

Algorithmic Lung Nodule Analysis in Chest Tomography Images

**A Statistical Extension of the Level
Set Method for Image Segmentation**

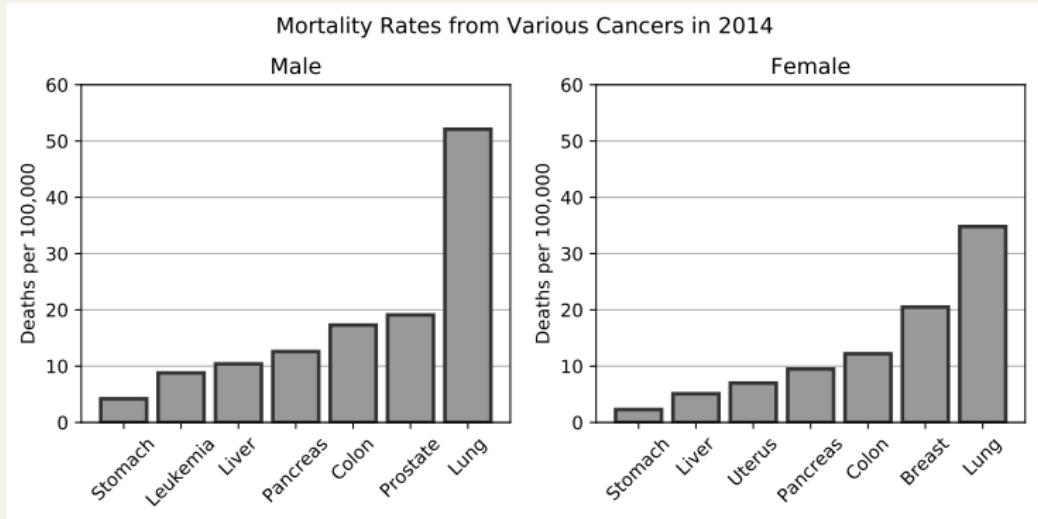
Matthew C. Hancock

Major Professor: Jerry F. Magnan

**Department of Mathematics
Florida State University**

November 9, 2017

Mortality Rates of Cancer by Anatomical Location



Medical Motivation for Computer-Aided Diagnosis

- ▶ Early detection of lung cancer increases survival rate [3]
- ▶ NLST [1] (study of 53,454 patients): low-dose CT screening reduced mortality rate by a relative reduction of 20% compared to traditional radiography screening.
- ▶ Lung CAD may aid in early detection and understanding of characteristics of malignant nodules to potentially improve survival rates
- ▶ Large dataset necessary for validating lung CAD methods prompts collection of LIDC dataset (1018 annotated chest CT scans) [2]

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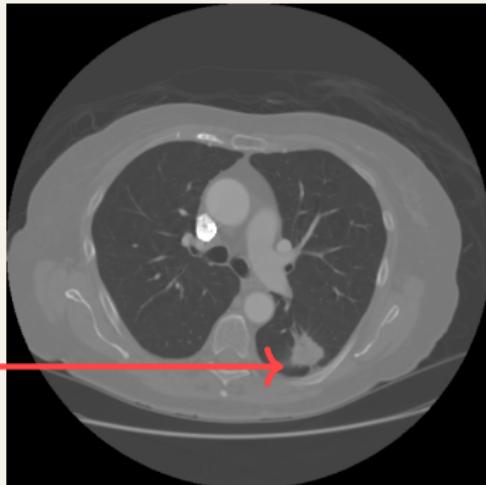
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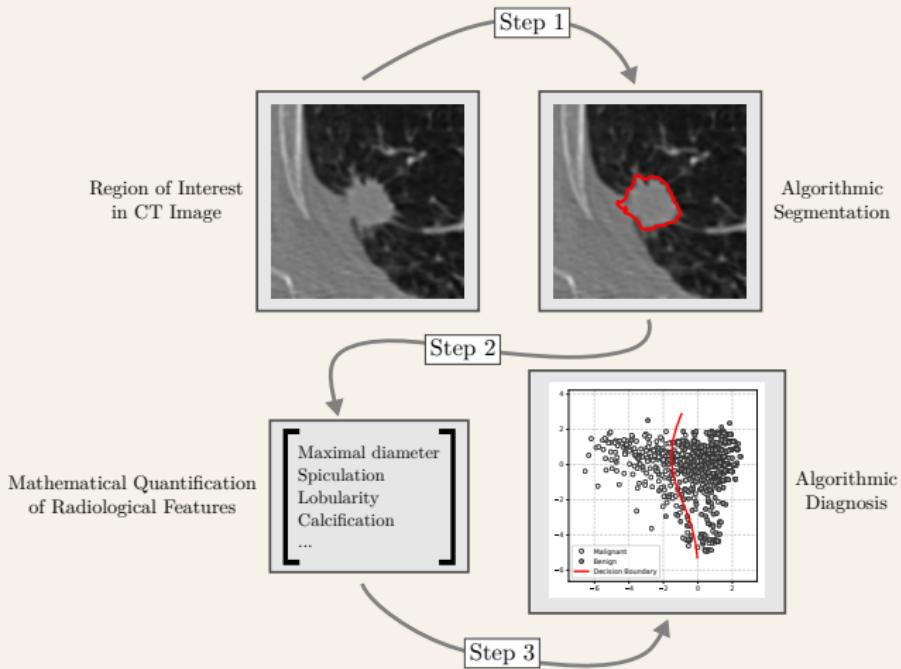
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Primary Object of Interest

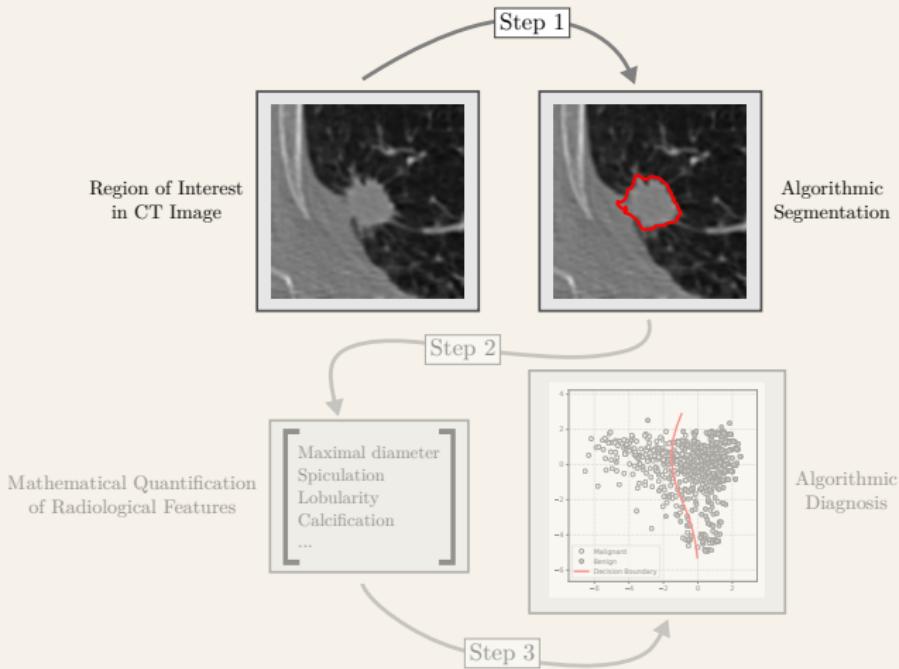


- ▶ **Lung nodule:** small to medium sized (roughly, 3 mm to 30 mm), abnormal region with a somewhat well-defined boundary [8]

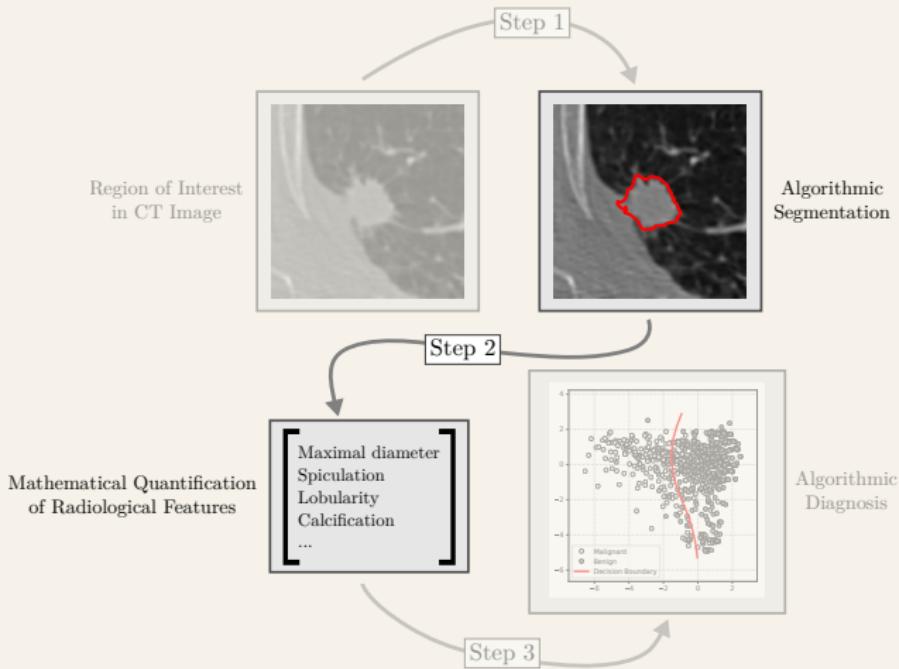
Lung CAD Paradigm for Nodules from Chest CT



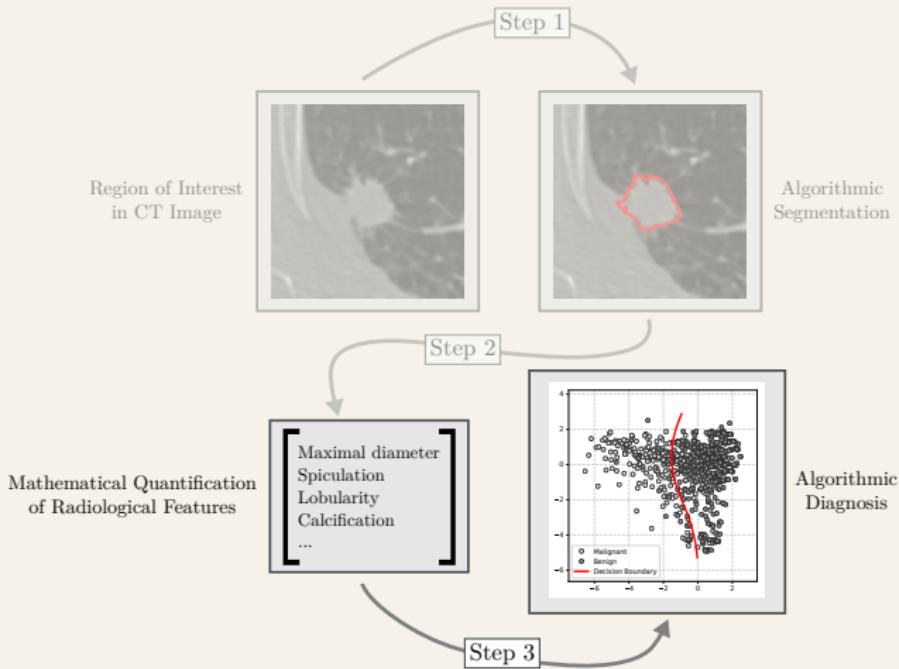
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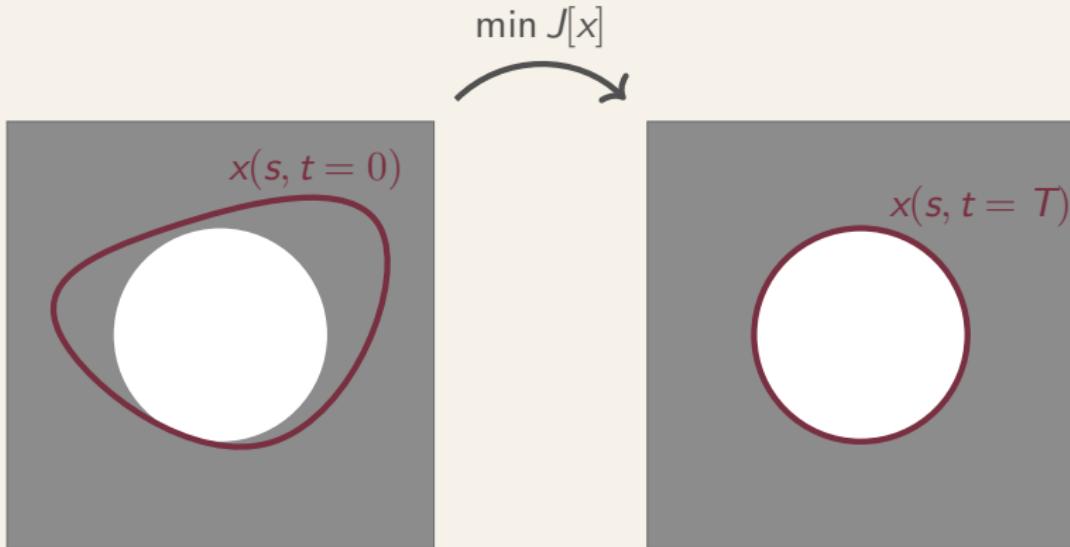


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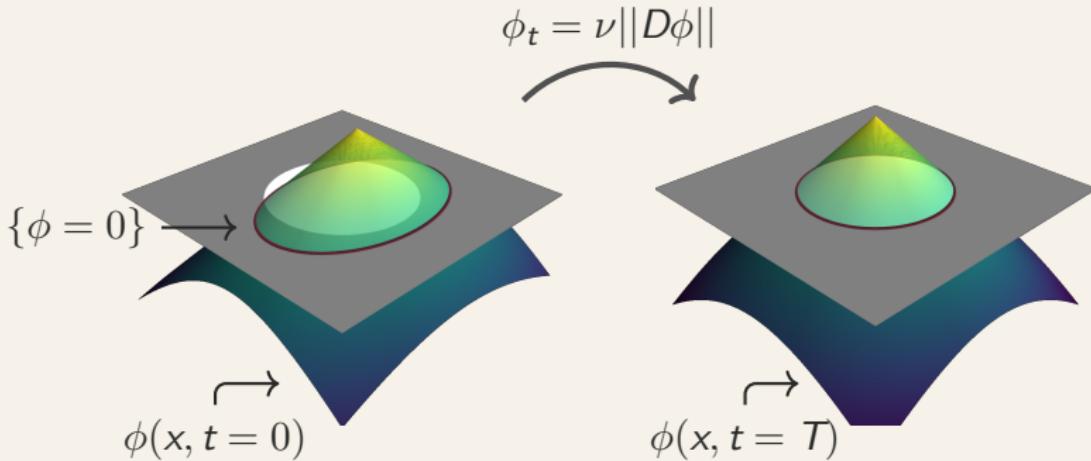
Active Contours

- ▶ Parameterized contour evolves to minimize shape and image functionals (Kass et al. [4])



Level Set Formulation

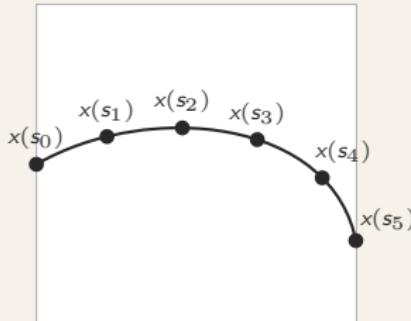
- ▶ Contour as zero level set of $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}$ (Malladi and Sethian [7])
- ▶ Enforce level curves evolve in normal direction with speed ν



Level Set Formulation: Advantages

- Discretization over regular spatial grid ϕ_{ij} , rather than discretized parametrized curve $x(s_i) = (x_1(s_i), x_2(s_i))$
- Extra book-keeping unnecessary for curves that merge / split
- Indifference to dimension

ϕ_{11}	ϕ_{12}	ϕ_{13}	ϕ_{14}	ϕ_{15}	ϕ_{16}
ϕ_{21}	ϕ_{22}	ϕ_{23}	ϕ_{24}	ϕ_{25}	ϕ_{26}
ϕ_{31}	ϕ_{32}	ϕ_{33}	ϕ_{34}	ϕ_{35}	ϕ_{36}
ϕ_{41}	ϕ_{42}	ϕ_{43}	ϕ_{44}	ϕ_{45}	ϕ_{46}
ϕ_{51}	ϕ_{52}	ϕ_{53}	ϕ_{54}	ϕ_{55}	ϕ_{56}
ϕ_{61}	ϕ_{62}	ϕ_{63}	ϕ_{64}	ϕ_{65}	ϕ_{66}



