac{1}{\sqrt{\left(1-\frac{\omega}{\omega_n}\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}} \qquad \alpha = -\tan^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}$$

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

resonant frequency

Resonant Peak Magnitude

$$M = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

⇒ The magnitude of the resonant peak gives an indication of the relative stability of the system.

↳ A large resonant peak indicates the presence of a pair of dominant closed-loop poles with small damping ratio, which will yield an undesirable transient response.

⇒ In practical design problems the phase margin and gain margin are more frequently specified than the resonant peak magnitude to indicate the degree of damping ~~of~~ in a system.

★ Correlation between Step transient Response and Frequency Response in the Standard Second Order System

⇒ The maximum overshoot in the unit-step response of the standard second order system can be exactly correlated with the resonant peak magnitude in the frequency response.

$$C(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$M_p = e^{-(\xi/\sqrt{1-\xi^2})\pi} \quad \left\{\text{Maximum overshoot}\right\}$$

⇒ This maximum overshoot occurs in the transient response that has the damped natural frequency $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$G(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)} \quad \left\{\begin{array}{l}\text{Open-loop transfer function}\\ \text{of second order system}\end{array}\right\}$$

for sinusoidal operation, the magnitude of $G(j\omega)$ becomes unity when

$$\omega = \omega_n \sqrt{\sqrt{1+4\xi^4} - 2\xi^2}$$

At this frequency phase angle of $G(j\omega)$ is

$$\angle G(j\omega) = -90 - \tan^{-1} \frac{\sqrt{\sqrt{1+4\xi^4} - 2\xi^2}}{2\xi}$$

So $\gamma = 180 + \angle G(j\omega)$

$$\boxed{\gamma = \tan^{-1} \frac{2\xi}{\sqrt{\sqrt{1+4\xi^4} - 2\xi^2}}}$$

★ Correlation between step transient Responce and Frequency Responce in Control System

⇒ The design of Control System is very often carried out on the basis of the frequency response.

↳ The main reason for this is the relative Simplicity of this approach compared with others.

1. The Value of M_r is indicative of the relative stability.

   ↳ Satisfactory transient performance is usually obtained if the value of M_r is in the range $1.0 < M_r < 1.4$ ($0\,dB < M_r < 3\,dB$), which corresponds to an effective damping ratio $0.4 < \xi < 0.7$.

2. The magnitude of the resonant frequency ω_r is indicative of the speed of the transient response.

   ↳ Rise time is inversely with ω_r.

3. The resonant peak frequency ω_r and the damped natural frequency ω_d for the step transient response are very close to each other for lightly damped system.

★ Cutoff Frequency and Bandwidth

The frequency ω_b at which the magnitude of the closed-loop frequency response is 3dB below its zero-frequency value is called the Cutoff frequency.

⇒ The closed-loop system filters out the signal components whose frequencies are greater than the cutoff frequency and transmits those signal components with frequencies lower than the cutoff frequency.

The frequency range $0 \leq \omega \leq \omega_b$ in which the magnitude of $C(j\omega)/R(j\omega)$ is greater than $-3dB$ is called the Bandwidth of the System.

↳ Bandwidth indicates the frequency where the gain starts to fall off from its low-frequency value.

⇒ Rise time & the bandwith ~~decreases with the increase in ξ~~ are inversely proportional to each other.

Cutoff Rate ⇒ It is the slope of the log-magnitude curve near the cutoff frequency.

↳ The cutoff rate indicates the ability of a system to distinguish the signal from noise.

