MAD2311.03 – Quiz 2 Name: ______________________

1. (5 points) Consider the equation:

$$e^{-y^2}\cos(x^2) = e^{-2}$$

a.) Find the corresponding y value(s) when $x = \sqrt{2\pi}$.

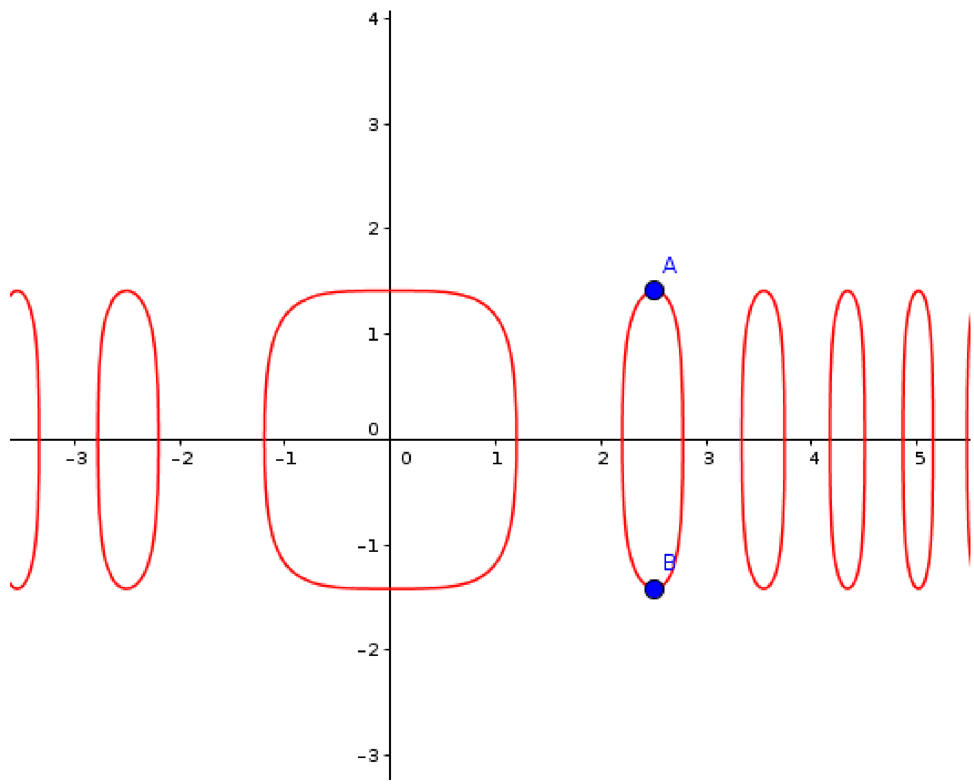
$$\begin{aligned} e^{-y^2}\cos((\sqrt{2\pi})^2) &= e^{-2} \\ e^{-y^2}\cos(2\pi) &= e^{-2} \\ e^{-y^2}\cdot 1 &= e^{-2} \\ e^{-y^2} &= e^{-2} \\ -y^2 &= -2 \\ y &= \pm\sqrt{2} \end{aligned}$$

b.) Use implicit differentiation to find y'.

$$\begin{aligned} \frac{d}{dx}\left(e^{-y^2}\cos(x^2)\right) &= \frac{d}{dx}\left(e^{-2}\right) \\ \frac{d}{dx}\left(e^{-y^2}\right)\cos(x^2) + e^{-y^2}\frac{d}{dx}\left(\cos(x^2)\right) &= 0 && \text{(LHS: product rule, RHS: derivative of constant is zero)} \\ e^{-y^2}\frac{d}{dx}\left(-y^2\right)\cos(x^2) - e^{-y^2}\sin(x^2)\frac{d}{dx}\left(x^2\right) &= 0 && \text{(derivative of exponential, cosine, and chain rule)} \\ -e^{-y^2}2yy'\cos(x^2) - e^{-y^2}\sin(x^2)2x &= 0 && \text{(chain rule)} \\ -2e^{-y^2}\left(yy'\cos(x^2) + \sin(x^2)x\right) &= 0 && \text{(factoring)} \\ yy'\cos(x^2) + \sin(x^2)x &= 0 && \text{(cancel factored terms (exp is never 0))} \\ y' &= -\frac{x\sin(x^2)}{y\cos(x^2)} && \text{(rearrange)} \end{aligned}$$

c.) Find the slope of the tangent line(s) when $x = \sqrt{2\pi}$.

The slope(s) are given by $y'|_{x=\sqrt{2\pi}, y=\pm\sqrt{2}}$, i.e., y' evaluated at $(\sqrt{2\pi}, -\sqrt{2})$ and $(\sqrt{2\pi}, \sqrt{2})$. In both cases, y' is zero, so the slope of the tangents at these points is zero. This function along with the two points of interest is plotted on the next page (not that you would be expected to plot this implicit function!). Notice how the slope of the tangent is horizontal at these points.



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