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Portfolios Under Construction

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DB Handbook of Portfolio Construction: Part 1

Part 1: Risk-Based Portfolio Construction Techniques

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Innovative techniques

The heart of portfolio construction is about how to achieve better diversification and risk reduction. In this paper, we introduce two innovative techniques to accomplish this goal. These techniques recognize the fact that asset returns are not normally distributed. The minimum tail dependence portfolio attempts to find assets that are less dependent to each other at the tail level to avoid crowded trades. The conditional value-at-risk (or CVaR) defines risk in a different way from volatility, by emphasizing tail risk. Portfolios constructed by minimizing CVaR are *ex ante* more conservative and have delivered the best *ex post* performance.

Encyclopedic coverage of risk-based portfolio construction for both multi-asset and equity portfolios

In this research, we conduct a comprehensive portfolio backtesting of a full suite of risk-based portfolio construction techniques, from simple (equally weighted and inverse volatility), traditional (risk parity, global minimum variance, and maximum diversification), to innovative (minimum tail dependence, minimum CVaR, robust minimum CVaR, etc.) for a broad range of contexts in multi-asset (asset allocation, bonds, commodities, alternative betas), country/sector (countries, economic low risk countries, global sectors, US sectors, European sectors, global industries, and regions x sectors), and equity portfolios (US, Europe, Asia ex Japan, EM, and global).

The superiority of risk-based allocations

In almost all contexts (both multi-asset and equities), risk-based allocations significantly outperform capitalization-weighted benchmarks, with higher Sharpe ratios, lower downside risk, better diversification, and less tail dependence, which makes the portfolio less likely to suffer in a liquidity crisis.

Source: gettyimages.com

A letter to our readers

Part I of DB Handbook of Portfolio Construction

From our extensive interaction with our institutional clients globally, we have noticed an interesting pattern. Most portfolio managers and research analysts tend to spend most of their time searching for alpha-related ideas[1], e.g., the next factor to predict stock returns, the next risk premium in commodities, the next global macro strategy, or the so-called the "Great Rotation" in asset allocation recently[2]. On the other hand, investors tend to rely on data vendors for their risk models, optimizers, and performance attribution tools. It is only in recent years since the global financial crisis that risk management and portfolio construction have been attracting more attentions than ever before. At the same time, most investors do not feel as comfortable in dealing with portfolio construction as alpha research, due to the lack of easily accessible, educational, and practical reading materials.

It is our intention to bring investors up to date with the most cutting edge portfolio construction techniques, in a non-technical manner. Rather than focusing on the mathematical details, we concentrate on the practical issues facing investors every day.

This is the first part of a series of handbooks that address this increasingly more important area of portfolio construction. In this paper, we focus on what-so-called "risk-based allocation" or "risk-based portfolio construction techniques". On the one hand, they are simpler – portfolios are constructed solely based on risk prediction. But risk, of course, can be defined in countless ways, so risk-based allocation can still be extremely complicated. On the other hand, risk-based allocation has become one of the hottest fields in both academia and investing in recent years. Again, this is not surprising given the prolonged global financial crisis started in 2008. Not only is managing risk becoming more paramount than outperforming a benchmark, but also risk-based allocation techniques actually do indeed outperform many (if not most) active strategies that require return prediction.

In a series of forthcoming research papers, we will discuss how to incorporate risk and return predictions when constructing portfolios. We will also address how to combine qualitative views with quantitative models in portfolio construction. In total, we hope these DB Handbooks of Portfolio Construction will help our readers better understand portfolio construction and use these techniques to better manage their portfolios.

It's all about diversification and risk reduction

Our regular readers would have noticed our serious interest in risk-based portfolio construction in both asset allocation and equity portfolio contexts[3]. In this paper, we

summarize the key theories and findings of our previous research. More importantly, we introduce a few new tools in portfolio analytics.

[1] This is indeed the case with us as well. In Luo, et al [2010]. "DB Quant Handbook", we almost exclusively discussed only alpha related topics.

[2] The term "Great Rotation" came to prominence in 2012, referring to the fact that bond yields are at record lows; therefore, bonds offer little upside. Investors are likely to rotate out of bonds and into other risky assets, especially stocks.

[3] See a complete list of our previous research in this space in Section XIV on page 104.

Portfolio construction itself does not introduce new assets (i.e., breadth) or new insights in return prediction (i.e., skill). The heart of portfolio construction is about diversification and risk reduction – both terms are yet to be defined.

We all know classic finance theory like Markowiz's mean-variance optimization heavily depends on the assumption that asset returns are jointly normally distributed. We also know that empirical evidence almost universally rejects the normality assumption. The traditional statistical tools (e.g., Pearson's correlation coefficient, portfolio volatility, etc.) are mostly based on this false assumption.

In this research, we first define an alternative approach to measure comovement in asset returns. Rather than relying on Pearson's correlation, we measure tail dependence between two assets using a Copula model. In Cahan, *et al* [2012], we demonstrated an interesting way to measure the crowdedness of plain vanilla types of low risk strategies in the equity space – median tail dependence. In this research, we extend the tail dependence concept by further constructing what we called weighted portfolio tail dependence (WPTD) by taking into account asset weights. More important, we design a strategy that proactively avoids crowded trades in what we call the minimum tail dependent portfolio (MinTailDependence).

Second, we give a practical introduction to conditional value at risk (also called expected shortfall) and how to construct a portfolio that minimizes expected CVaR, i.e., the MinCVaR portfolio. We further develop a new algorithm by combining robust optimization and CVaR optimization into what we call robust CVaR optimization, which shows great promise by outperforming all other portfolio construction techniques in terms of both Sharpe ratio and downside risk.

The MinTailDependence portfolio (along with maximum diversification) helps us better capture diversification benefits, while the MinCVaR strategy (along with global minimum variance portfolio) attempts to manage risk better.

Everything from asset allocation, multi-asset, to equity portfolios

In this paper, we empirically backtest seven risk-based allocations (equally weighted, inverse volatility/volatility parity, risk parity/equal risk contribution, global minimum variance, maximum diversification, minimum tail dependence, and minimum CVaR), compared to traditional capitalization-weighted benchmarks in different contexts from asset allocation, multi-asset (bonds, commodities, alternative betas), country/sector portfolios (MSCI ACWI, economically hedged country indices, global sectors, US sectors, European sectors, global industries, region x sector combinations), and equity portfolios (US, European, Asia ex Japan, Japan, emerging markets, and global equities).

Interestingly, in almost every single context, risk-based allocations significantly outperform traditional capitalization-weighted benchmarks, with higher Sharpe ratio, lower downside risk, better diversification, and less tail dependence.

Yin, Rocky, Miguel, Javed, John, and Sheng
Deutsche Bank Quantitative Strategy Team

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I. Performance measurement

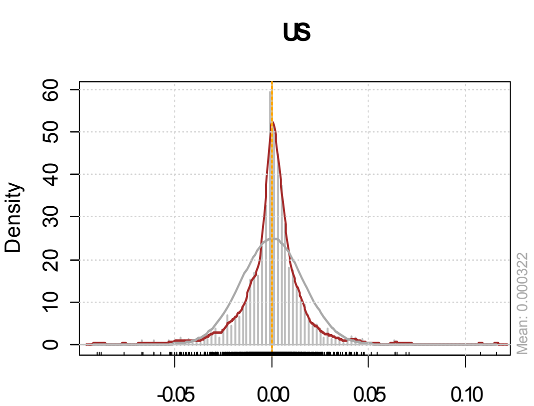
Regular readers of our research may have noticed that we are strong proponents of incorporating a full suite of performance measurement and attribution tools in investment management. In this research, we do not attempt to provide a complete coverage. A book-length introduction can be found in Bacon [2004]. Rather, we briefly summarize a few key metrics, which will be used extensively throughout the rest of the paper.

Return distribution

Traditional performance measures like the Sharpe ratio are only meaningful if the portfolio return follows a normal distribution. As we all know, asset returns are rarely normally distributed. As a first test, we need to assess if the normal distribution assumption is really valid. We can calculate skewness and kurtosis. We can further apply statistical normality tests, e.g., Jarque-Bera test, Shapiro-Wilk's test, Kolmogorov-Smirnov test, and D'Agostino normality test.

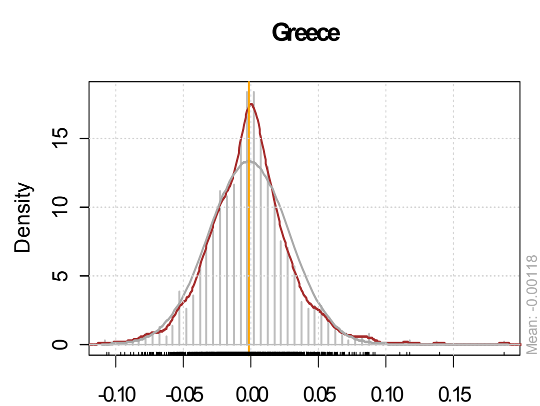
As shown in Figure 1 and Figure 2, the country returns for US and Greece clearly do not follow normal distribution – they both show negative skewness and excess kurtosis. Almost all the normality tests mentioned above easily reject the Null hypothesis of a normal distribution for US and Greece equity market returns.

Figure 1: Density plot – US



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Figure 2: Density plot - Greece



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

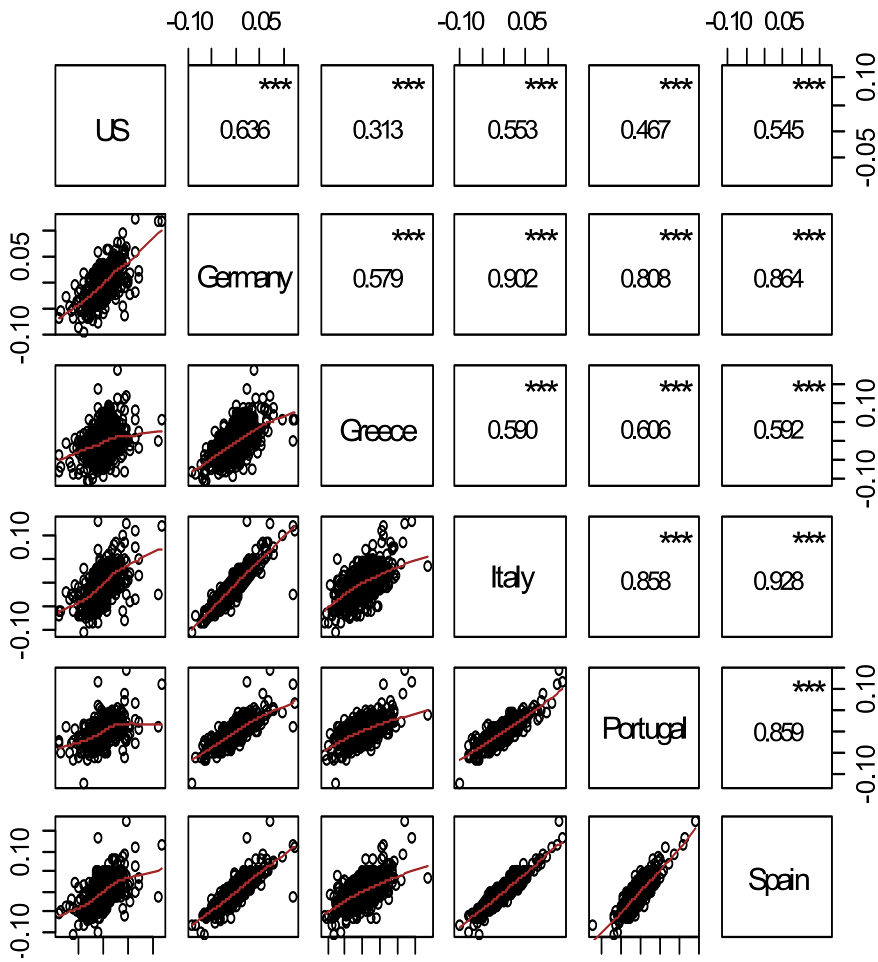
The traditional mean-variance portfolio optimization techniques also heavily depend on the assumption that asset returns jointly follow multivariate normal distribution. In portfolio construction, the joint multivariate distribution is far more important than univariate distribution.

Now, let's use a simple example of six countries (US, Germany, Portugal, Italy, Greece, and Spain) using the past five years of daily returns to demonstrate two main multivariate normality tests.

- **The multivariate Shapiro test** (see Royston [1982]). The multivariate Shapiro test shows a w-statistic of 0.82, corresponding to a p-value of less than 1%; therefore, the Null hypothesis of a multivariate normal distribution is rejected.
- **The nonparametric E-statistics test**, also called **energy test** (see Szekely [1989]). The E-statistic is 3.3×10^{34}, corresponding to a p-value of less than 1%, the Null hypothesis of a multivariate normal distribution is again easily rejected.

Figure 3 shows the scatterplot of the returns for the six countries. The correlations among all six countries appear to be very strong. In addition, we see some clear nonlinear relationships and some heavy tail dependence, especially among the peripheral European countries. In later sections, we will show how we can measure tail dependence, and more importantly, how to incorporate tail dependence in our portfolio construction process.

Figure 3: Scatterplot of six countries



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Risk analysis

The simplest and most traditional measure of risk is volatility. Similarly, for the relative risk to a benchmark, we typically use tracking error, i.e., the standard deviation of active returns. Volatility and tracking error are appropriate when the return distribution is normal. Empirically, as we know, the returns of most assets do not follow a normal distribution.

Value-at-risk (VaR)

Value at risk (or VaR) was developed in the early 1990s. For a given portfolio, probability and time horizon, VaR is defined as a threshold value such that the probability that the loss on the portfolio over the given time horizon exceeds this value is the given probability level. The typical VaR (also called "historical VaR") is based on the RiskMetrics (now part of MSCI) methodology. In our research, we focus on the so-called Modified Cornish-Fisher VaR (see Zangari [1996] for the original paper and the more efficient algorithm introduced by Boudt, Peterson, and Croux [2008]), which incorporates skewness and kurtosis via an analytical estimation using a Cornish-Fisher expansion.

The biggest problem when using VaR as a risk measure is that it is not coherent. Specifically, VaR is not sub-additive, meaning the VaR of a portfolio can be greater than the sum of the individual risks of each assets comprising the portfolio.

Conditional VaR/expected shortfall (ES)

At a preset confidence level denoted α, which typically is set as 1% or 5%, the CVaR/ES of a return series is the expected value of the return when the return is less than its α-quantile. Unlike VaR, CVaR has all the properties a risk measure should have to be coherent and is a convex function of the portfolio weights. With a sufficiently large data set, you may choose to estimate CVaR with the sample average of all returns that are below the α empirical quantile. If the return series is skewed and/or has excess kurtosis, Cornish-Fisher estimates of CVaR can be more appropriate (see Boudt, Peterson, and Croux [2008]).

CVaR can be interpreted as the average VaR when the loss is greater than VaR.

$$CVaR_{\alpha} = \frac{1}{1-\alpha}\int_{\alpha}^{1} VaR_{n}(L)du$$

where,

L is the loss function.

In a later section, we will show how to incorporate CVaR in the portfolio construction process.

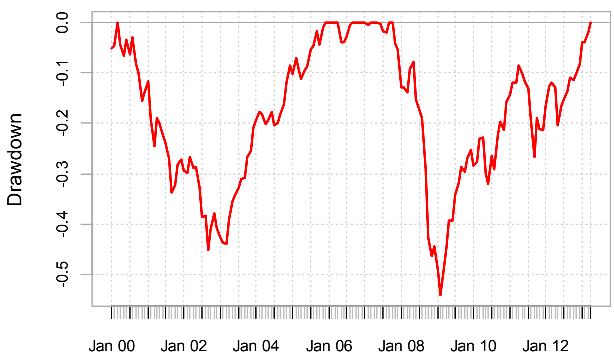
Drawdown measures

A widely used downside risk measure is maximum drawdown, defined as the worst cumulative loss ever sustained by an asset. The maximum drawdown measure is particularly popular in the world of commodities trading advisors (i.e., CTAs). For example, Figure 4 and Figure 5 show the drawdowns of the MSCI ACWI and our MinTailDependence[4] country portfolio. Our MinTailDependence portfolio has less severe drawdowns (i.e., maximum drawdown) and the length of each drawdown is shorter on average.

[4] The MinTailDependence portfolio seeks to build a portfolio of countries that are at least dependent from each other as possible, where dependence is defined as a copula-based tail dependent coefficient. The strategy will be fully defined in Section V on page 26.

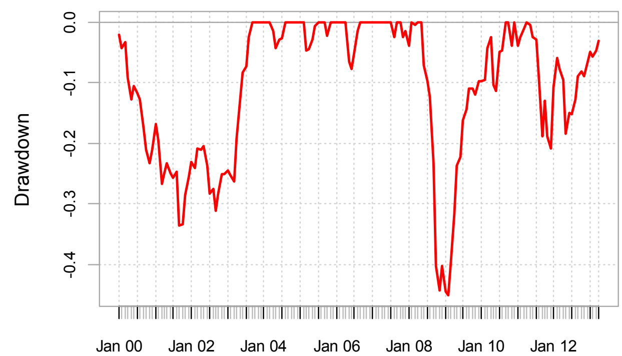
In an upcoming research paper, we will show how to build portfolios taking into account drawdown measures.

Figure 4: Drawdown – MSCI ACWI



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Figure 5: Drawdown – MinTailDependence portfolio



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Risk-adjusted performance

Sharpe ratio

The Sharpe ratio is defined as the mean return over volatility. The Sharpe ratio is an appropriate measure of performance only if the portfolio returns follows a normal distribution, which is typically not met in practice. Despite of the issues with the Sharpe ratio, it is still the most widely used performance metric today.

Adjusted Sharpe ratio

The adjusted Sharpe ratio was introduced by Pezier and White [2006] to account for skewness and kurtosis by incorporating a penalty factor for negative skewness and excess kurtosis:

$$AdjustedSharpeRatio = SharpeRatio \times \left[1 + \left(\frac{Skewness}{6}\right) \times SharpeRatio - \left(\frac{Kurtosis - 3}{24}\right) \times SharpeRatio^2\right]$$

Information ratio

The information ratio is the mean active return (i.e., return over benchmark) divided by active risk (also called tracking error).

Sortino ratio

The Sortino ratio is the mean return over downside deviation below a user-specified target (i.e., a required rate of return).

$$SortinoRatio = \frac{r_p - MAR}{DownsideDeviation}$$

where,

MAR or minimum acceptable return is the user-specified target return,

$$DownsideDeviation = \sqrt{\int_{-\infty}^{MAR} (MAR - x)^2 f(x) dx}$$, and

$f(x)$ is the pdf (probability density function) of the portfolio returns.

Portfolio diversification characteristics

In this paper, we focus on four measures of diversification: the diversification ratio, the concentration ratio, the weighted portfolio correlation (WPC), and the weighted portfolio tail dependence coefficient (WPTD). We will discuss this topic in detail in Section III: Strategy crowding and efficacy.

II. Traditional risk-based allocations

In this section, we briefly review the four traditional risk-based portfolio construction techniques. More details can be found in Alvarez, *et al* [2011] "Risk parity and risk-based allocation" and Luo, *et al* [2013] "Independence day".

Inverse volatility (InvVol)

The first allocation technique we explore is Inverse Volatility (InvVol), also called naïve risk parity or volatility parity. InvVol is easy to implement, in that portfolios are weighted inversely to their volatility. As such, more volatile assets are relatively down weighted. Mathematically, this can be expressed as follows:

$$\omega_{i,t} = \frac{1/\sigma_{i,t}}{\sum_{n} 1/\sigma_{i,t}}$$

Subject to:

$$\sum_{i=1}^{N} \omega_{i,t} = 1$$
$$\omega_{i,t} \geq 0$$

where,

$\omega_{i,t}$ is the weight allocated to asset i at time t,

$\sigma_{i,t}$ is the volatility of asset i at time t.

The main drawback of InvVol is that it does not take into account the correlation between assets. An asset may be unnecessarily penalized (i.e. down weighted) simply because it's relatively more volatile, while it may provide more diversification benefits should correlations also be considered.

Risk parity (RiskParity) or equal risk contribution (ERP)

Risk parity (RiskParity) or equal risk contribution (ERP) is similar to the InvVol allocation. It seeks to give equal risk budget to each asset in the portfolio (subject to constraints). The purpose of balancing risk equally across the assets in the portfolio is consistent with a "no-alpha" strategy in that it gives each asset "equal opportunity" to contribute to the portfolio's overall performance.

RiskParity takes into account the correlation between pairs of assets. And it can be implemented via the following optimization algorithm:

$$\omega_{i,t} = \arg\min_{w} \sum_{i=1}^{n} \sum_{j=1}^{n} [\omega_{i,t} \operatorname{cov}(r_{i,t}, r_{p,t}) - \omega_{j,t} \operatorname{cov}(r_{j,t}, r_{p,t})]^2$$

Subject to:

$$\sum_{i=1}^{N} \omega_{i,t} = 1$$
$$\omega_{i,t} \geq 0$$

where , $r_{i,t}$ and $r_{p,t}$ are the returns of asset i and portfolio p at time t.

One issue with RiskParity strategy is that, when the number of assets is large (e.g., greater than 100, as in most equity portfolios), it becomes increasingly similar to an EquallyWgted or an InvVol.

Global minimum variance portfolio (GlobalMinVar)

The next portfolio allocation method we look at is the popular global minimum variance portfolio (GlobalMinVar). The GlobalMinVar method aims to weight portfolios such that the overall portfolio risk is minimized without taking any particular view on expected returns. This can be implemented via the following optimization algorithm:

$$\arg\min_{\omega} \frac{1}{2} \omega_t' \Sigma_t \omega_t$$

subject to:

$$\omega_t' \iota = 1$$
$$\omega_t \geq 0$$

where,

ω_t is the vector of asset weights at time t,

ι is a vector of 1s, and

Σ_t is the asset-by-asset covariance matrix at time t.

GlobalMinVar portfolios are sensitive to our risk estimates and tend to be concentrated in a few assets.

Efficient frontier

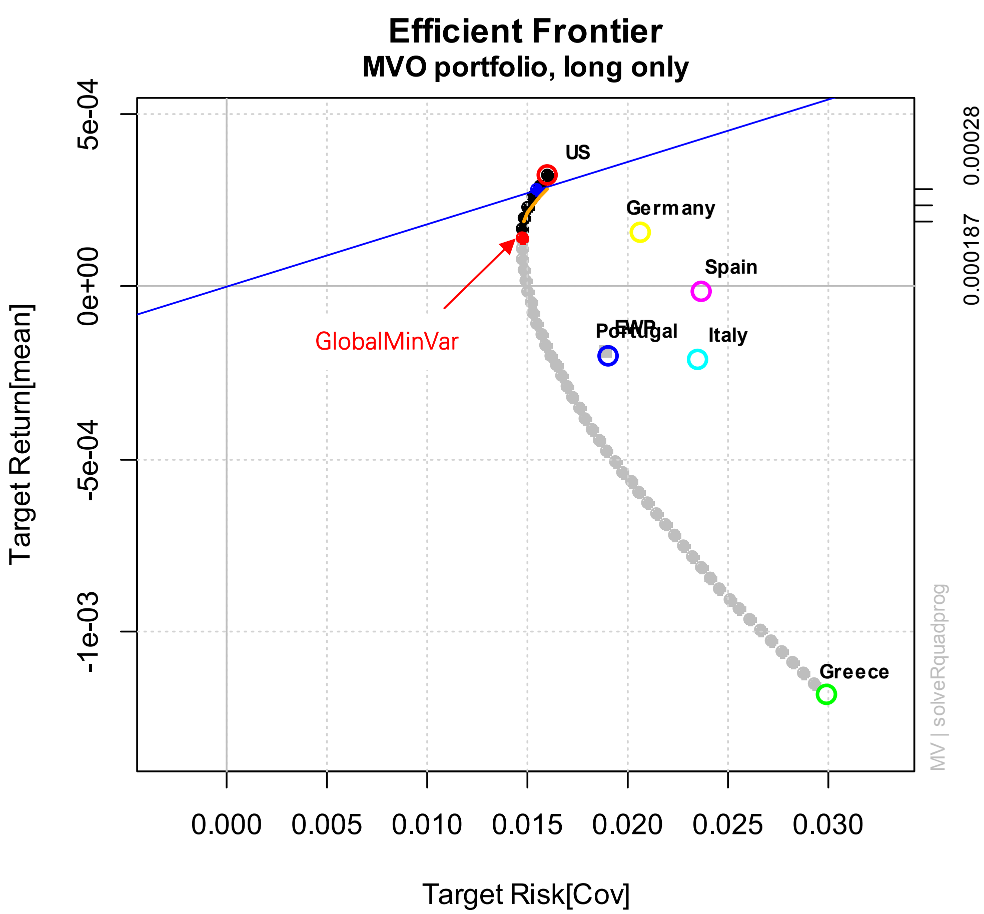
Portfolios on the efficient frontier are efficient, in the sense that they have the best possible expected return for their level of risk. Such portfolios (without including the risk-free asset) can be plotted in risk (i.e., volatility) – expected return space. The upward-sloped part of the left boundary of this region, a hyperbola, is then called the efficient frontier.

The efficient frontier together with the minimum variance locus (i.e., the bottom portion of the hyperbola) form the 'upper border' and 'lower border' lines of the feasible set. To the right the feasible set is determined by the envelope of all pairwise asset frontiers. The region outside of the feasible set is unachievable by holding risky assets alone. No portfolios can be constructed corresponding to the points in this region. Points below the frontier are suboptimal. Thus, a rational investor will hold a portfolio only on the frontier.

The GlobalMinVar portfolio, *ex ante*, is the portfolio with the lowest risk (subject to constraints) on the efficient frontier. In theory, it is also the portfolio with the lowest expected return. *Ex post*, however, as we will show you in later sections, it often tends to outperform many (if not most) active portfolios that try to take on more risk.

Using our example of six countries (US, Germany, Portugal, Italy, Greece, and Spain), Figure 6 shows the efficient frontier and the GlobalMinVar portfolio.

Figure 6: Mean-variance efficient frontier



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Maximum diversification portfolio (MaxDiversification)

Last, we look at the maximum diversification (MaxDiversification) approach. This allocation strategy attempts to create portfolios that are more diversified by maximizing the distance between the weighted average volatility of each underlying portfolio and the overall portfolio volatility. This can be shown in the following equation:

$$\arg\max_{\omega} \frac{\frac{\sum \omega_{i,t}\sigma_{i,t}}{n}}{\omega_t' \Sigma_t \omega_t}$$

subject to:

$$\omega_t' \iota = 1$$
$$\omega_t \geq 0$$

The equation above warrants some further explanation. The numerator is simply the weighted sum of the underlying asset volatilities. The denominator is the total portfolio volatility which takes into account the correlation between the underlying assets. The difference between the two is essentially the correlation terms. To maximize the overall ratio, the denominator containing the correlations must be minimized. This allocation strategy attempts to select assets that minimize the correlation between the underlying assets and hence "maximize diversification" as the name suggests.

Numerically, the MaxDiversification portfolio can be solved easily by minimizing $\psi' \mathrm{P} \psi$, where P is the correlation matrix. Therefore, the MaxDiversification optimization is almost the same as GlobalMinVar – the difference is to replace the covariance matrix with the correlation matrix. The final weights are then retrieved by rescaling the intermediate weight vector (optimized using the correlation matrix) with the standard deviations of the asset returns.

Step 1

$$\arg\min_{\omega} \frac{1}{2} \psi_t' \mathrm{P}_t \psi_t$$

subject to:

$$\psi_t' \iota = 1$$
$$\psi_t \geq 0$$

where,

ψ_t is the first intermediate vector of asset weights at time t, and

P_t is the asset-by-asset correlation matrix at time t.

Step 2

Then we need to rescale the first intermediate vector of asset weights ψ_t by each asset's volatility $\sigma_{i,t}$:

$$\xi_t = D_t^{-1/2} \psi_t \text{ or } \xi_{i,t} = {\psi_{i,t}}/{\sigma_{i,t}}$$

where,

ξ_t is the second intermediate vector of asset weights at time t, and

D_t is the diagonal matrix of asset variance at time t with $\sigma_{i,t}^2$ as its i,i th element and zero on all off-diagonal elements

Step 3

Finally, we rescale the second intermediate asset weight vector of the total weight, so the sum of the final weights equal to 100%, i.e., no leverage.

$$\omega_{i,t} = {\xi_{i,t}}/{\sum_{j=1}^{N} \xi_{j,t}}$$

III. Strategy crowding and efficacy

In Cahan, *et al* [2012], we attempted to measure the crowdedness of a strategy using two approaches: median pairwise correlation (MPC) and median pairwise tail dependence (MPTD). The rationale is that when investors increasingly trade a basket of stocks together, the MPC and MPTD of stocks within this basket will rise compared to the market. When the relative MPC (or MPTD) compared to the market is at certain level, it is an indication of strategy crowding. One of the key ingredients in the calculation is to use high frequency return data[5] to measuring the intraday trading behavior.

In this paper, we extend Cahan *et al* [2012] paper by taking into account asset weights. The other key difference is that we use daily return rather than high frequency return in our computation, for two reasons: 1) many of the assets (e.g., EM credit) in this paper do not have high frequency return data; and more importantly, 2) as we deal with global capital markets, we have to be aware of unsynchronized trading issues. Therefore, rather than measuring strategy crowding, the approaches introduced in this paper are more to measure the efficacy or potential diversification opportunity of a particular strategy. For example, if the weighted portfolio correlation of a risk-based strategy is approaching the average correlation of the market (i.e., a capitalization weighted benchmark index), it suggests the potential diversification benefit is shrinking and we are holding a portfolio that is increasingly similar to the index.

Weighted portfolio correlation

The median pairwise correlation (MPC) in Cahan *et al* [2012] does not take into account asset weights. The median pairwise correlation can remain low, but we could have owned a few heavily weighted and highly correlated assets in our portfolio, which may still expose our portfolio into a liquidity crisis. In this research, we develop a more refined measure which we call "weighted portfolio correlation" or WPC.

A portfolio's risk is not a simple weighted average of each asset's volatility. Similarly, we can't calculate a portfolio's weighted correlation as the weighted pairwise correlation, because the relationship is nonlinear.

The variance of a portfolio can be computed as:

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2\sum_{i=1}^{N-1} \sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}$$

where:

σ_p^2 is the variance of the portfolio

σ_i is the volatility of asset i

[5] Specifically, we use one-minute return data for US stocks and 15-minute return data for other regions.

ω_i is the weight of asset i

ρ_{ij} is the correlation coefficient between asset i and j

Now, let's assume $\rho_{average}$ (i.e., WPC) is the average pairwise correlation, then the above equation can be written as:

$$\sigma_p^2 = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} = \sum_{i=1}^{N} \omega_i^2 \sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j \rho_{average}$$

Therefore,

$$\sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} = \sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j \rho_{average} = \rho_{average} \sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j$$

And, finally,

$$WPC = \rho_{average} = \frac{\sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}}{\sum_{i=1}^{N-1}\sum_{j>i}^{N} \omega_i \omega_j \sigma_i \sigma_j}$$

Therefore, WPC is essentially the weighted pairwise correlation adjusted for asset volatility.

WPC and portfolio diversification

We can naturally expect a portfolio with a lower WPC to be more diversified. Indeed, as shown in Choueifaty and Coignard [2008], WPC is closely related to two other key concepts – diversification ratio (DR) and concentration ratio (CR).

First, let's quickly review the definition of DR:

$$DR = \frac{\sum_{i=1}^{N} \omega_i \sigma_i}{\sigma_p} = \frac{weightedAverageVolatility}{portfolioVolatility}$$

On the other hand, the CR is defined as:

$$CR = \frac{\sum_{i=1}^{N} \omega_i^2 \sigma_i^2}{\left(\sum_{i=1}^{N} \omega_i \sigma_i\right)^2}$$

The concentration ratio or CR is a simple measure of portfolio concentration that only takes into account the volatility of each asset. A fully concentrated long-only portfolio with only one asset has unit CR, while an InvVol portfolio has the lowest CR (which equals to the inverse of the number of assets it contains).

$$DR = \frac{1}{\sqrt{\rho_{average}(1-CR)+CR}}$$

Therefore, the diversification ratio can be improved by either reducing the concentration of the portfolio or decreasing the WPC.

The InvVol portfolio is by construction the minimum concentration portfolio, which is why it is also called volatility parity. MaxDiversification or more strictly speaking, the most diversified portfolio, is also the correlation parity portfolio. By construction, the MaxDiversification portfolio has the highest *ex ante* diversification ratio.

Weighted portfolio tail dependent coefficient (WPTD)

Similar to WPC, we can also define a portfolio tail dependent coefficient that takes into account of asset weights. However, the weighted portfolio tail dependent coefficient (WPTD) is more challenging than WPC. The reason is that we can't calculate a portfolio's tail risk (based on tail dependence) the same as we calculate a portfolio's variance. To follow a similar logic to how we define WPC, we define WPTD as:

$$\lambda_{average} = \frac{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\omega_i\omega_j\lambda_{ij}}{\sum_{i=1}^{N-1}\sum_{j>i}^{N}\omega_i\omega_j}$$

where,

λ_{ij} is the tail dependent coefficient between asset i and j.

Therefore, WPTD is essentially the weighted pairwise tail dependence, without taking into account asset volatility.

IV. Risk estimation and portfolio construction

Portfolio backtesting setup

For all portfolio backtestings in this paper, we follow the following procedures. All backtestings are completely out-of-sample.

All data used in this research are point-in-time. For example, we will use the point-in-time index constituents as our investment universe. All risk models are also point-in-time, either using commercial risk models or our own calculated covariance matrices.

All portfolios are rebalanced monthly. However, all risk models are computed daily, using typically a rolling one year (or five years) of daily returns. Portfolio performance is also measured daily, which is essential for downside risk metrics (e.g., tail dependence, CVaR, etc.) in particular.

All portfolios in this research paper are long only, fully invested. In some occasions, we build both portfolios without a maximum holding constraint (i.e., one or a few assets may dominate the portfolio) and with a maximum holding constraint. We will discuss the impact of constraints on portfolio performance.

Choices of risk models

Note that the above portfolio construction techniques require us to estimate the covariance matrix. In academic theory, covariance is typically assumed to be a given. In practice, we have to estimate the covariance matrix with limited data.

In most of our previous research, we use factor-based risk models in equities and some fairly naïve covariance matrices based on either the sample covariance matrix or the exponentially weighted moving averages for other asset classes. As discussed in Luo, *et al* [2010] and Luo, *et al* [2011], as the number of assets grows, the estimation error of the sample covariance matrix increases dramatically. Sample covariance matrices tend to suffer from too much estimation error, which could hurt portfolio performance.

In the equity portfolio management space, investors typically use factor models to predict the covariance matrix. There are three popular factor risk models: fundamental factor models (based on fundamental factors like value, momentum, quality, etc.), statistical risk models (based on statistic techniques like principle component analysis or factor analysis), and macroeconomic risk models (based on macroeconomic factors like inflation, industrial production, consumer confidence, commodities prices, etc.) In practice, most investors simply use a vendor supplied risk model, e.g., Barra, Axioma, Northfield, etc.

At the multi-asset portfolio level, however, using the sample covariance matrix (or some simple transformations like an exponentially weighted moving average) is still the common practice by many academics and practitioners. At the asset allocation level, we typically deal with a limited number of asset classes; therefore, the sample covariance matrix may not be as problematic as in the equity portfolios. In addition, factor-based risk models in the asset allocation space are not as well developed as in the equity space, with limited choice for vendor risk models. However, as shown in Luo, *et al* [2012], structured risk models can still reduce portfolio noise and improve performance.

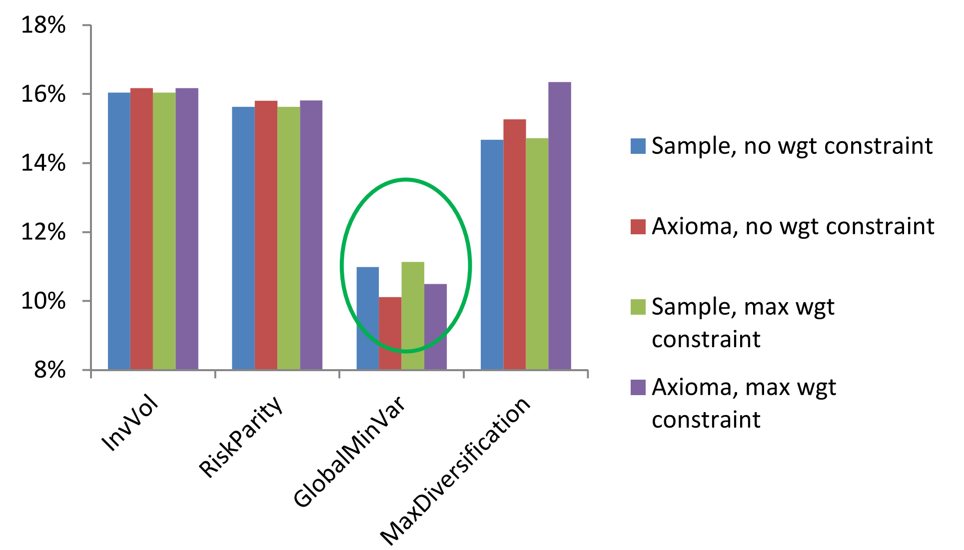
In this research, we make extensive comparisons between various risk models in both multi-asset and equity portfolio contexts.

Equity portfolios

This is where we would expect to see a clear benefit from a factor-based risk model[6]. Axioma's risk model shows much stronger numerical stability, especially for some of the more challenging optimizations like RiskParity. For most of the equity portfolios (e.g., US, European, Asia ex Japan, Japan, emerging markets, and global equities), we primarily use Axioma's medium horizon fundamental risk models for our portfolio backtesting in this paper[7].

Let's use US equities to demonstrate the impact of the risk models. Our investment universe is the MSCI USA index. We compare the performance of four risk-based allocations: InvVol, RiskParity, GlobalMinVar, and MaxDiversification using both the sample covariance matrix and Axioma's medium horizon fundamental risk model. As shown in Figure 7, the GlobalMinVar portfolio constructed using Axioma's risk model indeed delivers much lower realized volatility.

Figure 7: Realized volatility



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Country and sector risk models

When we construct country or sector portfolios, we can also leverage our equity risk models. Countries and sectors are nothing but portfolios of single stocks.

[6] This is also where factor-based risk models are most well developed and adopted.
[7] We also run the backtesting using sample covariance matrices. Please contact us for details.

Let's assume there are N_1 and N_2 stocks in country (or sector) i and country (or sector) j, respectively. Then the covariance matrix between country (or sector) i and country (or sector) j can be computed as:

$$\psi_{i,j} = (\omega_1, \omega_2, \cdots, \omega_{N1}) \begin{bmatrix} \sigma_{1,N1+1} & \sigma_{1,N1+2} & \cdots & \sigma_{1,N1+N2} \\ \sigma_{2,N1+1} & \sigma_{2,N1+2} & \cdots & \sigma_{2,N1+N2} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{N1,N1+1} & \sigma_{N1,N1+2} & \cdots & \sigma_{N1,N1+N2} \end{bmatrix} \begin{pmatrix} \omega_{N1+1} \\ \omega_{N1+2} \\ \cdots \\ \omega_{N1+N2} \end{pmatrix}$$

For real-world application, where we have multiple countries (or sectors), we can show that:

$$\psi_{i,j} = \omega_i' \Sigma_{i,j} \omega_j$$

where,

$\psi_{i,j}$ is the scalar covariance between country (or sector) i and country (or sector) j,

ω_i is the vector that contains the weight of each stock in country (or sector) i,

$\Sigma_{i,j}$ is the N_i (number of stocks in country/sector i) x N_j (number of stocks in country/sector j) block matrix from the big $N \times N$ stock-by-stock covariance matrix,

$N = \sum_{k=1}^{K} N_k$, and

K is the number of countries (or sectors) in our universe.

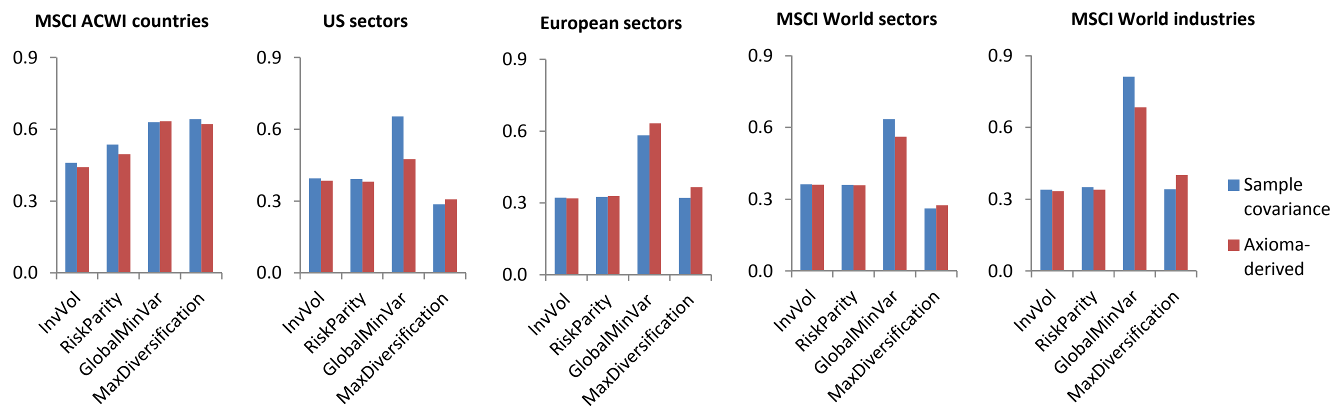
Performance comparison

In terms of Sharpe ratio, as shown in Figure 8, for country portfolios, the Axioma-based risk model and sample covariance matrix produce similar results. GlobalMinVar seems to be more sensitive to covariance matrix and benefit more from the sample covariance matrix.

