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SMART BETA THEORY MEETS PRACTICE

Special Edition

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SMART BETA THEORY MEETS PRACTICE

Special Edition

As smart beta strategies continue to gain momentum, asset owners and managers are shifting their attention to internal processes and focusing on portfolio construction. In this video, **Ruben Falk**—Senior Director, Investment Management at **S&P Global Market Intelligence**—discusses this trend and the reasons why smart beta works.

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Building Smart Beta Portfolios

Ruben Falk and Richard Tortoriello

Why is smart beta important? We believe that smart beta is continuing to gain momentum among a variety of constituencies, including ETF providers, asset managers and asset owners. Smart beta can be defined as a factor-based approach in which an index is constructed, not by traditional market cap weighting, but instead weighted based on share exposure to one or more "style" factors: dividend yield, price momentum, value, growth, etc. The goal is to outperform the benchmark index and/or reduce risk.

BACKGROUND

Over the last few years, there has been a flurry of new index and ETF introductions based on a transparent, rule-driven smart beta approach. The value proposition is simple – using a systematic approach based usually on a single factor, ETF providers can claim to capture a good amount of the active returns provided by mutual fund managers at a fraction of the price.[1]

According to Invesco,[2] smart beta ETFs accounted for 22% of US ETF inflows (but only 12% of assets) in 2015, with over 580 smart beta products ($485B AUM) up from 350 in 2014 ($230B). Nevertheless, we estimate that U.S. smart beta equity ETFs only comprise around 1% of traditional institutional U.S. equity manager assets.

As shown in Figure 1, smart beta ETFs are dominated by a few styles, with particular concentration in high-dividend yield strategies targeted at the retail market. Institutional investors appear not to be targeted, as traditional value and growth styles represent a very small share of the overall category.

We believe that asset managers are adopting smart beta strategies internally or through separately managed accounts. European and Canadian public pension funds have been increasingly relying on internalized smart beta, with the largest U.S. pension funds and endowments also adopting the approach.

A recent Russell survey[3] found that 46% of global asset owners with assets under management (AUM) greater than $10B have smart beta allocations, and among global asset owners, the most popular smart beta categories are low volatility, fundamental weighting, value, and multi-factor.

Asset managers are adopting smart beta processes in support of fundamental analysts and portfolio managers by employing quantitative screening and back-testing[4] or by developing more complex systematic products, with closer to active fees, where smart beta can potentially have greater pay-offs (e.g., for Europe, Australasia and Far East (EAFE) or emerging market mandates).

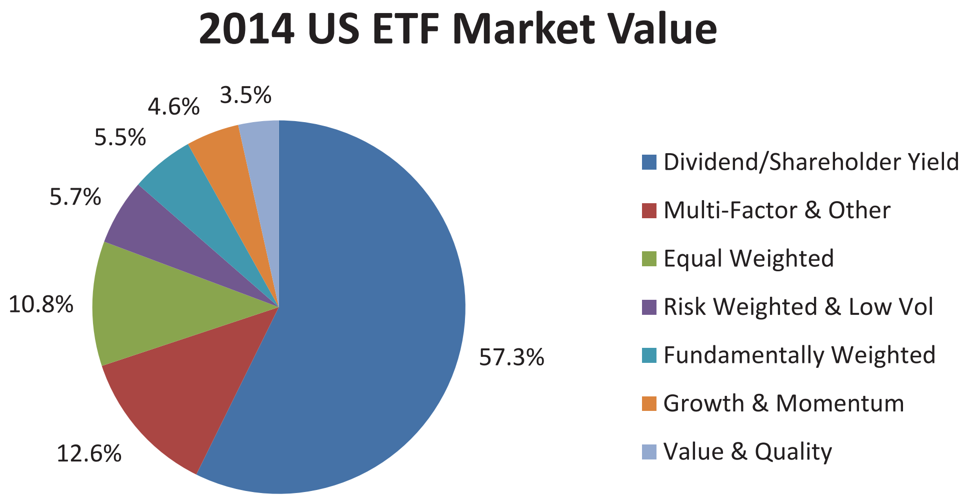
SMART BETA PORTFOLIO APPROACHES

Smart beta strategies begin with fundamental factor selection. Factors are generally intended to represent sources of both increased risk and increased return to compensate investors for the added risk.

Smart beta managers and index providers usually begin with the selection of style factor(s) combined with

F I G U R E 1

2014 Market Value of U.S. Smart Beta ETFs by Style Category



Source: How smart are "smart beta" ETFs? Denys Glushkov, University of Pennsylvania 2015. Data as of December 31, 2014.

a portfolio construction methodology. This methodology includes a view on whether the portfolio should be neutralized for exposures to other (unintended) factors than the target style factor(s). Neutralization to unintended factor exposures, if desired, can occur either as part of the factor definition process or during portfolio construction.

In terms of portfolio construction, a rules-based approach, used in most early smart beta products, benefits from being easy to explain and lends itself well to simple strategies such as equal or fundamental weighting. For implementing more complex portfolios and constraints, optimization-based construction may yield the best results.

PORTFOLIO CONSTRUCTION—OPTIMIZATION BASED

We'll now derive an investable strategy from a two-factor P/E and ROIC model for the S&P 1500 universe with realistic portfolio constraints. Let's assume that we want to construct quarterly rebalanced portfolios with the following characteristics:

A. Exactly 100 holdings, fully invested

B. Maximum 60% annual one-way turnover

C. Maximum trade size of 30% of average daily volume given a starting capital of $500 million[5]

D. Maximum 9% tracking error[6]

E. No single holding less than 0.25% of the portfolio

F. Holding size

1. Maximum 3% of portfolio, or
2. Maximum 5 times benchmark (B*5)

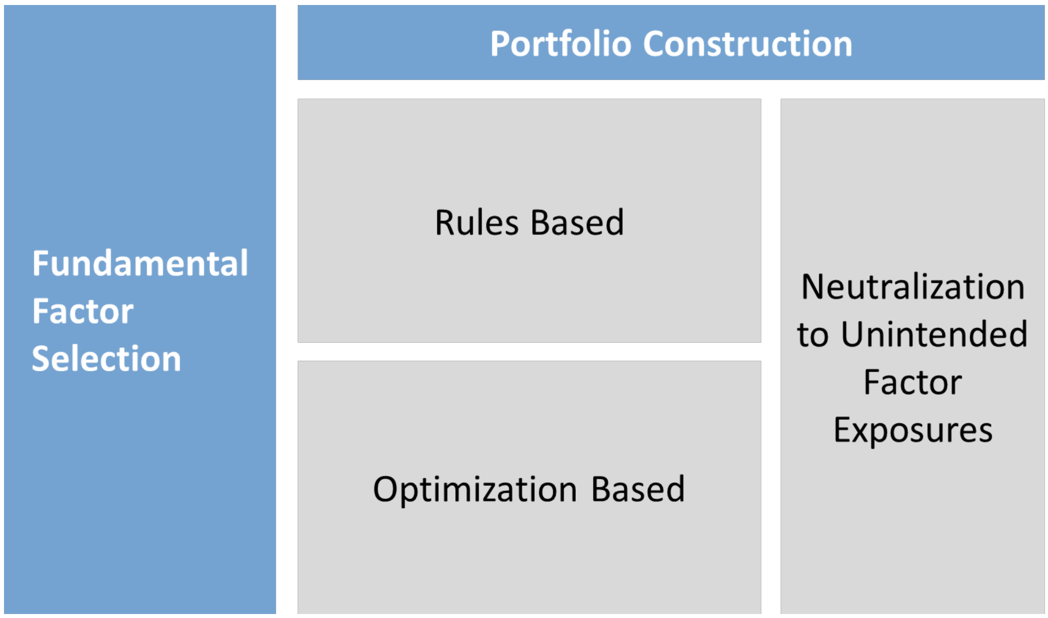
In an optimization based approach, it is straightforward to impose these constraints: At the end of every quarterly rebalancing period, we ask the optimizer to select 100 stocks such that the overall two-factor model score is maximized but subject to the full set of constraints (A through E, and F1 or F2) above.

Option F1 pushes the portfolio more toward equal weighting (small-caps over weighted) whereas option F2 pushes more toward capitalization weighting (large-caps over weighted). Because the constraints are not necessarily binding on every rebalancing, the average achieved characteristic across the entire simulation is likely to be less than the level at which the constraint is set, e.g. the average achieved tracking error was well below 9% in all cases.[7] In practice, many of the constraints chosen may be mandate-specific and a good optimizer[8] will allow for a variety of custom constraints.

Once all the constraints were applied, the return of the two-factor model was 12.7% and 12.2% for the small cap and large cap versions respectively which out-

FIGURE 2

A Smart Beta Process Flow Chart



performed the cap weighted benchmark by 3.3% and 2.8% respectively.

The next step we'll take is to neutralize exposures to unintended factor bets.[9] Strict sector neutralization (i.e., imposing portfolio sector weights equivalent to those of the benchmark) tends to simply push strategy risk and return toward overall benchmark risk and return characteristics.

Neutralization of unintended style factor bets is arguably more of an art than a science in that "styles" are subjectively defined and exposures to those styles can be measured in various ways. For this example, we use the style factor exposures as defined by the S&P Capital IQ Alpha Factor Library and U.S. Equity Risk Model from S&P Global Market Intelligence, which employ a market factor and eight style factors: price momentum, historical growth, analyst expectations, earnings quality, valuation, capital efficiency, size, and volatility.

We chose to only neutralize exposure to the market (beta) and historical growth factors, while imposing greater than or equal to benchmark exposures to valuation, capital efficiency, and earnings quality styles. The idea was to have the market and growth characteristics of our portfolio be similar to those of the benchmark (no unintended bets on market beta, market timing, or growth, since we are pursuing a value strategy). At the same time, consistent with our two-factor P/E and ROIC model, we'd like to emphasize exposure to valuation, capital efficiency, and earnings quality (arguably, our intended bets).

Again, when based on the absolute 3% holding constraint favoring small caps, the two-factor strategy performed quite well. However, when combined with the benchmark relative holding constraint which favors large caps, the imposition of style "neutrality" had a significant negative impact on performance. The implication is that, in all likelihood, there were some unintended bets in the original benchmark-relative holding-constrained strategy (e.g. higher market beta than the benchmark or well-timed growth tilts) which accounted for a significant part of the performance.

Said another way, in the large-cap biased strategy, exposures to other factors than P/E and ROIC accounted for most of the outperformance relative to the benchmark. Of course an investor might want these other exposures (e.g., high market beta) but that probably shouldn't be labeled as a strategy which takes advantage of attractive valuations (low P/E) and high capital efficiency (high ROIC).

FIGURE 3

P/E & ROIC Constrained Portfolio Results

	Sharpe Ratio	Annual Return	Tracking Error	Annual Turnover	Avg. P/E	Avg. ROIC	Avg. Mkt. Cap.($B)
Top 100, Max 3% Holding Size	0.61	12.7%	7.2%	55%	10.7	27.9%	27.7
Top 100, Max B*5 Holding Size	0.64	12.2%	6.0%	56%	12.6	25.1%	115.4
Benchmark (S&P 1500 Cap. Weighted)	0.49	9.4%	-	-	17.6	15.6%	79.5

Source: S&P Global Market Intelligence, ClariFI test dates Jan. 1, 1995 to July 31, 2016. For illustrative purposes only.

Note: Past performance is not an indication of future results. Indexes are unmanaged, statistical composites and it is not possible to invest directly in an index. These results are inherently limited because they do not represent the results of actual trading and were constructed with the benefit of hindsight. The returns shown do not reflect payment of any sales charges or fees an investor would pay to purchase the securities they represent. The imposition of these fees and charges would cause actual and back tested performance to be lower than the performance shown. Turnover is one-way.

IN CLOSING

As smart beta strategies continue to gain popularity, much effort is going into identifying factors likely to produce positive excess returns in the future. To avoid the pitfalls associated with data mining, it is always advisable to select factors that are based on economic theory. We believe that multifactor (i.e., two or more factor) models can potentially produce stronger risk-adjusted returns than single-factor models, particularly if the factors selected are complementary.

We also stress the importance of portfolio construction in controlling for liquidity and turnover, in order to build a strategy that not only works well on paper but can be implemented in reality. Risk tolerance assumptions should also be part of the portfolio construction process along with a view on what type of risk appetite the target investor is likely to have: relative risk (tracking error), absolute risk, or possibly a combination of both.

Finally, a view must be taken as to whether to neutralize for unintended factor bets which, as we mentioned, is more art than science as the list and definition of factors to be neutralized for varies, as does the measurement of exposure. Although a rules-based portfolio construction approach may work for a simple strategy, we believe that more complex strategies may benefit from the use of optimization in the portfolio construction phase.

FIGURE 4

P/E & ROIC, Constrained & Market Beta/Growth "Neutral"

	Sharpe Ratio	Annual Return	Tracking Error	Annual Turnover	Avg. P/E	Avg. ROIC	Avg. Mkt. Cap.($B)
Top 100, Max 3% Holding Size	0.62	12.6%	7.0%	54%	11.5	28.2%	27.4
Top 100, Max B*5 Holding Size	0.54	10.4%	4.9%	57%	12.9	25.1%	113.4
Benchmark (S&P 1500 Cap. Weighted)	0.49	9.4%	-	-	17.6	15.6%	79.5

Source: S&P Global Market Intelligence, ClariFI information Jan. 1, 1995 to July 31, 2016. For illustrative purposes only. Note: Past performance is not an indication of future results. Indexes are unmanaged, statistical composites and it is not possible to invest directly in an index. These results are inherently limited because they do not represent the results of actual trading and were constructed with the benefit of hindsight. The returns shown do not reflect payment of any sales charges or fees an investor would pay to purchase the securities they represent. The imposition of these fees and charges would cause actual and back tested performance to be lower than the performance shown.

ENDNOTES

[1]Mutual fund fees are falling, according to the Investment Company Institute: the average asset-weighted expense ratio for U.S. funds fell to 70 bps in 2014, from 99 bps in 2000. Average ETF expense ratios are 44 bps. Data as of December 31, 2014.

[2]"Smart Beta ETF Strategies: Expanding the Investor Toolbox" by Invesco Power Shares, 4/28/2016

[3]"Smart beta: 2016 global survey findings from asset owners" by FTSE Russell, May 2016

[4]A so-called "quantamental" process, which mixes quantitative analysis and qualitative/fundamental-based investing.

[5]Trades are assumed to be executed over several days.

[6]Tracking error is defined as the annualized standard deviation of the active (excess) portfolio returns relative to the benchmark.

[7]As an aside, this brief is not focused on traditional mean-variance optimization but instead on how to construct investable portfolios mainly by controlling for turnover and liquidity. The tracking error constraint is optional in this context and therefore set a level where it was typically not binding.

[8]We use the S&P Global Market Intelligence's ClariFI optimization module.

[9]As mentioned previously, this choice is up to the firm constructing the portfolio. In general, we believe that factor neutralization makes for a stronger and more defensible smart beta approach.

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What Is an Index?

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The standard definition of a financial index, such as the S&P 500 Index, is a market-capitalization-weighted average of a specific and relatively static list of securities. The financial index was first devised in the late 19th century as numerical shorthand for market activity. Today, indices serve many purposes. In addition to their original function of compressing information, indices act as indicators of time-varying risk versus reward and as a benchmark for performance evaluation, attribution, and enhancements. Since the advent of the capital asset pricing model (CAPM), indices have been used to construct passive investment vehicles and as building blocks for portfolio management.

However, recent technological advances in computing, trading, trade processing, telecommunications, and derivative securities have greatly increased the scope of possible financial products and services, including new forms of indices that have little resemblance to a static market-cap-weighted portfolio. In this article, I revisit the notion of an index in light of these new possibilities and propose a new perspective in which a broader definition offers many advantages but also some potential pitfalls.

Charles H. Dow published the first market index in the era of the telegraph, when markets moved unbelievably slowly by modern standards and a simple average of stock prices was enough to communicate important information about a market's health. Thanks to Moore's Law, technology has advanced exponentially since Dow's time, which has important implications for what we mean by an index. For example, consider the MSCI World Index, a market-cap-weighted average of over 1,600 stocks from what are sometimes called "developed markets," founded in 1969. Why does this index take this specific form? Market-cap weighting emerged largely because of older technological limits on trading and portfolio construction. It implies a buy-and-hold portfolio that does not need to be rebalanced, except when securities enter or leave the index list. It is a very specific and limiting way of constructing an index, an artifact of its financial and technological era.

Technology and the information revolution have changed so much of our daily lives that it should not be surprising that they have also changed the way we think about saving and investing. Modern trading technology opens up a whole new spectrum of possibilities for defining indices and creating financial products around them. Examples include target-date and life-cycle funds, which change their asset allocation characteristics as they approach their target dates; hedge-fund replication strategies, which replicate the betas of entire classes of hedge funds; trading-strategy indices, which use transparent, mechanical rules to implement specific trading strategies, such as currency carry trades or risk arbitrage; and "fundamental" indices, also called "smart beta" indices, where stocks within a portfolio are weighted according to their fundamentals or other non-market-cap factors. Mutual funds and active exchange-traded funds (ETFs) associated with these indices have already captured the imagination of many investors, with more than $544 billion invested in smart beta ETFs alone as of February 2015 (Evans [2015]). In the aftermath of the financial crisis, these new forms of investment resemble a Cambrian explosion of new species, an eruption of financial innovation and diversity after a long fallow period.

But this Cambrian explosion also contains some potential concerns. As many of its critics have commented, smart beta need not be smart at all. For the inexperienced investor, smart beta is often accompanied by "dumb sigma": unnecessary and unanticipated kinds of portfolio risk that do not carry a positive risk premium. One obvious example is the idiosyncratic risk of a highly undiversified portfolio, but there are other examples of risks that are not adequately rewarded, especially in the face of market distress.

We need a new framework for thinking about indices, indexation, and the distinction between active and passive investing that reflects the new reality of technology-leveraged investing. Although technology has made many useful new financial products and services possible, any form of leverage—including technology-leveraged investing—can create new and greater risks. In the financial industry, Moore's Law must be weighed against the technology-updated version of Murphy's Law: Anything that can go wrong, will go wrong, and will go wrong faster and bigger when computers are involved.

The starting point for this new framework is to generalize the definition of a financial index by focusing on its basic function. If an index is to be used as a benchmark against which managers are judged, it must have three key characteristics: it must be transparent, investable, and systematic. Traditional indices, such as the S&P 500, clearly satisfy this definition, but so do other portfolio strategies that involve more active trading, such as target-date funds and publicly disclosed, rules-based 130/30 strategies. Under this new definition, financial indices can vary greatly in complexity. To distinguish the more complex versions from the traditional market-cap-weighted indices, I will refer to the traditional indices as "static" and more complex indices as "dynamic."

Dynamic indices may contain more subtle risks, such as tail, illiquidity, or credit risk. As a result, these types of indices will require more sophisticated consumers, with the education and experience to properly assess the risks and use these indices responsibly. However, one the of most important implications of this new definition of an index is that investing and risk management can be decoupled: passive investing need not, and should not, imply passive risk-taking, as it currently does. The two pursuits are distinct, and it is important to separate them, especially during turbulent market conditions.

Moreover, due to their complexity and construction method, dynamic indices will be much more prone to backtest bias, so investors will need to use more sophisticated judgment to evaluate them. If used properly, dynamic indices can greatly benefit both investors and portfolio managers by letting them construct more highly customized portfolios that can achieve long-run investment objectives by managing short-run risks more effectively.

A BRIEF HISTORY OF INDICES AND INDEX FUNDS

Dow published the first U.S. stock market index in 1884, the Railroad Average, which still exists today as the Dow Jones Transportation Average (DJTA). The DJTA was a precursor to the Dow Jones Industrial Average (DJIA), which Dow began publishing in 1896. Today, the DJIA is one of the three most recognized indices in the world, along with the S&P 500 and the NASDAQ Composite. Dow created these averages to illustrate his theories in what is today called technical analysis (Lo and Hasanhodzic [2010, pp. 82–84]). Even practitioners who disdained Dow theory, however, found the Dow indices were tremendously useful for following the stock market as a whole.

The Dow indices were initially calculated as a simple average of stock prices. In fact, they were the earliest version of what is now called an equal-weighted index. In 1928, this method was revised to a price-weighting system in which each stock is weighted by its price, relative to the sum of all stock prices within the index. Even before then, however, economists had suggested that a market-cap-weighting system would be a more effective way to represent the overall market because splits or merges could move a stock's price with little or no financial effect, while market capitalization directly apportioned a stock's importance to its size in the market. Market-cap weighting (sometimes called value weighting) solved those problems. In 1923, the Standard Statistics Company used market-cap weighting, apocryphally at the urging of the economist Irving Fisher (Fox [2011, p. 27]), to compute an index to compete with the Dow, the precursor to today's S&P 500. Today, the large majority of financial indices follow a market-cap-weighting scheme.

These new indices, in turn, stimulated new thinking about their possible uses. In 1960, Edward Renshaw at the University of California and his graduate student Paul Feldstein first proposed the creation of index funds in their article, "The Case for an Unmanaged Investment Company" (Renshaw and Feldstein [1960]). Their work compared 89 diversified mutual fund returns with those of the DJIA, demonstrating that only 11 funds had higher returns than the DJIA. This was an idea slightly ahead of its time because there was as yet no compelling theoretical reason for their result to be more than a numerical coincidence—managers of the time remained confident in their ability to beat the market.

Two revolutionary financial theories catalyzed much broader acceptance of the index fund: the CAPM, introduced independently by Sharpe [1964] and Lintner [1964], both in 1964, and the efficient market hypothesis (EMH), independently proposed by Fama [1965] and Samuelson [1965] a year later. The CAPM allowed investors to construct a mean–variance-efficient portfolio simply by holding a basket of all stocks in proportion to their market capitalization, i.e., the market portfolio. The EMH, meanwhile, had an obvious corollary that, after accounting for transactions costs and fees, active investing could not outperform passive investing, on average. It is no exaggeration that, in combination, these two theories democratized personal investing by taking the reins of portfolio management from the active stock-picking gunslingers of the day and handing them over to a broadly diversified index fund that served as a proxy for the market portfolio. Almost overnight, typical retail investors were given new tools that allowed them to invest on their own, albeit passively.

Although academic research provided the seeds from which the index fund business grew, many credit John Bogle as the pioneering practitioner who planted these seeds and cultivated their first harvest in 1976: the Vanguard Index Trust. However, this was only the first index mutual fund; Bogle generously ascribes the roots of his business to others:

> The basic ideas go back a few years earlier. In 1969–1971, Wells Fargo Bank had worked from academic models to develop the principles and techniques leading to index investing. John A. McQuown and William L. Fouse pioneered the effort, which led to the construction of a \$6 million index account for the pension fund of Samsonite Corporation. With a strategy based on an equal-weighted index of New York Stock Exchange equities, its execution was described as 'a nightmare.' The strategy was abandoned in 1976, replaced with a market-weighted strategy using the Standard & Poor's 500 Composite Stock Price Index. The first such models were accounts run by Wells Fargo for its own pension fund and for Illinois Bell. (Bogle [1997])

The Wells Fargo group was directly connected to the innovations of academic finance: McQuown was friends with the Fama circle at the University of Chicago, while Fouse knew Sharpe personally and persuaded him to consult for Wells Fargo in the 1970s (Bernstein [1993, pp. 236–248]).

Although academic finance sowed the seeds of the index fund industry, it needed the proper environment to flourish—financial advances do not occur in a technological vacuum. Constructing cash portfolios of broad-based indices was an extremely difficult and costly task in the 1970s. It is easy to forget the formidable challenges posed by the back-office, accounting, and trade-reconciliation processes for even moderate-sized portfolios in the days before personal computers, FIX engines, and electronic trading platforms, now a distant memory to the managers of today's multi-trillion-dollar indexing industry. From the practitioner's perspective, fixing the set of securities and value-weighting them in an index reduced the amount of trading needed to replicate the index in a cash portfolio. Apart from additions and deletions to the index, a portfolio weighted by market capitalization never needs rebalancing because the weights automatically adjusted the proportions as market valuations fluctuated. These "buy-and-hold" portfolios were attractive not only because they kept trading costs to a minimum, but also because they were simpler to implement from an operational perspective.

Moreover, weighting stocks in proportion to their relative importance appealed to common sense.

The success of the index mutual fund led to an evolutionary explosion of financial innovation, centered on the concept of the index. Three different stock market index futures debuted in 1982, based on the New York Stock Exchange (NYSE) Composite, the S&P 500, and the Value Line index, respectively. Indices for each asset class emerged, as did additional index funds to track them: the first bond index fund for retail investors appeared in 1986, the first international share index funds in 1990, and the first ETF in 1993. ETFs were similar to index mutual funds in that they closely tracked an index but could be bought and sold throughout the day on exchanges. These served three broad purposes for the investor: they were performance indicators, vehicles for direct investment through the use of index funds, and vehicles for hedging, speculation, and investment through the use of derivative, index-based instruments.

At the same time, however, the concept of the index itself was evolving. Barr Rosenberg first proposed the notion of a "normal portfolio" in the late 1970s, and implemented it at BARRA in the 1980s (Kritzman [1987], Divecha and Grinold [1989], and Christopherson [1998]). This was an attempt to construct customized indices to describe the investment activities of more specialized managers, using a set of securities "weighted as the manager would weight them," (Christopherson [1998, p. 128]) in order to provide insight into their unique risk exposures. This was conceptually expanded in the 1990s, when Sharpe [1992] defined a new distinction between investment style and investment selection in performance attribution and measurement. Passive fund managers exposed the investor to an investment style, while the active fund manager provided both style and selection. Sharpe reasoned that an actively managed fund's performance should be determined by its selection return: the difference between the fund's return and that of a benchmark with the same style. Sharpe listed four conditions for a strong style portfolio, which he said should be "1) a viable alternative, 2) not easily beaten, 3) low in cost, and 4) identifiable before the fact."

Sharpe emphasized the difference between passive and active management in his exposition. However, with the proliferation of automated trading algorithms, it became clear that the fifth, implicit condition was not necessarily passivity. Benchmark algorithms for high-performance computing blurred the line between passive and active. The key distinction was a lack of human intervention and discretion. If a trading strategy could be codified into a set of transparent rules that gave consistent results on similar datasets, as did the benchmark algorithms used to test the machines that implemented them, how did this differ in spirit from an index constructed using a passive portfolio?

Academics and managers both questioned the original assumptions behind the CAPM portfolio. Merton [1973] extended the model inter-temporally, while Ross [1976] broke down the CAPM's beta into a multifactor structure. Some statistical tests suggested that market capitalization was not the most optimal weighting system in CAPM. Goldman Sachs even proposed an earnings-weighted portfolio in the early 1990s. More recently, Arnott, Hsu, and Moore [2005] followed this line of inquiry to create the original set of fundamental indices: portfolios of stocks weighted by book value, cash flow, revenue, sales, dividends, and employment, seeking to capture the value premium.

But perhaps the most obvious illustration of the changing nature of indices is the proliferation of target date and lifecycle funds. These funds are designed for specific cohorts of investors sorted by their planned retirement dates, changing their asset mix to become more conservative as they approach their target date (Bodie, Merton, and Samuelson [1992] and Shiller [2005]). Lifecycle funds are not static, but neither are they completely actively managed. Current trading and portfolio-management technology can create passive portfolios capable of capturing complex risk–return profiles that change through time, such as those of an aging population preparing for retirement.

Today, indices are at the forefront of the building tsunami of financial innovation. Just as the previous financial generation saw markets in everything, we currently see indices in everything, as well as funds and derivatives based on those indices. The technological environment facilitated this efflorescence, but these innovations would never have flourished had the investing public not found indices useful. What virtues does the modern index have that make it so attractive to investors?

WHAT IS AN INDEX?

Ideally, financial form should follow financial function; the proper definition of an index depends on its use. The traditional definition of an index as a market-cap-weighted basket of a fixed set of securities persists today not because of its inherent superiority or economy of implementation, but because its past success led to inertia in considering other alternatives. To understand how the index evolved, it is fruitful to adopt Merton's functional perspective and ask what functions an index serves (Merton [1989, 1995a, and 1995b] and Merton and Bodie [2005]). We can identify at least two distinct functions of a modern index. The first is largely informational: indices provide an aggregate measure of investment performance that abstracts from the vicissitudes of individual components to highlight economy-wide market drivers.

The second and more practical function is as a standard against which active managers can be compared, i.e., "This is how you would have performed, if you had invested in this particular passive manner." We see this explicitly in Rosenberg's normal portfolio and Sharpe's style return, as well as in how active managers construct their portfolios subject to tracking error constraints, relative to some index. For such comparisons to be economically meaningful, the index must be able to serve as the basis for an investment vehicle, a transparent portfolio with a plausible risk–reward profile, as McQuown, Fouse, Bogle, and other indexing pioneers envisioned.

To achieve this second function, we can reverse engineer three fundamental properties this form of index must have. First, it must be transparent, meaning that every aspect of the index must be public information and verifiable by any interested third party. Second, it must be investable, meaning that an investor should be able invest a large amount of capital in the portfolio over a short period of time and realize the return reported by the index. Finally, it must be systematic, meaning that the index's construction must be rules-based and not dependent on any discretion or human judgment. No alpha should be necessary to implement the index in a live portfolio; any investor should be able to do it (subject only to technological constraints).

This more general definition of an index may seem innocuous enough, but it does have several significant implications for how we think about indices and indexation. For example, this definition excludes certain well-known indices, such as the Case-Shiller Home Price Indices, as well as most hedge-fund indices (they are not based on liquid instruments and are, therefore, not investable in large size). However, these quantities still play important roles with respect to the first function, even if they are not investable. Moreover, they can often serve as the basis for other financial securities that *are* investable. For example, futures contracts on the Case-Shiller Indices do trade on the Chicago Mercantile Exchange, and real-estate investment trusts and hedge-fund beta replication funds are also offered through investable liquid securities.

Our new definition also includes all the traditional market-cap-weighted indices where the constituents are liquid securities, which we shall call "static indices" for the obvious reason. However, the main motivation for redefining an index is to cover the case of dynamic indices, which refers to all portfolios satisfying the three conditions of our new definition but which are not market cap weighted and, therefore, require more frequent rebalancing.

