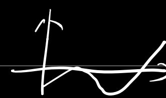
set $u(x_i) = \dfrac{f(x_i)}{f'(x_i)}$,



Then find. $x_{i+1} - \dfrac{u(x_i)}{u'(x_i)}$

Systems of non-linear equations

$$\begin{cases} x^2 + xy = 10 \\ y + 3xy^2 - 57 = 0 \end{cases}$$

$$u(x, y) = x^2 + xy - 10$$

$$v(x, y) = y + 3xy^2 - 57.$$

Multi-Dimensional 牛顿方法

$$u_{i+1} = u_i + \frac{\partial u_i}{\partial x}(x_{i+1} - x_i) + \frac{\partial u_i}{\partial y}(y_{i+1} - y_i)$$

$$v_{i+1} = v_i + \frac{\partial v_i}{\partial x}(x_{i+1} - x_i) + \frac{\partial v_i}{\partial y}(y_{i+1} - y_i)$$

当 $u_{i+1} = 0$

$v_{i+1} = 0$, 为函数的根

$$x_{i+1} = x_i - \frac{u_i \frac{\partial v_i}{\partial y} - v_i \frac{\partial u_i}{\partial y}}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}}.$$

$$y_{i+1} = y_i - \frac{u_i \frac{\partial v_i}{\partial x} - v_i \frac{\partial u_i}{\partial x}}{\underline{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}}}.$$

Jacobian