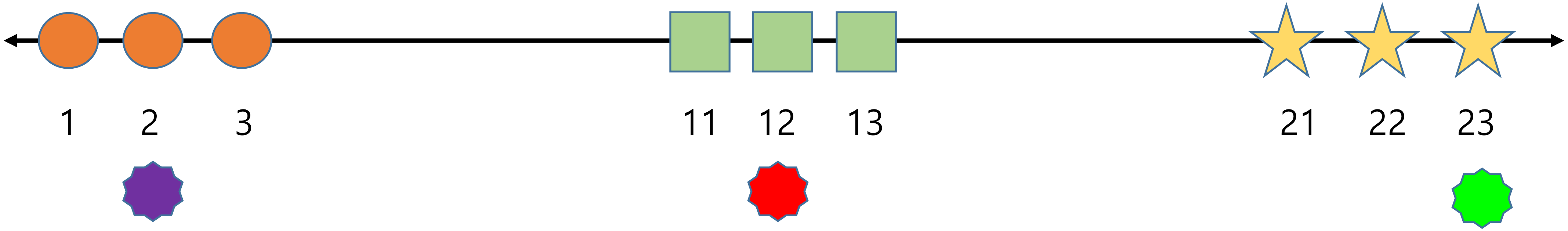

2.2 The k-means++ algorithm

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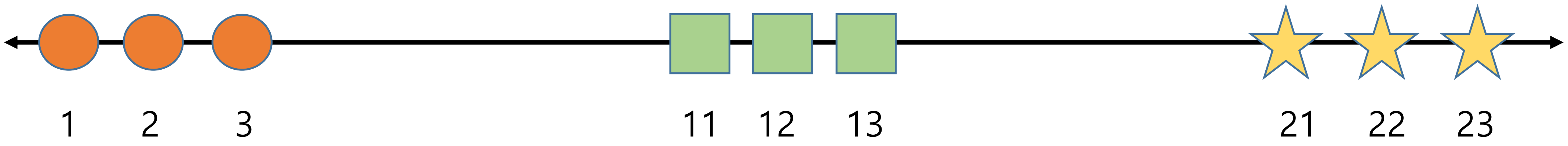
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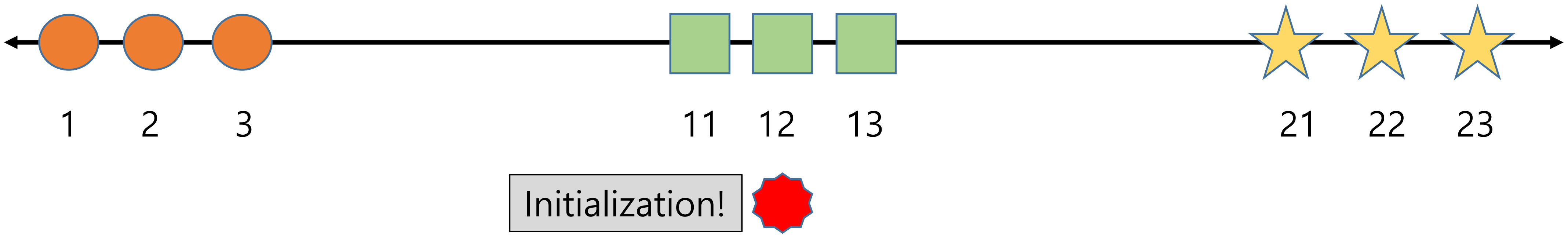
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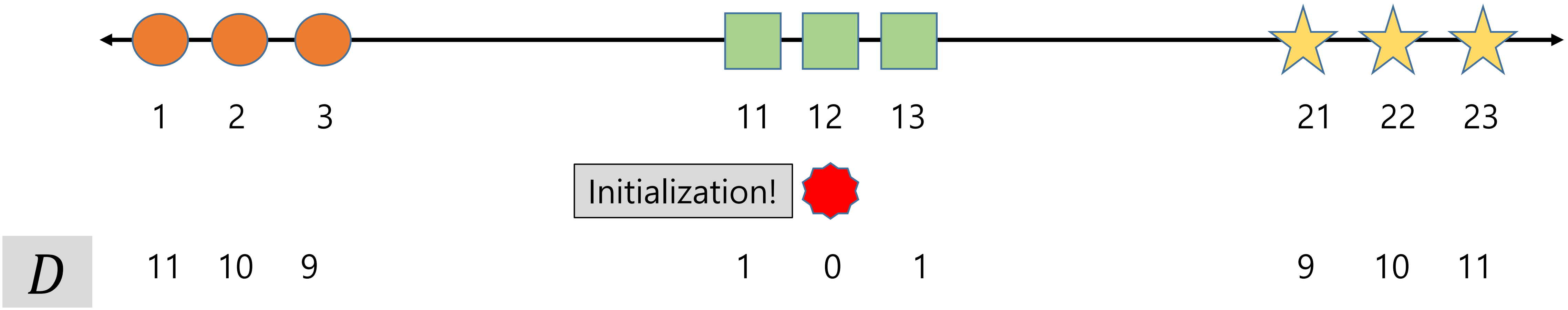
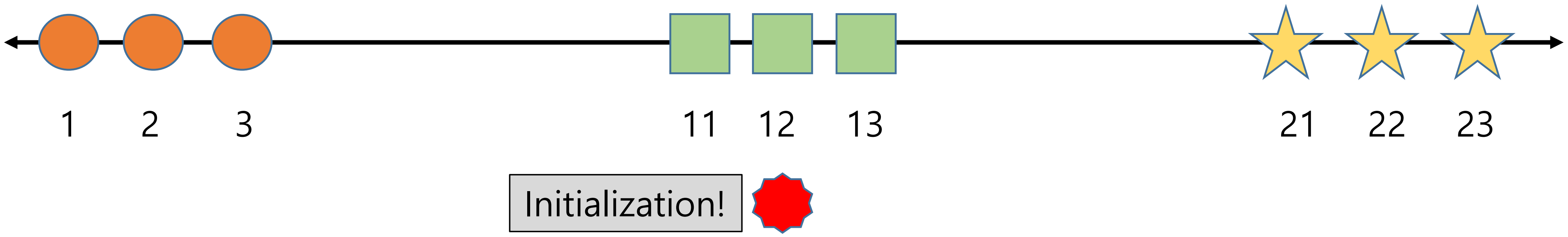
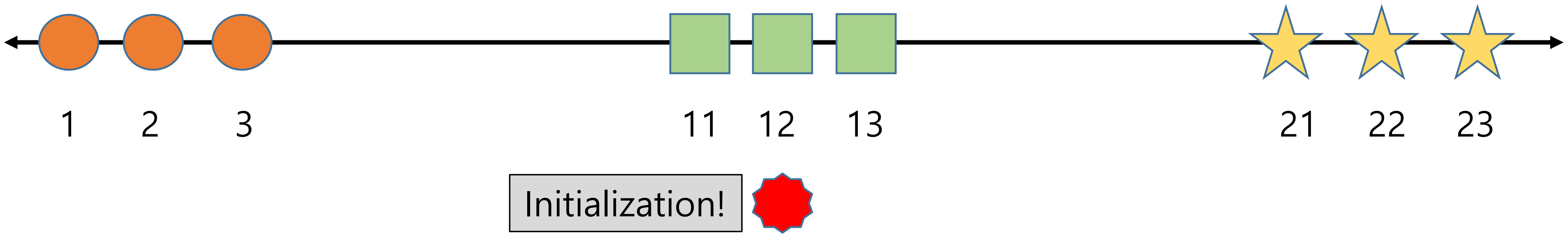
Initialization!


