

Q - Draw the logic Diagram for the following. Also make the truth table
 [Simplify the expression if needed]

a) $Q = (X+Y).Z + (ZX + ZY)$

b) $Q = \overline{XYZ} + (\overline{XY} + \overline{XZ} + \overline{YZ})$ (hint: use Demorgan's law to simplify)

c) $Q = (X \oplus Y)(X \oplus Z) + Y \oplus Z$

~~h~~
 d) $Q = \begin{cases} (X+Y).(\overline{XZ}) + \overline{Y}Z \\ (\overline{X}+Y)(XZ) + Y\overline{Z} \end{cases}$

← I have solved this as an example, but first try yourself.

; when $X=0$

; when $X=1$

e) $Q = (X+Y)\overline{Z} + \overline{X}.\overline{Y}.\overline{Z} + XYZ$

f) $Q = \begin{cases} X\overline{Y}Z + (X+Y)(X+Z) & ; Y=1 \\ XYZ + XY + \overline{Z}X & ; Y=0 \end{cases}$

Solve for (d)

$$Q = \begin{cases} (X+Y) \cdot \bar{X}Z + \bar{Y}Z & , \text{ when } X=0 \\ (\bar{X}+Y)(XZ) + Y\bar{Z} & , \text{ when } X=1 \end{cases}$$

Solution

First solving when $X=0$:

$\Rightarrow X=0$; so expression $(X+Y) \cdot \bar{X}Z + \bar{Y}Z$ can be simplified as;

$$\Rightarrow (X+Y)\bar{X}Z + \bar{Y}Z \Rightarrow Y \cdot Z + \bar{Y}Z \quad \left(\begin{array}{l} \because X=0 \\ \& \bar{X}=1 \end{array} \right)$$

$$\Rightarrow YZ + \bar{Y}Z$$

$$\Rightarrow Z(Y + \bar{Y})$$

; we know that $Y + \bar{Y} = 1$

$$\boxed{Q = Z}$$

Now Solving, when $X=1$:

$$\Rightarrow (\bar{X}+Y)(XZ) + Y\bar{Z}$$

$$\Rightarrow YZ + Y\bar{Z} \quad (\because X=1, \& \bar{X}=0)$$

$$\Rightarrow Y(Z + \bar{Z}) \quad (\because Z + \bar{Z} = 1, \text{ so we have})$$

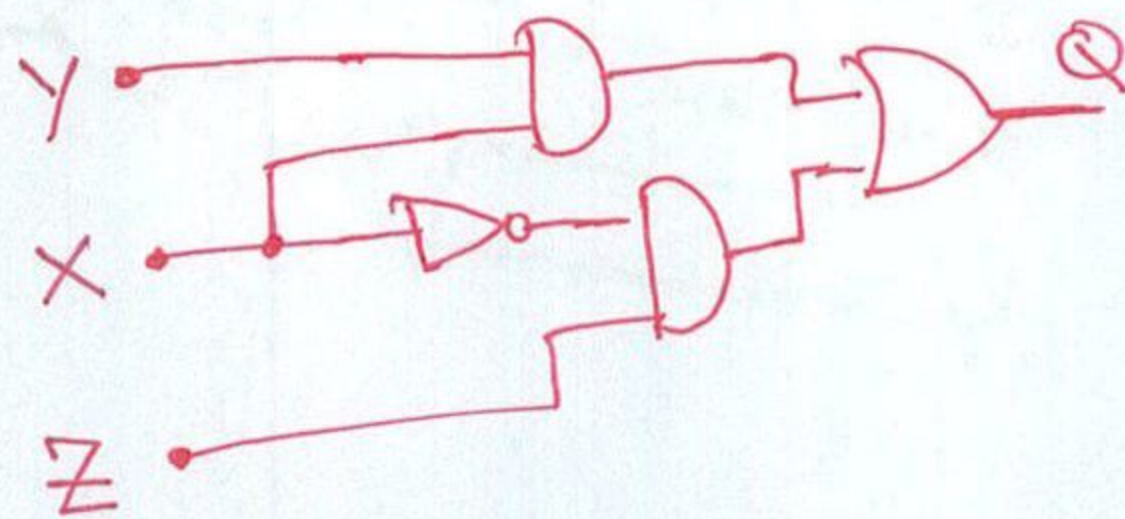
$$\boxed{Q = Y}$$

\Rightarrow Hence final ~~expression~~ logical expression is,

$$Q = \begin{cases} Z & \text{when } X=0 \\ Y & \text{when } X=1 \end{cases}$$

\Rightarrow logically, we can write it as-

$$\boxed{Q = \bar{X}Z + XY}$$



(Logic Diagram)

Solve (e) for the following Q

$$Q = (X+Y)\bar{Z} + \bar{X} \cdot \bar{Y} \cdot \bar{Z} + XYZ$$

Solution

$$Q = \bar{Z}[(X+Y) + \bar{X} \cdot \bar{Y}] + XYZ \quad \text{--- eq ①}$$

\Rightarrow (from Demorgan's law we know that)
 $\overline{a+b} = \bar{a} \cdot \bar{b}$

(we also know that $a = \overline{\bar{a}}$; i.e. if we twice inverse a variable it is equal to itself.)

Using the above two properties, we will solve eq ①

lets take $\overline{(X+Y) + \bar{X} \cdot \bar{Y}}$ from eq. ①

$$= \overline{\overline{(X+Y)} + \bar{X} \cdot \bar{Y}} \quad (\because \overline{\bar{a}} = a)$$

$$= \overline{\bar{X} \cdot \bar{Y}} + \bar{X} \cdot \bar{Y} \quad (\because \overline{a+b} = \bar{a} \cdot \bar{b})$$

Now, let consider $A = \bar{X} \cdot \bar{Y}$, so we can write above expression as

$$= \bar{A} + A = 1 \quad (\text{because of the property } \bar{a} + a = 1)$$

Hence we can put this value to eq ① now

$$Q = \bar{Z}[\cancel{(X+Y)} + \bar{X} \cdot \bar{Y}] + XYZ$$

$$\boxed{Q = \bar{Z} + XYZ}$$

