You write very well, ~~than~~ I don't have enough background to understand this except at a high level. Perhaps that is ok; I would think most readers of your reports will have the appropriate background.

# On SSB Modulation


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## Introduction

Single-sideband modulation (hereafter SSB modulation or just SSB) is a modulation scheme which decreases the bandwidth use of ordinary amplitude modulation (AM) by a factor of two [1]. Ordinary AM implemented using a single mixer produces two sidebands around the carrier frequency (optionally with a strong component at the carrier itself). Since the modulating signal is usually purely real, one of these sidebands is redundant and may be eliminated without loss of information. Additionally, SSB modulation often suppresses the carrier, fully or partially, reducing the amount of power needed to transmit the signal [2].

This report examines the mathematics behind AM & SSB, including a few properties of the Hilbert transform. Basic familiarity with Euler's formula is assumed. The treatment given here is specifically in light of the Hartley and Weaver architectures, and briefly discusses the advantages and disadvantages of each.

you might want to site these here (although I assume you do later)

## The Phasing Method

To illustrate the behavior of ordinary AM, consider a sinusoidal signal $s(t) = \cos \omega t$. Using Euler's formula, $e^{jt} = \cos t + j \sin t$, we may represent this as $s(t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$. Note that

What do these stand for?

a single real sinusoid consists of two complex sinusoids at positive and negative frequencies. If a baseband signal $\cos \omega_{\text{in}} t$ is mixed with a carrier signal $\cos \omega_{\text{lo}} t$, the result is

$$\begin{aligned}(\cos \omega_{\text{in}} t)(\cos \omega_{\text{lo}} t) &= \frac{1}{2}\left(e^{j\omega_{\text{in}} t} + e^{-j\omega_{\text{in}} t}\right) \cdot \frac{1}{2}\left(e^{j\omega_{\text{lo}} t} + e^{-j\omega_{\text{lo}} t}\right) \\ &= \frac{1}{4}\left(e^{j(\omega_{\text{lo}} \pm \omega_{\text{in}})t} + e^{-j(\omega_{\text{lo}} \pm \omega_{\text{in}})t}\right) \\ &= \frac{1}{2}\cos(\omega_{\text{lo}} \pm \omega_{\text{in}})t.\end{aligned}$$

The modulated signal contains frequency components both above and below the carrier, although these both have the same information content. The results of modulation are illustrated in figure 1.

F

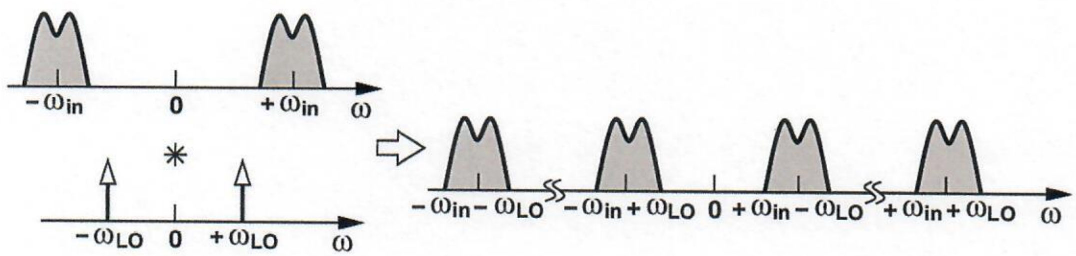


Figure 1: Amplitude modulation of a baseband signal [3].

I knew this stuff a long time ago! I'm only following at a high level right now.

One means of suppressing one of the sidebands, known as the *phasing method*, is done using the Hilbert transform. The Hilbert transform is equivalent to a filter with transfer function $H(\omega) = -j \operatorname{sign} \omega$. Applying this filter to a sinusoid $\cos \omega t = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t})$ yields $\frac{1}{2}(-je^{j\omega t} + je^{-j\omega t}) = \sin \omega t$ – in other words, taking the Hilbert transform of a signal shifts all real frequency components by $\frac{\pi}{2}$. For SSB, the baseband signal is modulated twice – once in its original form, and once after passing it through a Hilbert transformer [4].