1 2 3 11 12 13 21 22 23
Initialization!
D
11 10 9 1 0 1 9 10 11



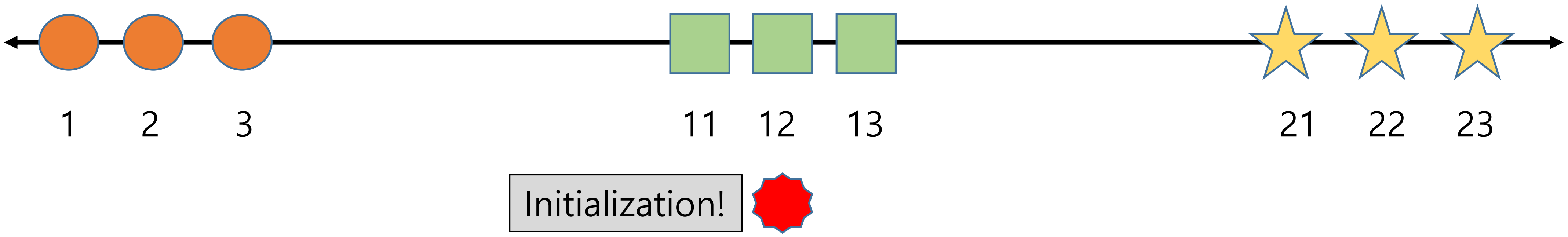
D	11	10	9	1	0	1	9	10	11
D^2	121	100	81	1	0	1	81	100	121



