MTH113TC Introduction to Probability and Statistics Tutorial 6

Year: 2020/21 Week: 5 **Based on Chapter 6**

1. State the null and alternative hypotheses to be used in testing the following claims and determine generally where the critical region is located:

   (i) The mean snowfall at Lake George during the month of February is 21.8 cm.

   (ii) The mean monthly household income is no more than \$8000/mth.

   (iii) The average rib-eye steak at the Longhorn Steak house weighs at least 340 g. .

   Answer: (i) $H_0 : \mu = 21.8, H_1 : \mu \neq 21.8.$ Critical region in both tails.
   (ii) $H_0 : \mu = 8000, H_1 : \mu > 8000.$ Critical region is right tail.
   (iii) $H_0 : \mu = 340, H_1 : \mu < 340.$ Critical region is left tail.

2. In a research paper, it is claimed that mice with an average life span of 32 months will live to be about 40 months old when 40% of the calories in their diet are replaced by vitamins and protein. Is there any reason to believe that $\mu < 40$ if 64 mice that are placed on this diet have an average life of 38 months? Assume a population standard deviation of 5.8 months. Use a p-value in your conclusion.

   Answer: p-value $= 0.0029$

3. It is claimed that automobiles are driven on average no more than 20,000 km/yr. To test this claim, 100 randomly selected automobile owners are asked to keep a record of the distance they travel. Would you agree with this claim if the random sample shows an average of 23,5000 km? Assume a population standard deviation of 3900 km. Use a p-value in your conclusion.

   Answer: p-value ≈ 0

4. A study was made to determine if the subject matter in a physics course is better understood when a lab constitutes part of the course. Students were randomly selected to participate in either a 3-semester-hour course without labs or a 4-semester-hour course with labs. In the section with labs, 11 students made an average grade of 85 with a standard deviation of 4.7, and in the section without labs, 17 students made an average grade of 79 with a standard deviation of 6.1 . It is claimed that the laboratory course increases the average grade by at least 8 points. Carry out hypothesis testing using p-value to conclude. Assume the populations to be normally distributed with equal variances.

   Answer: $0.15 < p$-value < 0.20

5. A study was conducted to determine if the “strength” of a wound from surgical incision is affected by the temperature of the knife. Eight dogs were used in the experiment. “Hot” and “cold” incisions were made on the abdomen of each dog, and the strength was measured. The resulting data is given below.

Dog	Knife	Strength
1	Hot	5120
1	Cold	8200
2	Hot	10,000
2	Cold	8600
3	Hot	10,000
3	Cold	9200
4	Hot	10,000
4	Cold	6200
5	Hot	10,000
5	Cold	10,000
6	Hot	7900
6	Cold	5200
7	Hot	510
7	Cold	885
8	Hot	1020
8	Cold	460

(i) Write an appropriate hypothesis to determine if there is a significant difference in strength between the hot and cold incisions.

(ii) Test the hypothesis using a paired t-test. Use a p-value in your conclusion.

Answer: (i) $H_0 : \mu_{\text{hot}} - \mu_{\text{cold}} = 0, H_1 : \mu_{\text{hot}} - \mu_{\text{cold}} \neq 0$ (ii) $0.3 < p\text{-value} < 0.4$

6. Aflotoxins produced by mold on peanut crops in Virginia are to be monitored. A sample of 64 batches of peanuts reveal levels of 24.17 ppm, on average, with a variance of 4.25 ppm. Test the hypothesis that $\sigma^2 = 4.2$ ppm against the alternative that $\sigma^2 \neq 4.2$ ppm. Use a p-value in your conclusion.

   Answer: p-value > 0.6

7. A study is conducted to compare the lengths of time required by men and women to assemble a certain product. Past experience indicates that the distribution of times for both men and women is normal but the variance of the times for women is less than that for men. A random sample of times for 11 men and 14 women produced the following data:

Men	Women
$n_1 = 11$	$n_2 = 14$
$s_1 = 6.1$	$s_2 = 5.3$

   Test the hypothesis that $\sigma_1^2 = \sigma_2^2$ against the alternative that $\sigma_1^2 > \sigma_2^2$. Use 0.05 level of significance in your conclusion.

   Answer: $f = 1.33 < 2.67 = f_{0.05}\,(10, 13)$

8. A die is tossed 180 times with the following results:

x	1	2	3	4	5	6
frequency	28	36	36	30	27	23

   Is this a fair die? Use a 0.01 level of significance.

   Answer: $\chi^2 = 4.47 < 15.086 = \chi^2_{0.01}$ with 5 degrees of freedom. Die is fair.

9. In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals:

	Non-smokers	Moderate Smokers	Heavy Smokers
Hypertension	21	36	30
No Hypertension	48	26	19

Test the hypothesis that the presence or absence of hypertension is associated with smoking habits. Use a 0.05 level of significance.

Answer: $\chi^2 = 14.60 > 5.991 = \chi^2_{0.05}$ with 2 degrees of freedom. Not independent.

10. Suppose that an allergist wishes to test the hypothesis that more than 30% of the public is allergic to some cheese products. Explain how the allergist could commit

(i) a Type I error;

(ii) a Type II error.

Answer: (i) Conclude that more than 30% of the public are allergic to some cheese products when, in fact, 30% or less are allergic.
(ii) Conclude that 30% or less of the public are allergic to some cheese products when, in fact, more than 30% are allergic.

11. A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles. What are the null and alternative hypotheses if she commits a

(i) Type I error by erroneously concluding that the training course is ineffective?

(ii) Type II error by erroneously concluding that the training course is effective?

Answer: (i) H_0 : The training course is effective. H_1 : The training course is ineffective.
(ii) H_0 : The training course is effective. H_1 : The training course is ineffective.

12. A random variable has a normal distribution with mean μ and a known variance, $\sigma^2 = 9$. The null hypothesis $H_0 : \quad \mu = 20$ is tested against the alternative hypothesis $H_1 : \quad \mu > 20$ using a random sample of size $n = 25$. It is decided the null hypothesis will be rejected if the sample mean is more than 21.4 .

(i) Obtain the probability of Type I error.

(ii) Obtain the probability of Type II error and the corresponding power of this test when, in fact, $\mu = 21$.

(iii) If we require the probability of Type I error to be 0.05 maximum, what is the new rejection rule?

(iv) What would be the probability of Type II error with the rule given in (iii) above when in fact $\mu = 21$? .

Answer: (i) 0.01 (ii) $0.75, 0.25$ (iii) 20.99 (iv) 0.49