MTH113TC Tutorial 5

Q1) Given: $n = 75$, $\bar{x} = 50$, $\sigma^2 = 100$

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $Z_{0.05} = 1.645$

So 90% C.I. for μ is $\bar{X} \pm 1.645\left(\frac{\sigma^2}{n}\right)^{1/2}$

$= 50 \pm 1.89948$

$= (48.1005, 51.8995)$

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Q2) $\sum x = 31.87$
$\sum x^2 = 169.7115$
$n = 6$

$\bar{x} = \frac{\sum x}{n} = \frac{31.87}{6} = 5.31167$

$s^2 = \frac{1}{n-1}\left[\sum x^2 - \frac{(\sum x)^2}{n}\right]$

$= \frac{1}{5}\left[169.7115 - \frac{31.87^2}{6}\right]$

$= 0.08574$

95% C.I. for μ is $\bar{x} \pm t_{0.025}\frac{s}{\sqrt{n}}$ where $t_{0.025} = 2.571$ with 5 degrees of freedom

$= 5.31167 \pm (2.571)\sqrt{\frac{0.08574}{6}}$

$= 5.31167 \pm 0.30734$

$= (5.0043, 5.619)$

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Assumptions: Random variables are i.i.d from a normal population with unknown mean and variance.

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Q3) Observations are <u>not</u> paired here.

Let population 1 be oak seedlings that received <u>no fertilizer</u>.

Let population 2 " " " " " <u>fertilizers</u>

Given: $n_1 = 10$ $\quad t_{0.025} = 2.101$ with 18 degrees of freedom

$n_2 = 10$

$$\bar{x}_1 = \frac{\sum x_{1i}}{n_1} = 0.399$$

$$\bar{x}_2 = \frac{\sum x_{2i}}{n_2} = 0.565$$

$$\left.\begin{aligned} S_1^2 &= \frac{1}{n_1 - 1}\sum (x_{1i} - \bar{x}_1)^2 = 5.2984 \times 10^{-3} \\ S_2^2 &= \frac{1}{n_2 - 1}\sum (x_{2i} - \bar{x}_2)^2 = 0.03487 \end{aligned}\right\} S_p = 0.1472$$

We are given $\sigma_1^2 = \sigma_2^2$, unknown.

95% C.I. for $\mu_2 - \mu_1$ is

$$(\bar{x}_2 - \bar{x}_1) \pm t_{0.025}\, S_p \sqrt{\frac{1}{10} + \frac{1}{10}}$$

$$= (0.565 - 0.399) \pm (2.101)(0.14172)\sqrt{\frac{1}{10} + \frac{1}{10}}$$

$$= 0.166 \pm 0.133 \text{, giving}$$

$$0.033 < \mu_2 - \mu_1 < 0.299 \quad \#$$

[You can consider $\mu_1 - \mu_2$, whose C.I. is $(-0.299, -0.033)$]

Q4) Given

$$\left.\begin{aligned} n &= 5 \\ \Sigma x &= 15 \\ \Sigma x^2 &= 48.26 \end{aligned}\right\} \quad \begin{aligned} \bar{x} &= \frac{15}{5} = 3 \\ s^2 &= \frac{1}{n-1}\left[\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right] \\ &= 0.815 \end{aligned}$$

Also, $\chi^2_{0.025} = 11.143$ and $\chi^2_{0.975} = 0.484$ with 4 degrees of freedom. So 95% C.I. for σ^2 is

$$\frac{(4)(0.815)}{11.143} < \sigma^2 < \frac{(4)(0.815)}{0.484}$$

$$\Rightarrow 0.293 < \sigma^2 < 6.736$$

Since this interval contains 1, it is reasonable to conclude using the observations that $\sigma^2 = 1$ subject to statistical error.

Hence the manufacturer's claim seem valid

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Q5) Observations are <u>not</u> paired here.

Given :

A	B
$n_A = 15$	$n_B = 15$
$\sum_i x_A = 57.3$	$\sum_i x_B = 74.1$
$\sum_i x_A^2 = 227.39$	$\sum_i x_B^2 = 374.01$

$$S_A^2 = \frac{1}{n_A - 1}\left[\sum_i x_A^2 - \frac{(\sum_i x_A)^2}{n_A}\right]$$

$$= 0.6074$$

$$S_B^2 = \frac{1}{n_B - 1}\left[\sum_i x_B^2 - \frac{(\sum_i x_B)^2}{n_B}\right]$$

$$= 0.5683$$

Also, $f_{0.025}(14, 14) = 2.98$ ← This is not found in our statistical table. You can obtain this value on Matlab using the command

$$\boxed{finv(0.975, 14, 14)}$$

$$\frac{S_A^2}{S_B^2}\frac{1}{f_{\frac{\alpha}{2}}(n_A-1, n_B-1)} < \frac{\sigma_A^2}{\sigma_B^2} < \frac{S_A^2}{S_B^2} f_{\frac{\alpha}{2}}(n_B-1, n_A-1)$$

$$\Rightarrow \left(\frac{0.6074}{0.5683}\right)\frac{1}{2.98} < \frac{\sigma_A^2}{S_B^2} < \left(\frac{0.6074}{0.5683}\right)2.98$$

$$\Rightarrow 0.359 < \frac{\sigma_A^2}{\sigma_B^2} < 3.186$$

Since the interval contains 1, it is reasonable to assume $\sigma_A^2 = \sigma_B^2$ based on our observations, subject to statistica error.

Q6) Given:

A	B
$n_A = 12$	$n_B = 12$
$\bar{x}_A = 36300$	$\bar{x}_B = 38100$
$S_A = 5000$	$S_B = 6100$

$$V = \frac{\left(\frac{5000^2}{12} + \frac{6100^2}{12}\right)^2}{\frac{(5000^2/12)^2}{11} + \frac{(6100^2/12)^2}{11}} = 21.1839$$

≈ 21 (round down)

with $t_{0.025} = 2.080$ with 21 degrees of freedom.

95% C.I. for $\mu_A - \mu_B$ is

$$(\bar{x}_A - \bar{x}_B) \pm t_{\frac{\alpha}{2}} \sqrt{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}$$

$$= (36300 - 38100) \pm 2.080 \sqrt{\frac{5000^2}{12} + \frac{6100^2}{12}}$$

$$= -1800 \pm 4736$$

i.e. $-6536 < \mu_A - \mu_B < 2936$

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Q7) Observations are <u>paired</u> here.

$n = 9$

difference $d_i = x_{2i} - x_{1i}$ is given below

Variety 2	Variety 1	$d_i = x_{2i} - x_{1i}$	d_i^2
45	38	7	49
25	23	2	4
31	35	-4	16
38	41	-3	9
50	44	6	36
33	29	4	16
36	37	-1	1
40	31	9	81
43	38	5	25
		$\sum d_i = 25$	$\sum d_i^2 = 237$
		$\bar{d} = \frac{25}{9} = 2.778$	$s^2 = \frac{1}{8}\left[237 - \frac{25^2}{9}\right]$
			$= 20.94$

$t_{0.025} = 2.306$ with 8 degrees of freedom.

95% C.I. for μ_D is

$$\bar{X}_D \pm t_{\alpha/2}\sqrt{\frac{s^2}{n}} = 2.778 \pm (2.306)\sqrt{\frac{20.94}{9}}$$

$$= 2.778 \pm 3.518$$

Therefore $-0.74 < \mu_D < 6.30$.

Pairing is necessary to factor out variability between the plots, e.g. locations, altitudes, climates and environmental conditions. //

[You may consider $d_i = x_{1i} - x_{2i}$, which gives $(-6.30, 0.74)$]