

COMPUTER FUNDAMENTALS

Number Systems :

- Decimal number → base :- 10 → (0 - 9) digits
- Binary num system → base : 2 → bits : 0 and 1
- Octal number system → base : 8 → digits : 0 to 7
group of 3 binary bits make 1 octal digit.
- Hexadecimal num. system → base : 16 → digits : 0-9 & A-F
where A = 10, B = 11, C = 12, ... F = 15
4 binary bits = 1 Hex. digit

> Conversions :-

- any base to decimal

multiply each digit by its corresponding power of base
and summing the result. (start from LSB (right side))

- decimal to any base

Divide the decimal number repeatedly by the target base
and note the remainders. Remainders in reverse order
will be the required conversion.

• ANY BASE to DECIMAL

1) Binary to decimal

multiply each bit by 2^n , starting from the right.

$$1) (1101)_2 = (\quad)_{10}$$

→ Binary : 1 1 0 1
 2^3 2^2 2^1 2^0

multiply each binary digit by 2 raised to its position
 and add them

$$\begin{aligned} &= 2^0 \times 1 + 2^1 \times 0 + 2^2 \times 1 + 2^3 \times 1 \\ &= 1 + 0 + 4 + 8 \\ &= 13 \end{aligned}$$

$$\therefore (1101)_2 = (13)_{10}$$

$$2) (101011)_2 = (\quad)_{10}$$

→ Binary : 1 0 1 0 1 1
 2^5 2^4 2^3 2^2 2^1 2^0

$$\begin{aligned} &= 2^0 \times 1 + 2^1 \times 1 + 2^3 \times 1 + 2^5 \times 1 \\ &= 1 + 2 + 8 + 32 \\ &= 43 \end{aligned}$$

$$\therefore (101011)_2 = (43)_{10}$$

$$3) (11000111)_2 = (?)_{10}$$

→ Binary : $\begin{array}{cccccccc} 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$

$$\begin{aligned} &= 2^7 \times 1 + 2^6 \times 1 + 2^5 \times 1 + 2^4 \times 1 + 2^3 \times 1 \\ &= 1 + 2 + 4 + 64 + 128 \\ &= 199 \end{aligned}$$

$$\therefore (11000111)_2 = (199)_{10}$$

④

2) Octal to decimal

multiply each digit by 8^n , [right → left]

$$1) (357)_8 = (?)_{10}$$

→ Octal : $\begin{array}{ccc} 3 & 5 & 7 \\ 8^2 & 8^1 & 8^0 \end{array}$

$$\begin{aligned} &= 1 \times 7 + 8 \times 5 + 64 \times 3 \\ &= 239 \end{aligned}$$

$$\therefore (357)_8 = (239)_{10}$$

$$2) (4725)_8 = (?)_{10}$$

→ Octal : $\begin{array}{cccc} 4 & 7 & 2 & 5 \\ 8^3 & 8^2 & 8^1 & 8^0 \end{array}$

$$\begin{aligned}
 &= 5 + 2 \times 8 + 7 \times 64 + 4 \times 512 \\
 &= 5 + 16 + 448 + 2048 \\
 &= 2517
 \end{aligned}$$

3) Hexadecimal to decimal

multiply each digit by 16^n
 replace (A to F) with its decimal equivalent

1) $(ABC)_{16} = (?)_{10}$

→ Hexadecimal : A B C
 16^2 16^1 16^0

where A = 10, B = 11, C = 12

$$\begin{aligned}
 &= 16^0 \times 12 + 16^1 \times 11 + 16^2 \times 10 \\
 &= 12 + 176 + 2560 \\
 &= 2748
 \end{aligned}$$

$\therefore (ABC)_{16} = (2748)_{10}$

2) $(4C)_{16} = (?)_{10}$

→ hex : 4 C
 16^1 16^0

where C = 12

$$\begin{aligned}
 &= 1 \times 12 + 16 \times 4 \\
 &= 76
 \end{aligned}$$

$\therefore (4C)_{16} = (76)_{10}$

3) $(4C8)_{16} = (?)_{10}$

→ Then : $\begin{array}{ccc} 4 & C & 8 \\ 16^2 & 16^1 & 16^0 \end{array}$

$$= 8 + 16 \times 12 + 256 \times 4$$

$$= 1224$$

$$\therefore (4C8)_{16} = (1224)_{10}$$

- DECIMAL to ANY BASE

- 1) Decimal to binary

divide by 2 and write down remainders. (↑)

1) $(125)_{10} = (?)_2$

→ Q R

$$125 \div 2 = 62 \quad 1$$

$$62 \div 2 = 31 \quad 0$$

$$31 \div 2 = 15 \quad 1$$

$$15 \div 2 = 7 \quad 1$$

$$7 \div 2 = 3 \quad 1$$

$$3 \div 2 = 1 \quad 1$$

$$1 \div 2 = 0 \quad 1$$

∴ $(125)_{10} = (1111101)_2$

2) $(1473)_{10} = (?)_2$

\rightarrow

Q	R
$1473 \div 2 = 736$	1
$736 \div 2 = 368$	0
$368 \div 2 = 184$	0
$184 \div 2 = 92$	0
$92 \div 2 = 46$	0
$46 \div 2 = 23$	0
$23 \div 2 = 11$	1
$11 \div 2 = 5$	1
$5 \div 2 = 2$	1
$2 \div 2 = 1$	0
$1 \div 2 = 0$	1

$\therefore (1473)_{10} = (10111000001)_2$

2) Decimal to octal

1) $(247)_{10} = (?)_8$

\rightarrow

Q	R
$247 \div 8 = 30$	7
$30 \div 8 = 3$	6
$3 \div 8 = 0$	3

$\therefore (247)_{10} = (367)_8$

2) $(1473)_{10} = (?)_8$

$\rightarrow +$

$$\begin{array}{r}
 & Q & R \\
 1473 \div 8 & = 184 & 1 \\
 184 \div 8 & = 23 & 0 \\
 23 \div 8 & = 2 & 7 \\
 2 \div 8 & = 0 & 2
 \end{array}$$



$\therefore (1473)_{10} = (2701)_8$

3) Decimal to hexadecimal

divide the decimal num. by 16 & write down the remainder. if the remainder is 10 to 15, convert it to A to F.

