



Anjuman-I-Islam's
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INTERNAL ASSESSMENT EXAMINATIONS - I & II

ROLL NO.	241215	CPRN NO.							DATE	2	2	0	8	2	0	2	5
BRANCH: Comp									SEMESTER: 03								
SUBJECT: DSGT									SUPERVISOR'S SIGN: <i>[Signature]</i> 8.15								
Q. NO.	1	2	3	4	5	6	TOTAL MARKS	SIGN OF EXAMINER									
MARKS OBTAINED	10	05	05	-	-	-	20	<i>[Signature]</i>									

START WRITING FROM HERE

Q1)

a) $p \rightarrow q$ (when prede-premise implies the result)

Inverse: Inverse is when the result implies to premise $[q \rightarrow p]$

01 Converse: Converse is when -ve premise implies to -ve result $\sim p \rightarrow \sim q$

Contrapositive: Contrapositive is when -ve result implies to -ve premise $\sim q \rightarrow \sim p$

b) Universal Set

Empty set

i) Universal set is a set that covers all the elements of a particular scenario. denoted as U

i) Empty set is a set that has no elements in it. denoted as $\{\}, \phi$

ii) It is a broader set that contains all the other sets

ii) Its value is null formula: ϕ

02 ex $U = \{1, 2, 3, \dots, 100\}$
Set of Natural numbers till 100

ex if $n = 0$
 $n(\{\}) = \{\phi\} = 1$ ($\because 2^0 = 1$)
if $n = 1$

12)

[Signature]

c) $P(S)$ is power set of set S

$$P(\phi) = \{\phi\}$$

$$\therefore n(P(\phi)) = 1$$

$$\dots (\because 2^0 = 1)$$

$$P(\{\phi\}) = \{\phi, \{\phi\}\}$$

(\because every element is a subset of itself)

$$\therefore n(P(\{\phi\})) = 2$$

$$\dots (2^1 = 2)$$

02

power set - Power set is defined as ^{sub}set of all the elements of a set

d) $a R b$ iff $a \equiv b \pmod{4}$

$$\therefore a - b = 4m \quad (\text{It means } a - b \text{ is divisible by } 4)$$

i.e. $a R b$ iff a is divisible by 4

$$\text{for } Z_4 \text{ i.e. } 0 R 1 = 4m \quad 0 R 15$$

$$\therefore \cancel{0 - 1 = 4m} \quad 0 - 1 = 4$$

$$Z_4 = \{$$

$Z_4:$

$$[0] = \{\dots, -12, -8, -4, 0, 4, 8, 12, 16, \dots\}$$

$$[1] = \{\dots, -11, -7, -3, 1, 5, 9, 13, 17, \dots\}$$

$$[2] = \{\dots, -10, -6, -2, 2, 6, 10, 14, \dots\}$$

$$[3] = \{\dots, -9, -5, -1, 3, 7, 11, 15, \dots\}$$

$$[4] = \{\dots, -12, -8, -4, 0, 4, 8, 12, 16, \dots\} \text{ same as } [0]$$

$$\therefore Z_4 = \{[0], [1], [2], [3]\}$$

02

e) Equivalence relation:-

If R is a relation on set A and if R is a reflexive relation (aRa), symmetric relation ($aRb \rightarrow bRa$) and as well as transitive relation ($aRb \& bRc \rightarrow aRc$). then R is an equivalence relation

Ex. $A = \{1, 2, 3\}$

$$R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$$

Reflexive:-

since $aRa \quad \forall a \in A$ it is reflexive relation
ex. $(1,1), (2,2), (3,3)$

Symmetric

Since $aRb \rightarrow bRa$, it is symmetric relation
ex. $(1,2), (2,1)$

Transitive

since $aRb \& bRc \rightarrow aRc$ it is transitive relation
ex. $(1,1)$

$\therefore R$ is Equivalence relation

Equivalence class:-

If R is a equivalence relation on set A , then equivalence class is the set of all the elements of A such for all the elements of a in R .

i.e:-

$$[a] = \{x \in A \mid (a, x) \in R\}$$

Ex: $A = \{1, 2, 3, 4\}$, Let \mathcal{Q} be equivalence class

$$R = \{(1,2), (1,3), (2,2), (2,4), (2,3), (3,4), (3,1)\}$$

$$[1] = \{2, 3\}$$

$$[2] = \{3, 4\}$$

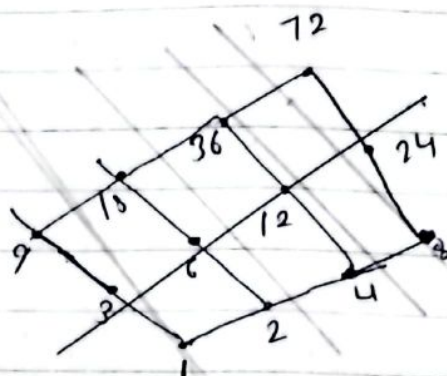
$$[3] = \{3, 4, 1\}$$

$$[4] = \emptyset$$

$$\therefore \mathcal{Q} = \{[1], [2], [3], [4]\}$$

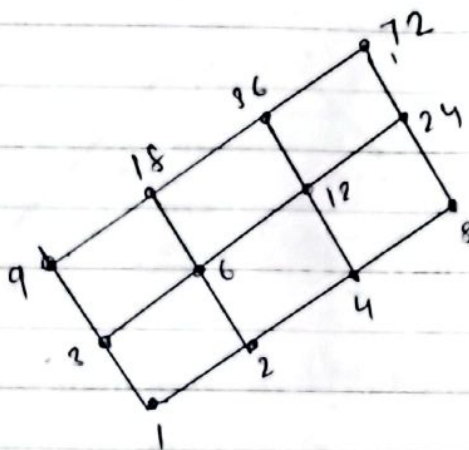
f)