This is more than a subtle semantic distinction. Just as a new formalism in mathematics can sometimes provide a mathematician more insight into proving a theorem, we can gain greater financial insight through grouping functionally similar financial ideas and concepts together, and exploring how they may interact in the financial system. Passive market-cap-weighted portfolios are the simplest form of index in common use, but it is clear how variations on this theme readily emerge from their conceptual potential. New trading technology has now given us the ability to create indices that are not necessarily market cap weighted, that are not necessarily even passive in the traditional sense. In the next section, we discuss one of these new variations: the strategy index.

THE BRAVE NEW WORLD OF STRATEGY INDICES

The strategy index is a dynamic index that embodies a particular investment strategy. Many of these dynamic indices are not typically considered strategies, such as life-cycle or target-date funds. However, consider the simplest version of such a fund, one that implements the "100-minus age" rule of thumb. It is entirely transparent—maintain a percentage equivalent to 100 minus the investor's age in a broad-based index fund and the remainder in bonds, and update this portfolio yearly. Its components are fully investable, and it is systematic in the sense that updating it requires no discretionary intervention. Much more complicated strategies, such as those underlying 130/30 funds, have been codified into dynamic indices (Lo and Patel [2008]). Meanwhile, dynamic indices for previously esoteric hedge-fund strategies such as merger arbitrage are now available to the average investor through rules-based portfolios that invest in publicly announced takeovers of certain pre-defined characteristics.

The theoretical underpinnings for these dynamic indices are straightforward enough and flow from variations on the original CAPM formulation. For example, CAPM can be generalized to multiple factors, such as Merton's intertemporal CAPM or Ross's arbitrage pricing theory. Meanwhile, the efficient frontier can be estimated using weightings other than market capitalization, as with Arnott, Hsu, and Moore's fundamental indices, equal-weight indices seeking to capture small-cap premia, or low-volatility indices looking to capture low-volatility premia. In this brave new multifactor, algorithmic world, nearly any plausible strategy can be broken down into components of investment style, weighting, and other conditions. In fact, the burgeoning literature and industry applications involving hedge-fund beta replication take this observation to its logical conclusion: if a hedge-fund strategy's returns contain common factors that can be cloned (i.e., identified, quantified, and replicated) using liquid futures contracts without the need for active management, why not use them as benchmarks for comparison (Hasanhodzic and Lo [2007])?

However, the key question for investors is whether a strategy index carries a sustainable risk premium, and if so, under what conditions. It is here that the other financial theory behind the modern index fund, the EMH, is relevant. The EMH implies that no investor should be able to generate a consistent return in the market above the risk–return relationship defined by the CAPM or a similar equilibrium asset-pricing model. Any sustainable risk premium above this benchmark should be arbitraged away by investors in pursuit of profit. However, there is compelling evidence that the EMH is only the limiting case of a more complex reality. The adaptive markets hypothesis (AMH) suggests that a sustainable risk premium may be available to investors for a period of time, given the financial environment and the market's population history (Lo [2004]).

As an illustration, the AMH explains that behavioral biases are likely responsible for many market anomalies and therefore are a possible source of risk premia for dynamic indices meant to exploit such anomalies. A naive critique of this possibility is that behavioral biases are often correctable: point out a behavioral bias to an individual, and that person will often stop exhibiting that bias. But the AMH is a hypothesis about marketplace dynamics, not statics. In fact, the "discipline of the market" should punish investors with this bias until they exit the market or adaptively change their strategy. If a behavior is innate, however, and a flow of new investors is coming into the market, then a behavioral bias premium may be sustainable. As P. T. Barnum almost said, "There's a new investor born every minute."

Although the question of sustainability is of prima facie importance to investors, the ultimate sources of expected return—risk premia or alpha—lie at the heart of this issue. Competition suggests that alpha should be capacity-constrained, hard to come by, and expensive. In theory, alpha is either competed away to nothing, or it becomes commoditized to a level at which the returns are just enough to compensate investors for the risks associated with the activity, i.e., beta, which should be less constrained, easy to come by, and cheap. The dynamic properties of these risk factors and their expected returns require a framework other than the EMH's static assumptions to interpret.

DISBANDING THE ALPHA BETA SIGMA FRATERNITY

The CAPM broke important new ground in teaching investors how to distinguish between unique investment acumen that justifies active management fees and commoditized risk premia that can be captured much more cheaply. But this dichotomy treats risk in a very rigid manner: active strategies manage risk actively and passive strategies do not manage risk at all. The reason for this distinction is largely historical—as described earlier in this article, passive investing became synonymous with market-cap-weighted indices to minimize the amount of trading needed to manage an index portfolio. Over time, the faithful reproduction of an index's returns by a portfolio of securities has become an index manager's overriding concern, irrespective of the rollercoaster ride that the index imposes on its investors. A more cynical perspective is that misery loves company: an index manager will not be punished for suffering losses if all index funds experience similar losses.

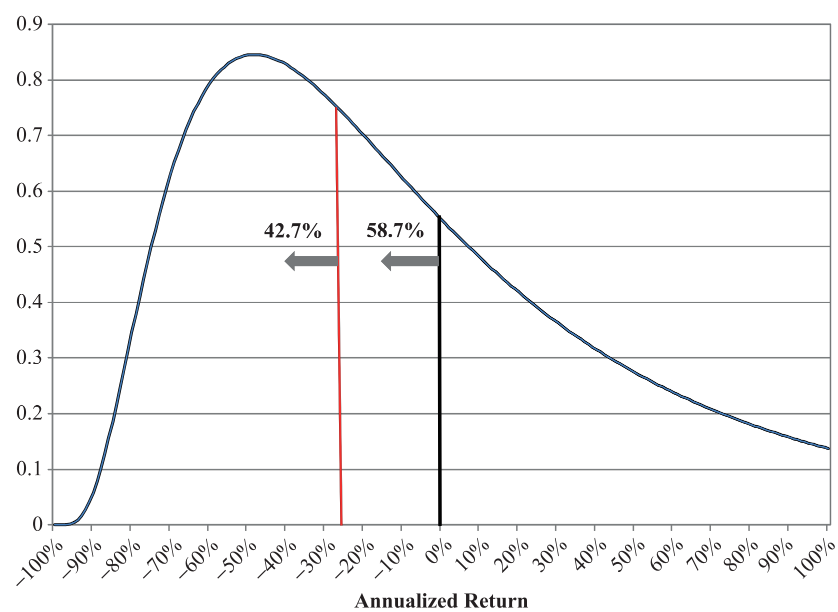
Although the manager may not be punished, the investor is not so fortunate. On October 24, 2008, when the S&P 500 volatility reached a record level of 89.53 as measured by the VIX index, passive investors in this benchmark were exposed to extraordinary amounts of risk that they surely did not intend to take. At an annualized volatility of 89%, the probability of loss is 58.7%, assuming log-normally distributed index returns with an annualized mean of 10%. The probability of losing 25% or more is 42.7% (see Exhibit 1).

One measure of extreme loss commonly used in the hedge-fund industry is the maximum drawdown (MDD), defined to be the largest percentage decline in a fund's net asset value over any investment period in the fund's entire history. The MDD of the S&P 500 from 2007 to the present occurred between October 9, 2007, and March 9, 2009, when the index declined by 56.8%, a loss not easily absorbed by any investor, especially investors who aren't regularly monitoring their portfolios.

Traditional passive investments are especially problematic when volatility can change dramatically, as it has over the last decade. Exhibit 2 illustrates the dynamic nature of volatility as measured by the VIX index's daily closing values from January 2, 1990, to March 20, 2015. Also plotted are the probabilities of a loss greater than 25%, assuming an annualized expected return of 10% and an annualized volatility equal to the VIX. Portfolio managers and financial advisers often chide retail investors for not focusing on the long run and being too sensitive to short-term fluctuations. Is it reasonable to expect any rational investor to stay the course in an investment that can lose more than half its value over a 16-month period and experience a volatility increase by a factor of 4 over 63 trading days?[1]

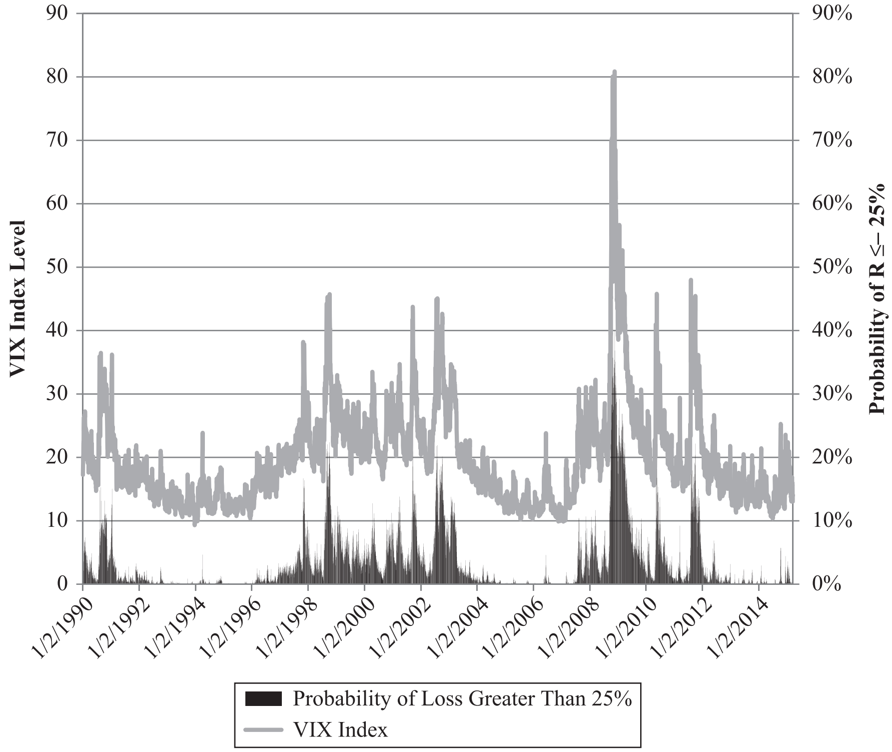
This lack of risk management—the fact that smart beta can be accompanied by dumb sigma—is perhaps traditional passive investing's greatest weakness. When market volatility is relatively stable, risk management may not be as important for static index products. Therefore, during the "Great Modulation" (Lo [2012])—the period from the 1930s to the early 2000s when U.S. stock market volatility was both low and relatively stable—it is not surprising that static index funds did an admirable job of letting investors capture the equity risk premium and manage their overall risk exposures by adjusting their asset allocation between a stock index, bonds, and cash. But when the volatility of volatility becomes significant, as it has over the past decade, forgoing risk management can be devastating to investors.

EXHIBIT 1
Log-Normal Distribution for Asset Returns



Notes: Assuming that simple (not continuously compounded) returns have an annualized mean and standard deviation of 10% and 89%, respectively. The probability of a negative realization is 58.7%, and the probability of a loss greater than −25% is 42.7%.

E X H I B I T   2
Daily Closing Values of the VIX Index



Note: From January 2, 1990, to March 20, 2015; assuming log-normally distributed returns with annualized expected simple return of 10% and volatility equal to the VIX.

Source: http://finance.yahoo.com (accessed March 22, 2015).

However, this is no longer a necessary evil of indexation, thanks to the many technological advances in algorithmic trading, securities exchanges, derivatives, telecommunications, and back-office and accounting systems infrastructure. What took McQuown and Fouse at Wells Fargo a month to implement in the 1970s can now be done almost instantaneously, and at much lower cost. Moreover, one of the most mind-numbing aspects of professional portfolio management—monitoring the portfolio in real time and deciding when to act in response to rapidly deteriorating market conditions—can be automated to a significant degree, especially for passive strategies that are dedicated to achieving an index's returns. The traditional pairing of active risk management with active investing, and passive risk management with passive investing can be severed.

One simple example of how to sever this link is to create a dynamic index fund that contains no alpha, but is actively risk-managed to a target level of volatility σ_o. This can be accomplished by investing a portion of the fund in cash if the estimated volatility $\hat{\sigma}_{t-q}$ of the index at date t exceeds σ_o, and investing more than 100% of the fund in the index (i.e., leveraging the fund) if $\hat{\sigma}_{t-q}$ falls below σ_o:

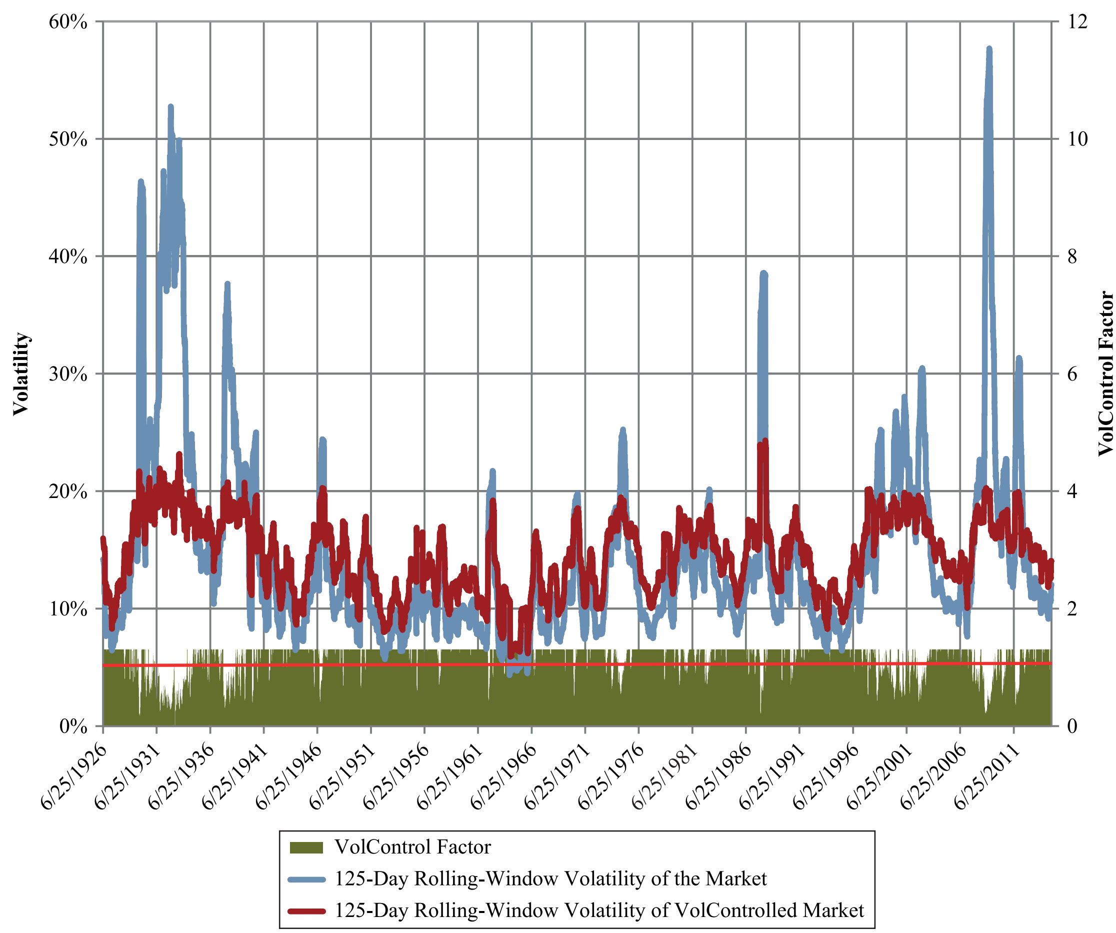
$$\tilde{R}_t = \kappa_t R_t,\ \kappa_t = \text{Min}\left[\frac{\sigma_o}{\hat{\sigma}_{t-q}}, \bar{l}\right], \qquad (1)$$
$$\hat{\sigma}^2_{t-q} = \frac{1}{n-1}\sum_{j=q}^{q+k-1}\left(R_{t-j} - \hat{\mu}_{t-q}\right)^2,\ q, \bar{l} \geq 1$$

where $\hat{\mu}_{t-q}$ is the rolling-window mean return between $t-q-k+1$ and $t-q$, and $\bar{l}$ is some fixed upper bound on the amount of usable leverage. By setting the leveraging/deleveraging factor to be the ratio of σ_o to $\hat{\sigma}_{t-q}$, and if $\hat{\sigma}_{t-q}$ is a reasonably accurate measure of short-term volatility, this algorithm will yield returns with volatility closer to the target σ_o than those of the static index. If σ_o is chosen with an investor's risk tolerance in mind, the actively risk-managed index $\tilde{R}_t$ will be more palatable as a long-term investment than the static index. Such a portfolio is passive in that there is no alpha, but it is actively risk-managed and not value-weighted.

To see how this volatility-control algorithm might work in practice, we apply it to daily CRSP value-weighted index returns from 1925 through 2014 using a 21-day rolling-window volatility estimator with one lag (i.e., $\hat{\sigma}_{t-1}$, and a value of 16.9% for the annualized volatility target level σ_o) which is the unconditional volatility of the CRSP value-weighted index returns over the entire sample period. Exhibit 3 contains a comparison of the volatility of the raw index (in gray) and the volatility-controlled index (in black), where we use 125-day rolling windows to estimate these volatilities. (We use a longer window for this comparison to show that the volatility control does have an effect, even outside the 21-day window used to scale the portfolio.) Comparing the two plots confirms the intuition that dynamically adjusting the portfolio as a function of short-term volatility does create a substantially less volatile time series of returns.

However, this stability comes at a cost. Scaling the portfolio on a daily basis requires monitoring short-term volatility and active risk management, that is, adjusting the portfolio's exposure either by trading the index constituents or (more likely) by implementing a futures

EXHIBIT 3
125-Day Rolling-Window



Note: Annualized volatility estimates for CRSP daily value-weighted index returns from June 25, 1926, to December 31, 2014, with and without volatility control using a scale factor $\kappa_t = Min\left[\frac{16.9\%}{\hat{\sigma}_{t-1}}, 1.3\right]$, *where* $\hat{\sigma}_{t-1}$ *is a 21-day rolling-window annualized volatility estimator.*

or forward contract overlay strategy to dynamically scale the index exposure up or down. The dark gray bar graph at the bottom of Exhibit 3 displays the amount of scaling κ_t involved; the straight horizontal dark gray line is set at a value of 1 for the scale factor. For much of the time, κ_t is at its upper bound of 1.3, implying that most of the time short-term annualized U.S. equity market volatility is less than 16.9/1.3 = 13% and the fund is 130% invested in the market. However, occasionally κ_t falls below the dark gray line, indicating that short-term volatility has exceeded the target level of 16.9% and a portion of the portfolio is switched into cash. We assume that the cash earns the yield on the one-month U.S. Treasury bill, and that all changes in portfolio weights incur transactions costs of 0.05% or 5 basis points of the trade. For the S&P 500, implementing the dynamic index (Equation (1)) using the Chicago Mercantile Exchange's E-Mini S&P 500 futures contract would yield considerably lower transactions costs than 5 basis points.[2]

By actively managing the risk of the fund, this algorithm reflects the typical investor's behavioral predilections—reducing market exposure when risk becomes too high and restoring it when risk returns to normal—but doing so more systematically and at a higher frequency than all but the most active traders can manage. As a result, investors are more likely to stay invested in this strategy, rather than exiting after a large loss and waiting too long before reinvesting. Exhibit 4 shows that staying invested in this fund is rewarded: $1 invested in 1926 becomes $11,141 in 2014, which compares favorably with the $4,162 from the unmanaged index. More importantly, the risk-managed strategy's MDD over this 89-year period is −72%, which is severe, but less so than the −84% of the raw index. The difference in excess kurtosis—a measure of the likelihood of tail events—also points to a substantial reduction in risk: 4.85 for the risk-managed fund versus 16.87 for the raw index, according to Exhibit 4. (By comparison, the excess kurtosis for a normal distribution is 0.)

EXHIBIT 4

Summary Statistics for Volatility Control Mechanism

	Raw Market Returns	Returns w/ VolControl	Stats for VolControl κ	
1926 to 2014 (Entire Sample)				
Mean	9.26%	10.41%	Mean	1.14
SD	16.86%	14.94%	SD	0.26
Sharpe	0.36	0.48	Min	0.17
Skew	–0.12	–0.54	Median	1.30
Kurt	16.87	4.85	Max	1.30
MDD	–84%	–72%		
CumRet	$4,162	$11,141		
1926 to 1935 (First 10 Years)				
Mean	3.5%	5.0%	Mean	0.92
SD	26.8%	17.2%	SD	0.36
Sharpe	0.06	0.18	Min	0.17
Skew	0.44	–0.15	Median	0.96
Kurt	9.64	3.14	Max	1.30
MDD	–84%	–72%		
CumRet	$1.51	$1.77		
2005 to 2014 (Most Recent 10 Years)				
Mean	7.7%	9.2%	Mean	1.07
SD	20.4%	15.7%	SD	0.30
Sharpe	0.31	0.50	Min	0.20
Skew	–0.18	–0.44	Median	1.26
Kurt	10.09	1.37	Max	1.30
MDD	–55%	–38%		
CumRet	$2.11	$2.42		

	Raw Market Returns	Returns w/ VolControl	Stats for VolControl κ	
2010 to 2014 (Most Recent 5 Years)				
Mean	14.1%	14.5%	Mean	1.13
SD	16.2%	15.7%	SD	0.25
Sharpe	0.87	0.92	Min	0.34
Skew	–0.40	–0.51	Median	1.30
Kurt	4.53	1.80	Max	1.30
MDD	–21%	–21%		
CumRet	$1.94	$1.98		
2012 to 2014 (Most Recent 3 Years)				
Mean	18.5%	20.3%	Mean	1.24
SD	11.8%	14.5%	SD	0.11
Sharpe	1.57	1.39	Min	0.81
Skew	–0.26	–0.32	Median	1.30
Kurt	1.09	1.17	Max	1.30
MDD	–10%	–13%		
CumRet	$1.67	$1.74		
2014 (Most Recent 1 Year)				
Mean	10.5%	9.4%	Mean	1.25
SD	11.3%	14.1%	SD	0.10
Sharpe	0.93	0.66	Min	0.91
Skew	–0.45	–0.57	Median	1.30
Kurt	1.19	1.39	Max	1.30
MDD	–8%	–10%		
CumRet	$1.11	$1.09		

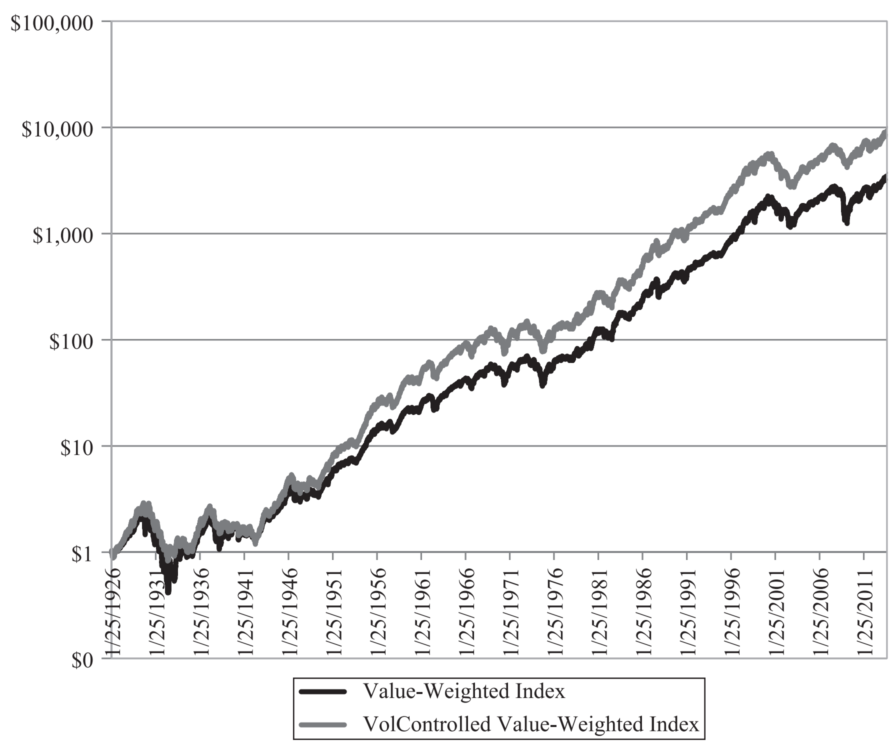
Notes: Applied to daily CRSP value-weighted index returns from January 25, 1926, to December 31, 2014, and for selected subperiods. The volatility control mechanism multiplies raw returns by a scale factor $\kappa_t = Min\left[\frac{16.9\%}{\hat{\sigma}_{t-1}}, 1.3\right]$, *where* $\hat{\sigma}_{t-1}$ *is a 21-day rolling-window annualized volatility estimator, and 16.9% is the annualized volatility of the index returns over the entire sample.*

This volatility-controlled dynamic strategy is reminiscent of portfolio insurance, except that the objective here is simpler: to maintain a more consistent volatility level so as to avoid triggering panic selling by investors. Typical portfolio insurance strategies such as that of Black and Pérold [1992] involve the dynamic replication of a put option on the portfolio's value, which involves reducing equity exposure as the value of equity declines. In the volatility control mechanism (equation (1)), equity exposure is reduced in response to increasing short-term volatility, not because of market direction. However, because stock prices and volatility are negatively correlated (Black [1976]), a strategy that reduces market exposure in response to increasing volatility will, on average, reduce equity exposure during declining markets and increase equity exposure during rising markets.

If the stock price–volatility relationship is persistent, such a volatility control mechanism may actually add to overall performance rather than subtracting from it, due to a partial investment in cash during periods when volatility exceeds the target. Exhibit 5 confirms this intuition, containing a comparison of the cumulative return of a $1 investment in the CRSP value-weighted index and the volatility-controlled index. Over the 89-year period, the volatility-controlled index is the winner by a factor of four. By reducing equity exposure when volatility is high, the risk-managed benchmark holds more cash when the equity risk premium is lower than average and holds more equity when the equity risk premium is higher than average, thereby exploiting the inverse relationship between stock prices and volatility Black [1976] documented more than four decades ago.

This simple example illustrates the potential benefits of separating active risk management from active investment management—one need not be tied to the other, given current trading technology, algorithmic overlay strategies, and a wide spectrum of liquid index futures

EXHIBIT 5
Cumulative Return of the CRSP Value-Weighted Index



Notes: With (gray) and without (black) volatility control from January 25, 1926, to December 31, 2014 (logarithmic scale). The volatility control consists of multiplying daily returns by a scale factor $\kappa_t = Min\left[\frac{16.9\%}{\hat{\sigma}_{t-1}}, 1.3\right]$, *where* $\hat{\sigma}_{t-1}$, *is a 21-day rolling-window annualized volatility estimator of the value-weighted index return.*

contracts. Moreover, there are many ways to improve upon the volatility control mechanism (equation (1)), leading to a host of new financial products and services that can be tailored to each individual investor's unique circumstances. This customization process is limited only by the imaginations of the portfolio manager and financial adviser, given the powerful trading and portfolio-optimization tools now at our disposal. By applying active risk management overlays to static indices, we can begin to harvest the benefits of smart beta without also suffering the consequences of dumb sigma.

BEWARE OF BACKTEST BIAS

The broader definition of an index that technology now supports has spawned a host of financial innovations that would have been impossible a decade ago. At the same time, it has also created some daunting new challenges for investors. Choosing among the dizzying array of financial products available today requires greater education and training—even for professional financial advisers—and typical retail investors may not be fully equipped to evaluate the potential risks and rewards of the various options with which they are bombarded. (Consider, for example, the sometimes counterintuitive behavior of double-leveraged inverse S&P 500 ETFs.) Moreover, the same technological advances that have brought us smarter betas also let us destroy wealth at the speed of light, as the shareholders of Knight Capital Group discovered on August 1, 2012.[3]

However, the single biggest challenge created by the smart beta revolution is the potential for misleading investors and portfolio managers through backtest bias. The problem is simple to state but devilishly difficult to address; in fact, it can be argued that backtest bias is an unavoidable aspect of any investment process. Suppose we wish to select the best of n investment opportunities, and each opportunity i can be evaluated by a single summary statistic θ_i such as its Sharpe ratio. Because θ_i is not directly observable, we must estimate it using whatever methods of evaluation we have at our disposal (including both qualitative and quantitative information), ultimately yielding estimators $\left\{\hat{\theta}_i = \theta_i + \epsilon_i,\ i = 1,\ldots,n\right\}$, where ϵ_i is the estimation error associated with our evaluation of investment i.

We would like to choose the investment i^* with the highest Sharpe ratio $\theta_{i^*} = \max_i \theta_i$, but because we only observe the estimated Sharpe ratios, it is tempting to select the investment i' with the highest estimated Sharpe ratio $\hat{\theta}_{i'} = \max_i \hat{\theta}_i$, and herein lies the problem. By selecting on the basis of imperfect estimates, we may be confounding genuine investment performance with random estimation error. In other words, by selecting the investment with the biggest $\hat{\theta}_i$, we hope to be getting the biggest θ_i, but we may, in fact, be getting the biggest ϵ_i instead. Given the inherent noisiness of even the best investment performance evaluation methods, it is impossible to completely eliminate ϵ_i.

However, there are several reasons why backtest bias is especially problematic for the burgeoning smart-beta industry. These reasons are related to the key drivers of backtest bias—other things being equal, backtest bias becomes more severe as 1) the number n of managers/models/track records grows, 2) the signal-to-noise ratio $\mathrm{Var}[\theta]/\mathrm{Var}[\epsilon]$ declines, and 3) decisions become more dependent on simulated performance statistics than on live track records. Although investors routinely face one or two of these issues in evaluating any investment opportunity, all three issues arise in the case of dynamic indices. The number of new products is growing rapidly, and because new products by definition do not have live track records, estimates of their performance can only be based on simulated returns and are, therefore, noisier than those of more-established products. Because simulations are, for many of these new products, the only way investors can develop intuition for the products' risk–return profiles, decisions tend to rely much more heavily on biased performance statistics.