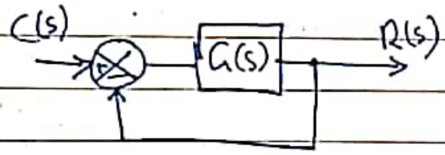
★ Closed-Loop Frequency Responce of unity feedback System

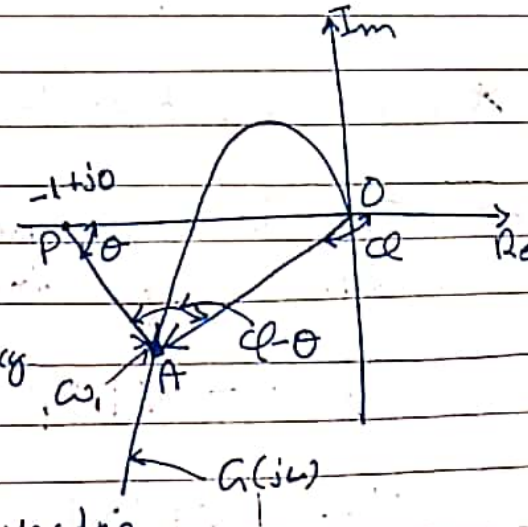
★ Closed loop Frequency Response

⇒ For a stable, unity-feedback closed-loop system, the closed-loop frequency response can be obtained easily from that of the open-loop frequency responce.

⇒ Consider a unity-feedback system ÷

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$





⇒ The vector $\vec{OA}$ represents $G(j\omega_1)$, where ω_1 is the frequency at point A.

⇒ The vector $\vec{PA}$, the vector from the $-1+j0$ point to the Nyquist locus, represents $1+G(j\omega_1)$.

⇒ Therefore ratio of $\vec{OA}$ to $\vec{PA}$ represents the closed loop frequency response.

$$\frac{\vec{OA}}{\vec{PA}} = \frac{G(j\omega_1)}{1+G(j\omega_1)} = \frac{C(j\omega_1)}{R(j\omega_1)}$$

⇒ The magnitude of the closed loop transfer function at $\omega=\omega_1$ is the ratio of the magnitudes of $\vec{OA}$ to $\vec{PA}$.

→ The phase angle of the closed loop transfer function at $\omega=\omega_1$ is the angle formed by the vectors $\vec{OA}$ to $\vec{PA}$ (ie $\phi-\theta$).

⇒ Let us define the magnitude of the closed loop frequency response as M and the phase angle as α.

$$\frac{C(j\omega)}{R(j\omega)} = Me^{j\alpha}$$

★ Constant-Magnitude Loci (M circle)

⇒ $G(j\omega)$ is a complex quantity and can be written as following:

$$G(j\omega) = X + jY \quad \{X \,\&\, Y \text{ are real quantity}\}$$

$$M = \frac{|X+jY|}{|1+X+jY|} \Rightarrow M^2 = \frac{X^2+Y^2}{(1+X)^2+Y^2}$$

$$\Rightarrow X^2(1-M^2) - 2M^2X - M^2 + (1-M^2)Y^2 = 0$$

If $M=1 \Rightarrow X = -\frac{1}{2}$ {Straight line parallel to Y-axis passing through the Point $(-\frac{1}{2}, 0)$}