1) $(199)_{10} = (?)_{16}$

$$\begin{array}{r}
 & Q & R & \text{Hex} \\
 \rightarrow 199 \div 16 & = 12 & 7 & 7 \\
 12 \div 16 & = 0 & 12 & C
 \end{array}$$

$\therefore (199)_{10} = (C7)_{16}$

2) $(2156)_{10} = (?)_{16}$

$$\begin{array}{r}
 & Q & R & \text{Hex} \\
 \rightarrow 2156 \div 16 & = 134 & 12 & C \\
 134 \div 16 & = 8 & 6 & 6 \\
 8 \div 16 & = 0 & 8 & 8
 \end{array}$$

$\therefore (2156)_{10} = (86C)_{16}$

Conversions

Decimal	Binary	Octal	Hexadecimal
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0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17
24	11000	30	18
25	11001	31	19
26	11010	32	1A

> Binary to octal

Group 3 binary bits from right to left
 If needed, add leading zeroes to make complete groups of 3
 convert each 3-bit group to its octal equivalent

$$1) (101011)_2 = (?)_8$$

→ Binary : 1 0 1 0 1 1
 ↓ ↓
 Octal : 5 3

$$\therefore (101011)_2 = (53)_8$$

$$2) (1101101)_2 = (?)_8$$

→ (add leading zeros)

∴ Binary : 001101101
 ↓ ↓ ↓
 Octal 1 5 5

$$\therefore (1101101)_2 = (155)_8$$

$$3) (110111000)_2 = (?)_8$$

→ Binary : 110 111 000
 ↓ ↓ ↓
 6 7 0

$$\therefore (110111000)_2 = (670)_8$$

> Octal to Binary

Convert each octal digit into its 3-bit binary number.

Combine all binary groups to get the final binary num.

$$1) \ (125)_8 = (\)_2$$

$$\rightarrow \text{Octal} : \begin{array}{ccc} 1 & 2 & 5 \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$\text{binary} : \begin{array}{ccc} 001 & 010 & 101 \end{array}$$

$$\therefore (125)_8 = (001010101)_2 \text{ or } (1010101)_2$$

$$2) \ (47)_8 = (\)_2$$

$$\rightarrow \text{Octal} : \begin{array}{cc} 4 & 7 \\ \downarrow & \downarrow \end{array}$$

$$\text{binary} : \begin{array}{cc} 100 & 111 \end{array}$$

$$\therefore (47)_8 = (100111)_2$$

$$3) \ (155)_8 = (\)_2$$

$$\rightarrow \text{Octal} : \begin{array}{ccc} 1 & 5 & 5 \\ \downarrow & \downarrow & \downarrow \end{array}$$

$$\text{Binary} : \begin{array}{ccc} 001 & 101 & 101 \end{array}$$

$$\therefore (155)_8 = (001101101)_2 \text{ or } (1101101)_2$$

> Binary to Hexadecimal

Group 4 binary bits, starting from right to left
 Add leading zeros if needed to complete a
 4-bit group.
 convert each group into its hex. equivalent.

$$1) (11110101)_2 = (?)_{16}$$

$$\begin{array}{r} \rightarrow \text{Binary: } 1111 \ 0101 \\ & \quad \downarrow \quad \downarrow \\ \text{Hex: } & F \quad 5 \end{array}$$

$$\therefore (11110101)_2 = (F5)_{16}$$

$$2) (1001101111)_2 = (?)_{16}$$

$$\begin{array}{r} \rightarrow \text{Binary: } 0010 \ 0110 \ 1111 \quad (\text{add leading zeros}) \\ & \quad \downarrow \quad \downarrow \quad \downarrow \\ \text{Hex: } & 2 \quad 6 \quad F \end{array}$$

$$\therefore (1001101111)_2 = (26F)_{16}$$

> Hexadecimal to binary

Write each hex digit as a 4-bit binary num.
combine all binary groups.

$$1) (3A7)_{16} = (?)_2$$

→ Hex : 3 A 7

binary : 0011 1010 0111

$$\therefore (3A7)_{16} = (001110100111)_2$$

$$2) (FAE1)_{16} = (?)_2$$

→ Hex : F A E 1

binary : 1111 1010 1110 0001

$$\therefore (FAE1)_{16} = (1111101011100001)_2$$

$$3) (7B6C)_{16} = (?)_2$$

→ Hex : 7 B 6 C

Binary : 0111 1011 0110 1100

$$\therefore (7B6C)_{16} = (0111101101101100)_2$$

any base → binary → any base

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> Octal to Hexadecimal

- 1) Convert octal to binary \Rightarrow each octal digit = 3 bit binary
- 2) Group the binary digits in 4s from right to left (add leading zeros if needed)
- 3) convert each 4-bit group to hex.

$$1) (157)_8 = ()_{16}$$

$$\begin{array}{ccc} \rightarrow & \text{Octal} & : \quad 1 \quad 5 \quad 7 \\ & \text{binary} & 001 \quad 101 \quad 111 \end{array}$$

$$\therefore (157)_8 = (00110111)_2$$

$$\begin{array}{lll} \text{Now, } \text{Binary} & : & 0000 \quad 0110 \quad 1111 \\ \text{Hex} & : & 0 \quad 6 \quad F \end{array}$$

$$\therefore (157)_8 = (6F)_{16}$$

$$2) (726)_8 = ()_{16}$$

$$\begin{array}{cccc} \rightarrow & \text{Octal} & : & 7 \quad 2 \quad 6 \\ & \text{Binary} & : & 111 \quad 010 \quad 110 \end{array}$$

$$(726)_8 = (111010110)_2$$

$$\begin{array}{lll} \text{Binary} & : & 0001 \quad 1101 \quad 0110 \\ \text{Hex} & : & 1 \quad D \quad 6 \end{array}$$

$$\therefore (726)_8 = (1D6)_{16}$$

Octal \rightarrow Hex \Rightarrow Oct \rightarrow Binary \rightarrow Hex
Hex \rightarrow Octal \Rightarrow Hex \rightarrow bin \rightarrow oct

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Hexadecimal to octal

Convert hex to binary
each hex digit = 4-bit binary

Group the binary digits in 3s from right to left.
convert each 3-bit to octal.