To illustrate the subtleties involved in evaluating simulated returns, consider the volatility control strategy proposed in the previous section and suppose that we make one small change in the algorithm (1):

$$\tilde{R}_t = \kappa_t R_t, \kappa_t = \mathrm{Min}\left[\frac{\sigma_o}{\hat{\sigma}_{t+2}}, \bar{l}\right], \qquad (2)$$
$$\hat{\sigma}^2_{t+2} = \frac{1}{n-1}\sum_{j=0}^{k-1}\left(R_{t+2-j} - \hat{\mu}_{t+2}\right)^2, \bar{l} \geq 1$$

How does Equation (2) differ from Equation (1)? The only change is that the short-term volatility estimator $\hat{\sigma}^2_{t+2}$ in Equation (2) now uses returns from $t-k+3$ to $t+2$ instead of from $t-k$ to $t-1$. The simulated performance of this version, summarized in Exhibit 6, is considerably: better than that of the strategy given by Equation (1): over the entire 89-year sample, the compound annual

Exhibit 6
Summary Statistics

	Raw Market Returns	Lagged Volatility Estimator: Returns w/ VolControl	Lagged Volatility Estimator: Stats for VolControl κ		Biased Volatility Estimator: Returns w/ VolControl	Biased Volatility Estimator: Stats for VolControl κ	
Mean	9.26%	10.41%	Mean	1.14	12.65%	Mean	1.15
SD	16.86%	14.94%	SD	0.26	13.78%	SD	0.26
Sharpe	0.36	0.48	Min	0.17	0.68	Min	0.14
Skew	–0.12	–0.54	Median	1.30	–0.21	Median	1.30
Kurt	16.87	4.85	Max	1.30	0.56	Max	1.30
MDD	–0.84	–0.72			–0.72		
CumRet	$4,162	$11,141			$73,626		

Notes: For look-ahead-biased volatility control mechanism applied to daily CRSP value-weighted index returns from January 25, 1926, to December 31, 2014. The volatility control mechanism multiplies raw returns by a scale factor $\kappa_t = Min\left[\frac{15\%}{\hat{\sigma}_{t+2}}, 1.3\right]$, *where* $\hat{\sigma}_{t+2}$ *is a 21-day rolling-window annualized volatility estimator that uses contemporaneous and two consecutive future daily returns to estimate short-term volatility.*

return is higher, the volatility is lower, hence the Sharpe ratio is higher, and the cumulative return of a $1 investment is $73,626, nearly seven times the amount generated by Equation (1) and more than 17 times the raw index's cumulative return.

How is this possible? The seemingly minor change in Equation (2) introduced look-ahead bias to the simulation by using contemporaneous returns to scale the portfolio's leverage. In the event of a large one-day decline in the market on dates t to $t + 2$, the short-term volatility indicator will increase, causing a decline in the scaling factor κ_t that multiplies the date-t return. This, of course, leads to improved performance, but it is completely spurious—in practice, we cannot scale down a losing position before incurring the loss, which is what Equation (2) implicitly assumes. To underscore the effect that this kind of bias can have, if we had used 5 days instead of 21 to compute volatility—which increases the relative importance of the contemporaneous and future returns in the short-term volatility estimator—a $1 investment in 1926 becomes $173,444 by 2014, more than 40 times the cumulative return of the value-weighted index.

Although an experienced quantitative portfolio manager can easily prevent this particular kind of bias, an investor may not detect it so easily. Even if this bias is avoided, many others can arise through the process of selecting "optimal" strategy parameters. For example, even the simple volatility control mechanism given by Equation (1) has several parameters: the rolling-window length k, the leverage upper bound $\bar{l}$, the number of lags q for the short-term volatility estimator $\hat{\sigma}_{t-q}$, and the transactions cost τ. If we consider optimizing this strategy over the parameter space of plausible values for each of these parameters—say $k = 2, \ldots, 125$, $\bar{l} = 1.00, 1.05, 1.10, \ldots, 2.00$, $q = 1, \ldots, 10$, $\tau = 1, \ldots, 10$ bps—we will be selecting the "best" among $124 \times 21 \times 10 \times 10 = 260,400$ models. Including other desirable features, such as a lower leverage limit, a stop-loss policy, and a turnover constraint to control trading costs, can yield a combinatorial explosion in the number of models. Searching over these many versions is likely to impart significant backtest bias, especially if the underlying signal of genuine performance is small relative to the statistical errors in estimating performance.

Because it is so easy to construct an attractive investment product on paper with the benefit of hindsight, and because such products rarely perform as well when they are implemented, regulators have developed strict rules to address this issue. Rule 206(4)-1 of the 1940 Investment Adviser Act states that:

> It shall constitute a fraudulent, deceptive, or manipulative act, practice, or course of business…for any investment adviser registered or required to be registered under [the Investment Adviser Act], directly or indirectly, to publish, circulate, or distribute any advertisement which refers, directly or indirectly, to any testimonial of any kind concerning the investment adviser or concerning any advice, analysis, report, or other service rendered by such investment adviser.

Although the term "testimonial" is never defined, the SEC clearly had in mind the pitfalls of backtest bias when it adopted this rule in 1961 because the commissioners wrote "… such advertisements are misleading; by their very nature they emphasize the comments and activities favorable to the investment adviser and ignore those which are unfavorable" (SEC Investment Advisers Act Rel. No. 121, Nov. 2, 1961, (adopting rule 206(4)-1)). The SEC routinely enforces this rule through cease-and-desist orders, criminal prosecutions, and severe financial penalties against unscrupulous managers using misleading simulations in their marketing efforts. For example, in 2012 the SEC issued a cease-and-desist order against a nationally syndicated radio personality and financial advice author who had been touting his "Buckets of Money" wealth management strategy in seminars he hosted for potential clients. According to Michele Wein Layne, regional director of the SEC's Los Angeles field office, "[the manager and his advisory firm] left their seminar attendees with a false sense of comfort about the Buckets of Money strategy…The so-called backtests weren't really backtests, and the strategy wasn't proven as they claimed."

However, even the SEC has acknowledged the necessity and value of simulating the performance of various portfolio strategies, and has provided guidance in the form of no-action letters that describe cases in which the SEC will not pursue enforcement action. The 1986 no-action letter in response to a request by Clover Capital Management is the most relevant for the use of simulated returns in marketing materials (SEC [1986]). This letter states:

> The staff no longer takes the position, as it did a number of years ago, that the use of model or actual results in an advertisement is per se fraudulent under Section 206(4) and the rules thereunder, particularly Rule 206(4)-1(a)(5). Rather, this determination is one of fact, and we believe the use of model or actual results in an advertisement would be false or misleading under Rule 206(4)-1(a)(5) if it implies, or a reader would infer from it, something about the adviser's competence or about future investment results that would not be true had the advertisement included all material facts. Any adviser using such an advertisement must ensure that the advertisement discloses all material facts concerning the model or actual results so as to avoid these unwarranted implications or inferences.

The letter then provides a number of specific examples of inappropriate practices, which include the advertising of simulated results that do not deduct trading costs and fees, highlight the potential for gains without also mentioning the risk of loss, fail to disclose material assumptions underlying the results and the inherent limitations of simulated returns, and so on. In short, the use of backtest results must satisfy the same standards of accuracy and disclosure as the use of live track records, and managers have an affirmative obligation to avoid any false or misleading statements about their simulations.

Seasoned investment professionals have long been aware of backtest bias and have learned to deal with it in several ways. The first and most obvious method is to treat all investment performance records with a healthy dose of skepticism and acknowledge that even a stellar track record contains some element of sheer dumb luck. How much historical success is luck versus skill is another way of asking how much of $\hat{\theta}_i$ is θ_i and how much is ϵ_i.

The second method is to use additional information to distinguish θ_i from ϵ_i. For example, if a manager claims to be a talented stock picker, we can check whether this manager's stock-picking success materially changed during bear markets; if so, then perhaps the manager's "skill" is more beta than alpha. If, on the other hand, the manager's stated strength is asset allocation, a standard performance attribution analysis lets us verify this claim by separating the manager's cumulative returns into market and asset-class timing, security selection, and other sources of value-added.

The third and most direct way to distinguish between θ_i and ϵ_i is to conduct live out-of-sample experiments. Follow the manager's performance over the course of the next year or two and evaluate the manager at the end of that period. By collecting new data on the manager's performance that are statistically independent of the past, we minimize backtest bias. If the manager's out-of-sample record is comparable to the backtest, then $\hat{\theta}_i$ may be more θ_i than ϵ_i.

In each of these three approaches, we seek additional information that can confirm the link between $\hat{\theta}_i$ and θ_i. If we can't find such information, then the more likely explanation for attractive historical performance $\hat{\theta}_i$ is lucky ϵ_i.

Simulations do play an important role in developing a deeper understanding of new investment products and we should not shun them. Although backtest bias is an unavoidable aspect of financial innovation, we can reduce its effect by practicing good statistical hygiene in generating and interpreting backtests.

CONCLUSION

A confluence of technological advances has caused tectonic shifts in the financial landscape, creating winners and losers overnight. The winners are technology-savvy investors who understand their own risk preferences and financial objectives and can appreciate the full spectrum of risks and rewards offered by today's dizzying array of smart-beta and index products. The losers are the technophobes and Luddites who don't know and don't care about investing—the investment ecosystem has become much more dangerous for them.

The traditional advice of "equities in the long run" and "buy and hold" worked well during the relative calm of the Great Modulation, and the equity risk premium was remarkably consistent and positive over this period. However, the same advice may not be as effective in the current environment of seesawing volatility and intense financial innovation. Dynamic indices have a great many benefits—more sources of diversification and risk sharing, cheaper ways to meet individual needs, and greater flexibility in reflecting investment views—but they come at a cost of potential pitfalls if abused. Hand-saws are not nearly as useful as chain-saws in clearing downed trees after a hurricane, but hand-saw accidents are not nearly as dangerous as chain-saw accidents. If investors do not adapt, but persist in treating modern financial tools as if they behaved like their less powerful predecessors, a series of chain-saw accidents will be the outcome. To Keynes' adage that "in the long run, we are all dead," we should append the further imperative to "make sure the short run doesn't kill us first."

ENDNOTES

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[1]In 2008, the VIX increased from 18.81 on August 22 to 80.86 on November 20.

[2]The contract value for the E-Mini S&P 500 is 50 times the index value, so an index level of 2,000 yields a contract value of $100,000. The bid/offer spread for this contract is typically one tick, which is $12.50 per contract, so the one-way cost can be approximated by half this amount, or 0.625 basis points. Additional fees for the E-Mini S&P 500 (commission, NFA, exchange, etc.) range from $1.87 to $2.46 per contract, depending on the method of execution, which amounts to 0.221 basis points on average, so the total cost of executing a single contract is slightly less than 1 basis point as of March 30, 2015.

[3]A programming error caused Knight Capital Group's automated trading system to enter into four million trades over a period of 45 minutes, resulting in a $440 million loss for the firm that ultimately led to its demise and acquisition by Getco, Inc.

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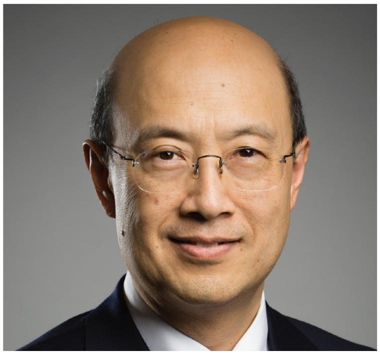
Practical Applications of

What Is an Index?



Author: Andrew W. Lo
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Overview

Recent advances in financial technology suggest that investors need to discard their conventional ideas on active versus passive investing. **MIT**'s **Andrew W. Lo** offers a new framework for thinking about indexation and active versus passive investing by distinguishing traditional or "static" indices from more complex or "dynamic" indices. Both types are transparent, investable and rules-based, but dynamic indices are not necessarily market-capitalization weighted and can be actively rebalanced to meet pre-specified risk–reward objectives.

Although dynamic indices can meet highly customized objectives and offer great diversification, investors should understand that they often come with complex risk exposures that need to be actively managed. Additionally, backtest bias can be an issue in assessing dynamic index performance, because these indices may have little or no live track records.

Practical Applications

- **Static versus dynamic**. Distinguish static portfolios from more complex, non-market-cap-weighted, dynamic portfolios that may be rebalanced frequently and are actively risk managed.
- **Dynamic indices often contain a greater variety of risk factors**. Investors need to understand their tail, illiquidity and credit risks, and they need to manage backtest bias.
- **Use volatility management**. One straightforward application is to actively manage the volatility of any traditional index to provide a relatively consistent level of risk regardless of how volatile or correlated the underlying assets become.

Practical Applications Report

The rapid proliferation of index products with risk–reward profiles more complex than those of conventional market-cap-weighted indices has blurred the line between active and passive investing. Advances in technology, computing and trading as well as product innovations have created a host of new types of index-related products,

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His awards include **Sloan** and **Guggenheim** Fellowships, the **Paul A. Samuelson Award**, the **Harry M. Markowitz Award**, the **CFA Institute's James R. Vertin Award**, and election to **Academia Sinica**, the **American Academy of Arts and Sciences**, the **Econometric Society**, and ***Time Magazine***'s ***2012 list*** of the "100 most influential people in the world." Andrew has also received teaching awards from the University of Pennsylvania and MIT. He received a BA in economics from Yale University in 1980 and an AM and PhD in economics from Harvard University in 1984.

such as target-date and lifecycle funds, trading-strategy indices, smart-beta indices and hedge-fund replication products.

Not only is the active–passive construct obsolete for defining indices, it also ignores the many different types of risk faced by so-called passive indices, says Lo, Charles E. and Susan T. Harris Professor at the **MIT Sloan School of Management**. Lo is also Director of the **MIT Laboratory for Financial Engineering** and Chief Investment Strategist at **AlphaSimplex Group**. The new world of technology-leveraged investing requires a new framework that offers a functional definition of index products, which Lo presents in ***What Is an Index?*** in ***The Journal of Portfolio Management***'s Spring 2016 issue.

FUNCTIONAL PURPOSE

There are two distinct functions of a modern index, Lo says. The first is informational: To provide an aggregate measure of investment return, not for individual securities but for sector- or market-wide drivers. The second is to provide a standard with which to compare active managers' performance, he explains. To be meaningful on both counts "the index must be able to serve as the basis for an investment vehicle, a transparent portfolio with a plausible risk–reward profile."

To meet those functional goals, Lo proposes a dynamic versus static framework for defining indices. He defines *all* indices as having three properties:

1. they are transparent (all aspects are public and verifiable);
2. investable (a large amount of capital can be easily committed in a short period);
3. systematic (construction is entirely rules-based and not dependent on any discretionary human judgment).

The main differentiator is that static indices—such as the traditional cap-weighted indices consisting of liquid securities—are buy-and-hold portfolios and are not expected to be rebalanced very often, he says. But dynamic indices may rebalance frequently to meet complex risk–reward objectives (e.g., trend-following subject to volatility, leverage, and liquidity constraints).

The **capital asset pricing model** (**William Sharpe**, 1964) introduced the distinction between active managers' alpha generation versus commoditized risk premia, but it also contributed to a rigid view on risk. "Active strategies manage risk actively and passive strategies do not manage risk at all," Lo says, pointing to the historical development where "passive investing became synonymous with market-cap-weighted indices to minimize the amount of trading needed to manage an index portfolio." But traditional passive investing doesn't protect investors from severe volatility spikes and loss (as occurred during the 2008 financial crisis), he notes. This is where dynamic indices can add substantial value, especially when the volatility of volatility is high.

Dynamic indices often contain subtle and varied risks, such as tail, illiquidity or credit risk, Lo says. Because these highly customizable portfolios typically do not have lengthy track records, investors must also be able to understand, and properly evaluate, backtest bias, he adds. "Investors need to become more educated because the kinds of risks are more complex in dynamic indices," Lo says. For instance, hedge-fund replication strategies—which aim to replicate the betas of broad

Key Definitions

Dynamic index

Any portfolio that is rules-based, transparent to all investors and investable.
—Andrew W. Lo

Static index

Any portfolio that is based on conventional market-capitalization (buy-and-hold) weighting.
—Andrew W. Lo

Watch the Video

"Investors need to become more educated because the kinds of risks are more complex in dynamic indices."

> “Think of volatility management as having cruise control in your car, which maintains a consistent speed. You don’t need to be an engineer to use it.”

categories of hedge-fund strategies—are by their nature more complex and risky than, say, a long-only equity fund.

The good news for investors in the new world of dynamic indexing is that risk management has become increasingly sophisticated, Lo points out. “Think of volatility management as having cruise control in your car, which maintains a consistent speed. You don’t need to be an engineer to use it.”

To show that passive strategies can be actively risk managed, Lo presents an example of a dynamic index fund that contains no alpha but maintains a target level of volatility. He proposes a specific volatility-control algorithm and applies it to a daily value-weighted index of all US stock returns from 1925 to 2014. This fund outperforms the unmanaged index, and as Lo explains, its objective is simple: “to maintain a more consistent level of volatility so as to avoid triggering panic selling by investors.”

BACKTEST BIAS IS HARD TO OVERCOME

The biggest challenge, Lo warns, is backtest bias, which is unavoidable when it comes to dynamic indices because the number of new products is growing, and their performance can only be based on simulated returns. “With dynamic indices, there’s usually no track record,” Lo says. “By selecting on the basis of simulated return statistics, we may be confounding genuine investment performance with random estimation error,” he notes, adding that a manager may end up focusing only on those simulations that are attractive.

Of course, backtest bias isn’t new, and there are ways that portfolio managers deal with it, Lo says. The first, obvious way is to be skeptical of all performance records and acknowledge that some portion is due to sheer luck. Second, investors can be more diligent and use additional information to verify performance claims (e.g., by conducting a performance attribution analysis that examines market and asset-class timing, security selection and other sources of alpha vs. beta). Third, before committing capital, investors can follow a manager’s performance for a year or two out of sample, so that estimation errors will be independent of the past.

How to deal with backtest bias:

1. Be skeptical.
2. Use additional information to verify performance claims.
3. Follow a manager’s performance out of sample before committing capital.

A BRIEF HISTORY

Lo says his interest in redefining indices came from his broader interest in markets in an evolutionary context, within his adaptive markets framework (see ***The Adaptive Markets Hypothesis*** in JPM’s ***20th Anniversary Issue***, 2004). In this article, he provides an insightful history of index products, which he said gave him a better understanding of their origins: the impact of Sharpe’s CAPM theory in 1964 and **Eugene Fama** and **Paul Samuelson**’s **efficient market hypothesis** in 1965, their practical application in the first index products by a **Wells Fargo** team and later by **Jack Bogle** and the subsequent proliferation of index funds by asset class, style, investor age and so on.

“This deep dive into history got me to understand the institutional constraints and the functional nature of these products,” Lo says, quoting **Robert Merton**’s frequently cited observation that “financial functions tend to be more stable than financial

Lo—"I'm an avid, but mediocre, squash player."

institutions." That is, focusing on the functional purpose of indices, to provide ways to reduce risk, is more useful than their specific form/structure.

"Today, indices are at the forefront of the building tsunami of financial innovation. Just as the previous generation saw 'markets in everything,' we currently see 'indices in everything,' as well as funds and derivatives based on those indices," says Lo in his recent article. In this new financial landscape, the winners will be technology-savvy investors who understand today's "dizzying array of smart-beta and dynamic-index products," he concludes.

Lo continues to expand the ideas from his adaptive markets hypothesis to implement it in practical ways. One area he's working on is finding ways to model the emotional reactions of investors that can be counterproductive to investing. Using insights from artificial intelligence, this research looks at the question of how we can algorithmically capture emotional reactions so as to help people be smarter investors.

When asked what he does to relax, Lo says: "I'm an avid, but mediocre, squash player."

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A New Metric for Smart Beta: *Factor Exposure per Unit of Tracking Error*

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Smart beta, an industry label for rules-based, non-market-capitalization-weighted investing, has become its own phenomenon in recent years. But what does success look like for smart beta strategies? Not surprisingly, given its newness, there remains a lack of clarity surrounding what makes a smart beta strategy a good one. Success for low-volatility equity strategies is reasonably straightforward—lower risk than the benchmark and downside protection. But success for value, momentum, quality, and other strategies is more nebulous. Investors are generally seeking superior risk-adjusted returns versus a market-cap-weighted equivalent. And in gauging the success of a strategy, the focus has centered in large part on performance.

To understand what success looks like, we should remind ourselves of the purpose of smart beta. In our view, smart beta is a way for investors to capture exposure to some underlying factor(s). Because performance of a factor can vary over any short period of time, we propose that smart beta investors should define success as the ability of that strategy to provide a strong and consistent level of exposure to the targeted factor(s). Moreover, exposure should be achieved in a risk-aware way. If an investor is taking on more risk relative to a cap-weighted benchmark, the investor should be compensated with a greater exposure to their targeted factor. In short, we suggest that investors should focus on their *factor exposure per unit of tracking error.*

The article is divided into two main parts. In the first section, we discuss why exposure per unit of tracking error (active risk) should be front and center as a metric for evaluating various smart beta strategies. Index researchers should use this metric to evaluate the efficacy of their portfolio construction decisions, and investors should use this metric when comparing different options. Then, in the rest of the article, we analyze the way in which portfolio construction decisions impact this new metric. For index researchers, this section provides a helpful guide to making portfolio construction decisions. For investors, we show how index methodology decisions link directly to an index's returns, exposure, tracking error, and exposure per unit of tracking error.

A NEW METRIC: FACTOR EXPOSURE PER UNIT OF TRACKING ERROR

In theory, all smart beta offerings have the potential to provide substantial benefits to investors by allowing efficient access to sources of active return using low-cost passive implementation. Investors can approach investing in smart beta from a variety of angles, depending on their investment objective. Some view it as a replacement for active investing, some as a replacement for passive.

Some use it to hedge existing exposures or to complete their portfolios. Some view it as a strategic long-term allocation, some as a short-term tactical investment. But in general, smart beta tends to be used to meet one or more of the following objectives:

- *Return enhancement*: The investor seeks to improve returns by explicitly taking exposure to a factor(s) that he or she believes will deliver excess returns over time.
- *Risk mitigation*: The investor seeks to reduce or minimize some measure of risk by explicitly taking exposure to risk-reducing assets.
- *Exposure management*: The investor seeks to target some level of exposure to a source of return because he or she believes it will improve the portfolio's overall risk and return characteristics.

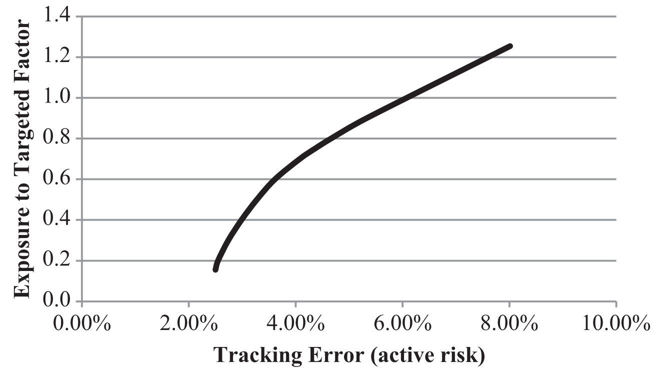
In order to achieve any of these objectives, the smart beta strategy must effectively capture the factor exposure(s) sought. For this reason, we believe the adoption of smart beta should proceed in two steps. First, investors should form an investment belief around the source of return they seek to capture and how it will help them achieve their investment objective. Second, they should invest in the smart beta strategy that delivers most consistent exposure to their desired source of return, subject to their preferences around tracking error, return, concentration, and liquidity.

Importantly, investors should evaluate smart beta strategies by considering how much factor exposure is delivered relative to the amount of tracking error (TE) taken. A modified efficient frontier is shown in Exhibit 1, analogous to the traditional Markowitz mean–variance efficient frontier. Total risk is replaced by tracking error, because investors are making an active decision to tilt toward some factor relative to the cap-weighted benchmark. Return is replaced by factor exposure, because the portfolio's objective is to give investors exposure to that factor (which indirectly will deliver return if the decision to invest in that factor is successful). Locating the point on the factor efficient frontier is straightforward if the investor has a preferred level of tracking error or exposure. (one possible exception is the case where risk mitigation is the sole objective, since tracking error becomes less relevant. This case deserves separate consideration.)

Note that portfolios that maximize exposure per unit of tracking error will deliver the strongest return per unit of tracking error (e.g., information ratio), if that factor performs well (and there are no incidental bets that drag down returns). In that sense, this framework is not that dissimilar from the traditional one. The crucial difference is that if the factor performs poorly, these same portfolios will deliver significant underperformance.

EXHIBIT 1

A New Efficient Frontier for Factor Portfolios (stylized illustration)



HOW TO BUILD PORTFOLIOS WITH HIGH FACTOR EXPOSURE PER UNIT OF TRACKING ERROR

As we have established, effective smart beta strategies should deliver high factor exposure per unit of tracking error. But how do we, as index researchers, achieve that? Recall that, once the factors are defined, there are three key decisions made in the construction of a smart beta factor portfolio:

1. *Security selection*: Which stocks will we include in the portfolio?
2. *Weighting*: How will we weight the stocks in the portfolio?
3. *Rebalancing frequency*: How often will we rebalance the portfolio?[1]

Traditional stock pickers pick stocks and decide how much of each they want to hold. Quant managers use optimizers and algorithms to pick stocks and weights simultaneously to balance various competing objectives (return, risk, constraints, etc.). In smart beta strategies, a set of rules must be created to select stocks and weight them.

There is an inherent set of trade-off in security selection and weighting. Reducing the number of names or increasing the degree to which the weights depart from market-cap weights will generally provide higher exposure to the factor in question but at the expense of liquidity, increased exposures to unintended bets, larger sector and country active weights, and greater domination of stock-specific risk and return.[2]

To keep the discussion tractable, we focus on a framework for building rules based portfolios that we have espoused in earlier research (see Bender and Wang [2015]). The universe is divided into subportfolios and the weighting scheme is a function of a starting weight (usually market cap weight) times a multiplier. Security screening and weighting are easy to understand in this framework; selecting which subportfolios to include is our security selection decision and selecting the multiplier scheme is our weighting decision. A summary of this tilted framework approach appears in Appendix A.

USING SECURITY SELECTION TO ACHIEVE HIGH LEVELS OF EXPOSURE PER UNIT OF TE

It is well known that the fewer stocks a portfolio has, the higher its tracking error (TE) to broad-based benchmarks will be. This is confirmed in Exhibit 2 using a value factor portfolio for a developed market global universe. There, we successively remove subportfolios (from our original set of 20), starting with the lowest ranked subportfolios by valuation.[3] The remaining securities are cap weighted,[4] and the portfolios are rebalanced annually. As we remove subportfolios, the tracking error to the original cap-weighted universe increases, nearly monotonically.

What happens to exposures? We find that as we remove subportfolios (reduce the number of securities in the portfolio), the exposure also increases, nearly monotonically, as shown in Exhibit 3. (Exposures are estimated by running regressions on the Fama–French [1992, 1993] factors.[5]) This direct link between security selection and exposure has several important implications. First, it illustrates that exposure can be controlled quite easily in rules based portfolios. Second, it shows that unless the methodology is overly engineered, portfolios with fewer stocks will tend to have higher exposures along with higher tracking error.

What was the impact on realized return as we successively screened out more stocks? Because this period was one in which value performed well, the realized return also improved as we successively removed subportfolios (see Exhibit 4).

In a period in which value did not perform well, the more concentrated portfolios still have higher

EXHIBIT 2

Tracking Error Increases with Fewer Stocks (value portfolio)

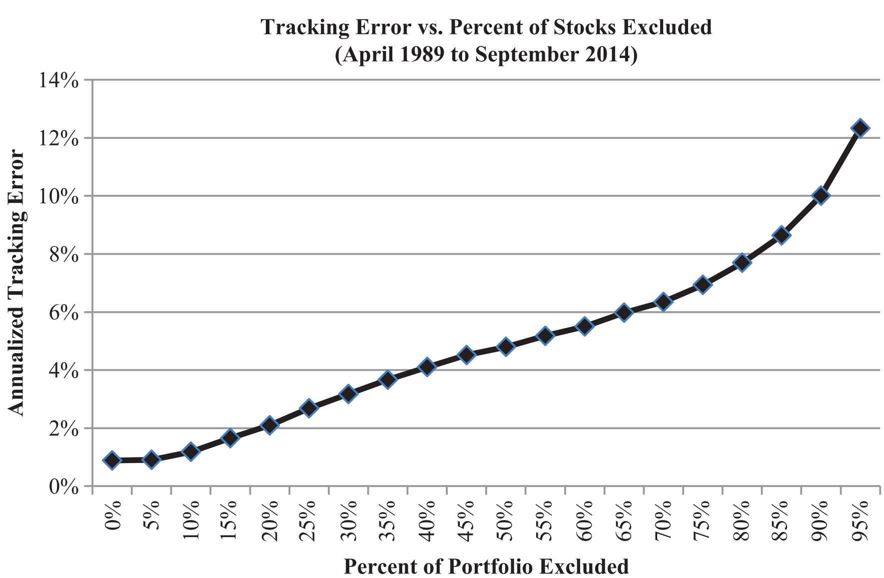


Exhibit 3

Factor Exposure Increases with Fewer Stocks (value portfolio)

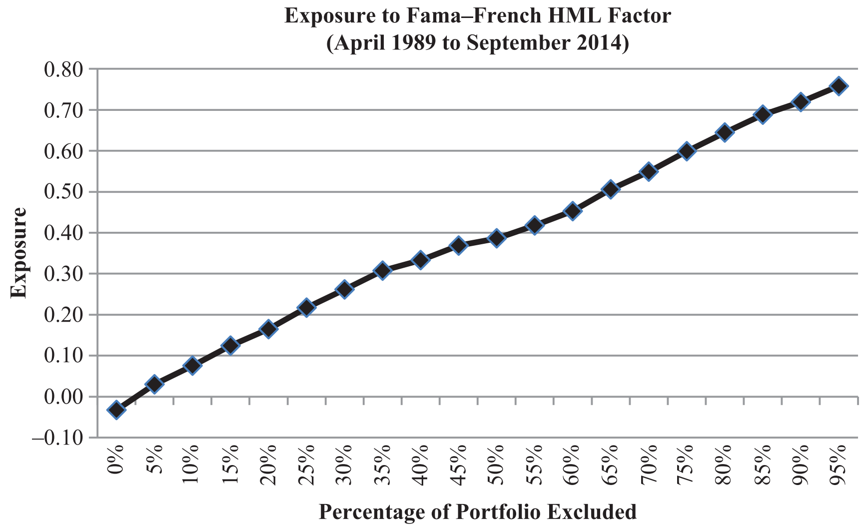
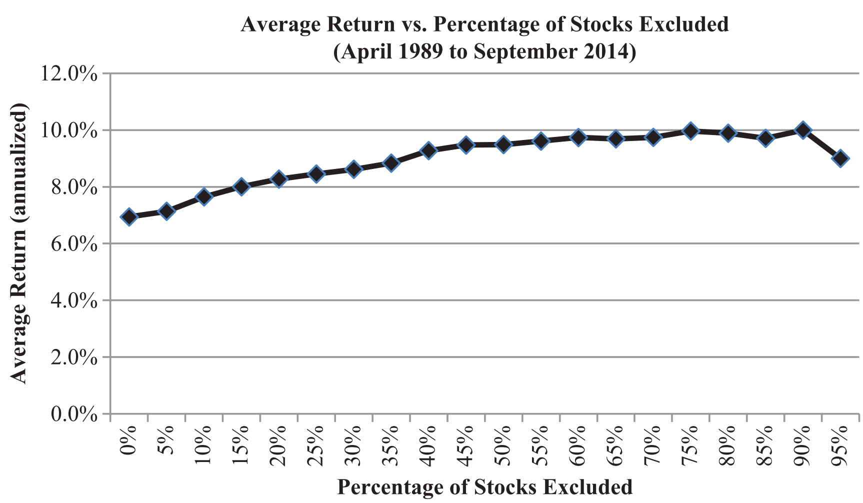


Exhibit 4

Average Return Increases with Fewer Stocks as Long as the Underlying Factor Performed Well During That Period: Example with Value Portfolio



tracking error and higher exposure but now greater underperformance, as shown in Exhibit 5.

