

A Sample Exam in L^AT_EX

This is a template you can use to create exams in L^AT_EX. I hope it is helpful!

The exam consists of 6 pages, not including this cover page. Please go through your copy to make sure that all pages have been printed.

The first part of the exam consists of short questions. No partial credit will be given. The second part of the exam consists of longer questions. Partial credit will be given for correct reasoning. Show all work for the longer questions.

Good luck!

Name: _____

ID Number: _____

Page	Points	Score
1	10	
2	10	
3	15	
4	10	
5	20	
6	5	
Total:	70	

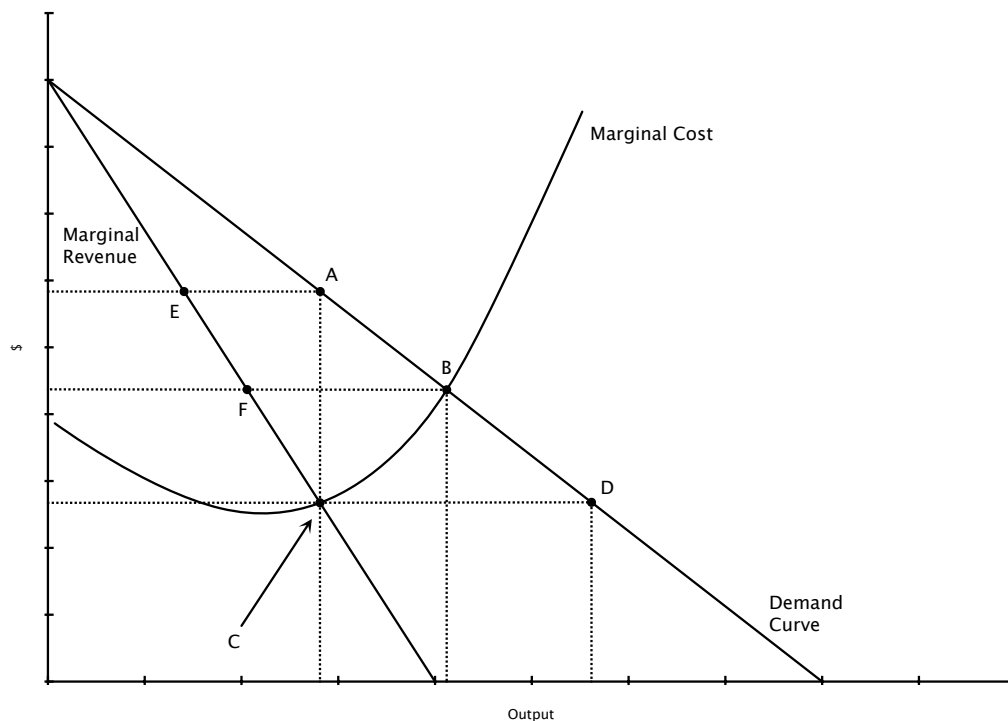
Short Questions

No partial credit will be awarded. Clearly circle *one* choice for each of the multiple-choice questions.

1. (5 points) A small coffee company roasts coffee beans in its shop. The *total* cost of roasting coffee beans is $3q^2$ when q pounds are roasted. The smell of roasting beans imposes costs on the company's neighbors. The *total* amount that neighbors would be willing to pay to have the shop stop roasting altogether is $\frac{3}{2} \cdot q^2$. The company sells its output in a competitive market at \$36 per pound, so that its total revenue is $36q$. What is the socially efficient amount of coffee for the company to roast?
 - A. 2 pounds.
 - B. 4 pounds.**
 - C. 6 pounds.
 - D. 8 pounds.
 - E. 10 pounds.
2. (5 points) A monopolist has discovered that the inverse demand function of a person with income M for the monopolists product is $p = 0.002M - q$. The monopolist is able to observe the incomes of its consumers and to practice price discrimination according to income. The monopolist has a total cost function, $c(q) = 100q$. The price it will charge a consumer depends on the consumer's income, M , according to the formula
 - A. $p = 0.002M - 100$
 - B. $p = M^2$
 - C. $p = 0.01M^2 + 100$
 - D. $p = 0.001M + 50$**
 - E. None of the above.

3. (5 points) An industry consists of two duopolist firms choosing their output simultaneously. They decide to enter into a secret agreement to maximize their joint profits. The *total* output of the industry then goes $[X]$ and the price goes $[Y]$.
- A. $X = \text{up}, Y = \text{down}$
 - B. $X = \text{up}, Y = \text{up}$
 - C. $X = \text{stays the same}, Y = \text{up}$
 - D. $X = \text{down}, Y = \text{down}$
 - E. $X = \text{down}, Y = \text{up}$

The following two questions refer to the figure below. The figure depicts cost curves for a firm. To answer the questions, simply refer to points labeled by capital letters on the graph.



4. (5 points) If the firm behaves competitively, taking price as given, what level of output and price does it choose?

4. _____

Solution: Bundle B .