In terms of downside risk and diversification benefit, the Axioma-based risk model appear to be able to deliver lower CVaR (see Figure 9) and comparable tail dependence (see Figure 11). However, portfolios constructed with sample covariance matrix seem to achieve higher diversification ratios consistently (see Figure 10).

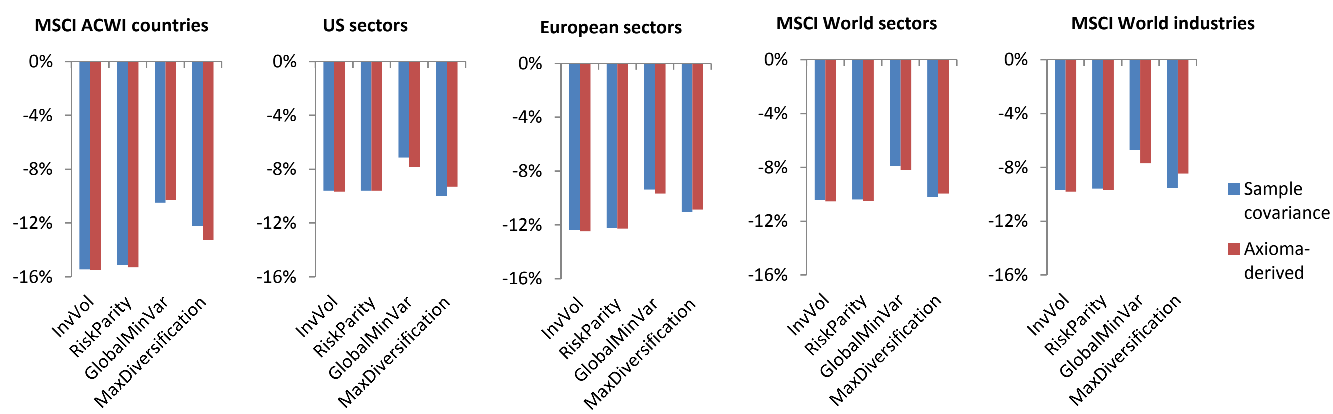
Since most macro investors do not have access to a factor-based stock risk model, for the rest of the paper, we use sample covariance matrices for all country/sector/industry portfolios. The results of using Axioma's risk model are available upon request.

Figure 8: Sharpe ratio



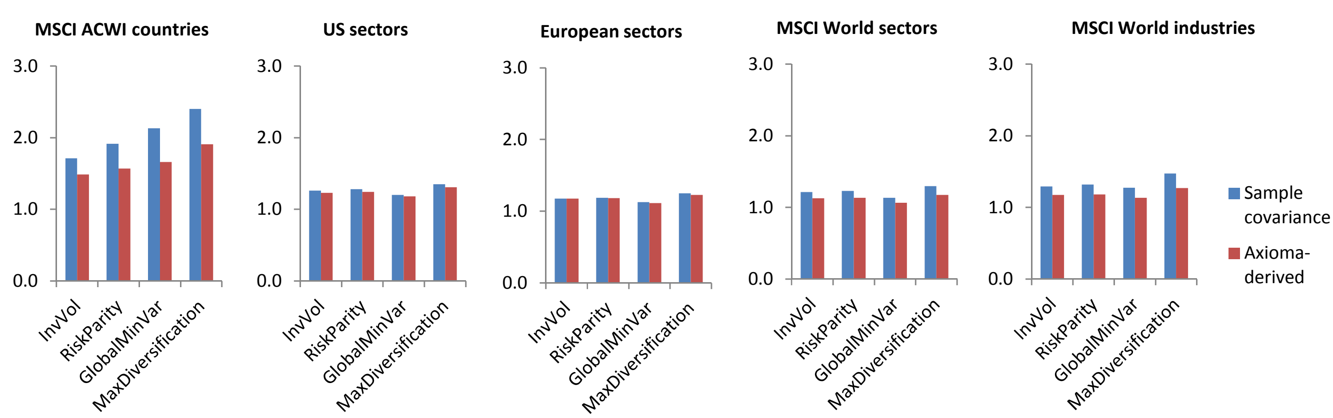
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 9: CVaR/expected shortfall



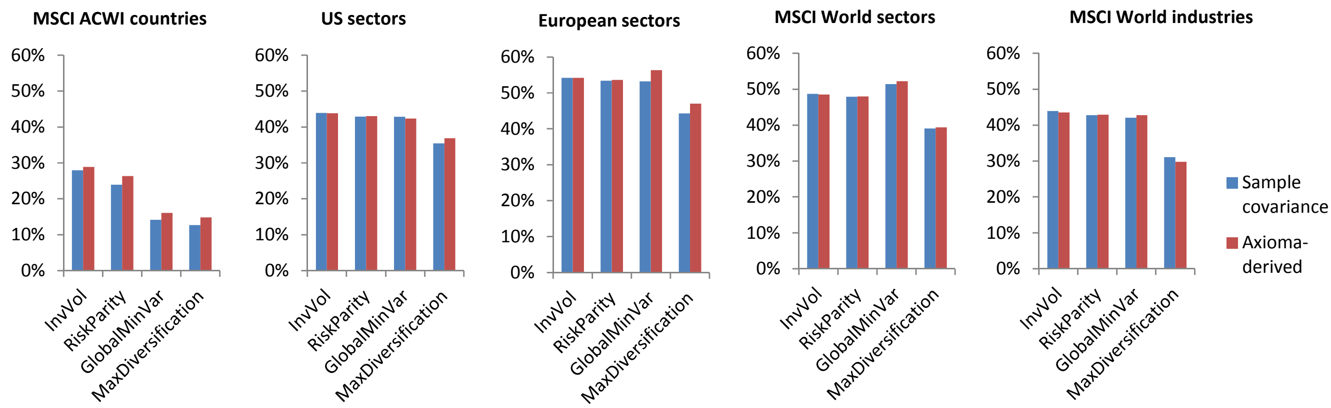
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 10: Average diversification ratio



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 11: Average weighted portfolio tail dependence



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Robust covariance estimation techniques

As alternatives to the sample covariance matrix and factor-based risk models, in this paper, we investigate five robust techniques to estimate the covariance matrix: minimum volume ellipsoid estimator (MVE), minimum covariance determinant estimator (MCD), orthogonalized Gnanadesikan-Kettenring estimator (OGK), shrinkage estimator (SHRINK), and bootstrap aggregation (BAGGED).

- Minimum volume ellipsoid (MVE) is definied in Venables and Ripley [2008].
- Minimum covariance determinant estimator (MCD) is defined in Rousseeuw [1985] and Rousseeuw and van Driessen [1999].
- Orthogonalized Gnanadesikan-Kettenring estimator (OGK) is defined in Gnanadesikan and Kettenring [1972] and Maronna and Zamar [2002].
- Shrinkage estimator (SHRINK) is defined in Schaefer, Opgen-Rhein, and Strimmer [2008], Ledoit and Wolf [2003].
- Bootstrap aggregation (BAGGED) is defined in Breiman [1996].

Figure 12 shows one simple example of the six countries (US, Germany, Greece, Italy, Portugal, and Spain). The lower triangle is based on sample covariance and correlation, while the upper triangle is based on the MVE robust covariance and correlation. We can see some notable differences. For example, the difference in correlation coefficient between US and Italy can differ by more than 10%.

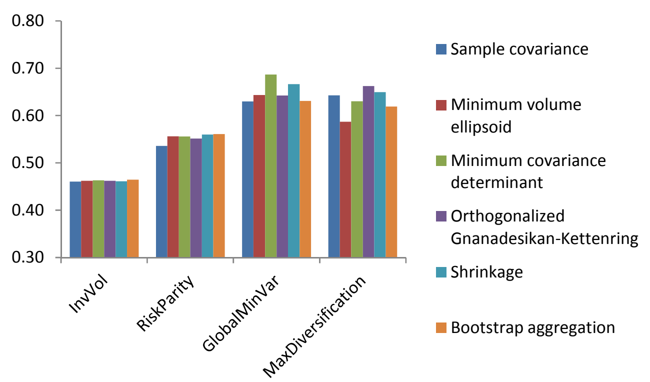
Figure 12: Correlation matrix (lower triangle = sample/upper triangle = MVE)

	US	Germany	Greece	Italy	Portugal	Spain
US	100%	70%	31%	65%	52%	60%
Germany	64%	100%	51%	90%	78%	86%
Greece	31%	58%	100%	53%	52%	55%
Italy	55%	90%	59%	100%	82%	90%
Portugal	47%	81%	61%	86%	100%	82%
Spain	54%	86%	59%	93%	86%	100%

Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

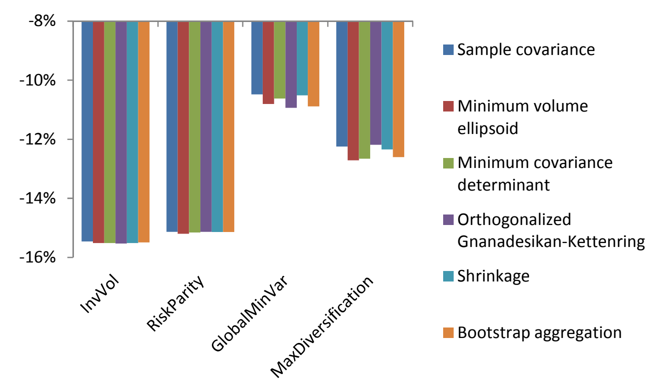
Let's use our MSCI ACWI country portfolios as an example to show the impact of robust covariance matrices in portfolio performance. As shown in Figure 13, the MCD and Shrinkage covariance matrices seem to produce the highest Sharpe ratio, especially for the GlobalMinVar portfolio. Almost all robust covariance matrices outperform the sample covariance matrix in terms of Sharpe ratio for InvVol, RiskParity, and GlobalMinVar. Robust covariance matrices also help us build more diversified portfolios (see Figure 15). On the other hand, robust covariance matrices do not seem to help to reduce downside risk (see Figure 14) or tail dependence (see Figure 16). Compared to InvVol and RiskParity, GlobalMinVar shows the best performance, but also seems to be more sensitive to covariance estimation.

Figure 13: Sharpe ratio



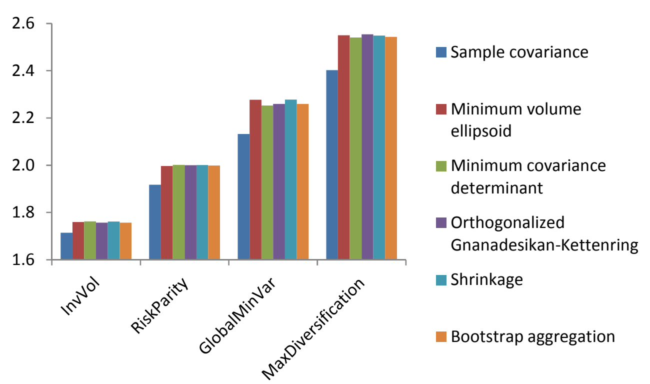
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 14: CVaR/expected shortfall



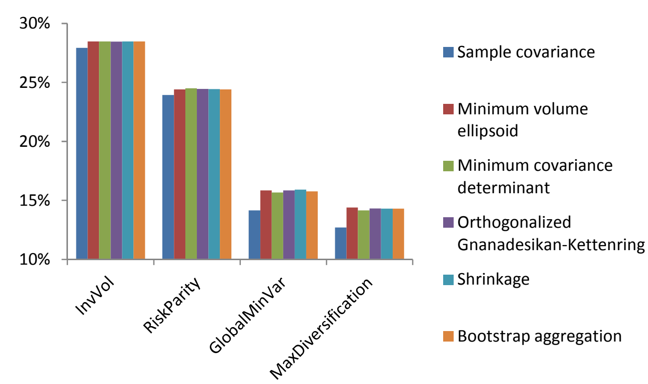
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 15: Average diversification ratio



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 16: Average weighted portfolio tail dependence



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

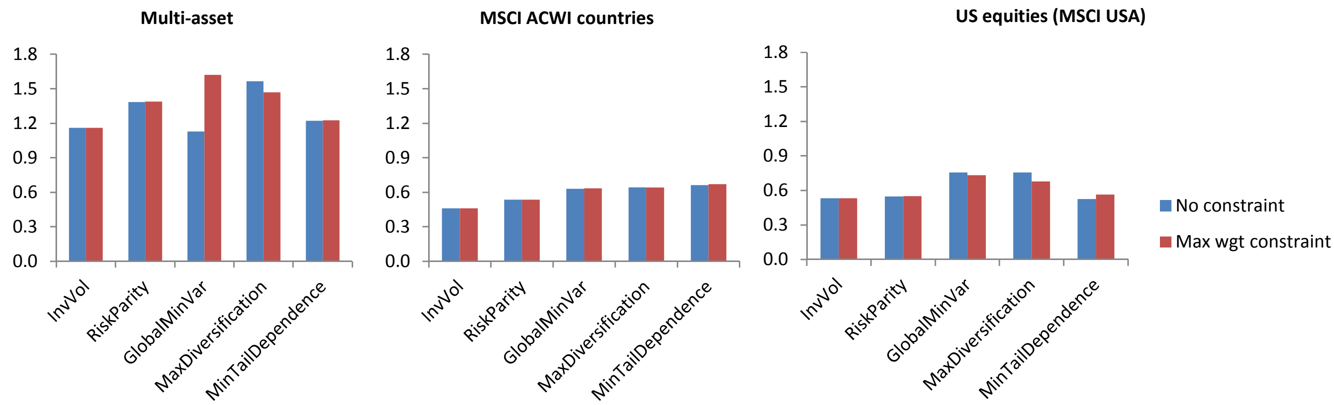
Constraints in portfolio construction

In all portfolio simulations in this research, portfolios are assumed to be long only and fully invested.

One of the most common constraints used in practice is a maximum holding constraint, i.e., the maximum weight of any single asset can't exceed a certain limit. The purpose of the maximum holding constraint is to reduce the exposure of a few highly concentrated positions. In this section, we use three simple examples to show the impact of the maximum holding constraint. We compare five portfolio construction techniques (InvVol, RiskParity, GlobalMinVar, MaxDiversification, and MinTailDependence) on three universes (multi-assets, countries, and equities).

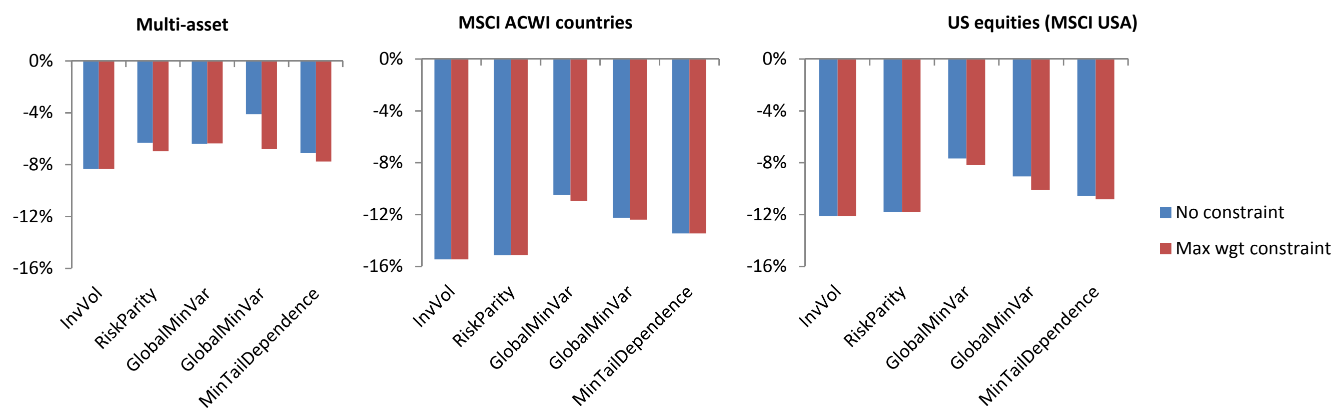
As show from Figure 17 to Figure 20, there is no clear benefit or damage in imposing a maximum holding constraint. Therefore, rather than data mining the "optimal" constraint, we focus on strategies that produce more "diversified" or less crowded portfolios for the rest of the research.

Figure 17: Sharpe ratio



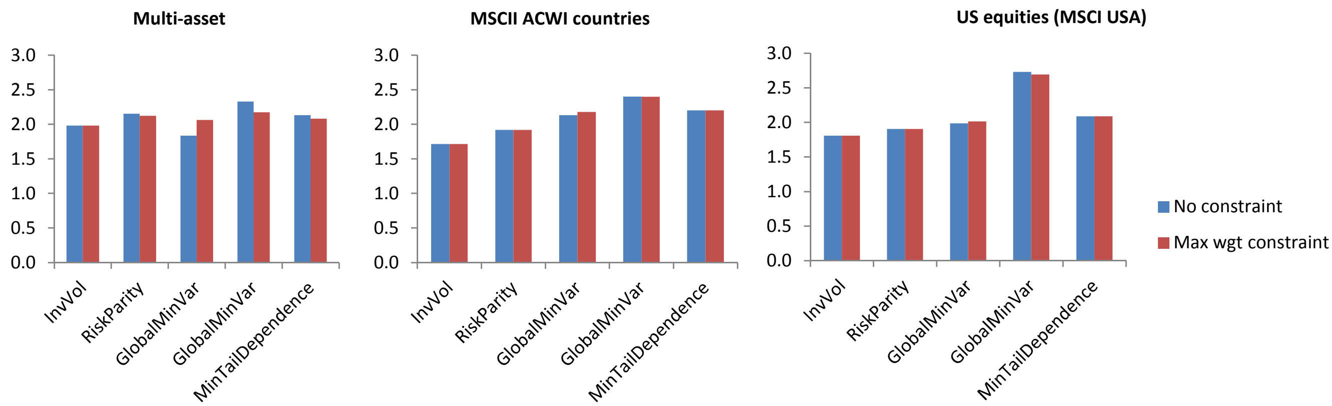
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 18: CVaR/expected shortfall



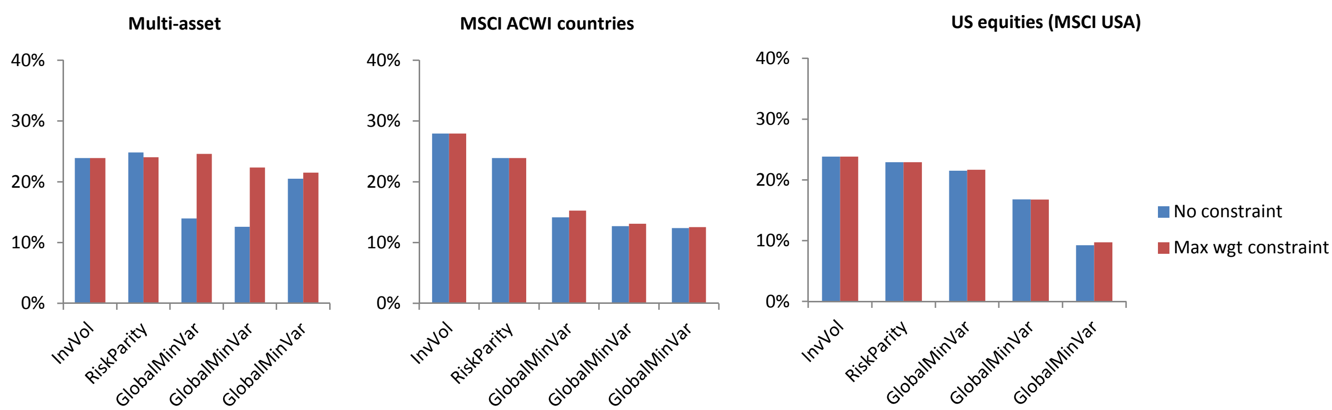
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 19: Average diversification ratio



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 20: Average weighted portfolio tail dependence



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

V. Minimum tail dependence portfolio

The purpose of MaxDiversification is to build a portfolio that is as diversified as possible, where diversification is measured by Pearson's correlation. The Pearson correlation coefficient only measures the dependence between two random variables correctly if they are jointly normally distributed. Empirically, asset returns are almost never jointly multivariate normally distributed. One may argue that the dependence in the left tail (e.g., the chance of both assets suffering extreme losses at the same time) is more relevant in risk management and portfolio construction.

Pfaff [2012] first introduced the minimum tail dependence portfolio (MinTailDependence) concept. Similar to MaxDiversification, with MinTailDependence, we try to build a portfolio that is as "diversified" as possible, where "diversification" is measured by tail dependence. In Cahan, *et al* [2012], we show how to use a copula model to measure tail dependence.

Introducing the copula model

The copula model was first introduced by Sklar [1959]. A more recent textbook explanation can be found in McNeil, *et al* [2005]. A copula models the dependence between assets in a multivariate distribution. Copula models allow for the combination of multivariate dependence with univariate marginals. Copula models gained popularity in the 2000s. For example, Li [2000] developed a well-known but controversial model for credit risk using exponentially distributed default times with a Gaussian copula, which was subsequently blamed for the 2008 financial crisis (see Salmon [2009]). Li's [2000] paper and the copula models used in the 2000s are more based on the Gaussian copula, which assumes no tail dependence[8]. Ironically, in the early days of applying copula models in credit risk, the real beauty of a copula (i.e., modeling tail dependence) was ignored, because the wrong type of copula (i.e., Gaussian copula) was chosen. The Gaussian copula assumes no tail dependence, which defeats the purpose of using a copula model in the first place.

In a non-technical sense[9],

$$\text{Joint Distribution} = \text{Copula} + \text{Marginal Distribution}$$

Therefore, a copula model gives us the flexibility to model joint asset return distributions. For example, we could fit an exponential GARCH model for each asset's marginal distribution, while at the same time, model the joint distribution using a t-copula model.

Figure 21 shows the difference between Pearson correlation and copula-based tail dependence. Tail dependence coefficients among our six countries are clearly higher than Pearson correlation coefficients – as expected, assets are more likely to fall at the

[8] More formally speaking, for any Gaussian copula where correlation coefficient less than 100%, tail dependence equals to zero.

[9] See Meucci, A. [2011], A short, comprehensive, practical guide to copulas, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1847864

same time than the average. Some differences are strikingly large. For example, the Pearson correlation between Portugal and Spain is only 56%, which is quite normal compared to other pairs of countries. However, the tail dependent coefficient is 86%, which is clearly on the high end.

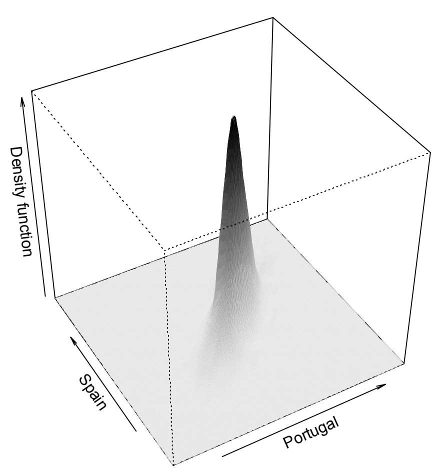
Figure 21: Correlation versus tail dependence (lower triangle = correlation/upper triangle = copula tail dependence)

	US	Germany	Greece	Italy	Portugal	Spain
US	100%	64%	31%	55%	47%	54%
Germany	53%	100%	58%	90%	81%	86%
Greece	25%	33%	100%	59%	61%	59%
Italy	44%	72%	36%	100%	86%	93%
Portugal	28%	53%	36%	58%	100%	86%
Spain	44%	64%	36%	81%	56%	100%

Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

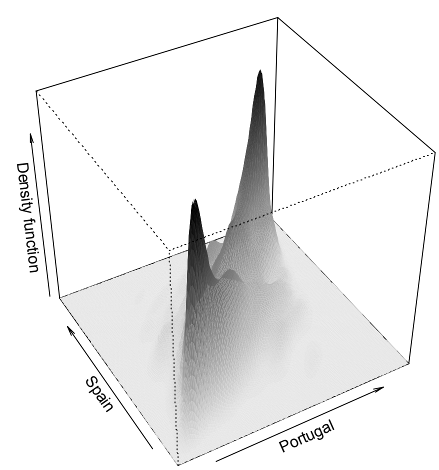
To visually examine how Pearson's correlation coefficient underestimates the true dependence, let's compare the theoretical bivariate normal distribution between Portugal and Spain (see Figure 22), with the empirical distribution (see Figure 23). The empirical distribution between Portugal and Spain is clearly bimodal with two distinct peaks (or modes), i.e., the probabilities of these two countries both move higher or fall lower are much higher than any other combinations.

Figure 22: Theoretical bivariate normal distribution



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Figure 23: Empirical distribution



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Minimum tail dependence portfolio optimization algorithm

Numerically, the MinTailDependence portfolio can be solved easily by minimizing $\psi' \mathrm{T} \psi$, where T is the tail dependence matrix. Therefore, the MinTailDependence optimization is almost exactly the same as MaxDiversification, by replacing the correlation matrix with tail dependence matrix. The final weights are then retrieved by rescaling the intermediate weight vector (optimized using the tail dependence matrix) with the standard deviations of the assets' returns.

Step 1

$$\arg\min_w \frac{1}{2}\psi_t' \mathrm{T}_t \psi_t$$

subject to:

$$\psi_t' \iota = 1$$
$$\psi_t \geq 0$$

where,

ψ_t is the first intermediate vector of asset weights at time t, and

T_t is the asset-by-asset tail dependence matrix at time t.

Step 2

Then we need to rescale the first intermediate vector of asset weights ψ_t by each asset's volatility $\sigma_{i,t}$:

$$\xi_t = D_t^{-1/2}\psi_t \text{ or } \xi_{i,t} = {\psi_{i,t}}/{\sigma_{i,t}}$$

where,

ξ_t is the second intermediate vector of asset weights at time t, and

D_t is the diagonal matrix of asset variance at time t with $\sigma_{i,t}^2$ at its i,i element and zero on all off-diagonal elements

Step 3

Finally, we rescale the second intermediate asset weight vector of the total weight, so the sum of the final weights equal to 100%, i.e., no leverage.

$$\omega_{i,t} = {\xi_{i,t}}/{\sum_{j=1}^{N}\xi_{j,t}}$$

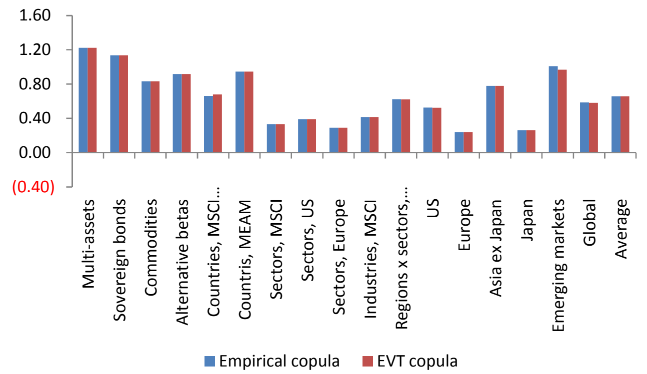
The choice of copula model – does it really matter?

There is a long list of copula models proposed in the academic literature. In this research, we test two copula models:

- Non-parametric estimation using the empirical tail copula
- Stable tail function

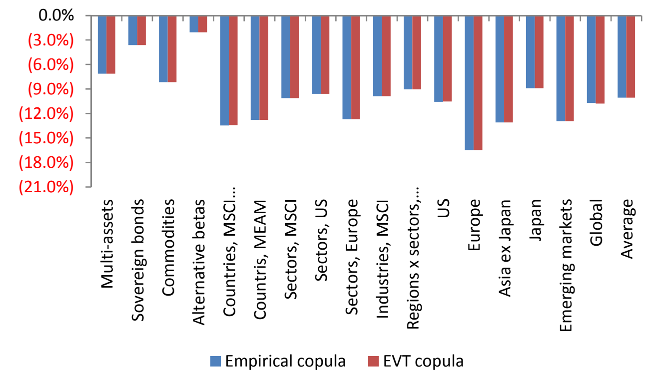
Interestingly, the results of the two copula model are almost identical (see Figure 24 and Figure 25). For the rest of this paper, we use the empirical tail copula as our main approach.

Figure 24: Sharpe ratio



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 25: CVaR/expected shortfall



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

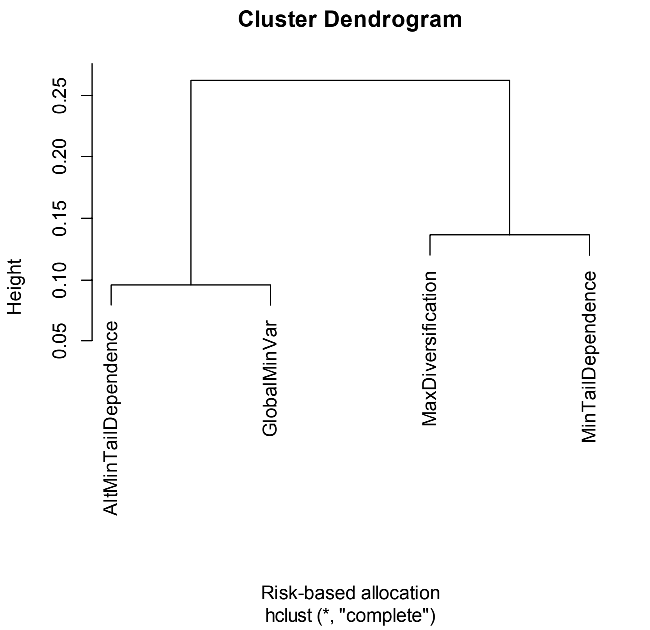
An alternative minimum tail dependence strategy

In this research, we also propose another form of MinTailDependence (hereafter called AltMinTailDependence). In the AltMinTailDependence set up, it is related to both MinTailDependence and GlobalMinVar. Essentially, we take the variance matrix (a diagonal matrix) and the tail dependence matrix (similar to correlation matrix, but replace pairwise correlation coefficient with tail dependent coefficient) to compute the covariance-tail dependence matrix. Finally, we perform a GlobalMinVar optimization using the covariance-tail dependence matrix.

As shown in Figure 26 and Figure 27, the MinTailDependence portfolio is similar to MaxDiversification, while the AltMinTailDependence approximates the GlobalMinVar portfolio.

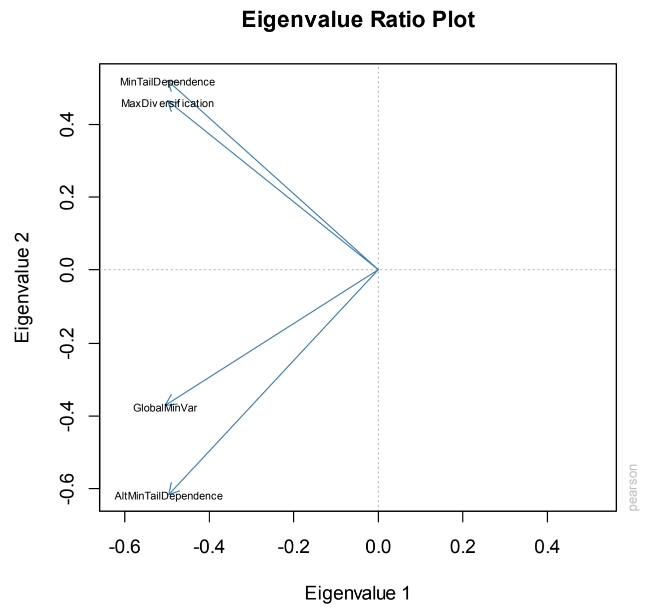
The AltMinTailDependence strategy offers higher Sharpe ratios (see Figure 28) and lower downside risk (see Figure 29). However, the MinTailDependence, which follows the similar philosophy as MaxDiversification, does offer better diversification, as measured by higher diversification ratio (see Figure 30) and lower tail dependence (see Figure 31). For the rest of the paper, we mainly use MinTailDependence.

Figure 26: Cluster analysis



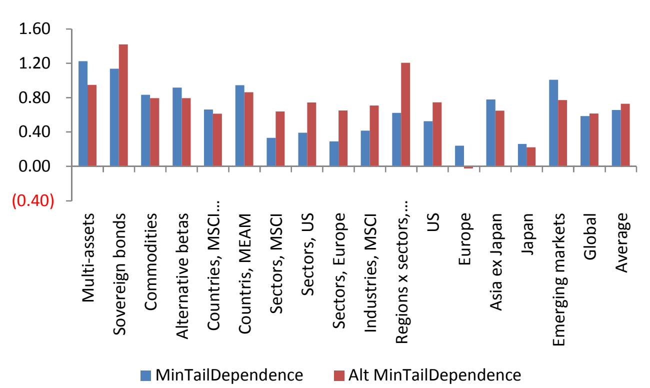
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 27: Eigenvalue ratio plot



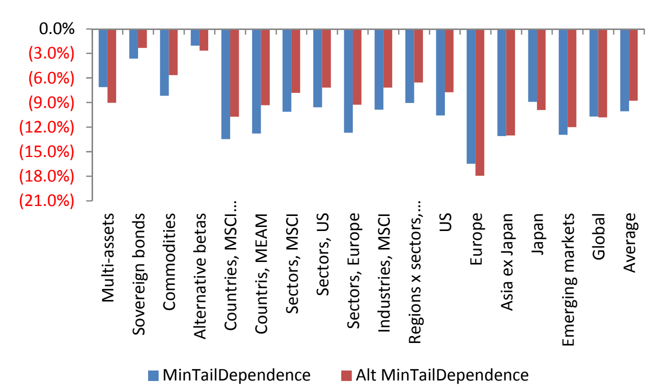
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 28: Sharpe ratio



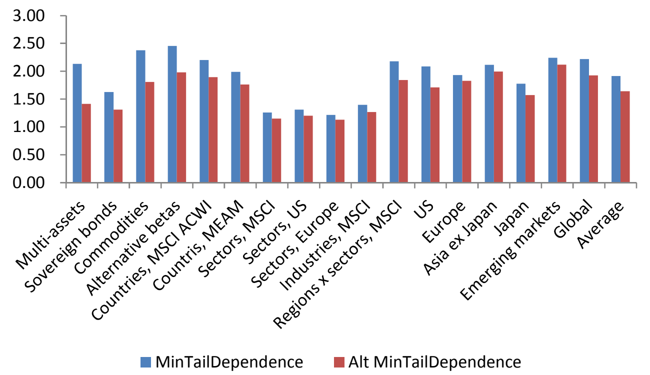
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 29: CVaR/expected shortfall



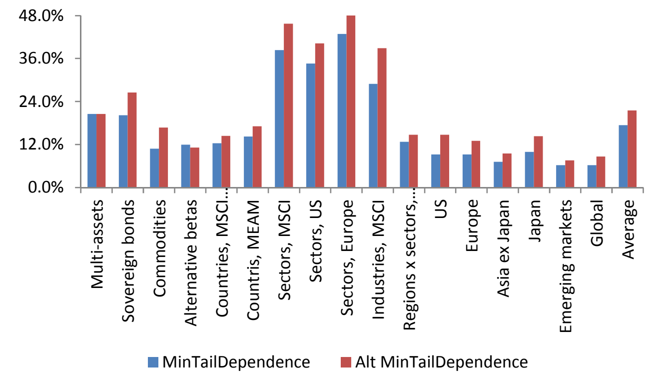
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 30: Average diversification ratio



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 31: Average portfolio tail dependence

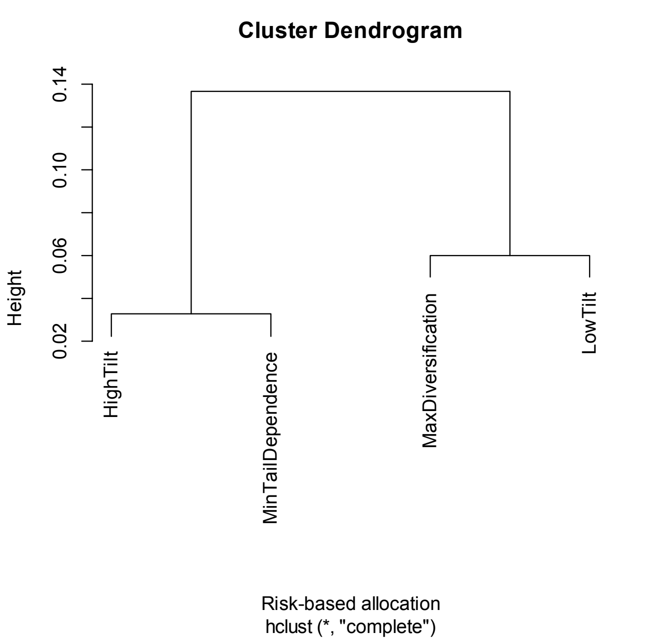


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Tail-dependence constraint

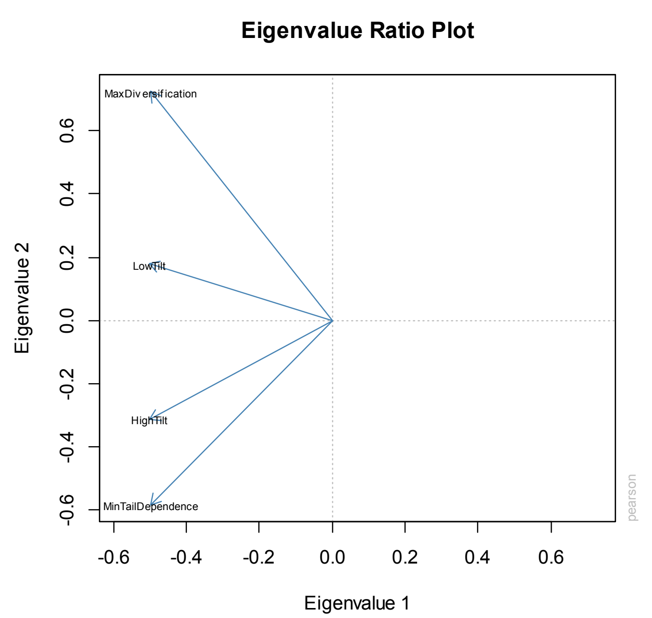
We don't have to strictly construct a MinTailDependence portfolio. Rather, we could add tail dependence as a constraint to, for example, the MaxDiversification portfolio. As a demonstration, we add a low (and a high) tilt in the MaxDiversification optimization towards tail dependence. As shown in Figure 32 and Figure 33, the more tilt we add towards the tail dependence, the constrained MaxDiversification portfolio more behaves like the MinTailDependence portfolio.

Figure 32: Cluster analysis



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 33: Eigenvalue ratio plot



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

VI. Minimum CVaR portfolio

CVaR or conditional value at risk is a statistical measure of tail risk, measured by assessing the likelihood (at a specific confidence level) that a specific loss will exceed the VaR. Mathematically speaking, CVaR is derived by taking a weighted average between the VaR and losses exceeding the VaR.

Rockafellar and Uryasev ([2000], [2001]) first introduced portfolio optimization with CVaR or conditional value at risk. In the risk management literature, VaR or value at risk is criticized as being an incoherent risk measure. On the other hand, CVaR, which is also referred as expected shortfall, is a coherent risk measure. In portfolio optimization, CVaR is a convex function, while VaR is not necessarily convex[10]. More importantly, as shown in Rockafellar and Uryasev [2001], CVaR optimization can be transformed into linear optimization, which tends to be easier to solve.

Despite the theoretical soundness of the CVaR methodology, in practice, we have to estimate CVaR empirically and face all the usual problems with estimation errors. It is the same trade-off between model error versus estimation error, i.e., we could have a better model, but may have more estimation error.

Please note that CVaR optimization is very flexible and powerful. In this research, we only explore a simple application of a minimum CVaR portfolio. In upcoming research, we will apply CVaR optimization in a more alpha-seeking context, i.e., maximizing expected returns subject to CVaR constraints.

CVaR optimization theory

Here we will only give a very brief description of the CVaR optimization problem and corresponding algorithm. A more detailed exposure can be found in Rockafellar and Uryasev ([2000], [2001]).

Basic definition of CVaR optimization

In the CVaR optimization setup, let's define ω as the vector of asset weights (i.e., our decision variable). The asset return distribution is defined by vector r. The loss function is further defined as $f(\omega, r)$. We assume random vector r has a probability density function of $p(r)$. The cumulative distribution function of the loss associated with a weight vector ω is therefore:

$$\Psi(\omega, r) = \int_{f(\omega, r) \leq \gamma} p(r) dr$$

For a given confidence level α, the VaR_α associated with portfolio W is:

$$VaR_\alpha = \min\{\gamma \in R : \Psi(\omega, r) \geq \alpha\}$$

Once VaR_α is defined, we can further define $CVaR_\alpha$ as:

[10] The problem with non-convex optimization is that we may get local optima instead of global optima. Therefore, a global optimizer is typically required, while global optimizations tend to be very slow.

$$CVaR_{\alpha}(\omega)=\frac{1}{1-\alpha}\int_{f(\omega,r)\leq VaR_{\alpha}(\omega)} f(\omega,r)p(r)dr$$

The minimum CVaR optimization can therefore be defined as:

$$\arg\min_{\omega} CVaR_{\alpha}(\omega)$$

Subject to:

$$\omega_t' \iota = 1$$
$$\omega_t \geq 0$$

The solution to CVaR optimization

First, let's develop a simple auxiliary function:

$$F_{\alpha}(\omega,\gamma)=\gamma+\frac{1}{1-\alpha}\int_{f(\omega,r)\geq\gamma}(f(\omega,r)-\gamma)p(r)dr=\gamma+\frac{1}{1-\alpha}(f(\omega,r)-\gamma)^{+}p(r)dr$$

where,

$$(f(\omega,r)-\gamma)^{+}=\max((f(\omega,r)-\gamma),0)$$

The $CVaR_{\alpha}$ optimization can be done by optimizing the $F_{\alpha}(\omega,\gamma)$ with respect to the weight vector ω and VaR_{γ}.