Which portfolio achieves the highest exposure per unit of tracking error? Because tracking error and exposure are affected similarly by screening decisions (i.e., the more stocks screened out, the higher the tracking error and the higher the exposure), the exposure per unit of tracking error should be relatively stable. Empirically, this relationship generally holds, however, it may vary depending on the time period. For instance, in Exhibit 6, which considers the results for the longest period we have, April 1989 to December 2014, exposure per unit of TE is high and stable across most portfolios, no matter how many stocks are excluded. However, in Exhibit 7, which considers a shorter period, June 2007 to December 2014, there appears to be a single portfolio that has higher exposure per unit of TE than all the others.

Exhibit 5

More-Concentrated Portfolios Experience Greater Return Drag when the Underlying Factor Does Poorly (value portfolio, April 1989 to September 2014)

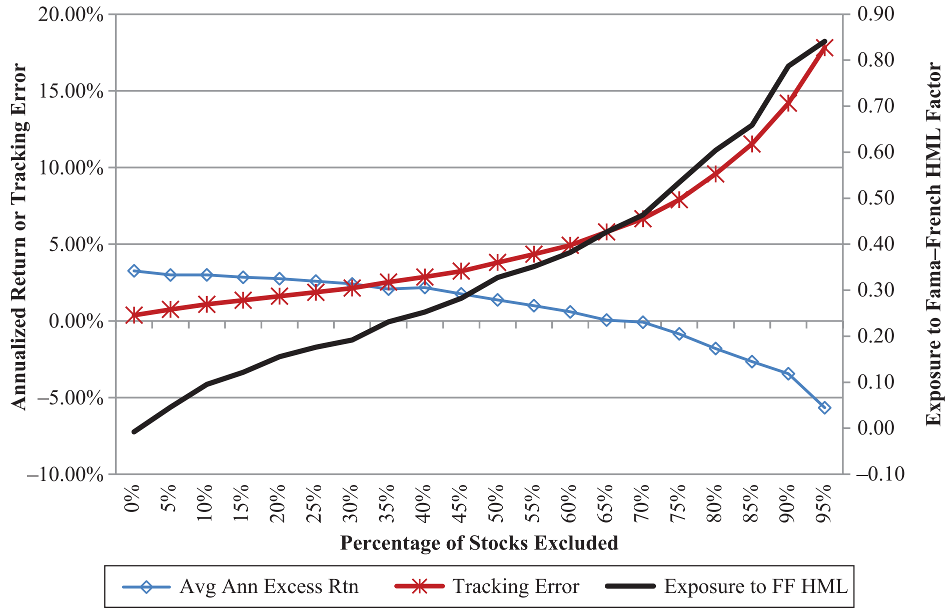
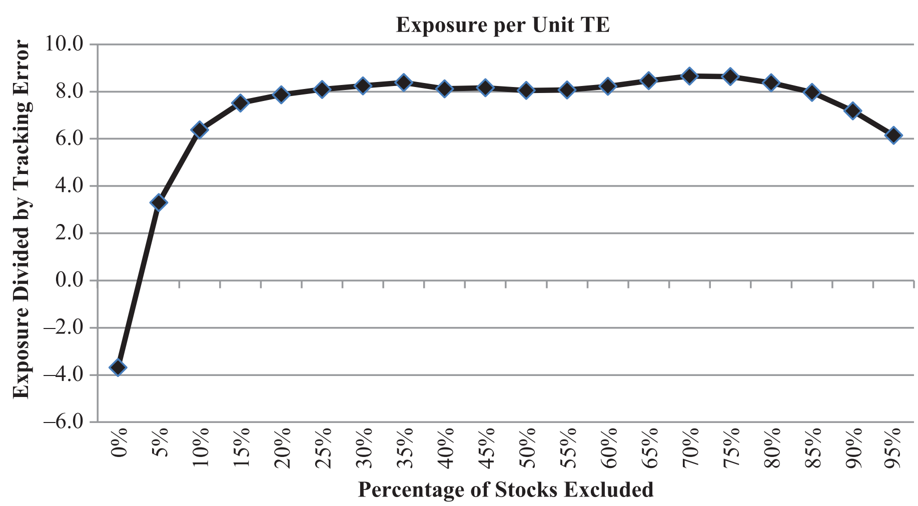


Exhibit 6

The Impact of Security Selection on Exposure per TE (example with a value portfolio, April 1989 to September 2014)



USING SECURITY WEIGHTING TO ACHIEVE HIGH LEVELS OF EXPOSURE PER UNIT OF TE

Security weighting has an impact that in many ways is similar to security selection—in general, a more aggressive weighting scheme leads to larger factor exposure and higher tracking error. To analyze the impact of varying the weighting scheme, we employ a kinked linear multiplier scheme, described in Appendix B, which allows us to directly vary the maximum effective multiplier.[6] The maximum effective multiplier is the largest effective multiplier in the portfolio—that is, the weight of

Exhibit 7

The Impact of Security Selection on Exposure per TE (example with a value portfolio, June 2007 to September 2014)

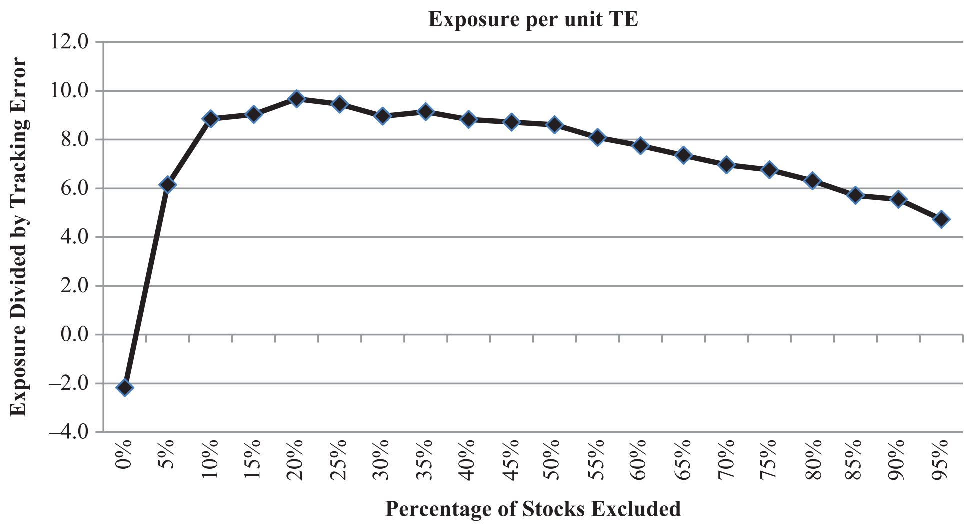
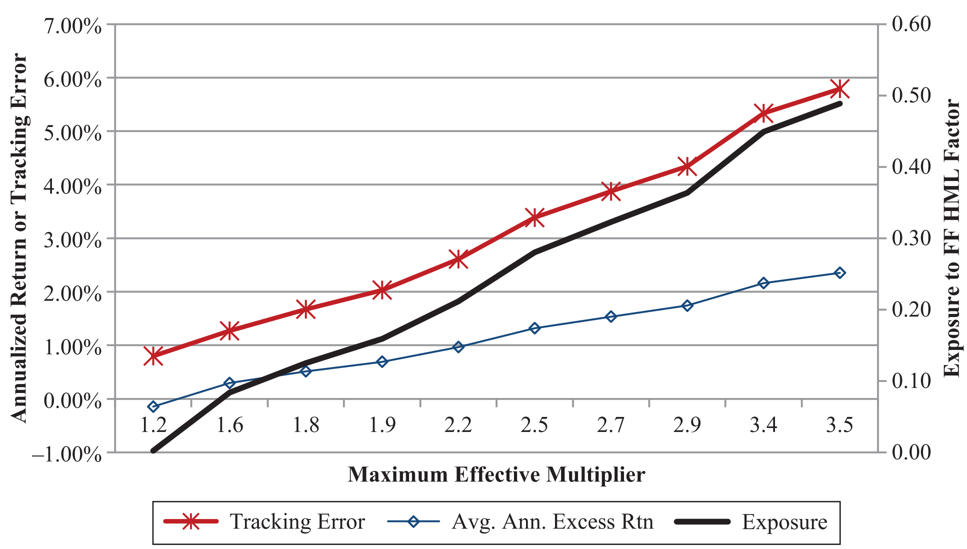


Exhibit 8

The Impact of Changing the Weighting Scheme (value portfolio, April 1989 to September 2014)

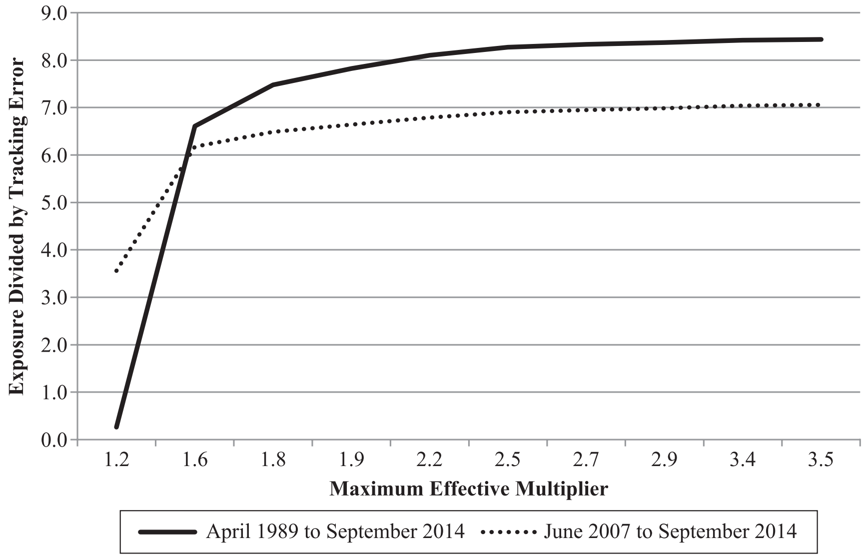


the security in the portfolio with the largest weight relative to its market-cap weight, divided by its market-cap weight. As it increases, the weighting scheme increasingly departs from market-cap weight, that is, becomes more aggressive. As shown in Exhibit 8, increasing the maximum effective multiplier drives tracking error, exposure, and return higher. This is consistent with the security selection results for the same period.

What happens to the highest exposure per unit of tracking error as we vary the maximum effective multiplier? The results are surprisingly consistent with the previous ones for security selection. As shown in Exhibit 9, above a certain level of aggressiveness, the exposure per unit tracking error remains fairly stable. This is true for both the full period, April 1989 to September 2014, and the subsample June 2007 to September 2014.

E x h i b i t 9

The Impact of Weighting Decisions on Factor Exposure per Unit of Tracking Error (value portfolio)



Similar to the security selection results, both exposure and tracking error increase as we increase the aggressiveness of the multiplier scheme, so the factor exposure per unit of tracking error will on average be relatively stable past a certain point. Although maximizing the factor exposure per unit of tracking error is certainly an option (directly via optimization), our results are comforting for those smart beta indexes employing transparent rules-based weighting and screening decisions. Weighting and screening decisions move the tracking error and exposure in the same direction. Thus, unless there are complex refinements on top of these two decisions (for instance, country and sector and liquidity refinements), investors should be reassured that high exposure per unit of tracking error will naturally fall out of the process.

Exhibit 10 shows the factor exposure versus tracking error trade-off for a selection of value indexes over the period. All have reasonably high exposure per unit of tracking error.

FURTHER CONSIDERATIONS

Our analysis until now highlights very straightforward relationships between security selection/weighting and resulting tracking error, return, and exposure. But there are two main caveats. The first caveat concerns the cases where the investor wants to target a factor that is modeled typically through regressions. These models attempt to account for interaction effects between factors, so that the "pure" return to the factor can be identified. Barra and Axioma are well-known examples. These pure factors are difficult to capture in a long-only, rules-based framework and generally require optimizers because exposures to non-targeted factors must be neutralized. That said, there are a few ways that rules-based methods can be used to neutralize certain active bets. (For instance, country neutralization can be done by applying security selection/weighting decisions within countries and cap weighting the country subportfolios).

The second caveat concerns whether it is sufficient to strictly target high consistent factor exposure per unit of tracking error. The answer, of course, is no. Smart beta should be evaluated in the same way other investments are evaluated—across a range of characteristics including costs, liquidity, capacity, and concentration. Exhibit 11 details a number of metrics an investor might consider in evaluating smart beta strategies. These metrics include not just the factor exposure per unit of tracking error metric we have argued for here but also the stability of the factor exposures over time and the unintended exposures that are not targeted. We generally would not recommend a very high concentration portfolio in practice. More concentrated portfolios have greater active country and sector weights and more of the return risk comes from the stock specific portion. Thus there is a natural limit to how concentrated smart beta portfolios can be.

EXHIBIT 10

Exposure vs. Tracking Error for Select Value Indexes (August 1996 to May 2014, annualized monthly returns, U.S. indexes)

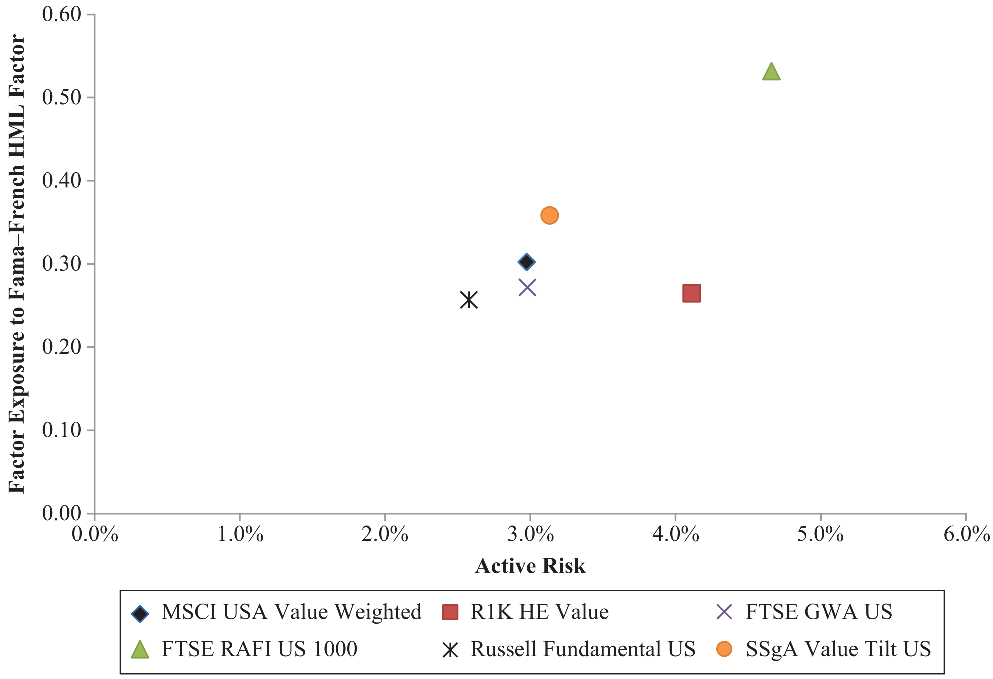


EXHIBIT 11

Metrics for Assessing Smart Beta Offerings

Active Risk	Liquidity/Capacity	Factor Exposure	Performance
• Tracking error • Concentration • Sector active weights • Country active weights	• Turnover • Number of names • Effective number of names • Weighted ADV • Market cap coverage • Top 10 security weights • Weighted average days to trade • Stock ownership as % of free float • Effective multipliers to cap weight	• High factor exposure per unit of tracking error • High stability of factor exposures • Limited factor exposure for non-targeted factors	• Absolute return and risk • Sharpe ratio • Relative return and risk • Information ratio • Downside risk/extreme risk • Drawdown

CONCLUSION

Although performance may be a significant motivator behind investing in smart beta strategies, we do not believe it should be the sole metric or even the most important metric for evaluating the success of smart beta. Factor returns vary over time, and for any given time period, a factor may underperform a market-cap-weighted benchmark; this does not mean the smart beta factor portfolio has "failed" in its objectives. Rather, we advocate that the success of a smart beta portfolio should be measured by the strategy's ability to achieve an effective factor exposure in a risk-aware way. An investor should be compensated with more factor exposure if that investor is taking on more risk relative to the benchmark. We suggest that if there is one metric that deserves more attention than any other metric, it should be factor exposure per unit of tracking error.

APPENDIX A

SUMMARY OF TILTED FRAMEWORK

The portfolio construction approach used in the study is summarized here. We rank stocks annually based on exposure to the targeted factor. We create 20 subportfolios of stocks each containing 5% of the market-cap weight of the entire universe. Subportfolio 20 has the stocks with the highest exposures to the factor, and subportfolio 1 has the stocks with the lowest exposure. Multipliers are assigned based on the subportfolio and applied to security cap weights. Larger multipliers are assigned to the subportfolios that deliver more exposure to the factor. Smaller multipliers are assigned to the subportfolios that deliver negative exposure to the factor.

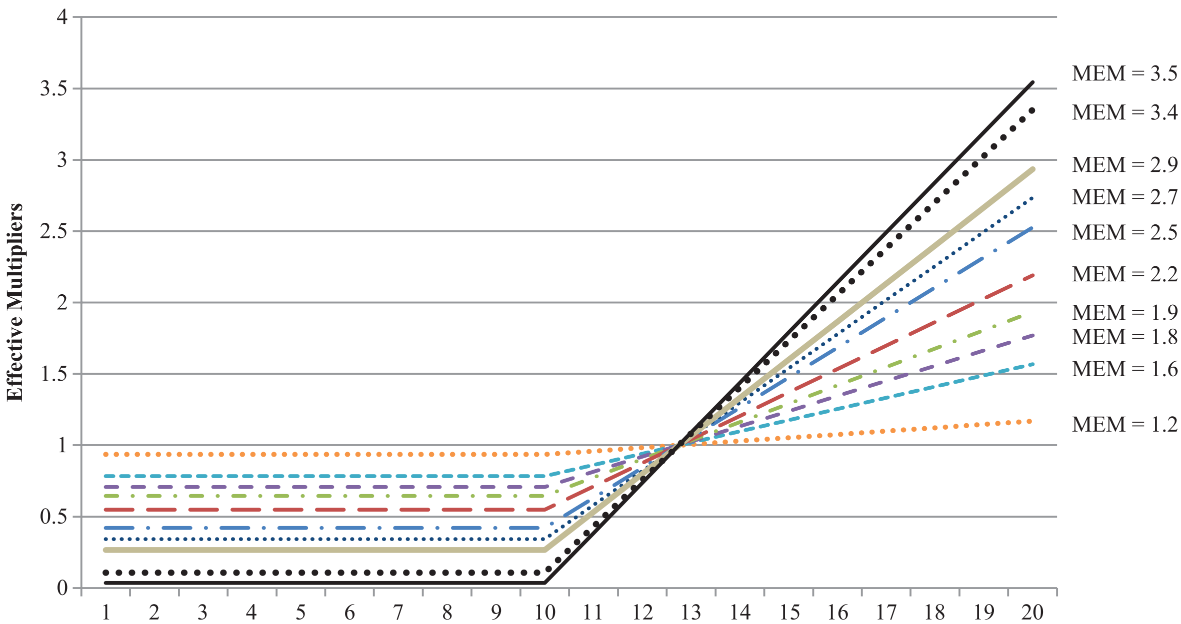
APPENDIX B

KINKED MULTIPLIER SCHEME

We employ a kinked multiplier scheme in the security weighting section of our analysis. This allows us to control the maximum effective multiplier (MEM) in an intuitive way. Many linear weighting schemes have a maximum effective multiplier of 2 (see Bender, Sun, and Wang [2016]), while a nonlinear convex or concave weighting scheme does not allow for easy calibration. The kinked multiplier schemes, along with their maximum effective multipliers, are shown in Exhibit B1.

EXHIBIT B1

Effective Multipliers Used for Security Weighting Analysis



ENDNOTES

[1]The optimal rebalancing scheme is driven by the factor's natural horizon or decay rate, in other words, how much the factor itself needs to turn over to express itself. Slow-moving factors with long decay rates include value and quality. Fast-moving factors with short decay rates include momentum and sometimes volatility. The rebalancing frequency should be tailored to the factor. The goal is to keep factor exposure high without generating unwarranted amounts of turnover. As relates to our discussion here, rebalancing frequency does not have a systematically directional impact on tracking error in the same way security selection and weighting do. And its impact is not so much on the average level of exposure as it is on the variability of the exposure. Thus, if the goal is to ensure a strong consistent level of exposure per unit of tracking error, it makes sense to first use security selection and weighting to attain the desired level of exposure per unit of tracking error and, second, calibrate the rebalancing frequency to keep this measure relatively stable over time.

[2]It is well known that as the number of stocks increases in a portfolio, the portion of the portfolio's stock-specific risk decreases and its systematic risk increases. We define systematic risk to include market risk and all risk that can be attributed to additional common factors, as in Rosenberg and Marathe [1972]. Because factor indexes are meant to capture these factors, then by default, a pure factor portfolio will hold many names and will have zero specific return and risk.

[3]Our definition of value is an equal-weighted blend of five fundamentals to price. The fundamentals are book value, earnings, sales, cash flow, and dividends.

[4]By construction, the portfolio that excludes no securities will have zero tracking error because the securities are all cap weighted.

[5]The choice of which reference factor portfolios to use is an important one, and deserves further discussion outside the scope of this study. Other candidates include commercially available factors provided by Barra, Axioma, Northfield, and other vendors, or custom factors that use the same definitions but orthogonalized for other factors.

[6]It is not feasible to use a linear multiplier scheme, keeping security selection constant, to vary the maximum effective multiplier. This is because there is a limit to the maximum effective multiplier of 2 in the absence of screening. The mathematical proof of this is available in Bender, Sun, and Wang [2016].

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Practical Applications of

A New Metric for Smart Beta: *Factor Exposure per Unit of Tracking Error*

Overview

With the proliferation of smart beta methodologies, underlying indices and investment products, it's difficult for investors to evaluate and choose. "There are more than a dozen value indices from different providers, and I have sympathy for asset owners and investors who have to differentiate among them," says **Jennifer Bender** of **State Street Global Advisors** (SSGA). But this concern got Bender and her colleagues **Xiaole Sun** and **Taie Wang** to think about what really makes a good smart beta portfolio.

The simple answer is its ability to provide a strong and consistent level of exposure to a desired factor or set of factors. "And that exposure should be achieved in a risk-aware way," says Sun. Risk taken relative to the capitalization-weighted benchmark is active risk and investors should be compensated with greater exposure to their targeted risk factors. Because screening and weighting decisions move tracking error versus the cap-weighted benchmark and exposure in the same direction, most rules-based indices naturally provide reasonably strong exposure per unit of tracking error—up to a point. Bender and her colleagues provide guidance on how indices can be built with this measure in mind.

Practical Applications

- **Factor exposure per unit of tracking error is an excellent way to measure a smart beta strategy.** Screening and weighting decisions at the security level move factor exposure and tracking error levels in the same direction. Methodologies with more complexity may result in portfolios that do not capture the desired factor efficiently.
- **Portfolios across an array of concentration levels can deliver high exposure per unit of factor exposure.** Portfolios utilizing a wide range of screening and weighting decisions have surprisingly consistent factor exposures per unit of tracking error.
- **Investors should use this metric to evaluate their smart beta portfolios.** Portfolios should be evaluated based on their ability to deliver a level of factor exposure commensurate with the level of tracking error taken.

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"There are more than a dozen value indices from different providers, and I have sympathy for asset owners and investors who have to differentiate among them."

—Jenn Bender

Practical Applications Report

Is the *factor exposure* commensurate with the level of tracking error taken?

Since 2008, there has been a virtual explosion of smart beta methodologies and products. This has led a lot of asset owners and institutional investors to ask themselves how best to differentiate across all of them. "Initially, we thought that if you're getting exposure to the desired factor, that's probably all you need," says Sun. The early indices used straightforward rules to tilt a portfolio toward a desired factor through security screening and weighting.

SSGA developed its own tilted framework in the mid-2000s, designed for broad universes and low tracking error portfolios. Early adopters of smart beta, back in the mid-2000s, were comfortable with these types of simple, rules-based methods. For a value factor index, for example, stocks are ranked by their value characteristics and larger multipliers are applied to those at the top. Over recent years, however, providers brought to market more-concentrated indices with higher tracking error, and methodologies became increasingly complicated. "We realized that some of these indices aren't really capturing the intended factors in the most efficient manner," she says. That led Bender and her colleagues to explore exactly what happens to factor exposure at various levels of screening and weighting.

FACTOR EXPOSURE PER UNIT OF TRACKING ERROR

"There are several ways to construct a factor portfolio," says Wang. For example, some can be very high conviction with just a concentrated number of those stocks that exhibit the factor characteristics. Another methodology might construct a portfolio in a more diversified, less aggressive way. "One isn't inherently better than the other, but in order to evaluate them, we used the idea of factor exposure per unit of tracking error," she says. Within the tilted framework, SSGA ranks stocks by their factor scores and allocates them to 20 subportfolios with a market-cap weighting. The subportfolios range from stocks with the highest factor scores to the lowest, and each represents 5% of the cap-weighted benchmark. Then they apply a multiplier to each subportfolio. The subportfolio with the higher ranking factor score receives a multiplier of two or three to boost its weight higher than that of market cap. The subportfolio with the lower ranking factor scores receives a multiplier of around 0.2, lightening its weight versus market cap. "We developed this methodology because it's flexible—you can adjust the magnitude of the tilt by the size of the multiplier, and you can adjust screening by adding or subtracting subportfolios from the final portfolio," Wang says.

"We developed this methodology because it's flexible—you can adjust the magnitude of the tilt by the size of the multiplier, and you can adjust screening by adding or subtracting subportfolios from the final portfolio,"

—Taie Wang

Their research found that tracking error and factor exposure rises with the exclusion of lower-ranked subportfolios. "By doing so, you're building a concentrated portfolio that offers higher factor exposure," says Wang. "However, the relationship between tracking error and exposure is close to monotonic." At a certain point, it becomes stable, meaning that excluding additional subportfolios does not affect the factor exposure per unit of tracking error. Changing the multiplier produced similar results. By using a higher multiplier, or more aggressive tilting, factor exposure rises, as does tracking error or active risk. But again, after a certain point, no matter how aggressively the portfolio is tilted, the factor exposure per unit of tracking error remains stable. An important caveat is that for portfolios that are very concentrated,

> “Our big takeaway is that asset owners should be more skeptical of some of the really complicated methodologies that are coming out today.”
>
> **—Jenn Bender**

you end up with large positions in certain countries, sectors or individual stocks. These attributes may not necessarily be desired even though you may be getting high factor exposure per unit of tracking error.

“What’s interesting about our finding is that if you’re just using simple screening and weighting rules, you will get reasonably high exposure and high exposure per unit of tracking error in the way that you expect,” says Bender. Screening out more stocks and increasing weights of those with high factor scores will increase the level of exposure to a desired factor. That’s good news for institutional investors, because most of them are still using frameworks based on screening and weighting. “Our big takeaway is that asset owners should be more skeptical of some of the really complicated methodologies that are coming out today,” she says. Overly engineered methodologies in smart beta indices could actually reduce the factor exposure per unit of tracking error metric that can define a strategy’s success.

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Jenn is Managing Director at **State Street Global Advisors** and Director of Research for its Global Equity Beta Solutions Group. She is responsible for developing innovative research and promoting SSGA's thought leadership across key areas of passive investing, including smart beta, thematic investing and evolutions in global equity investing. Jenn and her colleagues develop proprietary investment strategies, particularly in the area of smart beta, and custom equity solutions for SSGA clients.

Previously, Jenn spent eight years as a vice president on the index and analytics research teams at MSCI. She was responsible for research on equity index–related topics, including asset allocation and index fund management. Prior to that, Jenn worked in the Barra Research group, focusing on portfolio construction and risk modeling. She worked previously at State Street Associates and began her career as an economist at DRI.

Jenn holds a PhD and an MS in economics from Brandeis University. Her work has been published extensively in peer-reviewed journals and compendiums.

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Xiaole is a Principal at **State Street Global Advisors** and a Research Analyst in its Global Equity Beta Solutions group. She is responsible for the development of smart beta strategies in collaboration with her colleagues.

