MAC2311.03 – Quiz 3 Name: ______________________________

1. (5 points) Recall that the the area of a circle is $A = \pi r^2$. Suppose that the radius of the circle (and hence the area) changes with respect to time.

a.) Relate the instantaneous rate of change of the area of the circle with respect to time to the instantaneous rate of change of the radius of the circle with respect to time.

$$\begin{aligned} A(t) &= \pi[r(t)]^2 \\ \frac{d}{dt}\left(A(t)\right) &= \frac{d}{dt}\left(\pi[r(t)]^2\right) \\ A'(t) &= 2\pi r(t) r'(t) \end{aligned}$$

b.) Suppose during the interval $0 \leq t \leq \frac{3}{4}$, the radius is given by $r(t) = \sqrt{t(1-t)}$. Find the global minimum and the global maximum for **the area of the circle** during this interval by testing all appropiate points in time.

To find the global minimum and maximum, we must evaluate $A(t)$ at all critical numbers and at the end points. To find the critical numbers of $A(t)$, we have to find all t in the domain of $A(t)$ where $A'(t)$ is zero or where $A'(t)$ is undefined. Because we're given $r(t)$, and we know the relationship between $A'(t)$, $r(t)$, and $r'(t)$, we just need to find $r'(t)$.

$$r'(t) = \frac{1}{2}\frac{1-2t}{\sqrt{t(1-t)}}$$

So,

$$A'(t) = \pi(1-2t)$$

$A'(t)$ is zero when $t = 1/2$. We have then that:

$$\begin{aligned} A(0) &= 0 \\ A(1/2) &= \frac{\pi}{4} = \frac{4\pi}{16} \\ A(3/4) &= \frac{3\pi}{16} \end{aligned}$$

So the global minimum of $A(t)$ is 0 and occurs when $t = 0$, and the global maximum is $\pi/4$ which occurs when $t = 1/2$