5. (5 points) If the firm acts as a monopolist, what level of output and price does it choose?

5. _____

Solution: Bundle *A*.

6. (5 points) Suppose that a consumer considers coffee (C) and tea (T) to be perfect substitutes, but he requires two cups of tea to give up one cup of coffee. This consumer's budget constraint can be written as $2.5C + T = 10$. What should the consumer buy?
- A. 2 cups of tea and no coffee.
 - B. 10 cups of tea and no coffee**
 - C. 2.5 cups of coffee and no tea.
 - D. 4 cups of coffee and no tea.
 - E. none of the above.
7. (5 points) The market demand curve is $D(p) = 20 - 16 \cdot p$. When the price is \$1, what is the elasticity of demand?

7. _____

Solution: First, we calculate the elasticity of demand using the usual formula:

$$\frac{\partial D(p)}{\partial p} \cdot \frac{p}{D(p)} = -16 \cdot \frac{p}{20 - 16p}.$$

If we plug in $p = 1$, we find that the elasticity of demand is $-16 \cdot \frac{1}{20-16} = -4$.

Problems

For the following problems, show all your work. Partial credit will be awarded for correct reasoning.

8. Suppose that a consumer likes apples (a) and bananas (b), and has utility function $u(a, b) = a^{2/10} \cdot b^{8/10}$. Suppose that apples cost \$10 a pound and that the consumer has \$100 to spend. Denote the price of bananas as p .

(a) (5 points) What is the consumer's budget constraint?

Solution: The consumer's budget constraint is $10a + pb \leq 100$.

- (b) (5 points) What is the consumer's marginal rate of substitution. In 1–2 sentences describe what this quantity represents, aside from just “the slope of the indifference curve.”

Solution: The consumer's marginal rate of substitution is

$$-\frac{\partial u / \partial a}{\partial u / \partial b} = -\frac{(2/10) \cdot a^{-8/10} \cdot b^{8/10}}{(8/10) \cdot a^{2/10} \cdot b^{-2/10}} = -\frac{2}{8} \cdot \frac{b}{a}.$$

Note that this “assumes” we're graphing apples on the horizontal axis.

- (c) (5 points) Setting the MRS equal to the slope of the budget constraint, find the consumer's optimal choice of apples in terms of bananas.

Solution: If we graph apples on the horizontal axis, then the MRS is as above, and the slope of the budget constraint is $-10/p$. So at the consumer's utility-maximizing choice of goods, we must have

$$\frac{10}{p} = \frac{2}{8} \cdot \frac{b}{a},$$

which in turn implies that $a^* = \frac{2p}{80} \cdot b$.

- (d) (5 points) Plug this result into the budget constraint to find the consumer's demand for bananas.

Solution: Plugging in, we have

$$10 \cdot \frac{2p}{80} \cdot b + pb = 100,$$

which implies that

$$b^* = \frac{100}{p \cdot \left(\frac{20}{80} + 1\right)} = \frac{80}{p}.$$

9. Suppose that a competitive firm has production function $f(K, L) = \sqrt{KL}$, in which K denotes capital and L denotes units of labor. The firm must pay its workers sixteen dollars an hour, and must pay r dollars per unit of capital.

- (a) (5 points) Does the firm have constant, increasing, or decreasing returns to scale? Justify your answer.

Solution: Constant returns to scale:

$$f(2K, 2L) = \sqrt{2 \cdot K \cdot 2 \cdot L} = 2\sqrt{K \cdot L} = 2 \cdot f(K, L).$$

- (b) (5 points) Calculate the firm's technical rate of substitution (TRS). In 1–2 sentences, describe what this quantity represents, aside from just “the slope of the isoquant.”

Solution: The firm's technical rate of substitution is

$$\frac{\partial f(K, L) / \partial K}{\partial f(K, L) / \partial L} = \frac{\frac{1}{2} K^{-0.5} L^{0.5}}{\frac{1}{2} \cdot K^{0.5} L^{-0.5}} = \frac{L}{K}.$$

Note this assumes that capital is on the horizontal axis.

The TRS measures the firm's ability to substitute capital for labor while holding output constant.

- (c) (5 points) Find the firm's conditional demand for labor, $L^*(r, y)$ in terms of the price of capital and the firm's choice of output. (Hint: set the TRS equal to the slope of an iso-cost curve, and solve for capital in terms of labor. Then plug that result into the production function.)

Solution: The firm's iso-cost line is $C = 16L + rK$. If capital is on the horizontal axis, then $L = \frac{C}{16} - \frac{r}{16}K$. So at the optimum level of inputs, we observe

$$\frac{r}{16} = \frac{L}{K}.$$

This implies that $K = \frac{16L}{r}$. Plugging this into the production function leads to $y = \sqrt{\frac{16L}{r}} \cdot L = 4L \cdot r^{-1/2}$. So the conditional demand curve for labor is

$$L^* = \frac{y}{4} \cdot \sqrt{r}.$$