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HW4 Solution

7.5 $\max_{\boldsymbol{x}\in\mathbb{R}^2,\xi\in\mathbb{R}} \boldsymbol{p}^\top \boldsymbol{x} + \xi$ can be expressed as

$$\boldsymbol{c}^\top \boldsymbol{u} = \begin{bmatrix} p_0 & p_1 & 1 \end{bmatrix} \boldsymbol{u}$$

where the last component of $\boldsymbol{u}$ is ξ. Now, from the constraints $-\xi \le 0, x_0 \le 0$ and $x_1 \le 3$, we can find $\boldsymbol{A}\boldsymbol{u} \le \boldsymbol{b}$.

$$\boldsymbol{A}\boldsymbol{u} \le \boldsymbol{b} \text{ such that } \boldsymbol{b} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \text{ and } \boldsymbol{A}\boldsymbol{u} = \begin{bmatrix} x_0 \\ x_1 \\ -\xi \end{bmatrix}$$

Thus,

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \boldsymbol{u} = \begin{bmatrix} x_0 \\ x_1 \\ \xi \end{bmatrix}$$

Therefore, the standard linear program is

$$\max_{\boldsymbol{u}\in\mathbb{R}^3} \boldsymbol{c}^\top \boldsymbol{u}$$

$$\text{subject to } \boldsymbol{A}\boldsymbol{u} \le \boldsymbol{b}$$

7.6 For a linear program

$$\max_{\boldsymbol{x}\in\mathbb{R}^3} \boldsymbol{c}^\top \boldsymbol{x}$$

$$\text{subject to } \boldsymbol{A}\boldsymbol{x} \le \boldsymbol{b},$$

we can write the dual optimization problem as

$$\begin{aligned} \max_{\boldsymbol{\lambda}\in\mathbb{R}^m} \quad & -\boldsymbol{b}^\top \boldsymbol{\lambda} \\ \text{subject to} \quad & \boldsymbol{c} + \boldsymbol{A}^\top \boldsymbol{\lambda} = \boldsymbol{0} \\ & \boldsymbol{\lambda} \ge \boldsymbol{0}. \end{aligned}$$

from the Lagrangian $(\boldsymbol{c} + \boldsymbol{A}^\top \boldsymbol{\lambda})^\top \boldsymbol{x} - \boldsymbol{\lambda}^\top \boldsymbol{b}$ and its derivative $\boldsymbol{c} + \boldsymbol{A}^\top \boldsymbol{\lambda} = \boldsymbol{0}$. The problem gives

$$\boldsymbol{c} = -\begin{bmatrix} 5 \\ 3 \end{bmatrix}$$
$$\boldsymbol{b} = \begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix}$$
$$\boldsymbol{A} = \begin{bmatrix} 2 & 2 \\ 2 & -4 \\ -2 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$$

The dual linear program is expressed as

$$\begin{aligned} \max_{\boldsymbol{\lambda} \in \mathbb{R}^5} - &\begin{bmatrix} 33 \\ 8 \\ 5 \\ -1 \\ 8 \end{bmatrix}^\top \boldsymbol{\lambda}, \\ &-\begin{bmatrix} 5 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 & 2 \\ 2 & -4 \\ -2 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}^\top \boldsymbol{\lambda} = \boldsymbol{0}, \\ &\boldsymbol{\lambda} \geq \boldsymbol{0}. \end{aligned}$$