1) $(2F3)_{16} = (?)_8$

\rightarrow Hex : 2 F 3
Binary: 0010 1111 0011

$\therefore (2F3)_{16} = (0010\ 1111\ 0011)_2$

Binary : 001 011 110 011
Octal : 1 3 6 3

$\therefore (2F3)_{16} = (1363)_8$

2) $(A9B)_{16} = (?)_8$

\rightarrow Hex : A 9 B
Binary: 1010 1001 1011

$\therefore (A9B)_{16} = (1010\ 1001\ 1011)_2$

Binary : 101 010 011 011
Octal : 5 2 3 3

$\therefore (A9B)_{16} = (5233)_8$

#. Binary addition :-

A	B	$A + B$	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1
1	1	+1	1

Ex: 1) $10101 + 11001 = 10110$ 2) $\begin{array}{r} \boxed{1} \\ 10101 \\ + 11001 \\ \hline \boxed{1}01110 \end{array}$

\rightarrow

$\Rightarrow 11011$

#. Binary subtraction :-

A	B	$A - B$	Borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

1) $\begin{array}{r} \boxed{0}10 \\ 1 + 0 0 \\ - 1010 \\ \hline 0010 \end{array}$

2) $\begin{array}{r} \boxed{1}\boxed{0}\boxed{1}0 \\ 10 + 0 \\ - 0011 \\ \hline 0111 \end{array}$

3) $\begin{array}{r} \boxed{1}0\boxed{1}0 \\ 11 + 0 \\ - 1101 \\ \hline 0001 \end{array}$

Binary num. Representation

1) Unsigned numbers

- Represents only +ve integers
- All bits are used to represent the value (magnitude)
- No sign bit is used.

Range : 0 to $(2^n - 1)$ For n bits [0 to 15]
For 4-bit

Ex : 1111 = 15 , 0001 = 1

2) Signed - Binary Repres. (Sign-magnitude)

- used to represent +ve & -ve numbers.
- The MSB (Most Significant Bit) is the sign bit:
 - 0 → +ve
 - 1 → -ve
- The remaining bits represent the magnitude (value).

Ex :

Binary	Sign	Magnitude	decimal
0101	0	101	+5
1101	1	101	-5

Range : - $(2^{n-1} - 1)$ to $(2^{n-1} - 1)$ [-7 to +7]
For 4 bit

- a) 1's complement
- b) 2's complement

a) 1's complement

- Negative num. are represented by inverting all bits of the +ve binary num.
- MSB acts as sign bit.
- Has two zeros : +0 & -0

Flip all bits ($0 \rightarrow 1$, $1 \rightarrow 0$)

Range : $-(2^{n-1} - 1)$ to $(2^{n-1} - 1)$ -- For n-bits
 -7 to $+7$ for 4-bits

Ex : $+5 = 0101$

$-5 = 1010$ (in 1's comp.)

$+0 = 0000$

$-0 = 1111 \rightarrow$ (1's comp. of 0000)

Hence, 1's comp. has two representations for zero.

b) 2's complement

- Negative num. are represented by :
 - Taking the 1's complement, and
 - Add 1 to the result.

Range : $-(2^{n-1})$ to $(2^{n-1} - 1)$ for n-bits
 $\text{For 4-bits } -8 \text{ to } +7$

Ex : Represent -7 in 2's comp. (4-bit)

$\rightarrow +7 = 0111$

1's complement $\rightarrow 1000$

$$\text{Add } 1 \rightarrow 1000 + 1 = 1001$$

$$\therefore -7 = 1001 \quad (\text{in 2's comp})$$

2) Represent -5 in 2's complement (4-bit)

$$\rightarrow +5 = 0101$$

$$1\text{'s comp} \rightarrow 1010$$

$$\text{add } 1 \rightarrow 1010$$

$$+ 1$$

$$\hline 1011$$

$$\therefore -5 = 1011 \quad (\text{in 2's comp})$$

• Single zero representation in 2's complement

$$+0 = 0000$$

$$1\text{'s comp} \rightarrow 1111$$

$$\text{add } 1 \rightarrow 1111$$

$$+ 1$$

$$\hline 10000 \quad (5\text{-bits})$$

discard the leftmost carry in 4-bit representation.

$$\therefore -0 = 0000, \text{ same as } +0$$

* Shortcut for 2's comp.

1) Start from LSB (right)

2) Copy all bits as they are until u copy the 1st 1.