D	11	10	9	1	0	1	9	10	11
D^2	121	100	81	1	0	1	81	100	121

$$sum(D^2) = 606$$

$$sum\left(\frac{D^2}{sum(D^2)}\right) = 1$$



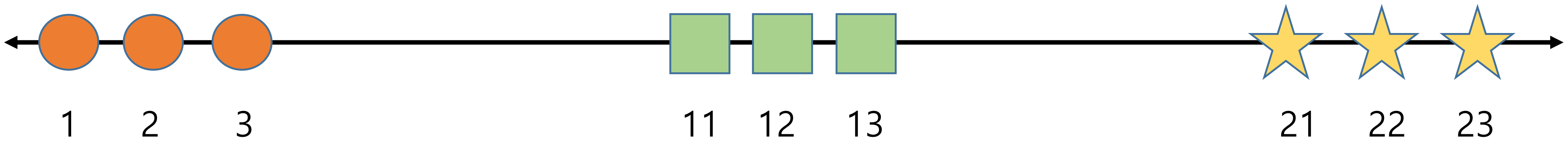
D	11	10	9	1	0	1	9	10	11
D^2	121	100	81	1	0	1	81	100	121

$$sum(D^2) = 606$$

$$sum\left(\frac{D^2}{sum(D^2)}\right) = 1$$

$$D^2/sum(D^2)$$

121/606	100/606	81/606	1/606	0/606	1/606	81/606	100/606	121/606



Initialization!

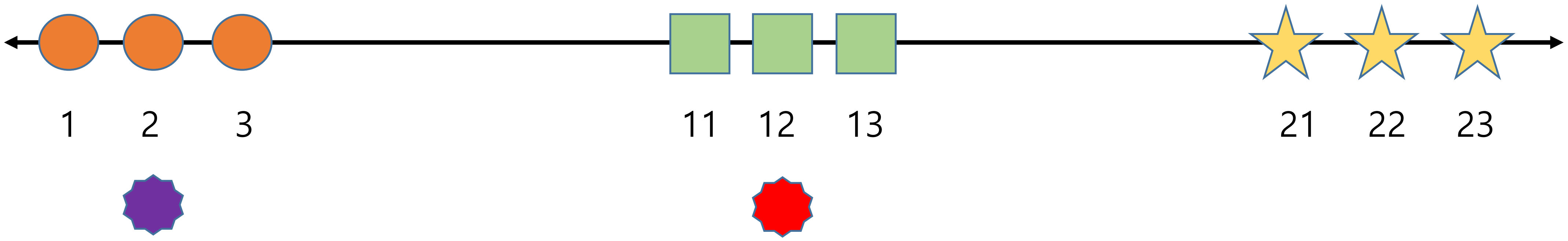
D	11	10	9	1	0	1	9	10	11
D^2	121	100	81	1	0	1	81	100	121

