MTH113TC Tutorial 6

Q1 (i) $H_0 : \mu = 21.8$

$H_1 : \mu \neq 21.8$

(ii) $H_0 : \mu = 8000$ ← comes from $\mu \leq 8000$

$H_1 : \mu > 8000$

(iii) $H_0 : \mu = 340$ ← comes from $\mu \geqslant 340$

$H_1 : \mu < 340$

For (i), critical region in <u>both tails</u>

(ii), critical region in <u>right tail</u>

(iii), critical region in <u>left tail</u>.

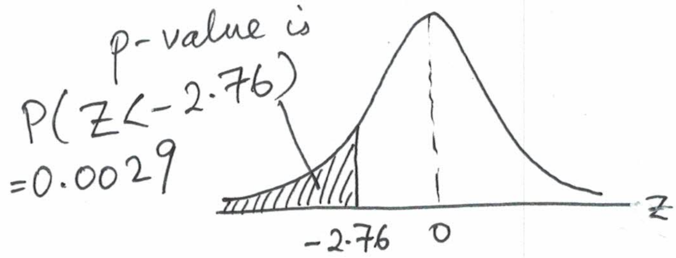
Q2) $H_0 : \mu = 40$

$H_1 : \mu < 40$

Given: $n = 64$, $\bar{x} = 38$, $\sigma = 5.8$

Test statistic $Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{38 - 40}{5.8/\sqrt{64}}$

$= -2.76$



There is moderate evidence against H_0 in favour of H_1. Hence there is indication from data that average life span is significantly less than 40 months.

Q3) $H_0: \mu = 20\ 000$

$H_1: \mu > 20\ 000$

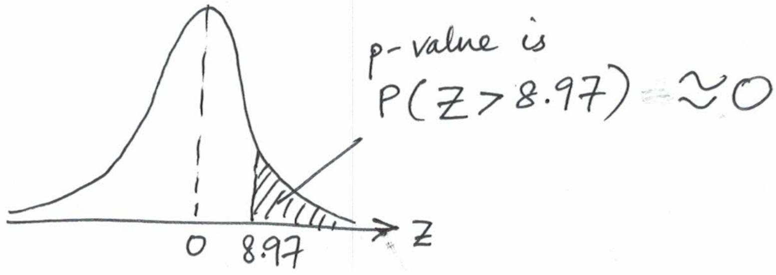
Given: $n = 100$, $\bar{x} = 23,500$, $\sigma = 3900$

Test Statistic $Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

$= \dfrac{23\,500 - 20\,000}{3900/\sqrt{100}}$

$= \dfrac{3500}{390}$

$= 8.97$



There is very strong evidence against H_0 in favour of H_1. Hence we conclude based on the observations that mean distance travelled is significantly more than 20 000 km #

Q4)

Population 1 (lab)	Population 2 (no lab)
mean μ_1	mean μ_2
variance σ_1^2	Variance σ_2^2
$n_1 = 11$	$n_2 = 17$
$\bar{x}_1 = 85$	$\bar{x}_2 = 79$
$s_1 = 4.7$	$s_2 = 6.1$

$H_0 : \mu_1 - \mu_2 = 8$

$H_1 : \mu_1 - \mu_2 < 8$

Assuming equal variances,

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$= \frac{(10)(4.7^2) + (16)(6.1^2)}{26}$$

$$= 31.395$$

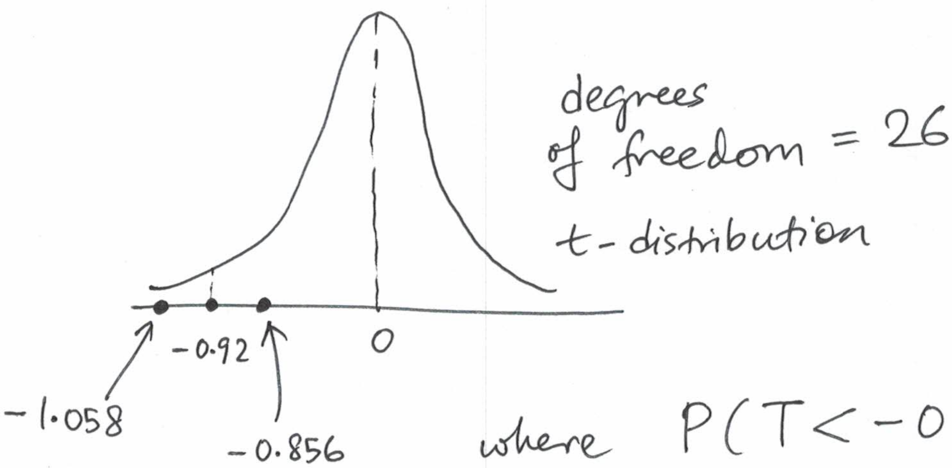
Test-Statistic $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{S_p \sqrt{1/n_1 + 1/n_2}}$ with 26 degrees of freedom