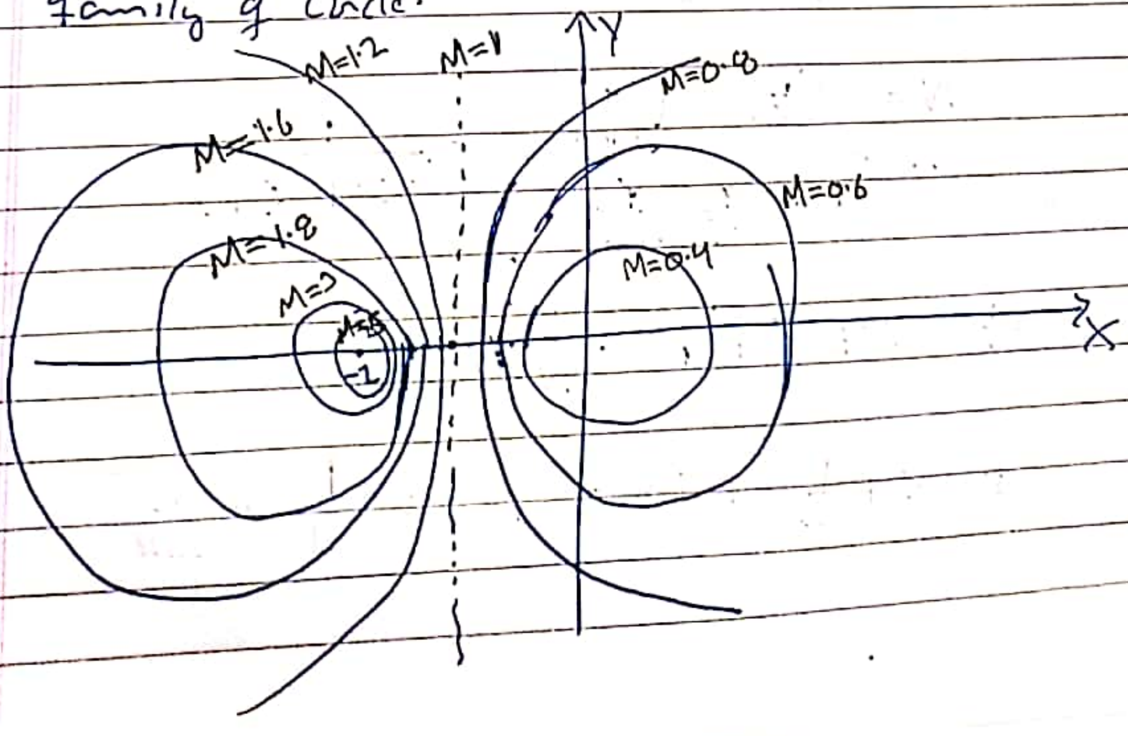
If $M \neq 1$

$$X^2 + \frac{2M^2}{M^2-1}X + \frac{M^2}{M^2-1} + Y^2 = 0$$

$$\Rightarrow \boxed{\left(X + \frac{M^2}{M^2-1}\right)^2 + Y^2 = \frac{M^2}{(M^2-1)^2}}$$

$\Rightarrow$ Above equation is of a Circle with center at $\left(\frac{-M^2}{M^2-1}, 0\right)$ and radius $\left|\frac{M}{M^2-1}\right|$.

$\Rightarrow$ The Constant M loci on the G(s) plane are thus a family of Circle.



⇒ It is seen that as M become larger compared with 1, the M circles ~~tie to the left of the -1+j0 point~~ become smaller and converge to the -1+j0 point.

⇒ Similarly as M become smaller compared with 1, the M circles ~~tie to~~ becomes smaller and converges to the origin.

⇒ The M circles are symmetrical with respect to the straight line corresponding to M=1 and with respect to real axis.

★ Constant-Phase-Angle Loci (N circle)

$$\angle e^{j\alpha} = \angle \frac{X+jY}{1+X+jY}$$

$$\alpha = \tan^{-1}\left(\frac{Y}{x}\right) - \tan^{-1}\left(\frac{Y}{1+X}\right)$$

Let $N = \tan\alpha$

$$N = \frac{\left(\frac{Y}{X}\right) - \left(\frac{Y}{1+X}\right)}{1 + \frac{Y}{X}\left(\frac{Y}{1+X}\right)} = \frac{Y}{X^2+X+Y^2}$$

$$\Rightarrow X^2 + X + Y^2 - \frac{1}{N}Y = 0.$$

$$\Rightarrow \left(X+\frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

⇒ This is ... $\left(-\frac{1}{2}, \ldots\right.$

⇒ Abou... $X = -1$ ... pass...

⇒ Con... ce...