$$sum(D^2) = 606$$

$$sum\left(\frac{D^2}{sum(D^2)}\right) = 1$$

$$D^2/sum(D^2)$$

121/606 100/606 81/606 1/606 0/606 1/606 81/606 100/606 121/606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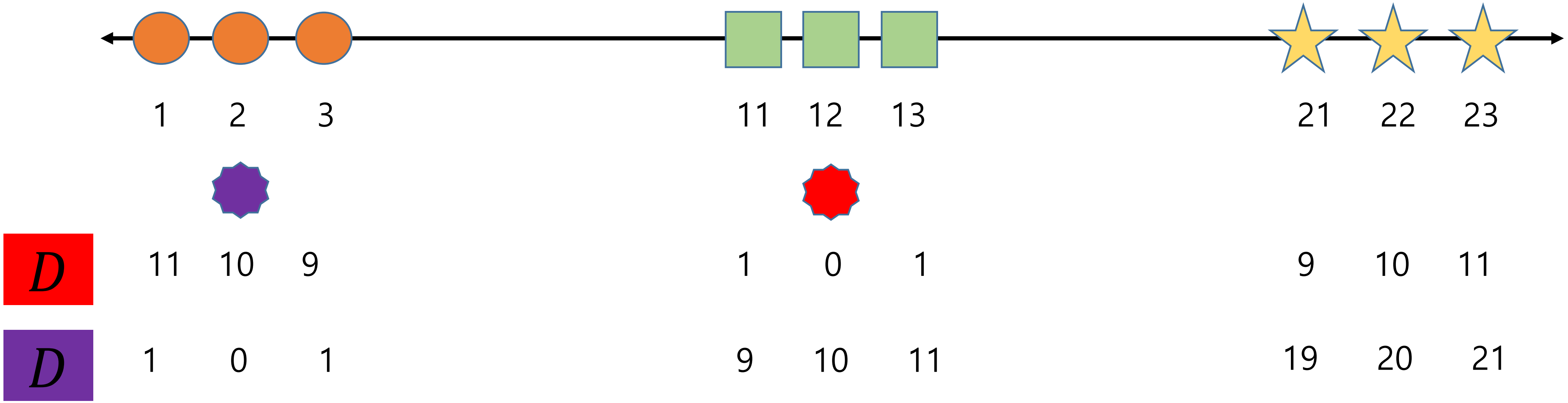
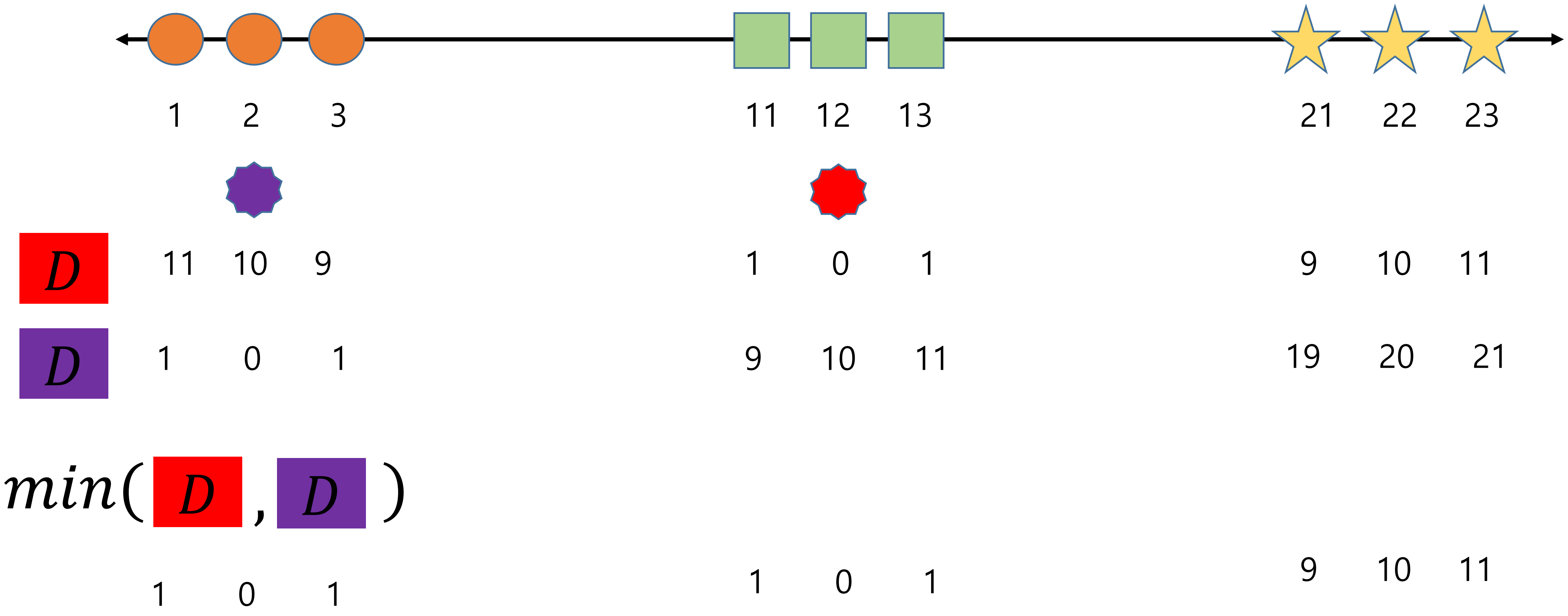
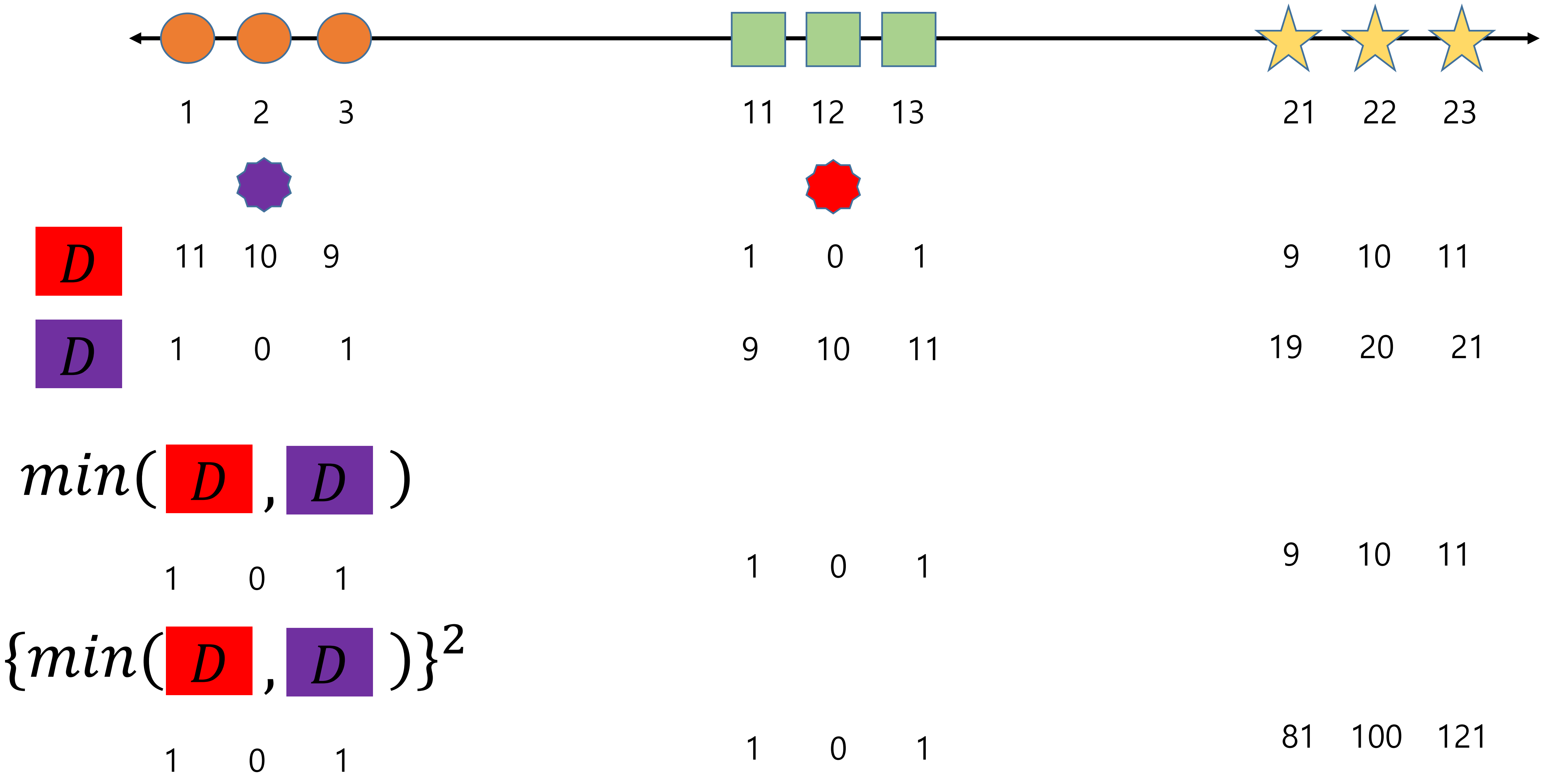
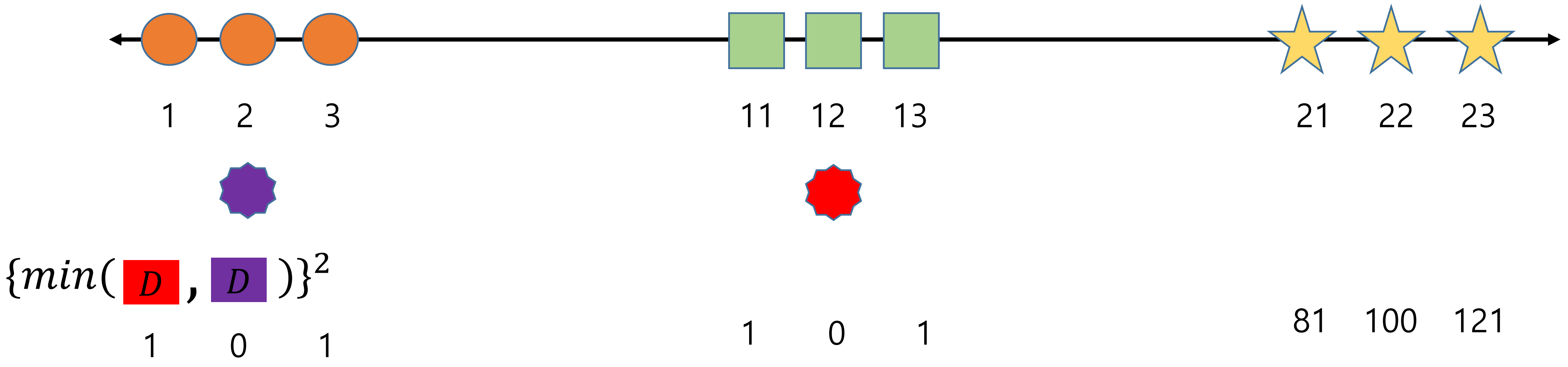