3) After that invert all remaining bits

$$+5 = 1010 \rightarrow \text{copy this as it is after that invert all}$$

$$-5 = 011$$

$$10100 \rightarrow 01100$$

\uparrow
after this invert

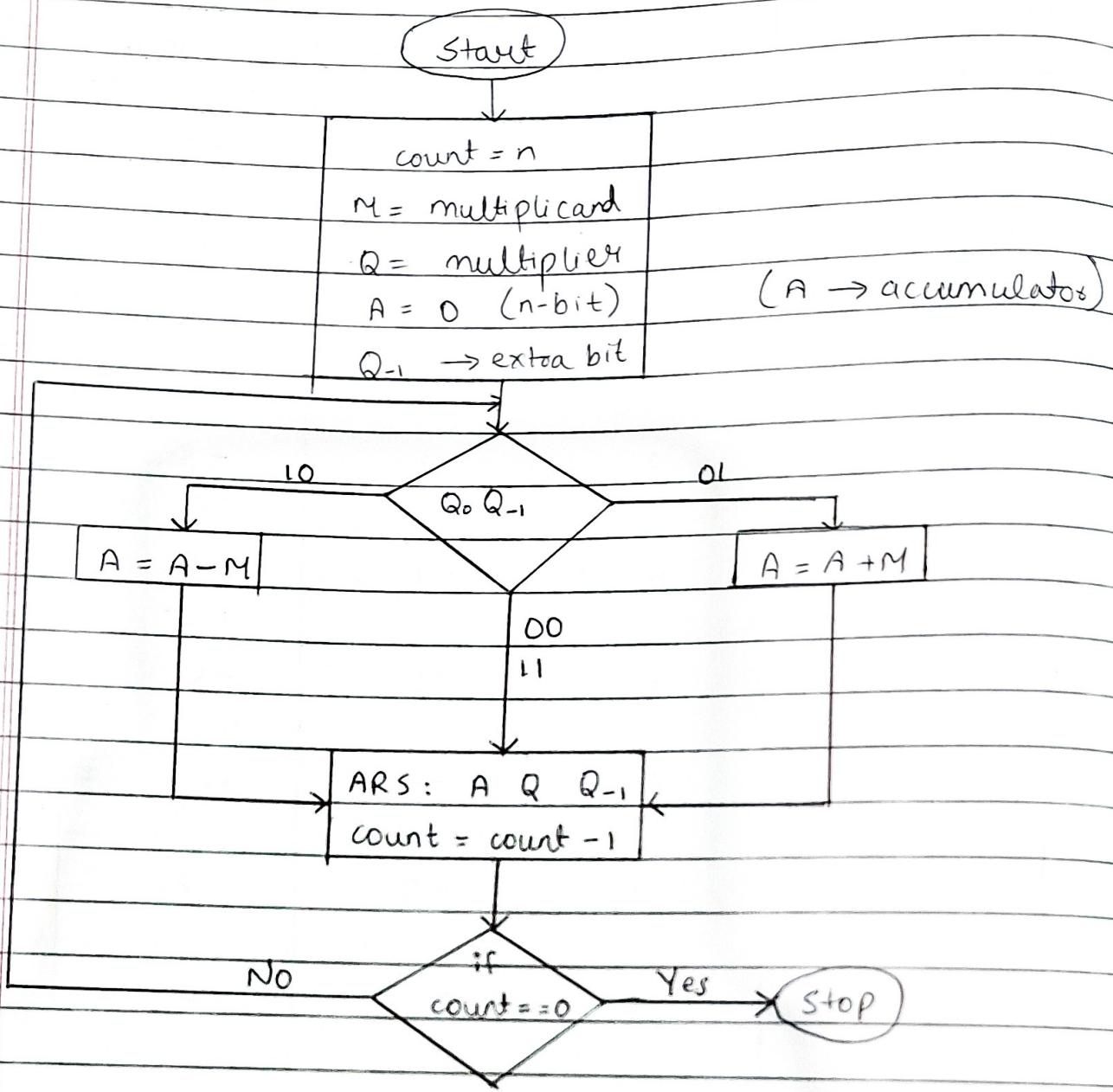
#. 4-bit binary representations :-

Binary	Unsigned	Signed	1's comp.	2's comp.
0000	0	+0	+0	+0
0001	1	+1	+1	+1
0010	2	+2	+2	+2
0011	3	+3	+3	+3
0100	4	+4	+4	+4
0101	5	+5	+5	+5
0110	6	+6	+6	+6
0111	7	+7	+7	+7
1000	8	-0	-0	-8
1001	9	-1	-6	-7
1010	10	-2	-5	-6
1011	11	-3	-4	-5
1100	12	-4	-3	-4
1101	13	-5	-2	-3
1110	14	-6	-1	-2
1111	15	-7	-0	-1

ARS → arithmetic right shift

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Booth's Algorithm (multiplication algo)



1) Solve: 5×3 by booth's multiplication

$$\rightarrow M = 5 \rightarrow 0101 \quad |, 4\text{-bit}$$

$$Q = 3 \rightarrow 0011$$

$$A = 0000$$

$$Q_{-1} = 0 \quad (\text{extra bit})$$

count = 4 (bcz we are using 4-bit nums)

$$-M = -5 \rightarrow 1011$$

count	A	Q	Q_{-1}	Operation
4	0000	0011	0	$A = A - M$
	1011	0011	0	$0000 + 1011$
	1101	1001	1	ARS
3	1110	1100	1	$A = A - M$ ARS
2	0011	1100	1	$A = A + M = 1110 + \dots$
	0001	1110	0	ARS
1	0000	1111	0	ARS

$$AQ = (00001111)_2 = (15)_{10}$$

2) Solve : $5 \times (-3)$ using booth's algo.

$$\rightarrow M \rightarrow 5 \equiv 0101, \quad -M = -5 \equiv 1011$$

$$Q \rightarrow -3 \equiv 0011, \quad Q = 1101$$

$$Q \rightarrow -3 = 1101, \quad \text{as } [+3 = 0011, -3 = 1101]$$

count	A	Q	Q_{-1}	Operation
4	0000	1101	0	
	1011	1101	0	$A = A - M$
	1101	1110	1	ARS

3	0010 0001	1110 0111	1 0	$A = A + M$ ARS
2	1100 1110	0111 0011	0 1	$A = A - M$ ARS
1	<u>1111</u>	<u>0001</u>	1	ARS

$$AQ = (11110001)_2 = (-15)$$

3) Solve: $(-5) \times 3$

$$\rightarrow M \rightarrow -5 \equiv 1011, \quad -M \rightarrow 5 = 0101$$

$$Q \rightarrow 3 = 0011$$

Count	A	Q	Q_{-1}	Operation
4	0000 0101 0010	0011 0011 1001	0 0 1	
3	0001	0100	1	ARS
2	1100 1110	0100 0010	1 0	$A = A + M$ ARS
1	<u>1111</u>	<u>0001</u>	0	ARS