If this process is applied to the same sinusoidal signals as above, yielding $s(t) = \cos \omega_{\text{in}} t$ and its Hilbert transform $\hat{s}(t) = \sin \omega_{\text{in}} t$. These signals are then modulated with $\cos \omega_{\text{lo}} t$ and

Something wrong with this sentence. 'If' it is applied, it yields? If what, then what?

$\sin \omega_{\mathrm{lo}} t$, respectively, to produce in-phase and quadrature components of the output:

I see you give formulas for these, but I don't have an intuitive understanding of what each is.

in-phase

quadrature

$$
\begin{aligned}
I(t) &= s(t) \cos \omega_{\mathrm{lo}} t \\
&= \frac{1}{4}\left(e^{j(\omega_{\mathrm{lo}} \pm \omega_{\mathrm{in}})t} + e^{-j(\omega_{\mathrm{lo}} \pm \omega_{\mathrm{in}})t}\right) \text{ by the above, and} \\
Q(t) &= \hat{s}(t) \sin \omega_{\mathrm{lo}} t \\
&= \frac{1}{2}\left(-je^{j\omega_{\mathrm{in}} t} + je^{-j\omega_{\mathrm{in}} t}\right) \cdot \frac{1}{2}\left(-je^{j\omega_{\mathrm{lo}} t} + je^{-j\omega_{\mathrm{lo}} t}\right) \\
&= \frac{1}{4}\left(-e^{\pm j(\omega_{\mathrm{in}} + \omega_{\mathrm{lo}})t} + e^{\pm j(\omega_{\mathrm{in}} - \omega_{\mathrm{lo}})t}\right) \text{ after distributing.}
\end{aligned}
$$

Note that, although expressed in terms of complex sinusoids, both $I(t)$ and $Q(t)$ are *real* signals; the process above may be implemented in analog hardware. Sideband selection is performed by either adding or subtracting $I(t)$ and $Q(t)$ – for example, adding yields

$$
\begin{aligned}
I(t) + Q(t) &= \frac{1}{4}\left(2e^{j(\omega_{\mathrm{lo}} - \omega_{\mathrm{in}})t} + 2e^{-j(\omega_{\mathrm{lo}} - \omega_{\mathrm{in}})t}\right) \\
&= \cos(\omega_{\mathrm{lo}} - \omega_{\mathrm{in}})t \text{ after canceling} \pm \sin,
\end{aligned}
$$

leaving only the lower sideband.

seems like odd wording.

Demodulation functions as modulation in reverse – the AM signal is multiplied by in-phase and quadrature components of a local oscillator, inverse Hilbert transformed, then added back together to form the baseband signal. The Hartley architecture is one example implementation, used since the early days of radio [3].

what is added to what?

## The Weaver Architecture

The phasing method allows SSB generation with a single modulation step and works with input signals of unknown bandwidth. The only difficulty with this method is implementation of an accurate wideband Hilbert transformer. Using DSP techniques on the original signal allows relatively easy generation of its Hilbert transform [4].

In analog circuits, however, the transform must be performed by a wideband phase shift network, which makes rejection ratios above 40 dB quite difficult to achieve. The Weaver architecture, reproduced in figure 2, offers an alternative method to generate SSB signals so long as the baseband signal is sufficiently bandlimited. The input signal is modulated four times – twice in both in-phase and quadrature – and lowpass filtered midway through. The main advantage of the Weaver method is that the requirements on the filters are quite lenient, easing analog implementation [5].