1 2 3 11 12 13 21 22 23
D 11 10 9 1 0 1 9 10 11
D 1 0 1 9 10 11 19 20 21



1 2 3
11 12 13
21 22 23
D 11 10 9 1 0 1 9 10 11
D 1 0 1 9 10 11 19 20 21
min(D , D)
1 0 1 1 0 1 9 10 11


1 2 3 11 12 13 21 22 23
D 11 10 9 1 0 1 9 10 11
D 1 0 1 9 10 11 19 20 21
min(D , D)
1 0 1 1 0 1 9 10 11
{min(D , D)}²
1 0 1 1 0 1 81 100 121


1
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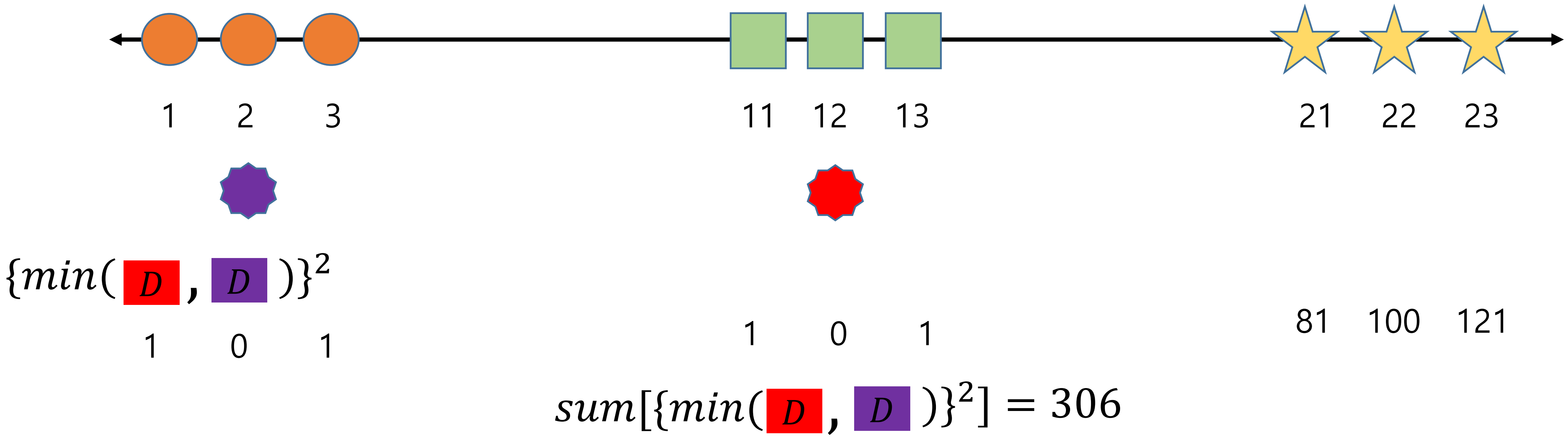
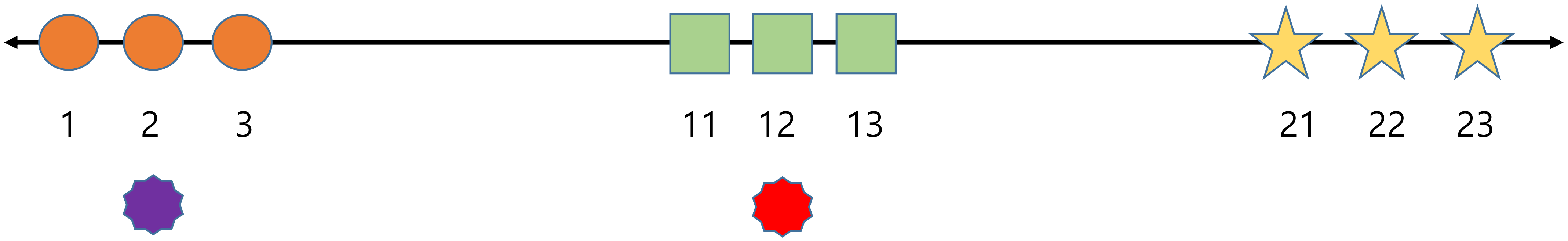
21
22
23
{min(D, D)}^2
1
0
1
1
0
1
81
100
121


1 2 3
11 12 13
21 22 23
{min(D , D)}²
1 0 1
1 0 1
81 100 121
sum[{min(D , D)}²] = 306



