MTH113TC Introduction to Probability and Statistics   Tutorial 5

Year: 2020/21   Week: 4   **Based on Chapter 5**

1. Suppose that the weight of a bag of potato chips (in grams) is a normal random variable with an unknown mean μ and a known variance $\sigma^2 = 100$. A random sample of 75 bags has mean $\bar{x} = 50$. Construct a 90% confidence interval for μ.

   Answer: $(48.1005, 51.8995)$

2. Calculate 95% confidence interval for the true mean density of the earth (in gm per cubic cm) from Cavendish's first six observations, namely,

   $$5.50, 5.61, 4.88, 5.07, 5.26, 5.55.$$

   What are the underlying assumptions?

   Answer: $(5.0043, 5.619)$.   Random variables are i.i.d. from a normal population with unknown mean and variance.

3. In a study conducted, 20 oak seedlings were planted in the same type of soil and received the same amount of sunshine and water. Half of which, serving as a control, received no fertilizer while the other half received 368 ppm of nitrogen in the form of sodium nitrate fertilizers. The stem weights, in grams, at the end of 140 days were recorded as follows.

No Fertilizer	Fertilizer
0.32	0.26
0.53	0.43
0.28	0.47
0.37	0.49
0.47	0.52
0.43	0.75
0.36	0.79
0.42	0.86
0.38	0.62
0.43	0.46

   Construct a 95% confidence interval for the difference in the mean stem weight between seedlings that receive no fertilizer and those that receive 368 ppm of nitrogen. Assume the populations to be normally distributed with equal variances.

   Answer: $(0.033, 0.299)$ or $(-0.299, -0.033)$

4. A manufacturer of car batteries claims that the batteries will last, on average, 3 years with a variance of 1 year. If 5 of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5 and 4.2 years, construct a 95% confidence interval for σ^2 and decide if the manufacturer's claim of $\sigma^2 = 1$ is valid. Assume the population of battery lives to be normally distributed.

   Answer: $(0.293, 6.736)$ Since this interval contains 1, it is reasonable to conclude using the observations that $\sigma^2 = 1$ subjected to statistical error. Hence the manufacturer's claim seem valid.

5. Two different brands of paints are being considered for use. Fifteen specimens of each type of paint were selected, and the drying times (in hours) were as follows.

Brand A	Brand B
3.5	4.7
2.7	3.9
3.9	4.5
4.2	5.5
3.6	4.0
2.7	5.3
3.3	4.3
5.2	6.0
4.2	5.2
2.9	3.7
4.4	5.5
5.2	6.2
4.0	5.1
4.1	5.4
3.4	4.8

Construct a 95% confidence interval for σ_A^2/σ_B^2. Is it justifiable to assume equality of population variances?

Answer: $(0.359, 3.186)$ Since the interval contains 1, it is reasonable to assume $\sigma_A^2 = \sigma_B^2$ based on our observations, subjected to statistical error.

6. A taxi company is trying to decide whether to purchase brand A or brand B tires for its fleet of taxis. To estimate the difference in the two brands, an experiment is conducted using 12 of each brand. The tires are run until they they wear out. The respective sample means and standard deviations are given below.

Brand A	Brand B
$\bar{x}_A = 36300$ km	$\bar{x}_B = 38100$ km
$s_A = 5000$ km	$s_B = 6100$ km

Construct a 95% confidence interval for $\mu_A - \mu_B$ assuming the populations to be normally distributed. You may not assume that their variances are equal.

Answer: $(-6536, 2936)$

7. Grants are awarded to the agricultural departments of 9 universities to test the yield capabilities of two new varieties of wheat. Each variety was planted on a plot of equal area at each university and the yields, in kg per plot, were recorded as follows.

					University				
Variety	#1	#2	#3	#4	#5	#6	#7	#8	#9
1	38	23	35	41	44	29	37	31	38
2	45	25	31	38	50	33	36	40	43

Find a 95% confidence interval for the main difference between the yields of the two varieties, assuming the difference of yields to be normally distributed. Explain why pairing is necessary in this problem.

Answer: $(-0.74, 6.30)$ or $(-6.30, 0.74)$ To factor out the variability between the plots, such as locations, altitudes, climates and environmental conditions.