Choosing the Speed Term $\nu(x)$

- ▶ Need:
 1. Contraction outside of object
 2. Expansion inside object
 3. $\nu(x) \rightarrow 0$ near border of object
- ▶ In general, depends on expected object shape and appearance
- ▶ Contract/expand where image dark/light. Stop at image edges:

$$\nu(x) = \text{sign}(\underbrace{M(x)}_{\text{Image}}) \exp\left(-\lambda \underbrace{\|D(M * G_\sigma)(x)\|}_{\text{Image edge strength}}\right)$$

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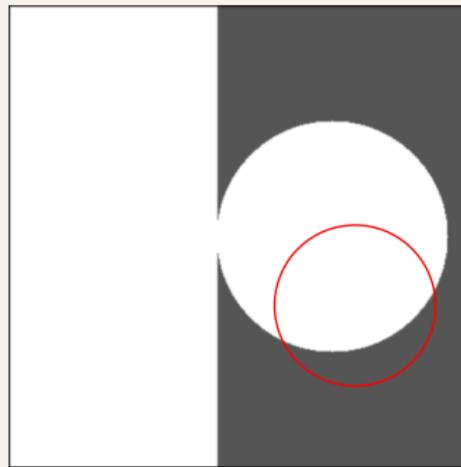
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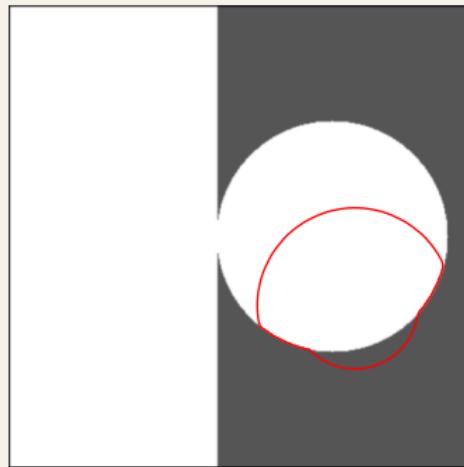
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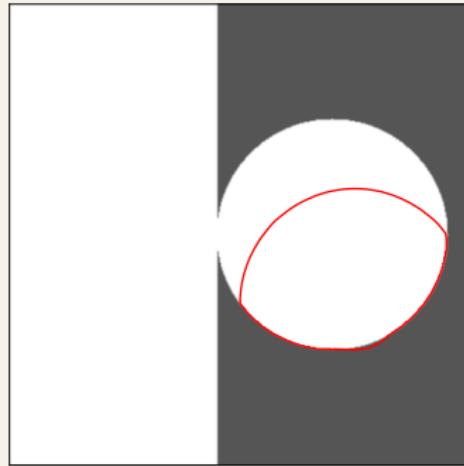
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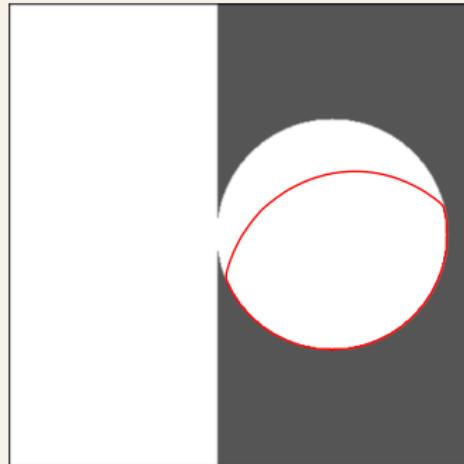
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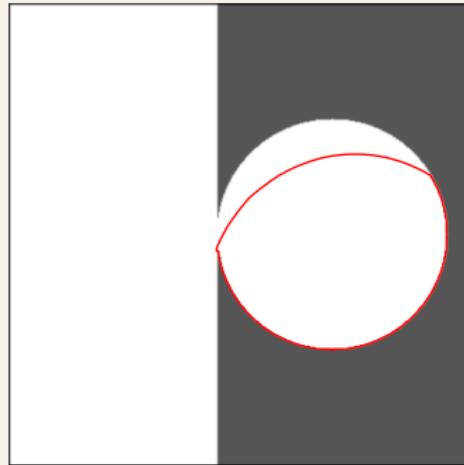
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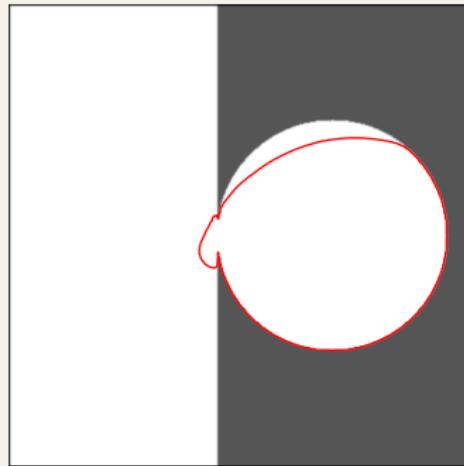
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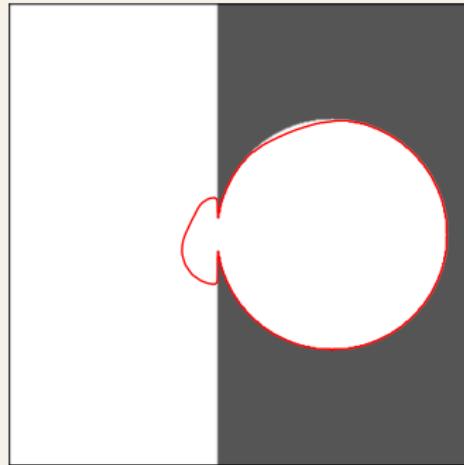
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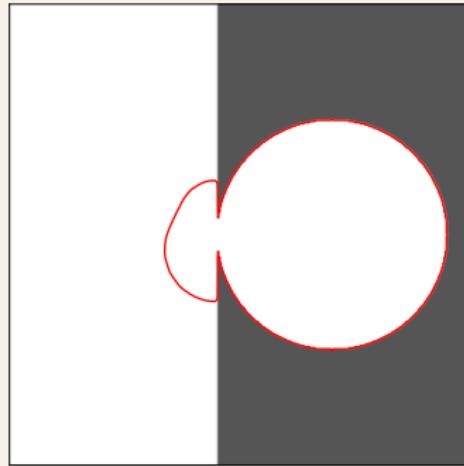
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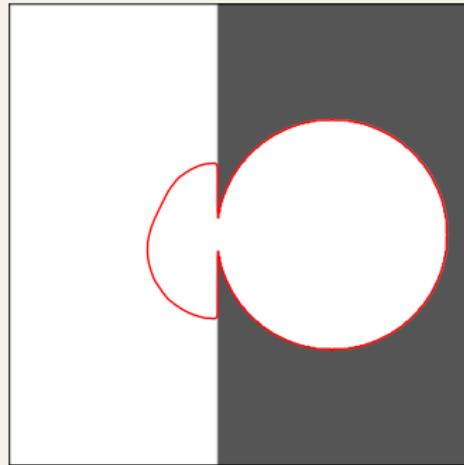
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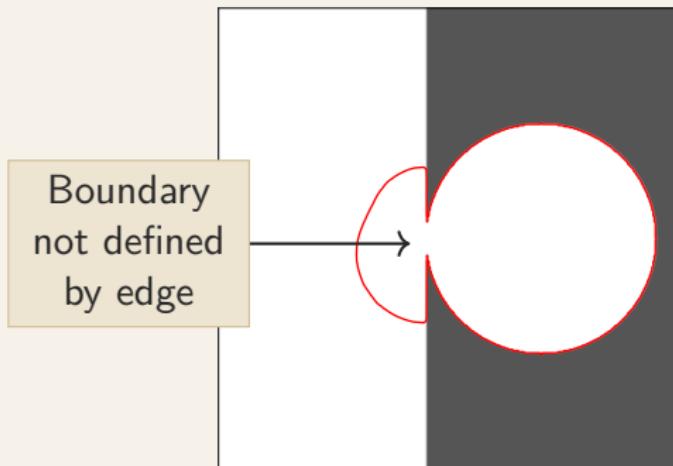
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Statistically-Calibrated Level Set (SCLS) Method

- ▶ Basic idea: use dataset of images and ground truth segmentations $\left(M(k), B(k) \right)_{k=1}^N$ to approximate $\nu(x)$
- ▶ How should data influence the estimate of $\nu(x)$?

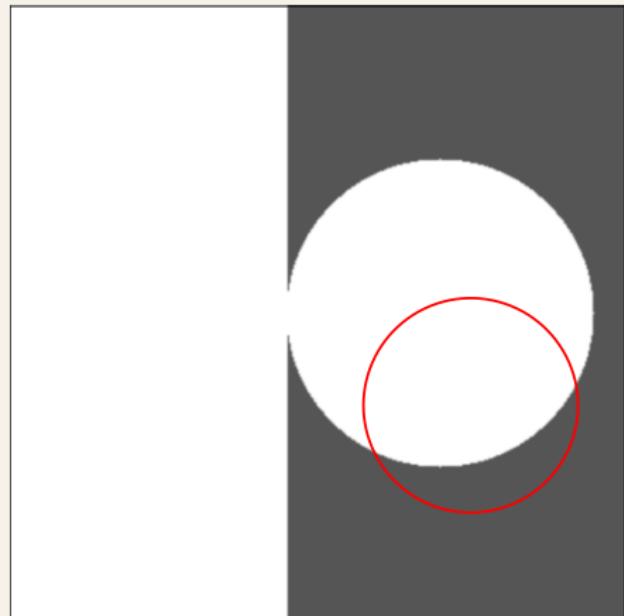
SCLS Method: Motivating Choice of $\nu(x)$

- Discretize the PDE:

$$\phi_{ij}^{n+1} = \phi_{ij}^n + \eta s_{ij} ||D\phi_{ij}^n|| \quad (1)$$

- If s_{ij} is a signed representation of the target segmentation B , then (1) converges to signed representation of B^*

*(with minor caveats on η and ϕ^0)