$$= \frac{(85-79)-8}{\sqrt{31.395}\sqrt{\frac{1}{11}+\frac{1}{17}}}$$

$$= -0.92$$

From statistical table,

$$0.15 < \underbrace{P(T < -0.92)}_{\text{p-value}} < 0.20$$



where $P(T < -0.856) = 0.20$

$P(T < -1.058) = 0.15$

Since $0.15 <$ p-value < 0.20

there is no evidence against H_0 in favour of H_1. We conclude lab course increases the average grade by at least 8 points. #

Q5) (i) Hypothesis testing on

$H_0: \mu_{hot} = \mu_{cold}$ (or $\mu_{hot} - \mu_{cold} = 0$)

$H_1: \mu_{hot} \neq \mu_{cold}$ (or $\mu_{hot} - \mu_{cold} \neq 0$)

(ii) Given: $d_0 = 0$, $n = 8$

Dog	HOT	Cold	difference d_i
1	5120	8200	-3080
2	10 000	8600	1400
3	10 000	9200	800
4	10 000	6200	3800
5	10 000	10 000	0
6	7900	5200	2700
7	510	885	-375
8	1020	460	560
			$\Sigma_i = 5805$

$\bar{d} = \frac{5805}{8}$

$= 725.625$

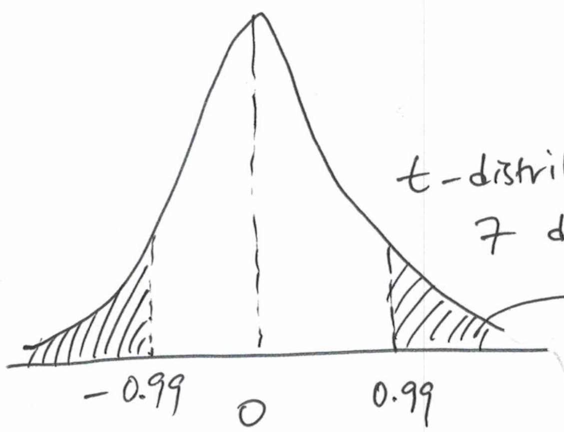
$S_d = \sqrt{\frac{1}{n-1} \Sigma (d_i - \bar{d})^2}$

$= 2072.2$

Test Statistic $t = \dfrac{\bar{d} - d_0}{s_d/\sqrt{n}}$ with $(n-1)$ degrees of freedom

$$= \frac{725.625 - 0}{\frac{2072.2}{\sqrt{8}}}$$ with 7 degrees of freedom

$$= 0.990$$



t-distribution
7 degrees of freedom

p-value
$= 2P(T > 0.99)$
is such that

$$2(0.15) < \text{p-value} < 2(0.20)$$

That is, $\quad 0.30 < \text{p-value} < 0.40$

There is no evidence against H_0 in favour of H_1.
Hence there is no indication from data that there is a significant difference in strength between hot and cold incisions.