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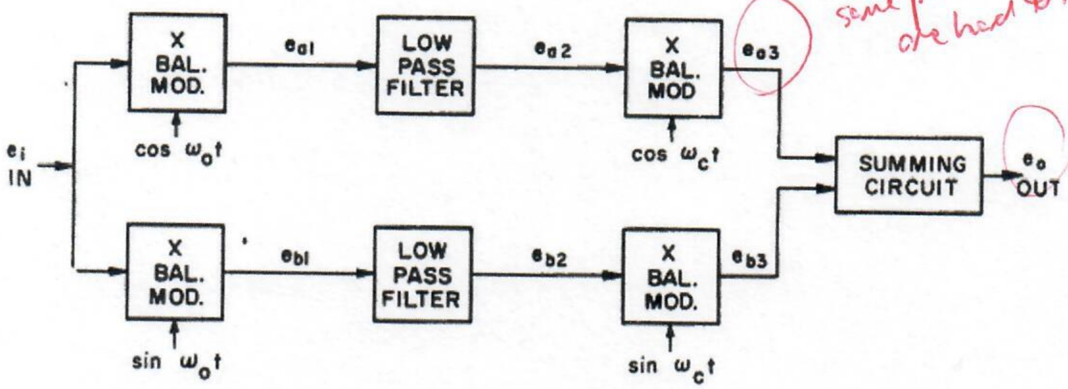


Figure 2: A Weaver modulator [5].

If the same sinusoid, $s(t) = \cos\omega_{\text{in}}t$, is modulated using the above schema, the first in-phase and quadrature signals $I_1(t)$ & $Q_1(t)$ (corresponding to $e_{a1}$ & $e_{b1}$ in figure 2) are

$$I_1(t) = \frac{1}{4}\left(e^{j(\omega_{\text{in}}\pm\omega_o)t} + e^{-j(\omega_{\text{in}}\pm\omega_o)t}\right) \text{ (as before), and}$$
$$Q_1(t) = \frac{1}{2}\left(e^{j\omega_{\text{in}}t} + e^{-j\omega_{\text{in}}t}\right)\cdot\frac{1}{2}\left(-je^{j\omega_o t} + je^{-j\omega_o t}\right)$$
$$= \frac{1}{2}\left(-je^{j(\omega_o\pm\omega_{\text{in}})t} + je^{-j(\omega_o\pm\omega_{\text{in}})t}\right).$$

Lowpass filtering the above signals with a (perfect) cutoff at $\omega_o$ yields

$$I_f(t) = \frac{1}{4}\left(e^{j(\omega_o-\omega_{\text{in}})t} + e^{-j(\omega_o-\omega_{\text{in}})t}\right) \text{ and}$$
$$Q_f(t) = \frac{1}{4}\left(-je^{j(\omega_o-\omega_{\text{in}})t} + je^{-j(\omega_o-\omega_{\text{in}})t}\right).$$

Note that the lowpass filter's transition band can be as much as $2\omega_{\text{in}}$ wide ~~–~~ (i.e.,) this band can be as wide as twice the lowest frequency component of the input signal while still preserving functionality [2]. After the second modulation stage and summation, we have

$$
\begin{aligned}
I_2(t) &= \frac{1}{4}\left(e^{j(\omega_o-\omega_{\text{in}})t} + e^{-j(\omega_o-\omega_{\text{in}})t}\right)\cdot\frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right) \\
&= \frac{1}{8}\left(e^{j(\omega_o-\omega_{\text{in}}\pm\omega_c)t} + e^{-j(\omega_o-\omega_{\text{in}}\pm\omega_c)t}\right), \text{ and} \\
Q_2(t) &= \frac{1}{4}\left(-je^{j(\omega_o-\omega_{\text{in}})t} + je^{-j(\omega_o-\omega_{\text{in}})t}\right)\cdot\frac{1}{2}\left(-je^{j\omega_c t} + je^{-j\omega_c t}\right) \\
&= \frac{1}{8}\left(e^{\pm j(\omega_o-\omega_{\text{in}}-\omega_c)t} - e^{\pm j(\omega_o-\omega_{\text{in}}+\omega_c)t}\right),
\end{aligned}
$$

so our transmission $T(t)$ is (after rearranging)

$$
T(t) = I_2(t) + Q_2(t) = \frac{1}{4}e^{\pm j(\omega_c-\omega_o+\omega_{\text{in}})t} = \frac{1}{2}\cos\left(\omega_c - \omega_o + \omega_{\text{in}}\right)t,
$$

an upper SSB signal centered at $\omega_c - \omega_o$.