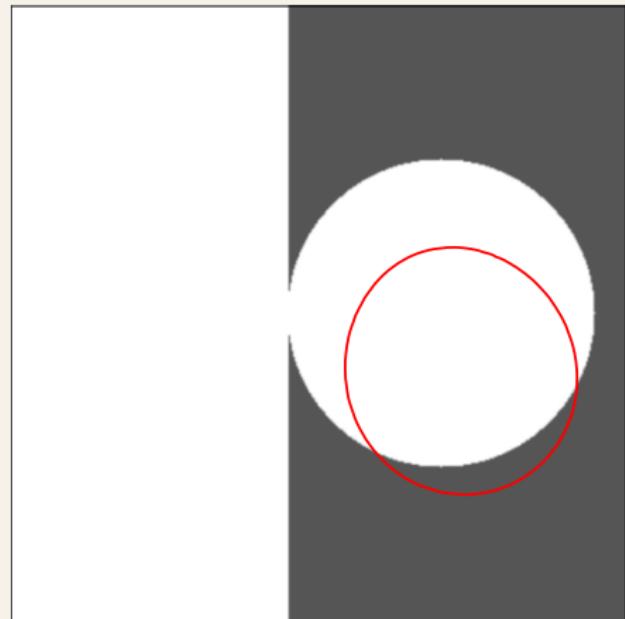
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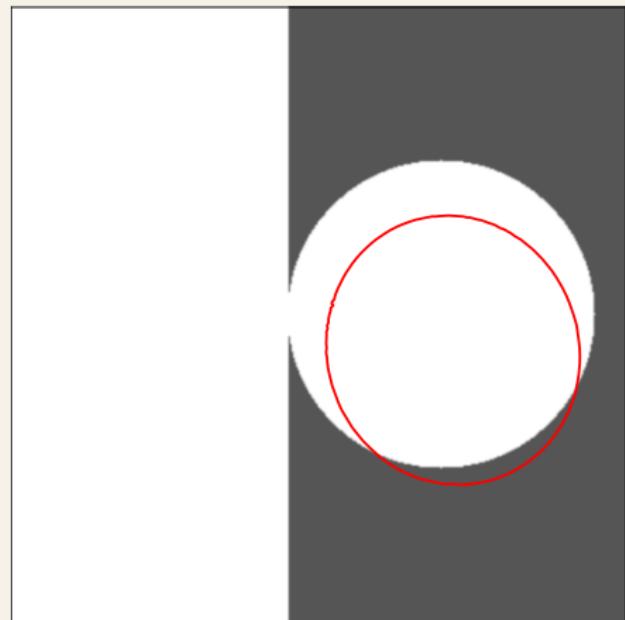
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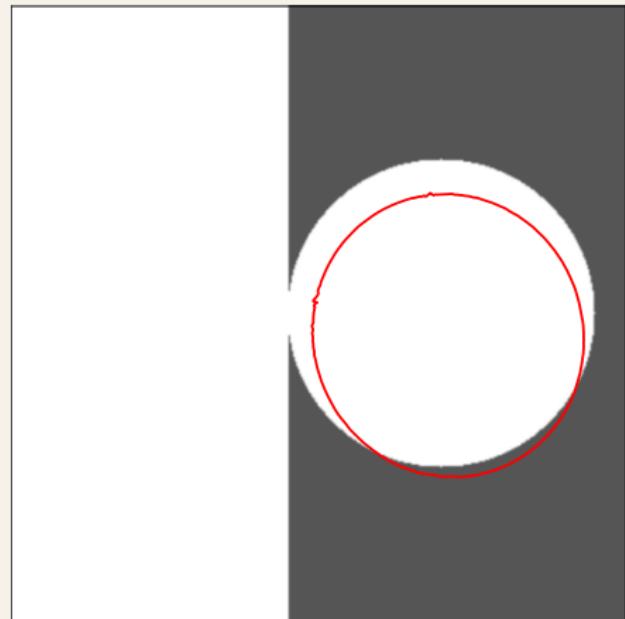
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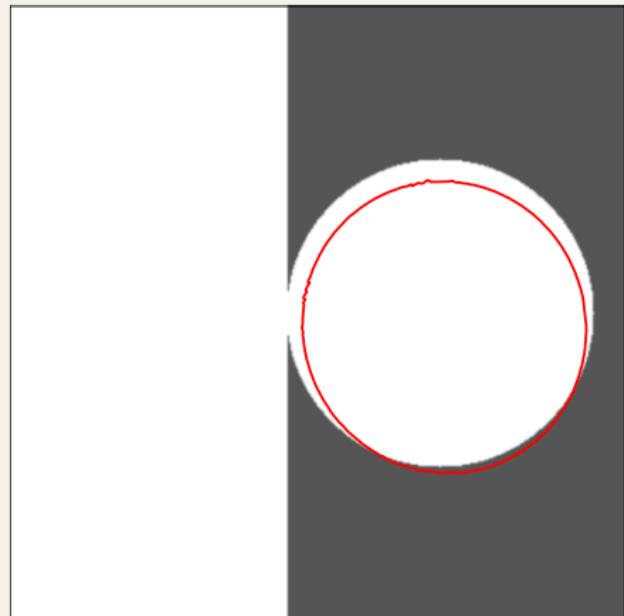
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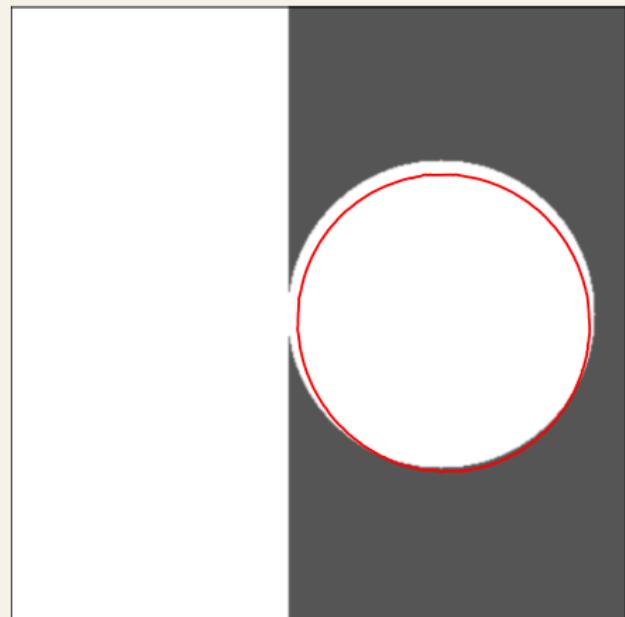
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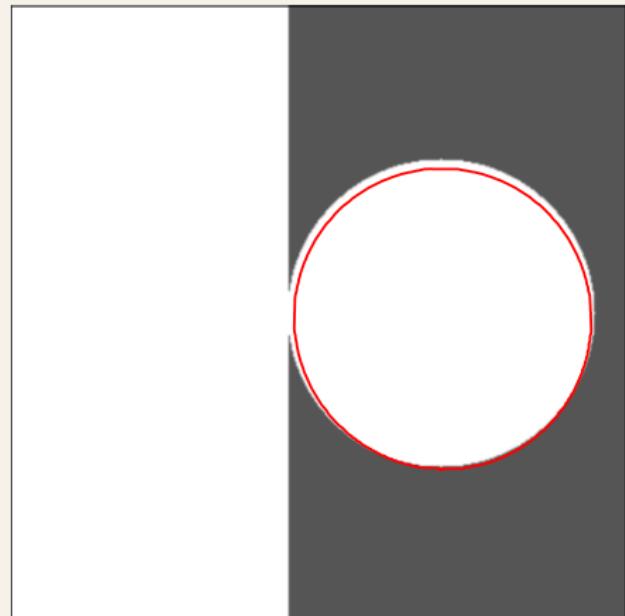
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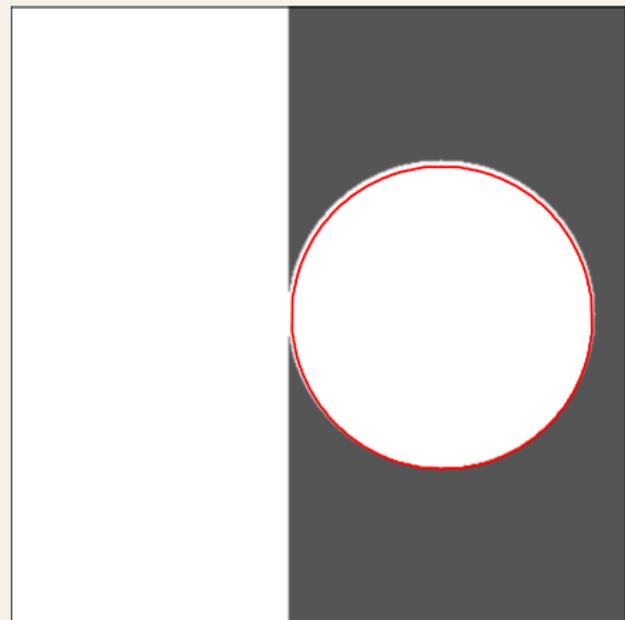
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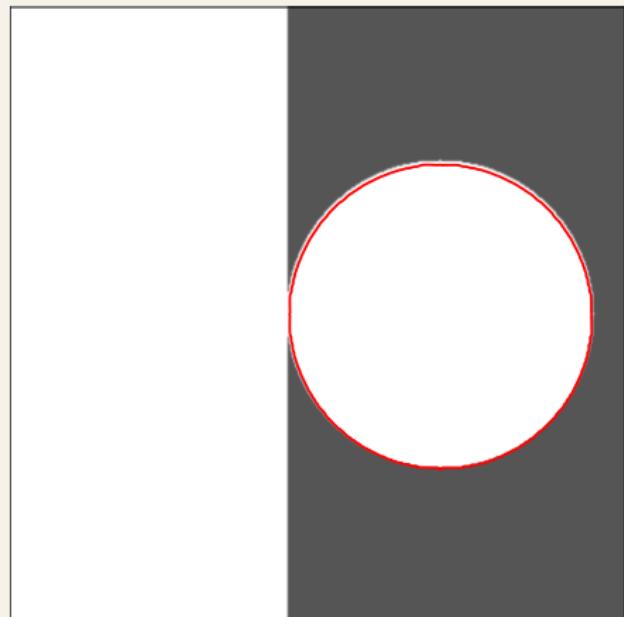
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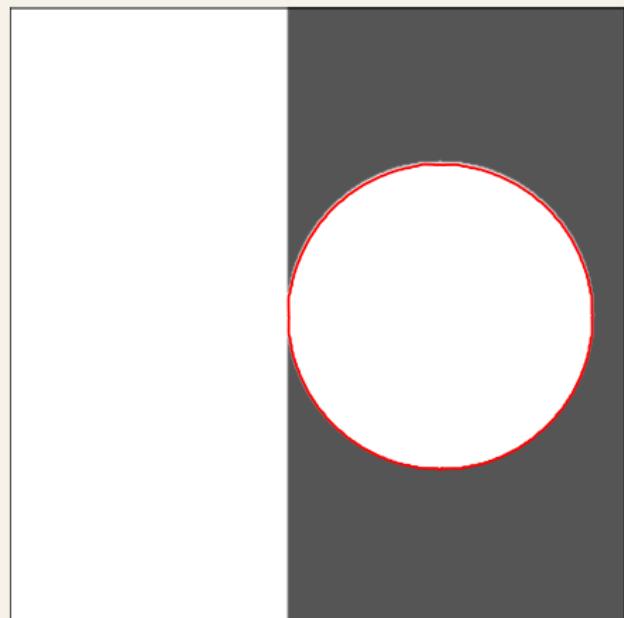
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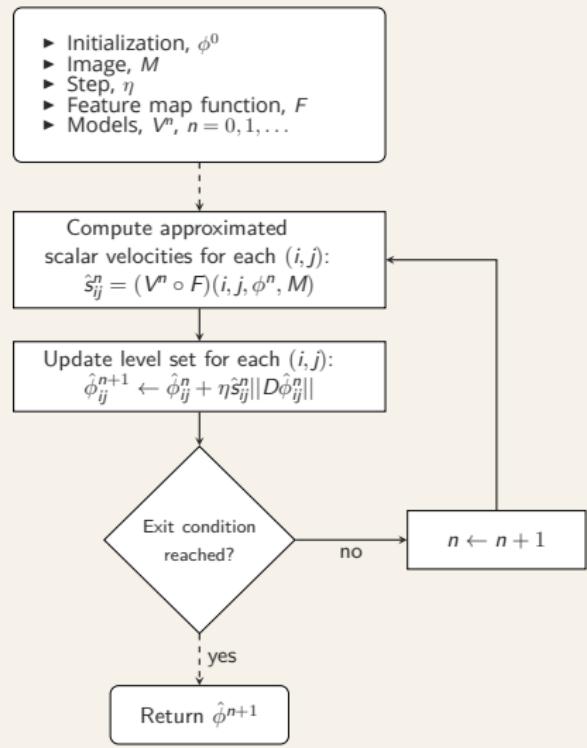
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SCLS Method: An Iterative Approximation Scheme

$$\phi_{ij}^{n+1} = \phi_{ij}^n + \eta s_{ij} \|D\phi_{ij}^n\|$$

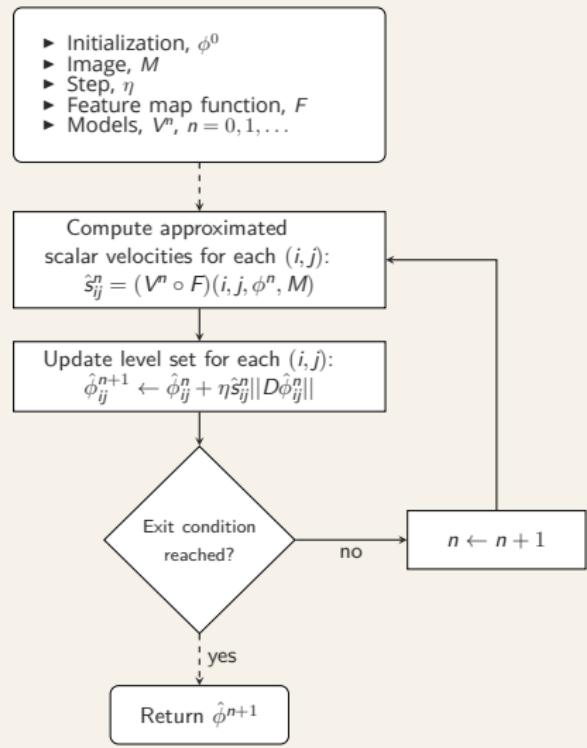
- ▶ Approximate $s_{ij} \approx \hat{s}_{ij}^n = V^n(F | \theta_n)$
- ▶ V^n is a regression model
- ▶ $F = F(i, j, \phi, M)$ is feature map
- ▶ V^n : Features \rightarrow Level set speed



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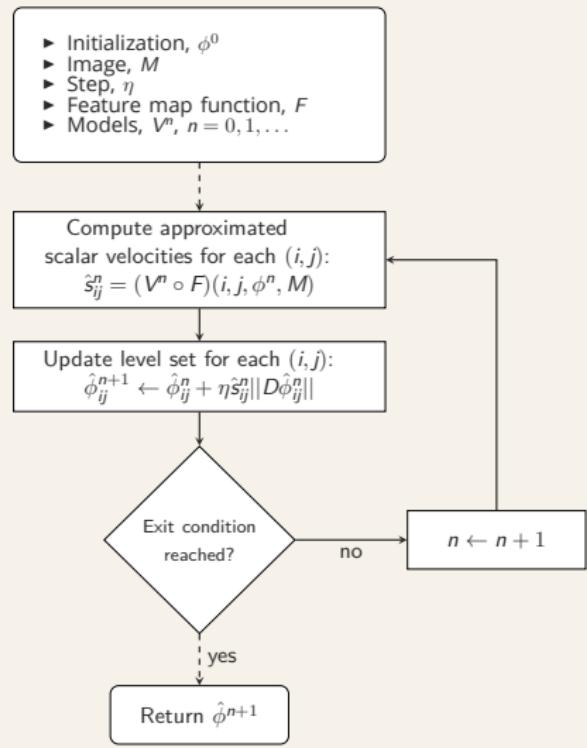
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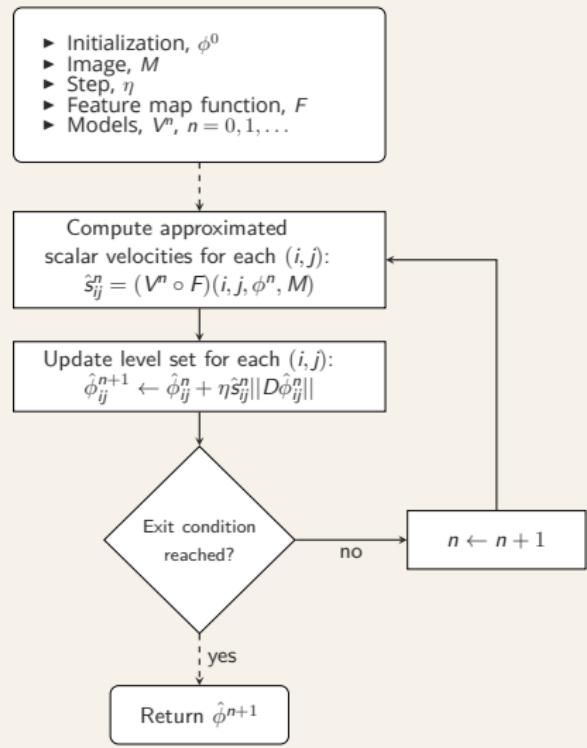
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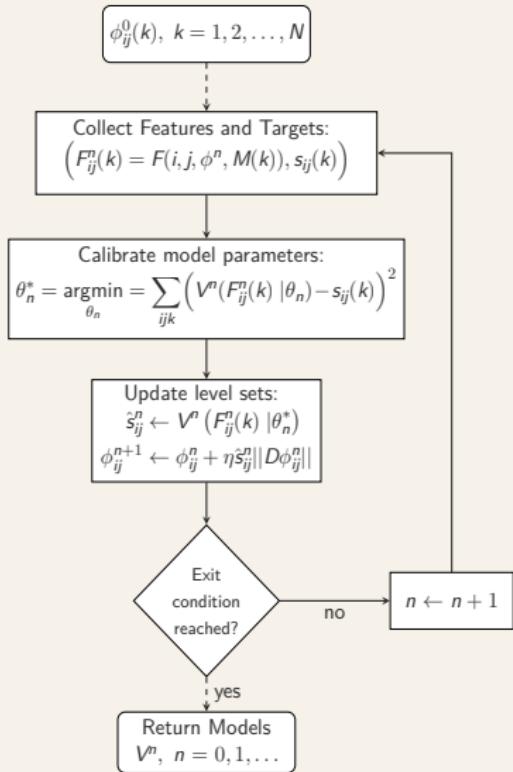
SCLS Method: Training Process

- $F = F(i, j, \phi, M)$ is a feature map of our choosing, e.g.,

$$F(i, j, \phi, M) = \begin{bmatrix} (G_\sigma * M)_{ij} \\ \sum_{uv} H(\phi_{uv}) \end{bmatrix}$$

- Parameters θ_n obtained via sequential optimization:

$$\theta_n^* = \operatorname{argmin}_\theta \sum_{ijk} (s_{ij} - V^n(F|\theta_n))^2$$



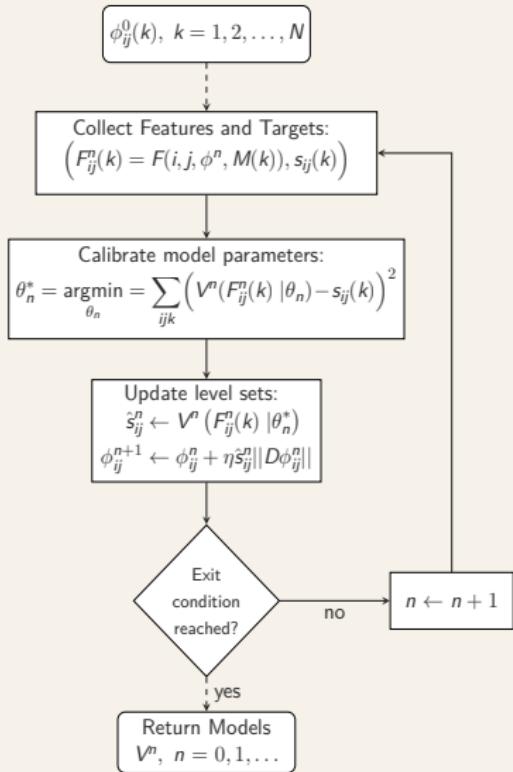
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SCLS: Details

- ▶ Choice of regression model: neural network
- ▶ Step size choice $\eta = 0.1$ works well empirically
- ▶ Initialization choice ϕ^0 is application dependent

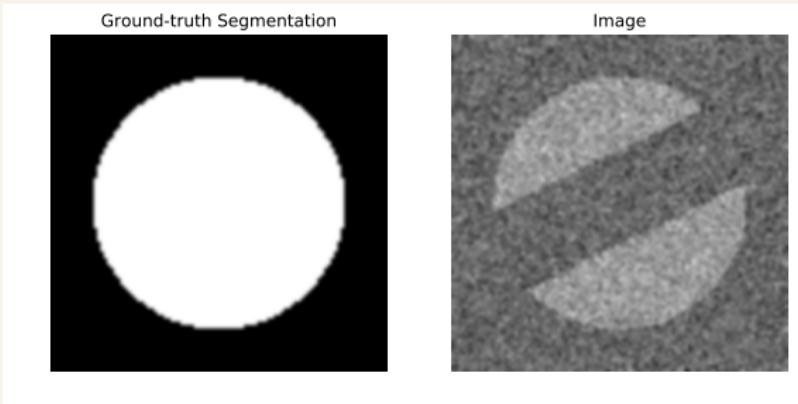
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Synthetic Data: Partitioned Circles



- ▶ 100/50/50 randomly generated image and segmentation pairs for training/validation/testing datasets