#

Q6) Given : $n = 64$, $\bar{x} = 24.17$, $s^2 = 4.25$

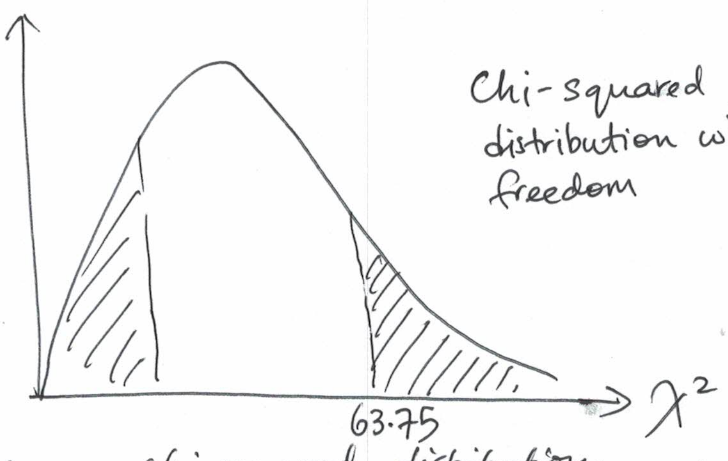
$H_0 : \sigma^2 = 4.2$

$H_1 : \sigma^2 \neq 4.2$

Test Statistic $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$ with $n-1$ degrees of freedom

$= \dfrac{(63)(4.25)}{4.2}$ with 63 degrees of freedom

$= 63.75$



Chi-squared distribution with 63 degrees of freedom

Based on Chi-squared distribution table with <u>60</u> degrees of freedom, ← no data for 63 degrees of freedom

$0.30 < P(\chi^2 > 63.75) < 0.50$

the p-value $= 2P(\chi^2 > 63.75) > 0.60$ for 60 degrees of freedom

So no evidence against H_0 in favour of H_1. Hence we conclude based on observations that σ^2 is not significantly different from 4.2 ppm.

Q7) population 1 (Men) | population 2 (Women)

$n_1 = 11$ | $n_2 = 14$

$S_1 = 6.1$ | $S_2 = 5.3$

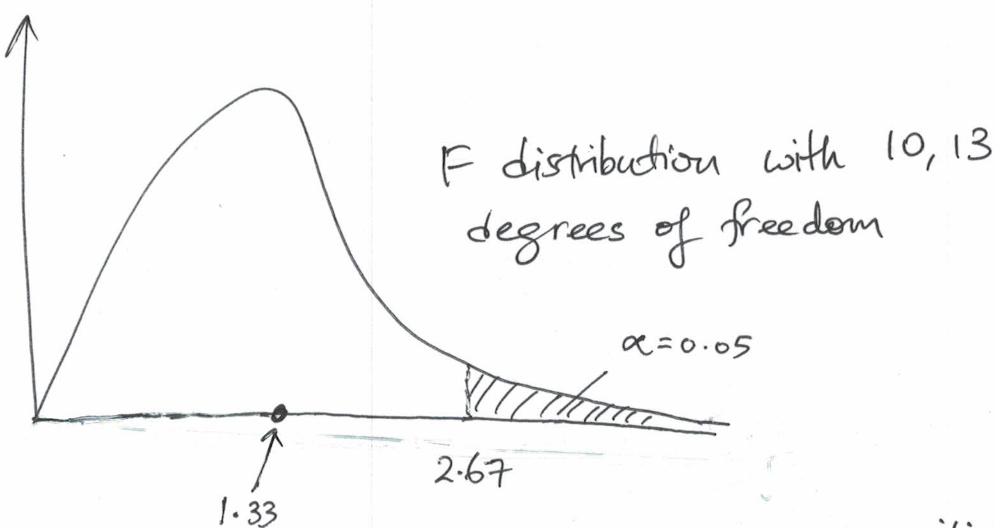
$H_0: \sigma_1^2 = \sigma_2^2 \quad \left(\text{or } \frac{\sigma_1^2}{\sigma_2^2} = 1 \right)$

$H_1: \sigma_1^2 > \sigma_2^2 \quad \left(\text{or } \frac{\sigma_1^2}{\sigma_2^2} > 1 \right)$

Test Statistic $f = \frac{S_1^2}{S_2^2}$ with $n_1 - 1, n_2 - 1$ degrees of freedom

$= \frac{6.1^2}{5.3^2}$ with 10, 13 degrees of freedom

$= 1.33$



Since the test statistic $f = 1.33$ is not in critical region, we do not reject H_0 at 0.05 level of significance. We conclude based on our observations that it is not significant that $\sigma_1^2 > \sigma_2^2$ //