$$D_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$$



$$D_{72} = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72\}$$

f) 1



Hasse - D_{72}
Diagram

(Q2)

b) $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1} \rightarrow p(n)$

Step 1:-

for $n=1$

$$P(1): \text{LHS} = \frac{1}{1(1+1)} = \frac{1}{2}$$

$$\therefore \text{RHS} = \frac{1}{1+1} = \frac{1}{2}$$

$\therefore p(n)$ is true for $n=1$

Step 2:-

Assume that $p(n)$ is true for $n=k$

$$p(k) = \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1} \quad \text{--- (1)}$$

Step 3:-

for $n=k+1$

$$\begin{aligned} p(k+1) &= \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} \\ &= \frac{k+1}{(k+2)} \end{aligned}$$

$$\begin{aligned} \text{LHS} &= \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} \\ &= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \quad \text{--- from (1)} \end{aligned}$$

$$= \frac{k}{(k+1)} \left(k + \frac{1}{k+2} \right)$$

$$= \frac{1}{(k+1)} \left(\frac{k^2 + 2k + 1}{k+2} \right)$$

$$= \frac{1}{(k+1)(k+2)} (k^3 + k^2 + k + 1)$$

$$= \frac{1}{(k+1)(k+1)} (k(k+1) + 1(k+1))$$

$$= \frac{(k+1)(k+1)}{(k+1)(k+1)}$$

$$= \frac{(k+1)}{(k+1)}$$

$$= \text{P.H.S}$$

$$\therefore \text{L.H.S} = \text{P.H.S}$$

05

\therefore It is proved using Mathematical induction that $p(n)$ is true for all $n \geq 1$

(Q2)

a)

$$n(U) = 100$$

$$\text{People eating fruits } n(F) = 37$$

$$\text{People eating vegetable } n(V) = 33$$

$$n(C \cap F) = 9$$

$$n(C \cap V) = 12$$

$$n(F \cap V) = 10$$

$$n(F \cap V \cap C) = 3$$

People eating cheese - $n(C)$

$$\text{Only cheese: } n(C - F - V) = 12$$

$$\text{i) Only cheese} = n(C) - n(C \cap V) - n(C \cap F) + n(C \cap F \cap V)$$

$$12 = n(C) - 12 - 9 + 3$$

$$\therefore n(C) = 30$$

\therefore 30 people eat cheese

05

$$\begin{aligned} \text{ii) } n(C \cup F \cup V) &= n(C) + n(F) + n(V) - (n(C \cap F) + n(C \cap V) + n(F \cap V)) \\ &\quad + n(C \cap F \cap V) \\ &= 30 + 37 + 33 - (9 + 12 + 10) + 3 \end{aligned}$$

$$n(C \cup F \cup V) = 72$$

$$\therefore n(C \cup F \cup V)^c = n(U) - n(C \cup F \cup V) = 100 - 72 = 28$$

\therefore 28 people do not eat any offering

(Q1)

9) xRy iff $2x+5y$ is divisible by 7

(i) Since $2x+5x = 7x$ is divisible by 7 ($\forall x \in \mathbb{Z}$)
 $\therefore xRx$

\therefore It is a reflexive relation.

ii) xRy i.e. $2x+5y = 7m \dots$ (given) — (1)

$\therefore yRx \Rightarrow 5x+2y$

Adding $(2y+5x)$ on both side in (1)

$$\therefore (2x+5y) + (5x+2y) = 7m + (5x+2y) \quad (2y+5x)$$

$$\therefore 7x+7y = 7m + (2y+5x)$$

$$\therefore 7y+7x = 7m + (2y+5x)$$

$$\therefore 7(y+x) = 7m$$

$$\therefore 7(y+x-m) = 2y+5x$$

as $(y+x-m)$ is constant / integer

$2y+5x$ is divisible by 7

$$\therefore yRx$$

As $xRy \rightarrow yRx$

It is a symmetric relation

iii) xRy i.e. $2x+5y = 7m_1 \dots$ (given) — (2)

yRz i.e. $2y+5z = 7m_2 \dots$ (given) — (3)

Adding (2) & (3)

$$2x+7y+5z = 7m_1+7m_2$$

$$\therefore 2x+5z+7y = 7(m_1+m_2)$$

$$\therefore 2x+5z = 7(m_1+m_2-y)$$

as (m_1+m_2-y) is constant / integer

$2x+5z$ is divisible by 7

$$\therefore xRz$$

As xRy & $yRz \rightarrow xRz$

It is a transitive relation