Synthetic Data: Partitioned Circles, Feature Maps Tested

Table: Feature Map 1

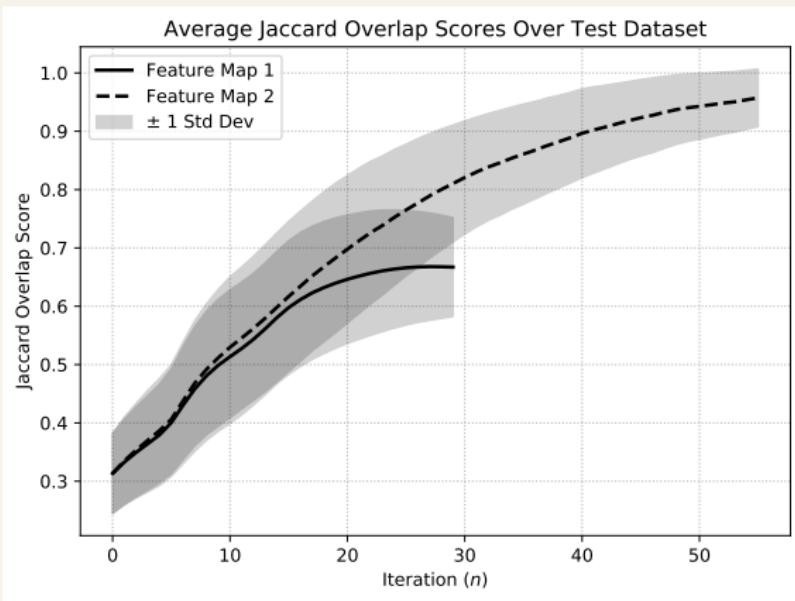
Feature	Local	Global
Image value	✓	
Image edge	✓	

Table: Feature Map 2

Feature	Local	Global
Area		✓
Boundary length		✓
Isoperimetric ratio		✓
Seg. moments, $p = 1, 2$		✓
Dist. from center of mass	✓	✓
Image value mean in seg.		✓
Image value std. in seg.		✓
Image value		✓
Image edge		✓

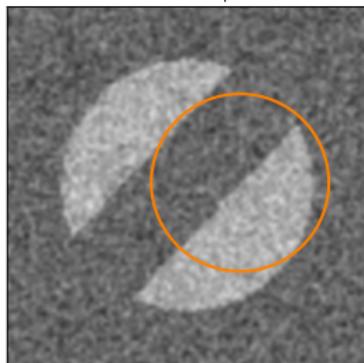
Image features are computed at four scales, $\sigma \in \{0, 1, 2, 3\}$

Synthetic Data: Partitioned Circles, Results



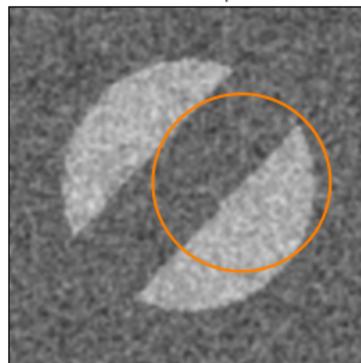
Synthetic Data: Partitioned Circles, Results

Feature Map 1



$n = 0$
Overlap = 0.4446

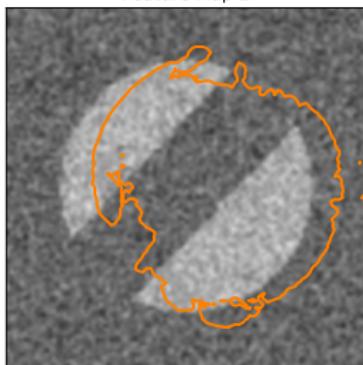
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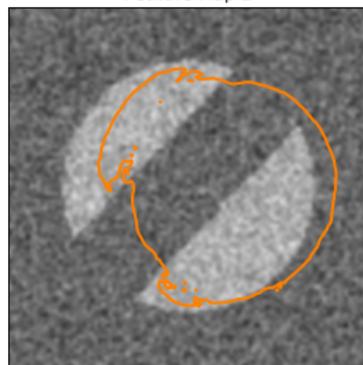
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Feature Map 1



$n = 9$
Overlap = 0.6635

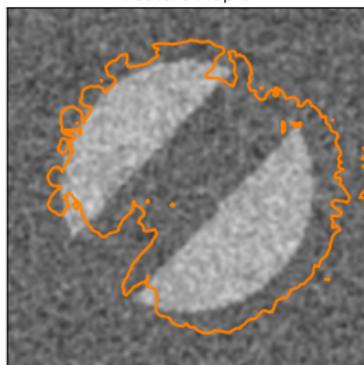
Feature Map 2



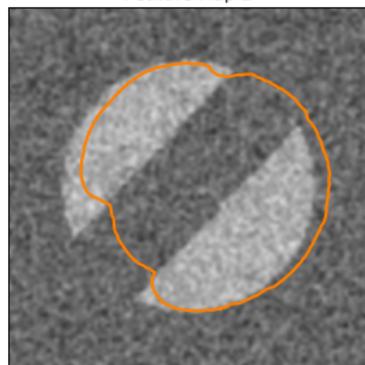
$n = 9$
Overlap = 0.6782

Synthetic Data: Partitioned Circles, Results

Feature Map 1

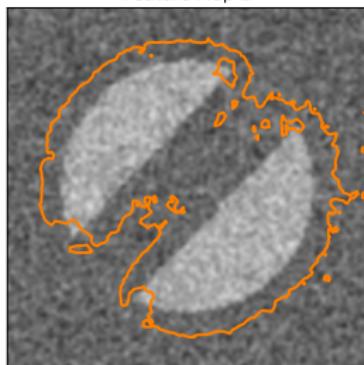


Feature Map 2



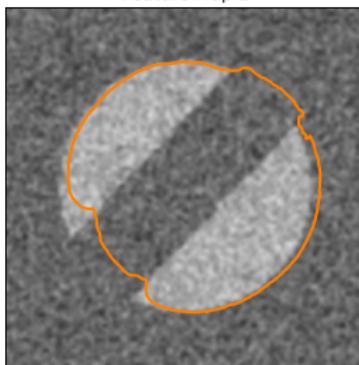
Synthetic Data: Partitioned Circles, Results

Feature Map 1



$n = 27$
Overlap = 0.6856

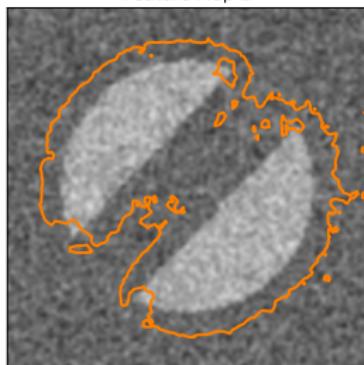
Feature Map 2



$n = 27$
Overlap = 0.8733

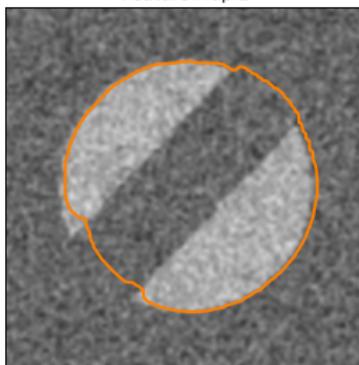
Synthetic Data: Partitioned Circles, Results

Feature Map 1



$n = 27$
Overlap = 0.6856

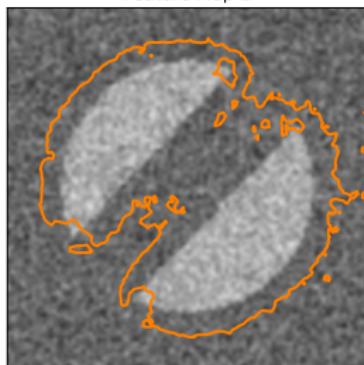
Feature Map 2



$n = 36$
Overlap = 0.9136

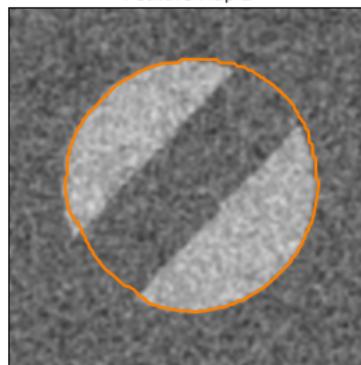
Synthetic Data: Partitioned Circles, Results

Feature Map 1



$n = 27$
Overlap = 0.6856

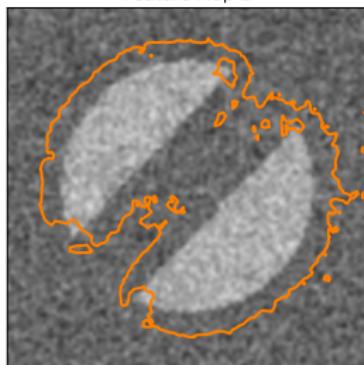
Feature Map 2



$n = 45$
Overlap = 0.9463

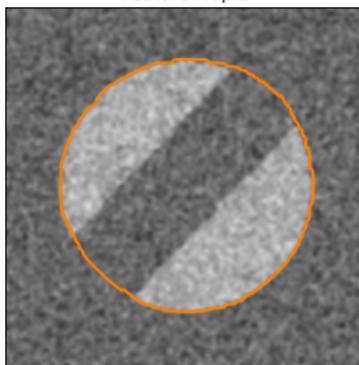
Synthetic Data: Partitioned Circles, Results

Feature Map 1



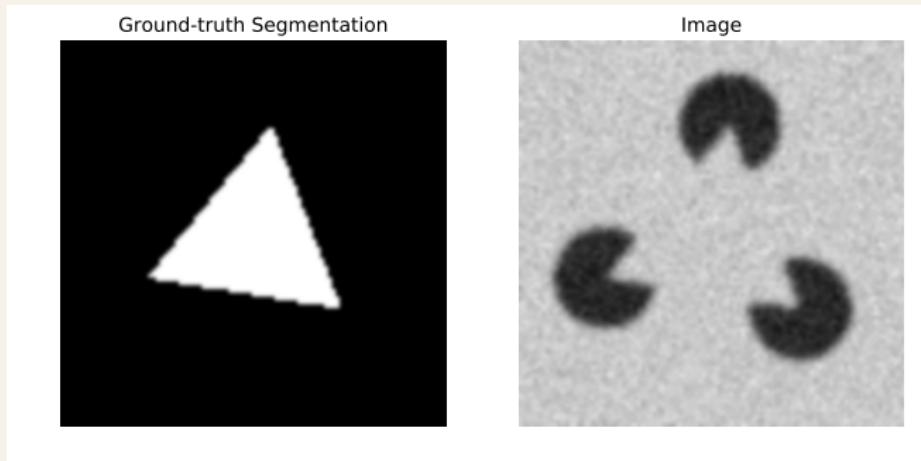
$n = 27$
Overlap = 0.6856

Feature Map 2



$n = 54$
Overlap = 0.9690

Synthetic Data: Illusory Triangles



- ▶ 100/50/50 randomly generated image and segmentation pairs for training/validation/testing datasets

Synthetic Data: Illusory Triangles, Feature Maps Tested

Table: Feature Map 1

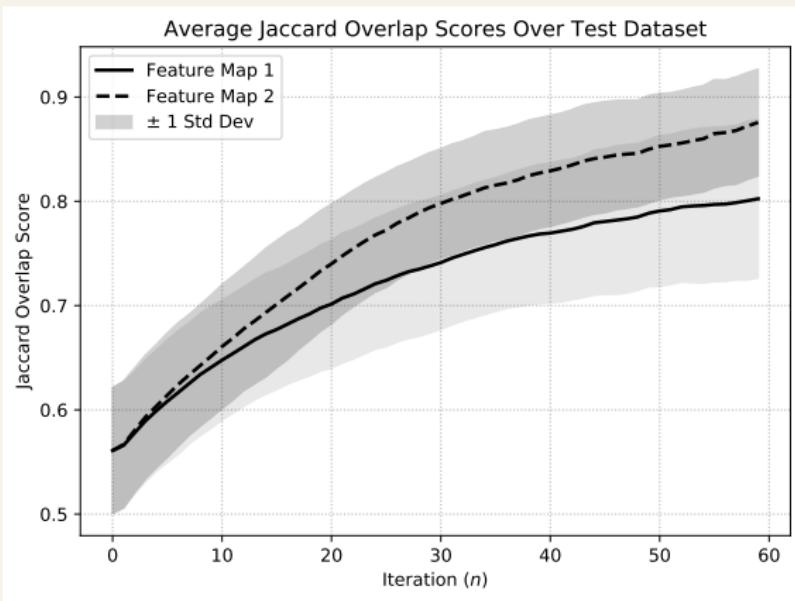
Feature	Local	Global
Area		✓
Boundary length		✓
Isoperimetric ratio		✓
Seg. moments, $p = 1, 2$		✓
Dist. from center of mass	✓	✓
Image value mean in seg.		✓
Image value std. in seg.		✓
Image value	✓	
Image edge	✓	

Table: Feature Map 2

Feature	Local	Global
Area		✓
Boundary length		✓
Isoperimetric ratio		✓
Seg. moments, $p = 1, 2$		✓
Dist. from center of mass	✓	✓
Image value mean in seg.		✓
Image value std. in seg.		✓
Image value		✓
Image edge		✓
Corner response		✓
Frenet frame		✓

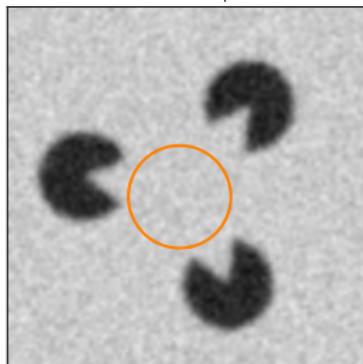
Image features are computed at four scales, $\sigma \in \{0, 1, 2, 3\}$

Synthetic Data: Illusory Triangles, Results

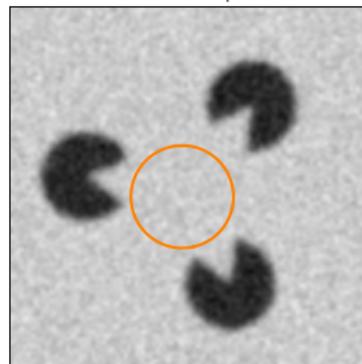


Synthetic Data: Illusory Triangles, Results

Feature Map 1

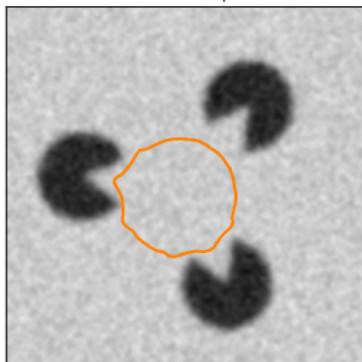


Feature Map 2

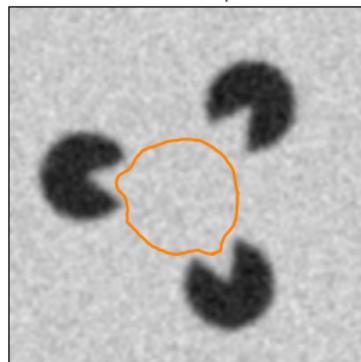


Synthetic Data: Illusory Triangles, Results

Feature Map 1

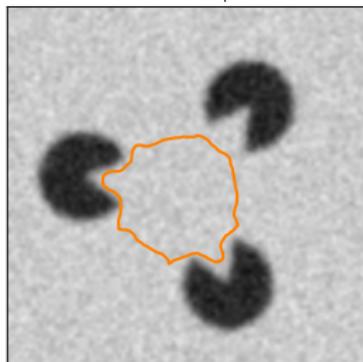


Feature Map 2



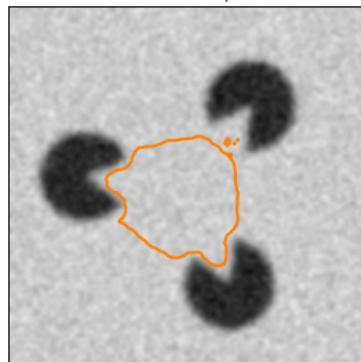
Synthetic Data: Illusory Triangles, Results

Feature Map 1



$n = 18$
Overlap = 0.6999

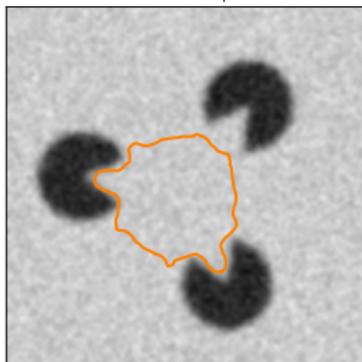
Feature Map 2



$n = 18$
Overlap = 0.7178

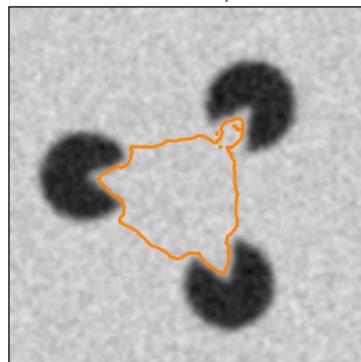
Synthetic Data: Illusory Triangles, Results