Previously, she was a research analyst at SSGA's Advanced Research Center, responsible for developing and enhancing active equity models for global and North American markets. Before joining SSGA, Xiaole conducted a professional development program rotation with State Street Global Markets' U.S. equity research team, where she prepared research publications, customized research reports and performed back-test analyses.

Xiaole has an ABD in international economics and finance from Brandeis University, and she received an MA and BA in economics from Peking University.

Taie Wang

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Taie is a Vice President at **State Street Global Advisors** and the Deputy Head of Research for smart beta in its Global Equity Beta Solutions Group. Her primary responsibilities include developing SSGA's proprietary smart beta strategies and overseeing the construction and reconstitution of the strategies managed by the passive portfolio management team. Based in Hong Kong, she also coordinates smart beta queries from SSGA's Asia-region clients.

Prior to joining SSGA, Taie worked in the portfolio construction group at Acadian Asset Management, where she was responsible for portfolio optimization, rebalancing and trading. Prior to that, Taie was a senior bond analyst at Lewtan Technologies, focusing on research and modeling of mortgage-backed securities. Taie also worked for BanK of America Securities and Koch Supply and Trading, conducting research on option pricing models and the behavior of commodity volatility smiles.

Taie holds an ABD in finance, an MS in finance and an MS in mathematics from the University of Illinois at Urbana-Champaign. She also received a BS in industrial economics and a BA in English from Tianjin University. Taie earned the Chartered Financial Analyst designation and is a member of the CFA Institute and the Hong Kong Society of Financial Analysts.

Smart Beta or Smart Alpha?

Kenneth Lillelund Winther and Søren Resen Steenstrup

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Smart beta investment products seem to offer the best of all worlds, with better risk-adjusted returns than traditional market capitalization indexes and lower costs than active management. Keeping in mind the often-cited underperformance of the majority of mutual funds relative to market capitalization indexes, it seems logical to consider shifting some investments from actively managed funds to smart beta products. However, is this logic transferable to a situation in which an institutional investor mimics the smart beta universe and asks the manager to be active?

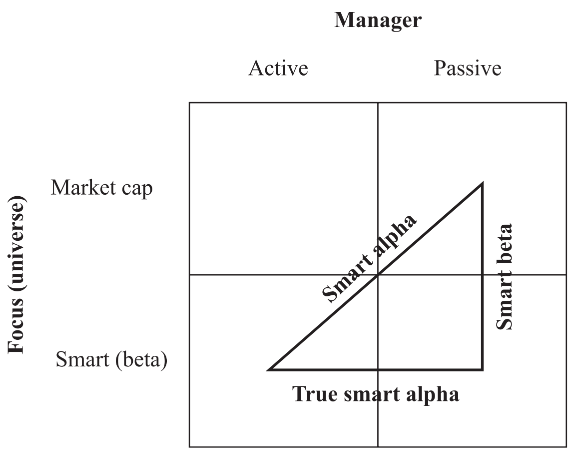
Exhibit 1 illustrates how an investment fund can focus on either a broad market capitalization-weighted or a smart (beta) universe while simultaneously seeking either an active or passive investment strategy. Investing passively in a market capitalization-weighted universe is equivalent to investing in a classic index fund, whereas *smart beta products* refer to a passive investment that targets rewarded risk premiums through other weighting schemes. By investing actively within the rewarded smart universe, it is possible to gain an excess return relative to the market capitalization index. We denote this excess return the *smart alpha*, as the return consists of both a smart beta and an alpha component. The alpha component is in excess of the smart beta return where the universe is similar to the opportunity space for the active manager, and we denote this excess return the *true smart alpha*.

To verify the existence of the smart alpha and true smart alpha, groups of actively managed funds, which each resemble a risk premium, were constructed and compared to both the traditional capitalization-weighted and smart beta benchmarks. The four most widely used risk premiums—value, size, low volatility, and momentum—were considered.

INTUITION BEHIND ACTIVE MANAGEMENT WITHIN RISK PREMIUMS

The academic literature has found that active management does not create value for the average retail investor (Jensen [1967], French [2008]), and although this is likely to be true, there is reason to believe that subsets of the industry generate value. One way to observe this is to separate retail money from institutional money because the latter is likely to be better managed, an assumption based on the fact that institutional investors are generally more thorough in analyzing the investment philosophy, process, and team behind the product when choosing a manager. Institutional investors also tend to evaluate historical returns and performance. Furthermore, institutional investors monitor the investment manager on an ongoing basis and will fire non-performing managers. This behavior

EXHIBIT 1
Smart Alpha Concept



is likely to result in the worst managed accounts not being offered to institutional investors or being terminated. Retail investors do not conduct due diligence but rather accept the recommendation of brokers, who often have incentives to recommend investments that are most profitable for themselves (Guercio and Reuter [2014]) instead of the best fund for the investor. Another important determinant is cost: The larger investment size of institutional investors makes the cost of investing lower, which is an important determinant for the probability of outperformance (Malkiel [1995]).

We see the motivation for active managers within the four smart beta universes as the following: The overarching theme of *value generation* for active managers is the *information premium*, that is, the value of finding or calculating information about investments by spending resources. Active managers can use knowledge that is relevant for an investment decision that smart beta products do not take into account—for example, with regard to investment flows and crowding of individual stocks or simply reacting to news relevant to a stock's fundamental value, whereas smart beta products have to wait until the next rebalancing time point. Within the *value premium*, the possibility of managers avoiding value traps is likely to be important. For the *size premium*, the relatively underanalyzed investment landscape enhances the probability of finding unexploited information. The *low volatility premium* is related to understanding the fundamental stability of a company rather than calculating the historical price swings, and the valuation overlay of low volatility or smaller companies also seems relevant. However, for *momentum funds* it is less clear to us why active managers should have an advantage over the smart beta benchmarks.

RISK PREMIUMS AND SMART BETA

Investment returns and risk are driven by systematic risk factors, some of which earn a positive premium to returns over time. Over the years, different risk premiums have been identified; the four most commonly mentioned are value, size, low volatility, and momentum (see, e.g., Haugen and Baker [1991]; Fama and French [1993]; Jegadeesh and Titman [1993]). These four will form the basis of our analysis. Other known premiums include *quality* and *high dividend yield*, which are omitted to avoid too much overlap between the risk premiums.

The next concept to address is *smart beta*, which collectively can be described as simple, transparent, and rule-based investment strategies that seek to provide exposure to risk premiums through weighting schemes that differ from the market capitalization. When choosing an appropriate benchmark, it is important that the benchmark universe resembles that of the active managers to provide a fair view of performance ability.

The MSCI World Factor indexes (seen in Exhibit 2) were used as benchmarks for each constructed risk premiums portfolio, as they may provide the best comparison. First, the indexes are constructed in a transparent manner, which we feel captures the underlying risk premium in a genuine way. Second, the parent index (i.e., MSCI World) is widely used by active managers as a benchmark, and the corresponding investment universe of many active managers is likely to resemble this index closely. Third, the indexes are constructed with a focus on investability, which increases the trustworthiness of their return histories.

EXHIBIT 2
Benchmarks

Portfolio	Benchmark
Market Capitalization	MSCI World
Value	MSCI World Value Weighted
Size	MSCI World Equal Weighted
Low Volatility	MSCI World Minimum Volatility
Momentum	MSCI World Momentum

The benchmarks have historically fared differently in terms of return over the time period, and it is important to note that all four of the individual smart beta benchmarks outperform the market capitalization-based benchmark in the sample period (see the Appendix, Exhibits A1 and A2). The outperformance relative to MSCI World is, however, not created gradually, and the value benchmark especially has exhibited a long streak of returns that do not outperform the benchmark.

DATA

Our data originated from the Morningstar Direct fund-database. This article distinguishes itself by using only funds from the separate account/collective investment trust (CIT) universe instead of only mutual funds or a combination of funds. This choice was made because there is reason to believe that institutional accounts are managed more efficiently than plain retail-only mutual funds. Monthly data from 1998M12 to 2013M12 were used, and closed funds were also included. Both gross and net returns were used in the analysis to capture both the gross value generation and value generation attributable to the investor (i.e., after cost). The U.S. three-month treasury yield was used as a proxy for the risk-free rate.

CONSTRUCTING THE ACTIVE RISK PREMIUM PORTFOLIOS

The analysis was conducted by constructing four portfolios, one each resembling the risk premiums value, size, low volatility, and momentum. (See the Appendix for a detailed description of the portfolio construction.) For the analysis, it was important that the constructed portfolios resemble the associated risk premiums (and smart beta benchmarks) as much as possible with regards to both the security universe and the manager strategy. The total number of funds was 1,880, distributed as follows: value (944), size (449), low volatility (220), and momentum (267).

The value portfolio was constructed by using the Morningstar Institutional Category Classification containing "value" in its name. Morningstar assigns the value category to funds via five variables (Morningstar [2008]): price-to-projected earnings, price-to-book, price-to-sales, price-to-cash flow, and dividend yield. These variables correspond well to the weighting variables of the MSCI Value Weighted index (MSCI [2013]). The size portfolio was constructed by including Morningstar Direct's midcap categories. The MSCI Equal Weighted index, as the name suggests, puts an equal weight on all the constituents of the parent index. The low volatility portfolio was constructed by using the bottom 20% of equity funds with regard to the standard deviation of three years of past returns for net returns. To maintain consistency in fund inclusion between the gross and net return dataset, the low volatility selection criteria were based on the net returns, and gross returns were subsequently found for the same funds. Although this approach does not ensure that the managers of the low volatility portfolio intend to construct a portfolio via their respective investment processes, there is a high degree of persistency within managers of the portfolio. The momentum portfolio was less direct in construction because there are no direct screening criteria to filter these funds; instead, the search criterion focused on funds with a "technical" investment process.

ANALYSIS

The analysis will focus on evaluating the funds in each of the portfolios, first relative to the common market capitalization-weighted index, MSCI World, and subsequently relative to their respective smart beta benchmarks. The individual funds within each of the portfolios have different lifetimes; for example, some funds existed at the start of the sample and were liquidated at a later point, whereas other funds were created late in the sample period. To make these comparable on an aggregate portfolio level, the individual funds were evaluated relative to the benchmark in their respective lifetimes.

To capture the chance of outperforming the benchmark at any point in time, the standard outperformance probability measure and the beta adjusted outperformance probability were used. Both measures can indicate the time needed for a fund to achieve outperformance, but the measures do not discriminate between the magnitudes of outperformance. The benchmarks were treated as costless in the analysis for simplicity's sake. Instead of assuming cost levels in the benchmark indexes, the fund evaluation considered both gross and net returns for the active funds. If the active funds do well when considering the net returns, this will speak in favor of the use of these kinds of active funds.

Evaluation of Smart Alpha

In Exhibit 3, the portfolio of active funds does well relative to the market capitalization-weighted index, MSCI World, with positive excess returns for all categories even after costs. The portfolios tend to have a lower standard deviation than the benchmark for all funds except the size portfolio, which seems consistent with the higher risk associated with investing in smaller companies. Lower risk is also reflected in the beta values, which are all less than one. The tracking errors appear moderate to high, resulting in positive information ratios and Sharpe ratios that are higher than the benchmark.

The outperformance probability in Exhibit 4 is above 50% and rising with a holding horizon, which is consistent with positive excess returns. Due to the different lifetimes of funds within each group and the potential positive relationship between fund performance and lifetime (i.e., "bad" funds are closed quickly), the slope of the outperformance probability curve should not be overstated. The trend is clearer when adjusting for beta in Exhibit 5.

The individual portfolios of active managers perform well relative to the traditional capitalization-weighted index via their ability to generate excess value with regard to return, even when risk-adjusting returns seems to be able to deliver value within relatively short time horizons.

Evaluation of True Smart Alpha

The same analysis was repeated, this time evaluating each portfolio relative to the respective smart beta index.

In Exhibit 6, excess returns, although still positive, are lower than those in the market capitalization analysis, except for the momentum category. The standard deviations are now generally higher, although the systematic risk, the beta, is still lower than one. The tracking errors are equal to or higher than in previous analysis, which has also been found by other studies, such as that by Brown, Harlow, and Zhang [2014]. Both the Sharpe and information ratios are lower than in previous analysis, but they are still higher than the benchmark and positive.

In Exhibit 7, the outperformance probabilities are still trending upward for longer holding horizons for gross returns, although the level is now lower. For net returns, the probabilities remain around 50% for size and low volatility, whereas momentum seems to underperform for horizons shorter than three years. The value portfolio still has historically been quite consistent at outperforming its smart beta index.

Another way to evaluate the performance is by using the Carhart four-factor model, which seeks to filter out the return generated from exposure to various systemic risk premiums. Interestingly, a clear majority of estimated alphas within each risk premiums portfolio are positive (Exhibit 8).

DISCUSSION

The preceding findings are relevant for several reasons. First, although the market, as a whole, will underperform as a logical consequence of costs, costs do not prevent individual investor segments from outperforming over time. The results here confirm that

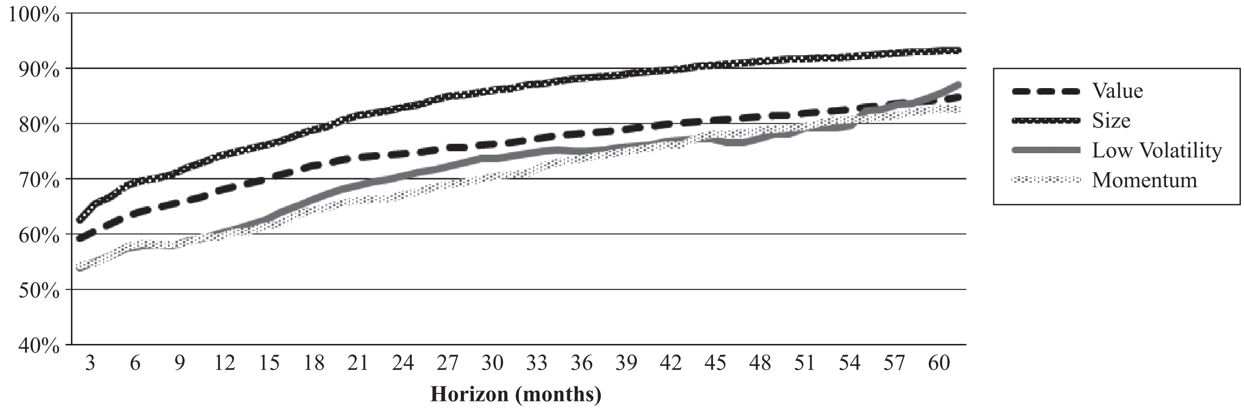
EXHIBIT 3
Smart Alpha (active funds vs. market capitalization index)

	Gross Returns				Net Returns			
Against MSCI World	**Value**	**Size**	**Low Vol**	**Momentum**	**Value**	**Size**	**Low Vol**	**Momentum**
Annualized Excess Return (Smart Alpha)	3.89%	5.93%	3.87%	3.01%	3.07%	5.02%	2.34%	2.11%
Annualized Std Deviation (Difference)	–0.09%	1.76%	–4.38%	–0.23%	–0.09%	1.76%	–4.38%	–0.23%
Sharpe Ratio (Difference)	0.16	0.23	0.30	0.12	0.10	0.17	0.18	0.06
Beta	0.91	0.98	0.66	0.87	0.91	0.98	0.67	0.87
Annualized Tracking Error	7.05%	8.58%	8.33%	7.86%	7.06%	8.56%	7.99%	7.91%
Information Ratio	0.58	0.69	0.45	0.40	0.46	0.58	0.30	0.27

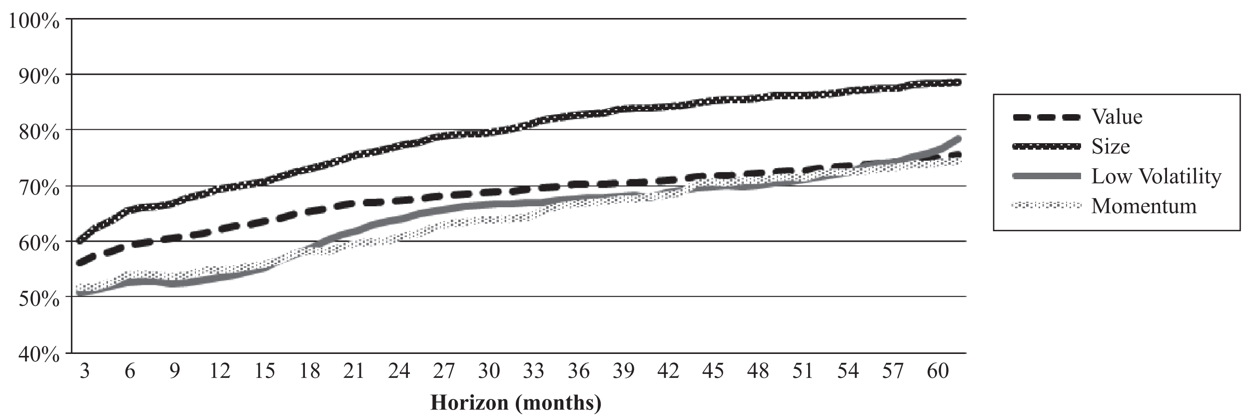
Note: Average values are shown for each group of funds.
Sources: Morningstar, MSCI, and Bloomberg.

Exhibit 4
Smart Alpha Strategies' Outperformance Probability

Panel A: Outperformance Probability vs. MSCI World (gross returns)



Panel B: Outperformance Probability vs. MSCI World (net returns)



Notes: Outperformance probability for each horizon was calculated by summing the number of times a fund's trailing return was higher than the benchmark in a given fund's lifetime. Outperformance = $r_{fund} - r_{benchmark}$.
Sources: Morningstar, MSCI, and Bloomberg.

putting effort into finding the right fund manager is rewarded, even after costs. The inherited benefits of size and a focus on finding the right managers, which institutional investors often possess, make them able to reap a better investment outcome. Choosing active managers who target a specific risk premium can further improve investment outcomes by focusing on an efficient part of the investment space (i.e., the rewarded risk premiums) and aiming at generating alpha on top of this (i.e., the true smart alpha).

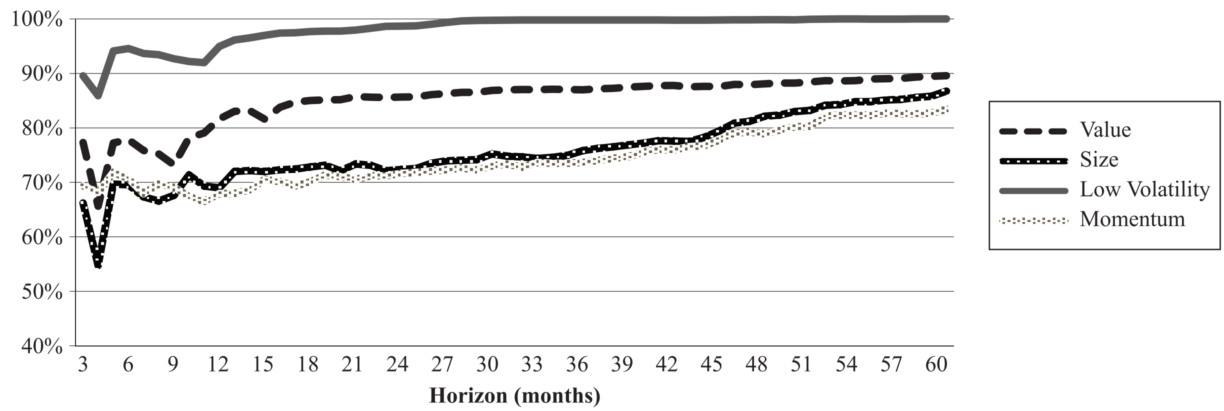
Furthermore, instead of focusing solely on whether actively managed or smart beta products are strictly better than the other, it is important to bear in mind that these products can serve different roles in a portfolio. Alpha is not generated evenly, but is increasingly likely to be unlocked as the investment horizon increases. Actively managed funds therefore seem to be a strong option for long investment horizons, whereas smart beta strategies could have more relevance on shorter horizons and seem more appropriate for dynamic allocation of factors (Winther and Steenstrup [2015]).

Finally, an aspect to consider is the different nature of the two types of products. Whereas actively managed funds often are managed by teams that make discretionary investment decisions, smart beta products are

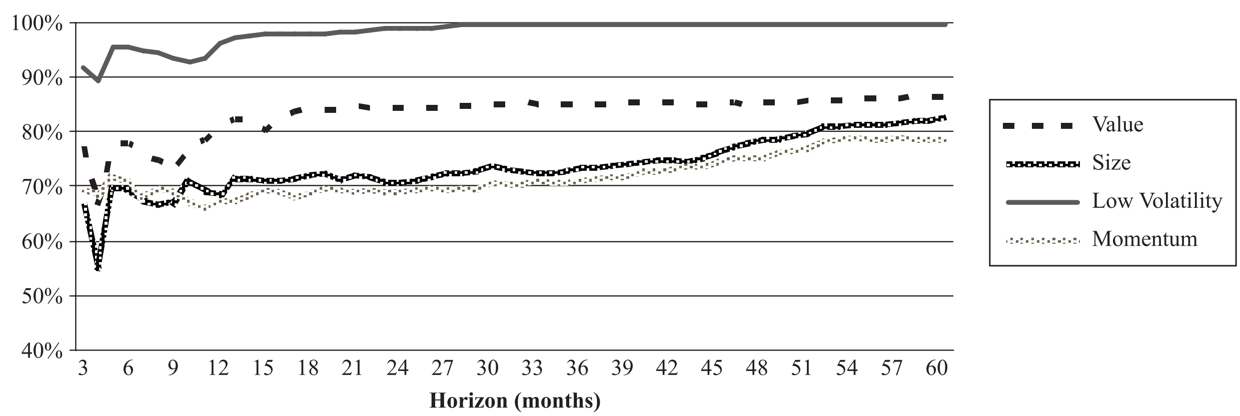
EXHIBIT 5

Smart Alpha Strategies' Beta Adjusted Outperformance Probability

Panel A: Outperformance Probability vs. MSCI World (gross returns)



Panel B: Outperformance Probability vs. MSCI World (net returns)



Notes: Outperformance probability for each horizon was calculated by summing the number of times a fund's trailing return was higher than the benchmark in a given fund's lifetime. Beta adjusted outperformance = $r_{fund} - \beta * r_{benchmark}$.

Sources: Morningstar, MSCI, and Bloomberg.

EXHIBIT 6

True Smart Alpha (active funds vs. smart beta)

	Gross Returns				Net Returns			
Against Smart Beta Benchmarks	**Value**	**Size**	**Low Vol**	**Momentum**	**Value**	**Size**	**Low Vol**	**Momentum**
Annualized Excess Return (true smart alpha)	2.57%	2.16%	1.96%	–0.02%	1.77%	1.31%	0.78%	–1.00%
Annualized Std Deviation (difference)	0.32%	0.18%	0.77%	0.20%	0.32%	0.18%	0.77%	0.20%
Sharpe Ratio (difference)	0.17	0.12	0.17	0.01	0.12	0.07	0.03	–0.06
Beta	0.93	0.89	0.90	0.81	0.93	0.89	0.93	0.81
Annualized Tracking Error	7.27%	8.96%	6.80%	10.35%	7.26%	8.94%	6.56%	10.34%
Information Ratio	0.37	0.24	0.30	0.00	0.26	0.14	0.12	–0.09

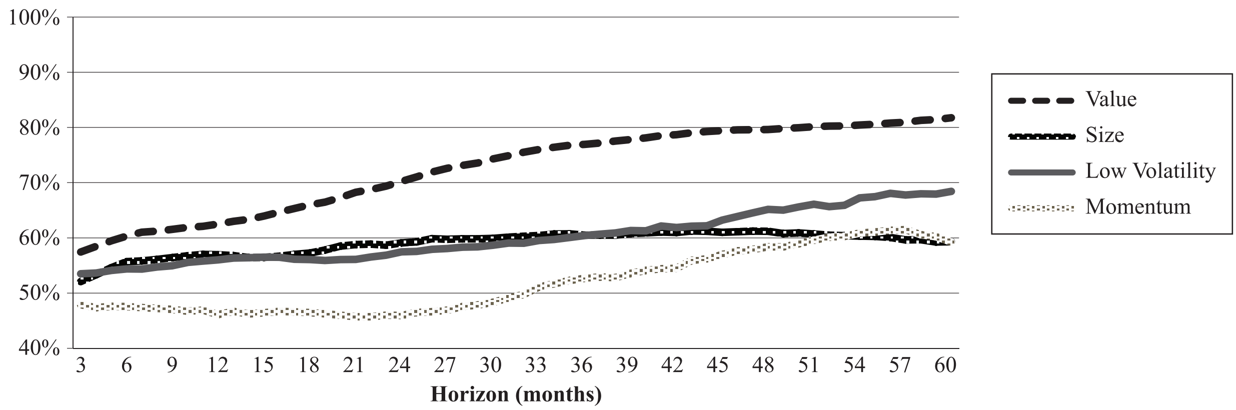
Note: Average values are shown for each group of funds.

Sources: Morningstar, MSCI, and Bloomberg.

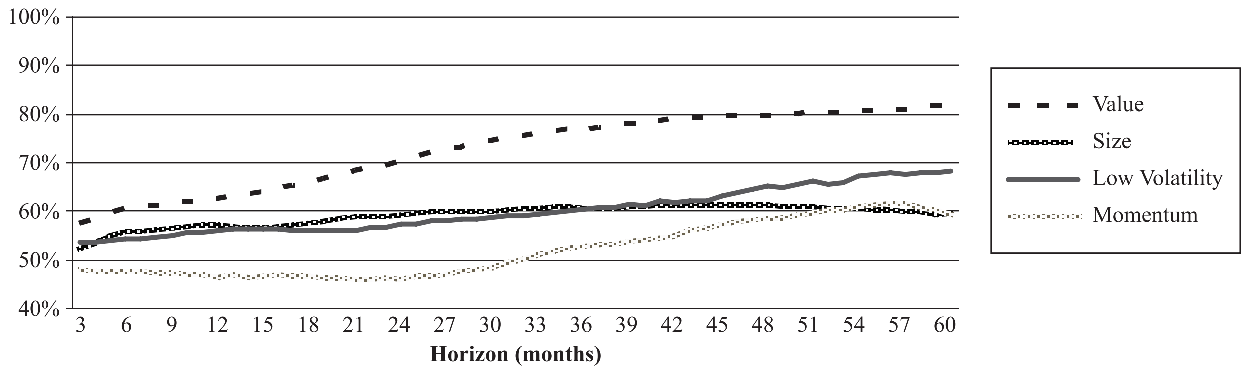
Exhibit 7

True Smart Alpha Strategies' Outperformance Probability

Panel A: Outperformance Probability vs. Smart Beta Benchmark (gross returns)



Panel B: Outperformance Probability vs. Smart Beta Benchmark (net returns)



Notes: Outperformance probability for each horizon was calculated by summing the number of times a fund's trailing return was higher than the benchmark in a given fund's lifetime. Outperformance = $r_{fund} - r_{benchmark}$.

Sources: Morningstar, MSCI, and Bloomberg.

Exhibit 8

Estimated Carhart Four-Factor Alphas for Each Risk Premium's Portfolio

	Gross Returns				Net Returns			
Against Carhart 4-Factor Model	**Value**	**Size**	**Low Vol**	**Momentum**	**Value**	**Size**	**Low Vol**	**Momentum**
Alpha > 0	93.18%	93.82%	96.30%	83.33%	86.83%	89.04%	93.18%	77.91%
Above 90% Significance Level	64.39%	63.84%	77.31%	46.90%	49.14%	50.46%	59.09%	30.62%
Above 95% Significance Level	54.87%	54.46%	72.22%	34.11%	37.37%	39.27%	50.91%	20.16%
Above 99% Significance Level	33.01%	32.72%	51.85%	15.89%	20.34%	20.55%	29.55%	7.75%

Notes: Values provide the percentage of funds with positive estimated alphas and different statistical significance. Significance level is derived from regression of Carhart four-factor model:

$R_{fund} = r_f + b_1 * (R_m - r_f) + b_2 * HML + b_3 * SMB + b_4 * MOM + \alpha$,

where R_{fund} *is the return for the individual fund;* r_f *is the risk-free rate;* $b_1 \ldots n$ *are factor loadings;* R_m *is the market return measured by the MSCI World Index; HML, SMB, and MOM are the Carhart–Fama–French factors; and* α *is the alpha.*

Sources: Kenneth French Data Library, Morningstar, MSCI, and Bloomberg.

often governed by strict rule-based investment decisions. Each of these carries its own benefits and drawbacks. For smart beta products, greater transparency and lower personnel risk are obvious benefits, but processes may fail to capture other important investment information that actively managed funds are able to find. As the smart beta investment funds grow in size, the information premium is likely to become even more relevant, and index arbitragers can extract return from the smart beta by frontrunning and thereby diminish the returns from the easy and transparent investment strategies.

