(B) Observable Canonical Form

Standard form

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \quad \text{—①}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ \vdots \\ b_1 - a_1 b_0 \end{bmatrix} u \quad \text{—②}$$

$$y = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + b_0 u \quad \text{—③}$$

Solution

⇒ Rearranging eq ① we get :-

$$s^n [Y(s) - b_0 U(s)] + s^{n-1} [a_1 Y(s) - b_1 U_s] + \cdots$$

$$+ s [a_{n-1} Y(s) - b_{n-1} U(s)] + [a_n Y(s) - b_n U(s)] = 0$$

⇒ Dividing by s^n and rearranging :-

$$\boxed{\begin{aligned} Y(s) = b_0 U(s) &+ \frac{1}{s}[b_1 U(s) - a_1 Y(s)] + \cdots \\ &+ \frac{1}{s^{n-1}} [b_{n-1} U(s) - a_{n-1} Y(s)] + \frac{1}{s^n}[b_n U(s) - a_n Y(s)] \end{aligned}} \quad \text{—④}$$

Now define state Variables as follows:-

$$X_n = \frac{1}{s}\left[b_1 U(s) - a_1 Y(s) + X_{n-1}(s)\right]$$

$$X_{n-1}(s) = \frac{1}{s}\left[b_2 U(s) - a_2 Y(s) + X_{n-2}(s)\right]$$

$$\vdots$$

$$X_2(s) = \frac{1}{s}\left[b_{n-1} U(s) - a_{n-1} Y(s) + X_1(s)\right]$$

$$X_1(s) = \frac{1}{s}\left[b_n U(s) - a_n Y(s)\right]$$

⇒ Using above definition of state Variable eq (4) Can be written as:-

$$Y(s) = b_0 U(s) + X_n(s) \quad \text{—(5)}$$

⇒ Putting Y(s) from eq (5) in state Variables we get:-

$$sX_n = X_{n-1} - a_1 X_n + (b_1 - a_1 b_0) U(s)$$

$$sX_{n-1} = X_{n-1} - a_2 X_n + (b_2 - a_2 b_0) U(s)$$

$$\vdots$$

$$sX_2(s) = X_1(s) - a_{n-1} X_n + \cancel{(b_{n-1})} (b_{n-1} - a_{n-1} b_0) U(s)$$

$$sX_1 = \cancel{x} - a_n X_n + (b_n - a_n b_0) U(s)$$

⇒ Taking Inverse Laplace transform of above Set of Equation we get:

$$\dot{x}_1 = -a_n x_n + (b_n - a_n b_0) u$$

$$\dot{x}_2 = x_1 - a_{n-1} x_n + (b_{n-1} - a_{n-1} b_0) u$$

$$\vdots$$

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$$\dot{x}_{n-1} = x_{n-2} - a_2 x_n + (b_2 - a_2 b_0)u$$

$$\dot{x}_n = x_{n-1} - a_1 x_n + (b_1 - a_1 b_0)u$$

$\Rightarrow$ The above set of equation gives eq ①
{Part 1 of solution}

$\Rightarrow$ Taking inverse Laplace transform of $Y(s) = b_0 U(s) + X_n(s)$
{eq ⑤}

$\Rightarrow$ $\boxed{y = x_n + b_0 u}$ —⑥

$\Rightarrow$ Equation ⑥ gives eq ② {Part 2 of solution}