Feature Map 1



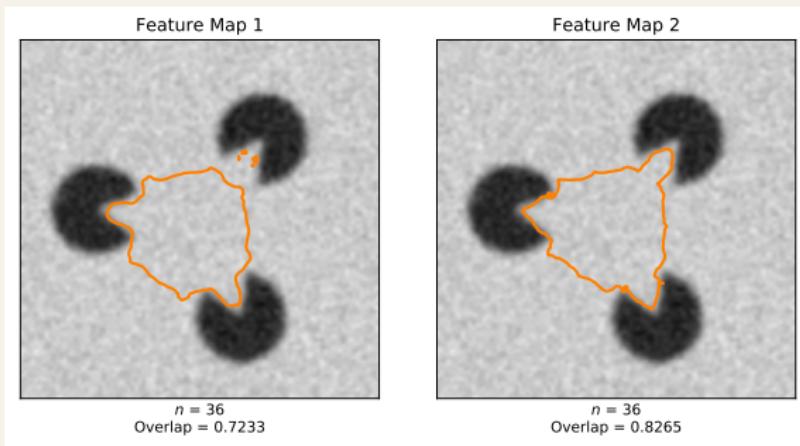
$n = 27$
Overlap = 0.7222

Feature Map 2



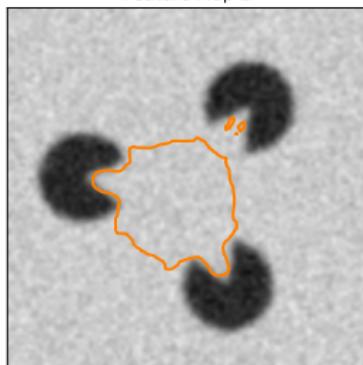
$n = 27$
Overlap = 0.7854

Synthetic Data: Illusory Triangles, Results



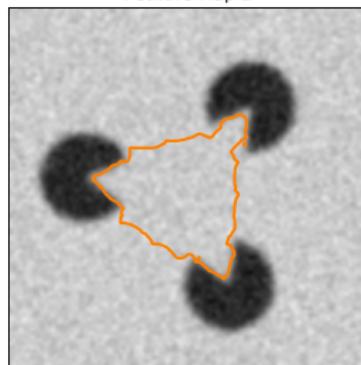
Synthetic Data: Illusory Triangles, Results

Feature Map 1



$n = 45$
Overlap = 0.7097

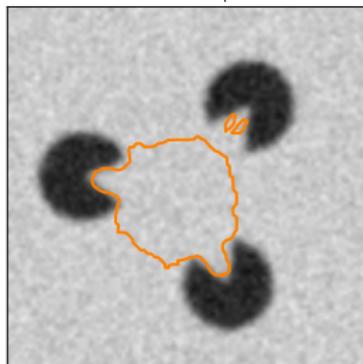
Feature Map 2



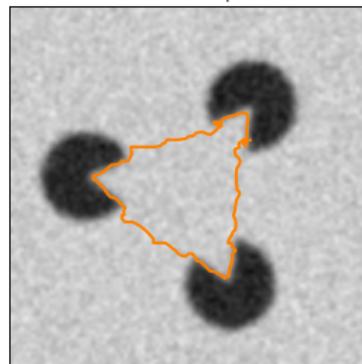
$n = 45$
Overlap = 0.8462

Synthetic Data: Illusory Triangles, Results

Feature Map 1



Feature Map 2



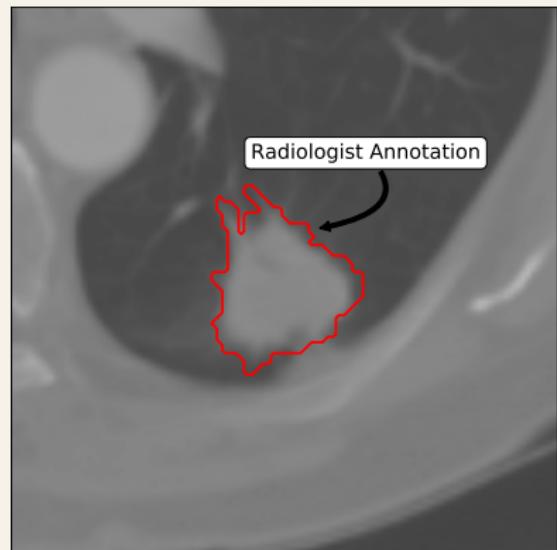
Lung Nodule Segmentation: LIDC Dataset

- ▶ 891 nodules where 4 annotators agree on existence of nodule at that location
- ▶ Sample 200/100/100 nodules for training/validation/testing
- ▶ Preprocess data:
 1. Transform annotation contours to boolean-valued images
 2. Interpolate images to uniform (1 mm) pixel spacing
- ▶ For each nodule 10 random seed points chosen



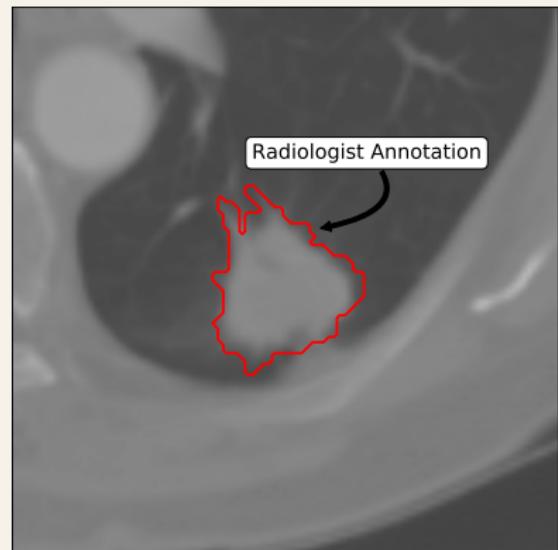
Lung Nodule Segmentation: LIDC Dataset

- ▶ 891 nodules where 4 annotators agree on existence of nodule at that location
- ▶ Sample **200/100/100** nodules for **training/validation/testing**
- ▶ Preprocess data:
 1. Transform annotation contours to boolean-valued images
 2. Interpolate images to uniform (1 mm) pixel spacing
- ▶ For each nodule 10 random seed points chosen



Lung Nodule Segmentation: LIDC Dataset

- ▶ 891 nodules where 4 annotators agree on existence of nodule at that location
- ▶ Sample **200/100/100** nodules for **training/validation/testing**
- ▶ Preprocess data:
 1. Transform annotation contours to boolean-valued images
 2. Interpolate images to uniform (1 mm) pixel spacing
- ▶ For each nodule 10 random seed points chosen

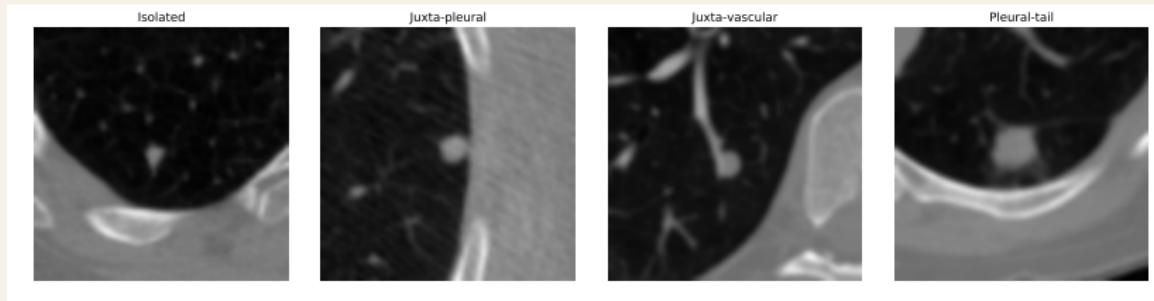


Lung Nodule Segmentation: LIDC Dataset

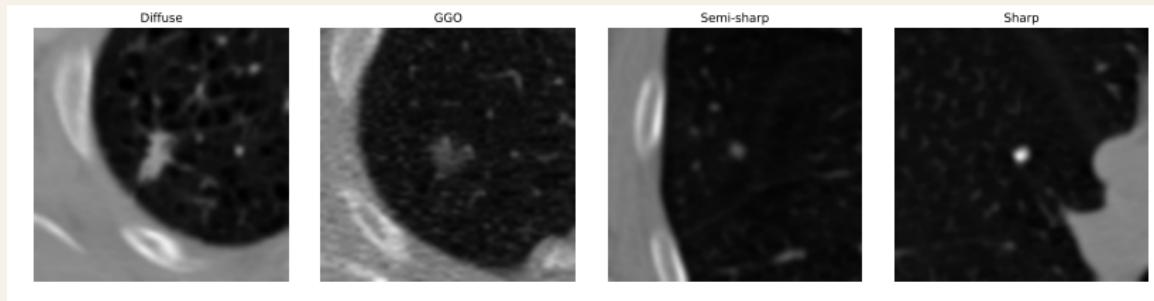
- ▶ 891 nodules where 4 annotators agree on existence of nodule at that location
- ▶ Sample **200/100/100** nodules for **training/validation/testing**
- ▶ Preprocess data:
 1. Transform annotation contours to boolean-valued images
 2. Interpolate images to uniform (1 mm) pixel spacing
- ▶ For each nodule 10 random seed points chosen



Lung Nodule Segmentation: LIDC Dataset

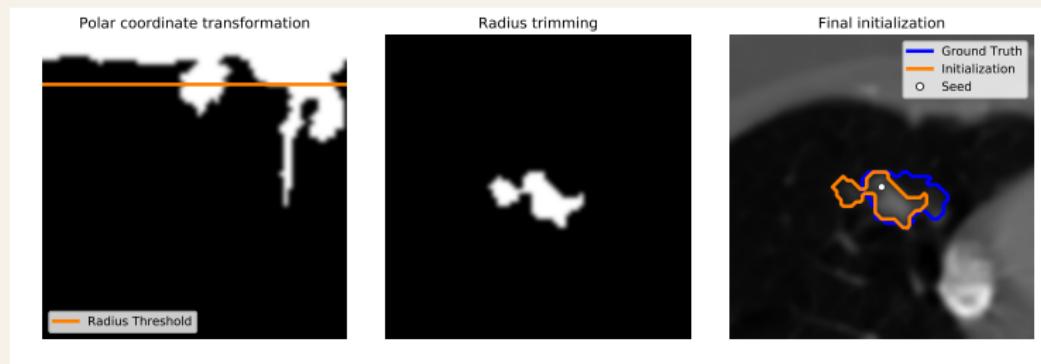


(a) Example nodules of various anatomical location types.



(b) Example nodules of various boundary density types.

Lung Nodule Segmentation: Initialization Routine



Lung Nodule Segmentation: Feature Maps Tested

Table: Feature Map 1

Feature	Local	Global
Area		✓
Boundary length		✓
Isoperimetric ratio		✓
Seg. moments, $p = 1, 2$		✓
Dist. from center of mass	✓	✓
Image value mean in seg.		✓
Image value std. in seg.		✓
Image value	✓	
Image edge	✓	

Table: Feature Map 2

Feature	Local	Global
Area		✓
Boundary length		✓
Isoperimetric ratio		✓
Seg. moments, $p = 1, 2$		✓
Dist. from center of mass	✓	✓
Image value mean in seg.		✓
Image value std. in seg.		✓
Image value		✓
Image edge		✓
Average Image Edge		✓
Frenet frame		✓

Image features are computed at four scales, $\sigma \in \{0, 1, 2, 3\}$

Lung Nodule Segmentation: Results

Table: Results for Feature Map 1.

N_h	n^*	J
8	14	0.7128 (\pm 0.1708)
16	14	0.7127 (\pm 0.1680)
32	24	0.7219 (\pm 0.1591)

Table: Results for Feature Map 2.

N_h	n^*	J
8	18	0.7241 (\pm 0.1594)
16	22	0.7359 (\pm 0.1532)
32	30	0.7468 (\pm 0.1428)

Lung Nodule Segmentation: Results

Table: Performance of various lung nodule segmentation methods on the LIDC dataset (table of results reproduced from reference [14])

Authors	Year	Number of Nodules		Jaccard overlap
		Training	Testing	
Tachibana and Kido [11]	2006	-	23	0.5070 (± 0.2190)
Wang et al. [13]	2009	23	64	0.5800
Messay et al. [9]	2010	-	68	0.6300 (± 0.1600)
Kubota et al. [5]	2011	-	23	0.6900 (± 0.1800)
Tan et al. [12]	2013	-	23	0.6500
Lassen et al. [6]	2015	-	19	0.5200 (± 0.0700)
Messay et al. [10]	2015	300	66	0.7170 (± 0.1989)
Wang et al. [14]	2017	350	493	0.7160 (± 0.1222)
SCLS Method (our work) **	-	200 ($\times 10$)	100 ($\times 10$)	0.7468 (± 0.1428)

** Our work is presently in 2D; results in table are 3D

Lung Nodule Segmentation: Results

Table: Results by type for Feature Map 2.

Density	Type	Diffuse	GGO	Semi-sharp	Sharp					
Location	Type									
Isolated	(5)	0.7124	(5)	0.7267	(26)	0.7798	(14)	0.8507	(50)	0.7876
Juxta-pleural	(1)	0.8178	(2)	0.5730	(17)	0.6430	(7)	0.7960	(27)	0.6840
Juxta-vascular	(0)	-	(0)	-	(9)	0.7462	(4)	0.6552	(13)	0.7182
Pleural-tail	(2)	0.5539	(0)	-	(6)	0.7829	(2)	0.8436	(10)	0.7493
	(8)	0.6859	(7)	0.6828	(58)	0.7348	(27)	0.8070	(100)	0.7468

Lung Nodule Segmentation: Training Time / Run Time

Training times (in hours):

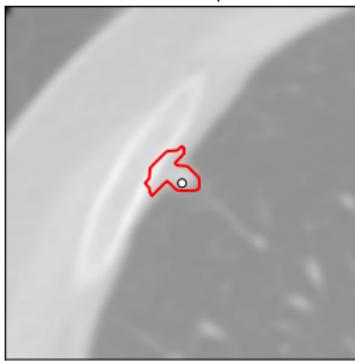
N_h	Feature Map 1	Feature Map 2
8	1.12	1.65
17	1.39	3.17
32	3.37	7.17
Total	5.88	12

**Average test time
(in milliseconds):**

Feature Map 1	Feature Map 2
270.20 (\pm 47.16)	545.09 (\pm 208.64)

Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 0$, Overlap = 0.3333

Feature Map 2



$n = 0$, Overlap = 0.3333

Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 4$, Overlap = 0.3387

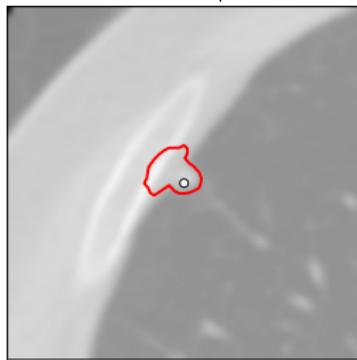
Feature Map 2



$n = 4$, Overlap = 0.3387

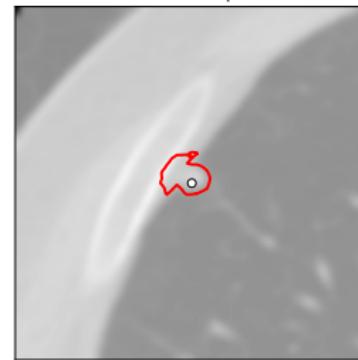
Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 9$, Overlap = 0.2785

Feature Map 2



$n = 9$, Overlap = 0.3729

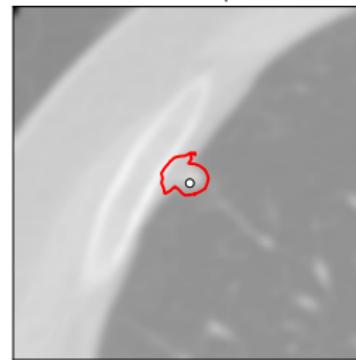
Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 13$, Overlap = 0.2619

