Test 3

6.11 Dual Simplex Method

- Ensure the LP is normal. Multiply any rows by -1 if needed.
- Solve a normal min problem by converting to a max problem
  - Multiply z row by -1 and maximize $-z$

1. If the RHS of each constraint is non-negative, current solution is optimal. If not, continue
2. Choose the most negative BV to leave the basis. The row in which the variable is basic is now the pivot **row**. To select the entering variable, we compute the following ratio for each variable x_j that has a *negative* coefficient in the pivot row:
   1. $\frac{\text{coefficient of } x_j \text{ in row } 0}{\text{coefficient of } x_j \text{ in pivot row}}$
   2. Choose the variable with the **smallest absolute value** in the ratio test to enter the basis. Use row operations to make variable basic.
3. If there is any constraint in which the right-hand side is negative and each variable has a non-negative coefficient, then the LP has no feasible solution. If no constraint indicating infeasibility is found, return to step 1.

9.3 Branch and Bound

- If x_i is not an integer in the optimal solution (PULP to find this), branch on x_i
  - If there are multiple non-integers that need to be integers, arbitrarily pick one (NOT both)
- Stop at infeasible or integer solutions
- Explore entire tree and pick best option
  - AB prune if you want

How to Determine Whether a Node Has a Completion Satisfying a Given Constraint

Type of Constraint	Sign of Free Variable's Coefficient in Constraint	Value Assigned to Free Variable in Feasibility Check
$\leq$	$+$	0
$\leq$	$-$	1
$\geq$	$+$	1
$\geq$	$-$	0

9.7 Implicit Enumeration

1. Test the absolute best case (values of every x_i that make z large/small) against all constraints at current node. If feasible, you're done at this node. If infeasible:
2. Test absolute best case for fulfilling each constraint. If you cannot fulfill a constraint, this node is infeasible. If you can fulfill all constraints, branch arbitrarily off of one of the remaining x_i.
3. Explore entire tree and pick best feasible z value.
   1. AB prune if you wanna be fancy

9.8 The Cutting Plane Algorithm

Step 1 Find the optimal tableau for the IP's linear programming relaxation. If all variables in the optimal solution assume integer values, then we have found an optimal solution to the IP; otherwise, proceed to step 2.

Step 2 Pick a constraint in the LP relaxation optimal tableau whose right-hand side has the fractional part closest to $\frac{1}{2}$. This constraint will be used to generate a cut.

Step 2a For the constraint identified in step 2, write its right-hand side and each variables's coefficient in the form $[x] + f$, where $0 \le f < 1$.

Step 2b Rewrite the constraint used to generate the cut as

$$\text{All terms with integer coefficients} = \text{all terms with fractional coefficients}$$

Then the cut is

$$\text{All terms with fractional coefficients} \le 0$$

Step 3 Use the dual simplex to find the optimal solution to the LP relaxation, with the cut as an additional constraint. If all variables assume integer values in the optimal solution, we have found an optimal solution to the IP. Otherwise, pick the constraint with the most fractional right-hand side and use it to generate another cut, which is added to the tableau. We continue this process until we obtain a solution in which all variables are integers. This will be an optimal solution to the IP.

Notes

- A cut is a constraint to be added to the optimal tableau
- Don't forget to convert to standard form in the new constraint
    - All added cuts add a row and a column