⇒ Th... th... th... C... -

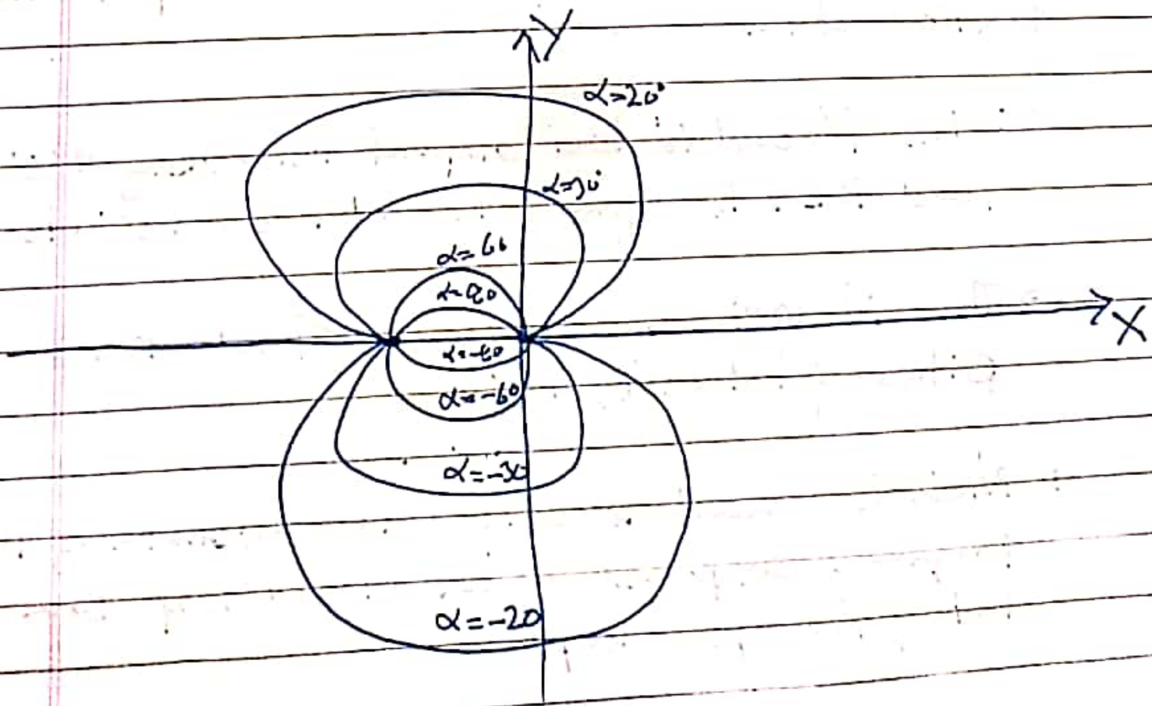
⇒ This is an equation of circle with center at $\left(-\frac{1}{2}, \frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4}+\frac{1}{(2N)^2}}$.

⇒ Above equation is satisfied when $X=Y=0$ and $X=-1, Y=0$ regardless of the value of N, each circle passes through the origin and the $-1+0j$ point.

⇒ Constant N locus for a given value of α is not actually the entire circle but only an arc.

{ i.e $\alpha = 30$ & $\alpha = -150$ are a part of same circle }

⇒ The use of the M & N circle enables us to find the entire closed-loop frequency response from the open-loop frequency response $G(j\omega)$ without calculating the magnitude and phase of the closed-loop transfer function at each frequency.



⇒ Graphically, the intersections of the G(jω) locus and M circle give the values of M at the frequencies denoted on the G(jω) locus.

⇒ The resonant peak value is the value of M corresponding to the M circle of smallest radius that is tangent to the G(jω). locus

★ Nichols Chart

⇒ In dealing with design problems, we find it convenient to construct the M & N loci in the Nichols plot.

↳ The chart is called Nichols chart.

⇒ The G(jω) locus drawn on the Nichols chart gives both the gain characteristic & phase characteristics of the closed-loop transfer function at the same time.

→ The M and N loci repeat for every 360° , and there is symmetry at every 180° interval.

→ The M loci are centered about the critical point (0dB, −180°)

→ Nichols chart is useful for determining the frequency response of the closed loop from that of the open loop.