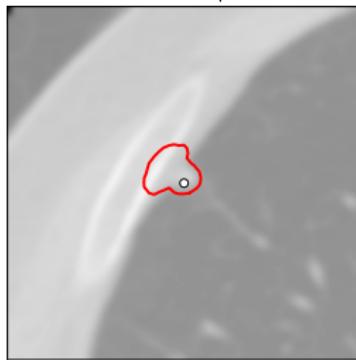
Feature Map 2



$n = 13$, Overlap = 0.3929

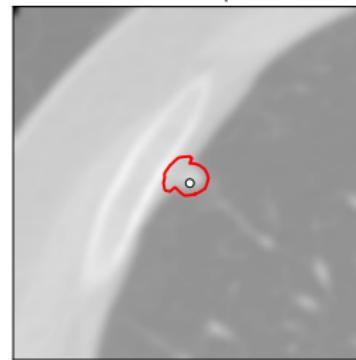
Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 17$, Overlap = 0.2558

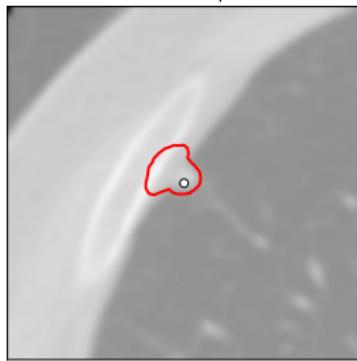
Feature Map 2



$n = 17$, Overlap = 0.4314

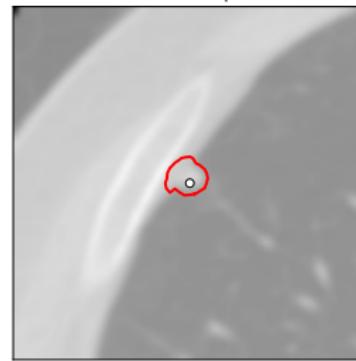
Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 21$, Overlap = 0.2619

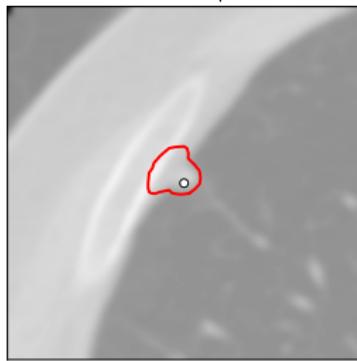
Feature Map 2



$n = 21$, Overlap = 0.4583

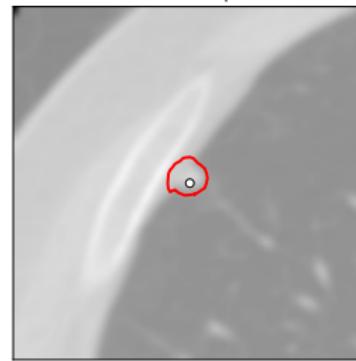
Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 24$, Overlap = 0.2750

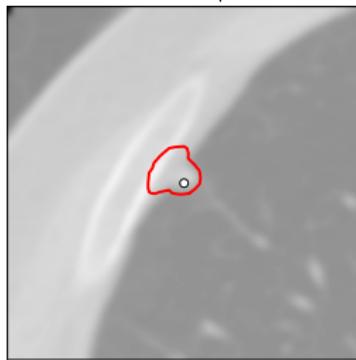
Feature Map 2



$n = 26$, Overlap = 0.4681

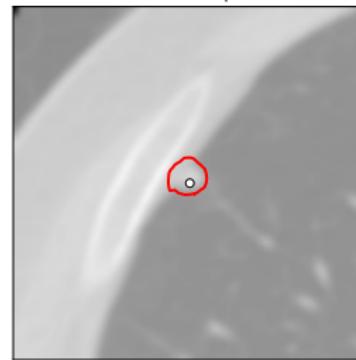
Lung Nodule Segmentation: Results, Example 1

Feature Map 1



$n = 24$, Overlap = 0.2750

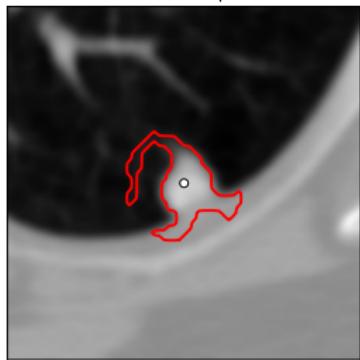
Feature Map 2



$n = 30$, Overlap = 0.4681

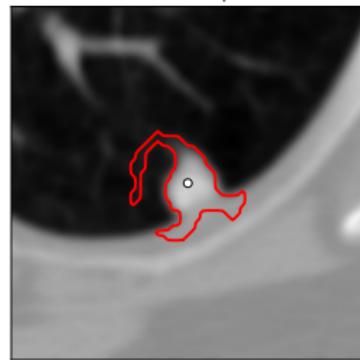
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 0$, Overlap = 0.5256

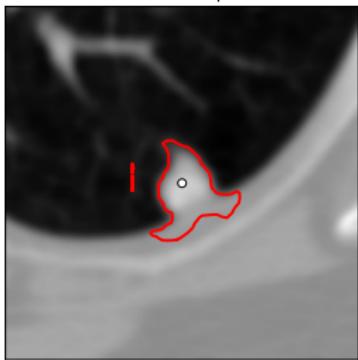
Feature Map 2



$n = 0$, Overlap = 0.5256

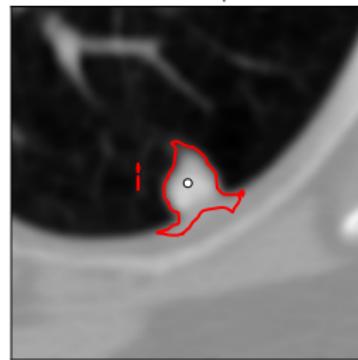
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 4$, Overlap = 0.5816

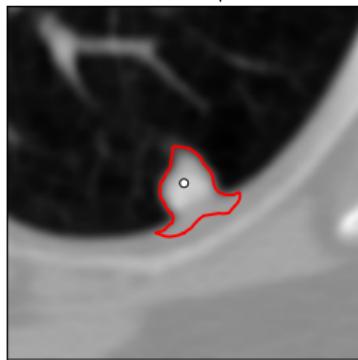
Feature Map 2



$n = 4$, Overlap = 0.6529

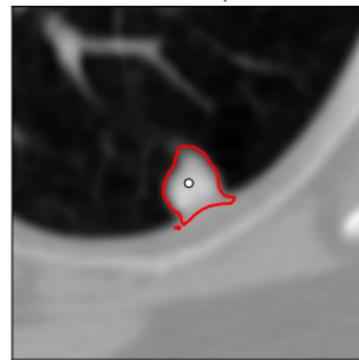
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 9$, Overlap = 0.6066

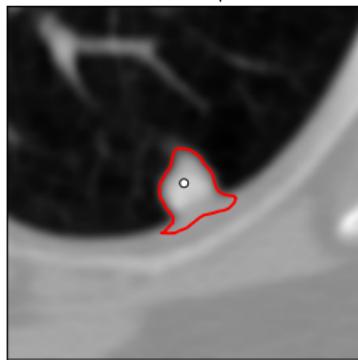
Feature Map 2



$n = 9$, Overlap = 0.7785

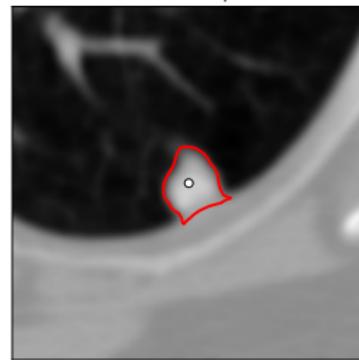
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 13$, Overlap = 0.6453

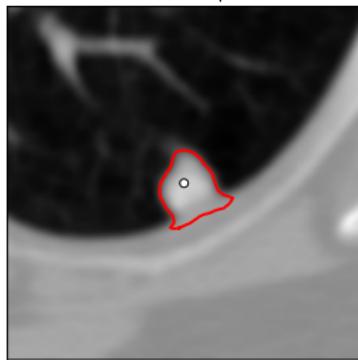
Feature Map 2



$n = 13$, Overlap = 0.8000

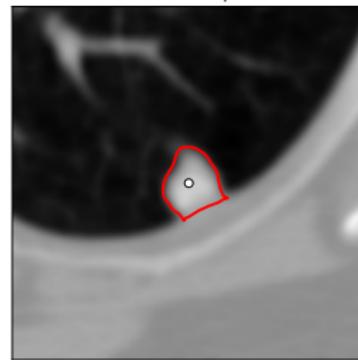
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 17$, Overlap = 0.6687

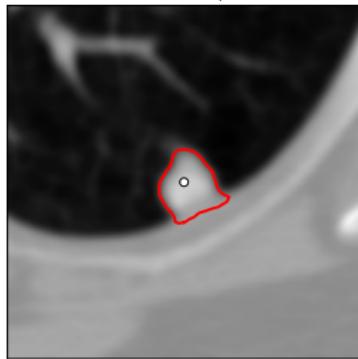
Feature Map 2



$n = 17$, Overlap = 0.8239

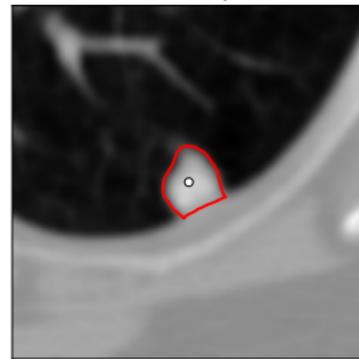
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 21$, Overlap = 0.7143

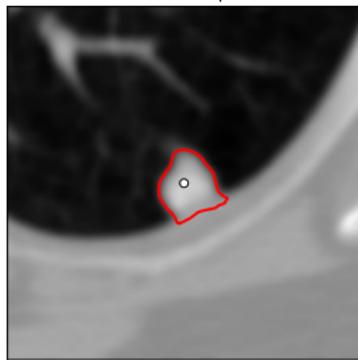
Feature Map 2



$n = 21$, Overlap = 0.8227

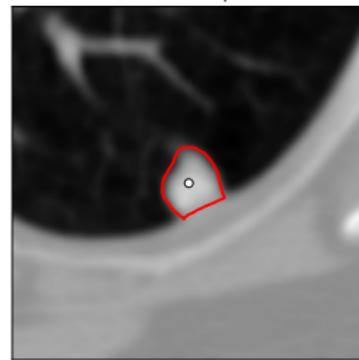
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 24$, Overlap = 0.7368

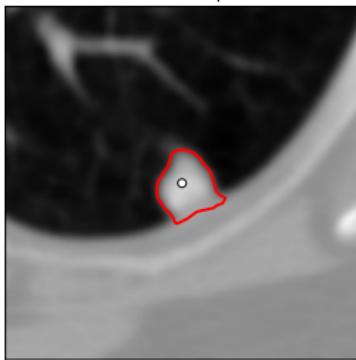
Feature Map 2



$n = 26$, Overlap = 0.8169

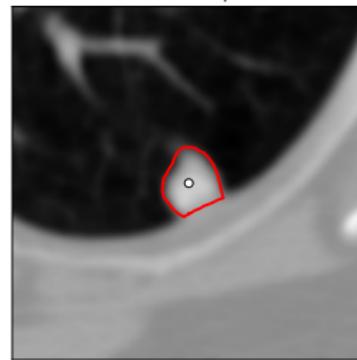
Lung Nodule Segmentation: Results, Example 2

Feature Map 1



$n = 24$, Overlap = 0.7368

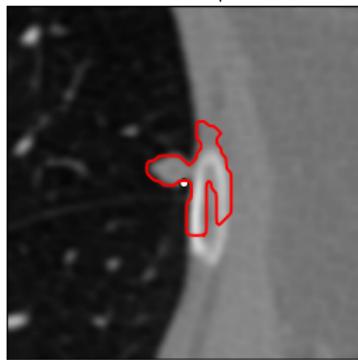
Feature Map 2



$n = 30$, Overlap = 0.8156

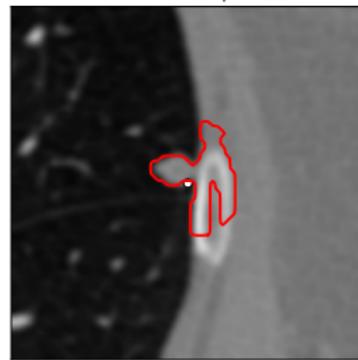
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 0$, Overlap = 0.2199

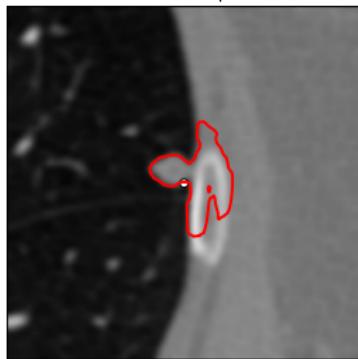
Feature Map 2



$n = 0$, Overlap = 0.2199

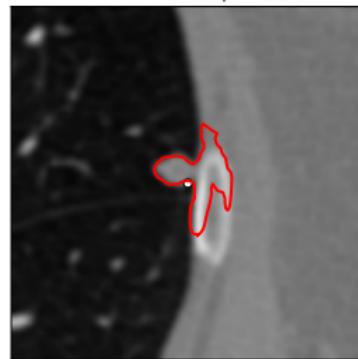
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 4$, Overlap = 0.2063

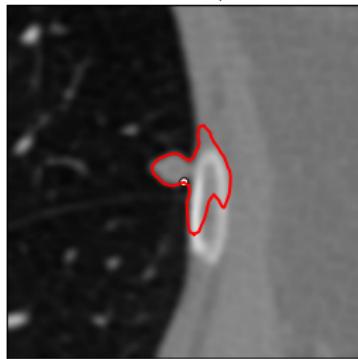
Feature Map 2



$n = 4$, Overlap = 0.2436

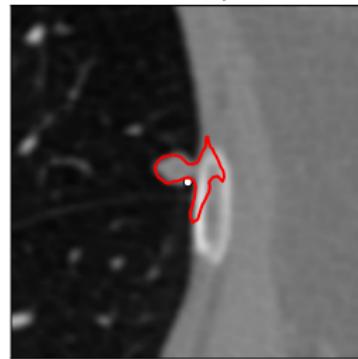
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 9$, Overlap = 0.2286

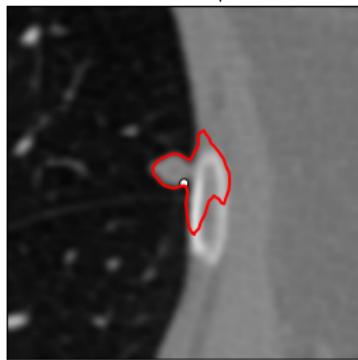
Feature Map 2



$n = 9$, Overlap = 0.3036

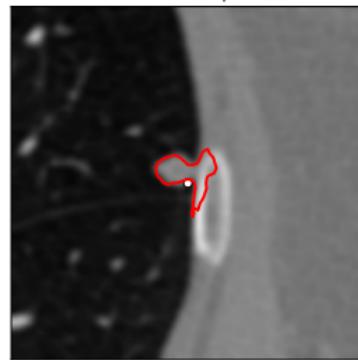
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 13$, Overlap = 0.2471

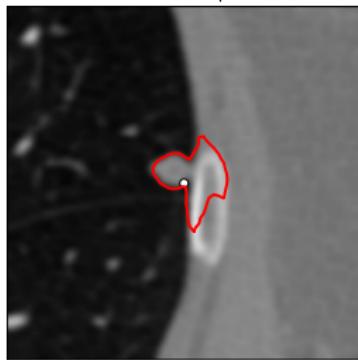
Feature Map 2



$n = 13$, Overlap = 0.3587

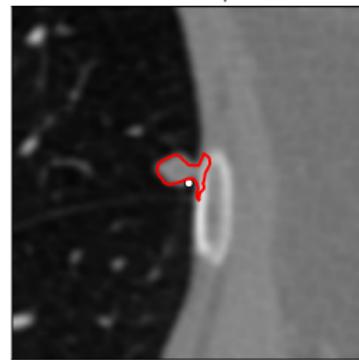
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 17$, Overlap = 0.2866

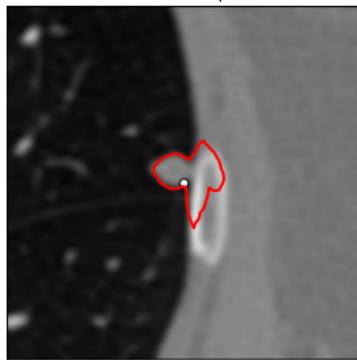
Feature Map 2



$n = 17$, Overlap = 0.4286

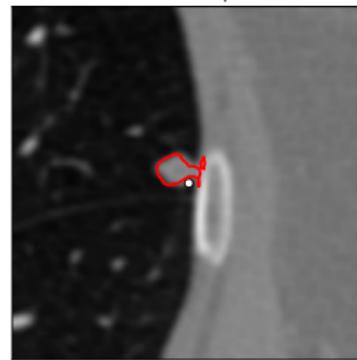
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 21$, Overlap = 0.3209