CONCLUSION

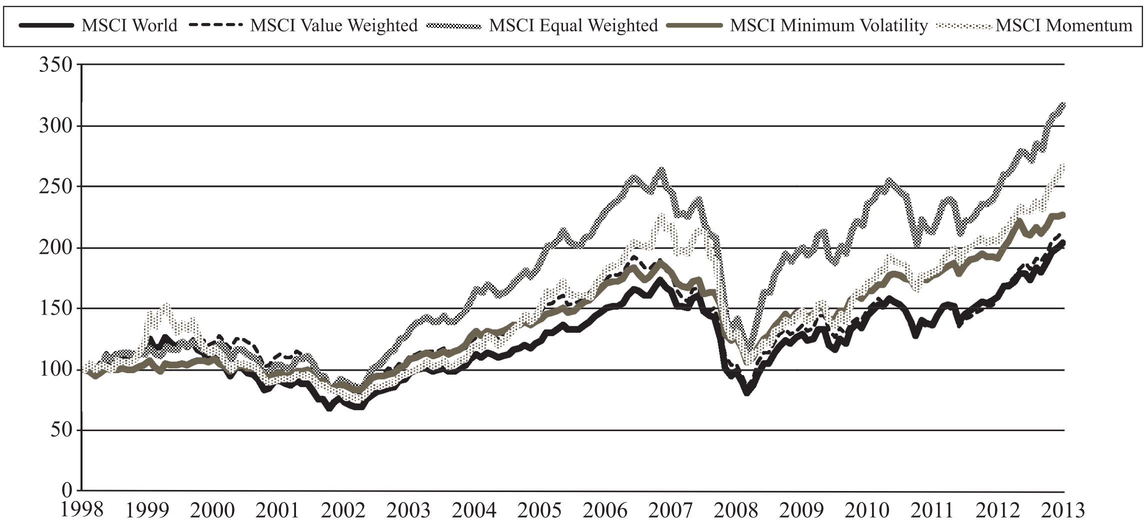
This article shows that actively managed funds that target known rewarded risk premiums have historically done well, not only against a traditional market capitalization-weighted benchmark, but also against their respective smart beta benchmarks. Using MSCI Factor indexes and separate account/CIT data from Morningstar as a proxy for institutional investors' actual equity investments, four groups of funds that each mimic a risk premium were formed. The four groups of funds outperformed the market capitalization-weighted benchmark, MSCI World, both before and after costs. The risk relative to benchmark is generally lower, which implies positive risk-adjusted performance. The outperformance probability is high and rises still higher with a longer investment horizon. When evaluating the true smart alpha, namely against the respective smart beta benchmarks, excess return still exists, and only the group of momentum funds struggle. The outperformance probability here hovers around 50% for all groups except value, but when adjusting for the lower level of systematic risk in the portfolios, the outperformance probability is high. The performance of the institutional actively managed investment funds makes the case less clear for indiscriminately switching to smart beta products and away from actively managed funds. Rather, an investment portfolio can benefit from using active managers with a smart beta benchmark on strategic equity exposure while smart beta investing on shorter time horizons, that is, from searching for the true smart alpha on the long term.

Appendix

BENCHMARKS AND DEVELOPMENT

Exhibit A1

Index Development of the Benchmarks

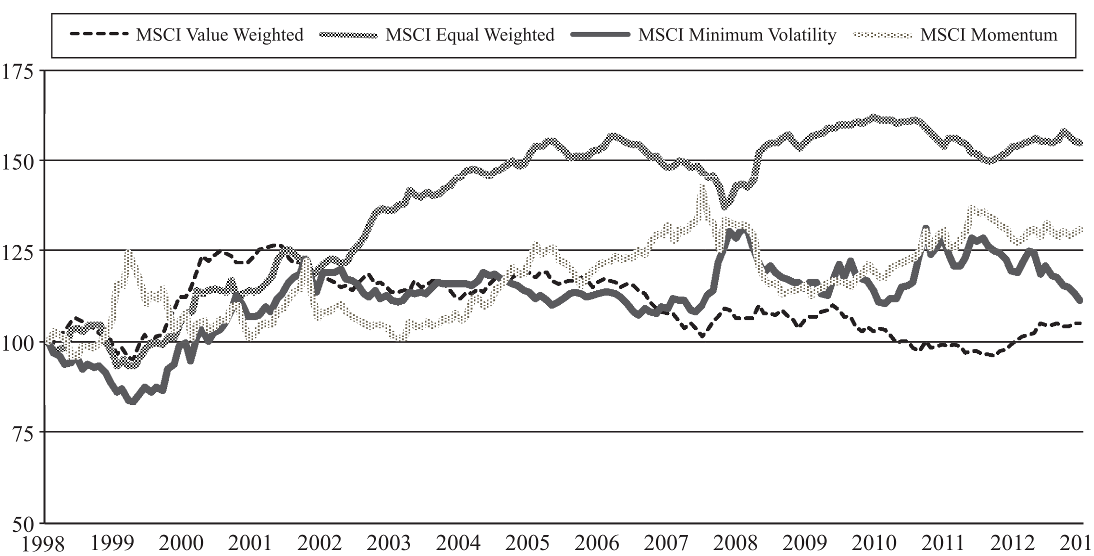


Note: 1998M12 = 100.

Sources: MSCI and Bloomberg.

Exhibit A2

Smart Beta Index Development Relative to MSCI World



Notes: 1998M12 = 100. The index expresses the development of the smart beta indexes relative to MSCI World. An upward movement implies outperformance relative to MSCI World.

Sources: MSCI and Bloomberg.

CONSTRUCTING THE DATASETS

All data came from the Morningstar Direct database. The separate account/CIT database was used in order to only consider institutional funds. All portfolios were constructed by unticking "Only Surviving Investments" to mitigate potential survivorship bias. Lastly, all portfolios were found by setting "Management Approach—Active" equal to true in order to remove passive and enhanced funds from the sample. Returns were calculated on a monthly frequency.

The *value* portfolio was created by including the following Morningstar Institutional Categories: "All-Cap Value," "Foreign Large Value," "Giant Value," "Large Cap Core Value," "Large Deep Value," "Mid Core Value," "Mid Deep Value," and "World Large Value."

The *size* portfolio was created by including the following Morningstar Institutional Categories: "Mid Core," "Mid Core Growth," "Mid Core Value," "Mid Core Deep Value," "Mid-Relative Value," "Mid-Valuation Sensitive Growth," and "World Mid Cap."

The *low volatility* portfolio was created by including all available equity funds under the "Global Broad Category Group" while excluding small cap categories. The funds were then sorted according to their three-year trailing return standard deviation. The funds with a standard deviation lower than or equal to the second decile were then included in the portfolio if more than 60% of their observations fell into the two bottom deciles. The portfolio was rebalanced monthly.

The *momentum* portfolio was created by including all available equity funds under the "Global Broad Category Group" equal to Equity and setting "Inv Analysis—Technical" equal to true.

ENDNOTE

We would like to thank Martin Dencker for helpful comments and suggestions for improvements of the article.

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Practical Applications of

Smart Beta or Smart Alpha?

Authors: Kenneth Lillelund Winther and Søren Resen Steenstrup
Source: *The Journal of Investing,* Vol. 25, No. 1.
Report Written By: Howard Moore
Keywords: Active Management, Active–Passive Debate, Factor Investing, Passive Investing, Smart Beta, Tryg

Overview

Investors have embraced smart beta, with its passive implementation, low fees, transparency and potentially better risk-adjusted returns relative to both traditional capitalization-weighted benchmarks and actively managed funds. As a result, many asset owners question the usefulness of active managers, and some are using smart beta strategies as a replacement. However, **Kenneth Winther of Tryg** in Denmark says, "We believe that many are coming to a conclusion too quickly regarding the effectiveness of active managers."

As part of a broader research effort to determine when active management and passive management each work best, published in ***Smart Beta or Smart Alpha?*** in the Spring 2016 issue of ***The Journal of Investing***, Winther and his co-author and colleague **Søren Steenstrup** constructed actively managed institutional equity portfolios designed to capture the most widely used risk premia in smart beta strategies: value, size, low volatility and momentum. They compare the returns and other performance measures to broad indices tilted toward those same factors. With the exception of the momentum portfolio, the actively managed portfolios generated superior absolute and risk-adjusted return measures not only to the common market-cap weighted indices but also to their smart beta factor-based benchmarks, even after costs. They call this smart alpha.

THINKING
of replacing your active managers with smart beta strategies?

CONSIDER
smart alpha instead.

Practical Applications

- **Active management works and institutional investors should consider smart alpha.** Actively managed institutional portfolios that seek to capture known risk premia can result in absolute and risk-adjusted returns that are superior to passively managed portfolios.
- **The exception is the momentum risk factor.** The actively managed momentum portfolio did not outperform the passive version, perhaps because of the available data.
- **Use smart beta to benchmark active managers.** Active managers that invest in factors should be evaluated and monitored against a corresponding smart beta index, not a passive, cap-weighted benchmark. Fees should be set according to true smart alpha contribution.

Practical Applications Report

Passive smart beta investment products seem to offer the best of all worlds, with better risk-adjusted returns than traditional cap-weighted indices and lower costs than active management. Furthermore, research has shown that the majority of actively managed mutual funds underperform cap-weighted benchmarks. Therefore, it seems logical to consider shifting some investments from actively managed funds to smart beta products. But what happens when an active manager mimics the smart beta universe?

> "You need to take specific active managers that target one risk premium and then see where they outperform the corresponding smart beta index."
>
> —Kenneth Winther

"We believe in simple, passive implementation of smart beta, because it has been documented in the academic world for decades," says Winther. However, Winther and Steenstrup had a harder time accepting that smart beta performs better than active management simply because active managers in aggregate have been shown to underperform market-cap indices. "Much of that research is focused on US retail investors, not institutional investors, so we didn't think it was representative," says Steenstrup. They believed intuitively that an active manager could add value to a factor investing strategy because a good one knows how to exploit inefficiencies in a given market. "Also, simple, rules-based implementation of a smart beta strategy makes it a little too simple to be the best possible investment style," says Winther.

SMART ALPHA ADDS VALUE

They chose the term "smart alpha" to denote that targeting risk factors through active management can draw on a number of advantages that a rules-based, transparent implementation cannot. "Smart beta's simplicity and transparency provide an opportunity to beat it because you know exactly what the smart beta is going to do," says Winther. Smart alpha uses additional information that can deliver extra value. For example, a successful value manager knows how to avoid value traps—stocks that are cheap for a reason—and a rules-based approach doesn't apply that judgment. "We know that skilled active value managers that consistently beat the benchmark exist," he says. In a low volatility strategy, the smart beta approach uses only the standard deviation over the last three years, but an active manager can better assess a company's true risk. "For example, an active manager knows if it has a sustainable business model or how changes in oil prices will affect their earnings," he says. Factors also can become crowded, driving up valuations, another situation that an active manager can avoid.

> An active manager can IDENTIFY and AVOID crowded factors.

Data for their factor portfolios came from the separate account/collective investment trust universe in the Morningstar Direct fund database. The momentum portfolio was less direct in construction because there are no direct screening criteria to filter these funds; instead, the search criteria focused on funds with a technical investment process. As a result, the actively managed momentum portfolio did not outperform the smart beta version. "One explanation for why it does poorly is because it's not really targeting the risk premium in as genuine a way as the others," says Steenstrup. "In addition, we couldn't see an obvious way that an active manager could construct a momentum portfolio any better than a rules-based, smart beta approach," he says.

For benchmarking, Winther and Steenstrup found they needed to measure active managers' performance directly against the comparable smart beta risk factor instead of a passive, cap-weighted index. “You need to take specific active managers that target one risk premium and then see where they outperform the corresponding smart beta index,” says Winther.

Institutional investors that are investing in smart beta—or considering it—should pay attention to Winther and Steenstrup’s research. “Our research is equally important to asset owners and the asset management industry, because the smart alpha concept is not as well known,” says Winther. “There is a lot of potential for the asset management industry to create actively managed funds using this smart alpha concept.”

Winther and Steenstrup have worked together for nine years, first in equity research and portfolio management, and for the last six and a half years at Tryg, Denmark’s largest nonlife insurer.

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How Different Are Alternative Beta Strategies?

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Alternative beta equity strategies give investors access to equity markets via non-capitalization weighted portfolios, offering reduced risk and/or enhanced investment return. These strategies are fully systematic and are embedded in indexes that can be passively replicated in transparent investment products, such as ETFs (exchange-traded funds). As such, alternative beta is also a field of intense industry and academic research. Many articles run exhaustive analyses of different alternative beta approaches (Hsu [2006]; Arnott et al. [2010]; Demey, Maillard, and Roncalli [2010]; Chow et al. [2011]; Lee [2011]; Monnier and Rulik [2012]; Clare, Thomas, and Motson [2015]; and Philips et al. [2015]).

The literature highlights that the bulk of long-term outperformance of alternative beta strategies with respect to the capitalization-weighted benchmark is related to their exposure to known risk factors such as size and value. Extra return might also come from periodic rebalancing of the strategy portfolios (Plyakha, Uppal, and Vilkov. [2012]), although this rebalancing bonus is not guaranteed (Rulik [2013a]). Another source of the residual alpha not explained by a static regression model can be the time-varying factor exposures (De Franco, Monnier, and Rulik [2013]).

Although individual alternative beta strategies are relatively well researched and increasingly understood by investors (Carhart [1997]; Fernholz [1999]; Clarke, De Silva, and Thorley [2006]; Choueifaty and Coignard [2008]; Demey, Maillard, and Roncalli [2010]; Monnier and Rulik [2011]; and Amenc et al. [2011]), choosing among different strategies and combining multiple alternative beta within a portfolio still represents a challenge. Alternative beta strategies has been shown to deliver a robust risk-adjusted benefit with respect to the market portfolio; however, no strategy has been shown to dominate its peers in all market conditions. As different alternative beta strategies follow different investment objectives, they are expected to have different statistical properties, such as risk, drawdowns, or exposure to equity market risk factors. Several providers of alternative beta solutions recently have begun promoting multi-strategy or multi-factor allocations, arguing that these are less risky, have higher investment capacity than individual strategies (Bender et al. [2013]), and allow harvesting factor premiums while time diversifying model risk present in the individual strategies (Amenc et al. [2015]).

Nevertheless, to our knowledge there is no formal framework for combining those strategies, and most of the multi-strategy solutions are driven by rules of thumb and common sense arguments. This leaves many important questions unanswered. Do investors have to include all alternative betas available on the market, or just some of them? How should we weight different alternative beta strategies within a multi-strategy portfolio? Should the allocation be static or dynamic, adjusting the strategy according to market conditions?

In this study we propose a quantitative method to approach these questions, based on statistical properties of strategy returns. We collect returns of 12 different portfolios, representing well-known strategy and factor approaches applied to large-capitalization U.S. stocks. After presenting descriptive statistics for each strategy's returns, we measure commonality among their returns through a series of statistical tests, including historical unconditional and tail correlations, principal component analysis (PCA), factor regressions, and minimum spanning tree graphs. The tests present sufficient evidence of redundancy in the set of alternative beta portfolios. In some cases the returns' similarity is in line with our expectations; this is the case, for example, for market capitalization weighting and diversity weighting. However, in other cases, the tests uncover that some strategies are statistically very similar despite following different weighting approaches (as is the case for equal-weighting and risk-efficient approaches).

We also find that there can be noticeable statistical differences among alternative beta portfolios pursuing similar investment objectives. For example, fundamental weighting and a basic version of value-weighting (based solely on book-to-market ratio) portfolios, both often associated with the same value investing factor, are quite distant in terms of realized long-term returns and risk allocations. The same is true for low-volatility and minimum-variance portfolios: Despite being perceived as interchangeable versions of low-volatility investing, they can be (relatively) distant in terms of correlations.

In order to give a structure to this statistical similarity, we construct two measures of distance between the strategies, one based on historical correlations and another based on systematic risk exposures. This allows us to apply a formal clustering technique to group different strategy portfolios into groups (clusters) in such a way that portfolios within one group are as similar as possible, while the groups themselves are as distant as possible. We use a method based on traditional K-means clustering technique with an additional term introduced to penalize the number of clusters.

The main findings related to this clustering exercise are the following: Consistent with the evidence collected from the correlation tests, we observe three persistent clusters centered around the market-cap portfolio, the equal-weight portfolio, and the low-volatility portfolios. The exact optimal number of clusters depends on the parsimony parameter chosen that roughly represents the distance at which investors look at alternative beta portfolios. For low values of the parsimony parameter even small differences are perceived as being important, while higher values of the parameter make the observer more tolerant to the differences among the strategies, and allow reducing the dimension of the cluster set. As the parsimony parameter increases, the clusters around the three previously mentioned centers begin to form and to grow bigger, merging eventually into a unique cluster, representing alternative beta from the point of view of an extremely distant observer.

We present several tests of multi-strategy allocations based on these statistical clusters rather than on individual strategy portfolios, with the clusters being built on the distance measure derived from pairwise correlations among the strategies. The first test represents an equal-weighted allocation constructed first from individual strategies and then from two different sets of strategy clusters. Cluster-based allocations exhibit better absolute and risk-adjusted returns, as well as a reduction in the maximal drawdown. One would logically attribute these gains to better diversification in the cluster-based allocations, as allocation building blocks have less redundancy. The complete picture is more nuanced though, as it appears that cluster-based allocations can have large exposures to individual strategies or factors that represent outliers and remain in separate clusters even for high values of the parsimony parameter (i.e., a small required number of clusters). Thus, such outliers can be significantly overweighted in a cluster-based allocation.

We then make a second test, using individual strategies and clusters in dynamic momentum-based allocations. We use past six-month returns to measure momentum and build multi-strategy portfolios that over-weight past winners and under-weight past losers. We show that cluster-based allocations have better performance and lower risk for this dynamic setup.

This article is organized as follows. After a short review of different alternative beta approaches, we evaluate individual statistical properties of several alternative beta portfolios, as well as their collective behavior. We then form clusters of strategies using two different definitions of statistical distance. Finally, we provide a simple application of our results, which proves how going through dependence structure analysis and considering clusters instead of individual strategies carries several advantages for investors who want to diversify their alternative beta strategies allocation.

ALTERNATIVE BETA STRATEGIES

This section is dedicated to a synthetic presentation of the alternative beta strategies used in this study. We provide standard definitions, the rationale behind the strategies, and their main properties. They are also contextualized with references to academic literature. In what follows, $\boldsymbol{w}$ denotes a column vector, $\boldsymbol{w}'$ its transposition, and Σ is a square matrix.

Equal Weighting

The equal-weight approach is without doubt the most natural way to build a portfolio. The idea of allocating the same amount to each asset is intuitive: The portfolio results in spreading the investment bets evenly across the investment universe so that the portfolio's return is an arithmetic average of all stock returns in the portfolio. Due to market movements, the portfolio is periodically rebalanced to restore equal weighting $w_i = 1/N$. The rationale for the equal-weighted portfolio is the search for maximal diversification in the absence of reliable information on the stocks' future risks and returns.

The strategy mitigates the large capitalization concentration bias of the capitalization-weighted portfolios and avoids trend-following behavior because the portfolio is periodically rebalanced. As far as risk is concerned, the equal-weighted portfolio is quite similar to the capitalization-weighted one. Equal weighting brings rather small changes to portfolio long-term volatility with respect to its market capitalization counterpart, and usually remains highly correlated to it. One of the main performance drivers of the equal-weight strategy is the correction of the "mega-cap" bias and the overweighting of small stocks, bringing in the premium attached to them, as was shown for the STOXX 600 Equal Weight Index over its market-cap counterpart over the 2003–2010 period (Monnier and Rulik [2011]).

Minimum Variance

Minimum variance is a portfolio construction technique that aims at selecting and weighting stocks, within a given investment universe, in order to minimize the total portfolio variance. The key input of this method is the covariance matrix of stock returns. As such, the problem does not require return forecasts that make the minimum-variance portfolio a purely risk-based solution. The weights, in the simplest case, are computed by solving the following optimization problem:

$$MV: \begin{cases} \min_w \ \boldsymbol{w}' \Sigma \boldsymbol{w} \ \ s.t. \\ \sum_i w_i = 1 \ \ and \ \ w_i \in [0,1] \end{cases} \tag{1}$$

where Σ is the covariance matrix. The minimum variance portfolio is optimal (in the mean–variance sense) under an assumption that returns are equal for all stocks. Knowing the difficulties when it comes to forecasting stock returns, this assumption does not seem too restrictive at all. Furthermore, according to Chopra and Ziemba [1993], the negative effects of the return forecasting errors are one of the main reasons for the disappointing performance of portfolios constructed with mean–variance optimization. As such, in the minimum-variance problem the weights are not affected by poor returns estimations. Although errors could also come from the estimation of the covariance matrix, these distortions would be much smaller compared to those coming from poor return forecasts.

The presence of estimation errors tends to give very concentrated portfolios. To solve this specific issue, it is very common to introduce constraints limiting the portfolio exposures to guarantee a desired level of diversification. (Such constraints might include maximal weight per stock, industrial sector, or country, or constraints neutralizing exposure to unwanted risk factors, limiting turnover, and targeting any quantitative measure of diversification). For a review of the impact of constraints in the minimum-variance portfolios, we refer to Rulik [2013b] and De Franco and Monnier [2014].

Behind the popularity of minimum-variance strategy and low-volatility strategy described later in this section, there is strong empirical evidence that, contrary to the conclusions of CAPM, minimum-variance strategies—mainly exposed to low volatility stocks—have been outperforming (risk-adjusted, or even in absolute terms) their capitalization-weighted benchmarks. This market inefficiency, which goes by the name of the "low-volatility anomaly," was discussed by Haugen and Heins [1975]; Haugen and Baker [1991]; Ang et al. [2006]; and Haugen and Baker [2010].

Risk Parity

Risk parity is a portfolio that weights constituents inversely proportional to their volatilities ($w_i \sim 1/\sigma_i$). The idea behind this weighting scheme is quite intuitive: every portfolio's assets are assigned a risk budget in terms of volatility, so that the components (under assumption of zero correlations) contribute equally to the total portfolio variance. As such, this allocation scheme is favored by investors comfortable with risk budgeting rules. Alternatively, the approach can also be modified to ensure parity among volatility of industrial sectors rather than individual stocks. Risk parity allocation requires as an input only the estimations of stock volatilities, and the problem is not distorted by return forecasting errors, similar to the minimum variance case. For more information on this weighting scheme, as well as its generalization, called equal risk contribution, we refer the reader to (Roncalli [2013, Ch. 2]).

The volatility of the risk parity portfolio is in between the volatilities of the equal-weight and the minimum variance portfolios. The low-volatility stocks tend to have more weight in this scheme but the overall volatility reduction is lower than the one of the minimum variance or low-volatility portfolios.

Diversity Weighting

Based on the diversity measure introduced by Fernholz [1999], this alternative allocation scheme aims at "smoothing" extreme bets in the market capitalization weighted portfolio. This is achieved by a power transformation of market capitalization weights. Mathematically, stock weights w are proportional to their tilted capitalizations c ($w_i \sim c_i^p$), where $p \in$ [0.1]. The case $p = 1$ leads to the capitalization-weight portfolio; the case $p = 0$ corresponds to the equal-weight portfolio. For intermediate values of p, this approach reduces the gap between the biggest and smallest weights of the capitalization-weight portfolio, and thus reduces the magnitude of errors caused by market misevaluations. Although it is possible to use any value for p within the range [0,1], the typical values encountered in practical applications are $p \geq 0.7$.

Maximum Diversification

The maximum diversification approach, also known as the most diversified portfolio (MDP), was proposed by Choueifaty and Coignard [2008]. The weights of MDP maximize the diversification ratio, which is the ratio of the weighted average of the volatilities of stocks to the volatility of the portfolio of these stocks. The MDP portfolio is given by the following optimization problem:

$$\text{MDP}: \begin{cases} \max_w \dfrac{w'\sigma}{w'\Sigma \mathrm{w}} \; s.\, t. \\ \sum_{\mathrm{i}} w_{\mathrm{i}} = 1 \;\text{ and }\; w_{\mathrm{i}} \in [0,1] \end{cases} \tag{2}$$

where σ is the vector of volatilities and Σ is the covariance matrix.

As Meucci [2009] pointed out, because the MDP portfolio is based on a differential diversification measure (i.e., based on a ratio of risk measures, and not on an absolute risk measure), it is difficult to tell how diversified this portfolio is, and whether it will have, for example, lower total portfolio variance or lower drawdown than the market-cap portfolio. Lee [2011] showed that the optimization problem can be reformulated as a mean–variance optimization problem, where expected returns of assets are assumed to be proportional to their volatilities. Thus, this portfolio would be mean–variance optimal in the case when all assets have identical Sharpe ratios. Seen from this perspective, the MDP approach has an objective similar to that of the risk efficient approach that will be discussed in the following section. For a detailed analysis of the properties of the MDP portfolio, we refer the reader to Choueifaty, Froidure, and Reynier [2011].

Fundamental Weighting

Fundamental weighting, also known as economic scale weighting, is a completely different approach to portfolio allocation (Arnott et al. [2005]). It is based neither on risk management considerations nor on portfolio optimization. It aims at adding value by relying on more efficient estimations of the companies' true economic value than those provided by market capitalizations. The noisy market hypothesis, which was proposed as a theoretical foundation of this approach in Siegel [2006], assumes that market valuations come with noise negatively affecting the ex post performance of cap-weighted portfolios (Hsu [2006] and Treynor [2005]). Therefore, a weighting scheme that is not dependent on market valuations is also free from market noise and, under certain assumptions, leads to outperformance of market-valuation-indifferent allocations.

Economic scale weighting uses company fundamentals—sales, cash flows, dividends, and book values—to build firm fair valuations, and to set the portfolio weights proportional to these valuations. As argued in Arnott et al. [2010], the errors in such weights will be independent from the errors present in stock prices, so when undervalued or overvalued stocks see their prices corrected, the fundamental index portfolio will not suffer whereas the capitalization-weighted will. Nonetheless, this argument is not without flaws, since outperformance of the Fundamental Index depends on the assumption that the errors in the "economic scale" valuations are independent of market weights. As Kaplan [2008] showed, this is not generally the case, and if omitting the assumption of errors' independence, the outperformance of the Fundamental Index portfolio is no longer guaranteed.

Risk Efficient

The risk-efficient strategy is an allocation technique that aims at maximizing a portfolio's Sharpe ratio, where the expected return is assumed to be dependent on the downside semi-deviation (Amenc et al. [2011]).

Being a downside risk measure, SEM represents a standard deviation below the mean of stock returns, or, equivalent, the dispersion of lower-than-average returns:

$$\mathrm{SEM} = \sqrt{E[\min(R - E[R], 0)^2} \tag{3}$$

The idea behind it is that investors require high returns for stocks with high downside risk. Mathematically,

$$RE: \begin{cases} \max_w \dfrac{w'\,\mathrm{SEM}}{w'\Sigma w} \; s.\,t. \\ \sum_i w_i = 1 \;\; and \;\; w_i \in [0,1] \end{cases} \tag{4}$$

where Σ is the covariance matrix that is estimated through a factor model.

Dividend

Dividend yield has been used in portfolio construction because dividend-paying stocks are attractive for income-seeking investors, and because the dividend yield factor is a widely regarded fundamental ratio. Assessing whether dividend yields do forecast future returns goes beyond the scope of this article. We just mention that modern theories use dividend yield as one of the quantitative measures when building equity valuation models. The established consensus, stemming from the seminal works of Modigliani and Miller [1958], states that dividends, and more generally capital structure, should not have an effect on expected returns. On the other hand, the predictive power of dividend yields on equity premium has been investigated, starting with the works of Fama and French [1988a and 1988b]. For example, Campbell and Shiller [1988] looked at the relationship between historical earnings and real dividends, and, as a consequence, at the relation between dividend yields and expected future returns over the long term. On the practical side, gaining exposure to dividend-paying stocks gives investors a way to get income from their portfolio while maintaining their exposure to equity markets.

Dividend-oriented strategy portfolios generally are constructed by selecting stocks of the top dividend-paying companies, subject to various screens of dividend quality and financial strength. The detailed description of dividend strategy portfolio used in this study is presented in Appendix A.

Factor Portfolios

The existence of systematic equity factors is widely accepted in the investment community, and shapes the way most investors think about investment strategies and managers. Fama and French showed in their seminal work (Fama and French [1992]) that both the market capitalization (size factor) and the ratio of book-to-market equity (value factor) explain a significant fraction of the cross-sectional difference in expected returns. A third significant factor, momentum, was later added by Carhart [1997], representing the excess performance of stocks with high past momentum over stocks with low past momentum. These factors, together with the market excess return, form the four-factor model widely used in academic research and industry applications.

Low volatility is another factor extensively covered in practitioner research over the past decades (Haugen and Heins [1975] and Baker, Bradley, and Wurgler [2011]). It measures the performance difference of stocks with low past volatility over stocks with high past volatility. Although the research factors are long–short portfolios, index providers promote mainly long-only factor versions that neglect the short leg of each factor. We construct here long-only factor-tilted portfolios representing tilts toward the following factors: value, momentum, and low volatility. We do not construct a size factor portfolio here because our investment universe, the S&P 500, is composed of large-cap stocks and does not allow for a meaningful size factor proxy. We consider the simplest and, to the extent a standard exists, most standard value, volatility and momentum measures, as detailed in Appendix A.

EMPIRICAL STUDY OF ALTERNATIVE BETA STRATEGIES

In this section, we study the empirical properties of alternative beta strategy returns over the past 25 years. We first examine descriptive statistics (performances, volatilities, and correlations). We then perform several statistical tests clearly showing that some alternative beta strategies have very similar behavior. By looking at the covariance structure, we propose different techniques to cluster them to reduce redundancy in the set of alternative beta strategies.

Data

Our dataset consists of 11 alternative beta strategies, along with the market-capitalization-weighted benchmark (MC). The strategies are listed in Exhibit 1. Data span from January 31, 1990, to December 31, 2014. All simulated strategies are based on the S&P 500 Index universe.[1]

For the Market Capitalization, the Equal Weighting, and the Dividend strategies we take the performance of official indexes; the rest of the strategies are simulated according to their respective methodologies using stocks belonging to the S&P 500 Index. Appendix A presents more details on the set of parameters used to simulate the portfolios' performances. Stock prices are taken from Datastream, while index levels come from Bloomberg. Finally, index levels are considered in their price return versions (dividends are not reinvested in the portfolios).

We also collect returns of Fama–French–Carhart factors: market (MKT), size (SMB), value (HML), and momentum (UMD). In this study the MKT factor is constructed from monthly returns of the market-cap portfolio (the S&P 500). The other factors are long–short portfolios that are long the stocks with low attributes such as market capitalization (size), book-to-market ratio (value), and past 12-month return (momentum), and short the stocks with high attributes, using tranches to minimize factor correlations. We refer to Fama and French [1993] and Carhart [1997], and Kenneth French's website, from which the factor returns are taken, for more details on the construction of these factors.

EXHIBIT 1
Alternative Beta Strategies

Alternative Beta	Portfolio	Label
Diversity Weighting	Simulated	**DW**
Dividend Investing	S&P 500 Dividend Aristocrat Index	**DI**
Equal Weighting	S&P 500 Equal Weight Index	**EW**
Fundamental Weighting	Simulated	**FW**
Low Volatility	Simulated	**LV**
Market Cap	S&P 500 Index	**MC**
Maximum Diversification	Simulated	**MD**
Minimum Variance	Simulated	**MV**
Momentum	Simulated	**MM**
Risk Efficient	Simulated	**RE**
Risk Parity	Simulated	**RP**
Value Weighting	Simulated	**VW**

Note: Refer to Appendix A for further details on the simulated alternative beta strategies.