Discretization

It is generally more desirable to approximate the continuous joint density function $p(r)$ with a number of discrete scenarios, e.g., r_s for $s=1,2,\cdots,S$, which typically represent historical returns. Then we can transform the $F_{\alpha}(\omega,\gamma)$ function into:

$$\hat{F}_{\alpha}(\omega,\gamma)=\gamma+\frac{1}{(1-\alpha)S}\sum_{s=1}^{S}(f(\omega,r_s)-\gamma)^{+}$$

Now, the minimum CVaR problem can be approximated by:

$$\arg\min_{\omega}=\gamma+\frac{1}{(1-\alpha)S}\sum_{s=1}^{S}(f(\omega,r_s)-\gamma)^{+}$$

Let's further introduce an artificial variable z_s to replace $(f(\omega,r_s)-\gamma)^{+}$. Then the minimum CVaR problem can be transformed into:

$$\arg\min_{\omega}=\gamma+\frac{1}{(1-\alpha)S}\sum_{s=1}^{S}z_s$$

subject to:

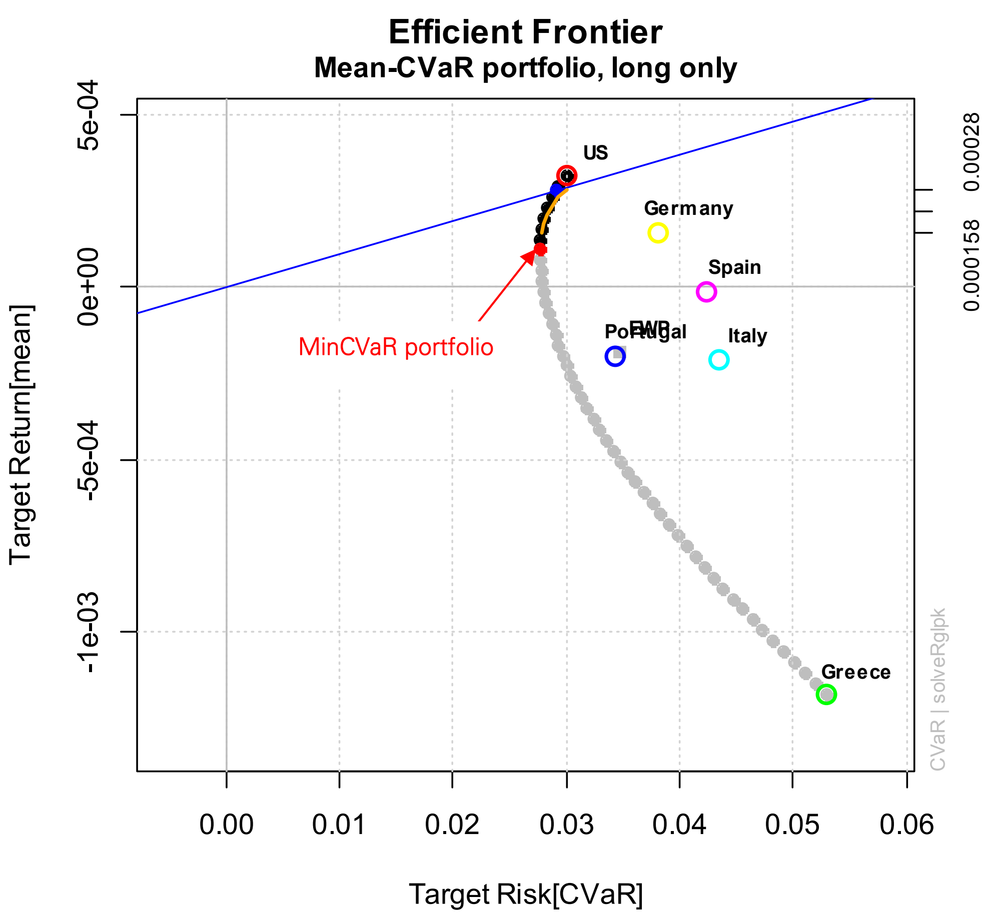
$$z_s \geq f(\omega,r_s)-\gamma$$
$$z_s \geq 0$$

It is typically further assumed that $f(\omega,r_s)$ is a linear function of ω. Then, the minimum CVaR optimization problem can be solved by linear programming algorithms.

Mean-CVaR efficient frontier and minimum CVaR portfolio

Similar to the mean-variance efficient frontier, we can define the mean-CVaR efficient frontier as a hyperbola containing portfolios with the following characteristics: for given level of risks (defined as CVaR), they have the highest expected returns. The portfolio that separates the efficient frontier from the lower border of the feasible set is the global minimum CVaR portfolio (i.e., the MinCVaR portfolio).

Figure 34: Mean-CVaR efficient frontier

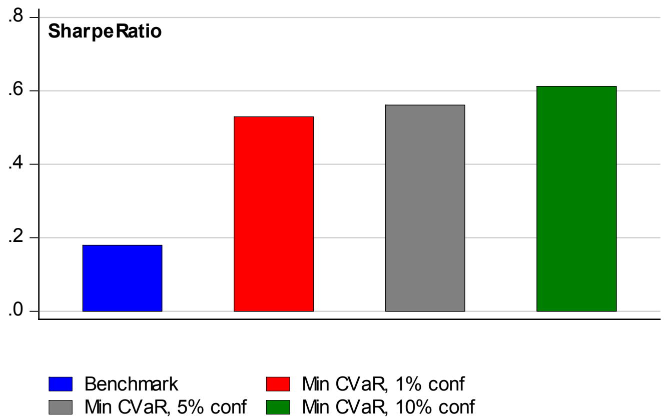


Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Choice of alpha parameter

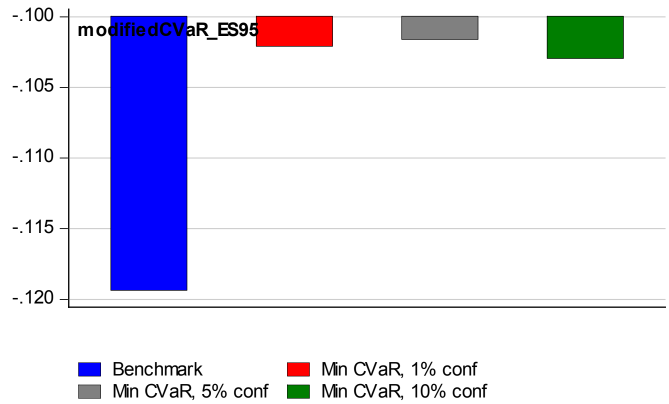
In MinCVaR optimization, one of the key input parameters is the confidence level α. The choice of α is more art than science. Therefore, let's first experiment with three different levels of α and study the impact of parameter sensitivity. We use our country portfolio as an example by investing in the 45 countries comprising the MSCI ACWI. We further set the α at 1%, 5%, and 10%. As shown in Figure 35 and Figure 36, the MinCVaR portfolio is not very sensitive to the choice of α. More importantly, all three portfolios significantly outperform the capitalization weighted benchmark.

Figure 35: Sharpe ratio



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Figure 36: CVaR/expected shortfall



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

Robust minimum CVaR optimization

In this paper, we propose a new optimization technique that we shall call "robust minimum CVaR optimization" or RobMinCVaR. To the best of our knowledge, this is probably the first serious attempt at combining the philosophy of robust optimization and CVaR optimization in one integrated framework. We borrow the ideas from robust optimization (see Michaud [1998]) and traditional CVaR optimization above.

Test of multivariate normal distribution

Despite the popularity of mean-variance optimization in academic research and professional investment management, one of the key assumptions under MVO (mean-variance optimization) is the multivariate normal distribution. As shown in previous sections, financial assets almost never follow a multivariate normal distribution. The mean-CVaR optimization relaxes the distribution assumption and therefore is far more flexible than MVO.

Fitting a multivariate skew-t distribution

To fully account for the nature of non-multivariate normal distribution in asset return data, we fit our 45-country return data at each month end, using five-years of rolling daily returns to a multivariate skew-t distribution. The family of multivariate skew-t distributions is an extension of the multivariate Student's t family, via the introduction of a shape parameter which regulates skewness. The fits are done using maximum likelihood estimation (see Azzalini and Capitanio [1999, 2003]).

Robust minimum CVaR optimization

We simulate 50 time series of the same five years of daily country returns with the above fitted multivariate skew-t distribution, for these 45 countries at each month end. Then we can construct 50 MinCVaR portfolios – one for each simulated data. We then further construct our final portfolio using three approaches:

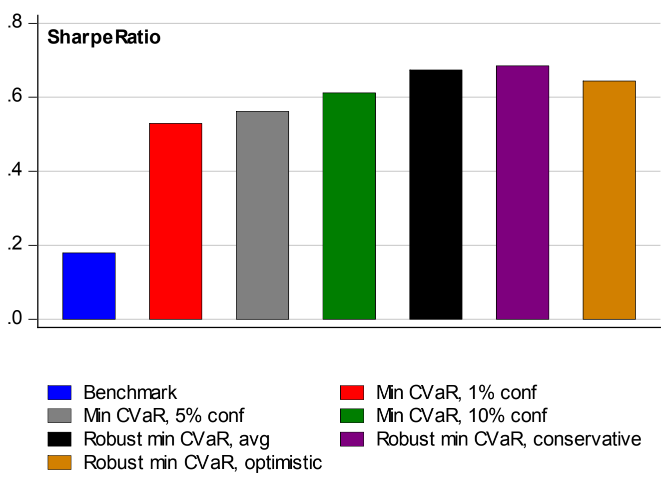
- Average (RobMinCVaR-Avg): we simply average the weights of the 50 asset weight vectors;
- The most conservative portfolio (RobMinCVaR-Conservative): for each of the 50 MinCVaR portfolios, we calculate the expected CVaR, then we take the portfolio with the lowest CVaR, i.e., the worst case scenario portfolio, as our final portfolio; and

- The most optimistic portfolio (RobMinCVaR-Optimistic): for each of the 50 MinCVaR portfolios, we calculate the expected CVaR, then we take the portfolio with the highest CVaR, i.e., the most optimistic case scenario portfolio, as our final portfolio

In the following simulation, we fix α at the 10% level. All three RobMinCVaR portfolios outperform the traditional MinCVaR strategy, with higher Sharpe ratios (see Figure 37) and slightly higher downside risks (see Figure 38). The RobMinCVaR-Conservative portfolio, in particular, shows a decent Sharpe ratio. Indeed, even compared to all other risk-based allocation techniques, the RobMinCVaR portfolio shows the highest Sharpe ratio (see Figure 39) and second lowest downside risk (see Figure 40).

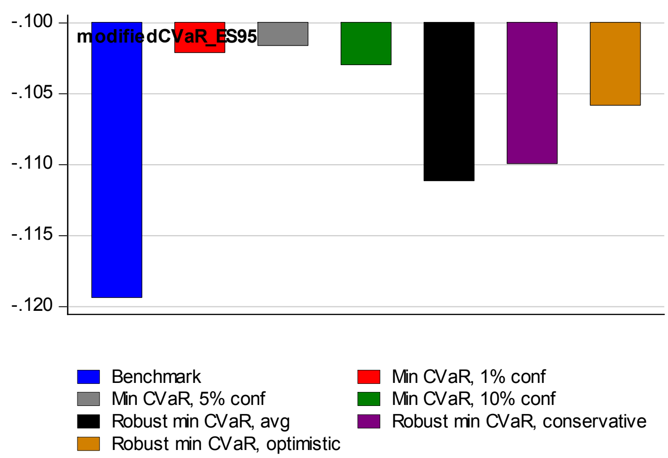
The biggest challenge in our RobMinCVaR optimization is computational speed. It can be slow even when the number of assets is modest. For example, with 45 countries, it takes about 30 seconds per period. If we run 50 simulations over the past 14 years, it can take around three days for a complete backtesting[11].

Figure 37: Sharpe ratio



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

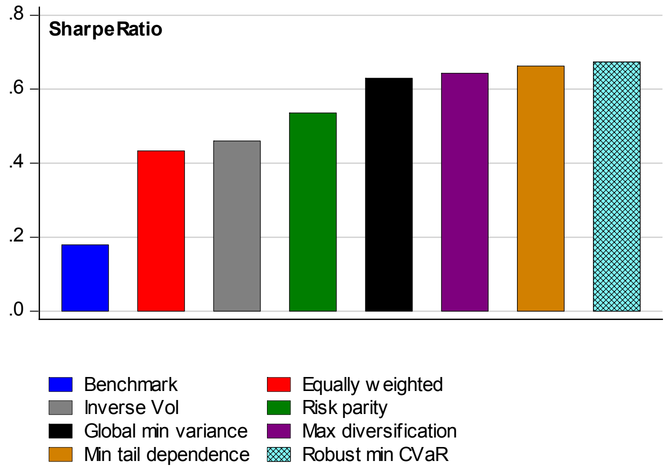
Figure 38: CVaR/expected shortfall



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

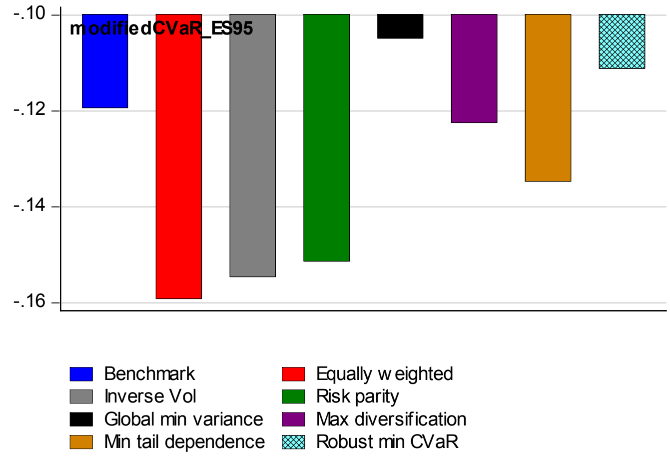
[11] Numerical and computational issues will be addressed in Section VII on page 40.

Figure 39: Sharpe ratio – risk-based allocations



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 40: CVaR/expected shortfall



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

How is RobMinCVaR related to other risk-based portfolio construction techniques

In this section, we introduce two interesting statistical tools that can help us better understand various portfolio construction techniques and see how they relate to each other.

Cluster analysis

Cluster analysis is a suite of statistical algorithms that divide data into groups (called clusters) in such a way that objects in the same cluster are more similar (in some sense or another) to each other than to those in other clusters.

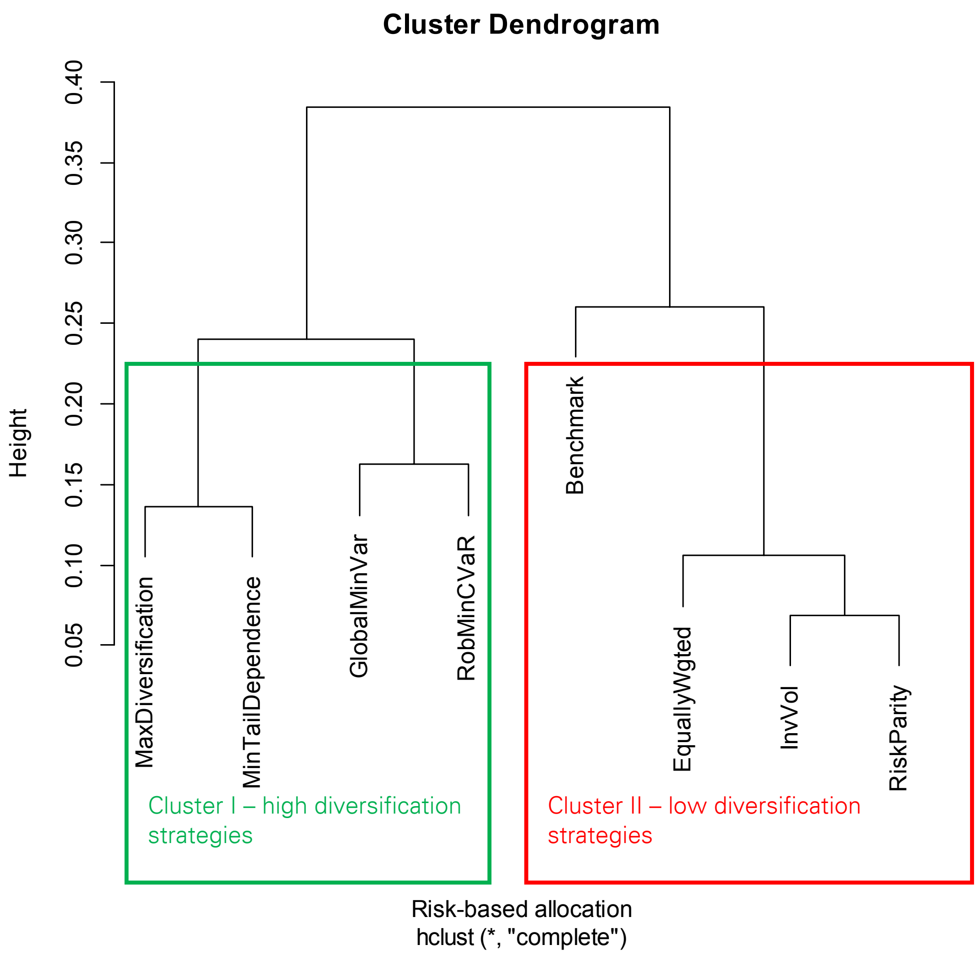
One particular form of clustering algorithm that is useful for our purpose is hierarchical clustering. The core idea is that objects are more related to nearby objects than to objects farther away. These algorithms connect "objects" to form "clusters" based on their distance. At different distances, different clusters will form to a dendrogram or an upside-down tree. In a dendrogram, the vertical-axis marks the distance at which the clusters merge, while the objects are placed along the horizontal-axis.

As shown in Figure 41, our clustering algorithm essentially divides our risk-based strategies (and benchmark portfolio) into two broad clusters:

- **Cluster I – low diversification strategies (red).** On the one hand, capitalization weighted benchmark (i.e., MSCI ACWI), EquallyWgted, InvVol, and RiskParity portfolios form one cluster. Within this cluster, clearly, InvVol and RiskParity are closer to each other, then joined with EquallyWgted, and finally joined with the benchmark.
- **Cluster II – high diversification strategies (green).** On the other hand, MaxDiversification and MinTailDependence portfolios are similar, which further joined by GlobalMinVar and RobMinCVaR portfolios.

It is also interesting to note that our RobMinCVaR is more closely linked to GlobalMinVar, as both try to achieve better risk reduction. RobMinCVaR emerges to deliver better *ex post* performance. On the other hand, MinTailDependence resembles MaxDiversification, since both attempt to obtain better diversification.

Figure 41: Cluster analysis



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

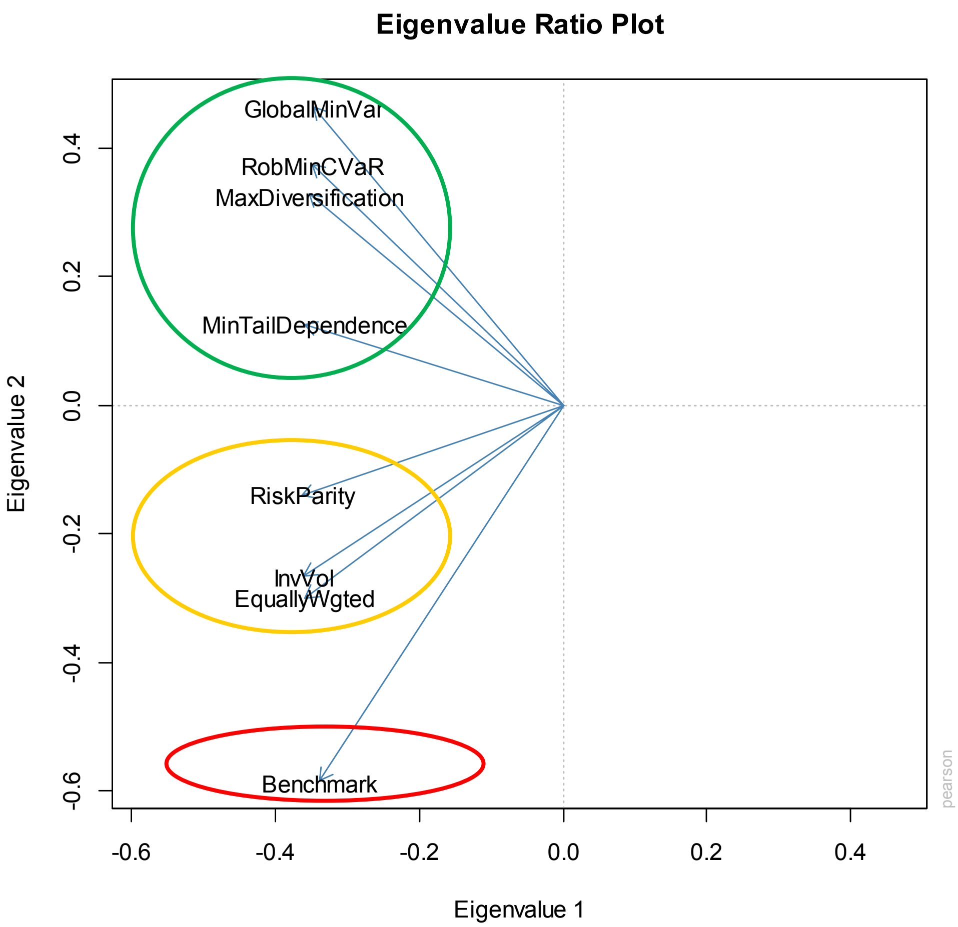
Eigenvalue ratio test

Another approach to group strategies together is based on eigenvalue analysis. We calculate the first two eigenvectors of the correlation matrix and then plot their components against the x and y directions.

Similar to the cluster analysis graph, the eigenvalue ratio chart in Figure 42 classifies our strategies into three buckets:

- **Cluster I – benchmark (red).** The capitalization weighted benchmark (i.e., MSCI ACWI) seems to form a cluster of its own.
- **Cluster II – low diversification strategies (yellow).** On the one hand, InvVol and EquallyWgted portfolios are closer to each other, then joined by the RiskParity portfolio.
- **Cluster III – high diversification strategies (green).** On the other hand, GlobalMinVar, RobMinCVaR, and MaxDiversification portfolios are similar, which further joined by MinTailDependence portfolio.

Figure 42: Eigenvalue ratio chart



Source: Bloomberg Finance LLP, MSCI, Deutsche Bank Quantitative Strategy

VII. Numerical and computational issues

Numerical and computational issues are rarely addressed in academic research. It is typically assumed that there are only a few assets involved in the optimization, data is well behaved, and there are no convergence issues.

In practice, for those researchers who do code their own optimization algorithms, we know it is a completely different matter. We have to choose a solver for a specific task (e.g., a linear programming solver, a quadratic programming solver, a nonlinear programming solver, a mixed integer programming solver, or a global optimization solver). Depending on the problem at hand, we may need to use a mix of different solvers. As discussed in Luo, *et al* [2011b], finding a set of good initial values (i.e., initial guess of what the weight vector looks like) is critical for both convergence and computing time.

Many of the optimization problems require an asset-by-asset covariance matrix (e.g., InvVol, RiskParity, GlobalMinVar, MaxDiversification). If we have to compute the covariance matrix using sample data, sometimes the covariance matrix is not positive definite. A simple solution is the find the closest matrix that is positive definite, using the Higham [1988] algorithm. A yet better approach is to use either a factor-based risk model or a robust covariance matrix.

Most academic research focuses on unconstrained optimization, while in practice, we typically have a maximum holding constraint. There could be more constraints, e.g., maximum turnover, minimum weight, maximum number of holdings. Some constraints are easier to handle. For example, maximum/minimum weight per asset, maximum/minimum weight by some grouping (e.g., sector, industry, country) are typically set as box constraints, which can be solved easily. On the other hand, the maximum turnover constraint is a nonlinear constraint and will require a nonlinear solver, which tends to be very slow. A maximum number of holdings constraint is even more difficult, because it requires a mixed integer solver.

The following table shows the approximate time for different number of assets with monthly rebalance over approximately 14 years, using eight-core 2.5G CPU for different optimization under Linux environment using multicore packages in R[12].

Figure 43: Numerical computing time (for complete 14 year backtest)

Pptimization	MaxDiversification	GlobalMinVar	MinTailDependence	RiskParity
500 assets	20 minutes	20 minutes	30 minutes	50 minutes
1600 assets	2.5 hours	2.5 hours	6.5 hours	14 hours

Source: Deutsche Bank Quantitative Strategy

Among the four optimizations (GlobalMinVar, RiskParity, MaxDiversification, and MinTailDependence), Risk Parity takes the most time compared to the other optimization techniques, MinTailDependence takes more time than the MaxDiversification and GlobalMinVar.

[12] See www.r-project.org

The computational time grows exponential with the number of assets. For example, for the Risk Parity optimization, 1600 assets take more than 15 times longer than 500 assets.

For the more computationally difficult problems, e.g., RiskParity, the computing time grows even faster as the number of asset increases. For example, RiskParity takes 2.5x longer than MaxDiversification for 500 assets, but takes more than 5x longer with 1600 assets.

The most time consuming optimization is CVaR optimization, especially our RobMinCVaR optimization, as portfolio resampling/simulation are required.

VIII. Country portfolios

Let's use country equity portfolio as our first example. Our sample includes 45 countries comprising the MSCI All Country World Index[13] (see Figure 44 for the full list of countries). We use daily total returns[14] in USD[15] to perform all portfolio backtests below. All strategies are monthly rebalanced.

Figure 44: MSCI ACWI

Developed countries		
Americas	**EMEA**	**Asia**
Canada	Austria	Australia
USA	Belgium	Hong Kong
	Denmark	Japan
	Finland	New Zealand
	France	Singapore
	Germany	
	Greece	
	Ireland	
	Israel	
	Italy	
	Netherlands	
	Norway	
	Portugal	
	Spain	
	Sweden	
	Switzerland	
	UK	
Emerging markets		
Americas	**EMEA**	**Asia**
Brazil	Czech Republic	China
Chile	Egypt	India
Colombia	Hungary	Indonesia
Mexico	Morocco	Korea
Peru	Poland	Malaysia
	Russia	Philippines
	South Africa	Taiwan
	Turkey	Thailand

Source: MSCI, Deutsche Bank Quantitative Strategy

Comparison of risk-based allocation

First, let's compare the performance of various risk-based portfolio construction techniques to the benchmark, MSCI ACWI[16]. In this section, we show all major

[13] This is further to extend our previous research in Luo, et al [2012]. "New insights in country rotation" and Luo, et al [2013]. "Independence day".
[14] Dividends are re-invested on the ex-dividend dates.
[15] The strategies are therefore currency unhedged.
[16] All analyses below use sample covariance matrix as the risk model. Results with other risk models are qualitatively similar and available upon request.

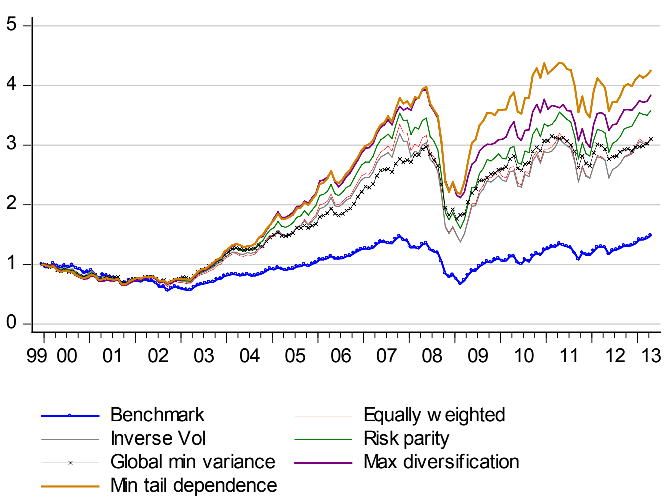
performance metrics: Sharpe ratio (see Figure 46), adjusted Sharpe ratio (see Figure 47), Sortino ratio (see Figure 48), worst monthly return (see Figure 49), value at risk (see Figure 50), and conditional value at risk/expected shortfall (see Figure 51), for completeness[17]. For most other portfolio backtests in the next few sections, we will only show some key statistics, e.g., Sharpe ratio, CVaR/expected shortfall.

For our universe of 45 countries, all three performance metrics (Sharpe ratio, adjusted Sharpe ratio, and Sortino ratio) are consistent – we see a nice monotonic increase from the benchmark portfolio, EquallyWgted, InvVol, RiskParity, to the more sophisticated techniques like GlobalMinVar, MaxDiversification, and MinTailDependence.

Interestingly, all four measures of downside risk are also consistent, indicating GlobalMinVar has the lowest tail risk.

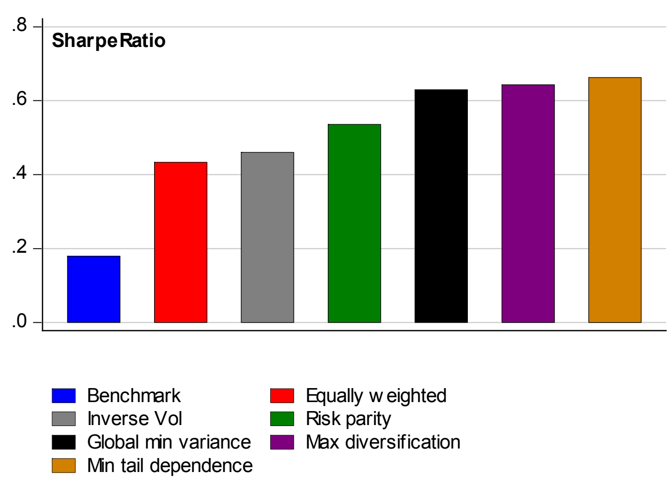
Figure 53 and Figure 54 show some basic statistics of portfolio concentration and diversification. More refined analysis about portfolio crowding and efficacy will be addressed at a later section.

Figure 45: Wealth curve



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

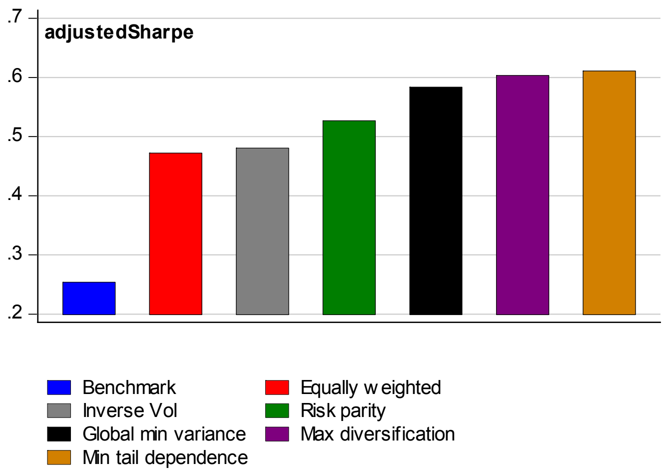
Figure 46: Sharpe ratio



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

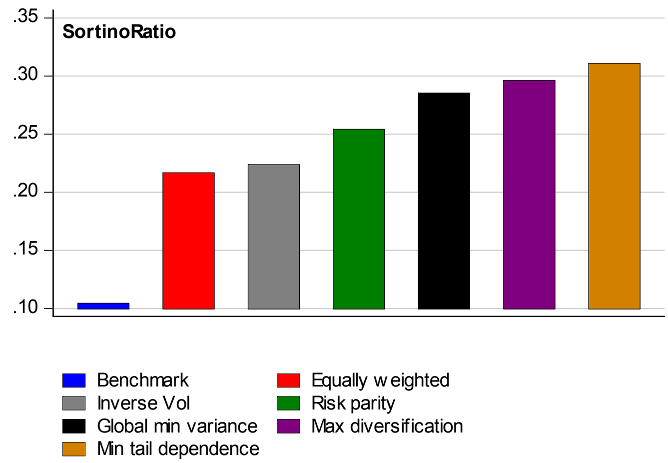
[17] We actually have a much more complete list of statistics for each strategies mentioned in this paper. Please contact us for details.

Figure 47: Adjusted Sharpe ratio



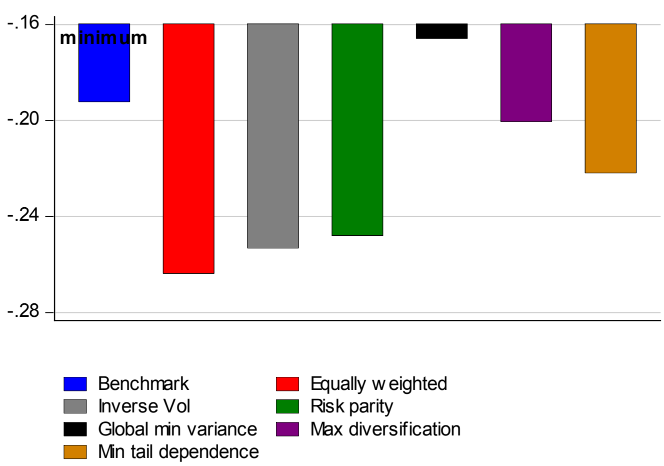
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 48: Sortino ratio



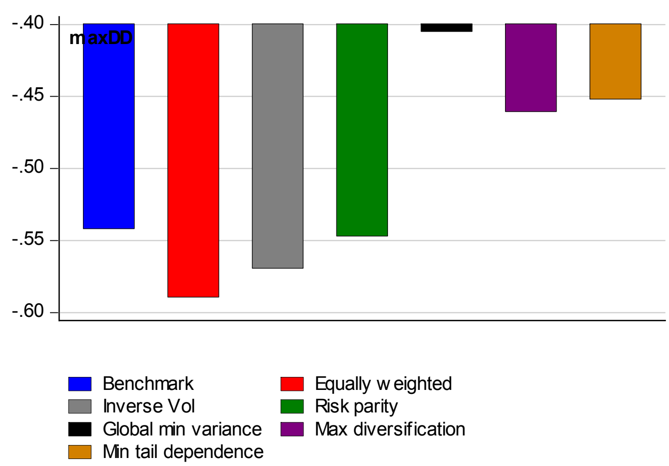
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 49: Worst monthly return



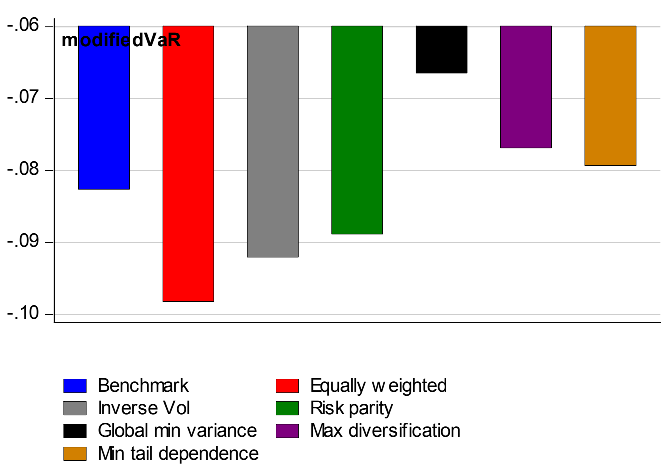
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 50: Maximum drawdown



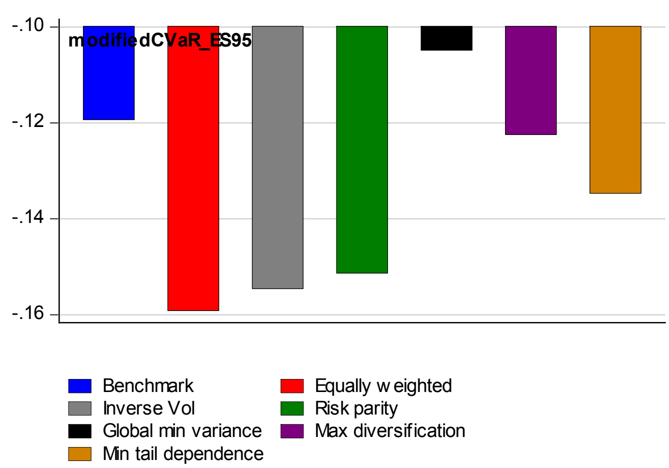
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 51: Value at risk



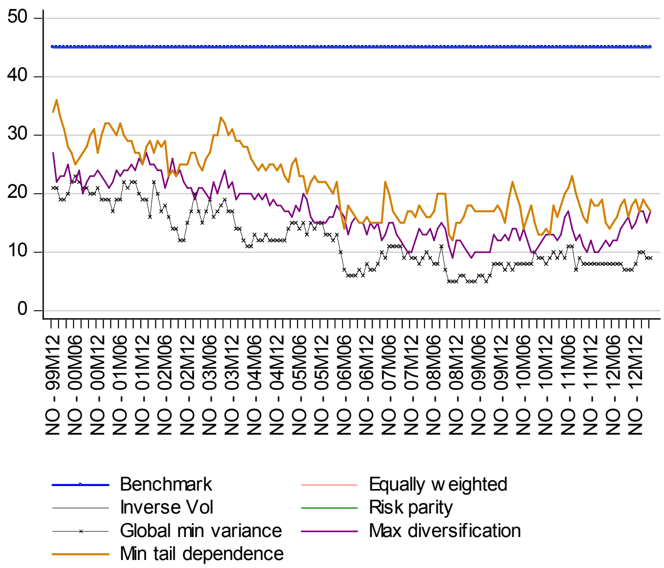
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 52: Conditional value at risk/expected shortfall



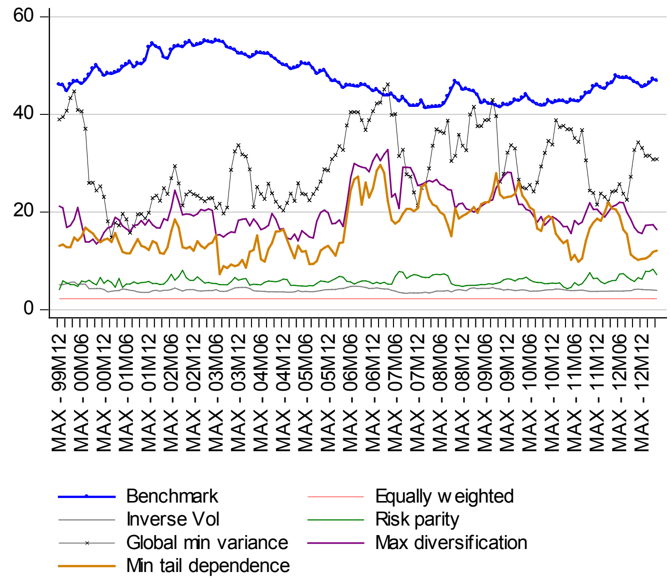
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 53: # of assets in the portfolios



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 54: Highest weight

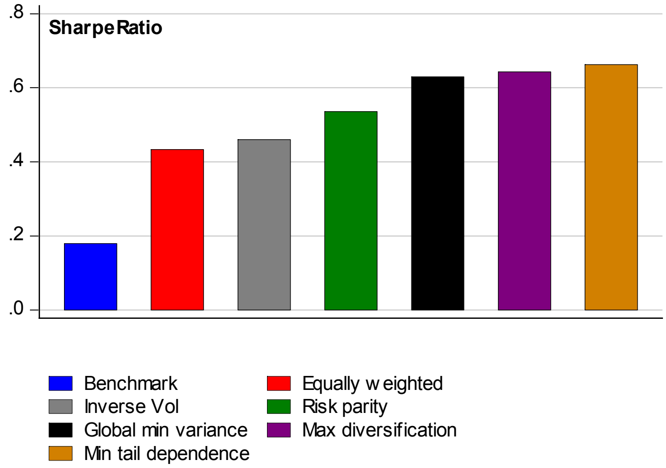


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Fully capturing the benefit of the low risk anomaly at the stock level

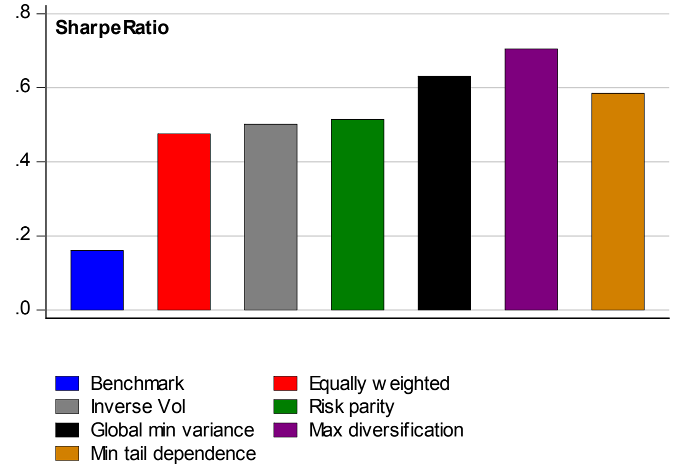
Interestingly, our risk-based country portfolios capture almost all the benefit of the so-called traditional low risk anomaly, which typically is constructed with single stocks. As shown from Figure 55 to Figure 58, the Sharpe ratio of our country portfolios is very much in line with the ones realized using global developed equities (i.e., MSCI World) and emerging markets (i.e., MSCI EM). Managing a portfolio of countries can be achieved by investing in country futures, ETFs, or custom swaps, which tends to be much easier (operationally simpler) than an equity portfolio. A related question is whether we can replicate the same low risk equity portfolios using sectors/industries. However, as shown in Figure 58 (and more details in Section X on page 70), sector/industry portfolios tend to be less effective than country portfolios. It appears that country effect still dominates the sector/industry effect.