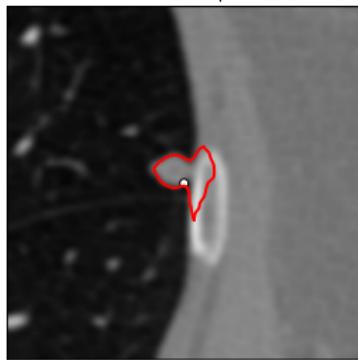
Feature Map 2



$n = 21$, Overlap = 0.4925

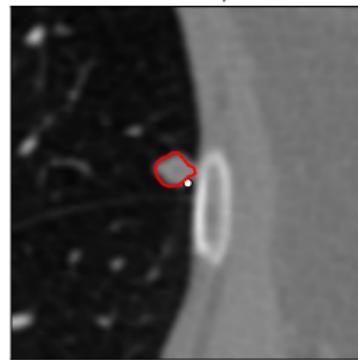
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 24$, Overlap = 0.3774

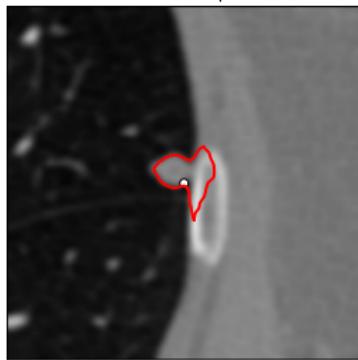
Feature Map 2



$n = 26$, Overlap = 0.6129

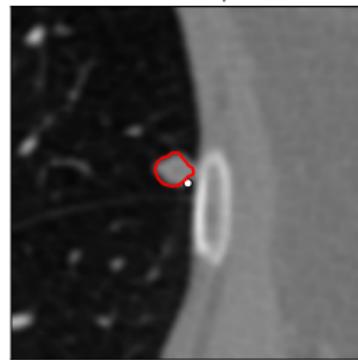
Lung Nodule Segmentation: Results, Example 3

Feature Map 1



$n = 24$, Overlap = 0.3774

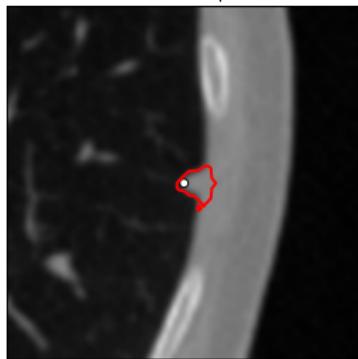
Feature Map 2



$n = 30$, Overlap = 0.5968

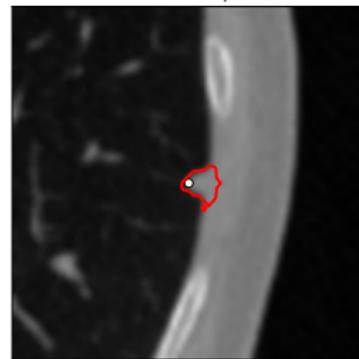
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 0$, Overlap = 0.5250

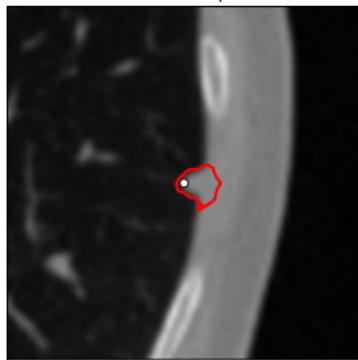
Feature Map 2



$n = 0$, Overlap = 0.5250

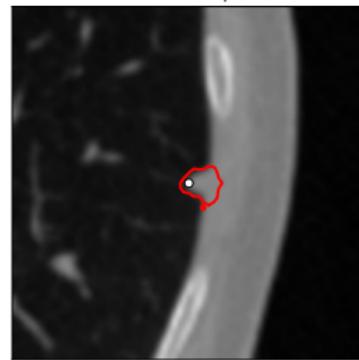
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 4$, Overlap = 0.4468

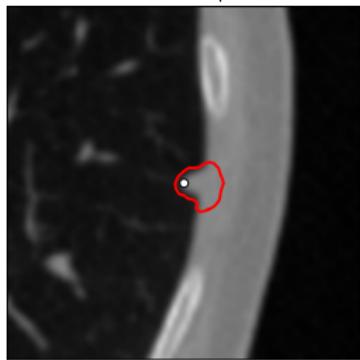
Feature Map 2



$n = 4$, Overlap = 0.5116

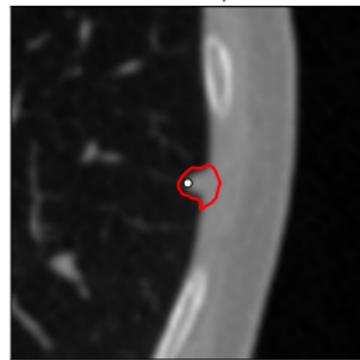
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 9$, Overlap = 0.3538

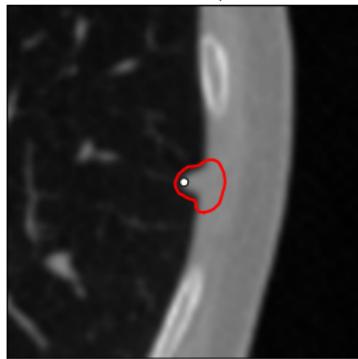
Feature Map 2



$n = 9$, Overlap = 0.4600

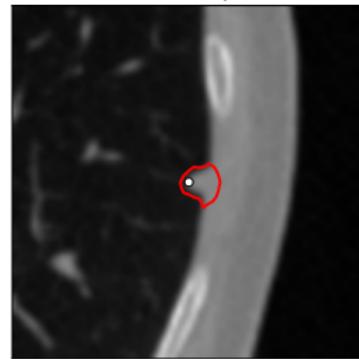
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 13$, Overlap = 0.3067

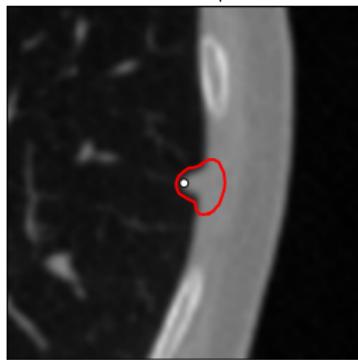
Feature Map 2



$n = 13$, Overlap = 0.4694

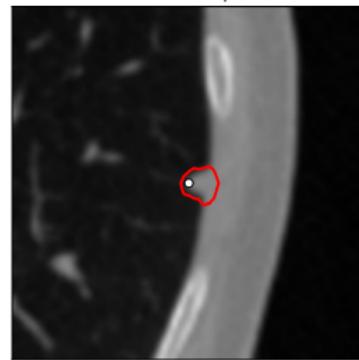
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 17$, Overlap = 0.2949

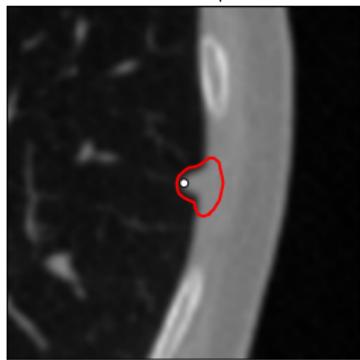
Feature Map 2



$n = 17$, Overlap = 0.5227

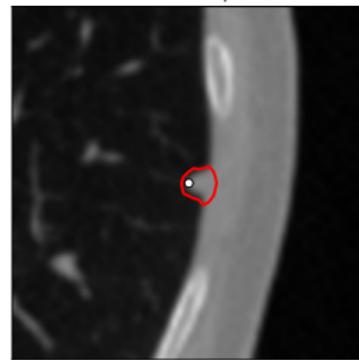
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 21$, Overlap = 0.3067

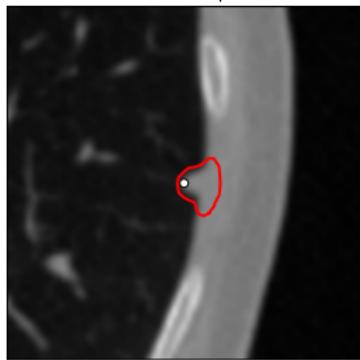
Feature Map 2



$n = 21$, Overlap = 0.5750

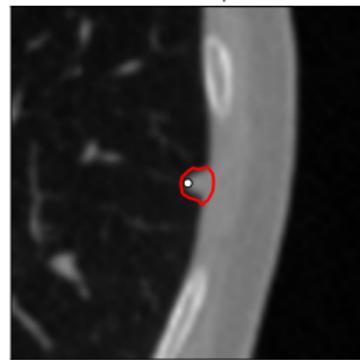
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 24$, Overlap = 0.3286

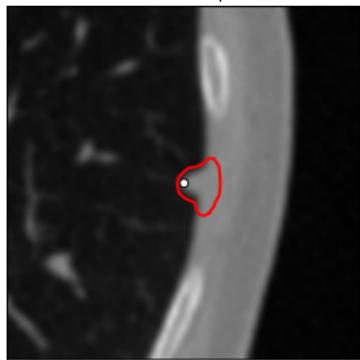
Feature Map 2



$n = 26$, Overlap = 0.5897

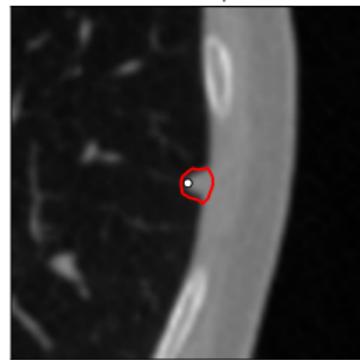
Lung Nodule Segmentation: Results, Example 4

Feature Map 1



$n = 24$, Overlap = 0.3286

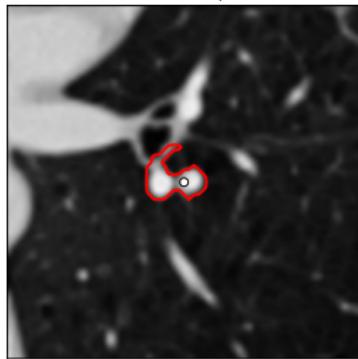
Feature Map 2



$n = 30$, Overlap = 0.6216

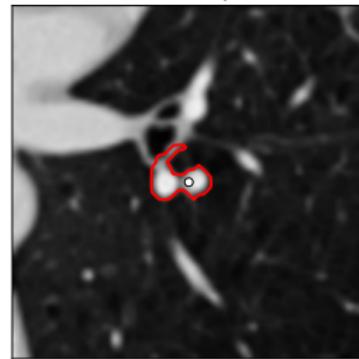
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 0$, Overlap = 0.3194

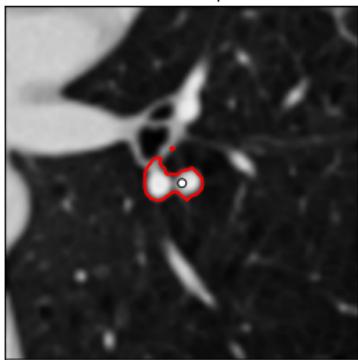
Feature Map 2



$n = 0$, Overlap = 0.3194

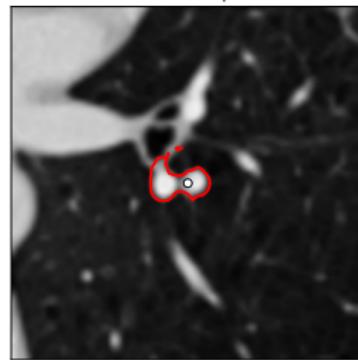
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 4$, Overlap = 0.3485

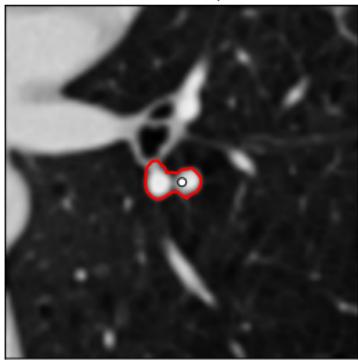
Feature Map 2



$n = 4$, Overlap = 0.3433

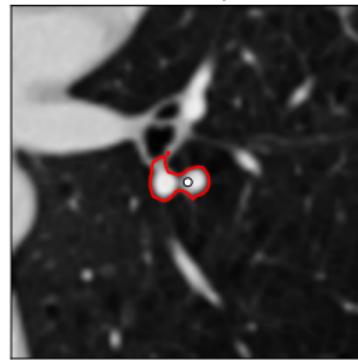
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 9$, Overlap = 0.3729

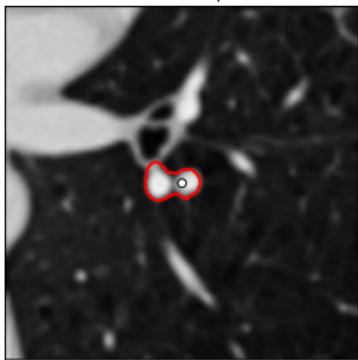
Feature Map 2



$n = 9$, Overlap = 0.3485

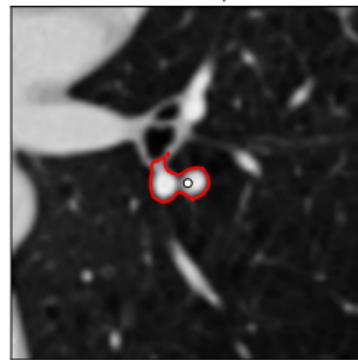
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 13$, Overlap = 0.3607

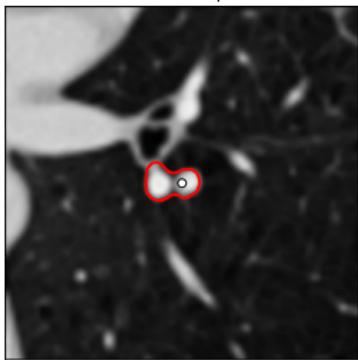
Feature Map 2



$n = 13$, Overlap = 0.3286

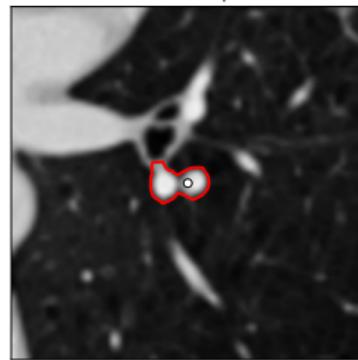
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 17$, Overlap = 0.3492

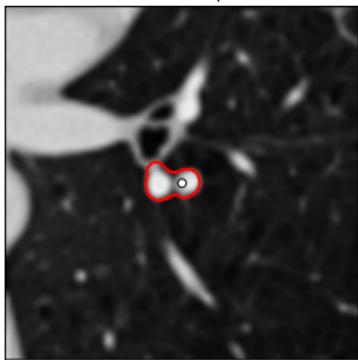
Feature Map 2



$n = 17$, Overlap = 0.3239

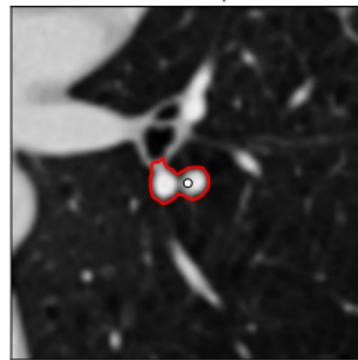
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 21$, Overlap = 0.3231

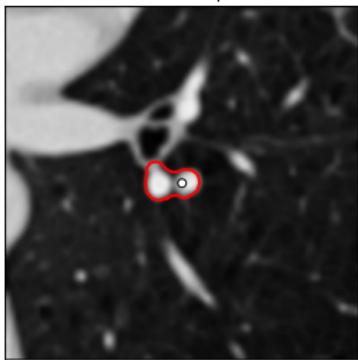
Feature Map 2



$n = 21$, Overlap = 0.3026

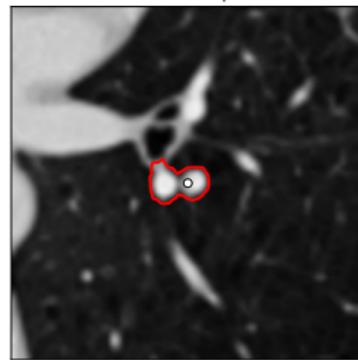
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 24$, Overlap = 0.3281