## Practicality

D. K. Weaver gives a practical implementation example (reproduced in F figure 3), which, though dated, serves to demonstrate the simplicity of the Weaver method. The circuit comprises sixteen diodes and eight transformers, four of which are air-core high-frequency types. Given its relative simplicity, the circuit is an attractive candidate for a simple analog SSB radio – which was its original intent. Both of the above methods of modulation are studied and used in both transmitters and receivers [1], [3].

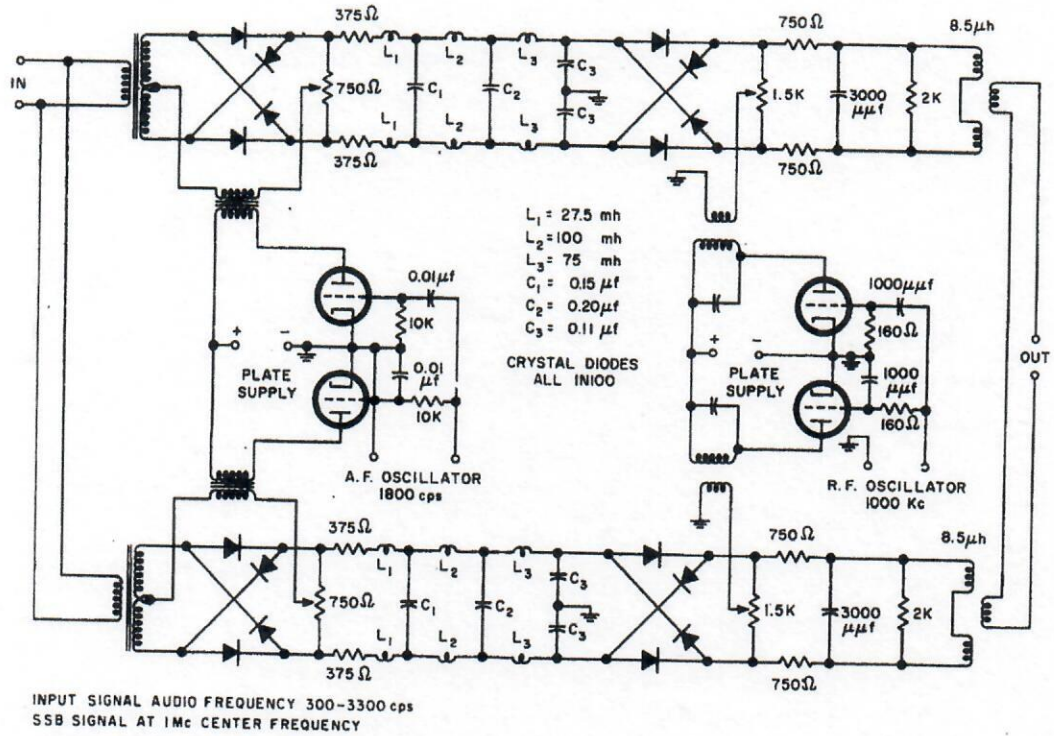


Figure 3: An implementation of a Weaver modulator [5].

## References


[1] T. A. Roppel, *SSB Generation - the Phasing Method*, Auburn University, 2009.

[2] D. Rowell, *Weaver SSB Modulation/Demodulation - A Tutorial*, PJRC, 2017.

[3] B. Razavi, *RF Microelectronics, 2nd Edition (Prentice Hall Communications Engineering and Emerging Technologies Series)*, 2nd ed. Prentice Hall, 2011.

[4] S. A. Tretter, *Single-Sideband Modulation (SSB) and Frequency Translation*, University of Maryland, 2023. [Online]. Available: `https://user.eng.umd.edu/~tretter/commlab/c6713slides/contents.html` (visited on 10/30/2023).

[5] D. K. Weaver, "A Third Method of Generation and Detection of Single-Sideband Signals," *Proceedings of the IRE*, pp. 1703–1705, 1956.