Figure 55: Sharpe ratio – country portfolios



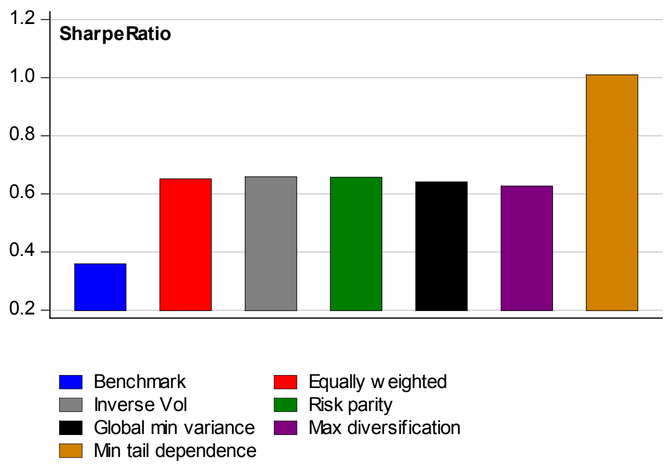
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 56: Sharpe ratio – global equities (MSCI World)



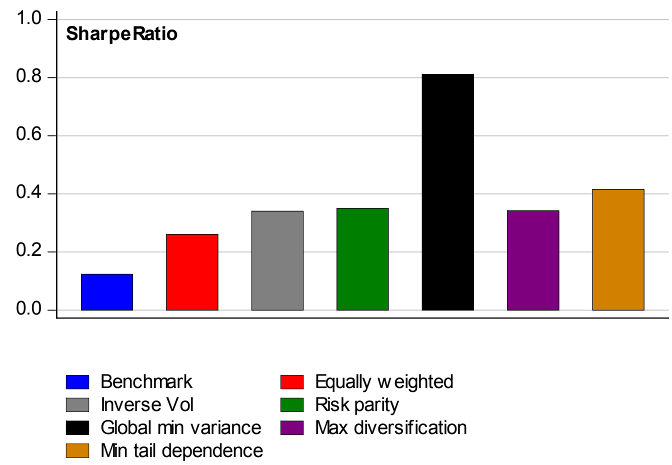
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 57: Sharpe ratio – emerging markets equities



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 58: Sharpe ratio – global industry groups



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Measure of crowdedness – portfolio correlation based

If we measure the crowdedness of a particular strategy based on weighted portfolio correlation (WPC), then the MaxDiversification is by definition (*ex ante*) the least crowded portfolio.

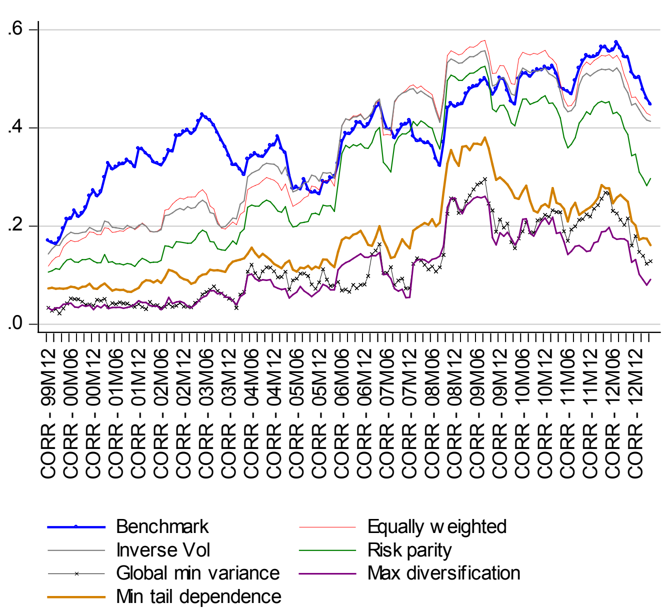
It is very interesting to see from Figure 59 to Figure 60 that the seven portfolios seem to form two groups.

- **Group I – low diversification strategies.** On the one hand, the capitalization weighted benchmark, EquallyWgted, and RiskParity form one group. Within this group, the weighted portfolio correlation is high and diversification ratio is low.
- **Group II – high diversification strategies.** On the other hand, the GlobalMinVar portfolio, the MaxDiversification portfolio, and the minimum tail dependence portfolio form another group, where the weighted portfolio correlation is low and diversification ratio is high.

The diversification ratio (actually, the square of DR – we call it DR2) is a very useful measure. Choueifaty, Froidure, and Reynier [2011] suggest that DR2 is equal to the number of independent risk factors (or degrees of freedom), represented in the portfolio. Therefore, as of April 30, 2013, the capitalization weighted benchmark (MSCI ACWI) had a DR2 of $1.35^2 = 1.82$, implying that a passive index investor would have been effectively exposure to 1.8 independent risk factors – a highly undiversified portfolio. On the other hand, the MaxDiversification portfolio had a DR2 of $2.27^2 = 5.15$, which suggests that the benchmark misses out on the opportunity to effectively diversify across more than three additional independent risk factors.

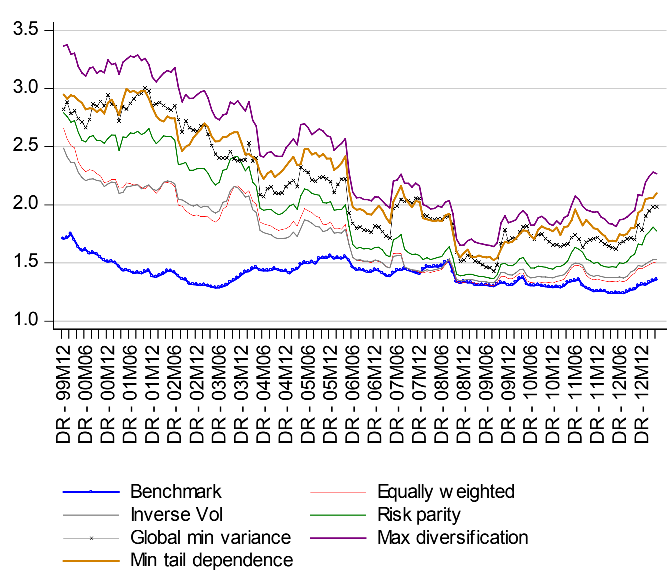
Figure 59 and Figure 60 show the weighted portfolio correlation (WPC) and diversification ratio (DR) of various risk-based strategies. The market's WPC and DR also changes over time. Therefore, another way to gauge the level of "crowdedness" or the potential of "diversification benefit" can be shown in Figure 61 and Figure 62, i.e., the relative WPC and DR to the benchmark. The Group II (i.e., MaxDiversification, GlobalMinVar, and MinTailDependence) portfolios clearly offer superior diversification benefit than the Group I (i.e., benchmark, EquallyWgted, InvVol, and RiskParity) strategies.

Figure 59: Weighted portfolio correlation



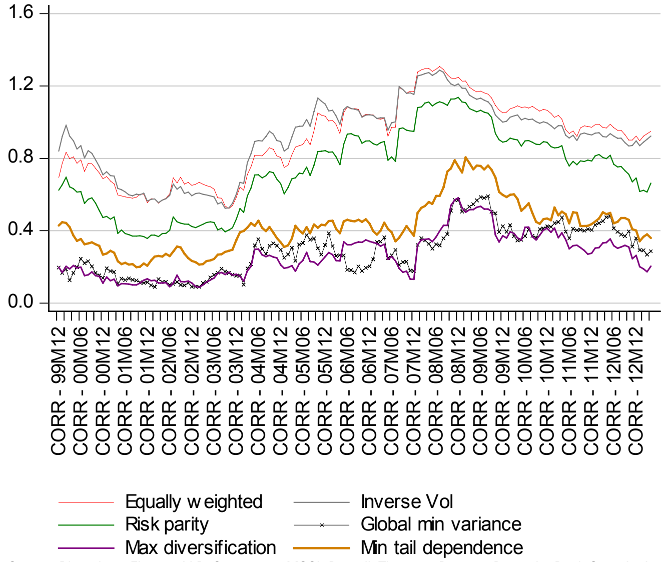
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 60: Diversification ratio



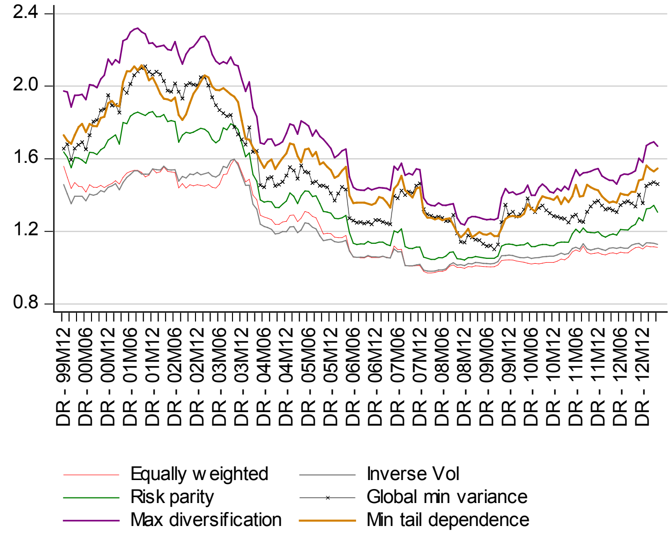
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 61: Relative WPC



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 62: Relative DR



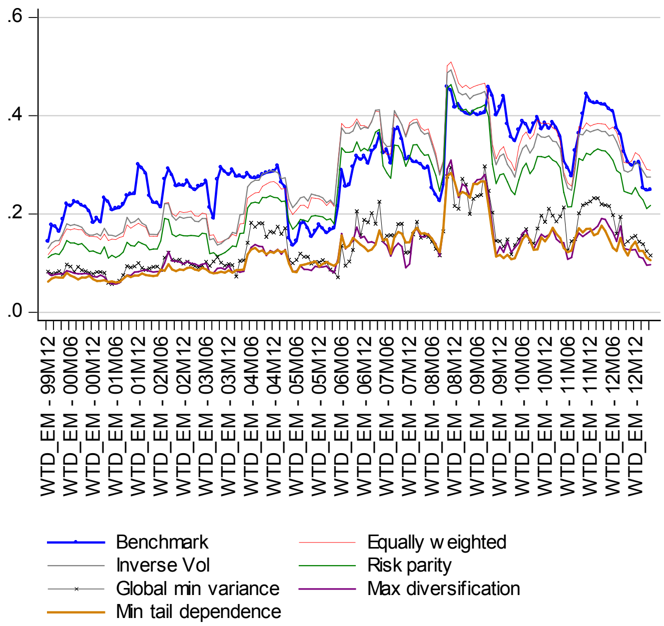
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Measure of crowdedness – portfolio tail dependence based

If we measure the crowdedness of a particular strategy based on the weighted portfolio tail dependence (WPTD), then the MinTailDependence portfolio is by definition (*ex ante*) the least crowded portfolio.

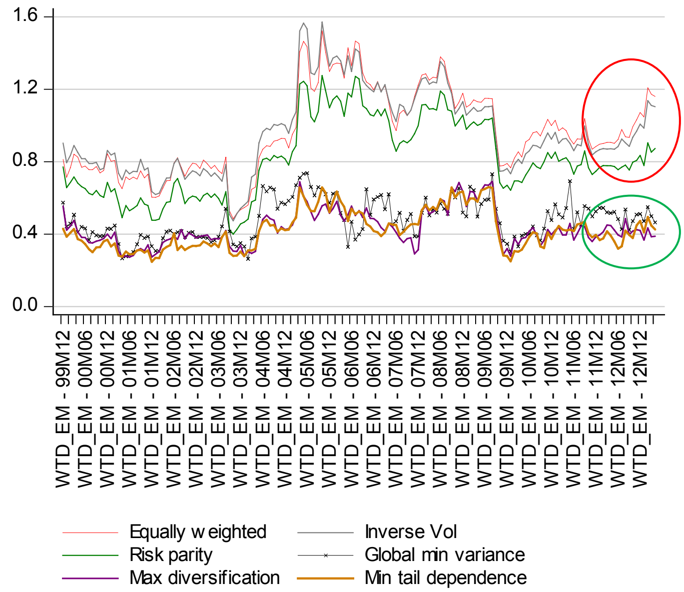
From Figure 63 and Figure 64, we see essentially the same two clusters as measured by WPC or DR. More interestingly, as shown in Figure 64, the EquallyWgted and InvVol at times had even higher tail dependence than the benchmark (the red circled strategies). The more diversified strategies (GlobalMinVar, MaxDiversification, and MinTailDependence portfolios) offer much better chance of avoiding crowded trades (the green circled strategies).

Figure 63: Weighted portfolio tail dependence



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 64: Relative WPTD



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

How to group these strategies

Holding correlation and overlap

Another way to investigate correlation is to see how their holdings are correlated, i.e., whether they hold the same assets with the same weights. Figure 65 shows the holding correlation as of April 30, 2013. Holding correlation seems to suggest that these strategies are quite different. There is some modest correlation among the risk-based allocations: RiskParity, GlobalMinVar, MaxDiversification, and MinTailDependence. However, these risk-based strategies seem to be uncorrelated to the capitalization weighted benchmark and inverse volatility portfolio.

Figure 65: Holding correlation

	Benchmark	InverseVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Benchmark	100.0%					
InverseVol	30.4%	100.0%				
RiskParity	12.8%	48.1%	100.0%			
GlobalMinVar	37.6%	56.6%	65.3%	100.0%		
MaxDiversification	38.9%	13.2%	79.4%	61.0%	100.0%	
MinTailDependence	32.1%	27.8%	73.0%	61.3%	81.8%	100.0%

Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

A more robust way to measure portfolio overlap is simply to see whether these strategies hold similar assets. Figure 66 shows the holding overlap as of April 30, 2013. As shown in Figure 66, the capitalization weighted benchmark, InvVol, and RiskParity portfolios hold essentially the same assets, but this fact is by construction, given the definitions of InvVol and RiskParity. The other risk-based strategies seem to hold very different assets.

Figure 66: Holding overlap

	Benchmark	InverseVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Benchmark	100.0%					
InverseVol	100.0%	100.0%				
RiskParity	100.0%	100.0%	100.0%			
GlobalMinVar	20.0%	20.0%	20.0%	100.0%		
MaxDiversification	37.8%	37.8%	37.8%	36.8%	100.0%	
MinTailDependence	37.8%	37.8%	37.8%	36.8%	54.5%	100.0%

Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

The holding based analysis can, however, be misleading. Holding different assets does not automatically indicate that their performances are uncorrelated.

Performance correlation

A more meaningful way to measure strategy correlation is to show how correlated these strategies perform over time (see Figure 67). Since they are all long-only/no-leverage portfolios, their performances are highly correlated. We also see that capitalization weighted benchmark, EquallyWgted, InvVol, and RiskParity portfolios seem to form one group, while GlobalMinVar, MaxDiversification, and MinTailDependence seem to form another group.

Figure 67: Performance correlation

	Benchmark	EquallyWgted	InverseVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Benchmark	100.0%						
EquallyWgted	95.1%	100.0%					
InverseVol	95.0%	99.8%	100.0%				
RiskParity	93.1%	99.3%	99.5%	100.0%			
GlobalMinVar	80.7%	87.8%	89.1%	91.2%	100.0%		
MaxDiversification	83.0%	91.6%	91.9%	94.7%	95.3%	100.0%	
MinTailDependence	87.1%	95.5%	95.7%	97.5%	93.3%	97.6%	100.0%

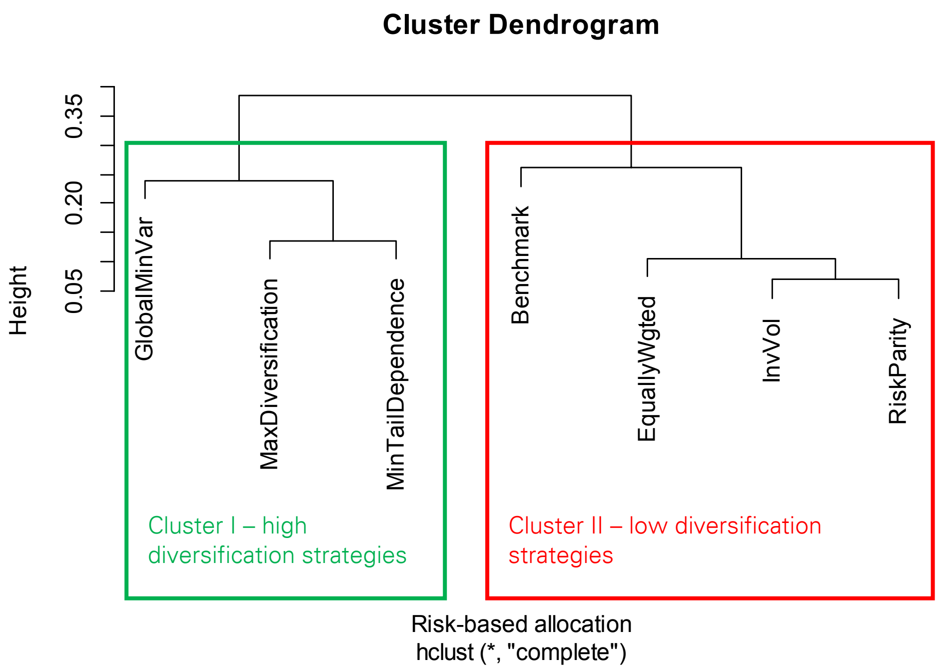
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Cluster analysis

As shown in Figure 68, our clustering algorithm essentially divides our risk-based strategies (and benchmark portfolio) into two broad clusters:

- **Cluster I – low diversification strategies (red).** On the one hand, capitalization weighted benchmark (i.e., MSCI ACWI), EquallyWgted, InvVol, and RiskParity portfolios form one cluster. Within this cluster, clearly, InvVol and RiskParity are closer to each other, then joined with EquallyWgted, and finally joined with the benchmark.
- **Cluster II – high diversification strategies (green).** On the other hand, MaxDiversification and MinTailDependence portfolios are similar, then further joined by the GlobalMinVar portfolio.

Figure 68: Cluster analysis



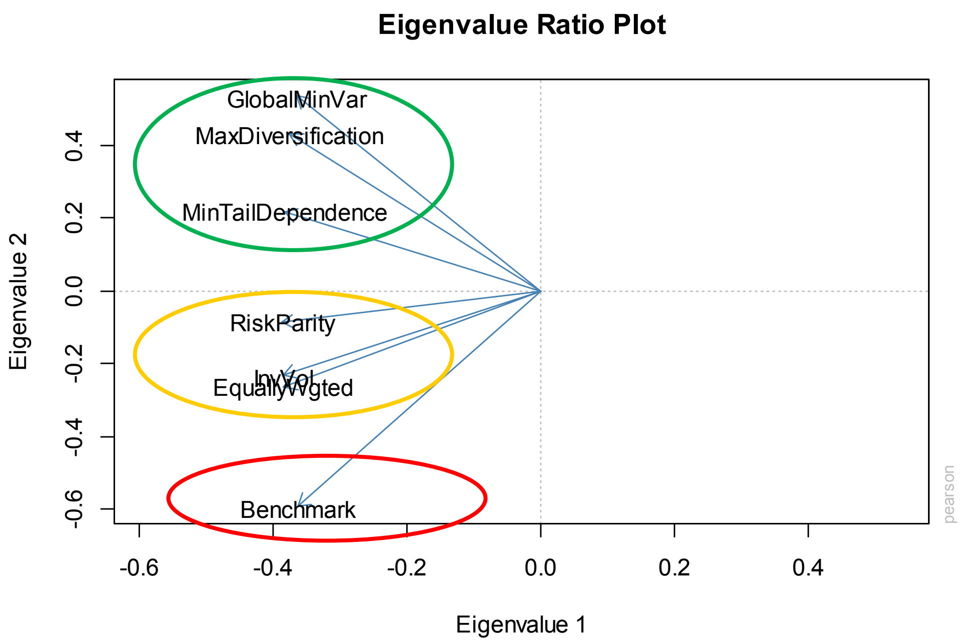
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Eigenvalue ratio test

Another approach to group strategies together is based on eigenvalue analysis. Similar to the cluster analysis graph, the eigenvalue ratio chart in Figure 69 classifies our strategies into three buckets:

- **Cluster I – benchmark (red).** The capitalization weighted benchmark (i.e., MSCI ACWI) seems to form a cluster of its own.
- **Cluster II – low diversification strategies (yellow).** On the one hand, InvVol and EquallyWgted portfolio are closer to each other, then joined with RiskParity portfolio.
- **Cluster III – high diversification strategies (green).** On the other hand, MaxDiversification and GlobalMinVar portfolios are similar, then further joined by the MinTailDependence portfolio.

Figure 69: Eigenvalue ratio chart

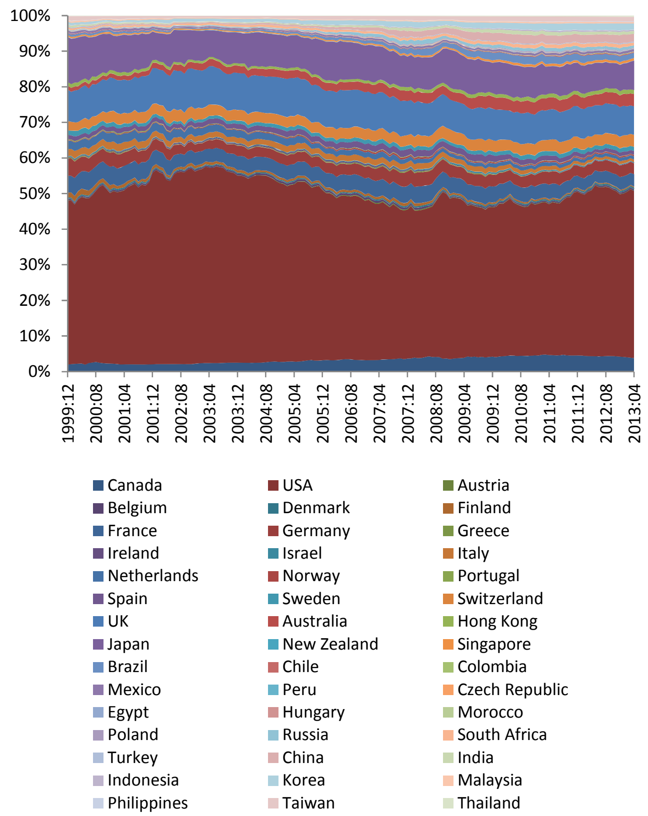


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Country weight

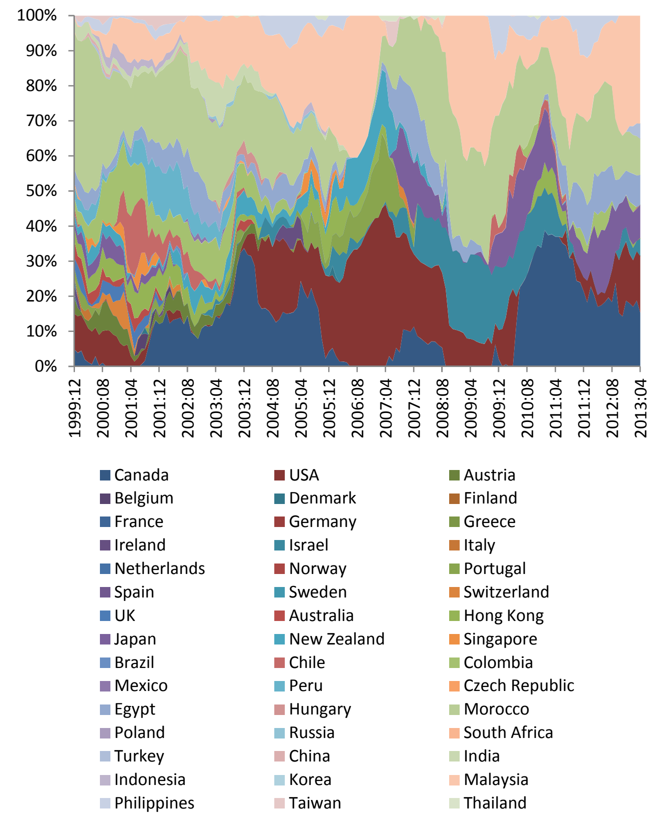
The country weight graphs from Figure 70 to Figure 73 again demonstrate that the capitalization weighted benchmark (i.e., MSCI ACWI) is overly concentrated – in fact, the US equity index dominates the benchmark throughout the backtest, followed by Japan and UK (see Figure 70). The GlobalMinVar portfolio is also highly concentrated with high turnover, Malaysia, Morocco, US, and Canada dominate the weights (see Figure 71). The MaxDiversification (see Figure 72) and MinTailDependence (Figure 73) portfolios are more diversified.

Figure 70: Benchmark – MSCI ACWI



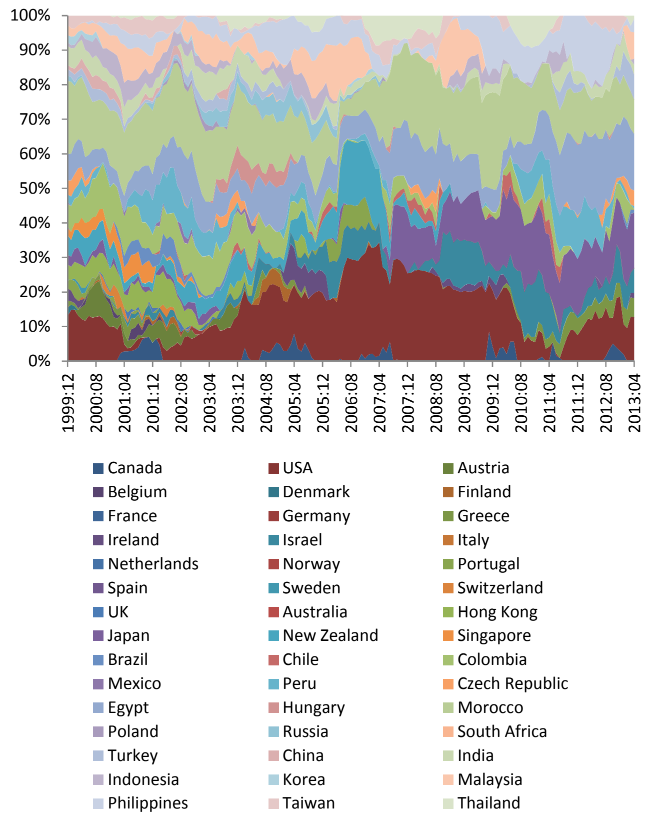
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 71: GlobalMinVar



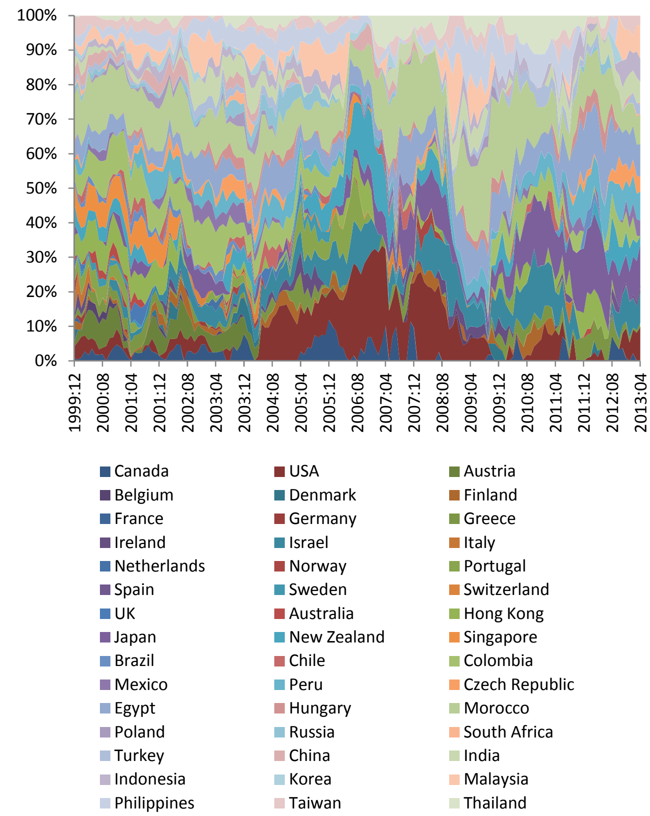
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 72: MaxDiversification



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 73: MinTailDependence



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

MEAM country equity portfolios

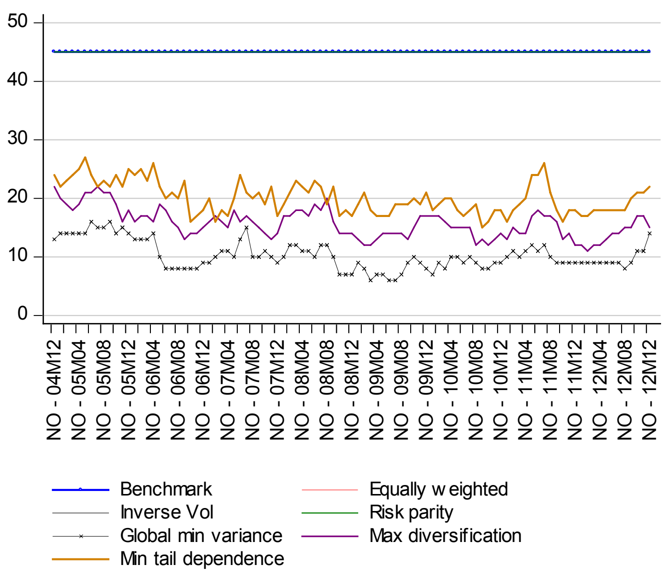
Investment universe

In Luo, *et al* [2013], we illustrate that avoiding economic uncertainties can significantly improve equity (and sovereign bond) performance for 45 countries in the MSCI ACWI. Specifically, we recommend investors to hedge their country equity (and bond) exposures on those days when certain economic indicators are expected to be released – these economic variables are expected to produce negative returns on their announcement days. We call this strategy the MEAM (macroeconomic announcement model). In this research, we want to understand the impact of constructing MEAM portfolios with further risk-based allocation techniques.

Our investment universe is the 45 MEAM country equity indices and our benchmark is based on the each country's weight in the MSCI ACWI (investing on each country's MEAM portfolio), rebalanced monthly.

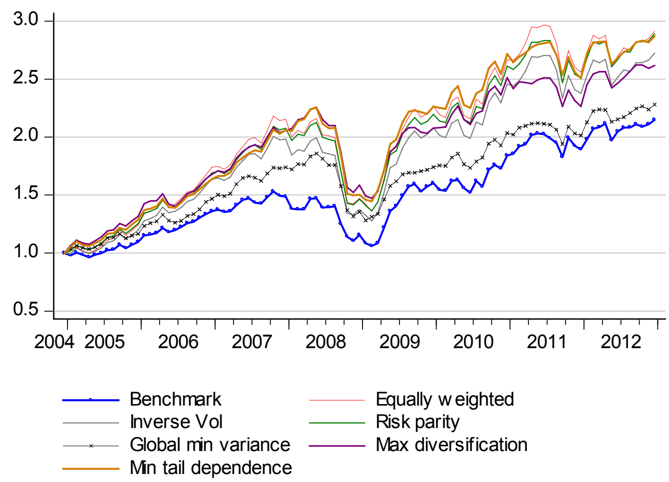
Similar to our country portfolio simulation, MinTailDependence is the clear winner, followed by GlobalMinVar, MaxDiversification, and RiskParity, as measured by the Sharpe ratio (see Figure 76). Interestingly, once economic risk is hedged, the additional risk-based portfolio construction techniques do not help to reduce downside risk (see Figure 77). MaxDiversification, MinTailDependence, and GlobalMinVar portfolios are more diversified by both diversification ratio and WPTD (see Figure 78 and Figure 79). Both cluster analysis algorithms also group MaxDiversification, MinTailDependence, and GlobalMinVar together (see Figure 80 and Figure 81).

Figure 74: Investment universe



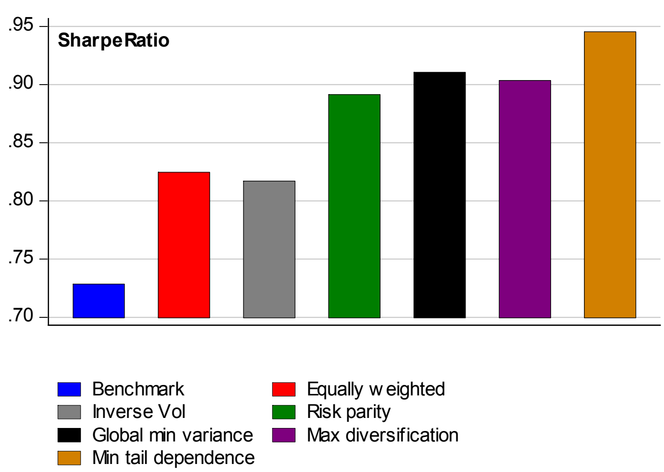
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 75: Wealth curve



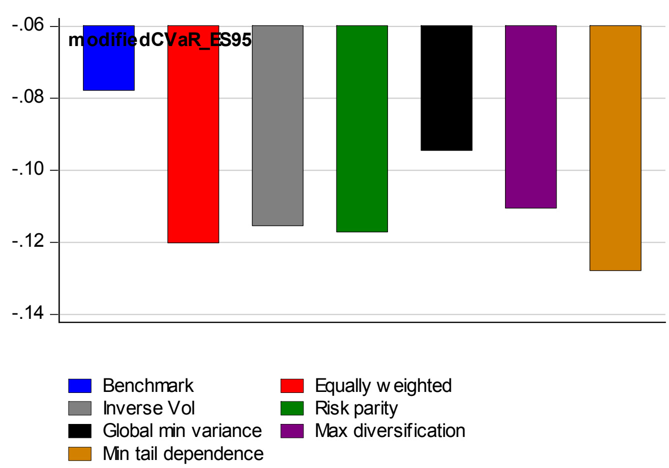
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 76: Sharpe ratio



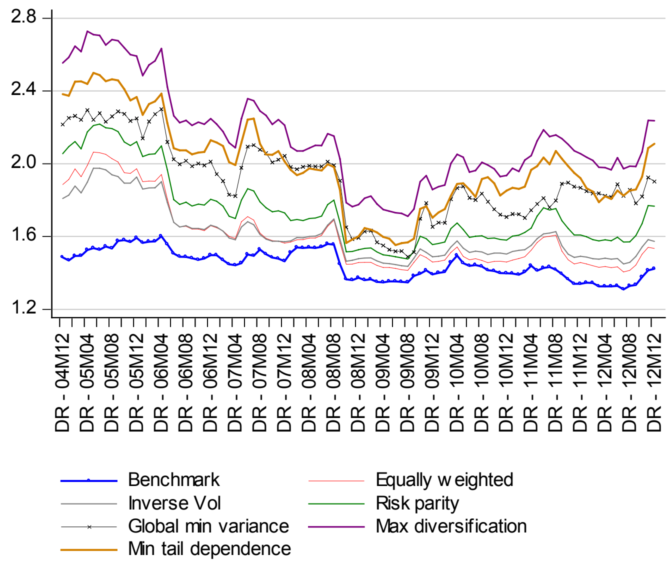
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 77: CVaR/expected shortfall



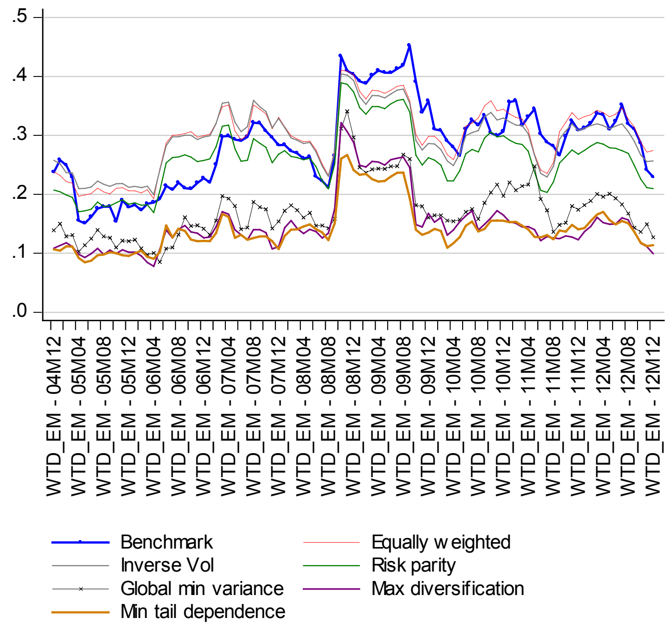
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 78: Diversification ratio



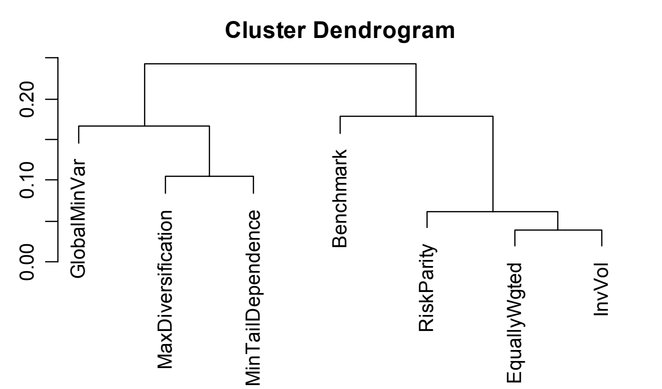
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 79: Weighted portfolio tail dependence



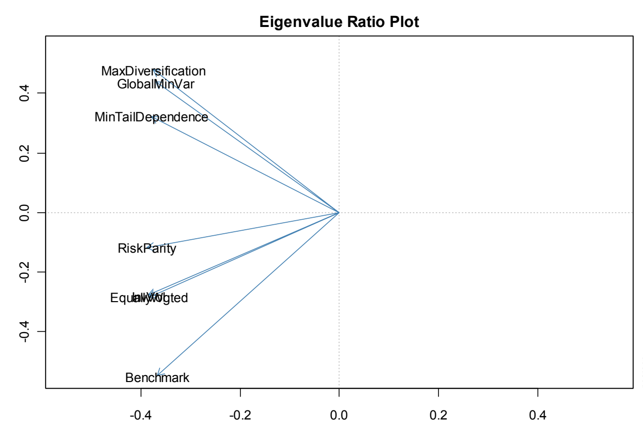
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 80: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 81: Grouping the strategies.



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

IX. Multi-asset portfolio

In this section, we lay out the impact of risk-based allocations in the multi-asset world. Specifically, we test our strategies in asset allocation, bonds, commodities, and alternative beta (also called risk premia or risk factors) space.

Asset allocation

Investment universe

We study 11 commonly used asset classes:

1. US large cap equity (Russell 1000)
2. US small cap equity (Russell 2000)
3. International equity (MSCI EAFE)
4. Emerging markets equity (MSCI EM)
5. REITs (S&P Global REITs)
6. US treasuries (Deutsche Bank US All Treasuries Index)
7. US high yield bonds (Deutsche Bank US High Yield Index)
8. Investment Grade Sovereign (Deutsche Bank USD Investment Grade Sovereign Bond Index)
9. EM credit (Deutsche Bank Emerging Markets Bond USD Index)
10. Commodities (S&P/GSCI)
11. Gold (front-end gold futures)

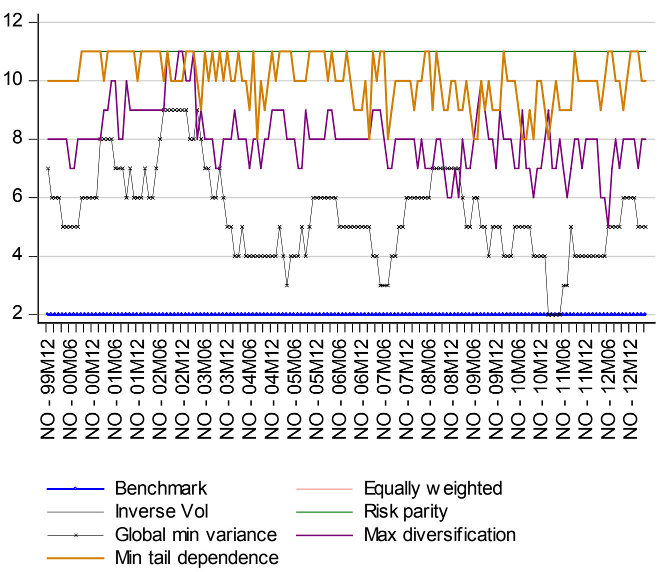
We set our investment benchmark as a traditional 60-40 portfolio, i.e., 60% global equities (MSCI World) and 40% of bonds (US treasuries).

As shown in Figure 82, the benchmark and GlobalMinVar portfolios are highly concentrated. A more sophisticated way to measure diversification and concentration is illustrated in Figure 86 using the diversification ratio (Choueifaty and Coignard [2008]). Benchmark, EquallyWgted, and GlobalMinVar are not as diversified as the RiskParity, MaxDiversification, and MinTailDependence portfolios. Figure 87 shows the weighted portfolio tail dependence (WPTD) – benchmark, EquallyWgted, InvVol, and RiskParity appear to be more exposed to strategy crowding than GlobalMinVar and MaxDiversification. Interestingly, the MinTailDependence portfolio, despite being the least crowded strategy *ex ante*, actually shows more tail dependence than GlobalMinVar and MaxDiversification *ex post*[18].

[18] This is most likely due to the estimation errors in tail dependent coefficient matrix.

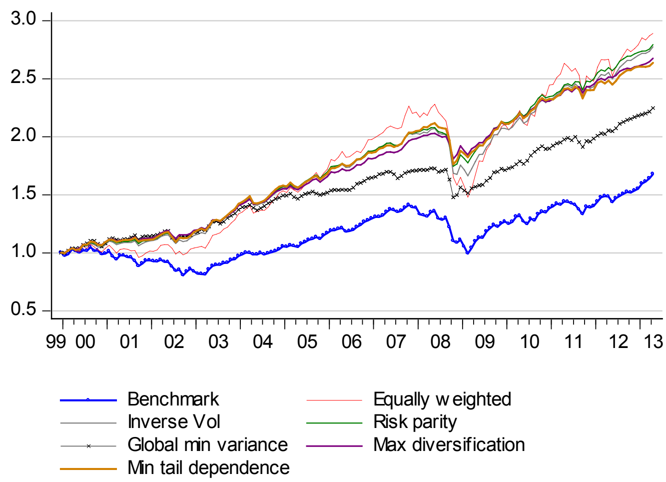
All risk-based allocations significantly outperform our 60-40 benchmark. MaxDiversification, RiskParity, and MinTailDependence have the best performance overall (see Figure 84 and Figure 85). They also seem to form one cluster (see Figure 88). Benchmark and InvVol portfolios show poor diversification (see Figure 89).