Descriptive Statistics of Alternative Beta Strategies

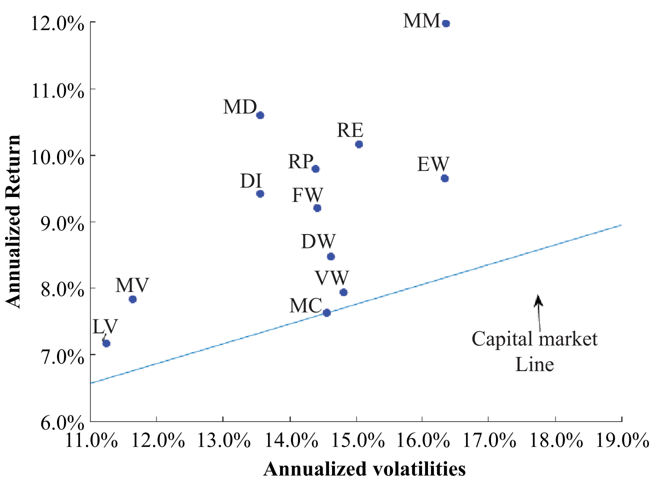
Let us first look at Exhibit 2, which shows the risk–return profile of alternative beta strategies compared to that of the market-capitalization portfolio. It is noticeable that almost all alternative beta strategies tend to outperform the benchmark in absolute terms. The same holds true for risk-adjusted returns: All alternative beta strategies lie above the capital market line, meaning that all of them outperform the benchmark in risk-adjusted terms. In Exhibit 3, we report the main characteristics of each strategy.

Momentum shows the highest annualized return and the largest volatility over the period, very similar to the EW portfolio. The Low Volatility strategy has the lowest volatility and Minimum Variance strategy has the lowest drawdown. All alternative beta strategies, except Low Volatility and Minimum Variance, show CAPM beta within the range [0.80, 1.06]. They all are leptokurtic with negative skewness indicating fat tails (especially the left one) compared with the normal distribution. All alternative beta strategies deliver a positive CAPM alpha over the MC benchmark.

Exhibit 4 shows correlations across alternative beta strategies' monthly returns.

EXHIBIT 2

Alternative Beta Strategies: Annualized Performances vs. Annualized Volatilities



Alternative beta portfolios are strongly correlated among themselves, with the average of pairwise correlations being 85.81% (excluding the market-cap portfolio). The average correlation of the strategies to the market portfolio is also very strong: 89.08%.

The lowest pairwise correlation in the dataset is between the Low Volatility and Momentum portfolios, at 57.77%. We clearly distinguish two groups among which pairwise correlation are particularly high:

- Diversity Weighting–Market Cap, and
- Equal Weighting–Risk Parity–Risk Efficient–Maximum Diversification.

Similar conclusions hold true in the left and right tails of the distribution, as one can see in Exhibit 5.

The two groups highlighted here have very similar behavior during extreme volatility regimes, with the tail correlation values being all highly significant.

The same groups highlighted before show very high correlations, especially in the left tail (which corresponds to high values of the VIX Index). Clearly not all alternative beta strategies provide diversification in times of crisis.

Return analysis shows some interesting patterns emerging from the risk/return profile of alternative beta strategies: some of them appear very similar when we look at returns, risk, CAPM betas, and maximum drawdowns.

EXHIBIT 3

Alternative Beta Strategies Statistics

	Ann. Return	Volatility	Skewness	Kurtosis	Sharpe Ratio	Max Drawdown	CAPM Alpha	CAPM Beta	CVaR 95%
DW	8.48%	14.62%	–0.65	4.49	0.35	–52.62%	0.84%	1	–9.76%
DI	9.42%	13.56%	–0.33	4.70	0.45	–47.18%	2.67%	0.8	–8.83%
EW	9.65%	16.34%	–0.51	5.21	0.39	–56.43%	1.76%	1.06	–10.88%
FW	9.21%	14.42%	–0.72	5.66	0.41	–56.54%	2.02%	0.9	–9.70%
LV	7.17%	11.24%	–0.69	4.40	0.34	–38.53%	1.33%	0.59	–7.22%
MC	7.63%	14.56%	–0.63	4.27	0.3	–52.56%	–	1	–9.70%
MD	10.60%	13.56%	–0.82	5.32	0.54	–45.72%	3.72%	0.83	–9.03%
MV	7.84%	11.64%	–0.68	4.58	0.39	–37.50%	1.74%	0.65	–7.73%
MM	12.00%	16.37%	–0.41	3.98	0.53	–47.45%	4.61%	0.94	–10.35%
RE	10.16%	15.04%	–0.63	4.99	0.46	–53.72%	2.62%	0.98	–10.04%
RP	9.79%	14.39%	–0.63	5.29	0.45	–51.51%	2.50%	0.92	–
VW	7.94%	14.81%	–0.93	5.70	0.31	–56.90%	0.57%	0.94	–10.60%

Notes: Data from January 31, 1990, to December 31, 2014. Sharpe ratios are computed with the return over U.S. risk-free rate, averaging 3.30% over the period. CAPM betas and alphas are computed with respect to the MC portfolio monthly returns.

At first glance, Diversity Weighting and Market Cap show very similar behavior, and the same can be said about Equal Weighting, Risk Efficient, and Risk Parity.

However, these similarities have been deduced from statistics that may be too reductive when it comes to describing the global behavior over long periods of time. To this effect, we use two common techniques that help to detect any common return drivers by making use of the entire structure of dependence rather than individual or average pairwise correlations: PCA and factor regressions.

EXHIBIT 4

Correlation Matrix Based on Monthly Returns, All Values Significant at 95% Level

	MC	EW	RE	RP	FW	VM	LV	MV	MD	MM	DI
DW	99.59	96.52	96.78	95.67	92.11	92.95	78.07	83.21	91.41	82.75	87.11
MC		94.49	94.66	93.34	90.56	92.19	76.18	80.77	88.83	83.79	85.53
EW			99.31	99.07	92.90	92.08	80.17	85.90	95.25	73.83	88.70
RE				98.62	92.69	91.59	80.71	87.49	96.80	75.44	87.90
RP					93.41	91.91	86.23	90.02	95.80	72.80	91.75
FW						93.42	79.47	83.02	89.42	72.39	85.31
VW							76.32	80.59	86.91	72.96	83.26
LV								91.75	83.82	57.77	87.38
MV									91.96	62.91	84.27
MD										71.97	84.53
MM											63.05

Notes: The correlation values are multiplied by 100. Data from January 31, 1990, to December 31, 2014.

Principal Component Analysis

The first test is a classic principal component analysis of the alternative beta strategies covariance matrix. The PCA allows transforming the set of correlated variables into a new set of uncorrelated variables, or principal components. Each principal component is a linear combination of original variables, and in our case these are long–short portfolios constructed as linear combinations of alternative beta strategies. The coefficients in these linear combinations indicate the relative importance of every original strategy in the corresponding principal component. And vice versa, the original strategies can be represented via linear combinations of principal components that play a role of statistical factors here.

The principal components can be ordered by their contributions to the total variation in the dataset. Generally, for equities is it sufficient to focus on a very small number of principal components (or statistical factors).

EXHIBIT 5

Alternative Beta Strategies Correlation Matrix

	MC	EW	RE	RP	FW	VW	LV	MV	MD	MM	DI
DW	97.90	93.65	93.87	96.54	88.66	93.94	76.95	77.57	76.23	58.35	78.31
MC		93.02	89.93	93.81	82.52	94.97	75.08	73.11	70.04	47.78	78.22
EW			95.92	97.68	76.95	90.25	65.56	73.35	76.42	46.52	75.64
RE				95.93	82.32	86.52	66.57	78.13	83.87	59.76	72.60
RP					85.43	93.93	76.91	81.28	80.65	50.48	79.90
FW						88.03	78.76	84.42	73.31	65.99	61.41
VW							82.89	81.45	71.70	42.56	77.32
LV								77.50	64.25	29.55	73.78
MV									85.63	44.13	68.80
MD										48.65	64.80
MM											11.91

	MC	EW	RE	RP	FW	VW	LV	MV	MD	MM	DI
DW	99.84	98.03	98.39	97.93	96.66	95.30	90.06	91.00	96.34	90.35	92.53
MC		96.83	97.28	96.75	85.87	94.97	89.60	90.44	95.18	91.40	92.27
EW			99.76	99.74	97.57	94.48	89.43	90.62	97.60	83.16	92.71
RE				99.65	97.36	94.56	90.48	91.87	98.43	84.73	91.95
RP					98.00	94.61	91.95	92.38	98.20	83.96	93.84
FW						96.45	91.88	91.60	95.93	85.40	93.62
VW							86.72	89.41	92.60	84.50	88.78
LV								95.11	92.41	83.27	91.05
MV									95.00	83.68	86.35
MD										85.16	89.20
MM											78.80

Notes: Left panel: Alternative beta strategies correlation matrix computed with left extreme data (VIX index below its lower 10% percentile; i.e., very low-volatility market regime). Only two pairwise correlations are not significant at 95% confidence level (the pairs MM–LV and MM–DI). Right panel: Alternative beta strategies correlation matrix computed with right extreme data (VIX index above its 90% percentile; i.e., very high-volatility market). All values are significant at the 95% confidence level.

Leaving aside the limitations of such a technique, we observe from Exhibit 6 that the first statistical factor explains the bulk of the variance among alternative beta strategies monthly returns. Although it is not easy to infer the economic meaning of the PCA factors, one can expect the first factor F1 to be a proxy of the broad market. The variance of several alternative beta strategies can be almost entirely explained by this factor. We note, however, that for Minimum Variance, Dividend Investing, Low Volatility, and Momentum, the total variance is not fully captured by the factor F1. The loadings of the first three factors of this PCA test are collected in Exhibit 7.

EXHIBIT 6

Proportion of Alternative Beta Strategy Total Variance Explained by the First Three PCA Factors

	F1	F2	F3	Sum
DW	95.47%	1.18%	0.53%	97.18%
DI	83.15%	4.28%	1.42%	88.85%
EW	96.48%	0.20%	1.26%	97.94%
FW	90.59%	0.08%	2.01%	92.68%
LV	73.81%	8.17%	12.88%	94.86%
MD	92.23%	0.68%	0.23%	93.14%
MV	81.41%	4.95%	7.05%	93.41%
MM	64.08%	33.16%	2.63%	99.87%
RE	97.09%	0.07%	0.59%	97.75%
RP	97.79%	0.95%	0.05%	98.79%
VW	88.62%	0.02%	4.24%	92.88%

EXHIBIT 7

Loadings of Alternative Beta Strategy Monthly Returns on the First Three PCA Factors

	F1	F2	F3
DW	0.32	0.14	0.14
DI	0.28	–0.25	–0.22
EW	0.36	–0.07	0.25
FW	0.31	–0.04	0.27
LV	0.22	–0.29	–0.54
MD	0.30	–0.10	–0.09
MV	0.24	–0.23	–0.42
MM	0.30	0.86	–0.36
RE	0.33	–0.04	0.16
RP	0.32	–0.13	0.04
VW	0.31	0.02	0.41

We notice that the first principal component F1 can indeed be related to the equity market factor, as all portfolios have positive significant loading for it. Analysis of the loadings to the two factors F2 and F3 shows that the Momentum strategy represents a clear outlier, having high positive loading to the factor F2, while all the other strategies have small or significant negative relation to this factor. One can also spot here a commonality among Minimum Variance, Low Volatility, and Dividend Investing strategy, as they have F1 loadings smaller than the rest of the group, significant negative F2 loading, and big negative F3 loading.

Fama–French–Carhart Regression of Alternative Beta Strategy Returns

One of the main limitations of PCA is the difficulty in relating the dominant principal components to the established equity and economic factors. To encompass this problem, we perform a classic regression based on Fama–French–Carhart factors, since they offer a more intuitive framework to explain the variations among strategies. We recall that the MKT factor corresponds to the monthly returns of the Market Cap portfolio. Exhibit 8 collects the results of the regressions of alternative beta strategy monthly returns using the following Fama–French–Carhart model:

$$R_t = \alpha + \beta^{MKT} \cdot MKT_t + \beta^{SMB} \cdot SMB_t + \beta^{HML} \cdot HML_t + \beta^{UMD} \cdot UMD_t + \varepsilon_t \quad (5)$$

where R is the monthly return of each alternative beta strategy and ε is a normalized white noise.

Again, for many alternative beta strategies we find that total variance is fully explained by the model, with levels of the R-squared coefficient being close to 1. Nonetheless, for Minimum Variance, Low Volatility, and Dividend Investing, the model leaves 1/5 to 1/3 of the total variations unexplained. All strategies except Momentum show significant value tilt, although for Diversity Weighting that tilt is rather small. If the majority of alternative beta strategies have a significant size exposure, this is not the case for Minimum Variance and Low Volatility, indicating that small-cap stocks tend to be more volatile than the market and thus do not represent a high proportion of low volatility portfolios. The same holds true for Dividend Investing because,

EXHIBIT 8
Fama–French–Carhart Regression Loadings

	α	MKT	SMB	HML	UMD	R^2
DW	0.001***	0.997***	0.040***	0.058***	–0.023***	99.47%
DI	0.002**	0.828***	–0.058*	0.307***	–0.042*	80.77%
EW	0.001**	1.032***	0.233***	0.269***	–0.136***	95.63%
FW	0.001*	0.908***	0.082***	0.334***	–0.079***	89.10%
LV	0.001	0.653***	–0.101***	0.341***	0.034*	71.12%
MD	0.003***	0.825***	0.204***	0.269***	–0.062***	85.02%
MV	0.002*	0.674***	0.013	0.280***	–0.024	72.08%
MM	0.001	1.063***	0.111***	–0.021	0.397***	87.30%
RE	0.002***	0.958***	0.199***	0.226***	–0.102***	94.55%
RP	0.002***	0.923***	0.141***	0.308***	–0.087***	93.67%
VW	0.001	0.928***	0.020	0.211***	–0.108***	89.29%

*Notes: Asterisks indicate loading significance: *** = 1%, ** = 5%, * = 10%, none = loading is not significant.*

structurally, this strategy looks at stocks with 25 years of increasing dividends, which is typically not the case for small-cap stocks. Finally, Diversity Weighting also has very small size bias; as per construction, the diversity power parameter was set equal to $p = 0.8$, implying just a moderate tilt of market-capitalization weights. In terms of factor exposure, again we note many similarities among Equal Weighting, Risk Efficient, and Risk Parity. To a lesser extent, we also find similarities between Fundamental Weighting and Value Weighting.

The statistical analysis presented in this section shows that diversification within the alternative beta family of strategies appears to be limited. The strategies are strongly correlated among themselves, as well as to the market portfolio. The strategies set is driven by very few statistical factors, and some strategies have very close factor loadings. This suggests that one can perform a dimensional reduction on this set of portfolios, restricting it to strategies or their linear combinations that have fairly different behavior.

Clustering

In what follows, we implement a statistical procedure to identify clusters in the set of alternative beta strategies according to some measure of distance. We consider two cases: the first where the distance is given by the pairwise correlations and the second where the distance is derived from Fama–French–Carhart factor loadings. The use of a correlation-based distance measure allows us to assess overall similarities in the behavior of alternative beta strategies, while the factor-based distance focuses on similarities in exposure to systematic risk factors, leaving aside the idiosyncratic risk.

The basic idea is to consider alternative beta strategies as points, more or less close to each other, where the degree of similarity can be derived by either correlation or factor exposure. We then build clusters of strategies so that 1) within each cluster, the strategies are as close as possible, and 2) the clusters are as distant as possible.

One can imagine a connected graph with each strategy being a node and the edges' length representing pairwise closeness, depending on either pairwise correlation or similarity in risk exposures. The graph will be very complex, since the majority of alternative beta strategies tend to resemble one another: as shown in Exhibit 4, correlations among them are all very high and exposures to the risk factor may be similar (Exhibit 8).

In order to clarify the picture of pairwise closeness and to obtain a readable chart, we derive a minimum spanning tree (MST) graph from the initial cloud. MST is a connected graph (i.e., any node can be connected with any other node by a path) that keeps only those edges between the nodes that give minimal total sum of the edges' weights (the weights are proportional to distances, in our case). For more details on MST, we refer to Cormen et al. [2001] and references therein.

The main advantage of using MST is that it reveals the closest links between the strategies, while it disregards less important similarities. The MST gives a first clue to the strategy grouping and efficiently determines the outlier strategies (located farthest from the crowd). Nevertheless, this technique is not able to determine the optimal number of clusters nor to give the exact composition of clusters. To define the clusters, we first apply the k-means algorithm (MacQueen [1967]), which allows determining an optimal cluster partition when the number of clusters is fixed. We then select the optimal number of clusters depending on a user-given parsimony parameter that specifies the desired degree of similarity among the strategies by penalizing the increase in the number of clusters. The lower the parameter value, the more attention is paid to differences among the strategies, treating each strategy as an individual cluster when the parameter value approaches zero. Vice versa, the higher the parsimony parameter value, the greater the granularity that is accepted, so that for very high parsimony values all the strategies are treated as one cluster. The clustering procedure is described in Appendix B.

Our procedure is summarized by the following work flow:

1. Define the set of strategies.
2. Define the measure of distance (based on correlations or factor exposures).
3. Derive an MST graph to have a first visual guess of clusters.
4. Optimize the number and composition of clusters for a range of values of the parsimony parameter.

In the next two sections we apply this work flow to identify clusters based on correlations (as given in Exhibit 4) and risk exposures (as given in Exhibit 8).

CLUSTERS BASED ON PAIRWISE CORRELATIONS

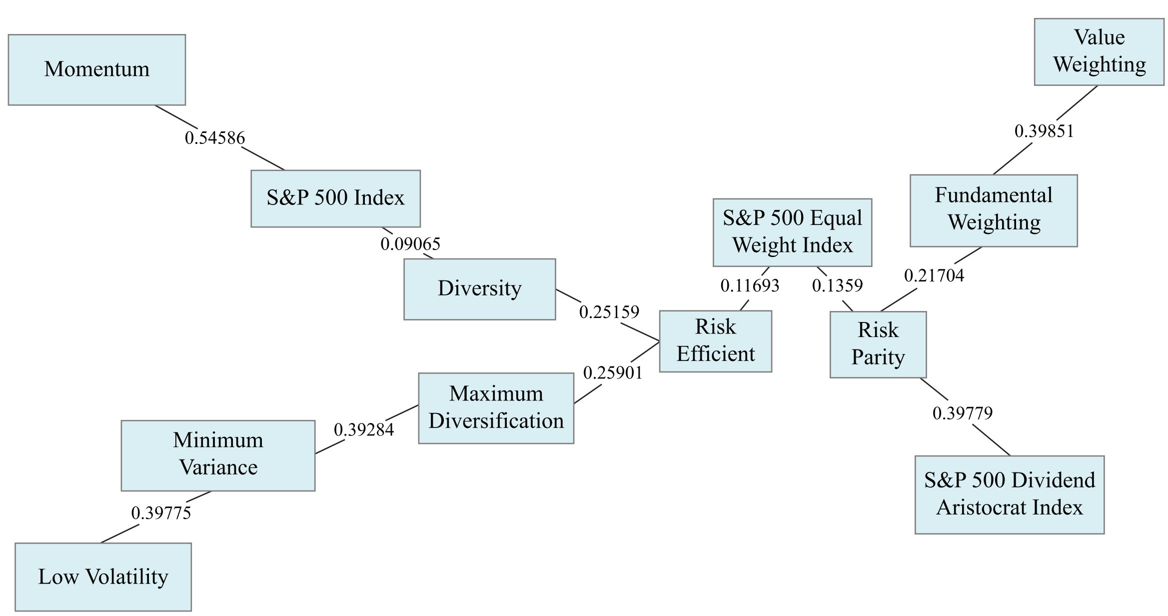
Let us define the degree of closeness between two alternative beta strategies i and j as follows:

$$d(i, j) = \sqrt{1 - \rho_{ij}^2} \tag{6}$$

where ρ_{ij} is the correlation coefficient between monthly returns of strategies i and j. For perfectly correlated strategies, the measure is equal to zero, meaning that the strategies are identical. For uncorrelated strategies, the measure is equal to 1. For intermediate correlation values, the more the strategies are correlated, the smaller is the weight of the respective edge on the graph. Exhibit 9 shows the MST graph associated with this measure.

EXHIBIT 9

Minimum Spanning Tree of the Alternative Beta Strategies Based on Pairwise Correlations, January 1990–December 2014



We distinguish at least four branches and a center on the MST graph. The center is formed by the equal-weight strategy and the two strategies closely related to it (in correlation sense): Risk Efficient and Risk Parity. The branches originate from the center and go in different directions. The first branch is driven by price momentum and, passing through Diversity and Market Cap, ends at Momentum. The second branch goes through Maximum Diversification and ends at the couple Minimum Variance–Low Volatility. The third branch is given by the Dividend Investing strategy, and, finally, the fourth one by two value-based portfolios, Fundamental Weighting and Value Weighting. In a second step, we apply the clustering procedure detailed in Appendix B to identify the optimal number and the composition of clusters for different values of the parsimony parameter. Results are shown in Exhibit 10.

When this parameter is set to zero, each strategy represents a separate cluster. For a very high level of parsimony we end up with only one cluster containing all the portfolios. For intermediate values of the parsimony parameter the optimal number of clusters changes, reflecting the degree of granularity dictated by the parameter. Until very high values of the parameter are reached, Momentum appears to be very far from other alternative beta strategies. Market Cap and Diversity Weighting are quickly grouped into one cluster, as both use market capitalization as a key ingredient for the weights. The choice of $p = 0.8$ (see Appendix A) obviously makes the diversity tilt relatively small, so that the strategies can be clustered.

The cluster made of Equal Weighting and Risk Efficient looks stable across a wide range of parsimony parameter values and, as the parameter value grows, it is very quickly joined by Risk Parity and Maximum Diversification. We notice that Minimum Variance and Low Volatility are not clustered for low values of the parsimony parameter, since they appear to be different enough; they are clustered only for medium to high levels of parsimony. For medium levels of parsimony a new cluster appears as the merger of Diversity Weighting–Market Cap and Fundamental Weighting–Value Weighting. Finally, for high levels of the parsimony parameter the majority of strategies are grouped under a large cluster containing Diversity Weighting–Market Cap–Fundamental Weighting–Value Weighting and Equal Weighting–Risk Efficient–Risk Parity–Maximum Diversification. At the same level of parsimony, a low-risk cluster stands apart, composed of Minimum Variance–Low Volatility, and joined eventually by Dividend Investing.

It should be noted that the parsimony parameter is a key ingredient, as it determines the number and composition of clusters. The described structure remains valid when we consider different time windows. Exhibits 11 and 12 show MSTs based on correlation matrixes computed respectively over the 1990–2002 and 2002–2014 subperiods. Although the weights of the edges have slightly changed because the correlations changed, the overall structure remains robust.

CLUSTERS BASED ON RISK EXPOSURES

In this section, we represent each strategy by a vector:

$$e_i = \left[\hat{\beta}_i^{MKT}, \hat{\beta}_i^{SMB}, \hat{\beta}_i^{HML}, \hat{\beta}_i^{UMD} \right] \tag{7}$$

where the loadings of strategy i are computed through a classic linear regression for which we use standardized

EXHIBIT 10

Cluster Identification Based on Correlations Depending on Value of the Parsimony Parameter

Parameter	Clusters											
0	DW	MC	FW	VW	EW	RE	RP	MD	MM	DI	LV	MV
0.5%	DW, MC		FW	VW	EW, RE		RP	MD	MM	DI	LV	MV
1%	DW, MC		FW, VW		EW, RE, RP, MD				MM	DI	LV, MV	
2.5%	DW, MC, FW, VW				EW, RE, RP, MD				MM	DI	LV, MV	
6.5%	DW, MC, FW, VW, EW, RE, RP, MD								MM	DI, LV, MV		
10%	DW, MC, FW, VW, EW, RE, RP, MD, MM									DI, LV, MV		
50%	DW, MC, FW, VW, EW, RE, RP, MD, MM, DI, LV, MV											

EXHIBIT 11

Minimum Spanning Tree of the Alternative Beta Strategies Based on Correlation, January 1990–December 2002

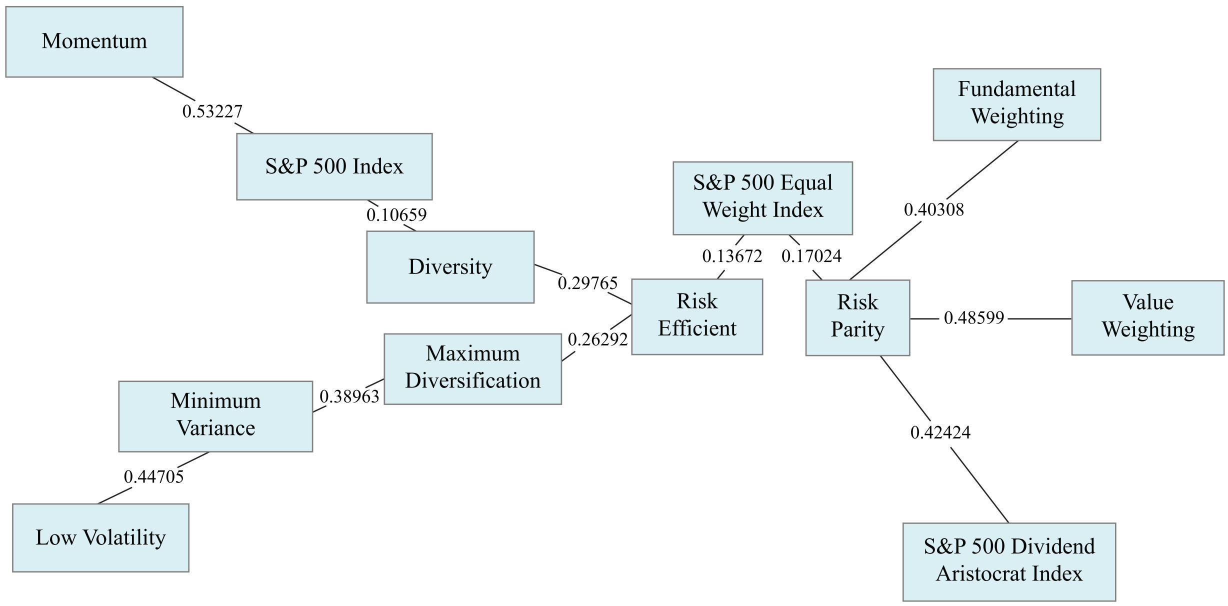
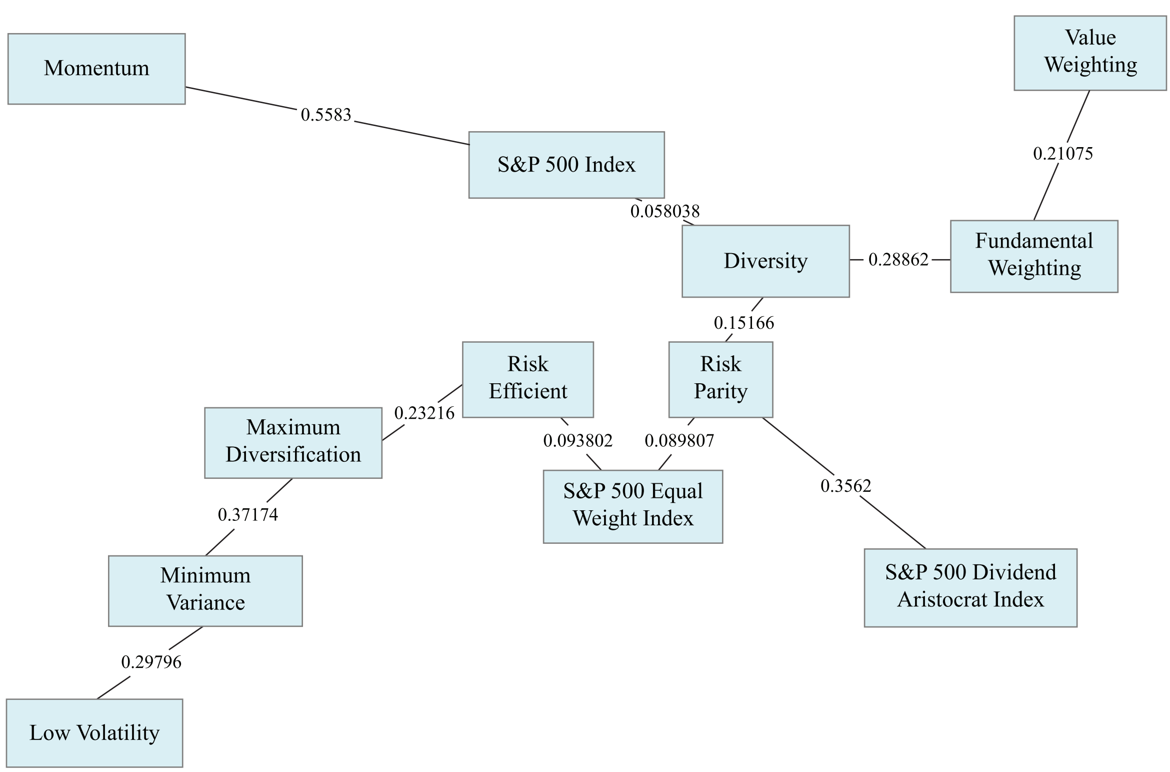


EXHIBIT 12

Minimum Spanning Tree of the Alternative Beta Strategies Based on Correlation, December 2002–December 2014



versions of the Fama–French–Carhart factors, scaling them in order to have the same factor volatility. We define the degree of closeness between two alternative beta strategies i and j as a Euclidean distance of the vector of their risk exposures e_i, e_j:

$$d(i,j) = \frac{1}{m}\sqrt{\sum_{k=1}^{4}(e_i^k - e_j^k)^2} \tag{8}$$

where $m > 0$ is a scaling constant. Although the results do not depend on m, we choose a scaling parameter so that the maximum distance across all alternative beta strategies is equal to 1: $\max_{ij} d(i,j) = 1$. Low distance between strategies means they have similar risk exposures. It is important to notice that, according to this metric, a strategy and a leveraged version of it will appear very different, whereas with the correlation-based measure they will appear to be the same. Furthermore, two strategies with similar risk exposures but very different unexplained parts of total variance (given by $1 - R^2$) will appear close even if one of them (or both) has a lot of idiosyncratic risk and could be rather weakly correlated. The MST graph derived from these distances is shown in Exhibit 13.

