Examples:

Solve $(D^3 - 4D^2D' + 4DD'^2)z = 2\sin(3x+2y)$

Solution: The auxiliary equation of

$$(D^3 - 4D^2D' + 4DD'^2)z = 0 \text{ is}$$

$$m^3 - 4m^2 + 4m = 0$$

$$\Rightarrow m(m^2 - 4m + 4) = 0$$

$$\Rightarrow m(m-2)^2 = 0$$

$$m = 0, 2, 2$$

$$C.F = f_1(y) + f_2(y+2x) + x f_3(y+2x)$$

$$P.I = \frac{1}{D^3 - 4D^2D' + 4DD'^2} 2\sin(3x+2y)$$

$$= 2 \cdot \frac{1}{D(D^2 - 4DD' + 4D'^2)} \sin(3x+2y)$$

$$= 2 \cdot \frac{1}{D(-9 - 4(-6) + 4(-4))} \sin(3x+2y)$$

$$= 2 \cdot \frac{1}{D(-9 + 24 - 16)} \sin(3x+2y)$$

$$= -\frac{2}{D} \sin(3x+2y)$$

$$= -\frac{2}{3}\left[-\cos(3x+2y)\right]$$

$$= \frac{2}{3}\cos(3x+2y)$$

General solution is

$$z = f_1(y) + f_2(y+2x) + x f_3(y+2x) + \frac{2}{3}\cos(3x+2y)$$

Example 2:

Solve $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{2x+y}$

Solution: The auxiliary equation of

$$(D^3 - 7DD'^2 - 6D'^3)z = 0 \text{ is } m^3 - 7m - 6 = 0$$

Now $m^3 - 7m - 6 = 0$

$$\Rightarrow m^3 + m^2 - m^2 - m - 6m - 6 = 0$$

$$\Rightarrow m^2(m+1) - m(m+1) - 6(m+1) = 0$$

$$\Rightarrow (m+1)(m^2 - m - 6) = 0$$

$$\Rightarrow (m+1)(m^2 - 3m + 2m - 6) = 0$$

$$\Rightarrow (m+1)\{m(m-3) + 2(m-3)\} = 0$$

$$\Rightarrow (m+1)(m-3)(m+2) = 0$$

$$m = -1, -2, 3$$

$$C.F = f_1(y-x) + f_2(y-2x) + f_3(y+3x)$$

$$P.I = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\left[\sin(x+2y) + e^{2x+y}\right]$$

$$= \frac{1}{D^2 D - 7DD'^2 - 6D'D'^2}\sin(x+2y)$$

$$+ \frac{1}{D^3 - 7DD'^2 - 6D'^3}e^{2x+y}$$

$$= \frac{1}{-1^2 \cdot D - 7D(-2^2) - 6D'(-2^2)}\sin(x+2y) + \frac{1 \cdot e^{2x+y}}{2^3 - 7 \cdot 2 \cdot 1^2 - 6 \cdot 1^3}$$

P-3

L-5 (PDE)

$$PI = \frac{1}{27D + 24D'} \sin(x+2y) + \frac{1}{8-14-6} e^{2x+y}$$

$$= \frac{1}{3} \cdot \frac{1}{9D + 8D'} \sin(x+2y) + \frac{1}{-12} e^{2x+y}$$

$$= \frac{1}{3} \cdot \frac{D}{9D^2 + 8DD'} \sin(x+2y) - \frac{1}{12} e^{2x+y}$$

$$= \frac{1}{3} \cdot \frac{D \sin(x+2y)}{9(-1^2) + 8(-2)} - \frac{1}{12} e^{2x+y}$$

$$= -\frac{1}{75} \cos(x+2y) - \frac{1}{12} e^{2x+y}$$