Figure 82: Investment universe



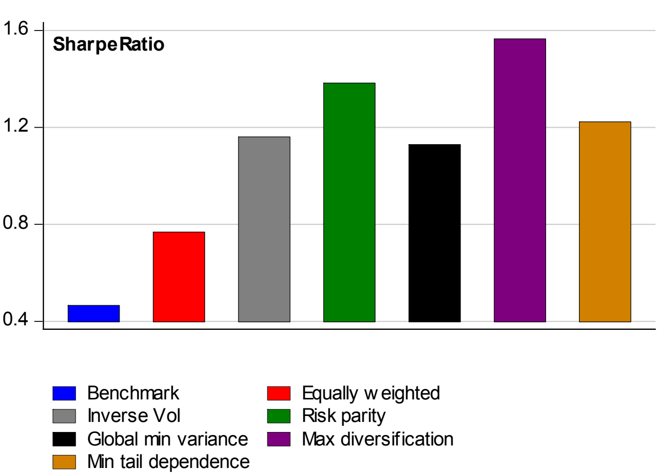
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 83: Wealth curve



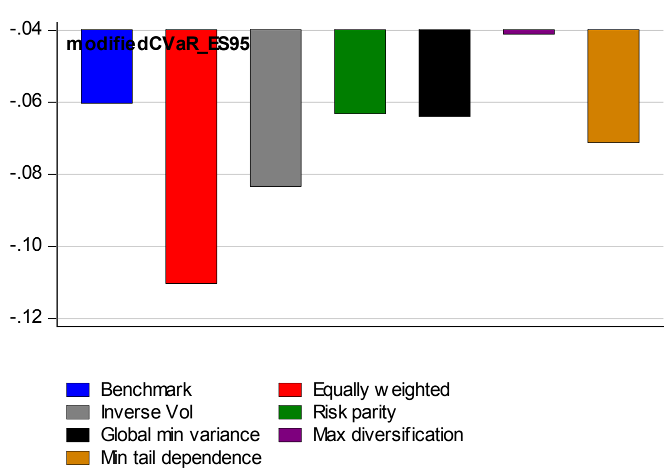
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 84: Sharpe ratio



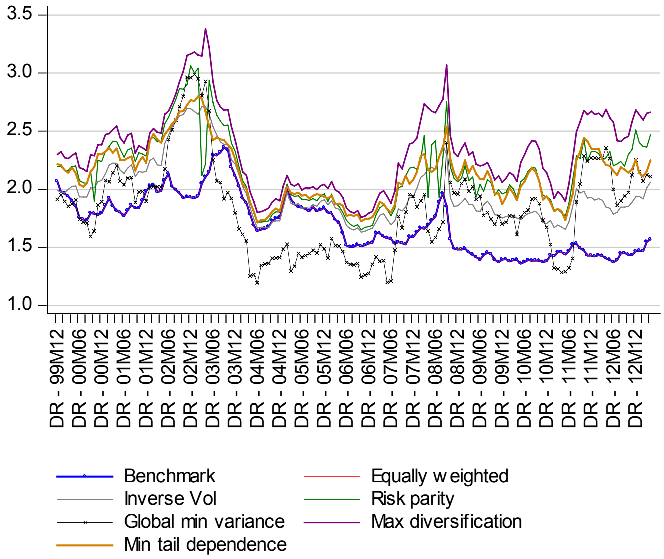
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 85: CVaR/expected shortfall



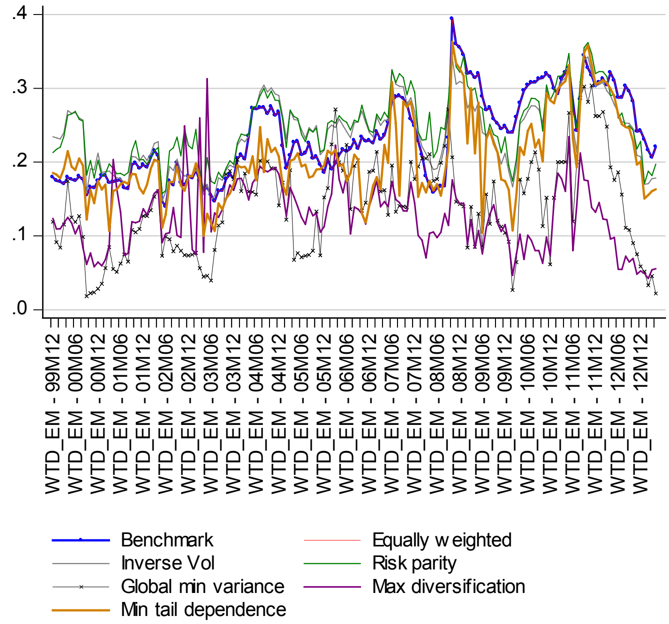
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 86: Diversification ratio



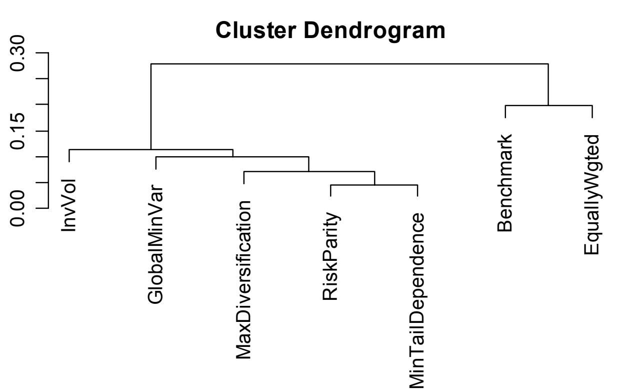
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 87: Weighted portfolio tail dependence



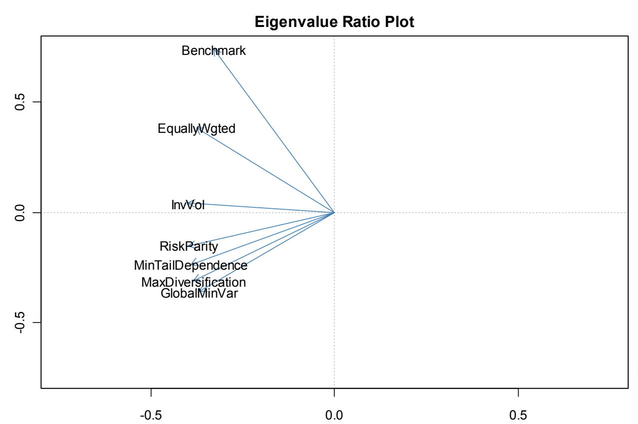
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 88: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 89: Grouping the strategies.

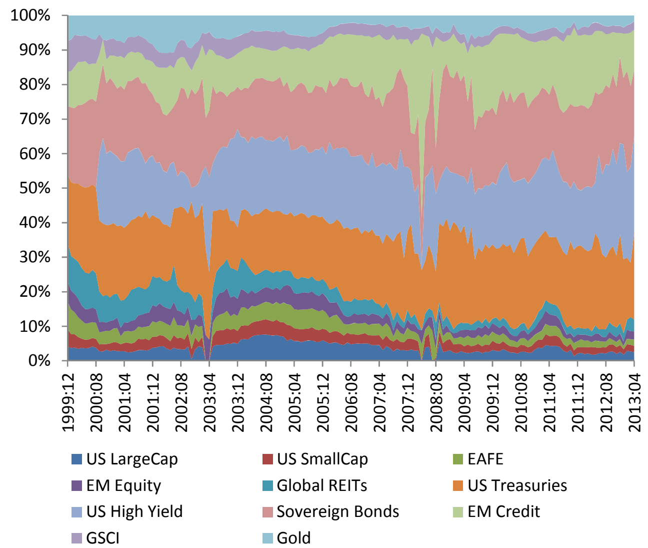


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Asset weight

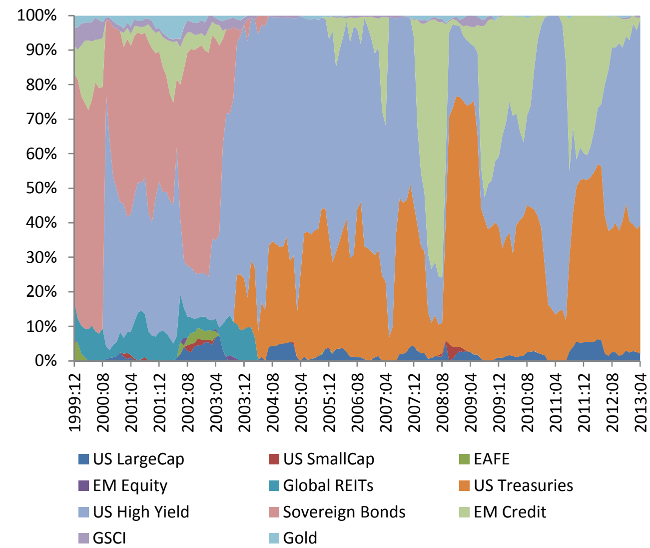
Figure 90 to Figure 93 lay out the asset weights over time for four of our strategies (RiskParity, GlobalMinVar, MaxDiversification, and MinTailDependence). All these strategies have been significantly overweighting bonds (US treasuries, US high yield, Sovereign bonds, and EM credit). RiskParity and MinTalDependence appear to be more balanced than GlobalMinVar and MaxDiversification.

Figure 90: RiskParity



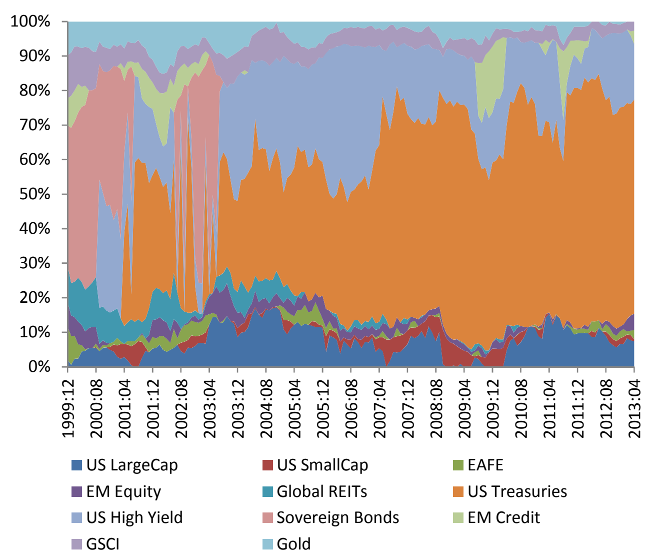
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 91: GlobalMinVar



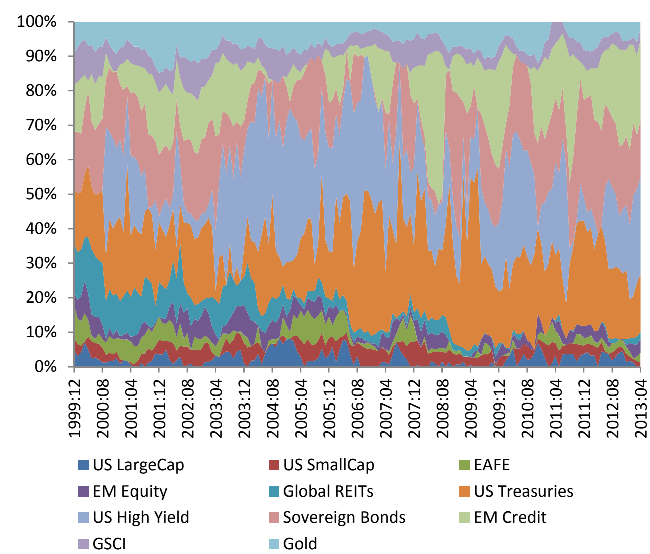
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 92: MaxDiversification



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 93: MinTailDependence

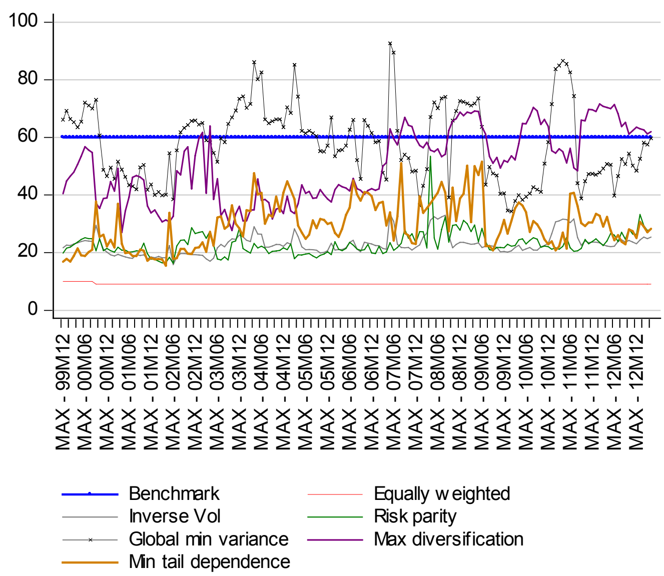


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Maximum weight constraint

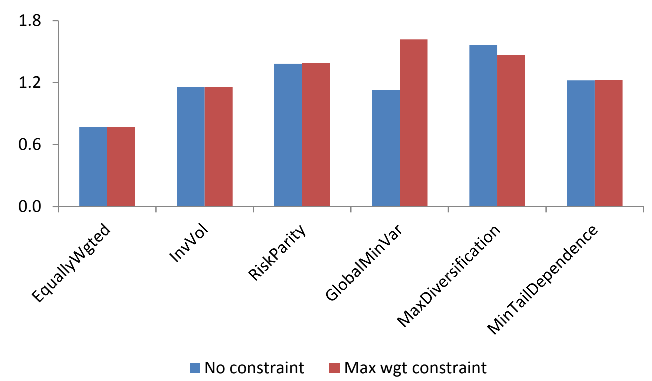
As shown in Figure 94, benchmark and GlobalMinVar portfolios are highly concentrated. The largest weight of a single asset can be over 90% at a given point in time. Now, let's re-run our portfolio simulation with a maximum weight constraint of 20%, i.e., no single asset weight can exceed 20%. As shown in Figure 95, adding a maximum weight constraint makes no difference to EquityWgted, InvVol and little difference to the RiskParity, MaxDiversification, and MinTailDependence portfolios. However, a maximum weight constraint does help GlobalMinVar portfolio, by reducing the concentration. In theory, unconstrained portfolios should produce the best results, because constraints reduce the transfer coefficient and are likely to push portfolios below the efficient frontier. In practice, almost everybody has some constraints. Constraints could be due to investment mandates. More importantly, portfolio managers realize that they have to estimate all the input parameters (e.g., expected returns and covariance matrix), which contains estimation errors. Constraints could be quite effective in reducing the impact of estimation errors on portfolio performance. Jagannathan and Ma [2003] showed that adding some constraints on weights is equivalent to using a shrinkage estimate of the covariance matrix. However, a maximum weight constraint is arbitrary. MaxDiversification, RiskParity, and MinTailDependence portfolios are clearly more robust than GlobalMinVar.

Figure 94: Maximum weight



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 95: Sharpe ratio comparison



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Sovereign bond portfolios

Investment universe

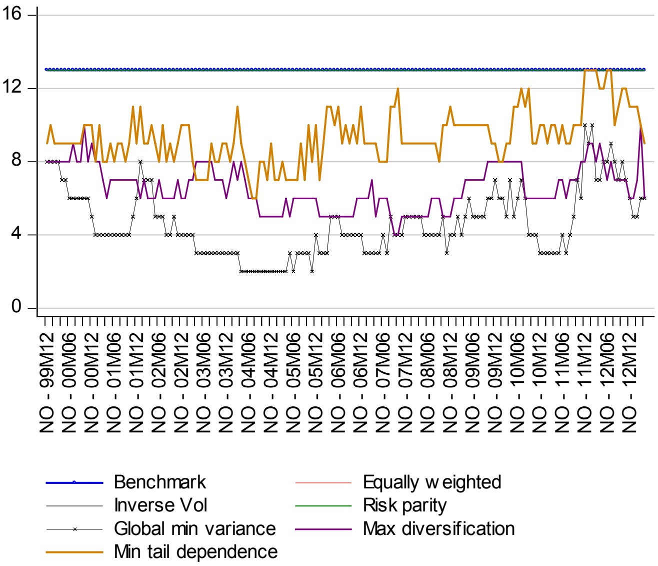
We use G13 long-term sovereign bond indices in the following 13 countries as our investment universe: US, Canada, Denmark, France, Germany, Italy, Norway, Sweden, Switzerland, UK, Australia, Japan, and New Zealand[19]. The returns are all based on USD; therefore, currencies are unhedged.

In the sovereign bond universe, GlobalMinVar outperforms the other risk-based allocation techniques, followed by MaxDiversification and MinTailDependence, as measured by Sharpe ratio (see Figure 98) and downside risk (see Figure 99). Interestingly, the three techniques all beat the more popular RiskParity scheme.

The GlobalMinVar, MaxDiversification, and MinTailDependence portfolios offer some diversification benefit (see Figure 100 and Figure 101). However, in the sovereign bond space, the diversification via risk-based allocation is likely to be limited.

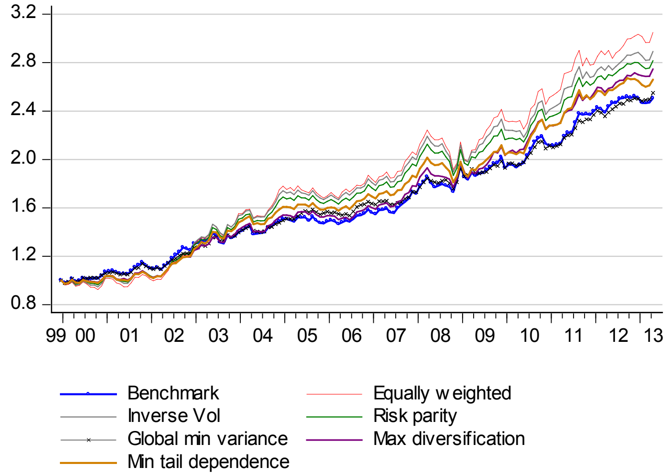
[19] We choose the same investment universe as in our previous research, e.g., Mesomeris, et al [2011], Jussa, et al [2012], and Luo, et al [2013].

Figure 96: Investment universe



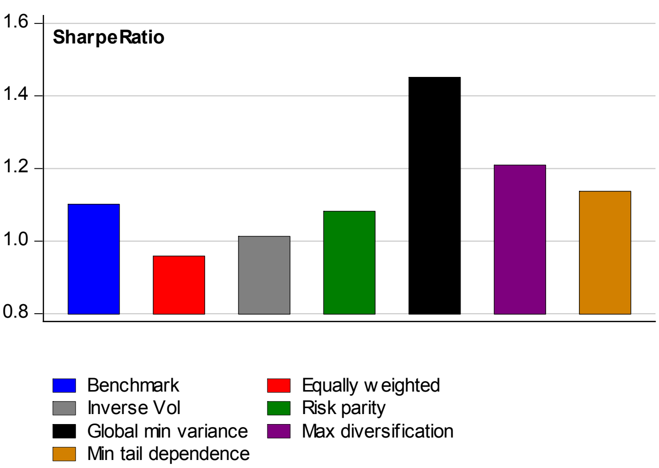
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 97: Wealth curve



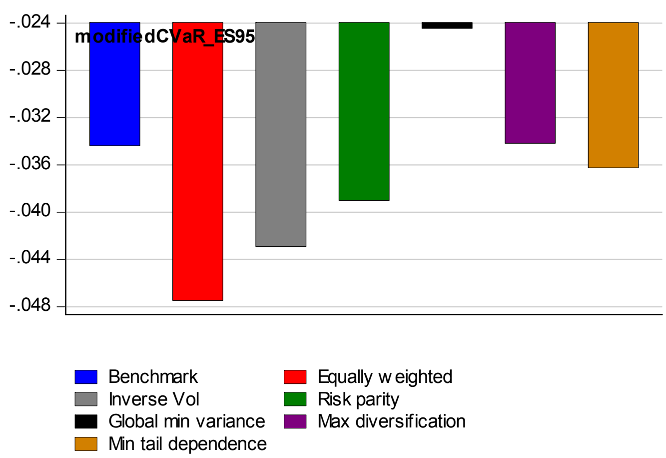
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 98: Sharpe ratio



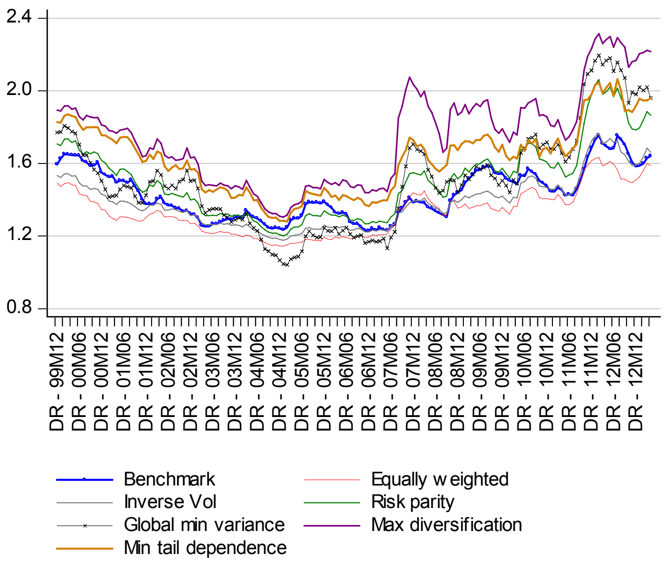
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 99: CVaR/expected shortfall



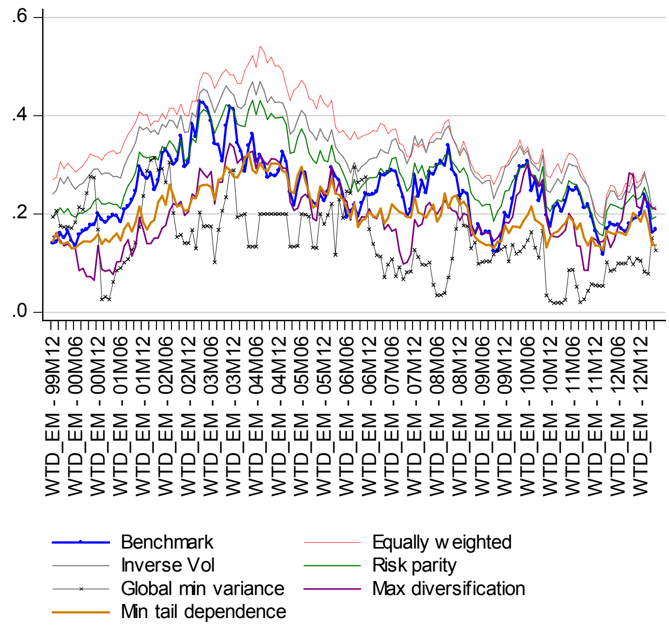
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 100: Diversification ratio



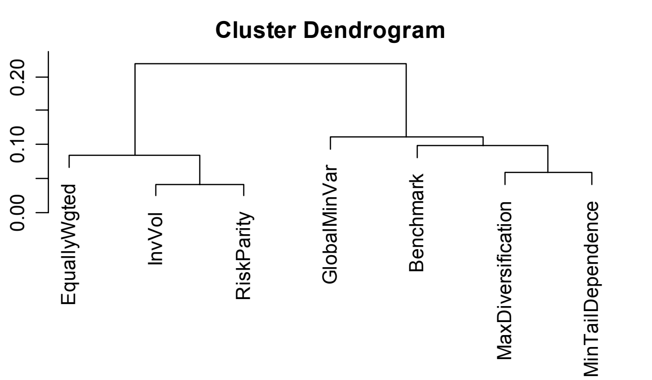
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 101: Weighted portfolio tail dependence



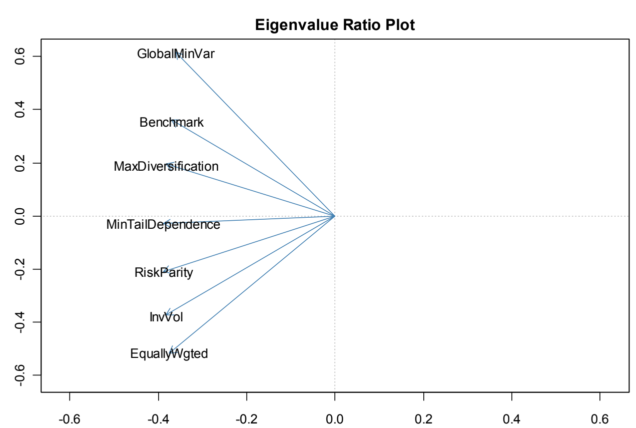
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 102: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 103: Grouping the strategies.



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Commodities portfolios

Investment universe

In this research, we use the same commodities universe as in Jussa, *et al* [2012], i.e., approximately 24 different commodities future indices (see Figure 104) from the DB Liquidity Commodities Indices Optimum Yield family (DBLCI-OY). The returns are all based on USD.

Contrary to conventional approaches of using front-end futures, DB Liquidity Commodities Indices Optimum Yield family offers a different approach to rolling over futures contracts. DBLCI-OY utilizes a rules-based approach when it rolls from one futures contract to another for each commodity in the index. This rule stipulates that each new commodity futures contract will be the one with the maximum "implied roll yield" based on the closing price for each eligible contract. Effectively, this enhanced rollover process is systematically optimizing the return generated by rolling a short-term contract into a longer-term contract and vice-versa.

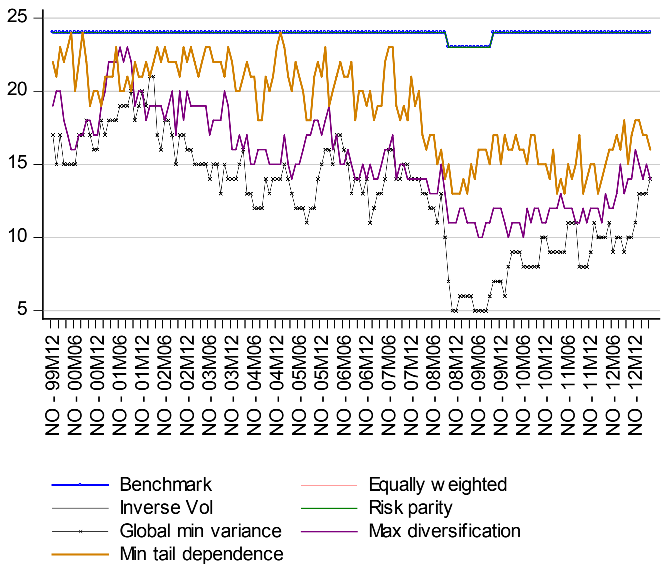
Figure 104: List of commodities futures

Category	Commodities	Bloomberg Ticker
Energy	Light Crude	DBLCOCLT
Energy	Heating Oil	DBLCOHOT
Energy	Rbob Gas	DBLCYTRB
Energy	Natural Gas	DBLCYTCO
Energy	Brent Crude Oil	DBLCYTCO
Energy	Gasoline	DBLCYTGO
Precious Metal	Gold	DBLCOGCT
Precious Metal	Silver	DBLCYTSI
Industrial Metal	Aluminium	DBLCOALT
Industrial Metal	Zinc	DBLCYTZN
Industrial Metal	Copper	DBLCYTCU
Industrial Metal	Nickel	DBLCYTNI
Industrial Metal	Lead	DBLCYTPB
Agriculture	Corn	DBLCOCNT
Agriculture	Wheat	DBLCOWTT
Agriculture	Soybeans	DBLCYTSS
Agriculture	Sugar	DBLCYTSB
Agriculture	Coffee	DBLCYTKC
Agriculture	Cotton	DBLCYTCT
Agriculture	Cocoa	DBLCYTCC
Agriculture	Kansas Wheat	DBLCYTKW
Livestock	Live Cattle	DBLCYTLC
Livestock	Lean Hogs	DBLCYTLH
Livestock	Feeder Cattle	DBLCYTFC

Source: Deutsche Bank

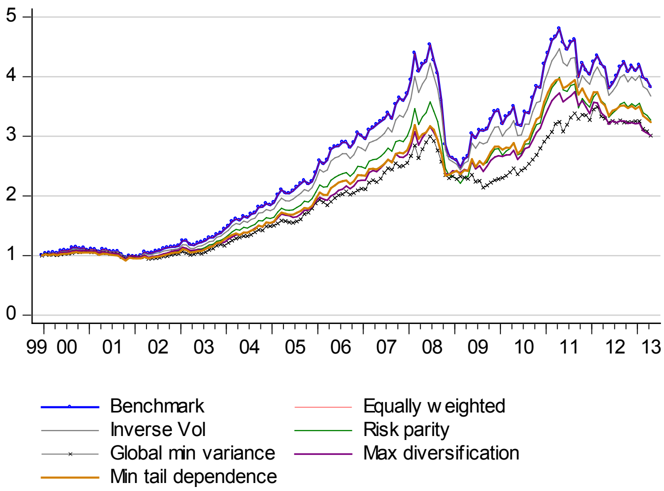
Similar to the bond investment universe, GlobalMinVar, MaxDiversification, and MinTailDependence produce higher Sharpe ratios (see Figure 107) and lower downside risks (see Figure 108) than other strategies. They also appear to form one cluster (see Figure 111 and Figure 112).

Figure 105: Investment universe



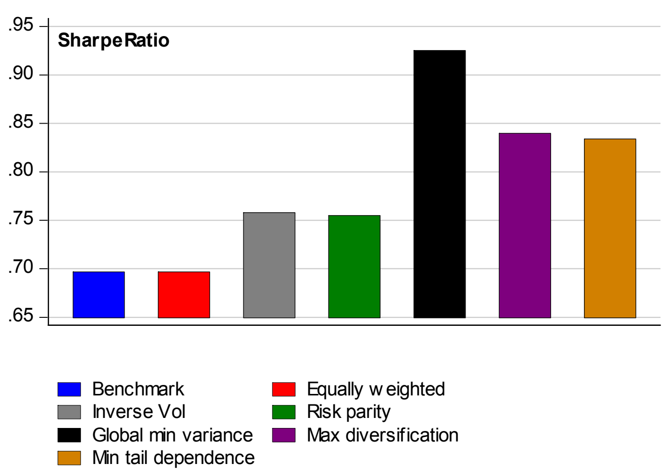
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 106: Wealth curve



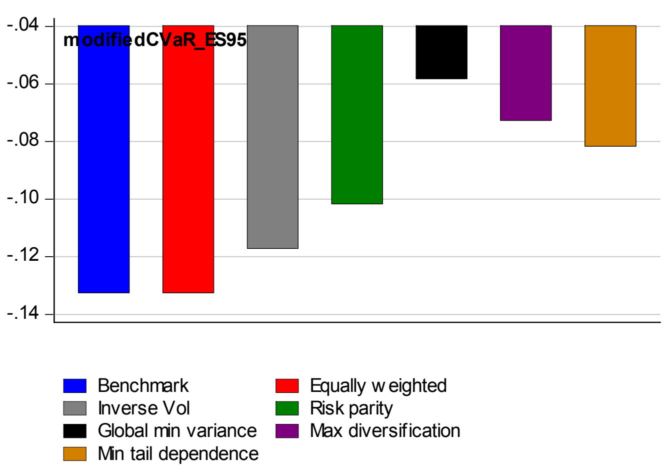
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 107: Sharpe ratio



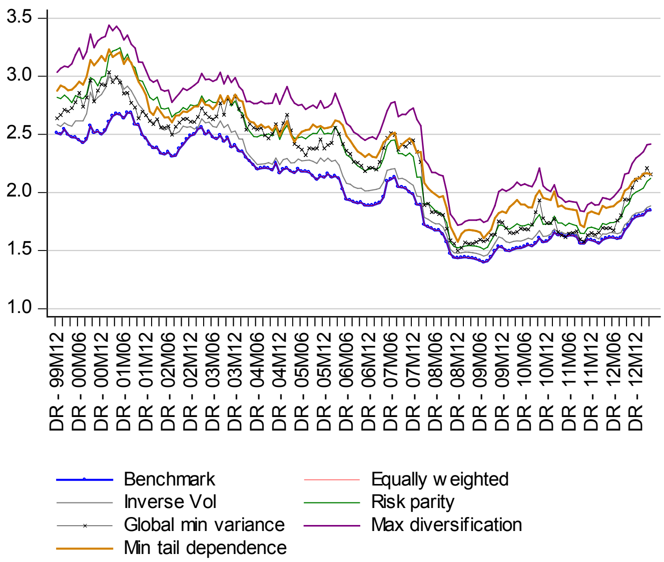
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 108: CVaR/expected shortfall



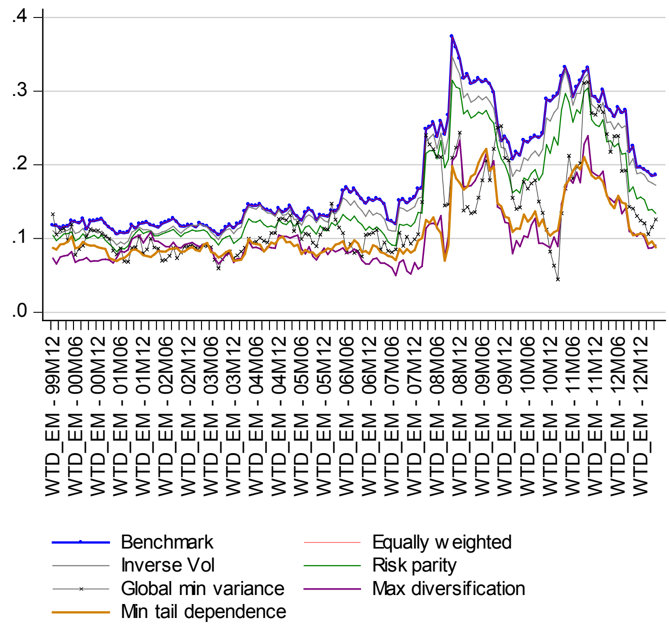
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 109: Diversification ratio



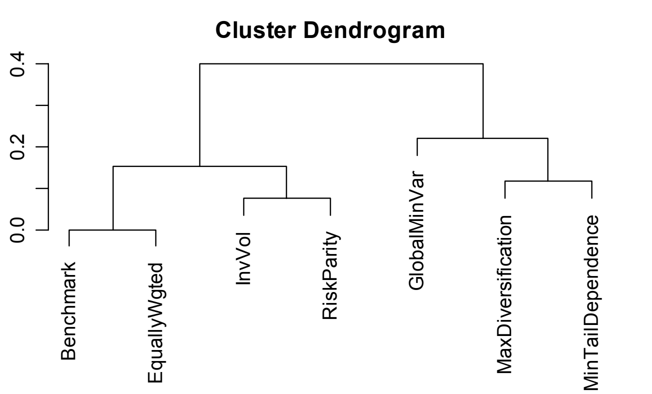
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 110: Weighted portfolio tail dependence



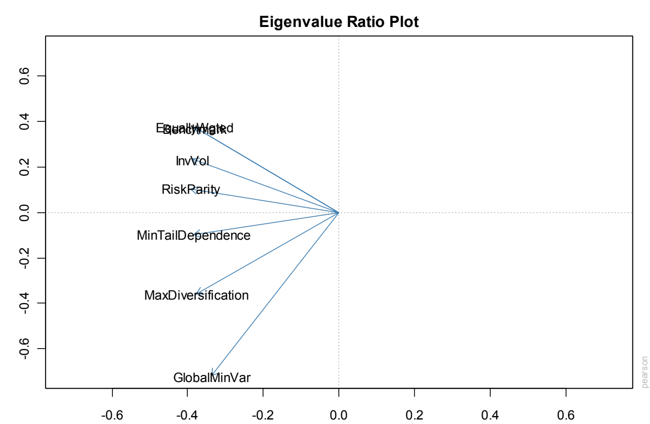
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 111: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 112: Grouping the strategies.



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Alternative beta portfolios

Investing along the alternative beta portfolios (also called risk premia or risk factors) has become more and more popular in recent years, especially among the pension/endowment/foundation/sovereign wealth funds. We have indeed also published our own research in this space (see Mesomeris, *et al* [2011] for details).

In this research, we define five simple alternative beta portfolios in global equities:

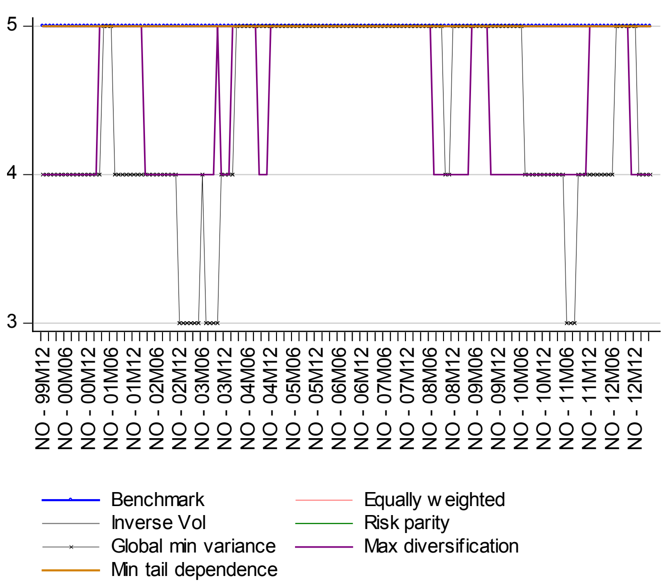
- Value, based on trailing earnings yield
- Momentum, based on 12-month total returns excluding the most recent one month
- Quality, based on return on equity

- Size. MSCI World SmallCap total return index – MSCI World LargeCap total return index
- Low volatility/low risk, based on trailing one-year daily realized volatility

All portfolios (other than size) are constructed for the MSCI World universe, in a regional and sector neutral way, by forming a long/short quintile portfolio, equally weighted within the long and the short portfolios. We divide the MSCI World into the following regions: US, Canada, Europe ex UK, UK, Asia ex Japan Developed, Japan, and Australia/New Zealand. We select equal number of stocks from each region in each of the 10 GICS sectors. For those region/sector buckets where we have less than five stocks, we do not invest in those buckets.

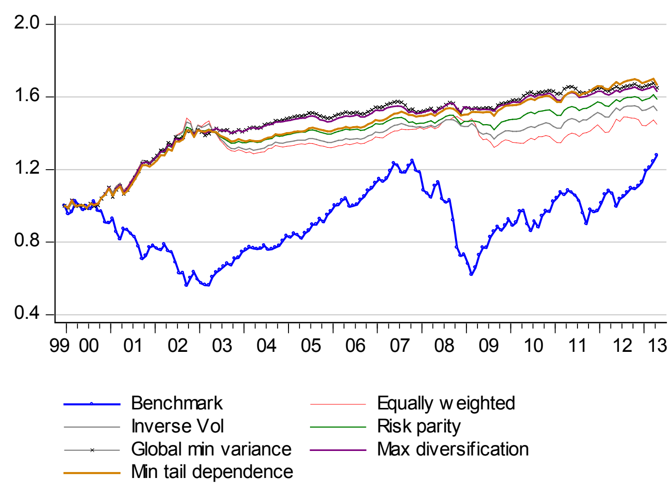
It is interesting to see that all portfolios constructed on alternative betas significantly outperform the benchmark MSCI World index, with much higher Sharpe ratios (see Figure 115) and much lower downside risks (see Figure 116). While our MinTailDependence strategy displays the highest Sharpe ratio and the lowest downside risk, the GlobalMinVar and RiskParity strategies are likely to be more diversified. Based on WPC and WPTD, we have not seen any sign of crowding yet.