$$AQ = (11110001)_2 = (-15)_{10}$$

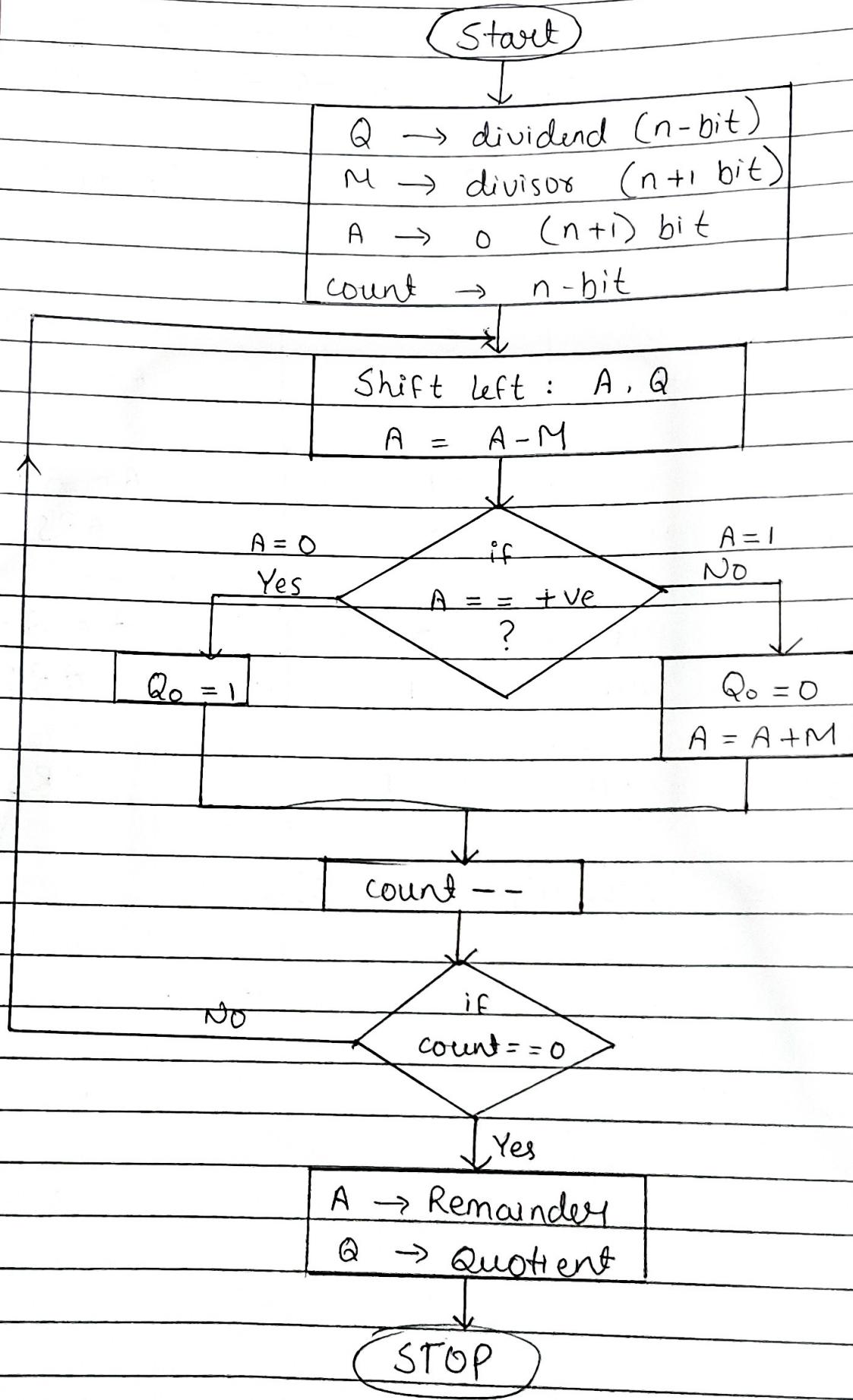
$$4) (-5) \times (-3)$$

$\rightarrow M \rightarrow -5 = 1011$, $-M \rightarrow 5 = 0101$
 $Q \rightarrow -3 = 1101$, $-Q \rightarrow 3 = 0011$

Count	A	Q	Q_{-1}	Operation
4	0000	1101	0	
	0101	1101	0	$A = A - M$
	0010	1110	1	ARS
3	1101	1110	1	$A = A + M + 1011$
	1110	1111	0	ARS
2	0011	1111	0	$A = A - M + 1010$
	0001	1111	1	ARS
1	0000	1111	1	ARS

$$\therefore A Q = (0000, 1111)_2 = (15)_{10}$$

#. Restoring division Algorithm



i) Solution : $6 \div 4$

$$\rightarrow Q \rightarrow 6 = 0110 \quad (\text{n-bit})$$

$$M \rightarrow 4 = 00100, \quad -M \rightarrow -4 = 11100 \quad [\text{s-bit}]$$

$$A = 00000 \quad [(\text{n+1}) \text{ bit}]$$

$$\text{count} = n = 4$$

count	A	Q	Operation
4	00000	0110	
	00000	110 <input type="checkbox"/>	Shift left
	11100	110 <input checked="" type="checkbox"/>	$A = A - M$
	00000	1100	$A = A + M$
3	00001	100 <input type="checkbox"/>	Shift left
	11101	100 <input checked="" type="checkbox"/>	$A = A - M$
	00001	1000 <input checked="" type="checkbox"/>	$A = A + M$
2	00011	000 <input type="checkbox"/>	Shift left
	11111	000 <input checked="" type="checkbox"/>	$A = A - M$
	00011	0000	$A = A + M$
1	00110	000 <input type="checkbox"/>	Shift left
	00010	000 <input checked="" type="checkbox"/>	$A = A - M$
	↓ remainder	↓ quotient	

$$\therefore \text{Quotient } Q = 0001 = (1)_{10}$$

$$\text{Rem. } A = 00010 = (2)_{10}$$

$$2) 7 \div 2$$

$\rightarrow Q \rightarrow 7 = 0111$ (4 bit), $A = 00000$ (5-bit)
 $M \rightarrow 2 = 00010$ (5 bit), $-M = 11110$