Q8) H_0: Die is fair

H_1: Die is not fair

Given $\alpha = 0.01$, $n = 180$

x	1	2	3	4	5	6
O_i	28	36	36	30	27	23
e_i	30	30	30	30	30	30

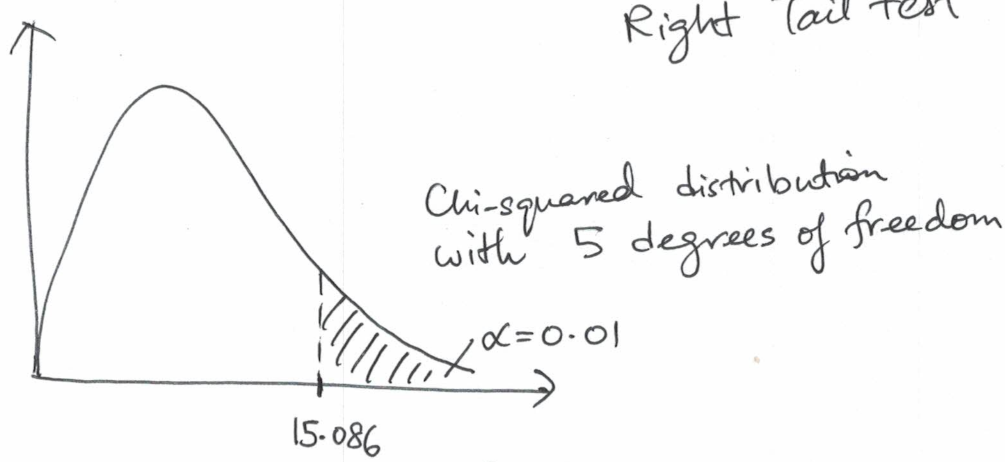
$e_i = \left(\frac{1}{6}\right)(180)$
$= 30$

Test Statistic $\chi^2 = \sum_i \frac{(O_i - e_i)^2}{e_i}$

$$= \frac{(28-30)^2}{30} + \frac{(36-30)^2}{30} + \cdots + \frac{(23-30)^2}{30}$$

$= 4.47$ with degrees of freedom $6-1 = 5$

Right Tail test



Since the test statistic $\chi^2 = 4.47$ is not in critical region, we do not reject Ho at 0.01 level of significance. We conclude based on our observations that it is not significant that die is unfair.

Q9) H_0: The two factors are independent
H_1: " " " " not independent

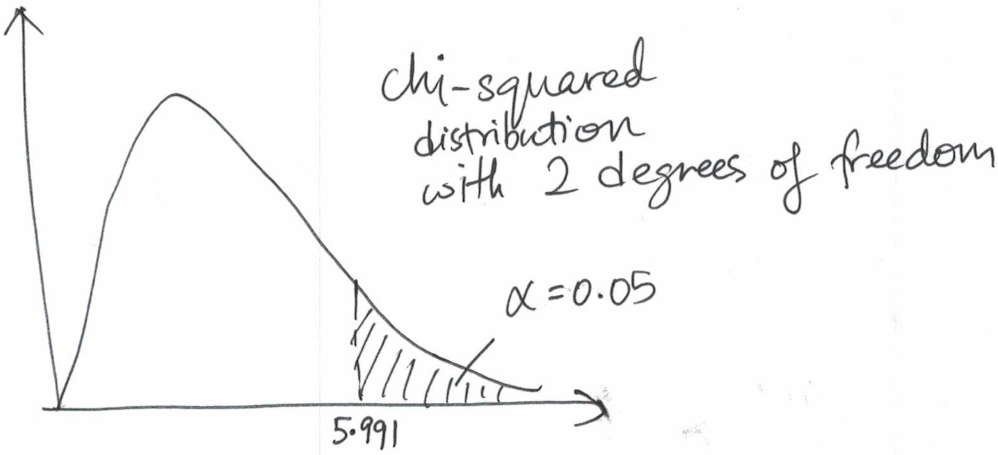
Given: $n = 180$, $\alpha = 0.05$

	Non-smoker	Moderate Smoker	heavy smoker	row sum
hypertension	21 (33.35)	36 (29.97)	30 (23.68)	87
No hypertension	48 (35.65)	26 (32.03)	19 (25.32)	93
Column sum	69	62	49	Grand Total = 180

Figures given in brackets are expected values.

