5
EKF SLAM

★ Defination of SLAM Problem

Given

① The robot's Controls

$$u_{1:T} = \{u_1, u_2, u_3 \cdots u_T\}$$

② Observations

$$z_{1:T} = \{z_1, z_2 \cdots z_T\}$$

Wanted

① Map of the Environment

$$m$$

② Path of the robot

$$x_{0:T} = \{x_0, x_1, \cdots x_T\}$$

Only this for Online SLAM

★ EKF Slam

Assumption : Known correspondences

⇒ State space (for 2D plane) is

$$x_t = (x, y, \theta, m_{1,x}, m_{1,y} \cdots m_{n,x}, m_{n,y})^T$$

Pose of robot

Positions of all the landmark

⇒ Map with n landmark : $(3+2n)$-dimensional.

$$\mu = \begin{pmatrix} x \\ m \end{pmatrix} \qquad \Sigma = \begin{pmatrix} \Sigma_{xx} & \Sigma_{xm} \\ \Sigma_{mx} & \Sigma_{mm} \end{pmatrix}$$

★ EKF SLAM : F

1. State Predict
2. Measurement
3. Measurement
4. Data asso
5. Update

★ Assumption

① Robot m
② Velocity -
③ Robot ob
④ Range - b
⑤ Known
⑥ Known

★ Initializa

⇒ Robot st
(all lan

$\mu_0 =$ (0

$\Sigma_0 =$

Scanned by CamScanner

* EKF SLAM : Filter Cycle

1. State prediction — {Prediction step}
2. Measurement prediction
3. Measurement
4. Data association
5. Update

{Correction step}

* Assumptions

① Robot moves in 2D plane
② Velocity-based motion model
③ Robot observes point landmarks (x,y)
④ Range-bearing Sensor
⑤ Known data asssociation
⑥ Known number of landmarks

* Initialization

⇒ Robot starts in its own reference frame (all landmarks unknown)

Step 1

$$\mu_0 = (0, 0, 0 \cdots 0)^T$$

$$\Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \infty & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \infty \end{pmatrix}$$

* Motion model (update step)

⇒ From the motion in the plane

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{V_t}{\omega_t}\sin\theta + \frac{V_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{V_t}{\omega_t}\cos\theta - \frac{V_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

⇒ to the 2N+3 dimensional Space:

$$\underbrace{\begin{pmatrix} x' \\ y' \\ \theta' \\ \vdots \end{pmatrix} = \begin{pmatrix} x \\ y \\ \theta \\ \vdots \end{pmatrix} + \overset{F_x^T}{\begin{pmatrix} \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{\text{Identity } (3\times3)} & \underbrace{\begin{matrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{matrix}}_{2N \text{ Cols}} \end{pmatrix}^T} \begin{pmatrix} -\frac{V_t}{\omega_t}\sin\theta + \frac{V_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ \frac{V_t}{\omega_t}\cos\theta - \frac{V_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}}_{g(u_t,\, x_t)}$$

Step 2

Step 3: $\overline{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

$$G_t = \begin{pmatrix} G_t^x & 0 \\ 0 & I \end{pmatrix}$$

G_t^x → Jacobian of the motion (3×3)

I → Identity (2N×2N)

$$G_t^x = \begin{pmatrix} 1 & 0 & -\frac{V_t}{\omega_t}\cos\theta + \frac{V_t}{\omega_t}\cos(\theta + \omega_t \Delta t) \\ 0 & 1 & -\frac{V_t}{\omega_t}\sin\theta + \frac{V_t}{\omega_t}\sin(\theta + \omega_t \Delta t) \\ 0 & 0 & 1 \end{pmatrix}$$

{for Velocity motion model}

{only change uncertainity in pose not in Landmark}

$$(F_x^T R_t^x F_x)$$

{Uncertainity of the motion model assumed to be given}

⇒ After putting everything in step 3 we get:

$$\bar{\Sigma}_t = \begin{pmatrix} G_t^x \Sigma_{xx} (G_t^x)^T & G_t^x \Sigma_{xm} \\ (G_t^x \Sigma_{xm})^T & \Sigma_{mm} \end{pmatrix} + R_t$$

★ EKF SLAM : Correction Step

→ Known data association.

→ $c_t^i = j$: ith measurement at time t observes the landmark with index j.

→ Initialize landmark if unobserved

→ Compute the expected observation

→ Compute the jacobian of h

→ Proceed with computing the Kalman gain.