count	A	Q	operation
4	00000	0111	
	00000	111 <input type="checkbox"/>	Shift left $A = A - M$
	11110	111 <input checked="" type="checkbox"/> 0	
	00000	111 0	$A = A + M$
3	00001	110 <input type="checkbox"/>	Shift left
	11111	111 <input checked="" type="checkbox"/> 0	$A = A - M$
	00001	111 0	$A = A + M$
2	00011	110 <input type="checkbox"/>	Shift left
	00001	100 <input checked="" type="checkbox"/> 1	$A = A - M$
1	00011	001 <input type="checkbox"/>	Shift left
	00001	0011	$A = A - M$
	↓ rem.	↓ Q	

$$\therefore \text{remainder} = (00001)_2 = (1)_{10}$$

$$\text{quotient} = (0011)_2 = (3)_{10}$$

$$3) \quad 11 \div 5$$

$$\rightarrow Q = 11 \rightarrow 1011$$

$$M = 5 \rightarrow 00101 \Rightarrow -M = 11011$$

coent	A	Q	Operation
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4	00000	1011	
	00001	011 □	SL
	11100	0110	$A = A - M$
	00001	0110	$A + = M$

3	00010	110 □	SL
	11101	1100	$A = A - M$
	00010	1100	$A = A + M$

2	00101	100 □	SL
	00000	1001	$A = A - M$

1	00001	001 □	SL
	11100	0010	$A = A - M$
	00001	0010	$A = A + M$

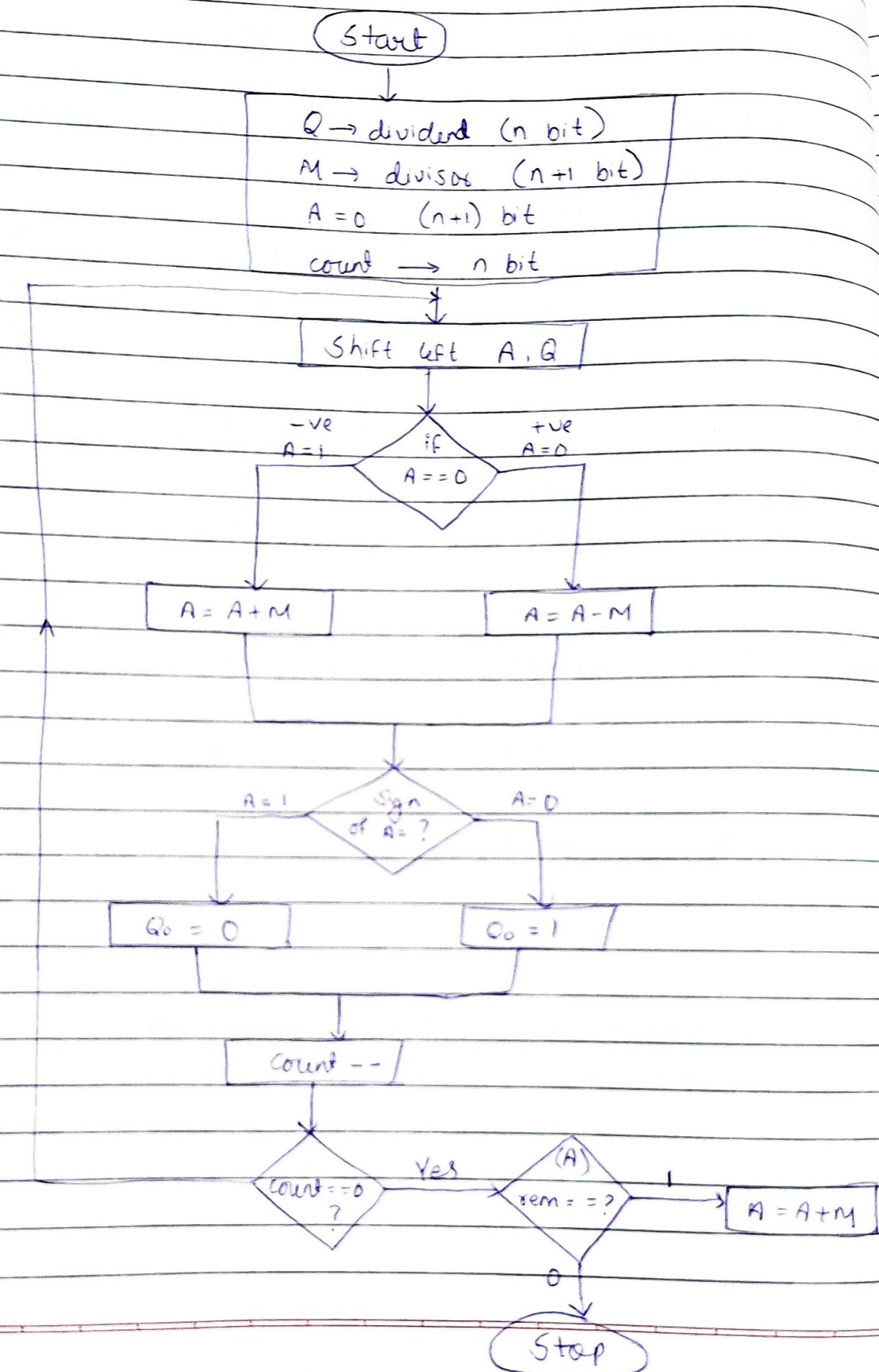
↓ ↓

rem . quot .

$$\therefore \text{remainder} = (00001)_2 = (1)_{10}$$

$$\text{quotient} = (0010)_2 = (2)_{10}$$

> Non-restoring



1) $8 \div 3$ by non-restoring --
 $\rightarrow Q \rightarrow 8 = 1000$
 $M \rightarrow 3 = 00011$, $-M \rightarrow 11101$
 $A = 00000$

count	A	Q	Operation
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4	00000	1000	
	00001	000□	SL
	11110	000□0	$A = A - M$