We find many similarities with Exhibit 11, especially the two branches made of Momentum on one side and Minimum Variance–Low Volatility on the other side. The center of the MST is now composed of value-based strategies (Fundamental Weighting and Value Weighting), while two new branches made of Maximum Diversification and Equal Weighting complete the MST on the right side. We now apply the procedure described in Appendix B to derive optimal clustering as a function of the parsimony parameter. Exhibit 14 collects the results.

The first cluster to appear is made of Fundamental Weighting and Risk Parity—when the parsimony parameter is set at 0.25%. When we increase this parameter to 0.5%, we note that Diversity Weighting and Market Cap are clustered together, and the same can be said for the couples Equal Weighting-Risk Efficient and Minimum Variance–Low Volatility.

For the parameter value of 3%, the number of clusters is reduced to five. Fundamental Weighting, Risk Parity, Maximum Diversification, and Value Weighting form the biggest cluster. Diversity Weighting and Market Cap form another cluster, and so does Equal

EXHIBIT 13

Minimum Spanning Tree of the Alternative Beta Strategies Based on Risk Exposures, January 1990–December 2014

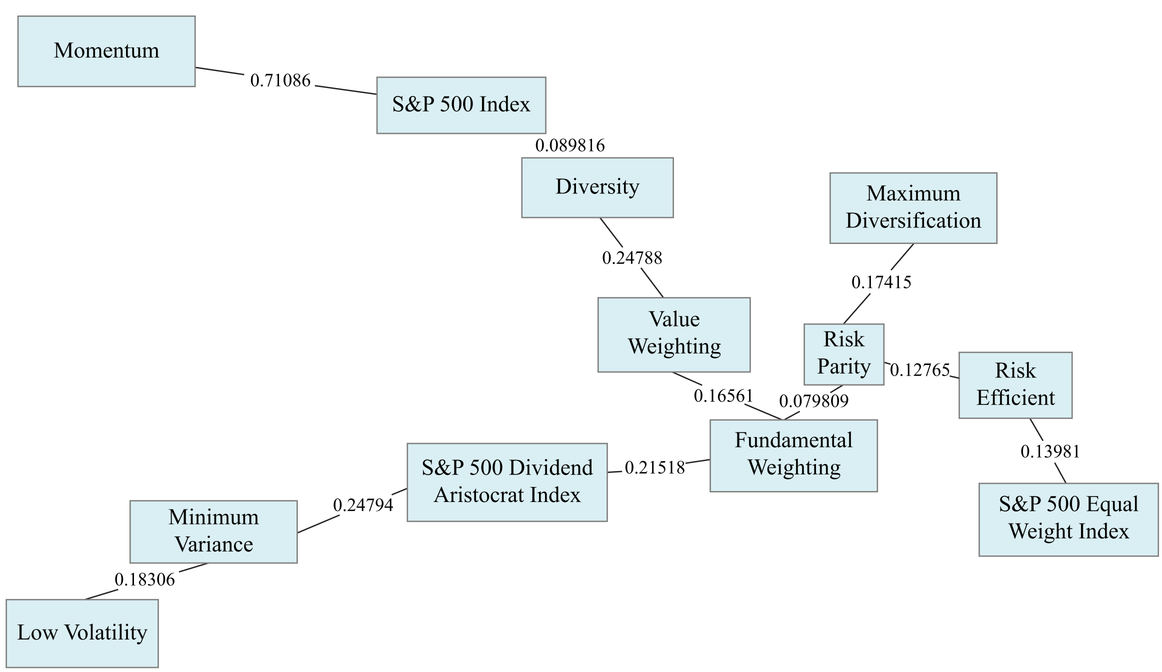


EXHIBIT 14

Cluster Identification Based on Risk Exposures Depending on the Parsimony Parameter

<table>
<tr><th>Parameter</th><th colspan="12">Clusters</th></tr>
<tr><td>0</td><td>DW</td><td>MC</td><td>EW</td><td>RE</td><td>FW</td><td>RP</td><td>MD</td><td>VW</td><td>DI</td><td>LV</td><td>MV</td><td>MM</td></tr>
<tr><td>0.25%</td><td>DW</td><td>MC</td><td>EW</td><td>RE</td><td colspan="2">FW, RP</td><td>MD</td><td>VW</td><td>DI</td><td>LV</td><td>MV</td><td>M</td></tr>
<tr><td>0.5%</td><td colspan="2">DW, MC</td><td colspan="2">EW, RE</td><td colspan="2">FW, RP</td><td>MD</td><td>VW</td><td>DI</td><td colspan="2">LV, MV</td><td>MM</td></tr>
<tr><td>1%</td><td colspan="2">DW, MC</td><td colspan="2">EW, RE</td><td colspan="3">FW, RP, MD</td><td colspan="2">VW, DI</td><td colspan="2">LV, MV</td><td>MM</td></tr>
<tr><td>3%</td><td colspan="2">DW, MC</td><td colspan="2">EW, RE</td><td colspan="4">FW, RP, MD, VW</td><td colspan="3">DI, LV, MV</td><td>MM</td></tr>
<tr><td>5%</td><td colspan="2">DW, MC</td><td colspan="6">EW, RE, FW, RP, MD, VW</td><td colspan="3">DI, LV, MV</td><td>MM</td></tr>
<tr><td>10%</td><td colspan="8">DW, MC, EW, RE, FW, RP, MD, VW</td><td colspan="3">DI, LV, MV</td><td>MM</td></tr>
<tr><td>30%</td><td colspan="12">DW, MC, EW, RE, FW, RP, MD, VW, DI, LV, MV, MM</td></tr>
</table>

Weighting–Risk Efficient. Dividend Investing now clusters with Minimum Variance–Low Volatility. We need to increase the risk–return parsimony parameter to 10% in order to get three clusters. Momentum is always clustered alone; as noted earlier, Momentum is effectively very far from the rest of the alternative beta strategies.

Concluding this section, we found that the set of alternative beta strategies can be structured in a number of distinct groups, which we call clusters. The exact grouping depends on the definition of distance and of the parsimony level required. We found that for two different definitions of distance, which represent strategy closeness in terms of correlations and in terms of common risk exposures, the results appear to be quite similar. In both cases the "central" block of strategies (Equal Weighting, Risk Efficient, Risk Parity, and Maximum Diversification) are more or less quickly grouped together. The result is especially striking for the couple Equal Weighting–Risk Efficient, as the two are quickly grouped together in spite of rather different investment approaches behind their two strategies. The appearance of a cluster of the two versions of low-volatility portfolios (Minimum Variance, Low Volatility) is possible starting from moderate values of the parsimony parameter, indicating that these strategies, with and without accounting for idiosyncratic risk, have enough differences in their behavior. Their cluster is joined eventually by the Dividend Investing strategy.

APPLICATIONS

This section is dedicated to a simple application of the clustering framework introduced in the previous section. We investigate here how clustering of strategies can be used in building multi-strategy portfolios. Given the redundancy among some strategies, investors who include all of them in a multi-strategy portfolio may introduce unwanted concentration on specific clusters. While investors believe they are diversified across different risk–return profiles, they are instead multiplying specific bets. This lack of diversification can be problematic during market downturns where, as we showed in Exhibit 5, similar alternative beta strategies tend to correlate in the tail more than expected. Clustering thus could be a way to improve diversification and reduce redundancy.

We test two kinds of portfolio allocations, equal-weighting and momentum, applying them on individual strategies as well as on clusters of strategies. Equal weighting is a simple static approach to portfolio allocation, giving the same weight to any portfolio constituent. Momentum is instead a dynamic portfolio allocation, where the weights of components are assigned depending on their recent performance. We consider strategy-based allocations as benchmarks for the performance of the respective cluster-based allocations. We simulate two families of cluster-based allocations, corresponding to two values of parsimony parameter (2.5% and 6.5%). The clusters are formed from the distances based on the total correlation measure (as stated in Exhibit 10). Very similar results hold true when we consider clusters made with the risk-based measure.

As one will see in the following examples, over the period of study and for these two different allocation schemes, a cluster approach brings an average decrease in annual volatility of around 50 basis points (bps) and an increase in the annualized alpha from 64 bps to 100 bps (here, alpha is measured with respect to the portfolios constructed from individual strategies).

Equal-Weight Allocation

Let us assume the investor wants to build an equal-weight allocation of all available alternative beta strategies (including as well the benchmark Market Cap portfolio). This portfolio of strategies is rebalanced monthly to restore equal weighting. We also consider alongside it an equal-weight portfolio of clusters based on pairwise correlations, as in Exhibit 10. The idea behind this is that equal weighing of alternative beta strategies will increase the effective weights of similar strategies, whereas the use of clusters will limit this effect. Within each cluster, we allocate the same weight for all the strategies of the cluster. We then compare the equally weighted allocation of strategies to the equally weighted allocation of clusters given by parsimony parameters 2.5% and 6.5%. We note that the equal weighting of individual strategies corresponds to a cluster-based allocation with parsimony parameter 0%. The clusters are as follows.

Parsimony parameter = 2.5%:

1. DW, MC, FW, VW
2. EW, RP, RE, MD
3. MM
4. DI
5. MV, LV

Parsimony parameter = 6.5%:

1. DW, MC, FW, VW, EW, RP, RE, MD
2. MM
3. DI, MV, LV

For example, the first portfolio (parameter = 2.5%) will invest, each month, 20% in MM and in DI, 10% (= 20%/2) in MV and LV, and 5% (= 20%/4) in other strategies. Exhibit 15 collects the basic statistics for the three portfolios.

We note that the overall performance is very similar over the long run, with cluster-based portfolios delivering slightly higher total returns. This is in line with our expectations, as the allocations are not too different, and clustering does not alter the overall risk exposure. Lower volatility and drawdowns are obtained for cluster-based portfolios compared with the $p = 0\%$ benchmark portfolio, indicating that one achieves better diversification using clusters. Finally, allocations based on clusters deliver positive and significant alphas over the strategy-based portfolio: 64 bps when the parsimony parameter is set at 2.5% and 100 bps when the parsimony parameter is set at 6.5%.

EXHIBIT 15
Basic Statistics for Three Equal-Weight Portfolios

Statistics/Parameters	0%	2.5%	6.5%
Return	9.31%	9.58%	9.83%
Volatility	13.38%	12.82%	12.85%
Sharpe Ratio	0.45	0.49	0.51
CAPM Beta	0.87	0.83	0.82
CAPM Alpha	2.22%	2.69%	2.97%
Max Drawdown	–50.11%	–48.21%	–47.77%
α over 0% Portfolio	–	0.64%	1.00%

*Notes: Data from January 1990 to December 2014. Returns, volatilities, and alphas are annualized. Sharpe ratios are computed using the return over U.S. risk-free rate, equal to 3.30% over the period. CAPM betas and alphas are computed with respect to the Market Cap monthly returns. Asterisks indicate loading significance: *** = 1%, ** = 5%, * = 10%, none = not significant.*

Momentum-Based Dynamic Allocation

Using the same clusters as described earlier, we now assume the investor wishes to have a multi-strategy allocation with an extra feature: the weights are dynamically tilted toward the best-performing portfolio components. More precisely, we implement an equal-weight static allocation scheme with an addition of a dynamic momentum tilt as follows:

- Each portfolio constituent is assigned a weight equal to $1/N$, where N is the number of constituents.
- Constituents are ranked according to their price momentum computed over the previous six months.
- The weights of two constituents with the highest momentum are tilted upward by 20%: $w_i = \frac{1}{N} \cdot (1 + 20\%)\cdot$
- The weights of the other constituents are tilted downward by 20%: $w_i = \frac{1}{N} \cdot (1 - 20\%)$.
- The tilted weights are normalized so their sum equals 1.

For example, for the parsimony parameter set at 0% (that corresponds to an equally weighted allocation of all strategies in the dataset, treating each as a separate cluster), the weight of each strategy is set initially to 8.33% (= 1/12). The weights of two strategies with the highest recent momentum will be tilted upward to 10% (= 8.33% * (1 + 20%)), while weights of other strategies will be lowered to 6.67% (= 8.33% * (1–20%)). Then all the weights are normalized to have their sum equal to 1, so that the first two strategies get the weights of 11.54% and the others the weights equal to 7.69%. The same methodology is applied to clusters (we only need to adjust the value N, which is the total number of clusters). Within each cluster the strategies are equally weighted. Exhibit 16 collects basic statistics for the three portfolios: a momentum-tilted portfolio constructed from alternative beta strategies (the benchmark), and two portfolios with the momentum tilt built on two clusters corresponding to the parsimony parameter of 2.5% and 6.5%.

Using clusters in a dynamic allocation brings even greater improvement in terms of annual performance, volatility, and maximal drawdown, with respect to a static allocation considered in the previous example. The annual alpha delivered by using clusters as building blocks of a momentum allocation is highly significant and positive: 88 bps for the parsimony parameter at 2.5% and 102 bps when the parameter is set at 6.5%.

The results are in line with our expectations. Indeed, although the momentum signal might be noisy, we expect it to be less so when using clusters, because the latter are more diversified than individual strategies and thus carry less idiosyncratic risk. Relative momentum estimated on very similar strategies might be very volatile, while the use of clusters makes the relative momentum estimation more robust, since differences between clusters tend to be more persistent. This demonstrates an additional advantage that a cluster-based allocation approach can bring: Besides the better exposure diversification that is achieved for a static allocation case, there might also be a potential benefit in the case of dynamic allocation due to more persistent relative differences among the clusters.

EXHIBIT 16

Basic Statistics for Three Momentum Portfolios

Statistics/Parameters	0%	2.5%	6.5%
Return	9.28%	9.81%	9.92%
Volatility	13.26%	12.73%	12.89%
Sharpe Ratio	0.46	0.52	0.53
CAPM Beta	0.86	0.82	0.82
CAPM Alpha	2.42%	3.14%	3.24%
Max Drawdown	–50.02%	–47.23%	–48.03%
α over 0% Portfolio	–	0.88%	1.02%

*Notes: Data from January 1990 to December 2014. Returns, volatilities, and alphas are annualized. Sharpe ratios are computed using the return over U.S. risk-free rate, equal to 3.30% over the period. CAPM betas and alphas are computed with respect to the Market Cap monthly returns. Asterisks refer to significance of loadings: *** = 1%, ** = 5%, * = 10%, none = not significant.*

CONCLUSIONS

We performed a quantitative assessment of similarities among different alternative beta and factor portfolios. This is, to our knowledge, the first attempt in the literature to classify such strategies by their statistical properties rather than by their investment approaches (such as diversification-based, risk-based, etc.). Our study confirms that, on average, alternative beta and long-only factor portfolios are fairly similar (average pairwise correlations of 85.81%), as they are all long-only equity portfolios with fairly diversified stock and sector exposure.

Using a measure of distance based on return correlations, we found three main groups of strategies, centered around market cap, equal weighting, and minimum variance–low volatility. As there is no optimal threshold that would allow us to determine the number of clusters, we proposed an optimization scheme that decides for the optimal number of clusters according to a chosen value of the parsimony parameter. Varying this parameter we found several possible structures of clusters, ranging from the case of a maximal number of clusters (where any difference in behavior is deemed significant and each strategy is treated as a separate cluster), to a case of one unique cluster (where all the differences are disregarded).

The higher the value of the parsimony parameter (i.e., the smaller the number of clusters), the more strategies join the three groups, except for Momentum, which is persistently accounted for as a separate cluster. The cluster around Market Cap is first joined by Diversity Weighting, then by Fundamental Weighting and Value Weighting. The cluster around Equal Weighting is first joined by Risk Efficient and then by Risk Parity and Maximum Diversification. Minimum Variance initially clusters with Low Volatility, and then both are joined by Dividend Investing. For very high values of the parsimony parameter, all the clusters eventually merge together.

Risk Efficient and Equal Weighting on one side and Diversity Weighting and Market Cap on the other side are among the strategies that are very quickly clustered together. If, for the latter pair, the result is obvious, as Diversity Weighting represents a tilted version of market-cap weighting, for the former pair of strategies this return similarity is somewhat surprising, given conceptual differences in their respective approaches (one is a simple diversification rule and the other is an optimized risk-based strategy). A possible explanation might be that portfolio constraints imposed for the Risk Efficient strategy induce a diversification similar to the equal-weight portfolio case.

Minimum Variance and Low Volatility strategies are, as expected, aggregated in one cluster, but the aggregation comes at a higher level of the parsimony parameter than in the case of the two other pairs of strategies mentioned here. This means that, despite the same objective of low portfolio volatility, the approaches of low volatility and minimum variance have some noticeable differences, creating a distance between these two portfolios in terms of correlation.

Our study also provides evidence of superior risk-adjusted performance achieved by cluster-based allocations compared with multi-strategy portfolios directly made of alternative beta strategies. Both in a static and in a dynamic allocation framework, our study highlights how using clusters could improve the efficiency of a multi-strategy portfolio, in terms of performance as well as risk.

APPENDIX A

SIMULATION PARAMETERS

For the Market Cap, Equal Weighting, and Dividend Investing strategies, we consider official index levels available on the respective index provider's website or in Bloomberg: SPX Index for Market Cap, SPW Index for Equal Weighting, and SP-DAUDP Index for Dividend Investing. Other alternative beta strategies are simulated according to their available methodologies. Unless specified otherwise, we assume the strategies are rebalanced quarterly in March, June, September, and December.

Diversity Weighting. The diversity strategy is simulated with power parameter value set at $p = 0.8$.

Risk Parity. The strategy assigns each stock a weight inversely proportional to its volatility. At each rebalancing date volatilities are computed on the previous 250 trading days, ending four days before the rebalancing date to insure replicability.

Maximum Diversification. Volatilities and correlations are computed over 750 trading days ending on the second Friday of the rebalancing quarter. The optimization is performed under constraints to keep the weights lower than the minimum of 1) 1.5% and 2) 20 times the market-capitalization weight.

Risk Efficient. The strategy is simulated according to the FTSE EDHEC-Risk Efficient Index family methodology. The covariance is estimated on weekly returns over two years ending on the first Friday of the rebalancing month. Expected returns are set to be proportional to the downside semi-deviation 25 estimated over the same period as the covariances. The theoretical portfolio (with weights proportional to $\Sigma^{-1}\mu$) is then modified to satisfy the strategy constraints. First, all negative weights are set to 0, and then the portfolio's weights are normalized by $1 - 1/\lambda$, with $\lambda = 3$. We then add $1/\lambda * N$ to each stock weight, N being the number of stocks in the universe. Stock whose weights exceed the upper bound λ/N are set to λ/N, and the remaining weight is reallocated pro rata to the rest of the stocks whose weights are not set by the lower bound, until all stock weights lie between the lower and the upper bounds. For liquidity reasons, stock weights are capped at 10 times the respective market-capitalization weights. We do not implement the turnover mechanism used in the original index, nor do we estimate the covariance matrix by factor model, to insure consistency with the rest of the alternative beta strategies simulated for this study. For the sake of simplicity, we also set the expected return proportional to the downside semi-deviation, whereas in the original index a smoothing procedure is implemented to insure coherence between deciles formed by grouping stocks according to semi-deviation measures.

Minimum Variance. The covariance matrix is computed over 750 trading days ending four days before the rebalancing date. Stocks with more than 10% missing data points over the estimation period are excluded from the optimization procedure. Stock weights are constrained at a maximum of 5%. Furthermore, each sector weight (given by GICS 1 classification) is constrained to be lower than 20%. The strategy is rebalanced monthly, each third Friday of the month.

Dividend Investing. The strategy underlying the S&P 500 Dividend Aristocrat index assigns equal weight to each component of the Market Cap that has increased, at the annual reconstitution, its dividend payouts over the previous 25 years. A stock diversification constraint (at least 40 stocks) and a sector diversification constraint (no more than 30% on each GICS 1 sector) are applied in order to improve portfolio diversification. The 25-year requirement is relaxed if the number of stocks with 25 years of increasing dividend figures to be lower than 40. For the full methodology, we refer to the S&P Index Guide.

Low Volatility. We simulate the index according to the methodology underlying the S&P 500 Low Volatility Index. Each third Friday of February, May, August, and November, we select 100 stocks with the lowest volatilities among the constituents of the S&P 500 Index. Stocks are assigned weights inversely proportional to their volatilities.

Volatility is computed over one year ending on the last trading day of, respectively, January, April, July, and October. We exclude stocks that have more than 10% of missing return observations over the calculation period.

Fundamental Weighting. The strategy reproduces the methodology that underlies the FTSE RAFI US Index (FR10 Index) on the S&P 500 universe. The fundamental weighting factors include dividends, book value, sales, and cash flows. The strategy is rebalanced yearly, each third Friday of March. The new weights are computed four weeks before the rebalancing date on the third Monday of March and the data for estimation are taken ending on the last trading day of January. Stocks with missing data are excluded from the universe. Each stock is assigned a score depending on book values, sales, dividends, and cash flows. A capping procedure depending on stock liquidity is also applied. Details on the parameters can be found in the official methodology of the FTSE RAFI US Index.

Value Weighting. The strategy reproduces the long-only leg of the classic Fama–French High-minus-Low (HML) factor. With monthly rebalancing, on the third Friday of each month, we form the eligible universe of stocks in the S&P 500 Index. We then select the top third of stocks with the highest book-to-price ratio. Stocks are weighted according to their free-float market capitalization. Prices and book values are taken four days before the rebalancing date.

Momentum. The strategy reproduces the long-only leg of the classical Fama–French momentum factor—Up-minus-Down (UMD). With monthly rebalancing, on the third Friday of the month, we form the eligible universe as the stocks in the S&P 500 Index with sufficient data (less than 10% of missing observation in the previous year). We then select the top 20% of stocks with the highest momentum over the previous year. Stocks are weighted according to their free-float market capitalization. Prices are taken four days before the rebalancing date.

APPENDIX B

CLUSTER OPTIMIZATION

To find the optimal cluster identification for the set $S = \{x_1, x_2, \ldots, x_n\}$, when the number of clusters is fixed, we optimize the following:

$$\min_C \sum_{i\le k} \sum_{x\in C_i} d(x, \mu_i)^2 \tag{9}$$

where d is a given metric, C is a partition of the set S, and μ_i is the barycenter of C_i. Note that μ_i, the barycenter of each cluster is a portfolio whose monthly return is given by the average of the returns of the alternative beta strategies that belong to the cluster.

Alternatively, it can be seen as an equal-weighted portfolio of alternative beta strategies that belong to the cluster, where equal weighting is restored every month. This problem can be solved with Lloyd's algorithm (Lloyd [1982]).

K-Means Algorithm with Distances Based on Pairwise Correlations

The set S is given by the alternative beta strategies monthly returns and d is the correlation-based distance (6).

For each $k = 1, \ldots, 12$, which is the number of portfolios in our dataset, we compute the optimal clustering $C(k)$.

To select the optimal number, we first compute, for a given k:

$$f(k) = \sum_{i\le k} \sum_{x\in C_i} \sum_{y\in S\setminus C_i} 1_{\rho(x,\mu_i) > \rho(y,\mu_i)} \tag{10}$$

This function gives a score to the clustering. We effectively assign higher scores to clusters able to identify strategies that appear more correlated to their barycenter than others do.

Note that without penalization, the function f achieves its maximum at k = 12, i.e., when each strategy represents a singleton cluster.

For the penalization we propose the following augmented objective function:

$$\varphi(k) = f(k) - \lambda \cdot \frac{n^2}{4} \cdot k^2 \tag{11}$$

where λ is a user-given parameter that drives the parsimony in the cluster formation and n is the size of S, in our case $n = 12$.

The optimal number of cluster k is chosen as to maximize φ, and the cluster identification is given by $C(k^*)$ previously identified.

K-Means Algorithm with Distances Based on Risk Exposures

The set S is given by the alternative beta strategies factor exposures and d is the standard Euclidean distance. For each $k = 1, \ldots, 12$, we compute the optimal clustering $C(k)$.

To select the optimal number of clusters, we first compute, for a given k:

$$f(k) = \sum_{i\le k} \sum_{x\in C_i} \sum_{y\in S\setminus C_i} 1_{\|x-\mu_i\| < \|y-\mu_i\|} \tag{12}$$

which can be seen as the potential of our clustering. The objective is to maximize this potential while penalizing for the number of clusters. Indeed, the maximum of the value of f is attained for $k = 12$, i.e., by considering each point as a singleton cluster. As before, we propose the following penalization (11), which allows us to find the optimal number of cluster k and the cluster identification $C(k)$.

ENDNOTE

[1]The S&P 500 composition data are courtesy of S&P.

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Practical Applications of

How Different Are Alternative Beta Strategies?

Authors: Carmine de Franco, Bruno Monnier, Johann Nicolle and Ksenya Rulik
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Report Written By: Howard Moore
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Overview

Alternative beta equity strategies provide access to equity markets through non-market-capitalization-weighted portfolios. They can offer reduced risk and enhanced investment returns compared with the overall market; however, no single strategy, or type of strategy, is proven to outperform in all market conditions. Although individual alternative beta strategies are relatively well researched and increasingly understood by investors, choosing among them and combining them for potential diversification benefits still represent a challenge.

Carmine de Franco and his colleagues **Bruno Monnier**, **Johann Nicolle** and **Ksenya Rulik** of **Ossiam** have devised a quantitative approach to compare the different alternative beta strategies based on statistical relationships among their returns. They found that the different alternative beta portfolios, designed to follow different investment objectives, are, on average, quite close to each other. They created clusters of alternative beta portfolios based on their statistical return characteristics and built multi-strategy and multi-factor portfolios based on the clusters, rather than individual investment objectives. By doing so, they found they were able to achieve better risk–return profiles.

Practical Applications

- **Alternative beta equity strategies are more similar to each other than one might think.** Portfolios of seemingly diverse investment objectives fall surprisingly close together, forming distinct clusters of similar risk and return profiles.
- **Investors should be careful when combining strategies.** Those that have different investment objectives actually may be statistically similar in terms of risk and return.
- **Use this technique to determine a strategy's diversification benefits.** Adding a new strategy to a portfolio may or may not provide the expected results, depending on what's already in a portfolio.

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Investors should consider the author's clustering technique.

Practical Applications Report

Most institutional investors understand how alternative beta works and are familiar with the behavioral characteristics of the individual strategies, such as equal weight, minimum variance, momentum, value and others. Each strategy can provide risk-adjusted returns superior to the market over the long term, but they have different performance characteristics in different market conditions. "Each alternative beta strategy has a unique risk and return profile, but there's no unique way to see if one strategy is better than another," says de Franco. Because different alternative beta strategies follow different investment objectives, they are expected to have different statistical properties, such as risk, drawdowns or exposure to equity market risk factors. "It's difficult to know which ones are truly effective," says Monnier.

> "Our results show that, when measured by return co-movements and common systematic risk exposures, different alternative beta portfolios are, on average, quite close to each other."
>
> —Carmine de Franco

Providers of alternative beta solutions offer multi-strategy and multi-factor portfolios, which can smooth returns while still harvesting factor premia. There are a number of ways to combine the individual strategies, but there is no formal framework in which to do so. "There is a lot of debate on how to combine them," says de Franco. "With so many providers introducing new multi-factor and multi-strategy products, we wonder if there could be redundancy and what you might end up with when you put them together." This leaves many important questions unanswered. Do investors have to include all alternative beta available on the market, or just some of them? How best to weight different alternative beta strategies within a multi-strategy portfolio? Should the allocation be static and rules based or dynamic to adjust to changing market conditions?

TAKE NOTE—STATISTICAL RELATIONSHIPS

De Franco and his team designed a quantitative method based on the statistical properties of the strategies' returns to answer these questions. The returns of 12 different portfolios, representing well-known strategies and factor allocations, from 1990 to 2014, were measured through a series of statistical tests. "Our results show that, when measured by return co-movements and common systematic risk exposures, different alternative beta portfolios are, on average, quite close to each other," says de Franco. Some have quite complicated methodologies, but their final results are often similar to very simple strategies. "Sometimes the complexity does not amount to much in terms of final results," says Monnier.

Equal–weighting and risk efficient portfolios: Different methodologies, statistically similar.

In some cases, results were in line with expectations, such as market-cap weighting and diversity weighting. "In other cases, the tests uncover some strategies that are statistically very similar despite following different methodologies," says de Franco. This is the case for the equal-weighting and risk-efficient portfolios. Conversely, there were significant differences between those portfolios that pursue similar investment objectives. "Surprisingly, the returns of some portfolios that have different strategic approaches can be closer than those of two portfolios that use different variations of the same approach," says Monnier. For example, fundamental weighting and value weighting based on book-to-market value are often associated with the same value factor, but their risk–return characteristics are, in fact, markedly different. The same is true for low volatility and minimum variance portfolios, despite often being perceived as interchangeable.

Given the potential redundancy of alternative beta, De Franco and his colleagues applied a clustering technique, which created three distinct groups of statistically similar strategy portfolios gathered around the market-cap-weighted, the equal-weight, and low volatility portfolios, respectively. They then tested multi-strategy allocations based on the statistical clusters versus the individual strategy portfolios.

Many investors believe they can achieve diversification by combining investment strategies that have different objectives. "The cluster-based allocations have less statistical redundancy, so there is better diversification," says Nicolle. An equal-weight allocation based on the clusters exhibits better absolute and risk-adjusted returns, as well as a reduction in drawdown. "We show that you have to differentiate the strategies according to their behavior and not by their methodology," says Monnier. "What may seem different at first may have similar results in the end."

De Franco, Monnier and Nicolle each have scientific and mathematic backgrounds. De Franco enjoys following economic developments, social trends and environmental issues. "I'm interested in the ways that social trends and environmental issues affect investing," he says. Monnier's interests are more technical. "I like to break down concepts and strategies that are complicated—or seemingly complicated—to find the key elements," he says. Nicolle, who worked on this study as an intern and joined Ossiam in May 2016, enjoys discovering new mathematical approaches and applying them to investment strategies. "It's motivating to work with high-caliber people," he says.

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