ANI
CANCEAL CRISTIAN GR. 1. 1.
CTI. ENG.

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The spring elastic constant evaluation

1. Purpose

The objective of the experiment is to determine the spring constant of a spiral spring using Hooke's law and the period of oscillatory motion in response to a weight.
Apparatus: A spiral spring, a set of known, rules weights, a weight hanger, a stop watch, and a lab scale.

2. Theory

A. Static method

We use a spiral spring with elastic constant k and undeformed length l_o and bodies with different mass.

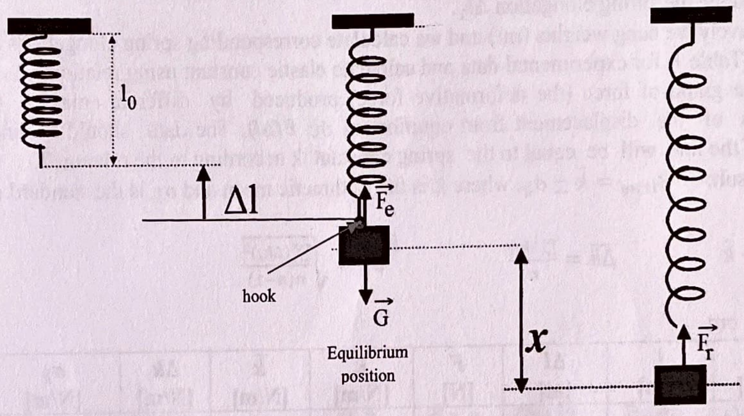

Fig. 1.

When a spring is stretched, according to Hooke's law, a restoring force F proportional to its elongation, x (or $\Delta l = l - l_o$) appears. Every spring obeys the Hooke's law if the deformation is not too great.

$$\vec{F_e} = -k\vec{\Delta l} \quad (1)$$

For the equilibrium

$$\Rightarrow mg = k\Delta l \quad (2)$$

$$k = \frac{mg}{\Delta l} \quad (3)$$

B. Dynamic method

When we move the body connected to a spring from its equilibrium position it starts to oscillate around the equilibrium position under the action of restoring (elastic) force. With x the distance from equilibrium position the Newton's second law is:

$$ma = -kx \quad (4)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \quad (5)$$

We note $\omega = \sqrt{\frac{k}{m}}$ and we call it the natural angular frequency. The second Newton law for the spring (5) is a differential equation having the solution

$$x(t) = A\sin(\omega t + \varphi) \tag{7}$$

The period for the harmonic oscillation is connected to the natural angular frequency throught:

$$\omega = \frac{2\pi}{T} \tag{8}$$

$$k = m\omega^2 = 4\pi^2\frac{m}{T^2} \tag{9}$$

3. What to do

Static method

1) We measure the undeformed spring length l_o.
2) We hang the weight hanger with mass m_l on the spring and we measure the deformed spring length l_l. We calculate the spring elongation Δl_1.
3) Successively we hang weights (m_i) and we calculate corresponding spring elongations Δl_i.
4) Use the Table 1. for experimental data and calculate elastic constant using relation (3).
5) Plot the graph of force (the deformative force) produced by different masses ($F=m\cdot g$) as a function of the displacement from equilibrium Δl: $\mathbf{F(\Delta l)}$. The data should be linear. Hence, the slope of the line will be equal to the spring constant k according to the relation 2.
6) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$\Delta k_i = k_i - \bar{k}$ $\overline{\Delta k} = \frac{\sum|\Delta k_i|}{n}$ sigma $\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n(\Delta k_i)^2}{n(n-1)}}$

Table 1.