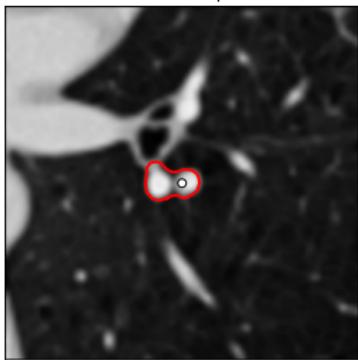
Feature Map 2



$n = 26$, Overlap = 0.2911

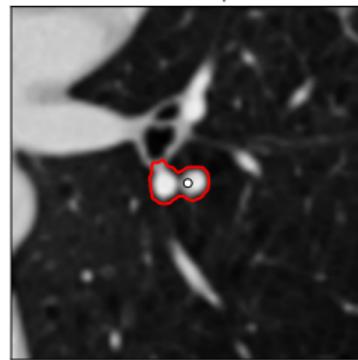
Lung Nodule Segmentation: Results, Example 5

Feature Map 1



$n = 24$, Overlap = 0.3281

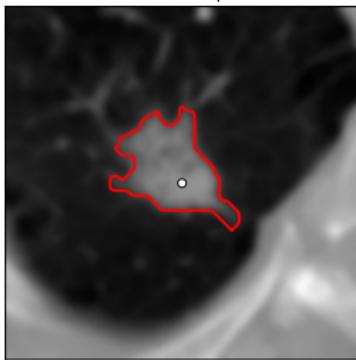
Feature Map 2



$n = 30$, Overlap = 0.2840

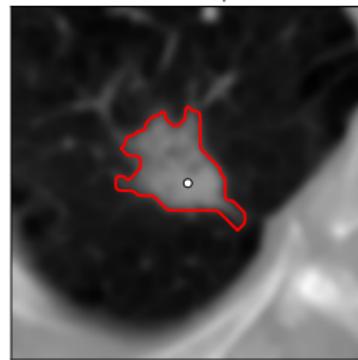
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 0$, Overlap = 0.6872

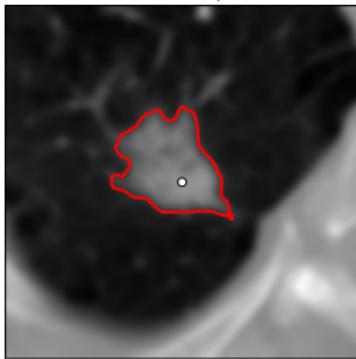
Feature Map 2



$n = 0$, Overlap = 0.6872

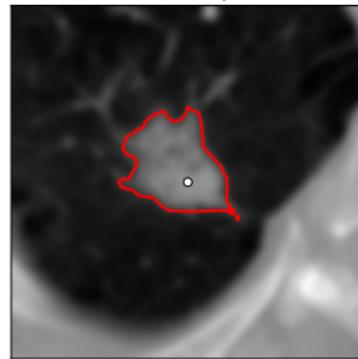
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 4$, Overlap = 0.7335

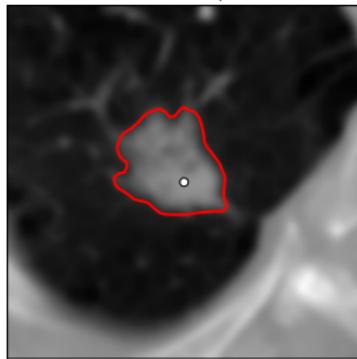
Feature Map 2



$n = 4$, Overlap = 0.7184

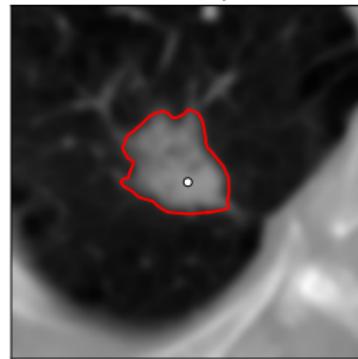
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 9$, Overlap = 0.7227

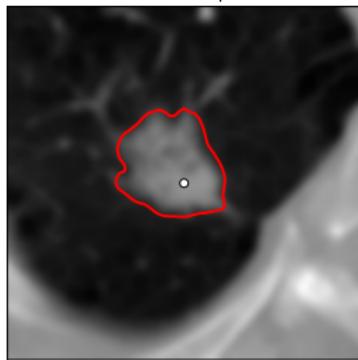
Feature Map 2



$n = 9$, Overlap = 0.6968

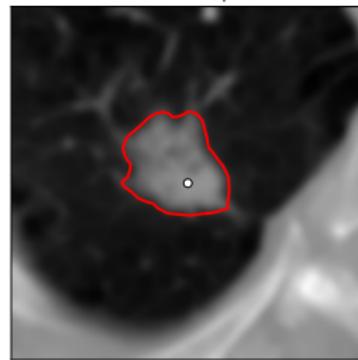
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 13$, Overlap = 0.7191

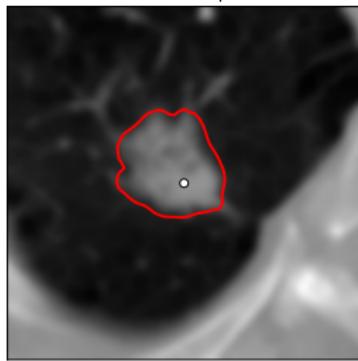
Feature Map 2



$n = 13$, Overlap = 0.7071

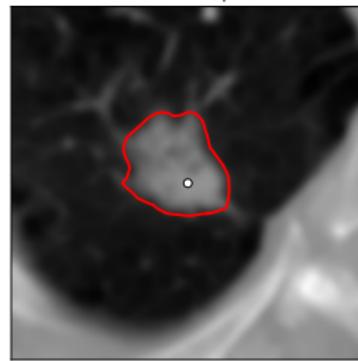
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 17$, Overlap = 0.7111

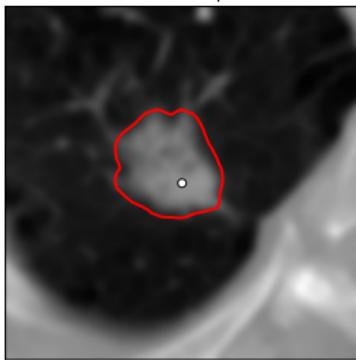
Feature Map 2



$n = 17$, Overlap = 0.7000

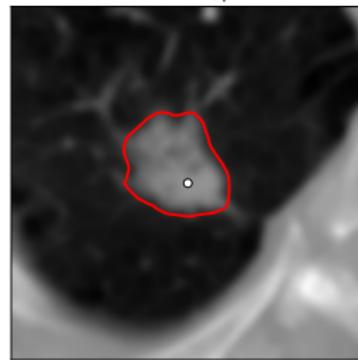
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 21$, Overlap = 0.7042

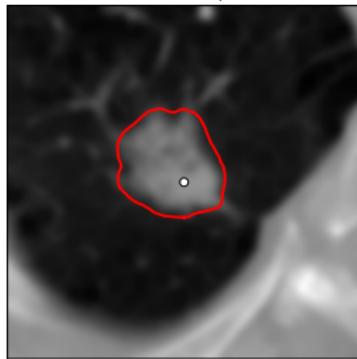
Feature Map 2



$n = 21$, Overlap = 0.6953

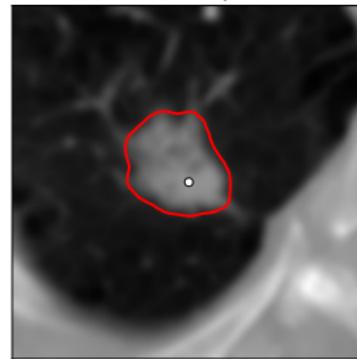
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 24$, Overlap = 0.7024

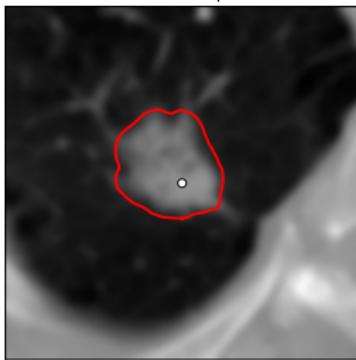
Feature Map 2



$n = 26$, Overlap = 0.6982

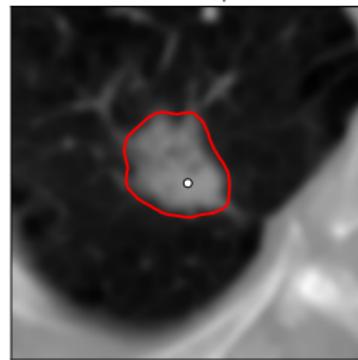
Lung Nodule Segmentation: Results, Example 6

Feature Map 1



$n = 24$, Overlap = 0.7024

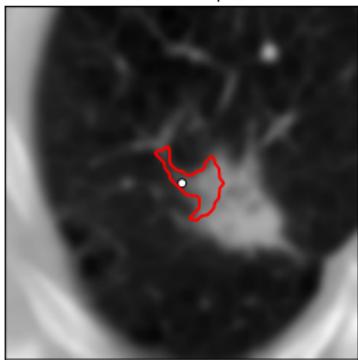
Feature Map 2



$n = 30$, Overlap = 0.6920

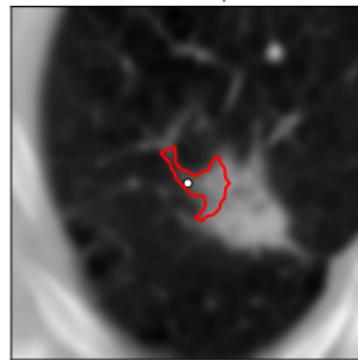
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 0$, Overlap = 0.1749

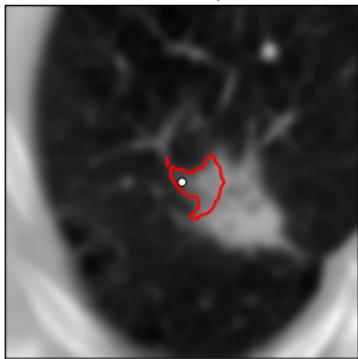
Feature Map 2



$n = 0$, Overlap = 0.1749

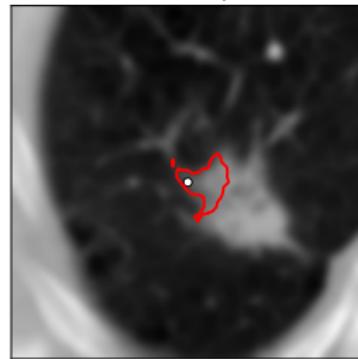
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 4$, Overlap = 0.1605

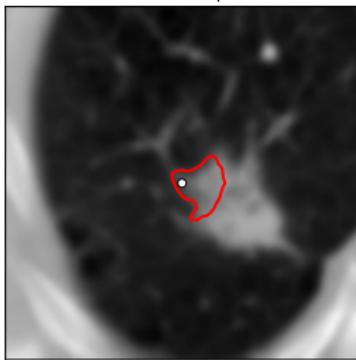
Feature Map 2



$n = 4$, Overlap = 0.1481

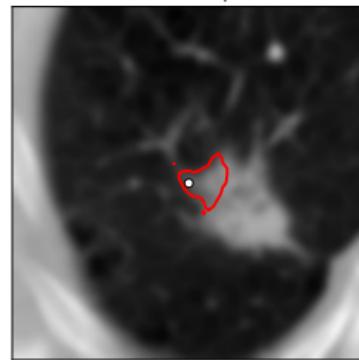
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 9$, Overlap = 0.1916

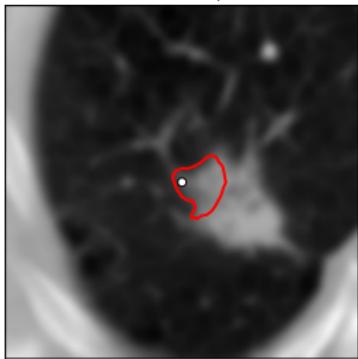
Feature Map 2



$n = 9$, Overlap = 0.1379

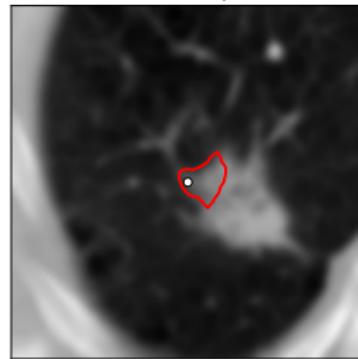
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 13$, Overlap = 0.1912

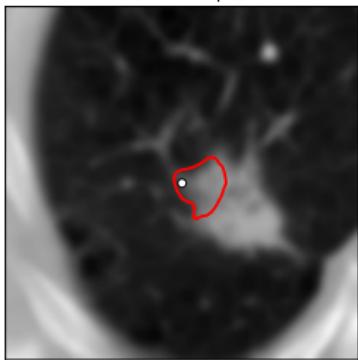
Feature Map 2



$n = 13$, Overlap = 0.1425

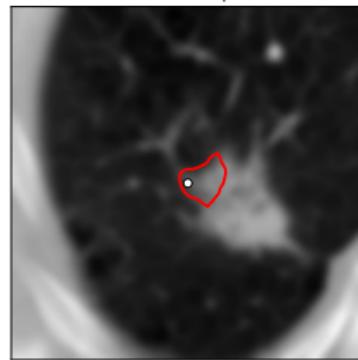
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 17$, Overlap = 0.1883

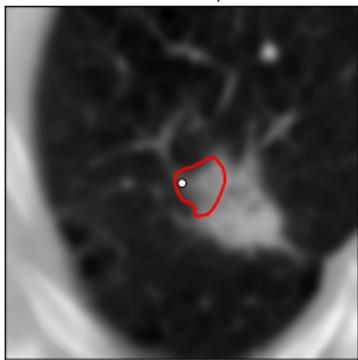
Feature Map 2



$n = 17$, Overlap = 0.1400

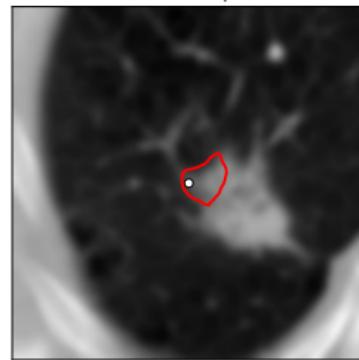
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 21$, Overlap = 0.1858

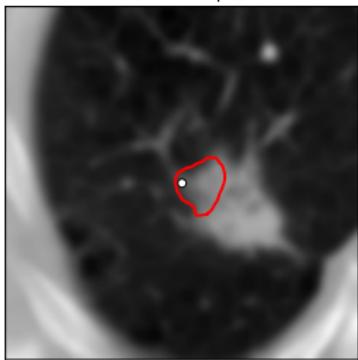
Feature Map 2



$n = 21$, Overlap = 0.1397

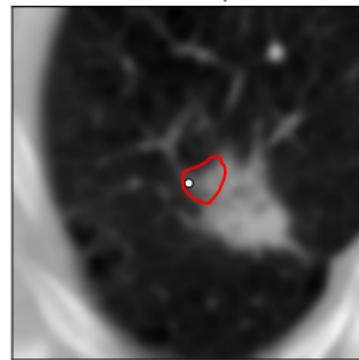
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 24$, Overlap = 0.1858

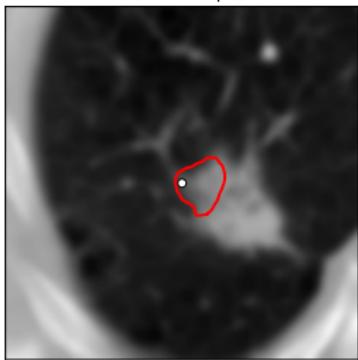
Feature Map 2



$n = 26$, Overlap = 0.1324

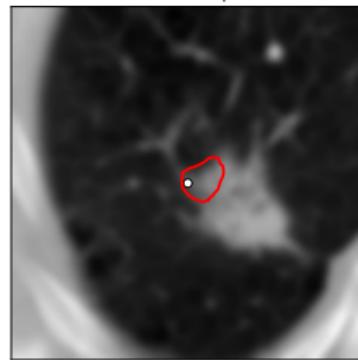
Lung Nodule Segmentation: Results, Example 7

Feature Map 1



$n = 24$, Overlap = 0.1858

Feature Map 2



$n = 30$, Overlap = 0.1247

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