Test Statistic $\chi^2 = \frac{(21-33.4)^2}{33.4} + \dots + \frac{(19-25.4)^2}{25.4}$

$= 14.60$ with $(2-1)(3-1)$
$= 2$ degrees of freedom.



Since the test statistic $\chi^2 = 14.60$ is in the critical region, we reject H_0 at 0.05 level of significance. We conclude based on our observations that the factors are not independent at 0.05 level of significance.

Q10) Let p be the proportion of the public allergic to some cheese products.

$H_0: p = 0.3$ ⟵ comes from consideration of $p \geq 0.3$.

$H_1: p < 0.3$

(i) Type I error : Rejection of H_0 when H_0 is true

So in this case it is "Concluding that fewer than 30% of the public is allergic to some cheese products when, in fact, 30% or more are allergic".

(ii) Type II error: Do not Reject H_0 when H_0 is false

So in this case it is "concluding that at least 30% of the public are allergic to some cheese products when, in fact, fewer than 30% are allergic".

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Q11) (i)

Type I error: "training ineffective when in fact, training is effective"

H_1 H_0

$\therefore$ She is testing H_0: training is effective
against
H_1: training is ineffective

(ii) Type II error: "training effective when in fact training ineffective"

H_0 H_1

$\therefore$ She is testing
H_0: training is effective
against
H_1: training is ineffective

\#

Q12 (i) $X \sim N(\mu, 9)$

$H_0: \mu = 20$

against

$H_1: \mu > 20$

Given: $n = 25$

$P(\text{Type I error}) = P(\bar{X} > 21.4 \text{ when } \mu = 20)$

$= P\left(\frac{\bar{X}-20}{3/\sqrt{25}} > \frac{21.4-20}{3/\sqrt{25}}\right)$

$= P(Z > 2.33)$

$= 1 - P(Z \leq 2.33)$

$= 1 - 0.9901$

$= 0.0099$

$= 0.01$ (to 2 dec. pl.)

ii) $P(\text{Type II error})$

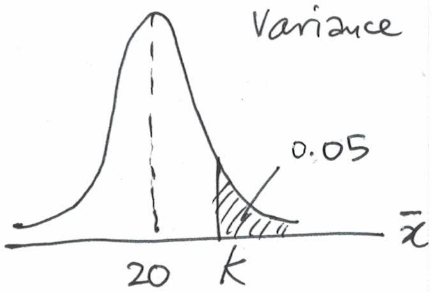
$= P(\bar{X} \leq 21.4 \text{ when } \mu = 21)$

$= P\left(\frac{\bar{X}-21}{3/\sqrt{25}} \leq \frac{21.4-21}{3/\sqrt{25}}\right)$

$= P(Z \leq 0.67) = 0.7486 = 0.75$ (to 2 dec. pl.)

Power $= 1 - 0.75 = 0.25$ (to 2 dec. pl.)

(iii)



Variance $\frac{3}{\sqrt{25}} = 0.6$

$P(\bar{X} \leq k) = 0.95$ where k is to be found

Under $H_0 : \mu = 20$,

$$P\left(\frac{\bar{X}-20}{0.6} \leq \frac{k-20}{0.6}\right) = 0.95$$

$$\Rightarrow P\left(Z \leq \frac{k-20}{0.6}\right) = 0.95$$

From statistical table, $\frac{k-20}{0.6} = 1.645$

$$\Rightarrow k = 20 + (0.6)(1.645)$$

$$= 20.987 = 20.99 \text{ (to 2 dec. pl.)}$$

The rejection rule is

H_0 is rejected if $\bar{X} > 20.9$ //

(iv) $P(\text{Type II error})$

$= P(\text{NOT reject } H_0 \text{ when } H_0 \text{ is false})$

$= P(\bar{X} \le 20.987 \text{ when } \mu = 21)$

$= P\left(\frac{\bar{X} - 21}{0.6} \le \frac{20.987 - 21}{0.6}\right)$

$= P(Z \le -0.02)$

$= 0.4920$ #

$= 0.49$ (to 2 dec.pl.)