Figure 113: Investment universe



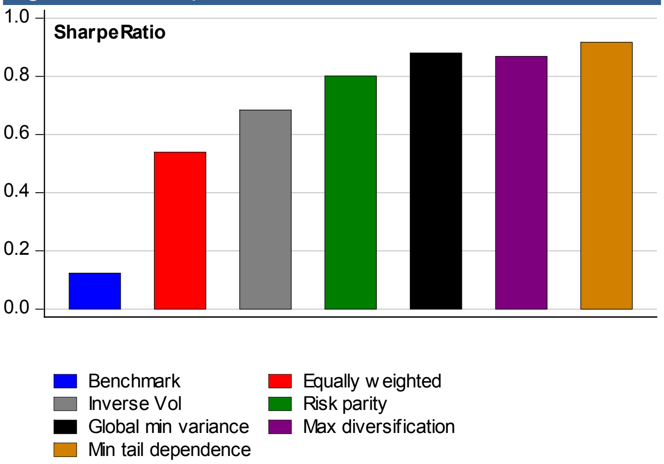
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 114: Wealth curve



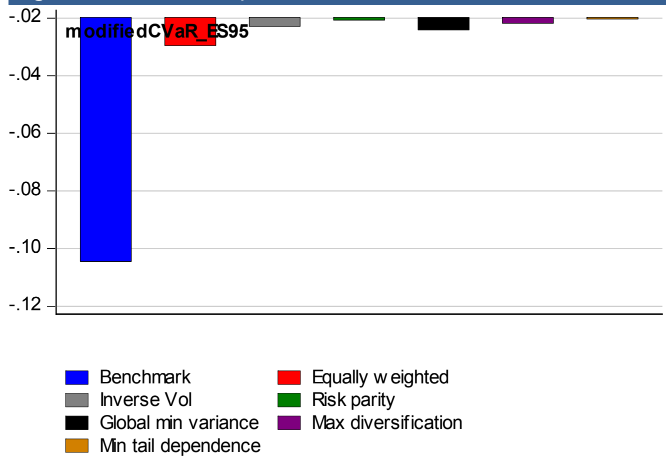
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 115: Sharpe ratio



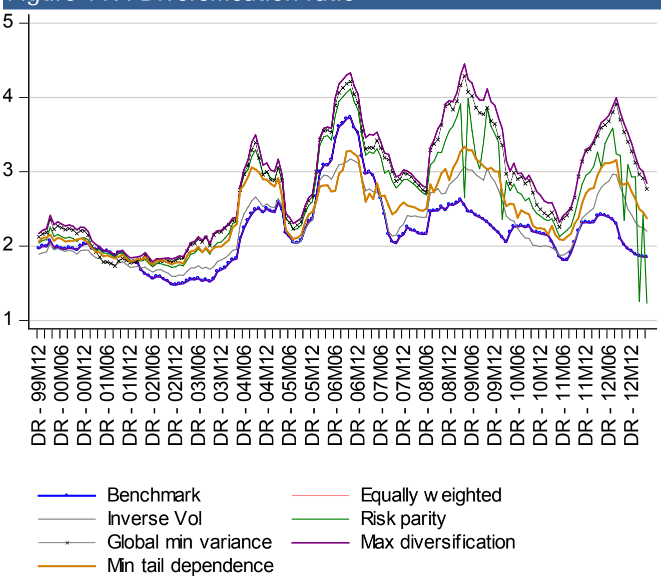
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 116: CVaR/expected shortfall



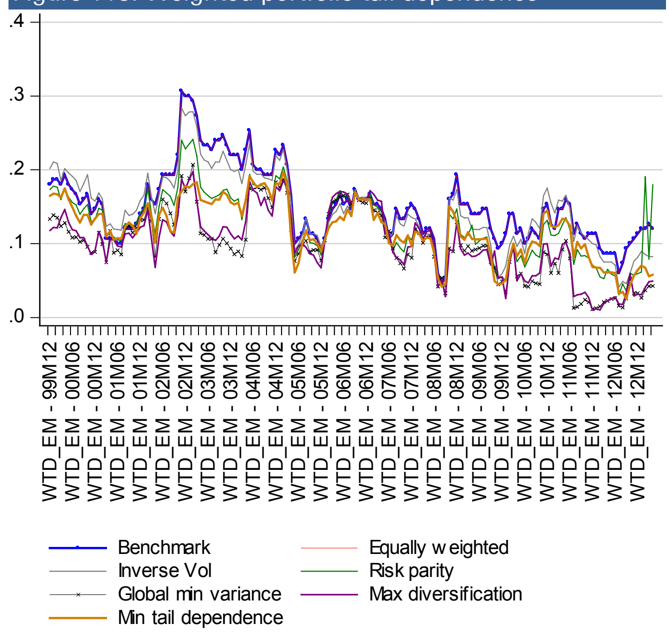
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 117: Diversification ratio



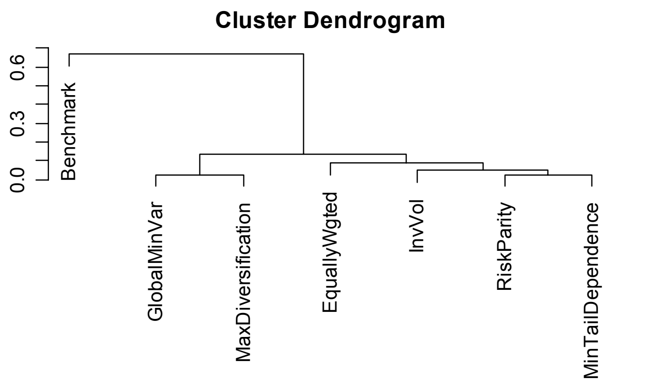
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 118: Weighted portfolio tail dependence



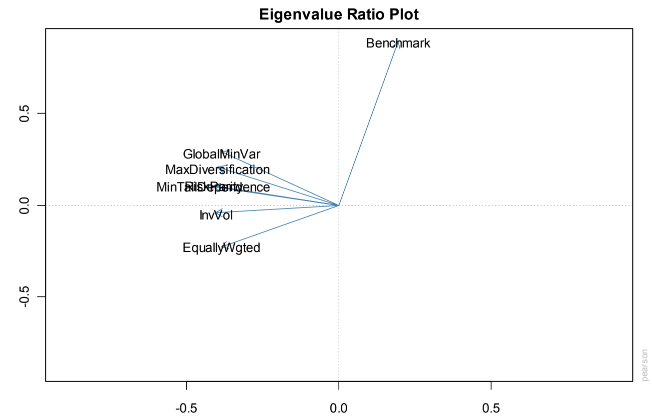
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 119: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 120: Grouping the strategies.



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

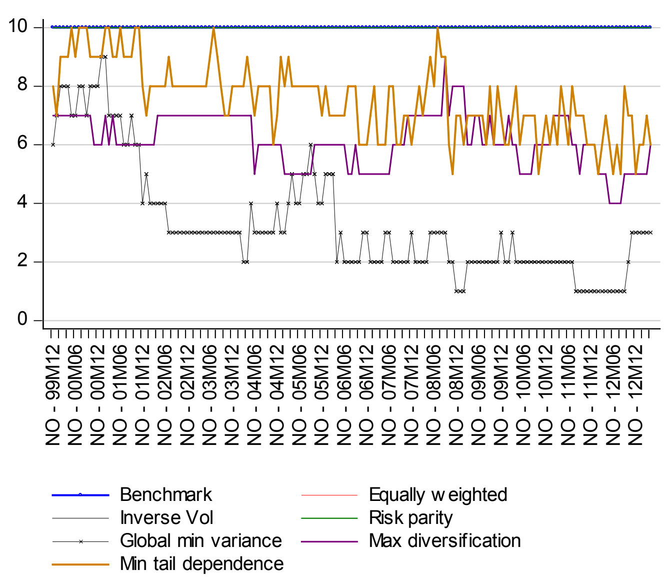
X. Sector/industry portfolios

Sector/industry rotation has gained a lot of attention in recent years, along with country rotation. In this section, we study five sector portfolios: MSCI World sectors, US sectors, European sectors, MSCI World industries, and finally, a combined region x sector portfolio. We clearly see less diversification benefit by investing in the sector/industry portfolios compared to the country portfolios

MSCI World 10 GICS sector portfolio

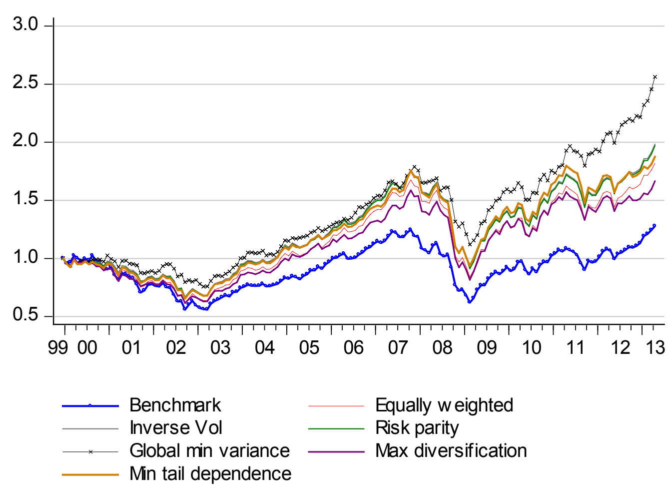
First, let's look at the 10 GICS sectors in the MSCI World universe. The GlobalMinVar is the clear winner in this simulation with the highest Sharpe ratio (see Figure 123) and lowest downside risk (see Figure 124) and forms a cluster of its own (see Figure 127 and Figure 128).

Figure 121: Investment universe



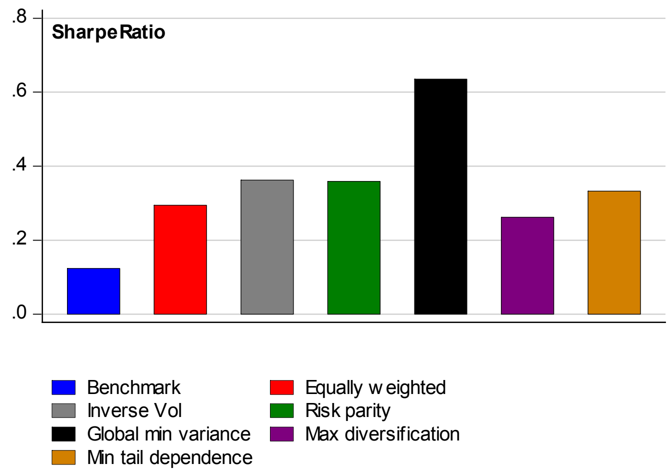
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 122: Wealth curve



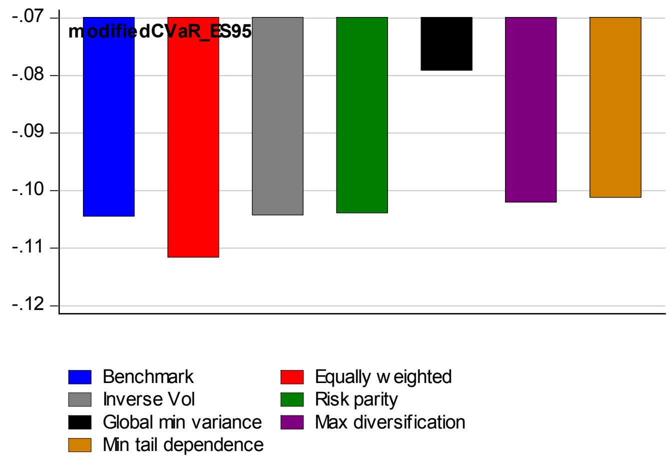
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 123: Sharpe ratio



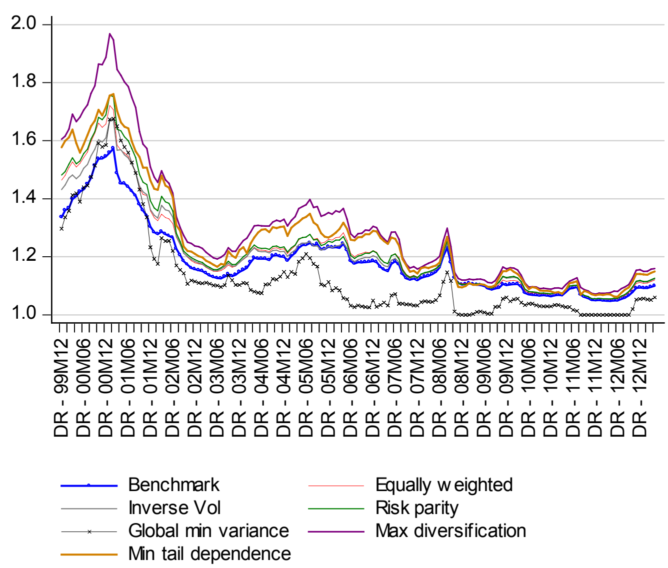
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 124: CVaR/expected shortfall



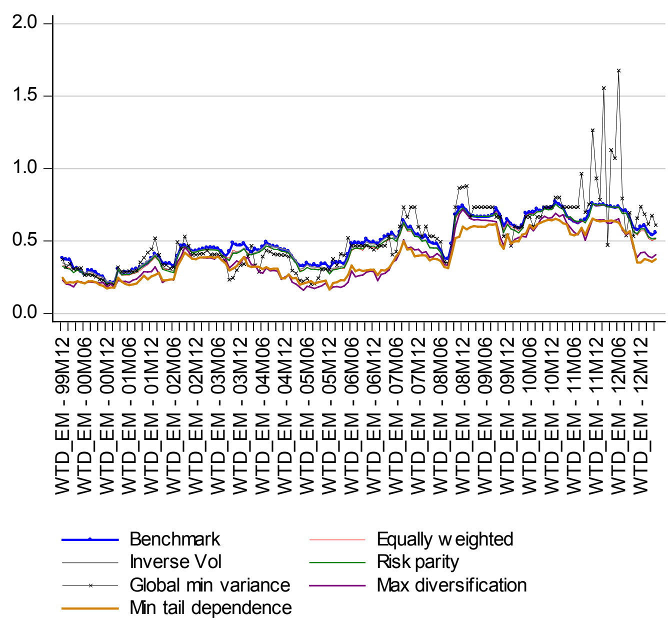
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 125: Diversification ratio



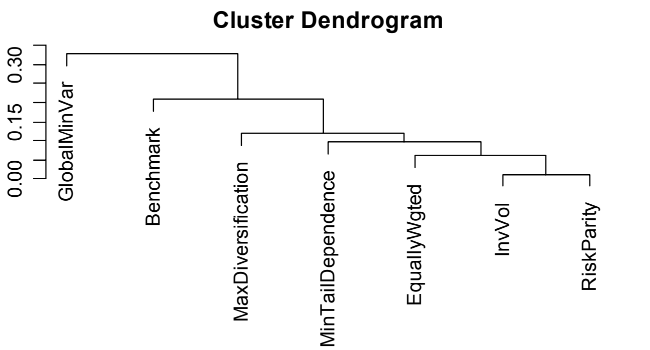
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 126: Weighted portfolio tail dependence



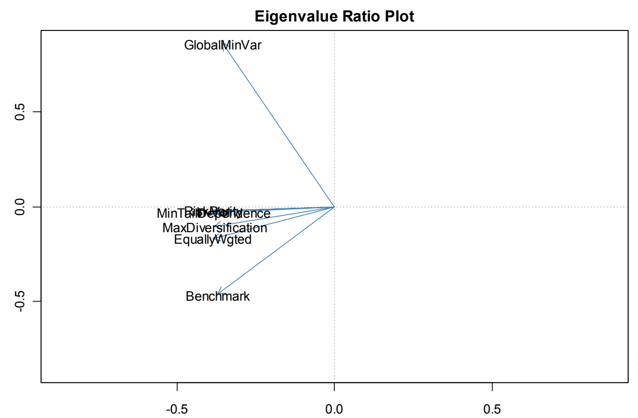
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 127: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 128: Grouping the strategies.

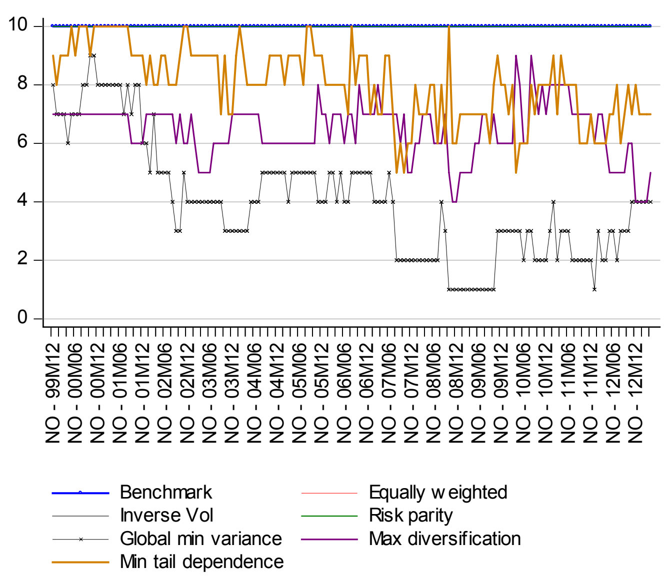


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

US S&P 500 10 GICS sector portfolio

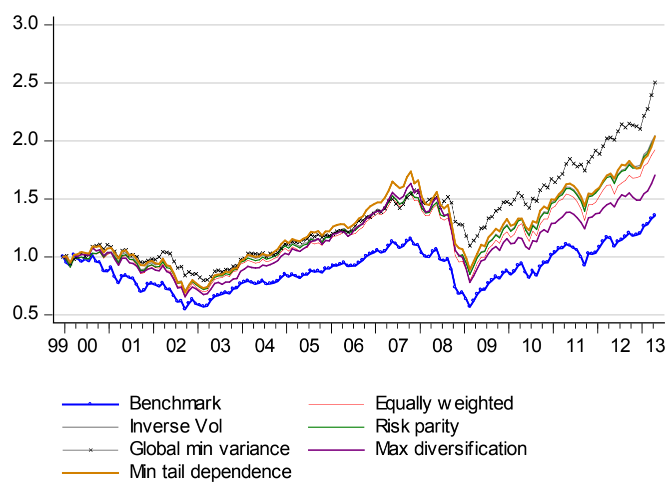
Now, let's use the 10 GICS sectors in the S&P 500 universe. Similar to global sector investment, in the US sectors, GlobalMinVar also dominates the other strategies (see Figure 131 and Figure 132) and forms its own cluster (see Figure 135).

Figure 129: Investment universe



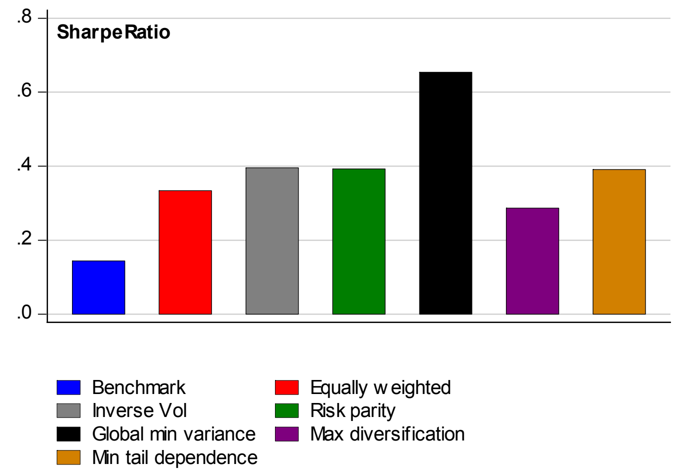
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 130: Wealth curve



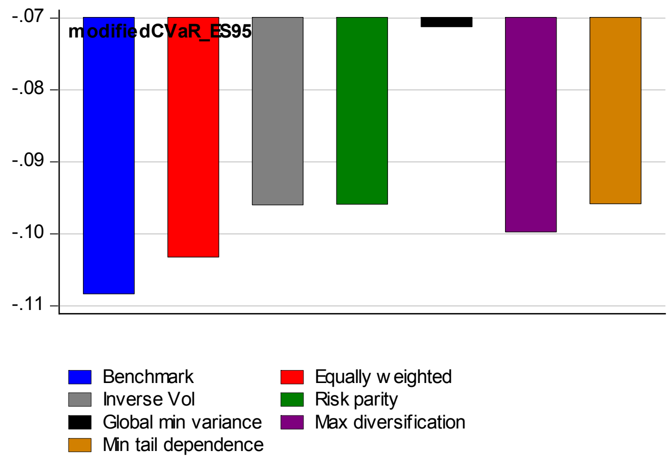
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 131: Sharpe ratio



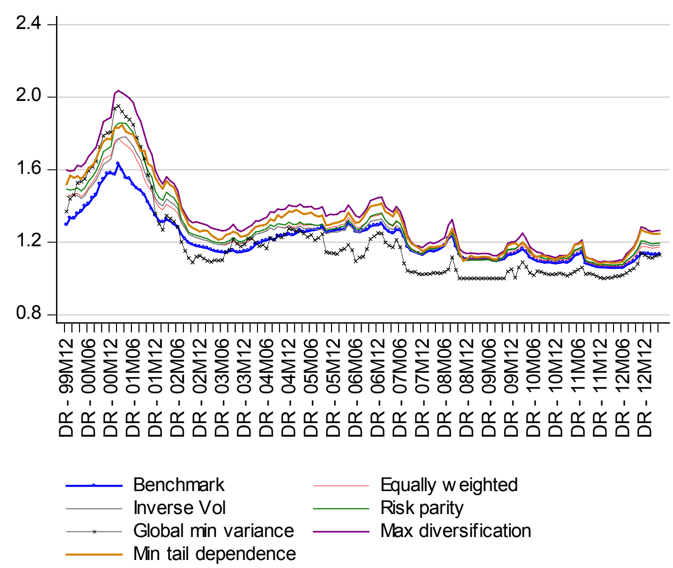
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 132: CVaR/expected shortfall



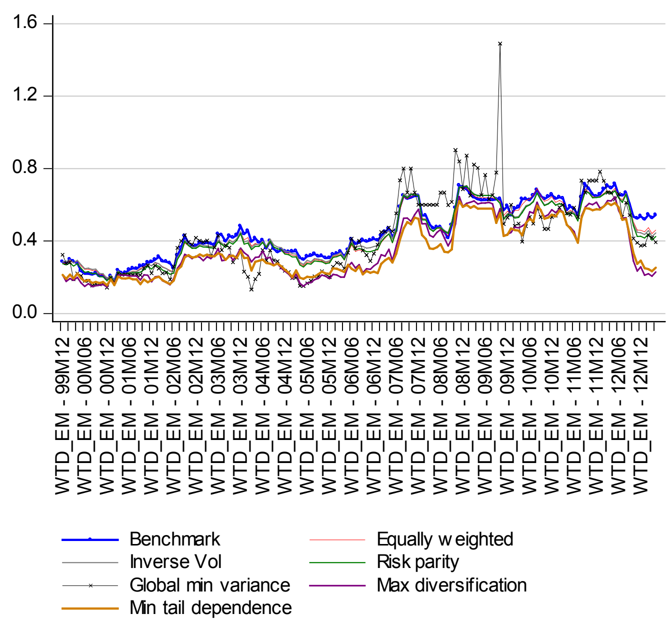
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 133: Diversification ratio



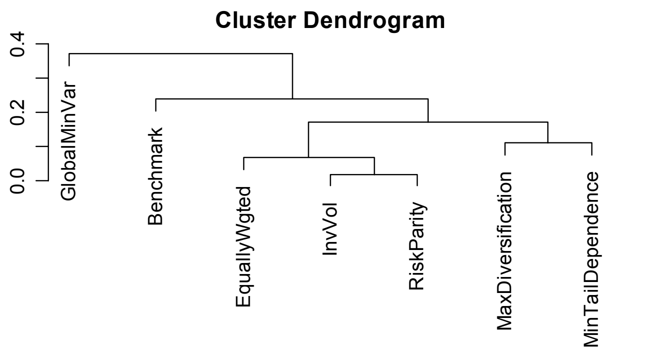
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 134: Weighted portfolio tail dependence



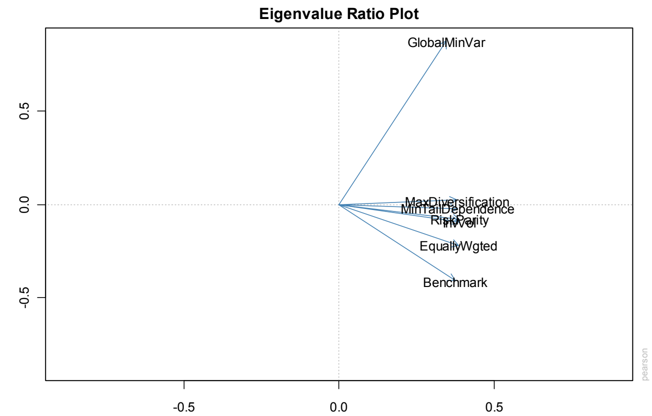
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 135: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 136: Grouping the strategies.

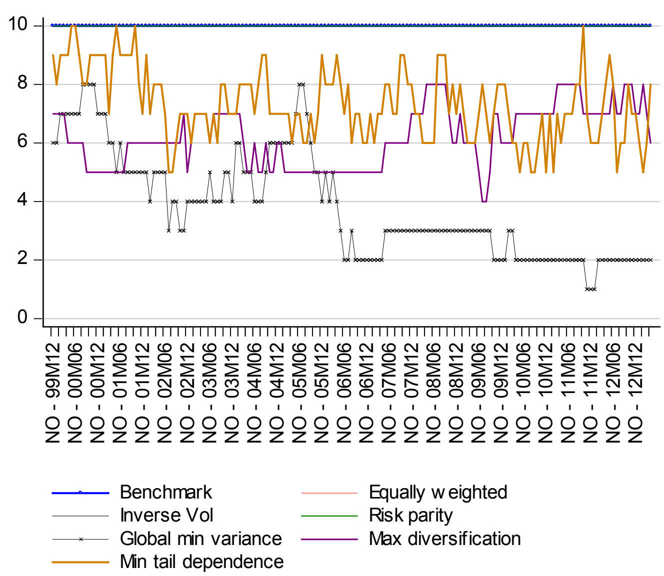


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

MSCI Europe 10 GICS sector portfolio

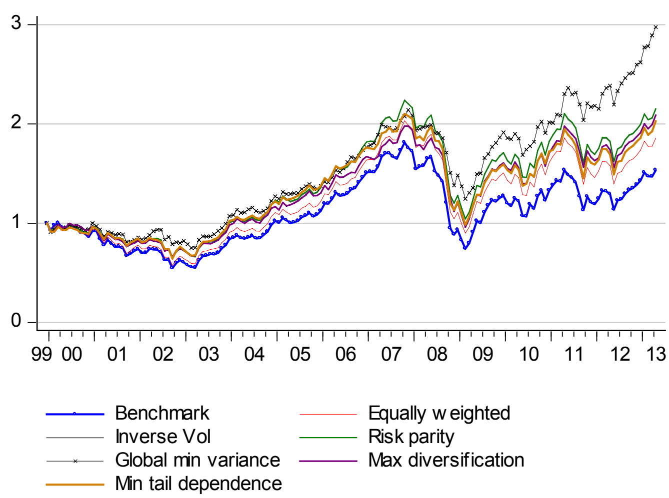
The results for the European 10 GICS sectors are similar to the results for global and US universes, with GlobalMinVar being the dominant strategy.

Figure 137: Investment universe



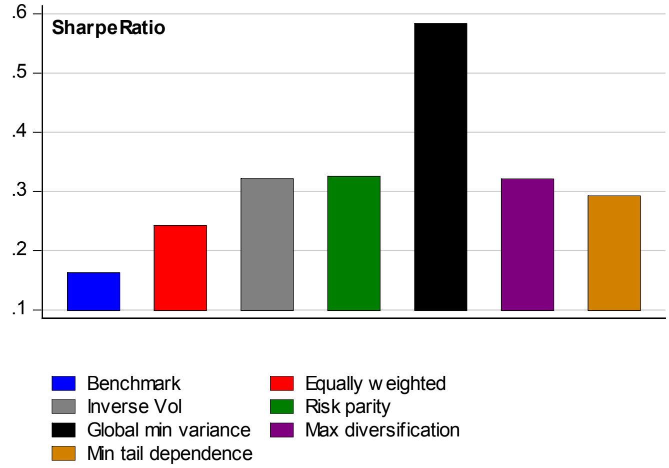
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 138: Wealth curve



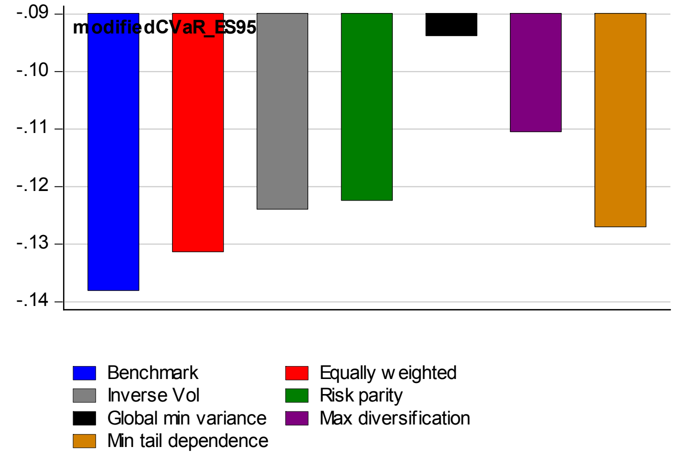
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 139: Sharpe ratio



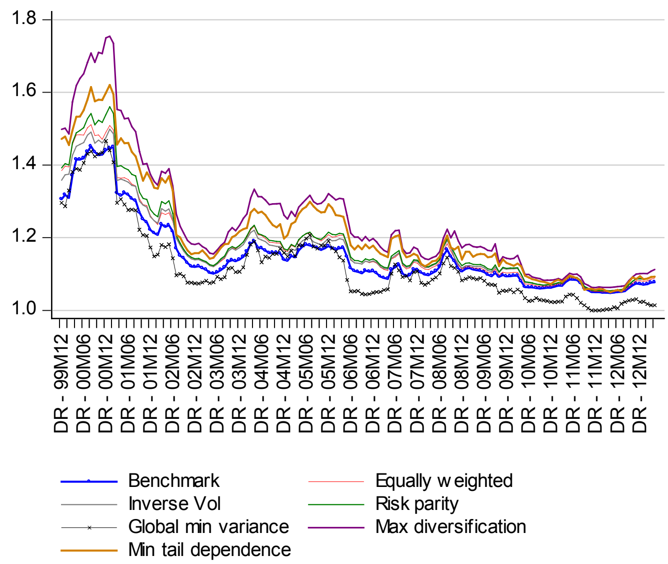
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 140: CVaR/expected shortfall



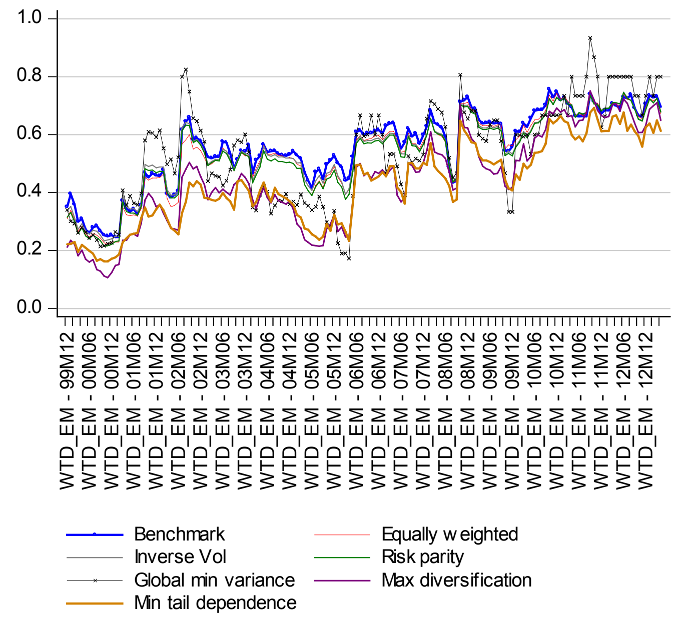
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 141: Diversification ratio



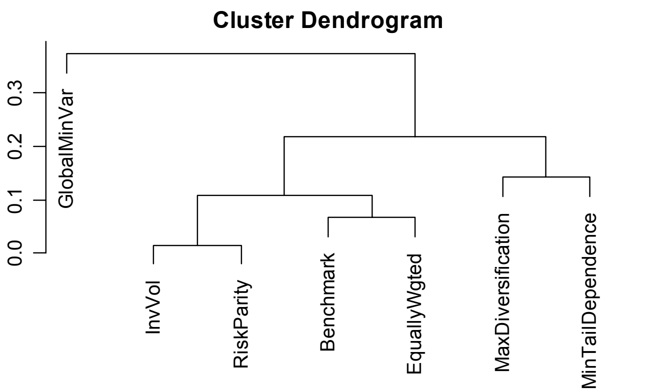
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 142: Weighted portfolio tail dependence



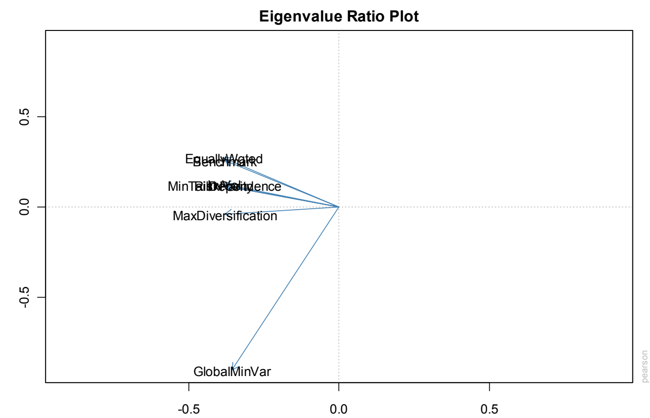
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 143: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 144: Grouping the strategies.

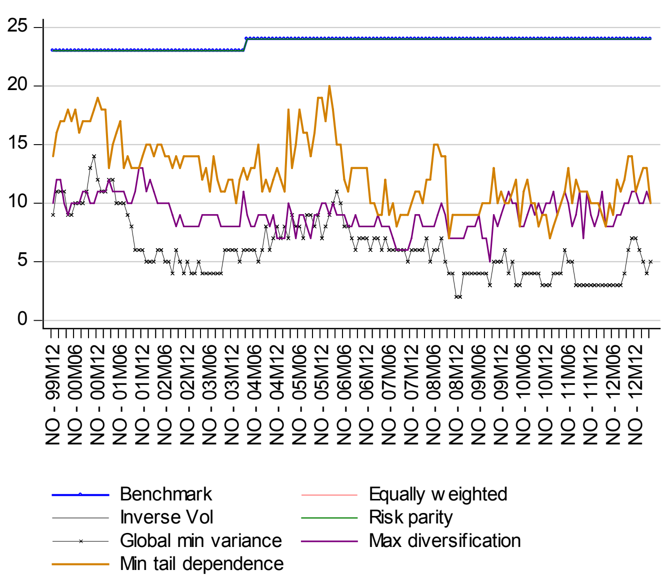


Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

MSCI World 24 GICS level 2 industry group portfolio

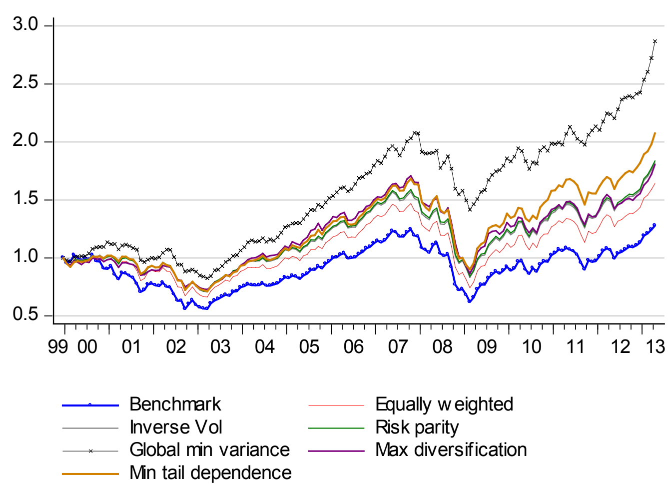
Even if we extend to the 24 GICS industry groups, the results remain the same – GlobalMinVar again dominates.

Figure 145: Investment universe



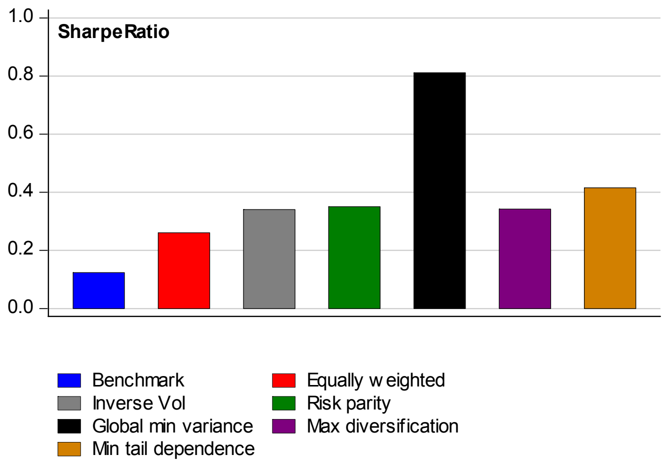
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 146: Wealth curve



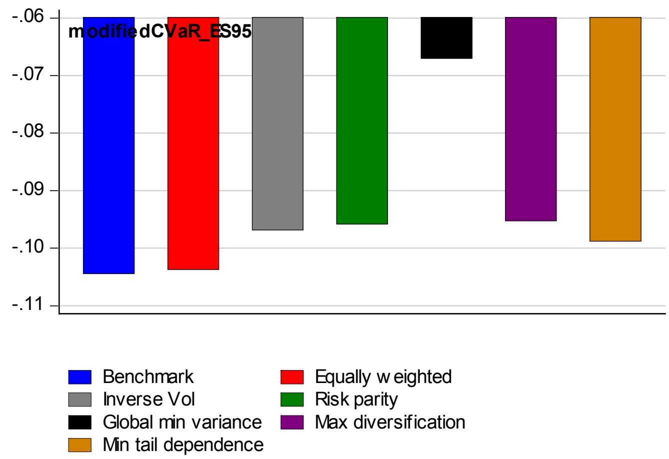
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 147: Sharpe ratio



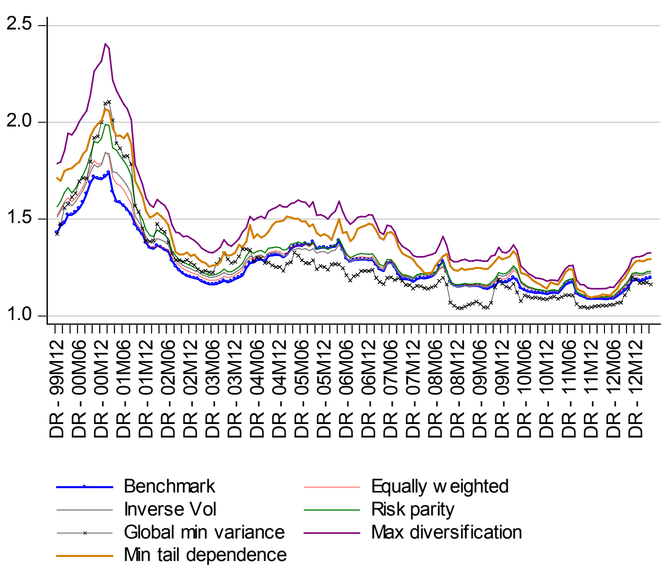
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 148: CVaR/expected shortfall



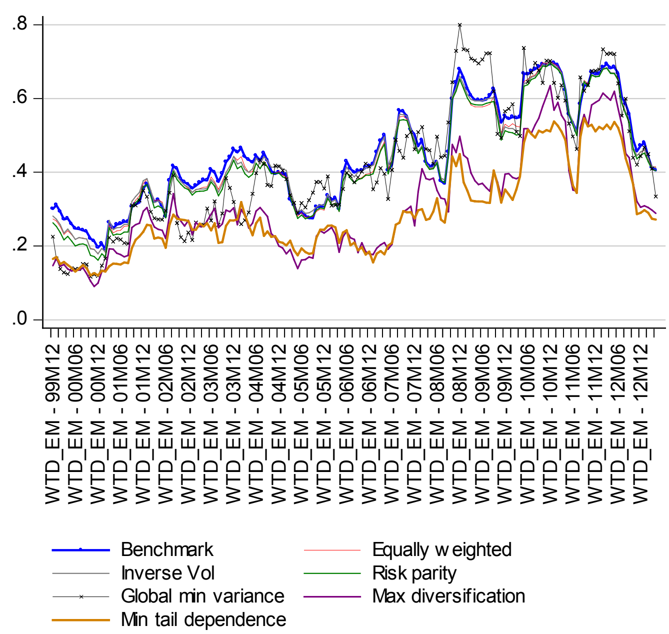
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 149: Diversification ratio



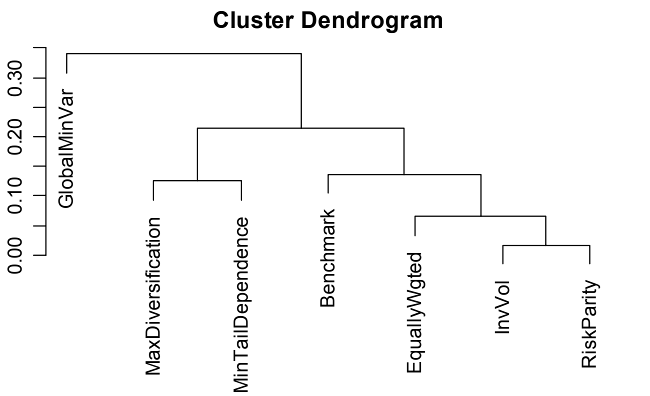
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 150: Weighted portfolio tail dependence



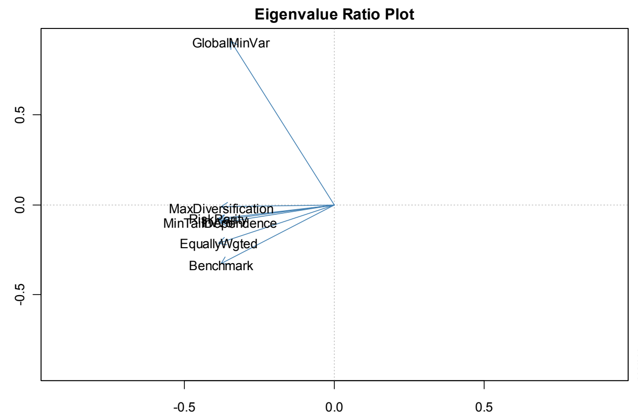
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 151: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 152: Grouping the strategies.



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

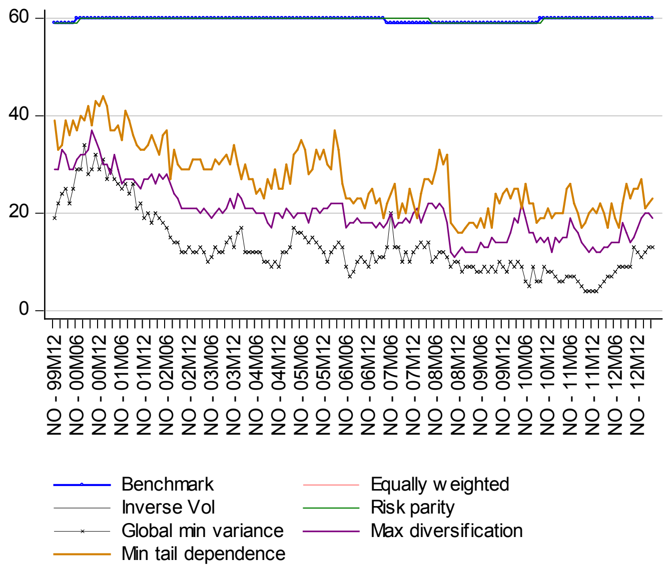
Region x sector portfolios

Investment universe

Following the same philosophy as in De Boer, Campagna, and Norman [2013], we backtest our risk-based allocation strategies in the country and sector two-dimensional grid. In De Boer, Campagna, and Norman [2013], the authors use the expansion of all countries on the 10 sectors. In our experience, for many countries, we have very limited sector coverage. Therefore, rather than using countries, we first break down the world (and here we focus on developed countries only, mostly for liquidity concerns) into six broad regions: US, Canada, Europe ex UK, UK, Pacific Free ex Japan (i.e., Hong Kong, Singapore, Australia, and New Zealand), and Japan. Then, within each of the six regions, we define the 10 GICS sectors. In the end, we have $6 \times 10 = 60$ region x sector assets, e.g., US consumer stables, Europe ex UK financials, Pacific Free ex Japan information technology, etc.