⇒ If the open-loop frequency response curve is superimposed on the Nichols chart, the intersection of the open-loop frequency-response curve $G(j\omega)$ and M and N loci give the value of the magnitude (M) and phase angle α of the closed-loop frequency response at each frequency point.

★ Experimental determination of Transfer Function

⇒ First step in the analysis and design of a Control System is to derive its mathematical model.
↳ Obtaining a model analytically may be quite difficult. We may have to obtain it by means of experimental analysis.

⇒ If the amplitude ratio and phase shift have been measured at a sufficient number of frequencies within the frequency range of interest, they may be plotted on the Bode diagram.
↳ Then the transfer function can be determined by asymptotic approximations.

Sinusoidal signal generators
↳ The Signal may have to be in mechanical, electrical or pneumatic form.

⇒ Frequency range needed:-

- Large time constant: 0.001 Hz to 10 Hz
- Small time Constant: 0.1 Hz to 1000 Hz

Determination of Minimum-Phase Transfer function from Bode Diagrams

⇒ To determine the transfer function experimentally, we first draw asymptotes to the experimentally obtained log magnitude curve.

→ If the slope of the experimentally obtained log-magnitude curve changes from -20dB to -40dB/decade at $\omega = \omega_1$, it is clear that a factor $1/[1 + j\omega/\omega_1]$ exist in the transfer function.

→ If the slope changes by -40 dB/decade at $\omega = \omega_2$, there must be a quadratic factor of the form

$$\frac{1}{1 + 2\xi\left(j\frac{\omega}{\omega_2}\right) + \left(j\frac{\omega}{\omega_2}\right)^2}$$

in the transfer function.

⇒ The undamped natural frequency of this quadratic factor is equal to the corner frequency.

⇒ The damping ratio ξ can be determined from the experimentally obtained log-magnitude curve by measuring the amount of resonant peak near the corner frequency ω_2.

⇒ Once the factors of the transfer function $G(j\omega)$ have been determined, the gain can be determined from the low-frequency portion of the log-magnitude curve.

↳ Since terms $[1 + j(\omega/\omega_1)]$ and $[1 + 2\xi(j\omega/\omega_2) + (\frac{j\omega}{\omega_2})^2]$ become unity at $\omega \to 0$, the sinusoidal transfer function $G(j\omega)$ can be written as:

$$\lim_{\omega \to 0} G(j\omega) = \frac{K}{(j\omega)^\lambda}$$

⇒ In many practical system $\lambda = 1, 2$ or 0.

1. $\lambda = 0$ {Type 0 System}

$$\lim_{\omega \to 0} G(j\omega) = K$$

⇒ Value of K can thus be found from this horizontal asymptote.

2. $\lambda = 1$ {Type 1 System}

$$G(j\omega) = \frac{K}{j\omega}$$

$$20 \log |G(j\omega)| = 20 \log K - 20 \log \omega$$

⇒ The frequency at which the low frequency asymptote intersects the 0-dB line is numerically equal to K.

3. For $\lambda = 2$ {Type 2 System}

$$G(j\omega) = \frac{K}{(j\omega)^2}$$

⇒ The frequency at which this asymptote intersects the 0dB line is numerically equal to $\sqrt{K}$.

⇒ The experimentally obtained phase-angle curve provides a means of checking the transfer function obtained from the log-magnitude curve.

⇒ If the experimentally obtained phase angle at very high frequencies is not equal to $-90(q-p)$, where p & q are the degrees of the numerator and denominator polynomial of the transfer function, respectively then the transfer function must be a nonminimum phase transfer function.

Nonminimum-Phase transfer function

⇒ If at the high frequency end, the computed phase lag is 180° less than the experimentally obtained phase lag, then one of the zeros of the transfer function should have been in the right-half S plane instead of the left half S plane.

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⇒ If the computed phase lag differed from the experimentally obtained phase lag by a constant rate of change of phase, then transport lag or dead time, is present.

↳ We assume the TF of the form

$$G(s)e^{-Ts}$$

where $G(s)$ is a ratio of two polynomial in s.

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