$\{min(D, D)\}^2$

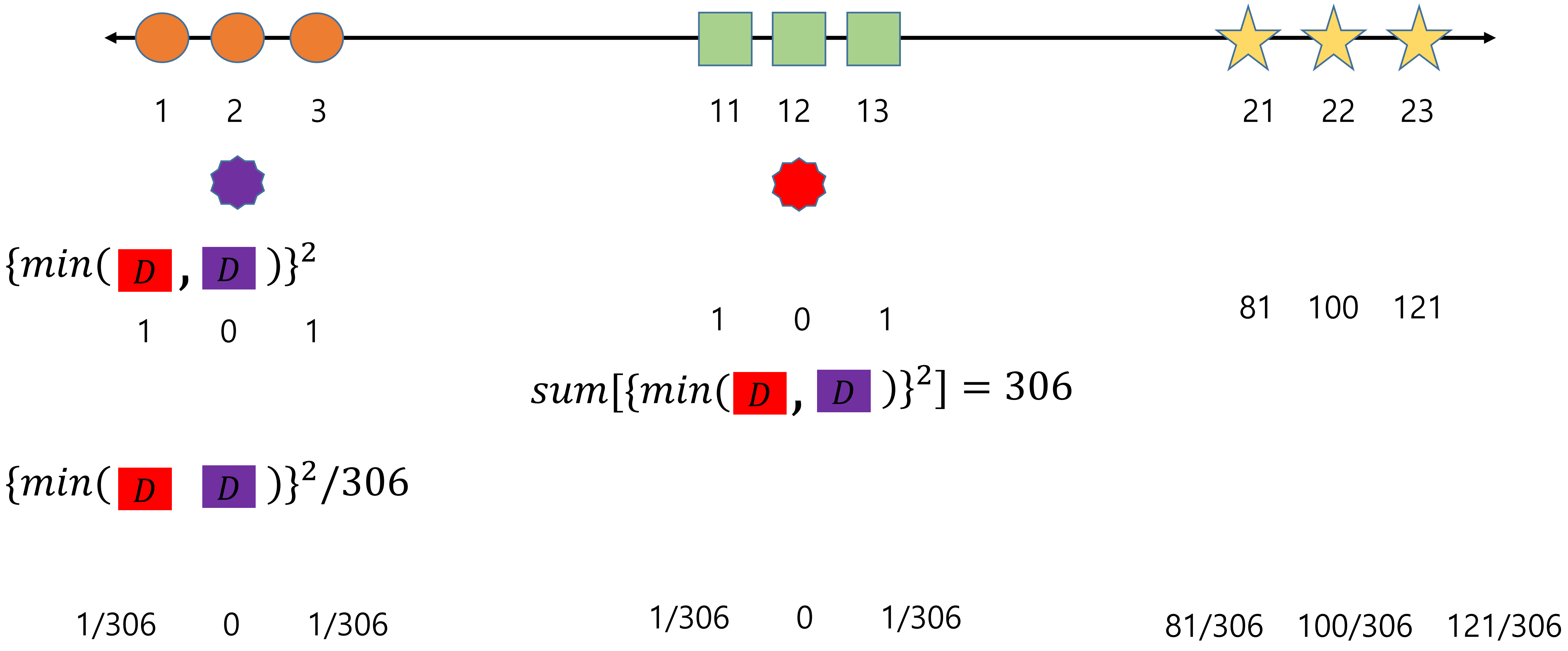
1 0 1

1 0 1

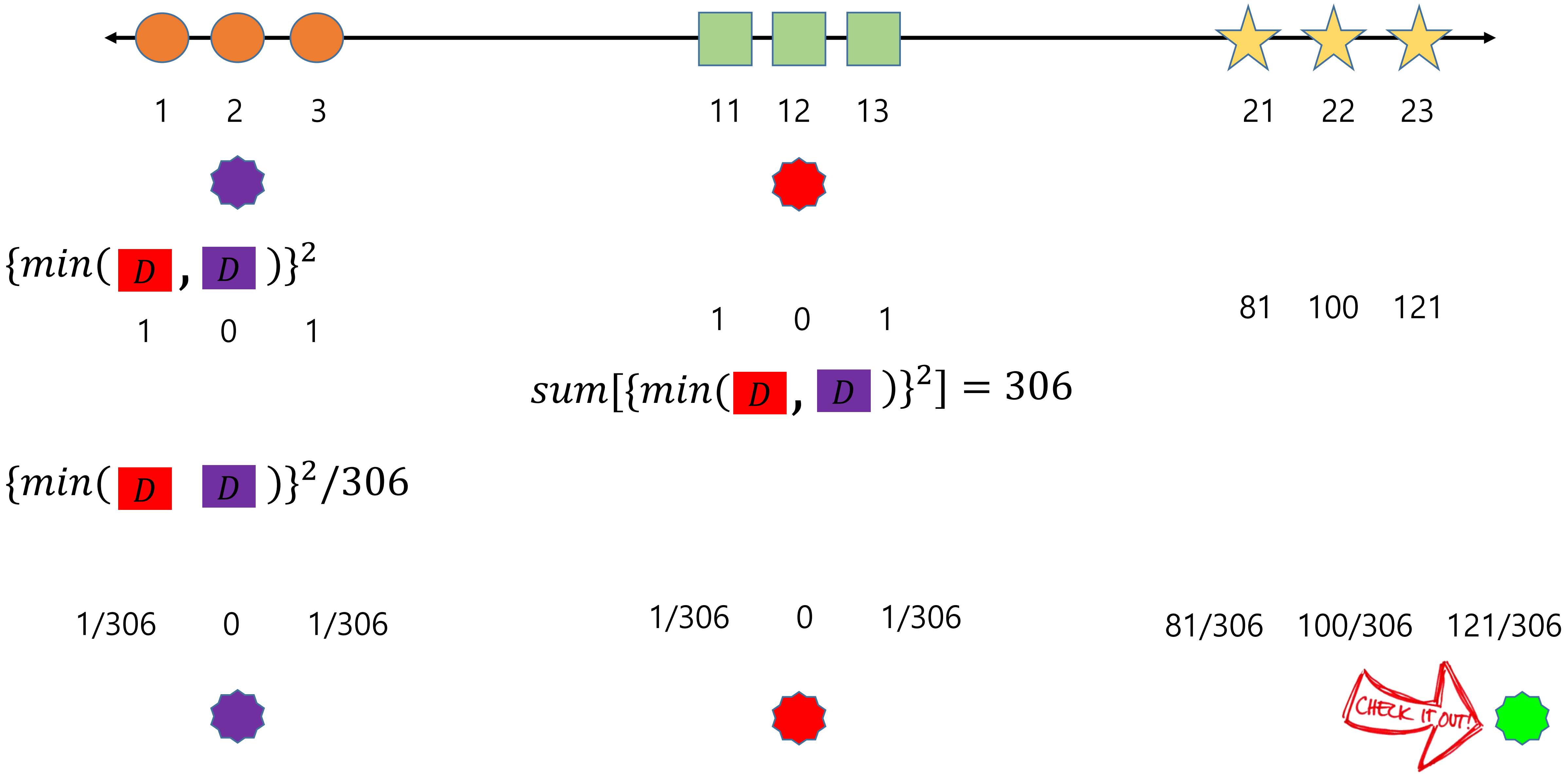
81 100 121

$$sum[\{min(D, D)\}^2] = 306$$

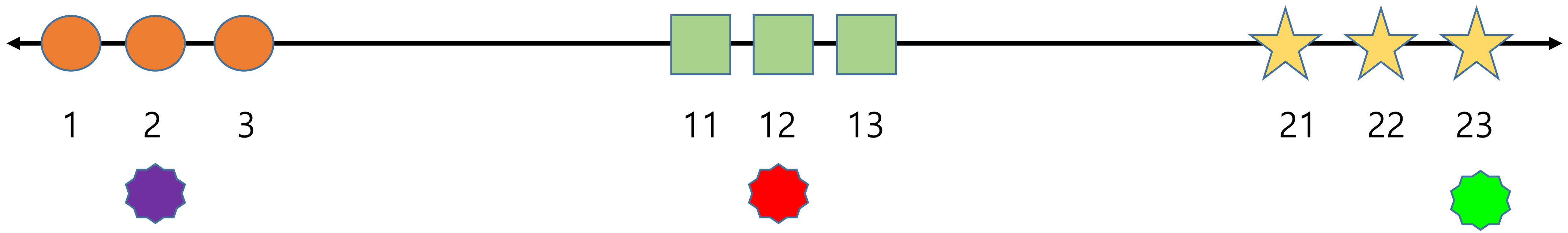
$\{min(D \quad D)\}^2/306$



1 2 3
11 12 13
21 22 23
$\{min(\boxed{D}, \boxed{D})\}^2$
1 0 1
1 0 1
81 100 121
$sum[\{min(\boxed{D}, \boxed{D})\}^2] = 306$
$\{min(\boxed{D}\ \boxed{D})\}^2/306$
