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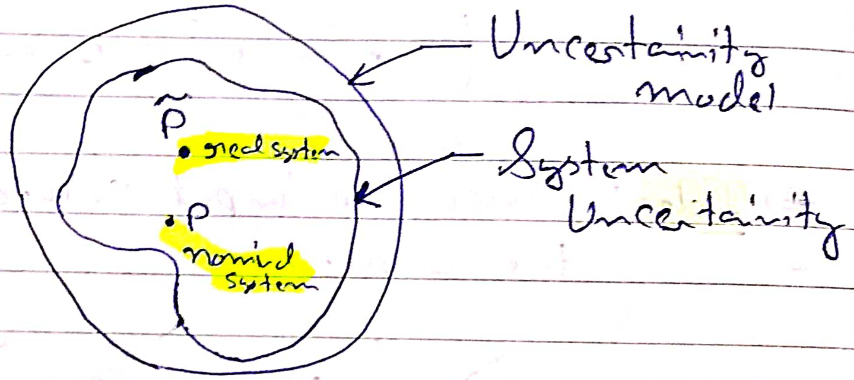
# Robustness & Implementation

## (1) ★ System uncertainity and uncertainty models




⇒ In order to take such uncertainity into account, we will first come up with an uncertainity model consisting of

- → A nominal model
- → A set of models that is guaranteed to contain the system uncertainty, and is easier to handle.

⇒ And then design a control system that meets the stability and performance specifications not only for P, but also for all other possible models in the uncertainity model.

⇒ We will aim at writing the transfer function of the "real" system in terms of the transfer function of a "nominal system" and of a unknown transfer function $\Delta$, representing the uncertainity as a perturbation on the nominal system.

⇒ The perturbation $\Delta$ is assumed to be a stable, minimum-phase transfer function such that it does not cancel unstable poles of the nominal system.

⇒ For convenience, we will also scale the perturbation $\Delta$ in such a way that

$$|\Delta(j\omega)| < 1 \quad \forall \omega$$

⇒ We can recover arbitrary magnitude/frequency models using another (known) transfer function $W_2$.

↳ In other words, the uncertainty will take the form

$$W_2(s)\,\Delta(s)$$

★ Multiplicative uncertainity

→ Multiplicative uncertainity models are of the form:

$$\tilde{P}(s) = (1 + W_2(s)\,\Delta(s))\, P(s)$$

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$$\Rightarrow \quad \frac{1}{W_2(s)}\left(\frac{\tilde{P}(s)}{P(s)} - 1\right) = \Delta(s)$$

$$\Rightarrow \quad \left|\frac{1}{W_2(j\omega)}\left(\frac{\tilde{P}(j\omega)}{P(j\omega)} - 1\right)\right| < 1$$

$$\Rightarrow \quad \left|\frac{\tilde{P}(j\omega)}{P(j\omega)} - 1\right| < |W_2(j\omega)|$$

⇒ So the multiplicative uncertainity model is a description of how much the ratio of the "real" and "nominal" transfer function is away from being equal to one.

⇒ Among other things, multiplicative uncertainity is useful when the gain of $\tilde{P}$ is uncertain

$$\tilde{P}(j\omega) = Y(\omega)\, G(j\omega)$$

$$Y(\omega) \in [Y_-(\omega), Y_+(\omega)] \;\; \forall \omega$$

where $G(s)$ is a known transfer function!

⇒ We can represent the same set of systems using a multiplicative uncertainity model with,

$$P(j\omega) = Y_o(\omega)\, G(j\omega) \quad , \quad Y_o(\omega) = \frac{Y_-(\omega) + Y_+(\omega)}{2}$$

and

$$W_2(j\omega) = \frac{Y(\omega) + Y_-(\omega)}{Y_+(\omega) + Y_-(\omega)}$$

## ② Computer implementation of a dynamic compensator

⇒ Compensator usually takes the form of a transfer function which can be written as

$$C(s) = K + \frac{c_{n-1} s^{n-1} + c_{n-2} s^{n-2} + \cdots c_0}{s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_0}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & 0 & & \ddots & \vdots \\ \vdots & \vdots & & & 1 \\ -a_0 & -a_1 & \cdots & \cdots & -a_{n-1} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}$$

$$C = [c_0 \; c_1 \; \cdots \; c_{n-1}] \qquad D = [K]$$

## ★ Pseudo code (Euler approximation)

1. Initialize the state (eg $x \leftarrow 0$)
2. Let dt be some small time interval.
3. loop
4. → Read the reference value r.
5. → Read the measured output value y.
6. → Compute the error $e = r - y$.
7. → Update the state as $x \leftarrow x + (Ax + Be)\,dt$
8. → Compute the output $u = Cx + De$
9. → Send the command u to the actuators.