Robot Dynamics

⇒ A path specifies the set of configurations a robot achieves as it moves from one configuration to another.

↳ Thus path planning is a Kinematic /geometric problem.

⇒ A path is not a complete description of the motion of a robot system.

↳ As the timing of the motion is not Specified

⇒ A trajectory is a path plus a specification of the ~~timing of the m~~ time at which each configuration is achieved.

↳ trajectory planning is not only a geometric Problem, but also a dynamic problem.

★ Lagrangian Dynamics

⇒ The equations of motion of a mechanical system can be generalized in a variety of ways.

↳ Hear we use a Lagrangian formulation, which is based on the Kinetic & potential energy of the system.

⇒ Lagrange's equations provides a straightforward recipe, amenable to computer implementation.

⇒ Let $q = [q_1, \cdots q_{n_Q}]^T \in R^{n_Q}$ be a vector of generalized coordinates representing the configuration of the system on the n_Q-dimensional configuration space.

⇒ Let $u = [u_1, \cdots u_{n_Q}]^T \in R^{n_Q}$ be the vector of generalized forces acting on the generalized coordinates.

⇒ The Lagrangian L of a mechanical system is written as the kinetic energy minus the potential energy.

$$\boxed{L(q, \dot{q}) = K(q, \dot{q}) - V(q)}$$

⇒ The Lagrangian equation of motion, also known as the Euler-Lagrange equation can be written as:

$$\boxed{\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{q}}\right) - \frac{\delta L}{\delta q} = u}$$

★ Standard forms of Dynamics

$$\boxed{u = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q)}$$

{Euler-Lagrange equation can be written in this form}

where,

$M(q)$ and $C(q,\dot{q})$ are $n_Q \times n_Q$ matrix

& $g(q) \in \mathbb{R}^{n_Q}$

⇒ Finally, mechanical system are often subjected to dissipative forces such as dry Coulomb friction.

→ These can be treated as external forces to be added after deriving the equations of motion using Lagrange's equations.

$$\boxed{u = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + b(q,\dot{q})}$$

★ Inertia Matrix

⇒ Kinetic energy of a mechanical system is determined by its inertia matrix, and can be written

$$\boxed{K(q,\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q}}$$

⇒ The fact that $M(q)$ is positive definite implies that the kinetic energy is positive for any nonzero $\dot{q}$.

★ Velocity Constraints

⇒ Suppose that the mechanical system is subjected to a set of K linearly independent constraints linear in velocity:

$$\boxed{A(q)\dot{q} = 0}$$

where, $A(q)$ is $K \times n_Q$ matrix

→ Such constraints are called Pfaffian constraints.

⇒ The constrained Lagrange's equation can then be written

$$\boxed{M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u + A^T(q)\lambda}$$

$$\boxed{A(q)\dot{q} = 0}$$

where,

$\lambda = [\lambda_1 \cdots \lambda_K]^T$ are Lagrange multipliers

⇒ The constrained Lagrange's equation yields $n_Q + K$ equations to be solved for the $n_Q + K$ variables.

⇒ If we are not interested in calculating the k constraint force, we can eliminate λ from constrained Lagrange equation.

$$\boxed{P_u(M\ddot{q} + C\dot{q} + g) = P_u u}$$

where, $P_u = I - A^T(AM^{-1}A^T)^{-1}AM^{-1}$

→ P_u is $n_a \times n_a$ matrix of rank $n_a - k$

↳ so not invertible.

⇒ Let $P = M^{-1}P_u M$, then constrained Lagrange equation can be re-arranged as:-

$$\boxed{P\ddot{q} = PM^{-1}(u - C\dot{q} - g)}$$