3	11100	000□	SL
	11111	000□0	$A = A + M$

2	11110	000□	SL
	00001	000□1	$A = A + M$

1	00010	001□	SL
	11111	0010	$A = A - M$

0	00010	0010	$A = A + M$
	↓ rem	↓ Quot.	

$$\text{remainder} = (00010)_2 = (2)_{10}$$

$$\text{Quotient} = (0010)_2 = (2)_{10} \quad //$$

2) $9 \div 5$, by non-restoring -----
 $\rightarrow Q \rightarrow 9 \rightarrow 1001$
 $M = 5 \rightarrow 00101$, $-M = 11011$

Count

A

Q

Operation

4

00000

1001

00001

001□

SL

11100

0010

 $A = A - M$

3

11000

010□

SL

11101

01010

 $A = A + M$

2

10010

100□

SL

11111

1000

 $A = A + M$

1

11111

000□

SL

00100

0001

 $A = A + M$

$$\text{remainder} = (00100)_2 = (4)_{10}$$

$$\text{quotient} = (0001)_2 = (1)_{10}$$

3) $-(8 \div 3)$

$\rightarrow Q \rightarrow 8 \rightarrow 1000$
 $M \rightarrow 3 \rightarrow 00011$, $-M = 11101$
 $A = 00000$

count

A

Q

operation

4

00000

1000

00001

000□

SL

11110 \rightarrow

000□0

A = A - M

3

11100

000 □

SL

11101

0000'

A = A + M.

2

11110

000 □

SL

00001

0001

A = A + M

1

00010

001 □

SL

11111

0010

A = A - M

0

00010

0010

A = A + M

$$\text{rem} = (00010)_2 = (2)_{10}$$

quotient \rightarrow take 2's complement

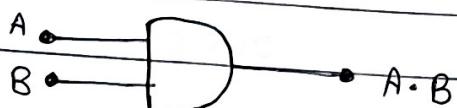
$$\therefore Q = 1110$$

#- ~~Bo Logic gates~~

> Basic gates

1) AND gate

O/p is high only if all inputs are high

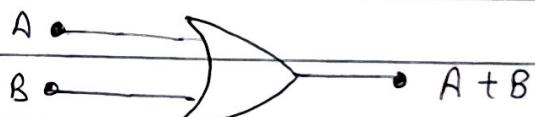


Boolean eqⁿ: $A \cdot B$

A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

2) OR gate

O/p is high if atleast one i/p is high

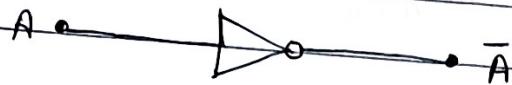


Boolean eqⁿ: $A + B$

A	B	$A + B$
0	0	0
0	1	1
1	0	1
1	1	1

3) NOT gate

it inverts the binary r/p
i.e o/p is the opposite of the i/p .



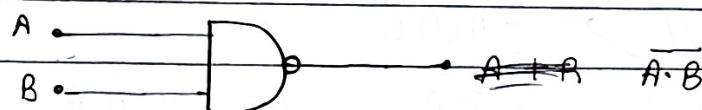
Boolean eqⁿ : \bar{A}

A	\bar{A}
0	1
1	0

> Universal gates

1) NAND gate

o/p is low only when all inputs are high .

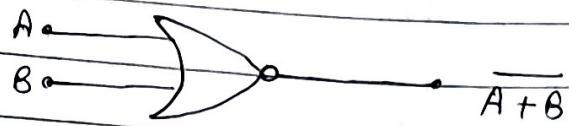


Boolean eqⁿ : $\bar{A} \cdot \bar{B}$

A	B	$\bar{A} \cdot \bar{B}$
0	0	1
0	1	1
1	0	1
1	1	0

2) NOR gate

o/p is high only when all inputs are low

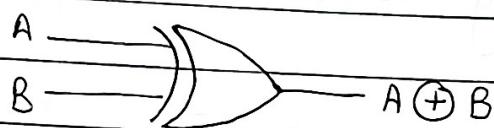


Boolean eqⁿ: $\overline{A+B}$

A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

> EX-OR gate (Exclusive OR) / XOR

o/p is high only when inputs are different.



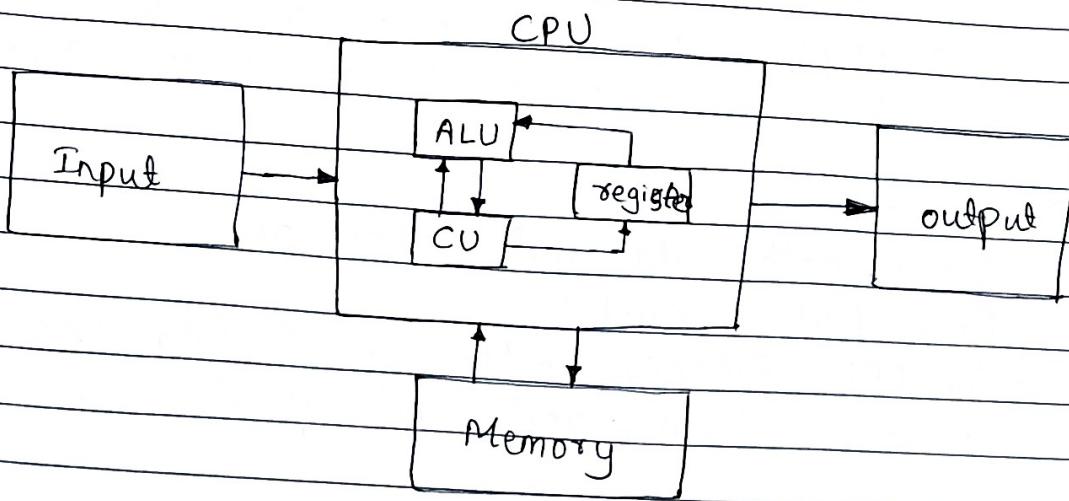
Boolean eqⁿ: $A \oplus B = A\bar{B} + \bar{A}B$

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

if $A = B \Rightarrow$ o/p ~~is~~ low

if $A \neq B \Rightarrow$ o/p high

#. Von Neuman Architecture



1) Input

- > The i/p unit provides data & instructions to the computer system.
- > Ex: keyboard, mouse, Scanner.
- > data goes from input to CPU

2) CPU (central Processing Unit)

The CPU is the brain of the computer. It is divided into different components as shown:

- > ALU (arithmetic logic unit)
 - Performs arithmetic operations (addition, subtraction, multiplication, division) and logical operations (comparison, AND, OR, NOT).
 - It is connected with control unit & registers
- > CU (control unit)
 - directs the flow of data & instructions b/w i/p, o/p, memory, and ALU.
 - Acts like the "traffic manager" of the CPU.
 - It fetches instructions from memory, decodes them, and tells the ALU / register what to do.