$l_0 = 44$ cm

Nr. crt	m [kg]	l [cm]	Δl [m]	F [N]	k [N/m]	$\bar{k}$ [N/m]	Δk [N/m]	$\sigma_{\bar{k}}$ [N/m]	k_{true} [N/m]
1	0.06	55,5	0.115	0.588	5.11	5.015	0.095	0.027	5.015± 0,027
2	0.075	59	0.15	0.735	4.9		-0.115		
3	0.095	63	0.19	0.931	4.9		-0.115		
4	0.110	65	0.21	1.078	5.13		0.115		
5	0.136	71	0.24	1.332	4.93		-0,085		
6	0.145	72,5	0.285	1.421	4.98		-0.035		
7	0.172	77	0.33	1.685	5.1		0.085		
8	0.205	84	0.40	2.009	5.0225		0.0075		
9	0.22	86,5	0.425	2.156	5.07		0.055		
10	0.23	89	0.45	2.254	5.008		-0.007		

Dynamic method

1) Hang the weight hanger with several weights (mass m_l) on the spring and set the equilibrium position of the system.
2) Pull the system out of its equilibrium position to make oscillations with 1-2cm amplitude.
3) Record the time for n=20 oscilations and find the period: $T = t/n$.
4) Repeat 1), 2) and 3) for different masses.
5) Complete the Table 2 using relation (9) for elastic constant.
6) Plot the graph of $T^2(s^2)$ as function of m(kg). The data should be linear. Find elastic constant from the slope (i.e. $T^2 = \frac{4\pi^2}{k}m \Leftrightarrow y = \text{slope}\cdot x$).
7) Final result: $k_{true} = \bar{k} \pm \sigma_{\bar{k}}$, where $\bar{k}$ is the arithmetic mean and $\sigma_{\bar{k}}$ is the standard deviation of the mean.

$\Delta k_i = k_i - \bar{k}$ $\overline{\Delta k} = \frac{\sum|\Delta k_i|}{n}$ $\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n(\Delta k_i)^2}{n(n-1)}}$

Table 2.

	Nr. crt.	m [kg]	t [s]	n	T [s]	T^2 (s^2)	k [N/m]	$\bar{k}$ [N/m]	Δk [N/m]	$\sigma_{\bar{k}}$ [N/m]	k_{true} [N/m]
70g	1	0.07	11.258	15	0.750	0.562	4.917		-0.067		
100g	2	0.1	13.401	15	0.893	0.797	4.953		-0.031		
	3	0.13	15.215	15	1.014	1.028	4.992		0.008		4.984 ±
	4	0.16	16.871	15	1.124	1.263	5.001	4.984	0.017	0.011	0.011
	5	0.19	18.391	15	1.226	1.503	4.990		0.006		
	6	0.22	19.75	15	1.317	1.734	5.008		0.024		
	7	0.25	21.085	15	1.405	1.974	4.999		0.015		
	8	0.28	22.281	15	1.485	2.205	5.013		0.029		

Compare the results obtained by the 2 methods, respectively by arithmetic and graphic mediation !!

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Static method:

$$F = m \cdot g$$

$$F_4 = 0.11 \cdot 9.8 = 1.078 \text{ N}$$

$$k_4 = \frac{F_4}{\Delta l_4} = \frac{1.078}{0.21} = 5.13 \text{ N/m}$$

$$\bar{k} = \frac{5.11 + 4.9 + \ldots + 5.07 + 5.008}{10} = 5.015 \text{ N/m}$$

$$\Delta k_4 = k_4 - \bar{k} = 5.13 - 5.015 = 0.115 \text{ N/m}$$

$$\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}} = \sqrt{\frac{0.06753025}{10 \cdot 9}} = \sqrt{0.0007503} = 0.027 \text{ N/m}$$

Dynamic method:

$$T_4^2 = (1.124)^2 = 1.263 \text{ s}^2$$

$$k = 4\pi^2 \cdot \frac{m}{T^2}$$

$$k_4 = 39.478 \cdot \frac{0.16}{1.263} = 5.001 \text{ N/m}$$

$$\bar{k} = \frac{4.917 + 4.953 + \ldots + 4.999 + 5.013}{8} = 4.984 \text{ N/m}$$

$$\Delta k_4 = k_4 - \bar{k} = 5.001 - 4.984 = 0.017 \text{ N/m}$$

$$\sigma_{\bar{k}} = \sqrt{\frac{\sum_1^n (\Delta k_i)^2}{n(n-1)}} = \sqrt{\frac{0.007481}{8 \cdot 7}} = \sqrt{0.0001335} = 0.011 \text{ N/m}$$