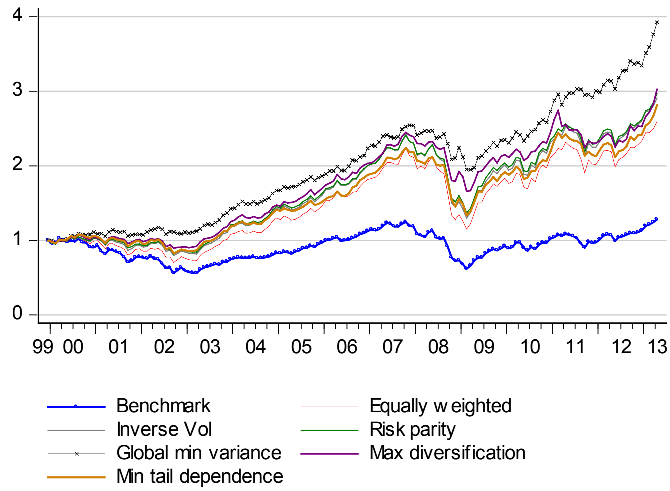
Interestingly, as long as we have sectors involved, GlobalMinVar wins again (see Figure 155 and Figure 156). As we add the country/region dimension, we see decent improvement in Sharpe ratio and reduction in downside risk, compared to pure sector/industry portfolios. More importantly, the potential diversification benefit grows significantly.

Figure 153: Investment universe



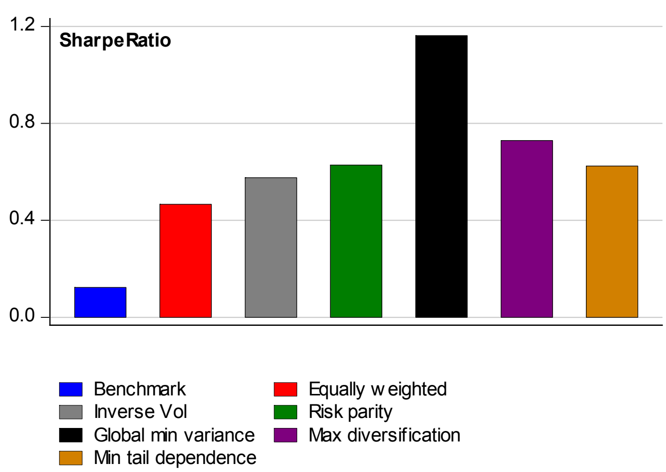
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 154: Wealth curve



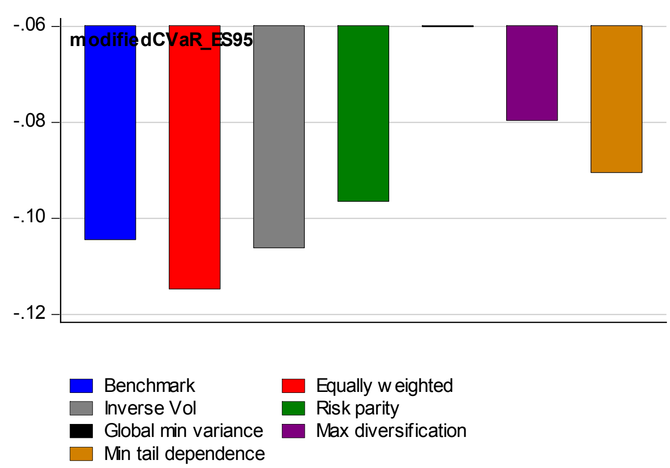
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 155: Sharpe ratio



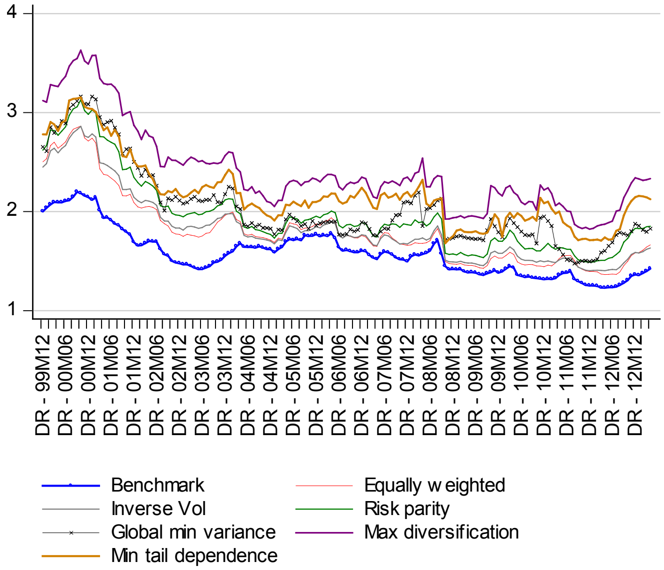
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 156: CVaR/expected shortfall



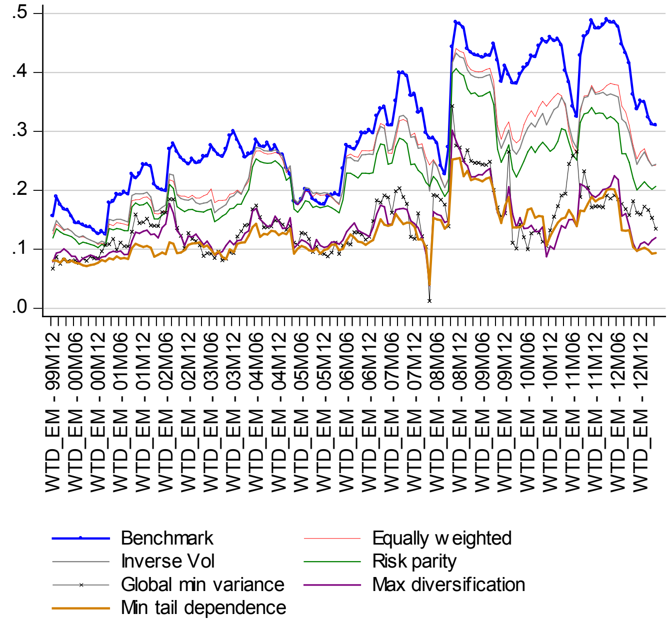
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 157: Diversification ratio



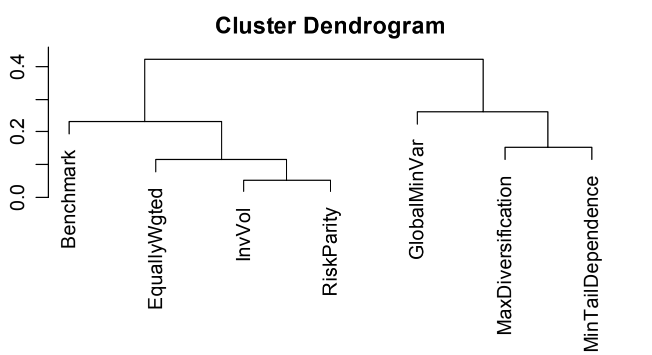
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 158: Weighted portfolio tail dependence



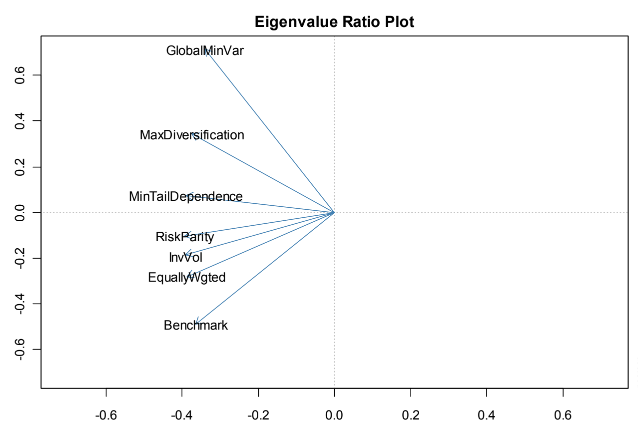
Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 159: Clustering the strategies



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 160: Grouping the strategies.



Source: Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

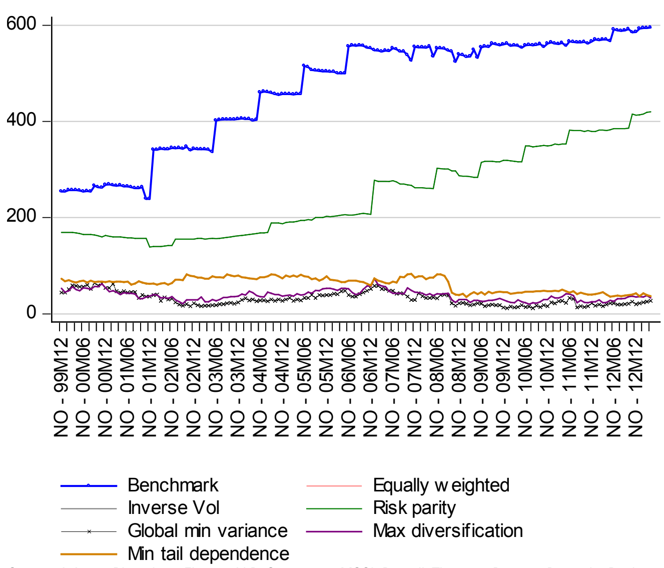
XI. Equity portfolios

Applying risk-based allocation in the equity space is probably where we see the most published research. In this section, we apply our risk-based allocations in six equity universes: US equities, European equities, Asia ex Japan equities, Japan equities, emerging markets equities, and finally global equities.

US equities

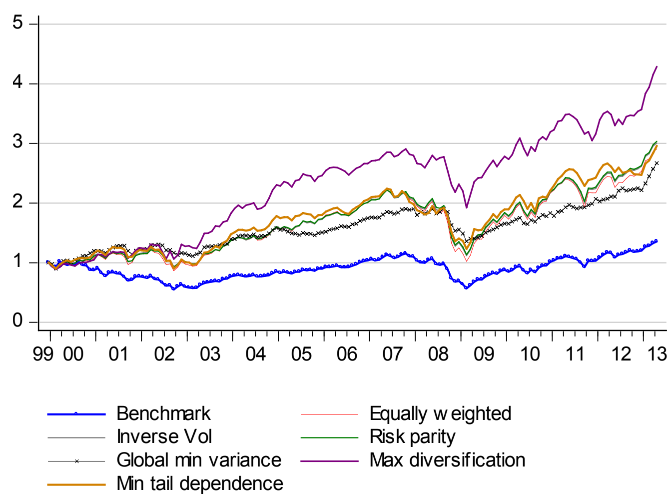
We use MSCI USA as our investment universe. For US equities, MaxDiversification and GlobalMinVar significantly outperform the other strategies, with higher Sharpe ratios (see Figure 163) and lower downside risks (see Figure 164). More importantly, all risk-based allocations considerably exceed the capitalization weighted benchmark. Consistently with what we found recently (see Cahan, *et al* [2013]), we start to detect increased level of crowding (or less chance of diversification) for EquallyWgted, InvVol, RiskParity, and GlobalMinVar strategies (see Figure 167 and Figure 168). If investors are worried about crowding, MinTailDependence and MaxDiversification strategies are great alternatives to the GlobalMinVar.

Figure 161: Investment universe



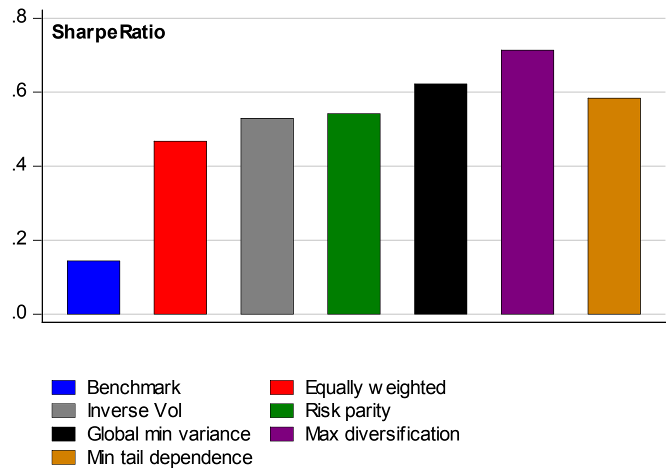
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 162: Wealth curve



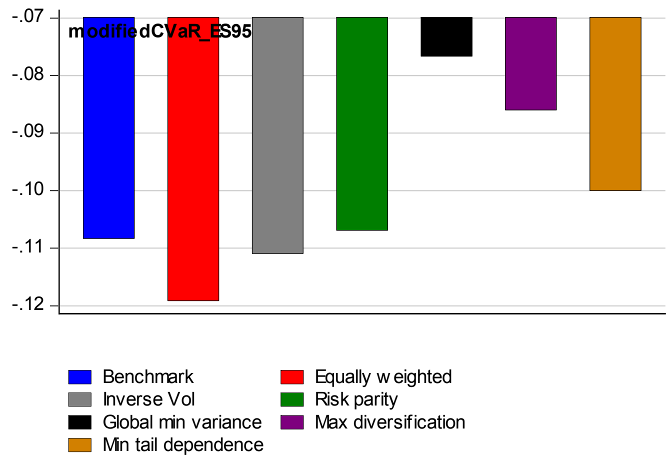
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 163: Sharpe ratio



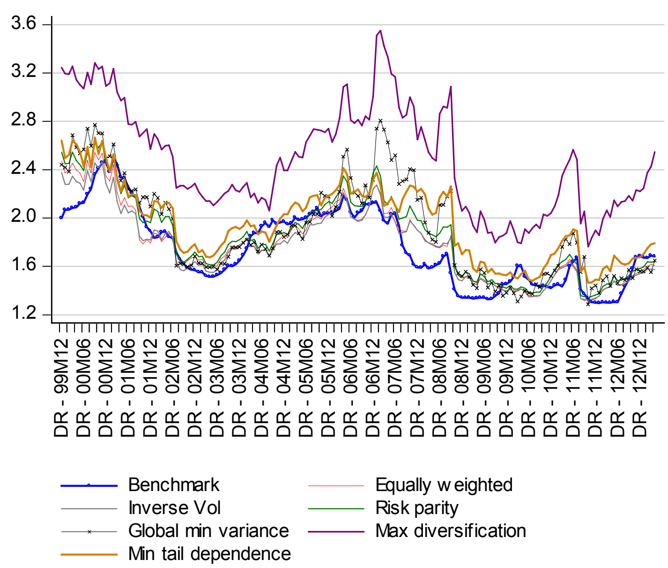
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 164: CVaR/expected shortfall



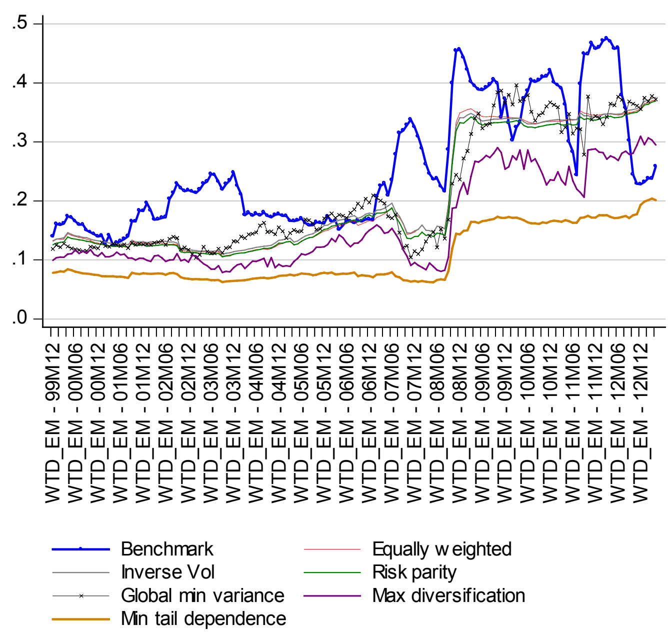
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 165: Diversification ratio



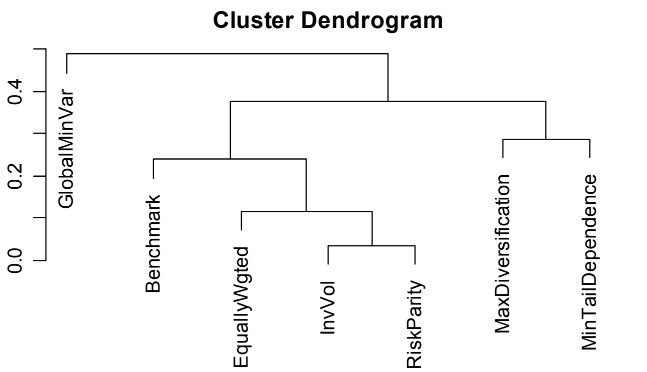
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 166: Weighted portfolio tail dependence



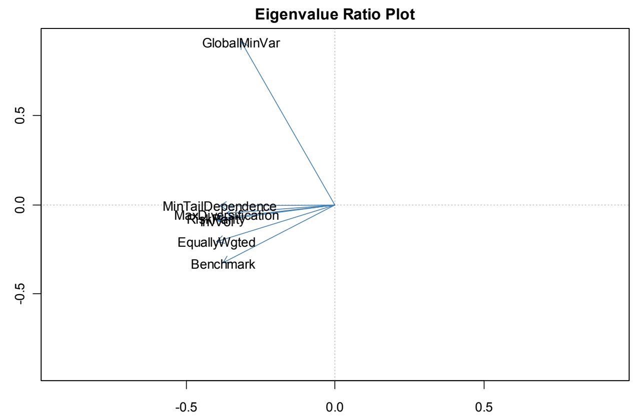
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 167: Clustering the strategies



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 168: Grouping the strategies.

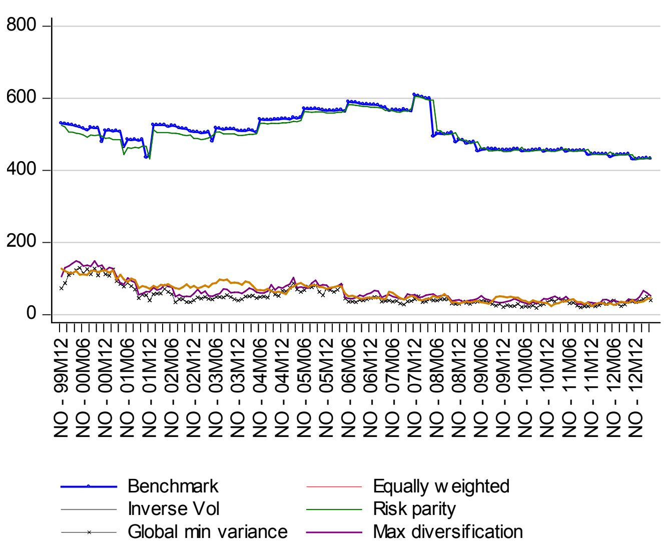


Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

European equities

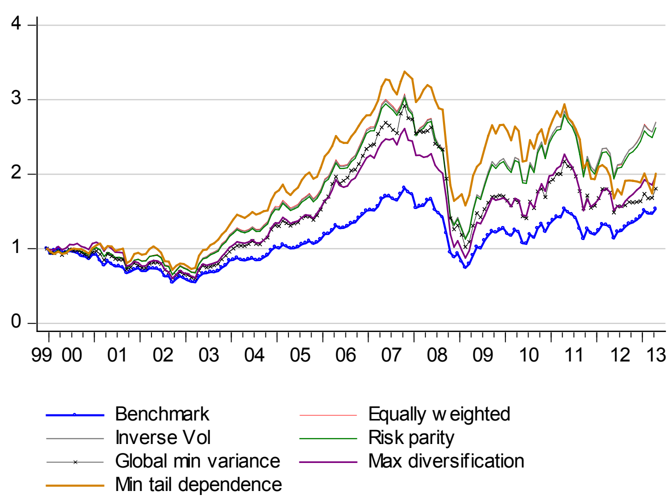
We use MSCI Europe as our investment universe (i.e., Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Israel, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and UK). Interestingly, in European equities, it is the simpler strategies that end up winning – EquallyWgted, InvVol, and RiskParity. All risk-based strategies substantially outpace the capitalization weighted benchmark, in terms of higher Sharpe ratios (see Figure 171) and lower downside risks (see Figure 172). We see much less level of crowding for risk-based strategies in European equities. In addition, our MinTailDependence portfolio again forms its own cluster and offers investors a great way to avoid crowded strategies like GlobalMinVar (see Figure 174).

Figure 169: Investment universe



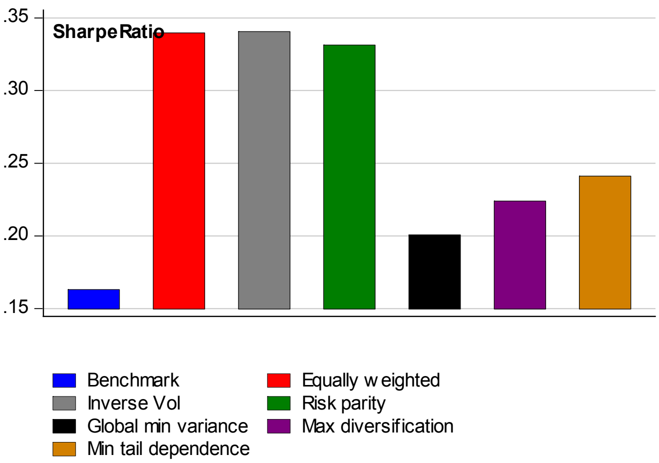
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 170: Wealth curve



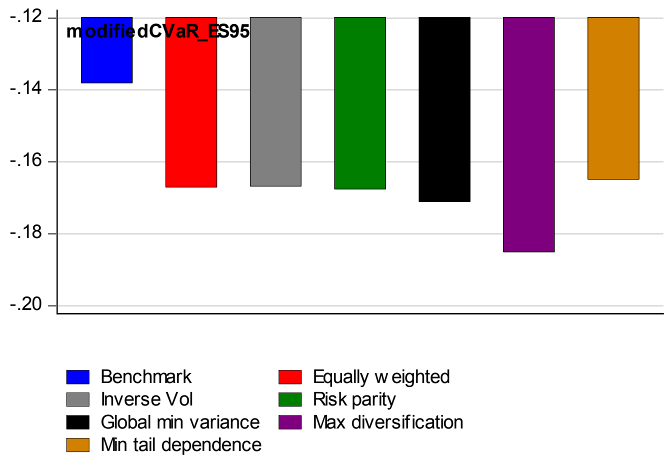
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 171: Sharpe ratio



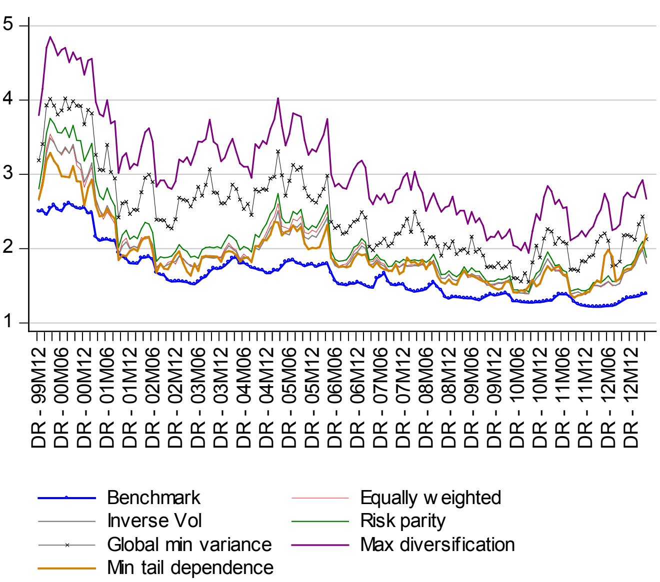
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 172: CVaR/expected shortfall



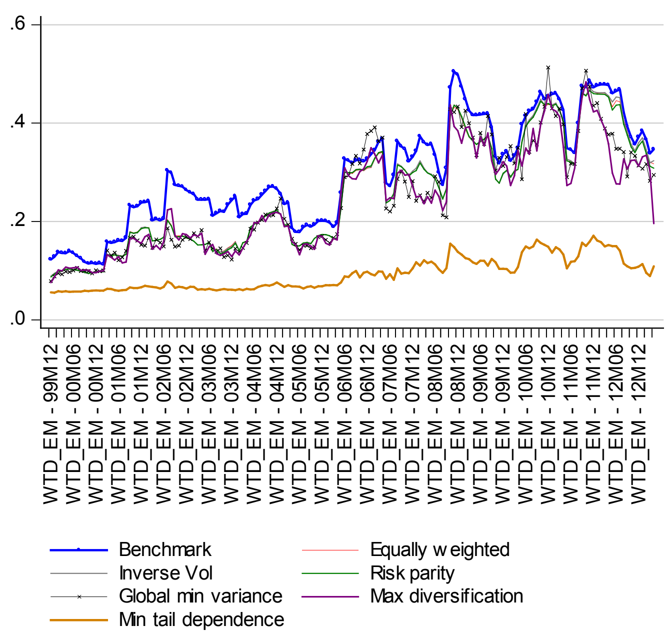
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 173: Diversification ratio



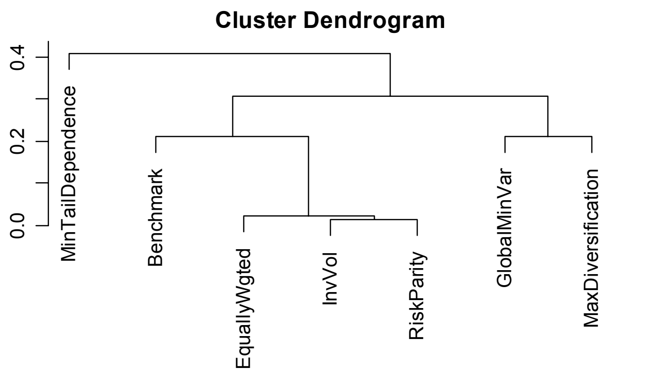
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 174: Weighted portfolio tail dependence



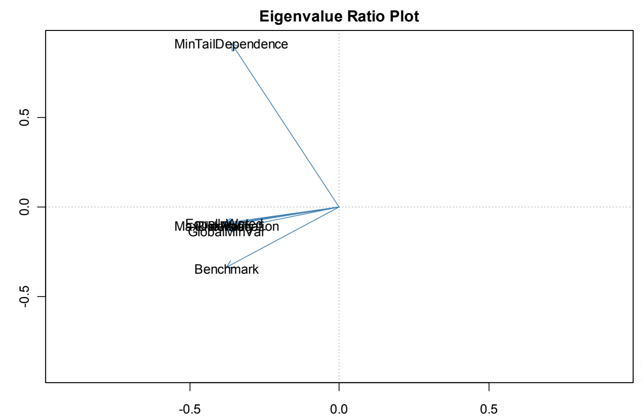
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 175: Clustering the strategies



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 176: Grouping the strategies.

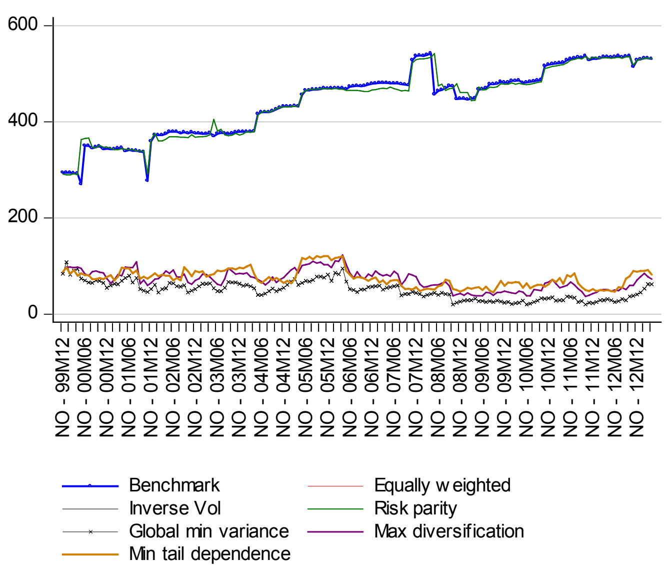


Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Asian equities

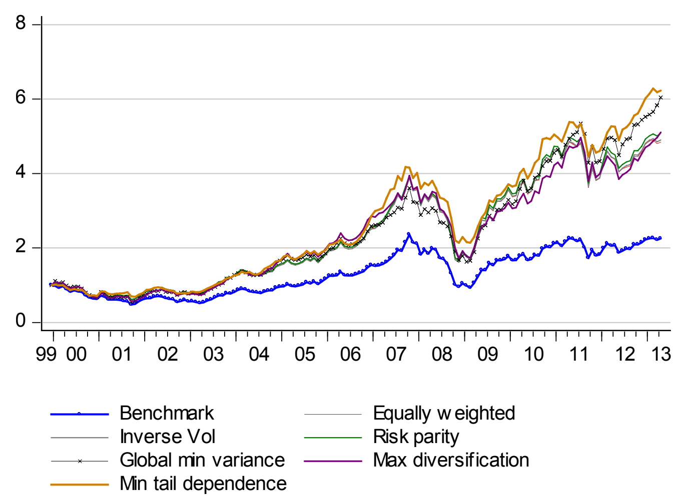
We use the MSCI Pacific ex Japan as our investment universe (i.e., Hong Kong, Singapore, China, India, Indonesia, Korea, Malaysia, Philippines, Taiwan, and Thailand). In this universe, MinTailDependence shows the highest Sharpe ratio (see Figure 179) and lowest downside risk (see Figure 180), followed by GlobalMinVar. Again, all risk-based allocations significantly beat the capitalization weighted benchmark. Similar to what we find in European equities, risk-based allocations still offer great diversification benefit in the Asia ex Japan universe. MinTailDependence again shows its uniqueness (see Figure 183) and is likely to avoid crowded trades (see Figure 182).

Figure 177: Investment universe



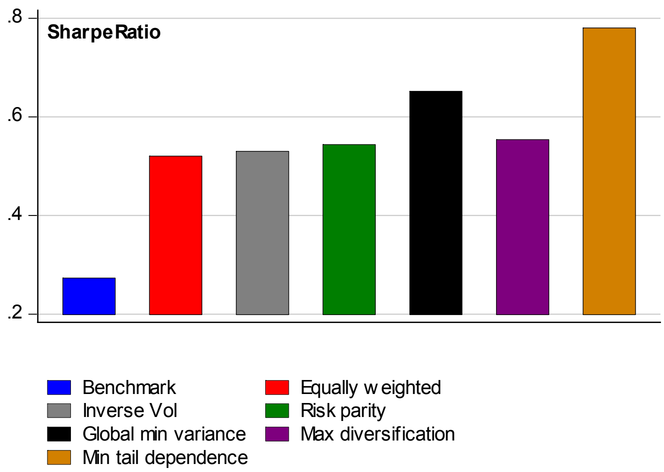
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 178: Wealth curve



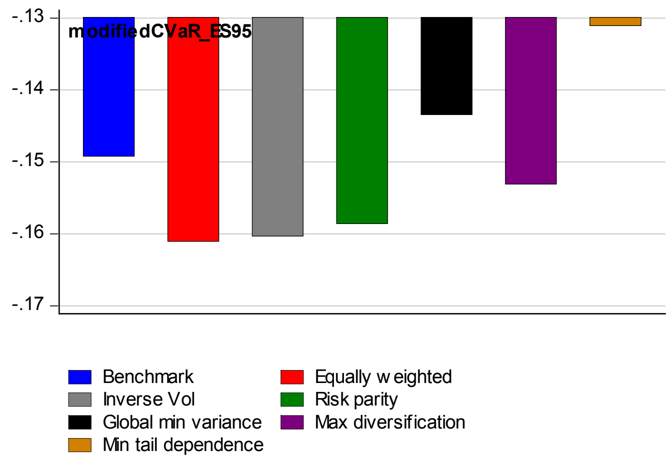
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 179: Sharpe ratio



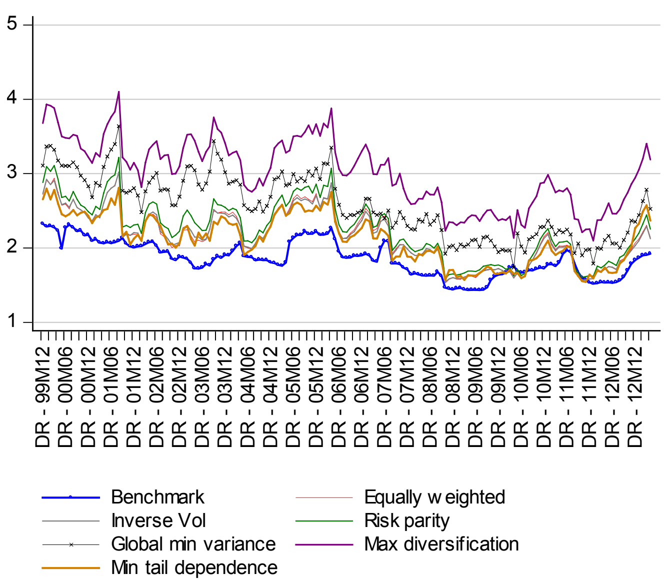
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 180: CVaR/expected shortfall



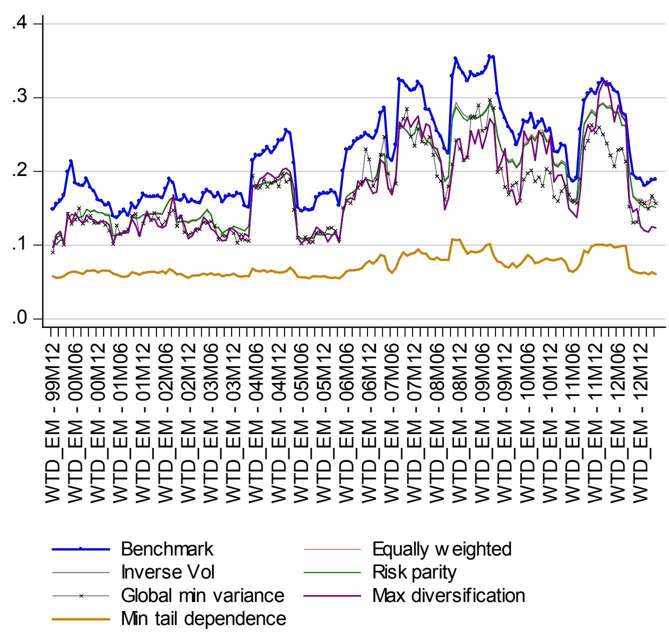
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 181: Diversification ratio



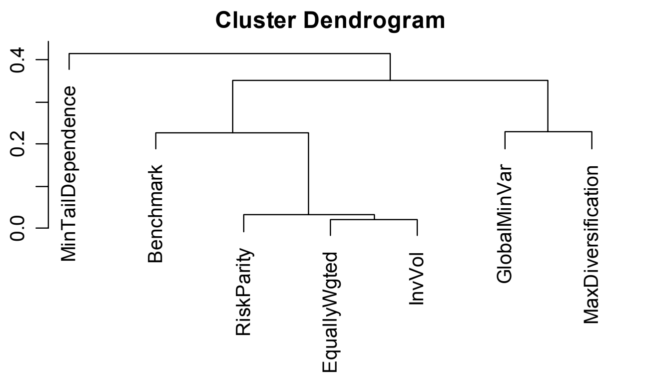
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 182: Weighted portfolio tail dependence



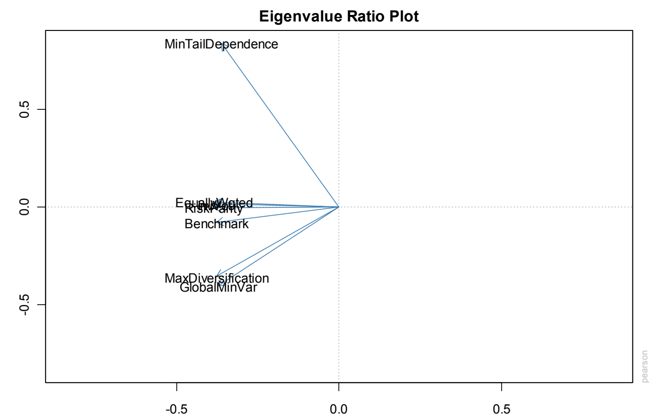
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 183: Clustering the strategies



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 184: Grouping the strategies.

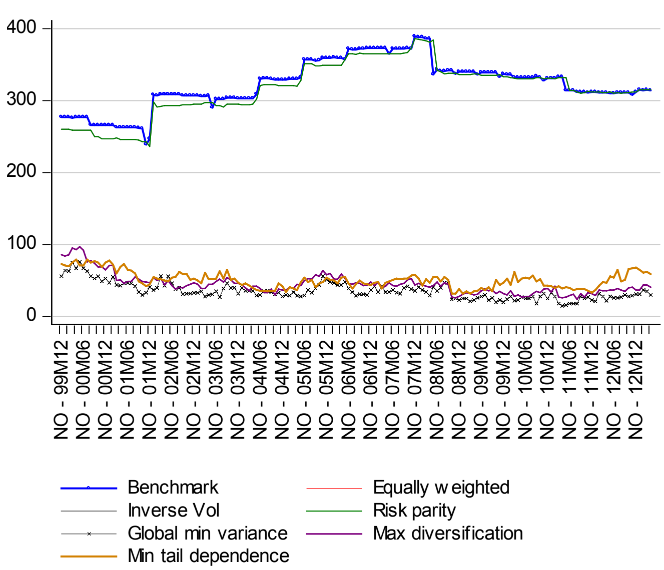


Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Japanese equities

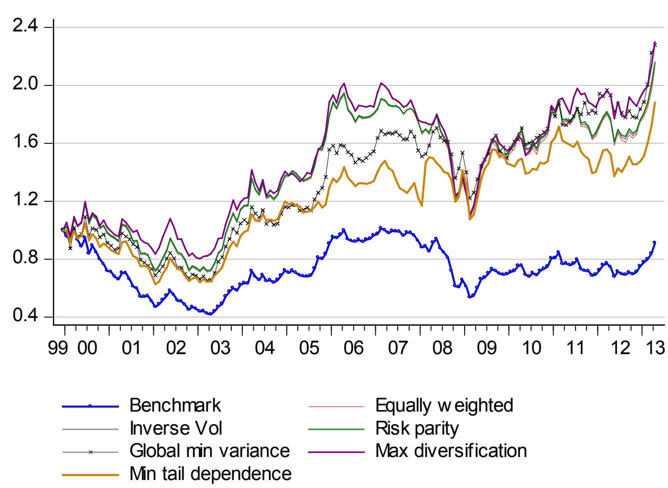
We use the MSCI Japan as our investment universe. GlobalMinVar has the highest Sharpe ratio (see Figure 187), while MinTailDependence has the lowest downside risk (see Figure 188). MinTailDependence again seems to be quite unique (see Figure 191), with significantly lower chance of crowding (see Figure 190), while the traditional risk-based strategies appear to be getting crowded in recent years.