> Registers

- Small, high-speed storage inside the CPU
- temporarily stores instructions, memory addresses, or intermediate results for faster processing.

3) Memory

- Stores both data and instructions
- Ex: RAM, Cache
- The CPU fetches instructions & data from memory, processes them, and sends results back if needed.

4) Output

- The o/p unit shows the processed data (results) to the user.
- Ex: Monitor, Printer, Speakers
- Data goes from CPU to o/p.

> Working Principle :-

- 1) Input gives data/instructions to CPU
- 2) Memory stores these data & instructions
- 3) Control unit (CU) fetches instructions from memory, decodes them, & co-ordinates with ALU & registers.
- 4) ALU performs required operations (arithmetic / logical)
- 5) Registers temporarily hold data & intermediate results.
- 6) CPU sends the final result to output unit.

. IEEE 754 floating point representation & conversion

it is a way to represent real numbers in binary .

1) Single Precision

- > total size : 32 bits
- > Sign bit : 1 bit
- > Exponents : 8 bits
- > Mantissa (fraction) : 23 bits
- > Excess - 127

Sign	Exponent	Mantissa
1-bit	8-bit	23 bits

∴ IEEE 754 has 8 exponent bits

$$2^8 = 256 \quad (\text{both +ve \& -ve})$$

$$2^7 = 128$$

∴ range : (0 - 127)

$$(2^{k-1} - 1 = 2^7 - 1)$$

.. Excess - 127

2) Double Precision

- > total size : 64 bits
- > Sign bit : 1 bit
- > Exponent : 11 bits
- > Mantissa (fraction) : 52 bits
- > Excess - 1023 ($2^{11-1} - 1 = 2^{10} - 1 = 1023$)

Sign	Exponent	Mantissa
1-bit	11-bits	52 bits

64 bits

$$1) (85.125)$$

→ Step 1: Convert 85 (integer part) to binary

2	85	1	↑
2	42	0	
2	21	1	
2	10	0	
2	5	1	
2	2	0	
	1	1	

$\therefore (85)_{10} = (1010101)_2$

Step 2: Convert 0.125 (Fractional part) to binary
Multiply by 2 repeatedly

$$0.125 \times 2 = 0.25 \rightarrow 0$$

$$0.25 \times 2 = 0.5 \rightarrow 0$$

$$0.5 \times 2 = 1.0 \rightarrow 1$$

$$\therefore (0.125)_{10} = (0.001)_2$$

$$\therefore (85.125)_{10} = (1010101.001)_2$$

Step 3:

Change the number in the form: $1.x \times 2^E$

$$\therefore 1010101.001 = 1.010101001 \times 10^6$$

$$\text{Mantissa} = 010101001$$

$$\text{Exponent } E = 6$$

Step 4: Find the exponent field (8-bits)
use excess $B - 127$ format

$$\text{Encoded exponent} = 6 + 127 = 133$$

2 133	1	
2 66	0	
2 33	1	
2 16	0	
2 8	0	
2 4	0	
2 2	0	
	1	1

$\therefore (133)_{10} = (10000101)_2$

Step 5: Sign bit

$\because 85.125$ is positive \Rightarrow sign bit = 0

\Rightarrow IEEE 754 - 32 bit (single Precision)

$85.125 = \boxed{0} \quad \boxed{10000101} \quad \boxed{010101001000000000000000}$
 1-bit 8-bits 23-bits

\rightarrow For double precision

as $(85.125)_{10} = (1010101.001)_2$

also $(1010101.001)_2 = 1.010101001 \times 10^6$

Mantissa = 010101001

Exponent E = 6

Exponent field (11-bits) \Rightarrow Excess - 1023

\therefore Encoded exponent = $6 + 1023 = 1029$

1029	2	1029	1
514	2	514	0
257	2	257	0
128	2	128	0
64	2	64	0
32	2	32	0
16	2	16	0
8	2	8	0
4	2	4	0
2	2	2	0
		1	1

$$\therefore (1029)_{10} = (10000000101)_2$$

IEEE - 754 : 64-bits

\Rightarrow

0	10000000101	010101001000 (52 bits)
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2) (74.625)

$$\rightarrow (74)_{10} = (1001010)_2$$

2	74	0	↑
37	2	1	
18	2	0	
9	2	1	
4	2	0	
2	2	0	
	1	1	

$$0.625 \times 2 = 1.250 \rightarrow 1$$

$$0.250 \times 2 = 0.5 \rightarrow 0$$

$$0.5 \times 2 = 1.0 \rightarrow 1$$

$$(0.625)_{10} = (0.101)_2$$

$$\therefore (74.625)_{10} = (1001010.101)_2$$

$$= 1.001010101 \times 10^6$$

\therefore Mantissa = 001010101
 $E = 6$

$$\text{Encoded exponent} = 6 + 127 = 133$$

$$(133)_{10} = (10000101)_2$$

→ Single Precision :

0	10000101	00101010100.....0
1 bit	8 bit	23 bit

For double precision,

$$E = 6$$

$$M = 001010101$$

$$\text{Excess - } 1023,$$

$$\text{Encoded-exponent} = 6 + 1023 = 1029$$

$$(1029)_{10} = (10000000101)_2$$

\therefore double precision:

0	10000000101	001010101000 00
1 bit	11 bits	52 bits