⇒ for range-bearing observation model

$$\underbrace{\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix}}_{\text{Observed location of landmark } j} = \underbrace{\begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix}}_{\text{Estimated Robot's Location}} + \underbrace{\begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}}_{\text{Relative Measurement}}$$

Predicted observation

$$z_t^i = (r_t^i, \phi_t^i)^T$$

⇒ Expected Observation

$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix} \qquad \text{Let } q = \delta^T \delta$$

$$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix}$$

$$h(\bar{\mu}_t)$$

* Jacobian for the Observation

$$^{low}H_t^i = \frac{\delta h(\bar{\mu}_t)}{\delta \bar{\mu}_t}$$

low-dim space $(x, y, \theta, m_{j,x}, m_{j,y})$

$$^{low}H_t^i = \frac{1}{q}\begin{pmatrix} -\sqrt{q}\,\delta_x & -\sqrt{q}\,\delta_y & 0 & \sqrt{q}\,\delta_x & \sqrt{q}\,\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{pmatrix}_{2\times 5}$$

$\Rightarrow$ Map it to the high dimensional space

$$H_t^i = {}^{low}H_t^i \, F_{x,j}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{2j-2} & 0 & 1 & \underbrace{0 \cdots 0}_{2N-2j} \end{pmatrix}$$

$\Rightarrow$ So we can now perform all the remaining steps. Step 4, Step 5, Step 6, Step 7

1 EKF_SLAM_Algorithm $(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t, c_t, R_t)$

2 $$F_x = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \underbrace{0 \; \cdots \; 0}_{2N \text{ Colums}} \end{pmatrix}$$

3 $$\bar{\mu}_t = \mu_{t-1} + F_x^T \begin{pmatrix} -\frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta}) + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta}) - \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ \omega_t \Delta t \end{pmatrix}$$

4 $$G_t = I + F_x^T \begin{pmatrix} 0 & 0 & -\frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta}) + \frac{v_t}{\omega_t} \cos(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & -\frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta}) + \frac{v_t}{\omega_t} \sin(\mu_{t-1,\theta} + \omega_t \Delta t) \\ 0 & 0 & 0 \end{pmatrix} F_x$$

5 $$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + \underbrace{F_x^T R_t^x F_x}_{R_t}$$

6 $$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$$

7 for all observed features $z_t^i = (r_t^i, \phi_t^i)^T$ do

10 $$\begin{pmatrix} \bar{\mu}_{j,x} \\ \bar{\mu}_{j,y} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{t,x} \\ \bar{\mu}_{t,y} \end{pmatrix} + \begin{pmatrix} r_t^i \cos(\phi_t^i + \bar{\mu}_{t,\theta}) \\ r_t^i \sin(\phi_t^i + \bar{\mu}_{t,\theta}) \end{pmatrix}$$

8 $j = c_t^i$

9 if landmark j never seen before

11 endif

12 $$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{j,x} - \bar{\mu}_{t,x} \\ \bar{\mu}_{j,y} - \bar{\mu}_{t,y} \end{pmatrix}$$

13 $$q = \delta^T \delta$$

14 $$\hat{z}_t^i = \begin{pmatrix} \sqrt{q} \\ \operatorname{atan2}(\delta_y, \delta_x) - \bar{\mu}_{t,\theta} \end{pmatrix}$$

$$F_{x,j} = \begin{pmatrix} 1 & 0 & 0 & \underbrace{0 \cdots 0} & 0 & 0 & \underbrace{0 \cdots 0} \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{2j-2} & 0 & 1 & \underbrace{0 \cdots 0}_{2N-2j} \end{pmatrix}$$

$$H_t^i = \frac{1}{q} \begin{pmatrix} -\sqrt{q}\,\delta_x & -\sqrt{q}\,\delta_y & 0 & \sqrt{q}\,\delta_x & \sqrt{q}\,\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & \delta_x \end{pmatrix} F_{x,j}$$

$$K_t^i = \bar{\Sigma}_t H_t^{iT} \left(H_t^i \bar{\Sigma}_t H_t^{iT} + Q_t \right)^{-1}$$

$$\bar{\mu}_t = \bar{\mu}_t + K_t^i (z_t^i - \hat{z}_t^i)$$

$$\bar{\Sigma}_t = (I - K_t^i H_t^i) \bar{\Sigma}_t$$

end for

$$\mu_t = \bar{\mu}_t$$

$$\Sigma_t = \bar{\Sigma}_t$$

return μ_t, Σ_t