Figure 185: Investment universe



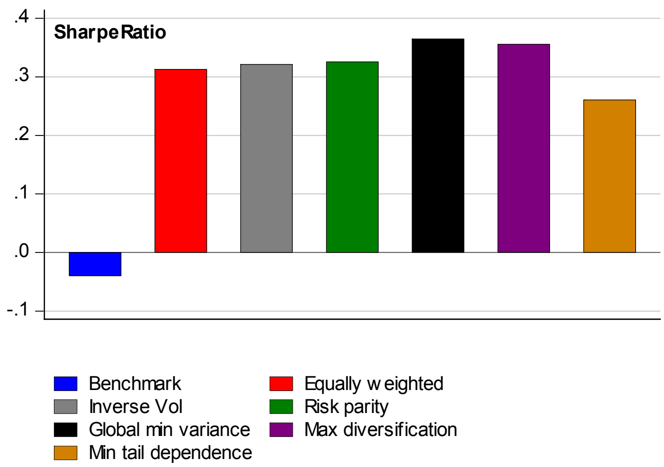
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 186: Wealth curve



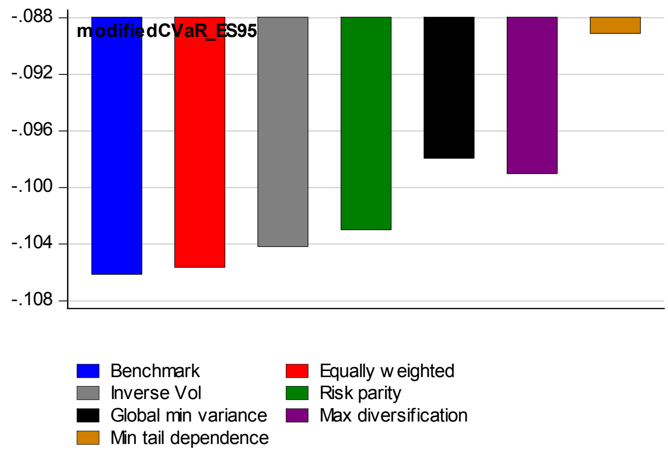
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 187: Sharpe ratio



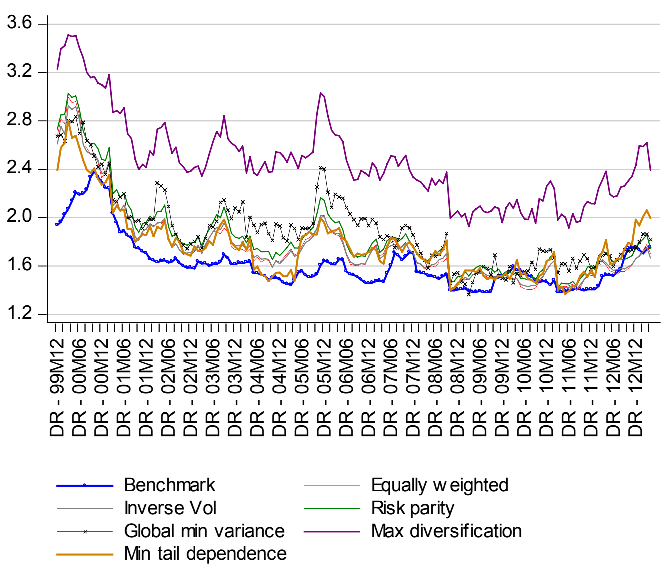
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 188: CVaR/expected shortfall



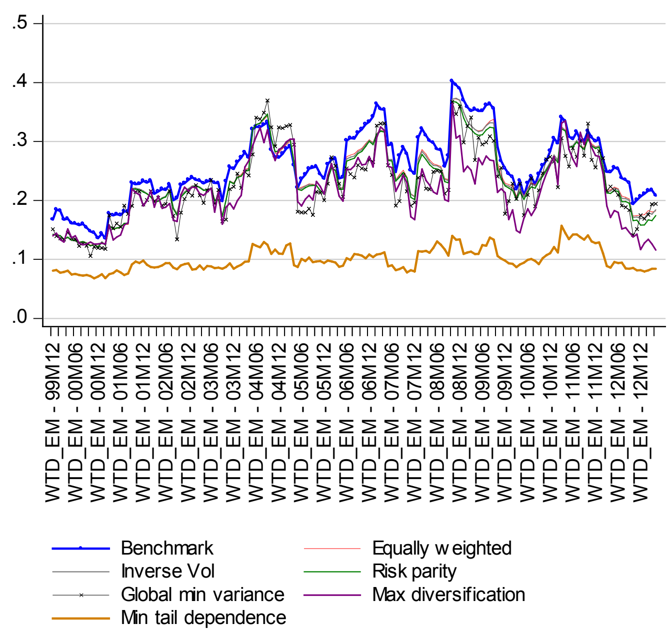
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 189: Diversification ratio



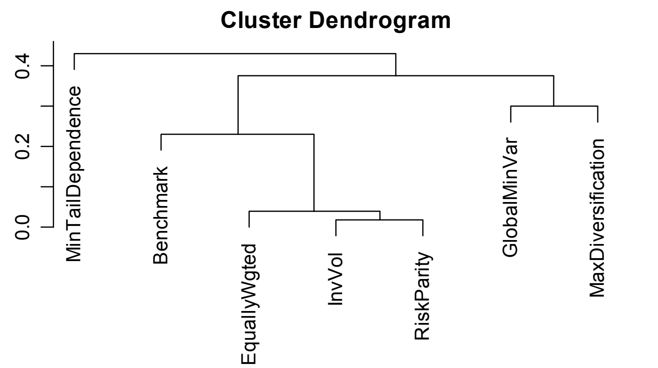
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 190: Weighted portfolio tail dependence



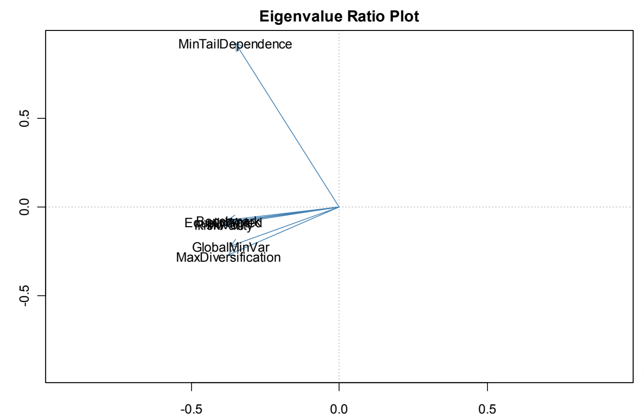
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 191: Clustering the strategies



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 192: Grouping the strategies.

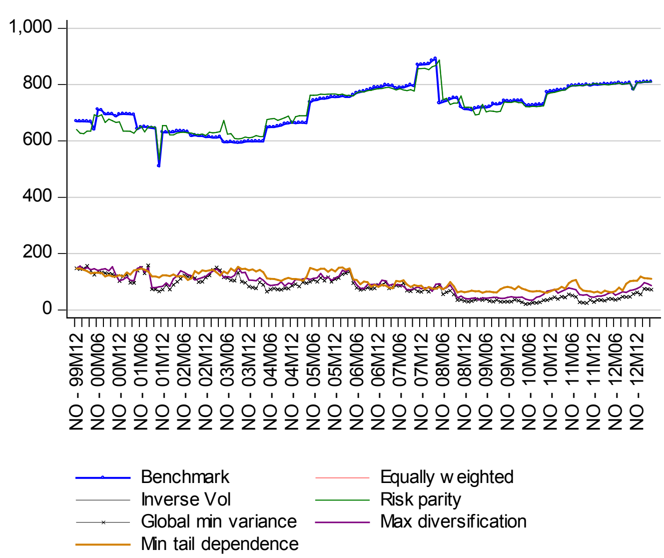


Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Emerging markets equities

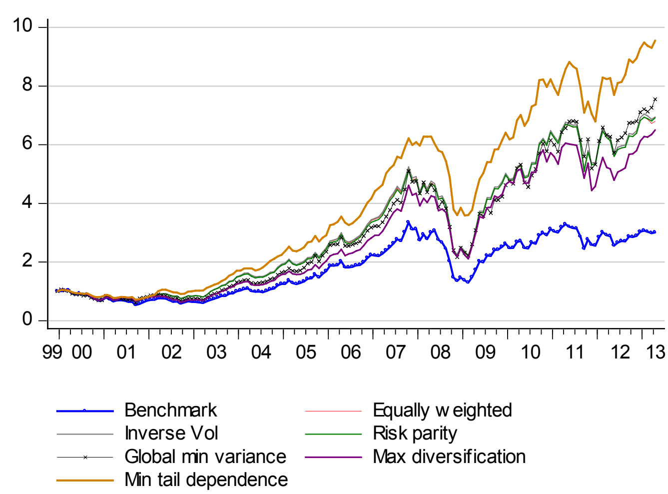
We use the MSCI EM as our investment universe. Similar to Asia ex Japan, our MinTailDependence portfolio is the clear winner, with substantially higher Sharpe ratio (see Figure 195) and lower downside risk (see Figure 196). It also appears to be highly unique (see Figure 199 and Figure 200), with much lower *ex post* tail dependence among its holdings (see Figure 198).

Figure 193: Investment universe



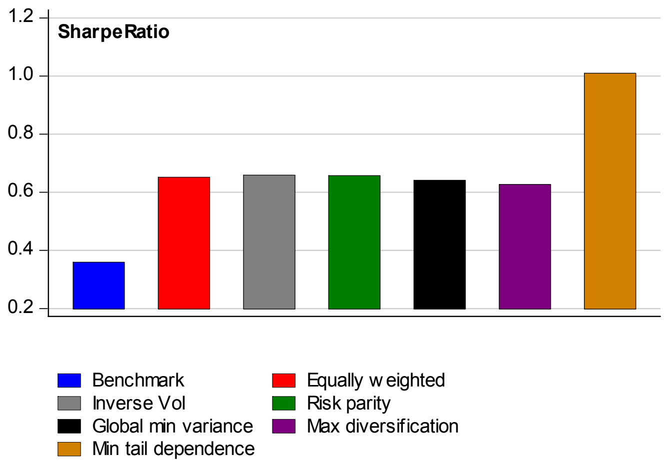
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 194: Wealth curve



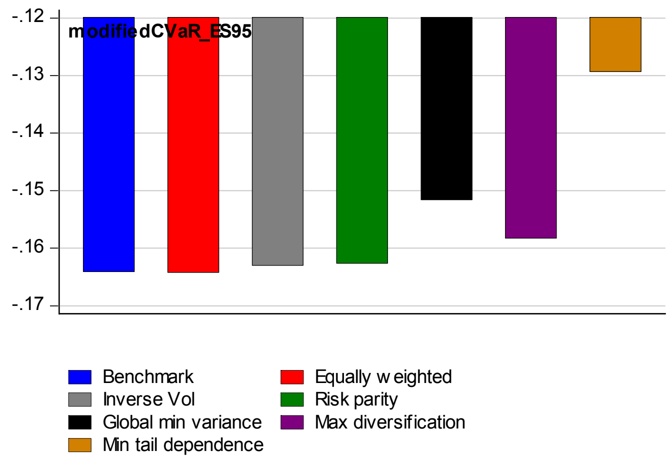
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 195: Sharpe ratio



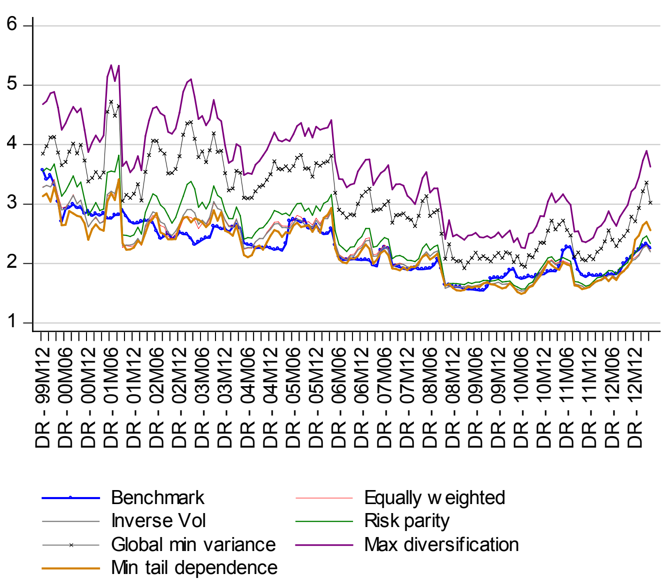
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 196: CVaR/expected shortfall



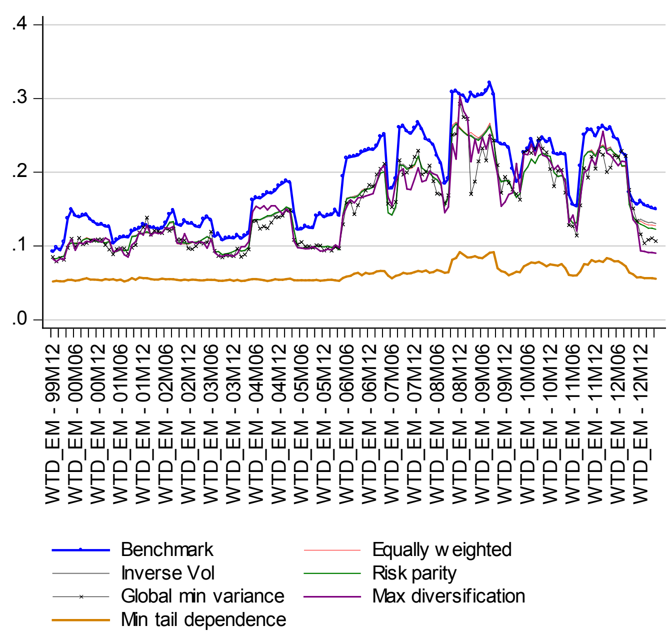
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 197: Diversification ratio



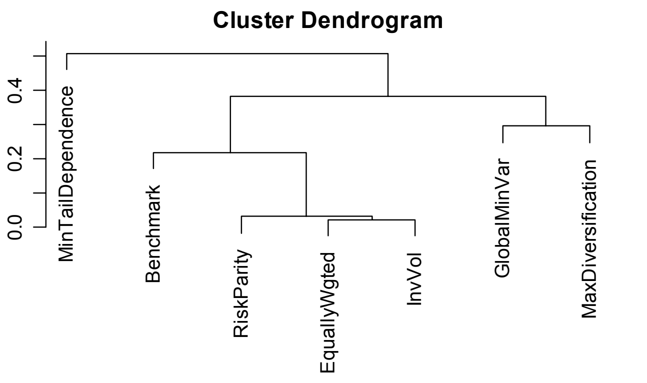
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 198: Weighted portfolio tail dependence



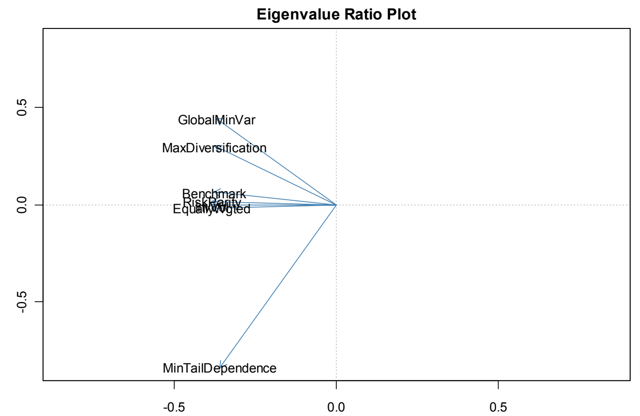
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 199: Clustering the strategies



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 200: Grouping the strategies.

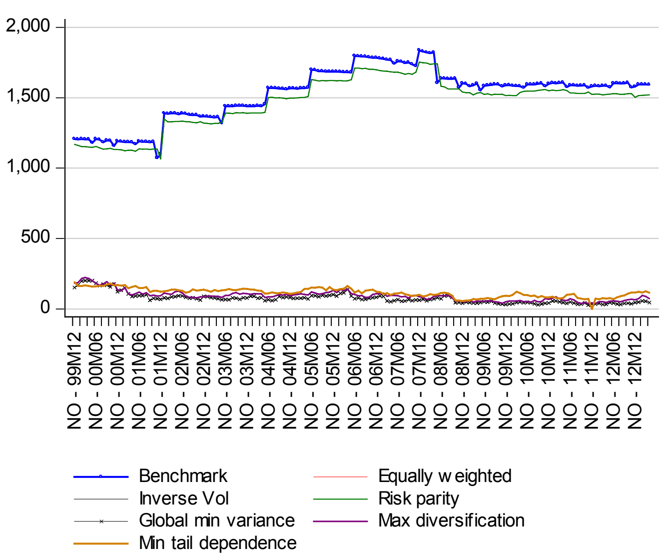


Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Global equity

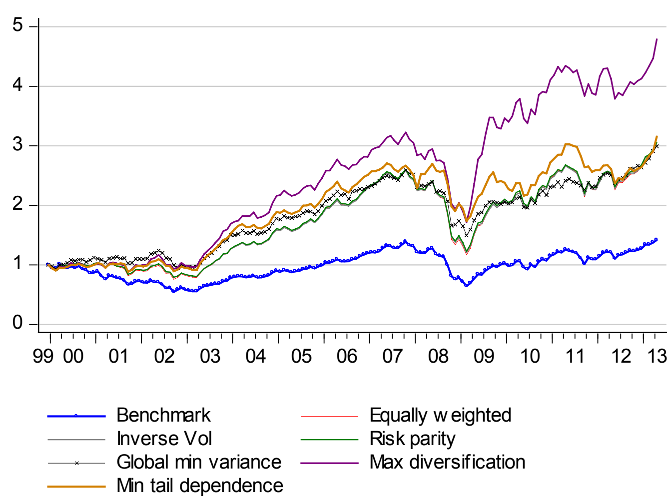
Our investment universe is the MSCI World Index, i.e., the developed countries. In the global equity space, MaxDiversification, GlobalMinVar, and MinTailDependence strategies appear to be very robust, with attractive Sharpe ratios (see Figure 203) and reasonable downside risks (see Figure 204). They also form a unique cluster (see Figure 207). MinTailDependence once again proves to be the least crowded strategy (see Figure 206).

Figure 201: Investment universe



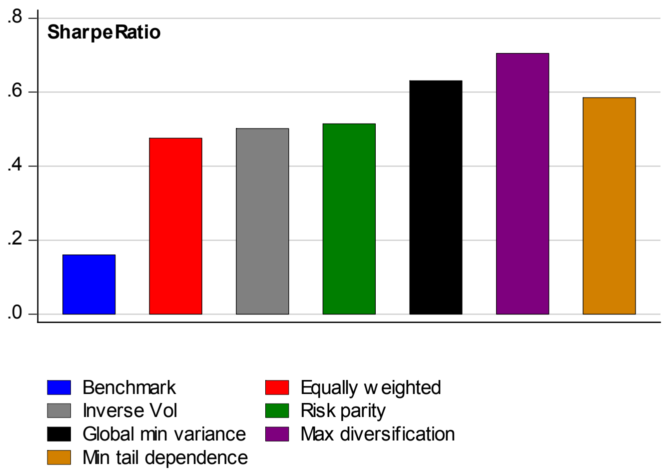
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 202: Wealth curve



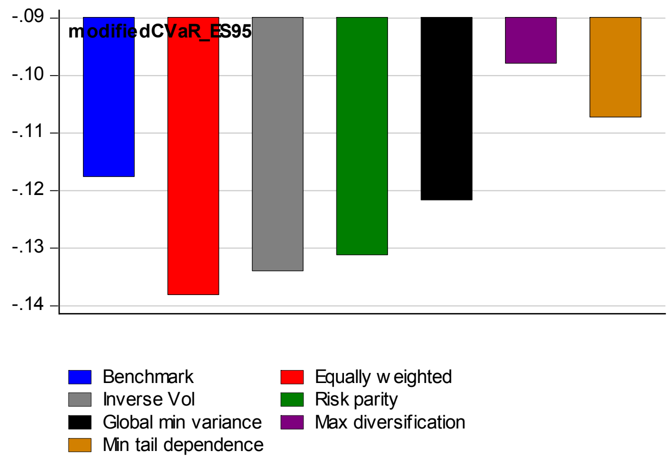
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 203: Sharpe ratio



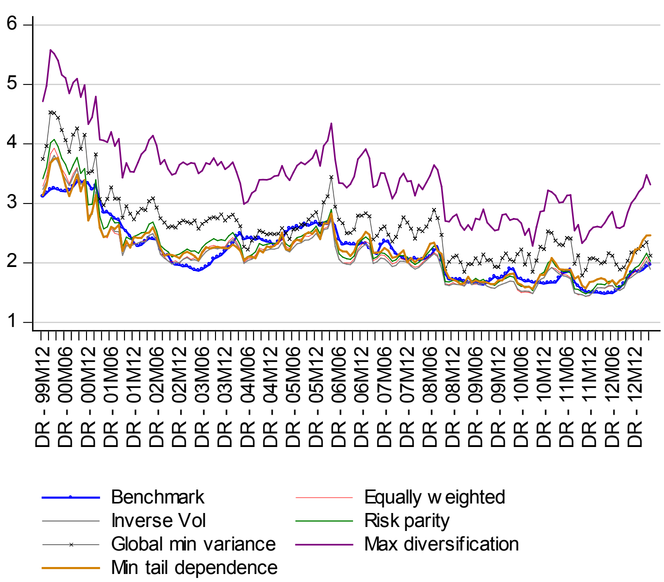
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 204: CVaR/expected shortfall



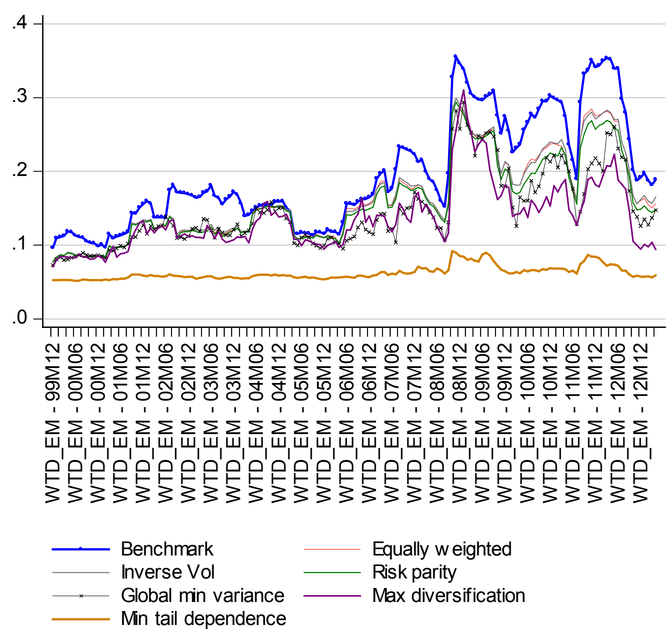
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 205: Diversification ratio



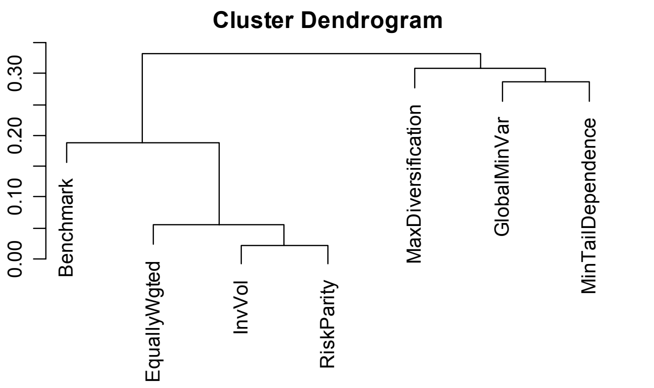
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 206: Weighted portfolio tail dependence



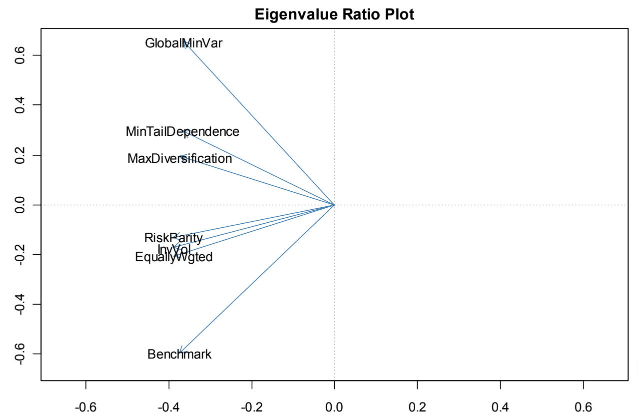
Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 207: Clustering the strategies



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Figure 208: Grouping the strategies.



Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

XII. Conclusion

The philosophy of portfolio construction

In summary, we survey seven risk-based allocations and compare their performance with the more traditional capitalization-weighted benchmark. Among the seven portfolio construction techniques, we can classify them into two categories, based on their main intended goals.

Diversification based

Diversification based strategies primarily try to diversify our investments. EquallyWgted is probably the most naïve way to reach this goal. InvVol or volatility parity further takes into account of each asset's risk in diversification. RiskParity brings it to the next level by incorporating correlation and asset specific risk. MaxDiversification, by construction is the most diversified strategy (*ex ante*). If we define asset dependence by tail dependence coefficient, to recognize that a linear correlation coefficient can't fully capture the full extent of comovement, then we can design our MinTailDependence strategy, which should deliver the highest tail diversification (*ex ante*).

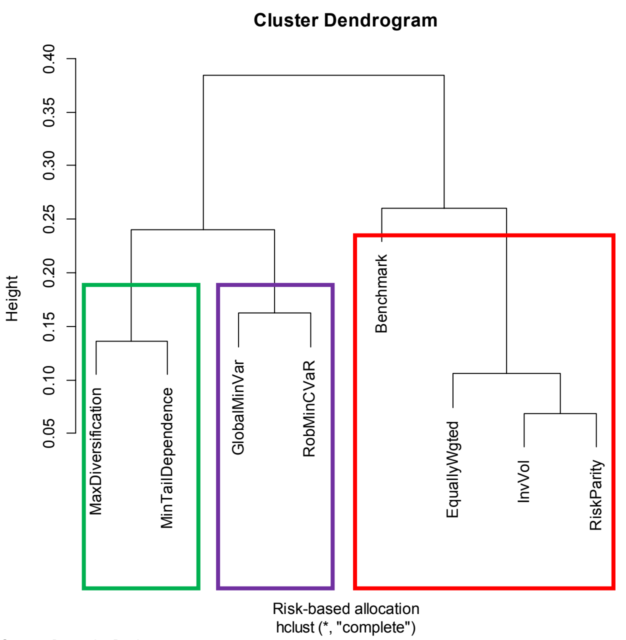
Risk reduction based

Strategies in this bucket attempts to reduce risk as the dominant goal. GlobalMinVar relies on volatility (or variance) as the definition of risk and aims to achieve risk reduction by minimizing a portfolio's expected variance. If we relax the assumption of multivariate normal distribution of asset returns (which is almost always violated with real life data), we can acknowledge that variance (or volatility) is not the only way to define risk. Conditional value at risk or CVaR is a more coherent definition of risk and has many nice properties. A strategy that tries to reduce CVaR risk is what we called MinCVaR[20].

Figure 209 and Figure 210 reproduce the cluster analysis graphs using our country allocation example in Section VI. Statistical cluster analysis confirms our argument that MaxDiversification and MinTailDependence are more designed to capture diversification, while GlobalMinVar and MinCVaR are more for risk control

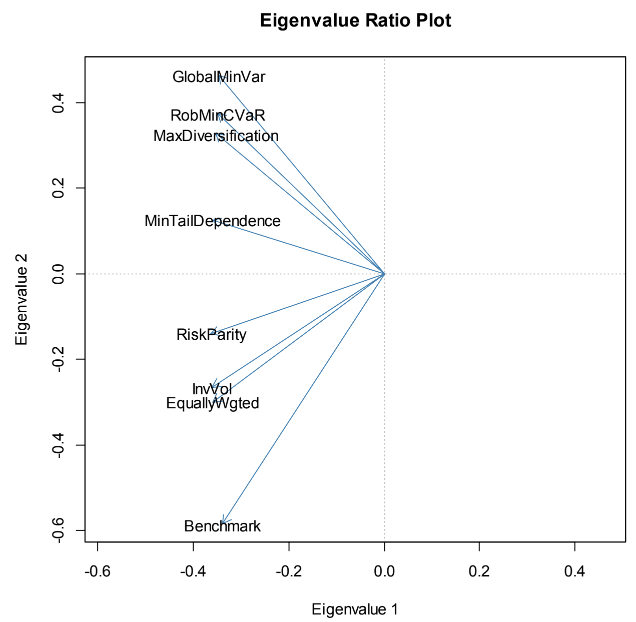
[20] Because our RobMinCVaR is very computationally intensive, we only show its application using our global country portfolio as an example. For this paper, we do not show RobMinCVaR in other contexts.

Figure 209: Cluster analysis



Source: Deutsche Bank

Figure 210: Eigenvalue ratio plot



Source: Deutsche Bank

A horse race of risk-based allocations

We apply our seven risk-based portfolio construction techniques in three different contexts:

- **Asset allocation:** multi-asset allocation, global sovereign bonds, commodities, and alternative betas
- **Country, sector, industry allocation:** global countries, economic risk hedged global countries, global sectors, US sectors, European sectors, global industries, and region x sector combinations
- **Equities:** US equities, European equities, Asia ex Japan equities, Japanese equities, emerging markets equities, and global equities

Sharpe ratio ranking

In terms of Sharpe ratio (see Figure 211), GlobalMinVar achieves the highest Sharpe ratio overall, followed by MinTailDependence, and MaxDiversification. Capitalization weighted benchmark delivers the lowest Sharpe ratio, followed by naïve EquallyWgted and InvVol strategies.

Figure 211: Sharpe ratio ranking

Sharpe ratio	Benchmark	EquallyWgted	InvVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Asset allocation, avg ranking	*6*	*7*	*5*	*4*	*2*	*1*	*3*
Multi-assets	7	6	4	2	5	1	3
Sovereign bonds	4	7	6	5	1	2	3
Commodities	6	6	4	5	1	2	3
Alternative betas	7	6	5	4	2	3	1
Country/sector allocation, avg ranking	*7*	*6*	*5*	*3*	*1*	*4*	*2*
Countries, MSCI ACWI	7	6	5	4	3	2	1
Countris, MEAM	7	5	6	4	2	3	1
Sectors, MSCI	7	5	2	3	1	6	4
Sectors, US	7	5	2	3	1	6	4
Sectors, Europe	7	6	3	2	1	4	5
Industries, MSCI	7	6	5	3	1	4	2
Regions x sectors, MSCI	7	6	5	3	1	2	4
Equities, avg ranking	*7*	*6*	*5*	*4*	*3*	*3*	*3*
US	7	6	5	4	2	1	3
Europe	7	2	1	3	6	5	4
Asia ex Japan	7	6	5	4	2	3	1
Japan	7	5	4	3	1	2	6
Emerging markets	7	4	2	3	5	6	1
Global	7	6	5	4	2	1	3
Overall ranking	***7***	***6***	***5***	***4***	***1***	***3***	***2***

Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Downside risk ranking

As shown in Figure 212, when we shift our focus to risk reduction, GlobalMinVar wins again and accomplishes its goal *ex post*. MaxDiversification and MinTailDependence portfolios are not far off, with comparable risk cutback. The naïve strategies (e.g., EquallyWgted, InvVol) and benchmark fall behind again with the highest downside risk.

Figure 212: Downside risk ranking

CVaR/expected shortfall	Benchmark	EquallyWgted	InvVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Asset allocation, avg ranking	*5*	*7*	*6*	*4*	*2*	*1*	*3*
Multi-assets	2	7	6	3	4	1	5
Sovereign bonds	3	7	6	5	1	2	4
Commodities	6	6	5	4	1	2	3
Alternative betas	7	6	4	2	5	3	1
Country/sector allocation, avg ranking	*6*	*7*	*5*	*3*	*1*	*2*	*4*
Countries, MSCI ACWI	2	7	6	5	1	3	4
Countris, MEAM	1	6	4	5	2	3	7
Sectors, MSCI	6	7	5	4	1	3	2
Sectors, US	7	6	4	3	1	5	2
Sectors, Europe	7	6	4	3	1	2	5
Industries, MSCI	7	6	4	3	1	2	5
Regions x sectors, MSCI	5	7	6	4	1	2	3
Equities, avg ranking	*4*	*7*	*6*	*5*	*2*	*3*	*1*
US	5	7	6	4	1	2	3
Europe	1	4	3	5	6	7	2
Asia ex Japan	3	7	6	5	2	4	1
Japan	7	6	5	4	2	3	1
Emerging markets	6	7	5	4	2	3	1
Global	3	7	6	5	4	1	2
Overall ranking	*5*	*7*	*6*	*4*	*1*	*2*	*3*

Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Diversification ranking

Now, let's move to diversification, defined as linear comovement. As shown in Figure 213, MaxDiversification also earns the highest diversification ratio *ex post*, followed by MinTailDependence and RiskParity. The benchmark, EquallyWgted, and InvVol have the lowest diversification.

Figure 213: Diversification ranking

Diversification ratio	Benchmark	EquallyWgted	InvVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Asset allocation, avg ranking	*6*	*7*	*5*	*3*	*4*	*1*	*3*
Multi-assets	6	6	4	2	5	1	3
Sovereign bonds	5	7	6	3	4	1	2
Commodities	6	6	5	3	4	1	2
Alternative betas	6	6	5	3	2	1	4
Country/sector allocation, avg ranking	*7*	*4*	*5*	*3*	*6*	*1*	*2*
Countries, MSCI ACWI	7	5	6	4	3	1	2
Countris, MEAM	7	6	5	4	3	1	2
Sectors, MSCI	6	4	5	3	7	1	2
Sectors, US	6	4	5	3	7	1	2
Sectors, Europe	6	4	5	3	7	1	2
Industries, MSCI	7	5	4	3	6	1	2
Regions x sectors, MSCI	7	6	5	4	3	1	2
Equities, avg ranking	*7*	*5*	*6*	*3*	*2*	*1*	*5*
US	7	5	6	4	3	1	2
Europe	7	4	5	3	2	1	6
Asia ex Japan	7	4	5	3	2	1	6
Japan	7	6	5	3	2	1	4
Emerging markets	6	4	5	3	2	1	7
Global	5	6	7	3	2	1	4
Overall ranking	***7***	***6***	***6***	***3***	***4***	***1***	***3***

Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Tail dependence/crowding ranking

Finally, we focus on strategy crowding and tail dependence (see Figure 214). Our MinTailDependence portfolio manages to deliver the best diversification *ex post*, especially in the country/sector and equity portfolios. MaxDiversification and GlobalMinVar also help avoid crowded trades. In US equities, where GlobalMinVar-type strategies have attracted great attention and money flow, we start to see signs of crowding. We do not observe similar patterns in other regions and asset classes yet. Our MinTailDependence strategy can be an effective diversifier.

Figure 214: Tail dependence/crowding ranking

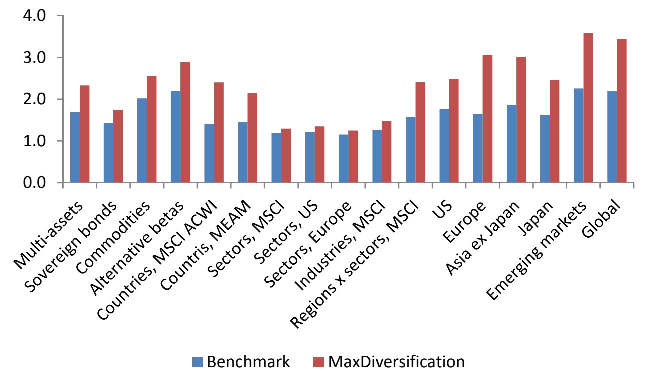
Weighted portfolio tail dependence	Benchmark	EquallyWgted	InvVol	RiskParity	GlobalMinVar	MaxDiversification	MinTailDependence
Asset allocation, avg ranking	*6*	*7*	*5*	*4*	*2*	*1*	*3*
Multi-assets	5	5	6	7	2	1	3
Sovereign bonds	4	7	6	5	1	2	3
Commodities	7	7	5	4	3	1	2
Alternative betas	7	7	5	4	1	2	3
Country/sector allocation, avg ranking	*7*	*5*	*6*	*4*	*3*	*2*	*1*
Countries, MSCI ACWI	7	6	5	4	3	2	1
Countris, MEAM	5	7	6	4	3	2	1
Sectors, MSCI	6	4	5	3	7	2	1
Sectors, US	7	5	6	4	3	2	1
Sectors, Europe	7	5	6	4	3	2	1
Industries, MSCI	7	5	6	4	3	2	1
Regions x sectors, MSCI	7	6	5	4	3	2	1
Equities, avg ranking	*7*	*5*	*6*	*4*	*3*	*2*	*1*
US	7	4	5	3	6	2	1
Europe	7	5	6	4	3	2	1
Asia ex Japan	7	5	6	4	2	3	1
Japan	7	6	5	4	3	2	1
Emerging markets	7	6	5	4	2	3	1
Global	7	5	6	4	3	2	1
Overall ranking	***7***	***6***	***5***	***4***	***3***	***2***	***1***

Source: Axioma, Bloomberg Finance LLP, Compustat, MSCI, Russell, Thomson Reuters, Deutsche Bank Quantitative Strategy

Potential diversification benefit

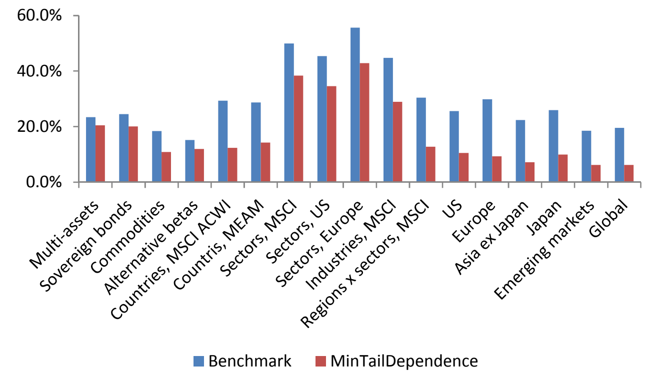
We would like to conclude this paper with a summary on potential diversification benefit of various investment universes. In Figure 215, we compare the best diversification strategy (MaxDiversification) with the least diversified portfolio (i.e., capitalization weighted benchmark) in our multi-asset, country/sector, and equity portfolios. It is interesting to note that overall multi-asset, country, and equity portfolios offer great diversification potential, while the opportunities in the sector/industry portfolios appear to be limited. Similarly, in Figure 216: Weighted portfolio tail dependence, we compare the weighted portfolio tail dependent parameter for the MinTailDependence and benchmark over the same investment universes. Again, multi-asset, country, and equity portfolios offer better diversification upside than sector/industry portfolios.

Figure 215: Diversification ratio



Source: Deutsche Bank

Figure 216: Weighted portfolio tail dependence



Source: Deutsche Bank

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XIV. Review of our previous research

We have been publishing quite intensively in the field of risk-based allocation. Below is a brief summary of each of our recent research.

Independence day, Luo, *et al* [2013], February 6, 2013

In all 45 countries in the MSCI ACWI, for both equities and bonds, we find significant positive (or negative) risk premia can be earned on the days when important economic indicators are released. We find low-risk investing by avoiding macroeconomic uncertainties is different from, additive to, and even dominating the traditional risk-based asset allocation techniques. We further demonstrate that the performance of GTAA and SAA can be improved significantly with the MEAM model.

Cross asset class momentum, Jussa, *et al* [2012], November 5, 2012

We study momentum risk premium in seven asset classes: equities, country/sector indices, hedge fund styles, currencies, commodities, and rates. The momentum risk premia across asset classes are not highly correlated. We then test a wide range of portfolio construction techniques, from naïve equally weighted, past return/Sharpe ratio weighted, to more sophisticated risk-based allocations (e.g., inverse volatility, risk parity, maximum diversification, and minimum variance). We also study a simple regime switching model.

Disentangling the downside, Cahan, *et al* [2012], September 20, 2012

Do investors get compensated for taking on higher risk through higher future returns? This is probably one of the greatest puzzles in finance. In this research, we surveyed a wide range of risk metrics. More importantly, we found an interesting risk-return relationship along the term structure – it seems that investors are only compensated in the longer term. In the end, we proposed a “smart” risk premium factor, by capturing the long-term risk premium, while avoiding the short-term underperformance that comes from holding risky stocks.

The risk in low risk?, Cahan, *et al* [2012], July 19, 2012

This paper tries to answer an important question whether low risk strategies are becoming crowded. We introduce a novel metric for measuring crowdedness called Median Pairwise Tail Dependence (MPTD). This is designed to measure the likelihood that stocks in a portfolio have simultaneous large negative drawdowns. Combined with a few other more conventional techniques, e.g., institutional ownership, we do not find low risk strategies are currently crowded. We also studied alternative low risk strategies in this paper.

A new asset allocation paradigm, Mesomeris, *et al* [2012], July 5, 2012

In this paper, we investigate asset allocation that transcends typical asset class silos to embrace uniquely identified return-producing units, or risk premia. The portfolio including risk factors alongside traditional asset class beta appears to be superior in terms of risk-adjusted return and maximum drawdown across all portfolio construction approaches we have investigated.

From Macro to Micro, Luo, *et al* [2012], May 2, 2012

We further expand our macro quant research (country, sector/industry, and style rotation). We expand our VRP concept globally and find VRP has strong predictive power in industry rotation. We study the “sun-of-the-parts” SOP method in country and sector rotation. We also link the country credit and equity markets. Lastly, we demonstrate how to proactively manage tail risk in country portfolios by employing the mean-variance-skewness (MVS) optimization.

New insights in country rotation, Luo, *et al* [2012], February 9, 2012
We study three sets of country selection signals: risk premium, bottom up, and top down. We find two risk premium factors are particularly interesting: Kelly’s tail risk and VRP. We further build a composite country rotation model. We also investigate a range of country risk models and portfolio construction techniques. More importantly, we demonstrate how equity portfolio managers and global macro/GTAA funds can both take advantage of a country rotation model. In the end, we study risk-based country allocation strategies.

Low risk strategies, Avettand—Fenoel, *et al*, [2011], November 16, 2011
Several methodologies are available to investors wishing to allocate money to low risk strategies. We showcase four risk-based portfolio construction techniques which either distribute risk or minimize some kind of risk metric: the dollar risk is distributed with Equally-Weighted (EW) portfolios, the volatility risk is distributed with Inverse Volatility (InvVol) and minimized with Minimum Volatility (MV), and the diversification risk is minimized with Maximum Diversification (MaxDiversification).

Risk parity and risk-based allocation, Alvarez, *et al* [2011], October 13, 2011
This research investigates the mechanics and efficacy of three popular risk-based asset allocation strategies – risk parity, minimum variance, and maximum diversification. We find risk parity and maximum diversification strategies are more robust to asset concentration and market environment. We also demonstrate how to incorporate alpha in risk parity, in the context of quantitative equity portfolio management.

Tail risk in optimal signal weighting, Luo, *et al* [2011], June 7, 2011
Traditional multi-factor stock selection models are built on mean-variance optimization and likely to expose to tail risk. We propose a few structured models to better estimate factor tail distribution. We then design an efficient optimization algorithm to balance the four conflicting goals of maximizing return/skewness and minimizing risk/kurtosis. Our MVSK models are aware of the underlying macroeconomic environment and effective reduce model downside risk, while maintain the same level of IR/Sharpe ratio.

Minimum variance: Exposing the “magin”, Alvarez, *et al* [2011], February 9, 2011
There are some nice properties for minimum variance portfolios, i.e., higher IR than the market portfolios, low turnover, and low correlation with traditional strategies. However, we find MVP is not necessarily a low-risk strategy. In the end, we propose a slight and simple enhancement to the strategy, which significantly improves MVP IR without increasing its risk. We also demonstrate that we can combine the MVP strategy with other active alpha models.

Robust factor models, Luo, *et al* [2011], January 24, 2011
Traditionally, managers focus on selecting factors, while using the sample factor covariance matrix in constructing multifactor models. We compare the performance of the sample factor covariance matrix with 12 structured models (constant correlation, single index, four Bayesian shrinkage estimators, and six multivariate GARCH models). Our backtesting suggests that robust factor models incorporating structured covariance matrices improve portfolio IR significantly.

Appendix 1

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