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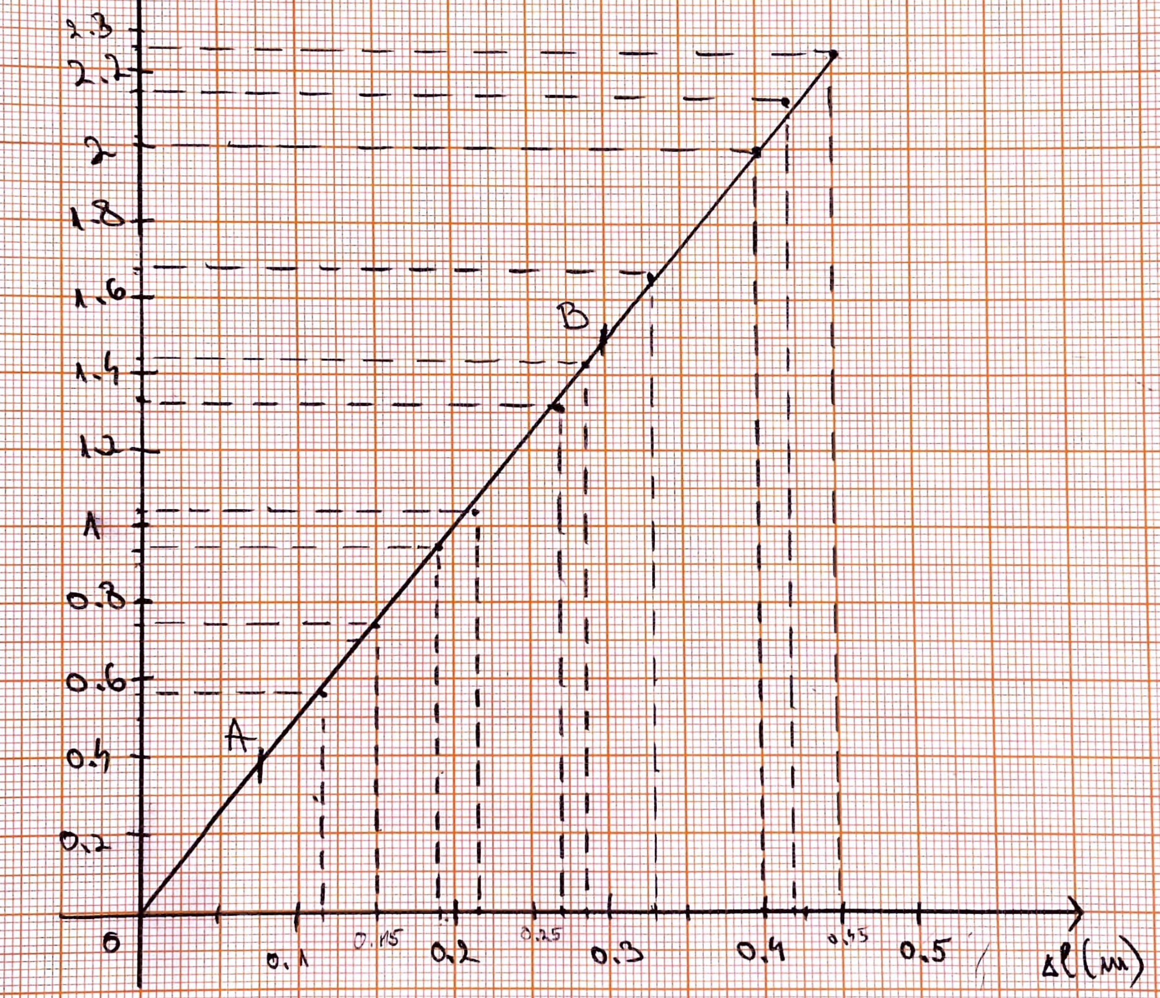
CTI-EN, year I, group 1.1

STATIC METHOD

A (0.05 m, 0.4 N)

B (0.3 m, 1.5 N)

$$slope = \frac{F_B - F_A}{\Delta l_B - \Delta l_A} = \frac{1.5 - 0.4}{0.3 - 0.05} = \frac{1.1}{0.25} = 4.4$$



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DYNAMIC METHOD