Tutorial - 1

1) Reduce the matrix to diagonal form

$$\begin{bmatrix} -3 & 2+2i \\ 2-2i & 4 \end{bmatrix}$$

→ characteristic equation is given as $|A - \lambda I|$

$$\therefore \begin{vmatrix} -3 & 2+2i \\ 2-2i & 4 \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix}$$

$$= \begin{vmatrix} -3-\lambda & 2+2i \\ 2-2i & 4-\lambda \end{vmatrix}$$

$$\therefore |A - \lambda I| = (-3-\lambda)(4-\lambda) - (2+2i)(2-2i)$$

$$= -12 + 3\lambda - 4\lambda + \lambda^2 - 4 + 4(-1)$$

$$= -12 - \lambda + \lambda^2 - 4 - 4$$

$$= \lambda^2 - \lambda - 20$$

$$\therefore \lambda = 5, \quad \lambda = -4$$

$$\therefore \text{Diagonal form} = \begin{bmatrix} 5 & 0 \\ 0 & -4 \end{bmatrix}$$

Q.2) Find Eigen values & Eigen vectors of $A^2 - 4A^{-1} + 6I$ for $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$

→ characteristic equation is given as $|A - \lambda I|$

$$\therefore \begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -5 & -2-\lambda \end{vmatrix}$$

$$\therefore |A - \lambda I| = \lambda^3 - 5\lambda^2 + (12 - 6 - 8 - 6 + 6 + 10)\lambda - 4$$

$\therefore |A - \lambda I| = \lambda^3 - 5\lambda^2 + 8\lambda - 4$

$\therefore \lambda = 1, 2, 2$

a) Let X be the eigen vector with eigen value as $\lambda = 2$

$\therefore [A - \lambda I] X = 0$

$$\therefore \begin{bmatrix} 2 & 6 & 6 \\ 1 & 1 & 2 \\ -1 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2R_2 - R_1$, $2R_3 + R_1$

$$\begin{bmatrix} 2 & 6 & 6 \\ 0 & -4 & -2 \\ 0 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 2 & 6 & 6 \\ 0 & -4 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank of matrix $(r) = 2$

no. of unknowns $(n) = 3$

$\therefore n - r = 3 - 2 = 1$ linearly independent variable

$$2x_1 + 6x_2 + 6x_3 = 0$$
$$-4x_2 - 2x_3 = 0$$

2 equations in 3 variable, assign 1 at random

$\therefore$ let $x_3 = t$

$\therefore x_2 = \dfrac{-t}{2}$ & $x_1 = \dfrac{-3t}{2}$

$$\therefore X = \begin{bmatrix} -3t/2 \\ -t/2 \\ t \end{bmatrix}$$

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b) let $\lambda = 1$ with eigen vector Y

$$\therefore \begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -5 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$3R_2 - R_1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 0 & 0 & 0 \\ -1 & -5 & -3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank = 2 , n = 3

$\therefore$ n - r = 1 linearly independent solution

$\therefore$ $3y_1 + 6y_2 + 6y_3 = 0$

$-9y_2 - 3y_3 = 0$

2 equations in 3 variables let 1 variable at random

$\therefore$ let $y_3 = s$

$\therefore$ $y_2 = -s/3$ & $y_1 = 4s/3$

$$Y = \begin{bmatrix} 4s \\ -s \\ 3s \end{bmatrix}$$

$\lambda = 1, 2, 2$

$\therefore A = 1, 2, 2$

$A^2 = 1, 4, 4$

$\therefore 4A^{-1} = 4, 2, 2$

$6I = 6, 6, 6$

$\therefore$ $A^2 - 4A^{-1} + 6I = 3, 8, 8$ are eigen values with eigen vector $\begin{bmatrix} -3t/2 \\ -t/2 \\ t \end{bmatrix}$ and $\begin{bmatrix} 4s \\ -s \\ 3s \end{bmatrix}$

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Q.3) Find Eigen values & eigen vectors of

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$

Hence Eigen values & Eigen vectors of $20 \operatorname{adj} A - 10A^{-1} + 6I$

→ Characteristic equation is given as $|A - \lambda I| = 0$

$$\therefore \begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 3-\lambda & -1 \\ 1 & -1 & 3 \end{vmatrix}$$

$$\therefore |A - \lambda I| = \lambda^3 - 9\lambda^2 + (9+9+9-(1+1+1))\lambda - 20 = 0$$

$$= \lambda^3 - 9\lambda^2 + 24\lambda - 20 = 0$$

$$\therefore \lambda = 2, 2, 5$$

a) $\lambda = 2$ let X be eigen vector $(A - 2I)X = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_1$, $R_2 + R_1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$r = 1$, $n = 3$ => 2 L.I sol^n

~~AM = 2~~

$$x_1 - x_2 + x_3 = 0$$

1 equation in 3 unknown assign 2 at random.

(1) Let $x_1 = 1$ & $x_2 = 0$ => $x_3 = -1$

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

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② Let $x_1 = 0$, $x_2 = 1$ $\quad x_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow x_3 = 1$

b) $\lambda = 5$ Let Y be eigen vector $(A - 5I)Y = 0$

$$\begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-2y_1 - y_2 + y_3 = 0$

$-y_1 - 2y_2 - y_3 = 0$

$$\frac{y_1}{\begin{vmatrix} -1 & 1 \\ -2 & -1 \end{vmatrix}} = \frac{y_2}{\begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix}} = \frac{y_3}{\begin{vmatrix} -2 & -1 \\ -1 & -2 \end{vmatrix}} = t$$

$y_1 = 3t$, $y_2 = -t$, $y_3 = 3t$

$$\therefore Y = \begin{bmatrix} 3t \\ -t \\ 3t \end{bmatrix}$$

$\lambda = 2, 2, 5$

$A = 2, 2, 5$

~~$A^2 = 4, 4, 25$~~

$20 \operatorname{adj} A = \frac{20|A|}{\lambda_1}, \frac{20|A|}{\lambda_2}, \frac{20|A|}{\lambda_3}$

$= 200, 200, 80$

$10A^{-1} = 5, 5, 2$

$6I = 6, 6, 6$

$20 \operatorname{adj} A - 10A^{-1} + 6I = 201, 201, 81$ //

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Q.4) Verify Cayley Hamilton theorem for

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

→ characteristic equation is $|A - \lambda I| = 0$

$$\begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$$

$\therefore |A - \lambda I| = \lambda^3 - 12\lambda^2 + 36\lambda - 32$

$\therefore \lambda = 8, 2, 2$

$\therefore A^3 - 12A^2 + 36A - 32 = 0$

$$\begin{bmatrix} 344 & -168 & 168 \\ -168 & 92 & -84 \\ 168 & -84 & 92 \end{bmatrix} - \begin{bmatrix} 528 & -240 & 240 \\ -240 & 168 & -120 \\ 240 & -120 & 168 \end{bmatrix} + \begin{bmatrix} 216 & -72 & 72 \\ -72 & 108 & -36 \\ 72 & -36 & 108 \end{bmatrix}$$

$$-32I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & & \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 0 & 0 \\ 0 & 32 & 0 \\ 0 & 0 & 32 \end{bmatrix} - 32I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence C.H Theorem is verified

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