1/306 0 1/306
1/306 0 1/306
81/306 100/306 121/306



1 2 3
11 12 13
21 22 23
{min(D , D)}²
1 0 1
1 0 1
81 100 121
sum[{min(D , D)}²] = 306
{min(D D)}²/306
1/306 0 1/306
1/306 0 1/306
81/306 100/306 121/306
CHECK IT OUT!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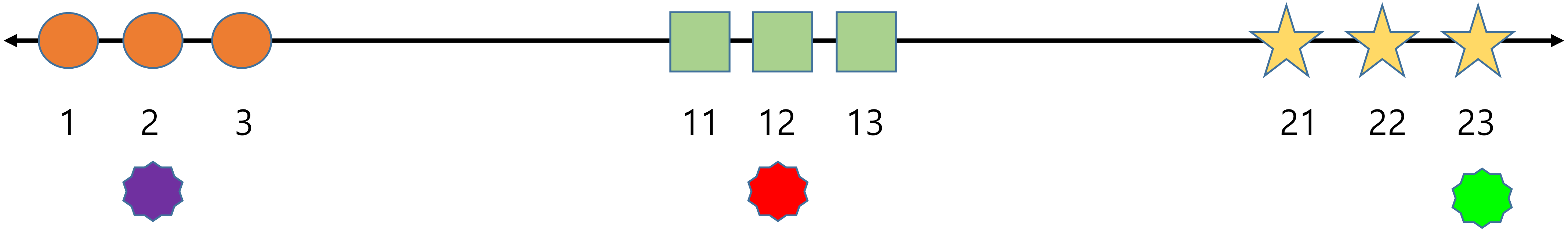

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